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Cyclic Maritime Inventory Routing With Variable Time Horizon

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Cyclic Maritime Inventory Routing With Variable Time Horizon

Master thesis written by

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Preface

This thesis is the final requirement for the degree Master of Science in Logistics. The thesis is written at Molde University College.

The work was supervised by Professor of Quantitative Logistics Lars Magnus Hvattum and Professor in Operations Research Sebastian Alberto Urrutia from Molde University College.

The author would like to thank Lars Magnus Hvattum and Sebastian Alberto Urrutia for guidance and advice during the this semester.

Summary

This thesis presents Maritime Inventory Routing Problems (MIRP), which seeks to find the optimal route for seagoing vessels. The aim of this thesis is to describe MIRP as a mathematical optimization problem, in which the time horizon is variable. The relationship between the total cost and time horizon is non-linear, which establish the main challenge of this thesis. The thesis presents three mathematical MIRP models. Model number one uses fixed time horizon. The second and third models present different ways of optimizing MIRP with a variable time horizon. The models are tested on different instances in order to present results in the computational study. The results show that choice of time horizon can have a significant impact on travel costs and operational costs.

Contents

1 Introduction

1.1 Maritime transport

Maritime transportation has become essential for global trade. Over 80% of all transported volume in the world comes from maritime transport, which makes it fundamental for a functioning global economy [1]. Access to the oceans has, in many ways, ensured the accumulation of wealth through importing scarce resources and exporting resources. Maritime transport also covers essential infrastructure and functions such as shipbuilding, logistics, and port construction. Due to the significant impact of shipping on global trade, research in this area spans many different disciplines such as environmental science, management, operations research, and social sciences [2].

In 2020, the corona pandemic spread rapidly worldwide and negatively affected the world economy. This led to a decrease in production and consumption, resulting in reduced supply, demand, and logistics services. Already in May 2020, volume from maritime transport had decreased by 12% compared with 2019 [1]. On the other hand, the impact of the corona pandemic on maritime transport was not as harmful as feared. In the fourth quarter of 2020, the volume had a decrease of 2% compared to 2019 [1]. The background was mainly on consumers' change in demand from services to traded goods. People worldwide had to spend more time at home due to the pandemic, and travel abroad was almost impossible. In Norway, the pandemic resulted in fewer flights to holiday resorts and a greater focus on household renovation projects. One can argue that this contributed to increased demand for traded goods among Norwegians, thus increasing volume in the tail part of 2020. Table 1 shows the rapid growth of maritime transport volumes since 2010.

Year	**Tanker trade**	**Main bulk**	**Other dry cargo**	**Total (all cargoes)**
2010	2752	2232	3423	8408
2011	2785	2364	3626	8775
2012	2840	2564	3791	9195
2013	2828	2734	3951	9513
2014	2825	2964	4054	9842
2015	2932	2930	4161	10023
2016	3058	3009	4228	10295
2017	3146	3151	4419	10716
2018	3201	3215	4603	11019
2019	3163	3218	4690	11071
2020	2918	3181	4 549	10648

Table 1: International maritime trade, 2010-2020 [1]

1.2 Thesis's purpose

This thesis builds on work done by former master's students at Molde University College. The main subject is Maritime Inventory Routing Problem and how to describe it as a mathematical optimization problem.

In work done in the past, the goal was mainly about expanding existing models of MIRP. In 2018, master students Line Eide and Gro Cecilie Håhjem Årdal [3] wrote that speed and load have a non-linear effect on shipping costs. To describe the problem linearly, Eide and Ådal used linear approximation, where each vessel is assigned speeds and a cost for the choice of speed.

In 2022, master students Amir Zojaji and Kiarash Soltaniani [4] modeled a Cyclic Maritime Inventory Routing Problem, where they also considered speed optimization, which Eide and Årdal modeled. A cyclic plan is created from a type of routing problem where the solution is a plan that works forever.

A cyclic plan implies the following:

- Consumption ports have equal inventory level in the beginning and the end of the time horizon.
- Production ports have equal inventory level in the beginning and the end of the time horizon.
- Positions of vessels are identical in the beginning and at the end of the time horizon.
- Vessels have equal load in the beginning and at the end of the time horizon

The mathematical model in the thesis by Zojaji and Soltaniani [4] uses a fixed time horizon for optimization with a cyclic solution. The issue with fixed time horizons is that the optimization needs to allow exploring the potential cost minimization of choosing a different time horizon. For example, Zojaji and Soltaniani [4] used 30 and 60 days planning horizons in their computational study. The main hunch is that there is a possibility of further minimizing costs by choosing the time horizon corresponding to the minimum average daily cost. To further minimize costs, it is essential to consider the average daily cost, and the following example explains why. Imagine two cyclic solutions with the following characteristics:

1. 30 day cyclic plan with total shipping and operating cost of 500 000 USD
2. 60 day cyclic plan with total shipping and operating cost of 800 000 USD

Also, consider that the vessels and production ports must serve the same amount of consumption ports in both solutions. At first glance, it is easy to consider that plan number 1 is better than number 2 because it is cheaper. However, solution number 2 is cheaper and can be explained by considering the average daily cost, which is described in the following way:

$$\text{Average daily cost} = \frac{\text{Shipping costs + operating costs}}{\text{Time horizon}}$$

Considering the formula above, the average daily cost of plan number 1 and 2 are 16 666.67 USD and 13 33.3 USD, respectively. Given that both plans are cyclic and have an average daily cost, it is interesting to see the total cost of both plans after 60 days have passed:

$$\begin{aligned}
\text{Total cost of plan 1 after 50 days} &= 16666.67 \cdot 60 = 1000000 \text{ USD} \\
\text{Total cost of plan 2 after 50 days} &= 1333.3 \cdot 60 = 800000 \text{ USD}
\end{aligned}$$

The calculations clearly show that plan number 2 is cheaper after 60 days. Performing the 60-day planning horizon once is 200 000 USD cheaper than repeating the 30-day optimal plan twice.

The example above was based on arbitrary numbers, but it explains the potential cost benefits of searching in a solution area where the time horizon is variable. This thesis takes a step further by minimizing average daily shipping and operational costs with a variable time horizon. The methods for minimizing average daily cost should also include cyclic solutions and speed optimization.

2 Literature review

The literature review chapter describes previous research relevant to this thesis. The chapter aims to pay tribute to the researchers who have contributed pioneering research in the routing field. The purpose of the chapter is to describe previous research that enables the writing of this thesis. Each chapter ends with a short paragraph explaining why the topic is relevant to the thesis. Note that the content of Sub-chapter 2.1 is abstract concerning the main topic of the thesis, but should give an overall picture of how logistics management and routing are connected.

The following topics are covered in the literature review:

1. Importance of logistics management
2. Vehicle routing problem
3. Inventory routing problem
4. Maritime inventory routing problem
5. End of horizon effect
6. Cyclic Inventory routing problem

2.1 Importance of logistics management

Throughout history, armies, aid organizations, and educational institutions have depended on functioning logistics to achieve their goals. The logistics are linked to specific tasks that the organization must carry out. For example, armies use logistics to position soldiers, aid organizations must ensure stock levels of medical equipment, and educational institutions must schedule exams. For companies, the ability to match supply and demand is the key element for success. In cases where companies struggle to manage logistics, it has often resulted in less income, lower service levels, damaged reputations, and reduced market share [5]. The challenge of managing logistics has become more significant due to the short life cycle of products. This leads to significant volatility and unpredictability in customer demand.

Management of logistics systems primarily deals with how to manage costs and uncertainty. The authors in [5] point out that challenges are related to the following two observations:

1. Managing cost efficiency and service level at a single facility entails many challenges. On the other hand, the degree of difficulty increases when you want to optimize the entire supply chain.
2. Supply chains often have a degree of uncertainty; challenges in forecasting future demand, uncertain journey times, and unforeseen machine downtime. It is essential to minimize uncertainty, in order to manage the remaining uncertainty.

The relevance of this sub-chapter is well documented above in the numbered list with two arguments. The first argument emphasizes that managing a supply chain has greater difficulty than managing one facility. The first argument is relevant because this thesis aims to manage the flow of products between production and consumption ports, a flow that resembles a supply chain. Furthermore, managing the flow between production and consumption ports also requires forecasting future demand. Therefore, knowledge about the demand is significant to make the routing successful.

2.2 Vehicle routing problem

Supply chains often involves functions that involve management of a fleet of vehicles used to traverse to customers. In order to operate the fleet cost efficient, the planner must frequently determine how much volume goes on each vehicle and which customers they serve. This type of planning falls in under Vehicle Routing Problems (VRP).

The most common version of the VRP is the Capacitated Vehicle Routing Problem (CVRP) [6]. It is described in the following way: A set of vehicles with capacity limits are located in one depot. The set of customers have to be served by the fleet of vehicles. The objective is to compute a feasible set of routes, where amounts transported does not exceed the capacity. The route of each vehicle starts in the depot, serves the customers along the route, and then returns back to the depot.

The precursor to the Vehicle Routing Problem was introduced in 1959 by Dantzig and Ramser [7]. The paper describes how to find the optimal routing for a set of vehicles using heuristics. The mathematical problem builds on a real-world problem, where a set of gasoline trucks has to serve several service stations from a bulk terminal. The shortest route between the nodes is given, and each service station has a demand for some product. The desired solution is a minimum total mileage for the set of trucks, given that demand at service stations is satisfied. In 1964, Clark and Wright [8] generalized the problem as a linear optimization problem, where a set of vehicles traverses to a set of customers in a geographic area. Golden et al. used the term "vehicle routing" for the first time in 1975 [9]. Since then, VRP has become a widely studied subject within operations research.

After the problem was first studied, researchers developed more complex models to cover the different scenarios where VRP is relevant. Examples of more complex models cover features such as pick-up and delivery with time windows or congestion. In 1979, Linstra and Renooy Kan [10] described VRP as NP-hard, which results in that exact optimization methods applying better to minor problems. However, in real life, companies often have to route trucks to thousands of customers from several terminals and are subject to many constraints. Thus, heuristics and metaheuristics are more suitable for real-world problems [11].

The amount of research related to VRP increased significantly during the 90s [12]. One of the primary drivers for this increase was the availability of computers, which allowed researchers to use complex search algorithms. For example, in 1994, Gendreau et al.[13] described a tabu search heuristic called Taburoute. Shortly explained, the algorithm uses predetermined solutions, repeatedly taking a vertex out of its initial route and inserting it into a new route. The algorithm also allows infeasible solutions, which are connected to penalty costs in the objective function. According to the author [13], penalty costs reduce the risk of obtaining a solution at a local minimum. Moreover, the heuristic can also be executed with feasible and infeasible initial solutions.

This sub-chapter explained the main characteristics of the Vehicle Routing Problem and mentioned influential researchers within the field. The VRP is relevant to this thesis because it optimizes vehicles and resources to deliver goods. By solving the VRP, a company can minimize transportation costs and utilize the vehicle fleet effectively. The sub-chapter also mentioned solution methods that build on heuristics, which is not a solution method used in this thesis. The solution method used for this thesis is exact optimization. However, mentioning heuristics is reflective because it allows for solving complex problems. Likewise, exact optimization methods may often not be capable of solving VRP with large complexity.

2.3 Inventory Routing Problem

The Inventory Routing Problem (IRP) typically arises when a vendor is responsible for replenishing customer inventory. According to Campell et al. [14], the IRP incorporates inventory control and vehicle routing, which are essential supply chain elements. They are often solved separately; however, combining elements can benefit the supply chain in some cases.

In order to use IRP in real life, vendors and customers must use a vendor-managed resupply policy (VMR) [14]. The principle of VMR is that the vendor chooses the delivery- time and size. In return, the vendor must ensure the product is always available when the customer needs it. Furthermore, this type of agreement can minimize customers' resources used for inventory control. This advantage comes from the warehouse being managed externally.

For IRP to be a successful routing method, the vendor must predict the customer's consumption. In cases where consumption could be better predicted, the customer may risk stockouts. In 2002, Jaillet et al. [15] developed an IRP model for commodities such as heating oil. The researchers argued that adopting real-time technology for inventory control was applicable; however, the technology could not quantify the cost benefits of long-term delivery plans [15]. As a result, a distributor cannot operate with long-term optimality if deliveries are always reactive to the current data. As a result, in some cases, it is more reasonable to base the long-term plans on expected consumption with a normal distribution.

As Chapter 2.1 mentioned, managing logistics is vital to companies' success. One of the main tasks of logistics management is to avoid excessive inventory and stockouts. With that in mind, Fedgruen and Simchi-Levi [15] emphasized that the safety stocks provided by using IRPs may contribute to minimizing inventory costs. They argued that the IRPs enable companies to meet a certain service level with an aggregate safety stock, which is often considerably smaller than the sum of the safety stocks of the individual customers [15].

The Inventory Routing Problem's main ideas apply to the problem this thesis aims to solve. Firstly, the IRP combines inventory control and vehicle routing, which are fundamental functions in a supply chain. Secondly, the IRP also uses a vendor-managed resupply policy. This type of policy lets the customer set forth delivery (demand) requirements. Likewise, this type of policy enables the vendor to optimize its routing and inventory control.

2.4 Maritime Inventory Routing Problem

Maritime Inventory Routing Problem (MIRP) deals with shipping one or more products between production and consumption ports with maritime modes of transport. Christiansen and Fagerholt [16] introduced various applications within MIRP in 2009. The authors pointed out that the problem arises when the ship operator is responsible for inventories in production- and consumption ports.

In 2010, Song and Furman [17] described a MIRP model with practical features, which included daily changeable production, draft limits, and a heterogeneous fleet. Draft limits are port restrictions and state that certain vessels cannot visit the port due to vertical space requirements, which is the space between the ship hull and the seabed. The solution method proposed by Song and Furman [17] was a large neighborhood search heuristic and a Branch & Cut algorithm. The main emphasis of the article was to obtain results within a reasonable time. However, it also highlighted that features such as variable production are essential for the progress of the research within MIRP.

In 2016, Agra et al. [18] introduced a local search heuristic and exact methods for solving a stochastic maritime inventory routing problem. The article emphasizes the impact weather conditions have on sailing times. Therefore, the sailing times between any two nodes were considered random and followed a log-logistic distribution. Furthermore, any violation of inventories was penalized. The researchers discovered that the exact optimization method was easy to use with small penalty costs. Conversely, when penalty costs were high, the integrality gaps increased in the decomposition procedure. As a result, the local search heuristic worked better with high penalties [18].

In 2022, Algendi et al. [19] introduced a model where the production rate can be selected from several alternative production levels. The researchers found that optimizing the production rate can significantly reduce costs [19]. However, a few delimitations could influence the usefulness of optimizing production rates. For example, the inventory holding cost was ignored and could significantly increase the total costs.

MIRP and IRP share the main feature that both incorporate different supply chain functions. However, the practical features can be very different between MIRP and IRP. For example, port draft limits would not be an issue in a typical IRP problem. Note that the practical features mentioned in this sub-chapter are abstract, considering the aim of this thesis. For example, weather conditions or draft limits are not included in any methods. Despite that, the thesis aims to solve a type of MIRP, with results obtained in a reasonable time. Note that reasonable time means a time limit that the author of this thesis predefines.

2.5 End of Horizon Effect

One of the common challenges with IRP is the End of Horizon Effect, which the authors address in [20]. The problem is that inventory levels for consumption- and production ports become low and high, respectively. Therefore, optimizing total cost in IRP, the optimal solution, does not necessarily respect acceptable inventory levels. However, a few methods have been tested in [21] to avoid the End of Horizon Effect. For example, specifying tight production- and inventory bounds at the end of the time horizon. Another method is to increase the minimum volume delivered each time a customer is served.

The next sub-chapter will address the type of routing problem which is used to avoid the consequences of the End of Horizon Effect.

2.6 Cyclic Inventory Routing Problem

Given a distribution center, a set of customers, and vehicles, the Cyclic Inventory Routing Problem determines a distribution plan that minimizes costs. The cost elements in the literature can vary greatly, but cost elements such as traveling and inventory costs are often used. The type of plan determined by CIRP is cyclic, which was briefly clarified in Chapter 1.2.

In 2006, Aghezzaf et al. [22] described a single product Cyclic Inventory Routing Problem as a mixed integer mathematical program. The model Aghezzaf et al. [22] presented uses the multi-tour concept. This concept allows the vehicles to undertake several tours during the time horizon, i.e., the vehicles leave the distribution centers to replenish customers, return to the distribution center for reloading and start a new tour to replenish a different set of customers. In 2009, Raa and Aghezzaf [23] described CIRP to find three-way cost trade-offs between vehicle fleet size-, distribution-, and inventory costs. The authors used a heuristic solution approach for the computational experiments.

The role of the Cyclic Inventory Routing Problem is that it only allows repeatable routing plans. In this thesis, this type of plan is called a cyclic solution and was briefly mentioned in Chapter 1.2.

3 Problem description

The problem description in this master thesis aims to describe what type of problem to solve. It sets the context for the research.

The problem of this thesis is set in a geographical area where vessels, ports, and a single product are the physical entities. The ports are divided into two main groups: Consumption- and production ports. The role of the production ports is to produce a product, and the consumption ports consume the product. The ports have fixed upper and lower bounds for the inventory. Considering the time horizon and bounds on inventory levels, each port can be visited many times. Each port must be visited at least once during the time horizon. The ports must have equal inventory in the beginning and end of the time horizon.

The fixed fleet of vessels is heterogeneous, meaning each vessel is equipped with different capacities. The role of the vessel fleet is to transport the product from the production ports to the consumption ports. The vessels can choose the start location and start load, which are considered variables. The load transported in the vessels must comply with the demand of consumption ports throughout the time horizon. Furthermore, each ship must return to the starting location, and the end load must equal the start load. This rule of equality enables the cycle to operate indefinitely. The vessels can also choose a speed when traveling between two nodes. At the end of the time horizon, the position of a vessel can be in the middle of the ocean. Therefore the vessel needs extra time to return to its previous position. The extra time is called delay time. The mathematical models are equipped with speeds. Each time any vessel travels between ports, the vessel is assigned a speed. When traveling between two ports, a vessel can choose different speeds for a fraction of the distance.

The activities provided by the ports and vessels are completed within a time horizon. In contrast to earlier research, the number of days in the planning horizon is unknown in the optimization moment. Therefore, the planning horizon is variable. All in all, the vessels and ports should operate with a time horizon that ensures the lowest possible daily cost.

4 Justification of the research

This section explains why focusing on variable time horizons is essential to enable cost optimization. The chapter deals with a computational experiment done early in the writing phase. This chapter's main point is to justify why research on variable time horizons is relevant.

The primary motivation for the thesis was discovered in a computational test of the model described by Zojaji and Kiarash [4]. The test consisted of 60 computations, where the time horizon increased by one day for each computation. Each time horizon requires a fixed set of maximum number of visits to each customer in order to obtain a feasible solution within the time horizon T.

1. For a feasible solution, instances with T between 11 and 35 days require a maximum of 2 visits
2. For a feasible solution, instances with T between 36 and 53 days require a maximum of 3 visits
3. For a feasible solution, instances with T between 55 and 60 days require a maximum of 4 visits

Figure 1 describes the physical entities of the data used in the computational test. Production ports $P1$ and $P2$ have a production rate of 6 per time unit. Consumption ports $D1$ and $D2$ have a consumption rate of 3 and 9 per time unit, respectively. Vessels $S1$ and $S2$ have capacities of 160 and 130, respectively. The arrows in Figure 1 represent the possible routes between the nodes. For readability and simplicity, the routes have arrows at each end of the lines, which means direct links between both nodes. The objective function is the sum of all transportation- and operational costs.

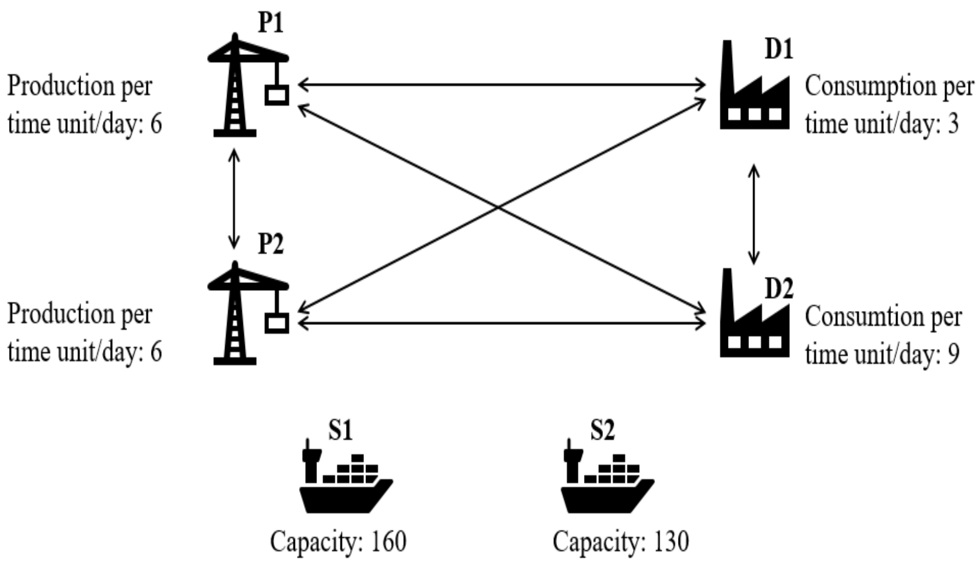

Figure 1: Computational test. [Made by the author]

Table 2 is an example of one cyclic solution from the computational test with a time horizon $T = 48$. Vessel $S1$ starts in $P2$ with a load of 144 and ends in $P2$ with a load of 144. Vessel $S2$ starts and ends in $D2$ with a load of 2. To summarise, the solution in Table 2 shows the main principle of how a cyclic plan looks like.

S1	**S2**
$P2 \rightarrow D1$. Start load = 144	$D2 \rightarrow P1$ Start load = 2
$D1 \rightarrow P2$ Load = 0	$P1 \rightarrow D2$ End load = 160 - 158=2
$P2 \rightarrow D2$ Load=160	
$D2 \rightarrow P1$ Load = 0	
$P1 \rightarrow D2$ Load = 160	
$D2 \rightarrow P2$ End load = 160-16=144	

Table 2: Solution with $T = 48$. [Made by the author]

,

The plot in Figure 2 displays the total cost of the computational test. The data used in the computational test is equal to the data shown in Figure 1. The only differences in the data between the instances are the fixed planning horizon and the maximum number of visits. Each dot represents the number of days in the planning horizon on the x-axis and the computed total cost on the y-axis. The statistics show multiple optimal cyclic solutions with a fixed time horizon. Also, according to Figure 2, it is safe to assume that the total cost increases when the time horizon increases. For example, the total cost of using a time horizon of 20 days is considerably lower than using 50 days. For example, 27 days have a higher total cost than 32 days. The reason is that the 27-day instance was not an optimal solution and had higher operational costs.

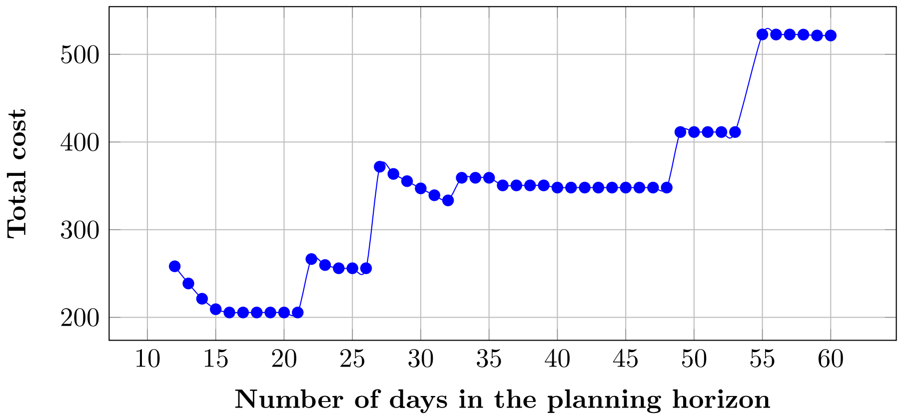

Figure 2: Total cost of n number of days in the planning horizon. [Made by the author]

The statistics in Figure 2 do not tell which planning horizon represents the minimum average daily cost. Recall that Chapter 1.2 states that the average daily cost is the total cost divided by the time horizon. With that in mind, all calculations in Figure 3 are the total cost from Figure 2 divided by the fixed time horizon. As mentioned, the total cost of a cyclic plan will always increase when the number of days in the planning horizon increases. However, the average daily cost can be significantly higher for a shorter planning horizon. The statistics in Figure 3 reflect that the correct choice of planning horizon can significantly affect the average daily cost. Therefore, the main argument is that shipping companies should execute a cyclic plan with a time horizon corresponding to the minimum average daily cost. For example, a shipping planner can choose any feasible planning horizon between 11 and 60 days. The planner's decision is based on the statistics in Figure 3. The planner's optimal choice would be a time horizon of 48 days with an average daily cost of 7,25.

All the optimization instances where done one by one in the described computational test. Each instance were optimized with a fixed time horizon of 11 to 60 days. By graphically displaying the solutions, the statistics show that the number of days in the planning horizon can greatly affect the average daily cost. Therefore, the computational experiment underpins the importance of research on this topic. It also illustrated that the time horizon could be unknown in the optimization moment.

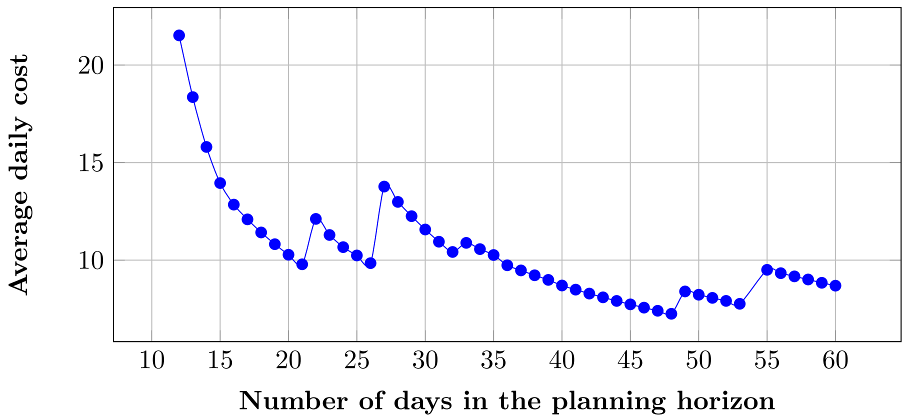

Figure 3: Average daily cost of n number of days in the planning horizon. [Made by the author]

Figure 4 displays the CPU time of each optimization instance. In order to find the optimal time horizon of 48 with the original model, it required computing all instances from 11 to 60 days separately. This process is very time-consuming, and the instances with over 55 days of planning horizon required over 90 minutes clock time. As a result, one of the main motivations was to develop a mathematical model that requires less solving time for the described computational test.

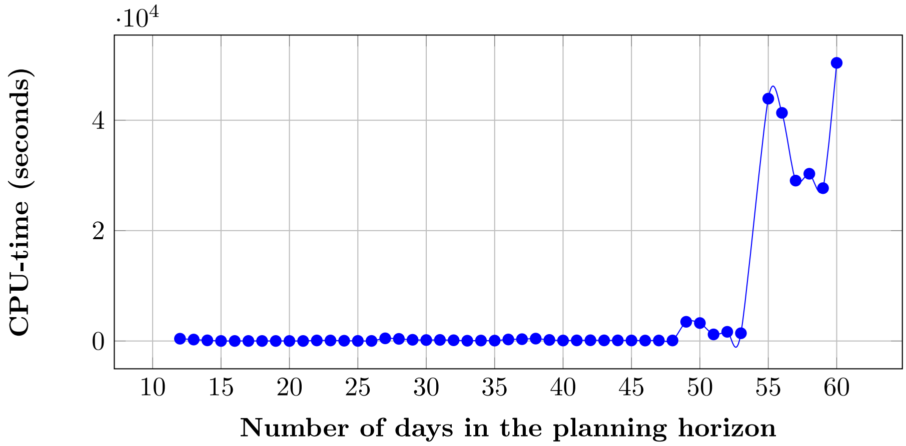

Figure 4: CPU-time (seconds) with n number of days in the planning period. [Made by the author]

5 Overview of the models presented

The last section explained the motivation, justification and reason why a variable time horizon is beneficial when solving MIRP. This subchapter gives a quick overview of how to solve the challenges discussed earlier. The challenges will be solved by editing the mathematical model Zojaji and Soltaniani [4] presented in their thesis.

The first mathematical model presented in this thesis is the original cyclic model with a fixed time horizon by Zojaji and Soltaniani [4].

The second model presented extends the original with a new set of available time horizons. The model can only choose one of the declared integer time horizons. Furthermore, the number of sailing cost matrices equals the number of available time horizons. Recall that the original model uses a fixed time horizon, so the model only required one sailing cost matrix. The presented models minimize the daily average cost and require an average sailing cost matrix for each available time horizon. However, the objective function can only consider the average sailing cost matrix corresponding with the chosen time horizon.

The third model uses a linear approximation to obtain a cyclic solution with a variable time horizon. Linear approximation is useful because it simplifies complex functions using linear functions instead. Recall that Chapter 1.2 expresses that the average daily cost equals the total cost divided by the time horizon. When the time horizon becomes variable, the total cost and time horizon relationship becomes non-linear. This relationship occurs when variables are above and below the fraction line. The way to deal with non-linear relationships is to use linear approximation.

The method for linear approximation is carried out by defining breakpoints for the time horizon and total cost. The solution aims to find the breakpoint between total cost and time horizon, corresponding with the minimum average daily cost. Figure 5 presents in a plot what a solution with breakpoints could look like. The weights are multiplied by the total cost break point and the breakpoints of the time horizons. Moreover, the solution is the cross where the red and blue lines meet. However, this is better described mathematically in this thesis's modeling section.

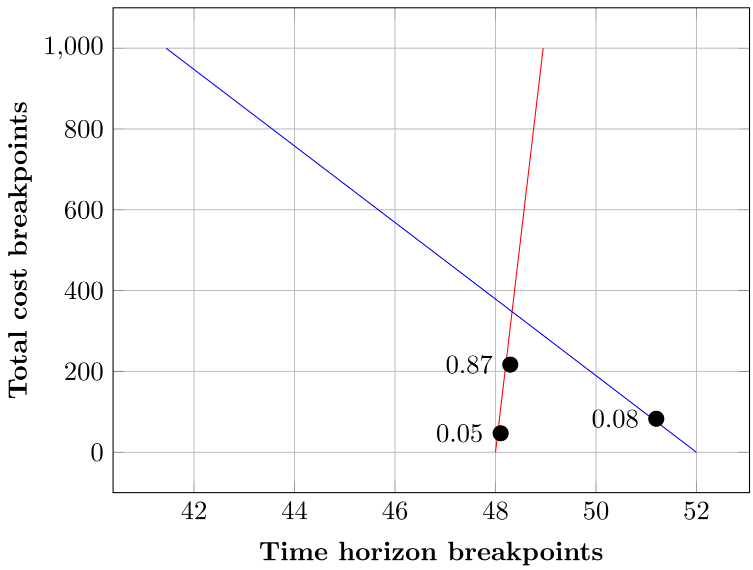

Figure 5: Example of linear approximation [Made by the author]

6 Mathematical modeling

In this chapter, all models are described mathematically. First, the chapter presents the original model with a fixed time horizon, by Zojaji and Soltaniani [4]. Afterward, the two developed models are presented in the same order as Chapter 5.

6.1 MIRP with fixed time horizon

This sub-chapter of the modelling section presents the model described by Zojaji and Soltaniani [4]. The model uses fixed time horizon.

SET

N : Set of production and consumption ports

V : Set of vessels

S^A : Set of possible nodes (i, m)

S^A_v : Set of nodes that can be visited by vessel v

S^X_v : Set of all possible visits (i, m, j, n) of vessel v

S^S_v : Set of breakpoints for the speed of vessel v, where $S^S_v = \{1, 2, .., U\}$

PARAMETERS

T: Number of days in the time horizon

J_i: $J_i = 1$ if i is a loading port and -1 if i is an unloading port. $i \in N$

D_i: Consumption or demand at port i per unit of time. $i \in N$

U^S_i: The upper bound on the inventory level at port i in time period t. $i \in N$

L^S_i: The lower bound on the inventory level at port i in time period t. $i \in N$

H_i: Minimum number of visits at port i. $i \in N$

M_i: Maximum number of visits at port i. $i \in N$

Q_i: Minimum load/unload quantity at port i. $i \in N$

K_i: Minimum time between two consecutive visits at port i. $i \in N$

L^R_v: The fixed amount ship v loads/discharges per time period. $v \in V$

T^Q_v: $\frac{1}{L^R_v} \rightarrow$ Time for unload/load each unit. $v \in V$

$S^P_{v,e}$: Speed e of ship v. $v \in V, e \in S$

C_v : Capacity of ship v. $v \in V$

$D^{PP}_{v,e}$: Daily traveling cost. $v \in V, e \in S$

$A_{i,m}$: Earliest time for starting visit m to port i. $i \in N, m \in 1..M[i]$

$B_{i,m}$: Equals the number of days in the planning period T. $i \in N, m \in 1..M[i]$

$U_{i,m}$: $MIN\{T; T + B_{i,m} \cdot U^S_i\} \rightarrow$ latest time for finishing visit

$D^I_{i,j}$: Port to port distance matrix. $(i,j) \in N$

$P_{i,v}$: Port cost at port i for vessel v. $i \in N, v \in V$,

$T^{PP}_{i,j,v,e}$: $\frac{D^I_{i,j}}{24 \cdot S^P_{v,e}} \rightarrow$ Time (number of units) required by ship v to sail from port i to port j (days). $(i,j) \in N, v \in V, e \in S$

$C^{PP}_{i,j,v,e}$: $D^{PP}_{v,e} \cdot T^{PP}_{i,j,v,e} \rightarrow$ Sailing cost from port i to port j with ship v. $(i,j) \in N, v \in V, e \in S$

VARIABLES

ROUTING VARIABLES

$w_{i,v}$: Equals 1 if vessel v starts from port i, 0 otherwise. $i \in N, v \in V$

$x_{i,m,j,n,v}$: Equals 1 if vessel v departs from node (i, m) to node (j, n).
$(i, m, j, n) \in S_v^X, v \in V : i <> j$

$x^O_{i,j,n,v}$: Equals 1 if vessel v departs from its initial position to node (i, m), 0 otherwise.
$i \in N, (j, n) \in S^v_A, v \in V$

$x^D_{j,n,i,v}$: Equals 1 if vessel v end its routes at node (i, m), 0 otherwise.
$(j, n) \in S_v^A, i \in N, v \in V$

$o_{i,m,v,r}$: Equals 1 if vessel v operates (i, m), 0 otherwise. $(i, m) \in S_v^A, v \in V, r \in R$

$y_{i,m}$: Equals 1 if there is a visit (i, m), 0 otherwise. $(i, m) \in S^A$

FLOW VARIABLES

$q_{i,m,v}$: The quantity loaded/unloaded at node (i, m) by vessel v.
$(i, m) \in S_v^A, v \in V$

$f_{i,m,j,n,v}$: Flow associated to arcs x (from port to port)
$(i, m, j, n) \in S_v^x, v \in V$: i<>j

$f^O_{i,j,n,v}$: Flow associated to arcs x^O (from intital position).
$i \in N, (j, n) \in S_v^A, v \in V$

$f^D_{j,n,i,v}$: Flow associated to arcs x^D. (to destination)
$(j, n) \in S_v^A, i \in N, v \in V$

TIME VARAIBLES

$t_{i,m}$: Start time of visit number m to port i. $(i,m) \in S^A$

t_v^S: The amount of delay that may occur for each vessel v. $v \in V$

STOCK VARIABLES

s_i^O: The stock level in port i at the beginning of the planning horizon. $i \in N$

$s_{i,m}$: Stock level at node (i,m). $(i,m) \in S^A$

$g_{i,m,j,n,v,e}$: ≥ 0 and $\leq 1 \rightarrow$
Auxiliary variable which determines the speed v when going from
node (i,m) to node (j,n), with e corresponding to a given choice of speed.
$(im, j, n) \in S_v^X, v \in V, e \in S_v^S$

$g_{i,j,n,v,e}^O$: ≥ 0 and $\leq 1 \rightarrow$
Auxiliary variable which determines the speed v when going from node
(j,n) to destination at port i, with e corresponding to a given choice of speed
$i \in N, (j,n) \in S_v^A, v \in V, e \in S_v^S$

$g_{j,n,i,v,e}^D$: ≥ 0 and $\leq 1 \rightarrow$
Auxiliary variable which determines speed v when going
from node (j,n) to destination at port i, with e corresponding to a given choice of speed
$j \in N, n \in 1..M_j, i \in N, v \in V, e \in S$

OBJECTIVE FUNCTION

Minimize

$$\sum_{i,m,j,n,v,e:i\neq j} C^{PP}_{i,j,v,e} \cdot g_{i,m,j,n,v,e} + \sum_{i,j,n,v,e} C^{PP}_{i,j,v,e} \cdot g^{o}_{i,j,n,v,e} + \sum_{j,n,i,v,e} C^{PP}_{i,j,v,e} \cdot g^{d}_{j,n,i,v,e} + \sum_{i,m,v} P_{i,v} \cdot o_{i,m,v} \quad (1)$$

Objective function minimizes average daily costs.

ROUTING CONSTRAINTS

$$\sum_{i\in N} w_{i,v} \leq 1 \qquad \forall v \in V \quad (2)$$

Shows that each vessel must have a maximum of one start position.

$$\sum_{(j,n)\in S^A_v} x^D_{j,n,i,v} = w_{i,v} \qquad \forall i \in N, v \in V \quad (3)$$

Illustrates that each vessel should return to the start position.

$$\sum_{(j,n)\in S^A_v} x^O_{i,j,n,v} = w_{i,v} \qquad \forall i \in N, v \in V \quad (4)$$

Illustrates each vessel should start from initial position.

$$o_{i,m,v} - \sum_{(j,n)\in S^A_v : i\neq j} x_{i,m,j,n,v} - \sum_{j\in N} x^D_{i,m,j,v} = 0 \qquad \forall v \in V, (i,m) \in S^A_v \quad (5)$$

Ensures that if a vessel is at node i it must either leave to another node or end.

$$o_{i,m,v} - \sum_{(j,n)\in S^A_v : j\neq i} x_{j,n,i,m,v} - \sum_{j\in N} x^o_{j,i,m,v} = 0 \qquad v \in V, (i,m) \in S^A_v \quad (6)$$

Defines that if a node is visited by vessel v, the vessel must either arrive at the node from the origin or from another node.

$$\sum_{v\in V} o_{i,m,v} = y_{i,m} \qquad \forall (i,m) \in S^A_v \quad (7)$$

Shows that a vessel can only visit (i,m) if there are at least m visits.

$$y_{i,m-1} - y_{i,m} \geq 0 \qquad \forall (i,m) \in S^A : H_i + 1 \leq m \leq M_i \quad (8)$$

Guarantees that if a port i is visited m times, then it also has been visited $m-1$ times.

$$y_{i,m} = 1 \qquad \forall (i,m) \in S^A : m \in \{1..H_i\} \quad (9)$$

Defines the number of mandatory visits for port i.

$$\sum_{e \in S_v^S} g_{i,m,j,n,v,e} = x_{i,m,j,n,v} \qquad \forall v \in V, (i,m,j,n) \in S_v^X \quad (10)$$

Enforces that speed of a vessel must be set for a route from node (i,m) to node (j,n) and only if the route exists.

$$\sum_{e \in S_v^S} g^O_{j,n,i,v,e} = x^O_{j,n,i,v} \qquad \forall v \in V, (j,n) \in S_v^A, i \in N \quad (11)$$

Enforces that speed of a vessel must be set for for a route from the origin to node (j,n) and only if that route exists.

$$\sum_{e \in S_v^S} g^D_{j,n,i,v,e} = x^D_{j,n,i,v} \qquad \forall v \in V, (j,n) \in S_v^A, i \in N \quad (12)$$

Enforce that speed of a vessel must be set for a route from node (j,n) to the destination if that route exists.

LOADING AND UNLOADING CONSTRAINTS

$$q_{i,m,v} \leq MIN\{C_v, U_i^S\} \cdot o_{i,m,v} \qquad \forall v \in V, (i,m) \in S_v^A \quad (13)$$

Ensures the quantity loaded/unloaded cannot exceed the vessel capacity or the maximum port capacity.

$$Q_i \cdot o_{i,m,v} \leq q_{i,m,v} \qquad \forall v \in V, (i,m) \in S_v^A \quad (14)$$

Shows that if a vessel visits the port, then the amount loaded/unloaded should be at least equal to the minimum quantity.

$$\sum_{(i,m) \in S_v^A} J_i \cdot q_{i,m,v} = 0 \qquad v \in V \quad (15)$$

Total amount loaded must be equal to total amount unloaded.

FLOW CONSTRAINTS

$$\sum_{i \in N} f^O_{i,j,n,v} + \sum_{(i,m) \in S^A_v : i \neq j} f_{i,m,j,n,v} + J_j \cdot q_{j,n,v} = \tag{16}$$

$$\sum_{(i,m) \in S^A_v : j \neq i} f_{j,n,i,m,v} + \sum_{i \in N} f^D_{j,n,i,v} \qquad \forall v \in V, (j,n) \in S^A_v$$

Sum of incoming flow from origin, particular ports and loaded/unloaded volume should be equal.

$$f^O_{i,j,n,v} \leq C_v \cdot x^O_{i,j,n,v} \qquad \forall v \in V, i \in N, (j,n) \in S^A_v \tag{17}$$

Ensures that flow from the initial position should be less or equal to the capacity of the vessel.

$$f_{i,m,j,n,v} \leq C_v \cdot x_{i,m,j,n,v} \tag{18}$$

$$\forall v \in V, (i,m) \in S^A_v, (j,n) \in S^A_v : j \neq i$$

Flow from a port to another port should not exceed the capacity of the vessel.

$$f^D_{j,n,i,v} \leq C_v \cdot x^D_{j,n,i,v} \qquad \forall v \in V, (j,n) \in S^A_v \tag{19}$$

Ensures that flow to the destination is less or equal to the capacity of the vessel.

TIME CONSTRAINTS

$$t_{i,m} - t_{i,m-1} - \sum_{v \in V} T^Q_v \cdot q_{i,m-1,v} - K_i \cdot y_{i,m} \geq 0 \tag{20}$$

$$\forall (i,m) \in S^A : m > 1$$

Enforces the minimum time period between two consecutive visits to port i.

$$t_{i,m} + \sum_{v \in V} T^Q_v \cdot q_{i,m,v} - t_{j,n} \tag{21}$$

$$+ \sum_{v \in V, e \in S} MAX\{U_{i,m} + T^{PP}_{i,j,v,e} - A_{j,n}; 0\} \cdot g_{i,m,j,n,v,e} \leq U_{i,m} - A_{j,n}$$

$$\forall (i,m) \in S^A, (j,n) \in S^A : i \neq j$$

Relates the start time associated with node (i,m) to the start time associated with node (j,n) when a vessel travels between ports i and j.

$$t_v^s + \sum_{j \in N} \sum_{e \in S} T^{PP}_{j,i,v,e} \cdot g^O_{j,i,m,v,e} \leq t_{i,m} + T \cdot (1 - o_{i,m,v}) \tag{22}$$

$$\forall v \in V, (i,m) \in S_v^A$$

Ensures that the travel time + delay time should not exceed the start time of the visit to the port.

$$t_{j,n} + T_v^Q \cdot q_{j,n,v} + \sum_{i \in N} \sum_{e \in S_v^S} T^{PP}_{j,i,v,e} \cdot g^D_{j,n,i,v,e} \leq T + t_v^S + T \cdot (1 - o_{j,n,v}) \tag{23}$$

$$\forall v \in V, (j,n) \in S_v^A$$

Expresses time at node (j, n) plus travel time for a vessel traveling from node (j, n) to destination at node i should not exceed the time horizon plus delay time.

$$t_{i,m} \geq A_{i,m} \qquad \forall (i,m) \in S^A \tag{24}$$

Expresses time windows for the start time of visits.

$$t_{i,m} \leq B_{i,m} \qquad \forall (i,m) \in S^A \tag{25}$$

Expresses time windows for the end time of visits.

INVENTORY CONSTRAINTS

$$s_{i,1} = s_i^o + (J_i \cdot D_i \cdot t_{i,1}) \qquad \forall i \in N \tag{26}$$

Defines the stock level at the start time of the first visit to a port

$$s_{i,m} = s_{i,m-1} - \sum_{v \in V} J_i \cdot q_{i,m-1,v} + J_i \cdot D_i \cdot (t_{i,m} - t_{i,m-1}) \tag{27}$$

$$\forall (i,m) \in S^A : m > 1$$

Ensures that the stock level at the start of the visit is set by the stock level at the start of the previous visit. The load or unload operation in the previous visit and the time elapsed between the two visits.

$$s_{i,m} \sum_{v \in V} q_{i,m,v} - \sum_{v \in V} D_i \cdot T_v^Q \cdot q_{i,m,v} \leq U_i^S \qquad \forall (i,m) \in S^A : J_i = -1 \tag{28}$$

Imposes limitation of the stock level at the end of a visit to a consumption port.

$$s_{i,m} - \sum_{v \in V} q_{i,m,v} + \sum_{v \in V} D_i \cdot T_v^Q \cdot q_{i,m,v} \geq L_i^S \qquad \forall (i,m) \in S^A : J_i = +1 \quad (29)$$

Imposes limitation of the stock level at the end of a visit to a production port.

$$s_{i,M_i} + \sum_{v \in V} q_{i,M_i,v} - (D_i \cdot (T - t_{i,M_i})) \geq L_i^S \qquad \forall i \in N : J_i = -1 \quad (30)$$

Imposes lower bound on the inventory level until the end of the time horizon for consummation ports.

$$s_{i,M_i} - \sum_{v \in V} q_{i,M_i,v} + (D_i \cdot (T - t_{i,M_i})) \leq U_i^S \qquad \forall i \in N : J_i = +1 \quad (31)$$

Imposes upper bound on the inventory until the end of the horizon for production ports.

$$s_{i,m} \geq L_i^S \qquad \forall (i,m) \in S^A : J_i = -1 \quad (32)$$

Enforces lower bound on the stock level at the start of each visit for consumption ports.

$$s_{i,m} \leq U_i^S \qquad \forall (i,m) \in S^A : J_i = +1 \quad (33)$$

Enforces upper bound on the stock level at the start of each visit for production ports.

$$\sum_{v \in V} \sum_{m \in \{1..M_i\}} q_{i,m,v} = T \cdot D_i \qquad \forall i \in N \quad (34)$$

Enforces that inventory level at the end of the planning horizon is equal to initial inventory level for all ports.

6.2 MIRP with variable time horizon as a set

This sub-chapter explains the model where the time horizon is modeled as a set. The model allows for integer variable time horizon. The description of the mathematical model shows new additions and changes to the model described in the Chapter 6.1.

New sets

R: Set of possible time horizons (days)

New parameters

T^H_r: Available number of days in the chosen time horizon. $r \in R$

$R^{PP}_{i,j,v,e,r}$: $\frac{C^{PP}_{i,j,v,e}}{T^H_r} \rightarrow$ Average daily sailing cost. $(i,j) \in N, v \in V, e \in S^S_v, r \in R$

$A^P_{i,v,r}$: $\frac{P_{i,v}}{TH_r} \rightarrow$ Average daily port operating cost $i \in N, v \in V, r \in R$

$B_{i,m,r}$: $T^H_r \rightarrow$ Latest time for starting visit m at port i with time horizon r.
$(i,m) \in S^A, r \in R$

$U_{i,m}$: $MAX\{T^H_r\} \rightarrow$ Latest time for finishing visit m at port i
with time horizon r. $(i,m) \in S^A$

New and modified variables

t^H_r: Equals 1 if time horizon r is chosen, 0 otherwise. $r \in R$

b: The variable b equals the number of days in the chosen time horizon.

$o_{i,m,v,r}$: Equals 1 if vessel v operates (i,m), 0 otherwise. $(i,m) \in S^A_v, v \in V, r \in R$

$g_{i,m,j,n,v,e,r}$: $>= 0$ or $<= 1 \rightarrow$
Auxiliary variable to determine the speed and time horizon of vessel v when going from node (i,m) to node (j,n), with s corresponding to a given choice of speed.
$(im,j,n) \in S^X_v, v \in V, e \in S^S_v, r \in R$

$g^{O}_{i,j,n,v,e,r}$: $>= 0$ or $<= 1 \rightarrow$

Auxiliary variable to determine the speed and time horizon of vessel v when going from node (j,n) to destination at port i, with s corresponding to a given choice of speed

$i \in N, (j,n) \in S^A_v, v \in V, e \in S^S_v, r \in R$

$g^{D}_{j,n,i,v,e,r}$: $>= 0$ or $<= 1 \rightarrow$

Auxiliary variable to determine speed and time horizon of vessel v when going from node (j,n) to destination at port i, with s corresponding to a given choice of speed

$j \in N, n \in 1..M_j, i \in N, v \in V, e \in S^S_v, r \in R$

New objective function

Minimize

$$\begin{aligned}
&\sum_{(i,m,j,n)\in S^X_v, v\in V, e\in S^S_v, r\in R: i\neq j} R^{PP}_{i,j,v,e,r} \cdot g_{i,m,j,n,v,e,r} && + \qquad (35)\\
&\sum_{i\in N,(j,n)\in S^A_v, v\in V, e\in S^v_S, r\in R} R^{PP}_{i,j,v,e,r} \cdot g^{O}_{i,j,n,v,e,r} && +\\
&\sum_{(j,n)\in S^A_v, i\in N, v\in V, e\in S^v_S, r\in R} R^{PP}_{i,j,v,e,r} \cdot g^{D}_{j,n,i,v,e,r} && +\\
&\sum_{(i,m) S^A_v \in, v\in V, r\in R} A^{P}_{i,v,r} \cdot o_{i,m,v,r}
\end{aligned}$$

Objective function minimizes average daily costs.

New and modified constraints

$$\sum_{r \in R} t_r^H = 1 \tag{36}$$

The chosen time horizon can only be one specific number of days.

$$\sum_{r \in R} T_r^H \cdot t_r^H = b \tag{37}$$

Sum of all time horizons multiplied by all chosen time horizons should equal the variable number of days in the planning horizon b.

$$\sum_{v \in V} \sum_{e \in S_v^S} g_{i,m,j,n,v,e,r} \leq t_r^H \qquad \forall (i,m,j,n) \in S_v^X, r \in R \tag{38}$$

Ensures that the auxiliary variable $g_{i,m,j,n,v,e,r}$ is equal to the chosen time horizon.

$$\sum_{v \in V} \sum_{e \in S_v^S} g_{i,j,n,v,e,r}^O \leq t_r^H \qquad \forall i \in N, (j,n) \in S_v^A, r \in R \tag{39}$$

Ensures that the auxiliary variable $g_{i,j,n,v,e,r}^O$ is equal to the chosen time horizon.

$$\sum_{v \in V} \sum_{e \in S_v^S} g_{j,n,i,v,e,r}^D \leq t_r^H \qquad \forall (j,n) \in S_v^A, i \in N, r \in R \tag{40}$$

Ensures that the auxiliary variable $g_{j,n,i,v,e,r}^D$ is equal to the chosen time horizon.

$$\sum_{v \in V} O_{i,m,v,r} \leq t_r^H \qquad \forall (i,m) \in S_v^A, r \in R \tag{41}$$

Ensures that the binary variable $O_{i,m,v,r}$ is equal to the chosen time horizon.

$$\sum_{r \in R} o_{i,m,v,r} - \sum_{(j,n) \in S_v^A : i \neq j} x_{i,m,j,n,v} - \sum_{j \in N} x_{i,m,j,v}^D = 0 \tag{42}$$

$$\forall v \in V, (i,m) \in S_v^A$$

Ensures that if a vessel is at node i it must either leave to another node or end. The equation is serves the same purpose as 5, but it has to sum over all $r \in R$.

$$\sum_{r \in R} o_{i,m,v,r} - \sum_{(j,n) \in S_v^A : j \neq i} x_{j,n,i,m,v} - \sum_{j \in N} x^O_{j,i,m,v} = 0 \qquad \forall v \in V, (i,m) \in S_v^A \tag{43}$$

Defines that if a node is visited by vessel v, the vessel must either arrive at the node from the origin or from another node. It is the equivalent of constraint 6, but sums over all $r \in R$.

$$\sum_{v \in V} \sum_{r \in R} o_{i,m,v,r} = y_{i,m} \qquad \forall (i,m) \in S_v^A \tag{44}$$

Shows that a vessel can only visit (i,m) if there are at least m visits. The equivalent of equation 7, but sums over all $r \in R$. The constraint ensures the same purpose as 7, but it must sum over all available time horizons ($r \in R$).

$$\sum_{e \in S_v^S} \sum_{r \in R} g_{i,m,j,n,v,e,r} = x_{i,m,j,n,v} \qquad \forall v \in V, (i,m,j,n) \in S_v^x \tag{45}$$

Enforces that speed and time horizon of a vessel must be set for a route from node (i,m) to node (j,n) and only if the route exists. The summation is performed for all $r \in R$, and serves the same purpose as 10.

$$\sum_{e \in S_v^S} \sum_{r \in R} g^O_{j,n,i,v,e,r} = x^O_{j,n,i,v} \qquad \forall v \in V, (j,n) \in S_v^A, i \in N \tag{46}$$

Enforces that speed and time horizon of a vessel must be for for a travel from the origin to node (j,n) and only if that route exists. The equivalent of equation 11, but sums over all $r \in R$.

$$\sum_{e \in S_v^S} \sum_{r \in R} g^D_{j,n,i,v,e,r} = x^D_{j,n,i,v} \qquad \forall v \in V, (j,n) \in S_v^A, i \in N \tag{47}$$

Enforce that speed and time horizon of a vessel must be set for a route from node (j,n) to the destination if that route exists. The constraint serves the same purpose as equation 12. The main difference is the summation of all $r \in R$.

$$q_{i,m,v} \leq MIN\{C_v, U_i^S\} \cdot \sum_{r \in R} o_{i,m,v,r} \qquad \forall v \in V, (i,m) \in S_v^A \tag{48}$$

Ensures the quantity loaded/unloaded cannot exceed the vessel capacity nor the maximum port capacity. The variable $O_{i,m,v,r}$ represents if vessel v operates in (i,m), but it can only operate with one time horizon. For this reason, the equation must sum over all $r \in R$. The equation has the same function as 13.

$$Q_i \cdot o_{i,m,v,r} \leq q_{i,m,v} \qquad \forall v \in V, (i,m) \in S_v^A, r \in R \quad (49)$$

Describes that if a vessel visits the port, then the amount loaded/unloaded should be at least equal to the minimum quantity. The equation has the same function as equation 14, but differs because it is subject to each available time horizon ($r \in R$).

$$t_{i,m} + \sum_{v \in V} T_v^Q \cdot q_{i,m,v} - t_{j,n} \quad (50)$$

$$+ \sum_{v \in V} \sum_{e \in S_v^S} (U_{i,m} + T_{i,j,v,e}^{PP}) \cdot g_{i,m,j,n,v,e,r} \leq U_{i,m}$$

$$\forall (i,m) \in S^A, (j,n) \in S^A, r \in R : i \neq j$$

Relate the start time associated with node (i,m) to the start time associated with node (j,n) when a vessel travels between ports i and j. It is the equivalent of equation 21, but is subject to $r \in R$.

$$t_v^S + \sum_{j \in N} \sum_{e \in S_v^S} T_{j,i,v,e}^{PP} \cdot g_{j,i,m,v,e,r}^O \leq t_{i,m} + U_{i,m} \cdot (1 - o_{i,m,v,r}) \quad (51)$$

$$\forall v \in V, (i,m) \in S_v^A, r \in R$$

Ensures that the travel time + delay time should not exceed the start time of the visit to the port. It is the equivalent of equation 22, but is subject to $r \in R$.

$$t_{j,n} + T_v^Q \cdot q_{j,n,v} + \sum_{i \in N} \sum_{e \in S_v^S} T_{j,i,v,e}^{PP} \cdot g_{j,n,i,v,e,r}^D \leq T_r^H + t_v^S + U_{j,n} \cdot (1 - o_{j,n,v,r}) \quad (52)$$

$$\forall v \in V, (j,n) \in S_v^A, r \in R$$

Expresses time at node (j,n) plus travel time for a vessel traveling from node (j,n) to destination at node i should not exceed the time horizon plus delay time. It is the equivalent of equation 23, but is subject to $r \in R$.

$$t_{i,m} \leq \sum_{r \in R} T_r^H \cdot t_r^H \qquad \forall (i,m) \in S_v^A \quad (53)$$

Expresses that start time in port (i, m) must be less or equal to the chosen time horizon. The equation is serves the same purpose as equation 25. The main difference is the variable time horizon on the right hand side.

6.3 MIRP with linear approximation

This sub-chapter presents a mathematical model which uses variable time horizons. The approach of this model is a linear approximation. A set of breakpoints for the upper/lower bound of the total cost and time horizon is defined. This model intends to choose the variable time horizon corresponding to the minimum average daily costs. The model in Chapter 6.2 must declare all available time horizons. In contrast, the model presented in this sub-chapter can declare ranges of time horizons. The selected variable time horizon can take both an integer number and a non-integer number of days. If the time horizon is a non-integer, it falls between the range of two time horizons (or breakpoints).

New sets

B^T: Set of breakpoints for the time horizon, where $B^T = \{1, 2, ..T^{MAX}\}$

B^C : Set of lower and upper bound for the total cost, where $B^C = \{1, 2\}$.

New parameters

T_k: The breakpoints for the time horizon. $t \in B^T$

C_c: The lower and upper bound for the total cost. $c \in B^C$

$Z_{k,c}$: $\frac{C_c}{T_k} \rightarrow$ Average daily cost, where lower and upper bound on total cost is divided by breakpoints in the time horizon. $t \in B^T, c \in B^C$

T^{MAX}: $MAX\{T_k\} \rightarrow$ The maximum allowed number of days in the planning horizon

T^{MIN}: $MIN\{T_k\} \rightarrow$ The minimum allowed number of days in the planning horizon

New variables

t^H: $\geq T^{MIN}$ or $\leq T^{MAX}$ $\rightarrow$ Determines the time horizon.

b_k: Equals 1 if breakpoint k is used, 0 otherwise. $k \in B^T$

$p_{k,c}$: ≥ 0 or ≤ 1 $\rightarrow$ Auxiliary variable to determine the time horizon and total cost between. the breakpoints. $k \in B^T, c \in B^C$

c^T: $\sum_{v\in V} \sum_{(i,m,j,n)\in S_v^X} \sum_{e\in S_v^S} C^{PP}_{i,j,v,s} \cdot g_{i,m,j,n,v,e} + \sum_{v\in V} \sum_{(j,n)\in S_v^A} \sum_{e\in S_v^S} \sum_{i\in N} C^{PP}_{i,j,v,s} \cdot g^O_{i,j,n,v,e} + \sum_{v\in V} \sum_{(j,n)\in S_v^A} \sum_{e\in S_v^S} \sum_{i\in N} C^{PP}_{j,i,v,e} \cdot g^D_{j,n,i,v,s} + \sum_{v\in V} \sum_{(i,m)\in S_v^A} P_{i,v} \cdot o_{i,m,v}$

$\rightarrow$ The variable c^T expresses total traveling costs and operational costs in all ports

New objective function

Minimize

$$\sum_{k\in B^T} \sum_{c\in B^C} Z_{k,c} \cdot p_{k,c} \tag{54}$$

Objective function minimizes average daily costs.

New constraints

$$\sum_{k\in B^T} \sum_{c\in B^C} T_k \cdot p_{k,c} = t^H \tag{55}$$

Ensures that breakpoints T_k multiplied by auxiliary variable p_{tc} is equal to the chosen time horizon t^H.

$$\sum_{k\in B^T} \sum_{c\in B^C} C_c \cdot p_{k,c} = c^T \tag{56}$$

Ensures that breakpoints for total cost C_c multiplied by auxiliary variable $p_{k,c}$ is equal to the total cost c^T.

$$\sum_{k\in B^T} \sum_{c\in B^C} p_{k,c} = 1 \tag{57}$$

The sum of the $p_{k,c}$-*variables*, which chooses the breakpoint in time horizon and total cost, must be equal to 1.

$$\sum_{k \in B^T} b_k \leq 1 \tag{58}$$

The sum of binary variable b_k, which chooses a breakpoint for the time horizon, must be less or equal to 1.

$$p_{k,c} <= b_k + b_{k+1} \qquad k \in B^T : k \neq MAX\{B^T\}, \forall c \in B^C \tag{59}$$

Constraint ensures that the chosen breakpoint for the time horizon equals for lower and upper breakpoint of the total cost.

$$p_{k,c} \leq b_k \qquad k \in B^T : k = MAX\{B^T\}, \forall c \in B^C \tag{60}$$

Ensures that the selected time horizon breakpoint converges with the time horizon and total cost breakpoints.

$$t_{i,m} \leq t^H \qquad \forall (i,m) \in S^A \tag{61}$$

The start time of visit (i, m) must be smaller or equal to the variable time horizon t^H. The purpose of this equation is the same as equation 25 from the original model.

$$s_{i,M_i} + \sum_{v \in V} q_{i,M_i,v} - (D_i \cdot (t^H - t_{i,M_i})) \geq L_i^S \qquad \forall i \in N : i = -1 \tag{62}$$

Imposes lower bound on the inventory level until the end of the time horizon for consummation ports. The equation functions in the same way as equation 30, but the fixed time horizon T is changed to the variable time horizon t^H on the left hand side.

$$s_{i,M_i} - \sum_{v \in V} q_{i,M_i,v} + (D_i \cdot (t^H - t_{i,M_i})) \leq U_i^S \qquad \forall i \in N : J_i = +1 \tag{63}$$

Imposes upper bound on the inventory on the inventory level until the end of the horizon for production ports. The equation functions in the same way as equation 31, but the fixed time horizon T is changed to the variable time horizon t^H on the left hand side.

$$\sum_{v \in V} \sum_{m \in \{1..M_i\}} q_{i,m,v} = t^H \cdot D_i \qquad \forall i \in N \tag{64}$$

Enforces that inventory level at the end of the planning horizon is equal to initial inventory level for all ports. The fixed time horizon T from equation 34 is changed to variable time horizon t^H on the right hand side.

7 Computational study

This section presents a computational study of how the developed mathematical models perform regarding results and interpretation. The computational results were run with the remote desktop of Molde University College. The processor is Intel Xeon Gold 6330 CPU and 24 GB RAM. The servers connected to the remote desktop manage these resources under a "fair share" policy. Due to this policy, most optimization instances ran in the night time. The models are coded in AMPL programming language, and CPLEX 20.1.0.0 is used as the solver. The data used for the computational study comes from two sources and are explained in the following two chapters.

7.1 Explanation of optimization instances

The first data group used for this thesis is taken from Agra et al. [24]. The datasets are artificial data called A, B, C, D, and F. Each optimization instance has a unique ID that comprises the following:

1. Dataset
2. Number of ports
3. Number of vessels

Instance ID	Dataset	Number of ports	Number of Vessels
A-4-1	A	4	1
B-3-2	B	3	2
C-4-2	C	4	2
D-5-2	D	5	2
F-4-3	F	4	3

Table 7: Optimization instances. [Made by the author]

The point of each optimization instance is to find the minimum average daily cost. The instances which use mathematical models SET and LA uses variable time horizon, which means that the models can choose a time horizon between 1 and 60 days in each optimization run.

The vessels are defined with further characteristics such as:

- Possible sailing speeds for the vessels (knot per hour)
- The fixed amounts a vessel loads/unloads per time period
- The time for loading/unload each unit
- Capacities of the vessel (tons)
- Daily traveling costs related to each speed (1000 USD per day)

- Operational costs for operating a vessel in a port (1000 USD)

Table 8 indicates the speed alternatives of the vessels in each dataset. The speeds are collected from the master thesis of Evsikova [25]. Table 9 indicates the maximum cargo capacity for the ships. Each capacity is stated in 1000 DWT. The abbreviation DWT stands for deadweight tonnage, and is the unit of measure for how much a ship can carry.

	Speeds (knots per hour)					
Dataset	**13,5**	**15**	**16.2**	**18**	**19**	**21**
A			S1	S1		S1
B	S1 & S2	S1 & S2			S1 & S2	
C	S1 & S2	S1 & S2			S1 & S2	
D	S1 & S2	S1 & S2			S1 & S2	
F	S1 & S2 & S3	S1 & S2 & S3			S1 & S2 & S3	

Table 8: Indicates the speed alternatives for each data dataset. [Made by the author]

	Capacity (In 1000 DWT)			
	100	**130**	**150**	**160**
A				S1
B	S1	S2		
C		S2		S1
D	S1		S1	
F		S2	S3	S1

Table 9: Indicates the capacites of the vessels for each dataset. [Made by the author]

7.2 Limitation on timelimit

Note that the clock time elapsed for an ORG instance is the sum of all 60 optimization runs, essential for comparison with SET and LA instances. Also, each of the 60 optimization runs in ORG has a time limit of 2 hours clock time. In order to make a comparison of time use between the models, the time limit for optimization with SET- and LA instances equals the sum of time used for the corresponding ORG instance. For example, "A-4-1-ORG" uses dataset A and requires 2362 seconds. Therefore, the corresponding instances "A-4-1-SET" and "A-4-1-LA" will have a time limit of 2362 seconds. However, due to computational complexity, some ORG instances require enormous amounts of time to finish all 60 optimization runs. It is subsequently unreasonable to set the time limit of corresponding SET- and LA instances equal to the time sum required for ORG. For example, "D-5-2-ORG" required 4 days to complete all 60 optimization runs. Due to the author's schedule, setting a time limit of 4 days for the corresponding SET- and LA instances is impossible. Therefore, if any ORG instance requires more than 6 hours of computational time, the corresponding SET- and LA instances will have a time limit of 6 hours.

7.3 Comparison of computational time

This subchapter explains and compares the computational time (seconds) between the models and instances. The numbers in Table 10 represents the computational clock time of the instances. The column group "Mathematical Model" comprises of the three presented mathematical models from Chapter 6.1, SET from Chapter 6.2 and LA from Chapter 6.3. Remember that each optimization with the ORG model has 60 optimization runs, with fixed time horizons from 1-60 days and a 2-hour time limit per optimization run.

The instances which use the mathematical model ORG optimize 60 times with a fixed time horizon ranging from 1 to 60. The optimal solution for ORG instances is found where the fixed time horizon corresponds to the minimum average daily cost. In other words, the optimization method for ORG instances equals the method described in Chapter 4.

	Mathematical Model		
Instance ID	**ORG**	**SET**	**LA**
A-4-1	2362	81	0.61
B-3-2	75661	21600	16362
C-4-2	85120	21600	267
D-5-2	355027	No solution	21600
F-4-3	135657	15026	450

Table 10: Computational clock time (seconds). [Made by the author]

Dataset A

Instance "A-4-1-ORG" needed 2362 seconds to finish and is the only instance with the original model which required less than 6 hours. Therefore, the corresponding models "A-4-1-SET" and "A-4-1-LA" has an equal time limit. The SET- and LA instances required 97 % and 99,98 % less time to solve to optimality, respectively.

Dataset B

With dataset B, the ORG and SET model used 6 hours or more computational time; therefore, it is difficult to compare the effectiveness in terms of computation time. However, the fact that SET required a maximum time limit was expected. The reason for this is that 15 % of the optimization runs in ORG used the maximum time limit. Likewise, when the SET model allows 60 different time horizons in one optimization run, the SET model inherits the complexity of the ORG model. On the other hand, the LA model solved dataset B to optimality primarily due to the simplified solution space.

Dataset C

Dataset C required over 6 hours or more computational time with ORG and SET models. Consequently, 12 % of the optimization runs in ORG required the maximum time limit, and the SET model was not optimally solved. The LA model solved dataset C to optimality.

Dataset D

Of all datasets, D required the most computational time. The level of complexity rises with D instances because 2 production ports and 2 vessels have to serve 3 consumption ports. The ORG model required 4 days of computation time, and 82 % of the 60 optimization runs did not prove optimality. Optimization with the SET model was impossible because the software failed to start the solution process with CPLEX. The LA model managed to obtain a solution but did not solve to optimality.

Dataset F

Model SET and LA managed to solve dataset F to optimality. This outcome was not expected because 22 % of the optimization runs with ORG did not prove optimality. When the percentage of non-proven optimal solutions is high in ORG, it is reasonable to expect that SET and LA models will struggle to obtain optimal solutions. However, compared to the other datasets, F has 3 vessels that serve 2 consumption ports. Thus, dataset F has more resources to complete the task of delivering the product to consumption ports. Although not mathematically proven, the extra vessel might explain why SET and LA obtain optimal solutions.

7.4 Cost Comparison

This sub-chapter aims to compare the costs of instances done with the developed models from Chapters 6.2 and 6.2 with instances done by Zojaji and Soltaniani [4] with the original model from Chapter 6.1. However, note that the original model's results are re-optimized by this thesis's author, but the instances are the same. The models developed for this thesis allow choosing any time horizon between 1 and 60 days, whereas Zojaji and Soltaniani used a 30 and 60-day fixed planning horizon. Zojaji and Soltaniani [4] used datasets A, B, C, D, F, and G for their computational study. Due to the tight schedule of the author of this thesis, dataset G is left out. This sub-chapter presents a cost comparison for each data set. Each comparison shows a plot, and the data of the blue cost curve in each plot are the results obtained for the 60 computations in each ORG instance. The AMPL solutions of ORG, SET and LA with data set C are presented in the appendix.

Table 11 requires an explanation from left to right:

- The columns "C30" and "C60" represent the average daily cost when running the original model of Zojaji and Soltaniani [4] (from chapter 6.1) with 30 day and 60 day time horizon, respectively.
- The column group SET comprises of the average average daily cost found with the mathematical model from Chapter 6.2. Columns "Change C30 (%)" and "Change C60 (%)" compare the the average daily cost of the SET model with using fixed time horizons of 30 and 60 days, respectively.
- Finnaly, the column group LA comprises of the average daily cost found with the mathematical model from Chapter 6.3. Columns Change C30 (%)" and "Change C60 (%)" compare the the average daily cost of the LA model with using fixed time horizons of 30 and 60 days, respectively.

In order to fully understand the cost comparison of Table 11, it is also necessary to visualize the numbers on an average daily cost curve. Each average daily cost curve follows the same example as shown in Chapter 4. One plot for each data set will be presented in the following chapters.

			SET			LA		
Instance	**C30**	**C60**	**C-SET**	**Change C30 (%)**	**Change C60 (%)**	**C-LA**	**Change C30 (%)**	**Change C60 (%)**
A-4-1	7.69	6.11	4.43	42.39	27.5	4.69	39.01	23.24
B-3-2	11.41	10.54	8.88	22.17	15.75	8,60	24,63	18.41
C-4-2	11.57	8.69	7.25	37.34	16.57	7.2	37.77	17.15
D-5-2	18.52	13.07	No solution	-		10.9	41.14	16.6
F-4-3	10.53	8.58	6.74	35.99	21.45	6.66	36.75	22.38

Table 11: Cost comparison. [Made by the author]

7.4.1 Cost analysis of data set A

The blue line in Figure 6 represents the average daily costs of dataset A, with the corresponding time horizon. The red points represent the time horizon on the x-axis and the average daily cost from Table 11 on the y-axis. The points on the curve in Figure 6 clearly show that using a variable time horizon reduces average daily cost. For example, by using the SET model, it is possible to reduce average daily costs by 42.39 % and 27.5 %, when comparing with C30 and C60, respectively.

Figure 7 presents the total cost curve and results with red marks. By comparing Figures 6 and 7, it is possible to see that the total costs hardly change and average daily costs are reduced gradually between time horizons 27 and 53. As a result, it is possible to execute routing plans with a long time horizon and a low average daily cost.

According to Figure 6, the SET- and LA are both solved optimally, but the SET solution gives a slightly better solution than LA. This happens primarily because cost elements in the objective function are defined differently in SET and LA models. The SET model requires declaring all integer time horizons between 1 and 60 to search for solutions. In contrast, the LA model allows declaring individual breakpoints of time horizons. For example, the LA model can choose any non-integer time horizon between 50 and 55, but the days between are not declared as parameters. As a result, the SET model has a more accurate area to search for the optimal solution, but the LA model works much faster.

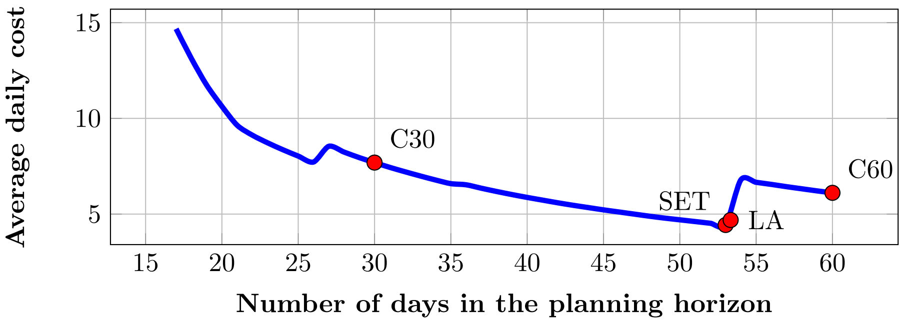

Figure 6: Average daily costs of dataset A: C30, C60, C-SET and C-LA. [Made by the author]

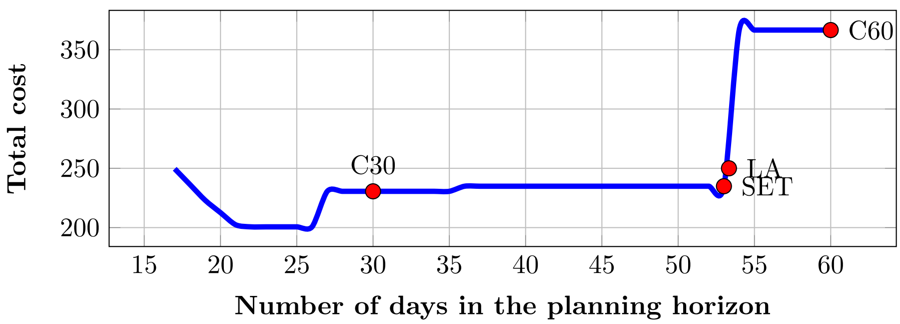

Figure 7: Total cost dataset A: C30, C60, C-SET and C-LA. [Made by the author]

7.4.2 Cost analysis of data set B

Figures 8 and 9 represent the average daily costs and total cost of data set B, respectively. Again, the red dots mark the results of each instance. The SET and LA models achieve feasible solutions with a lower average daily cost than C30 and C60. However, LA solved the model optimally, and SET did not because the computational clock time reached 6 hours.

Although the LA model solved the problem optimally, it is safe to say that the solution does not correspond with the minimum average daily cost. The results of SET are an average daily cost of 8.9 (1000 USD) with a time horizon of 46 days. The results of LA are an average daily cost of 8.60 (1000 USD) with a time horizon of 57.5 days. However, the minimum average daily cost on the blue curve in Figure 8 corresponds to 8.54 (1000 USD) with a time horizon of 59 days. It is also worth mentioning that optimality was not proven when solving the original model with just 59 as the fixed time horizon. Therefore, with the on-hand information, it is difficult to know how far the author is from an optimal solution.

In order to prove a new and better optimal solution, one measure would be to let LA include 59 and 60 as breakpoints to prove a better optimal solution. However, this is not a part of the experiment because extensive fine-tuning of data restricts the "competition" between the models.

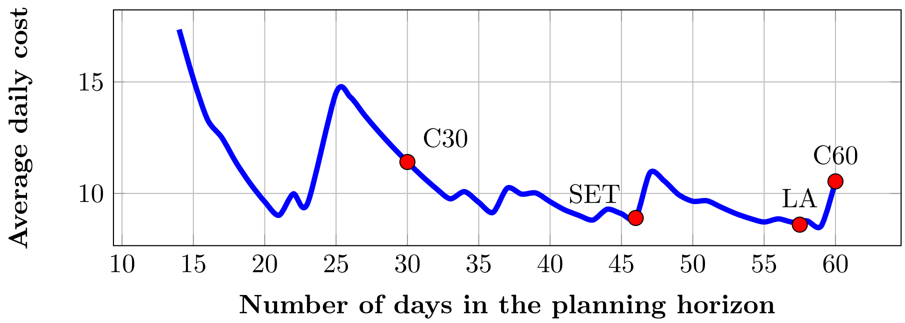

Figure 8: Average daily costs of dataset B: C30, C60, C-SET and C-LA. [Made by the author]

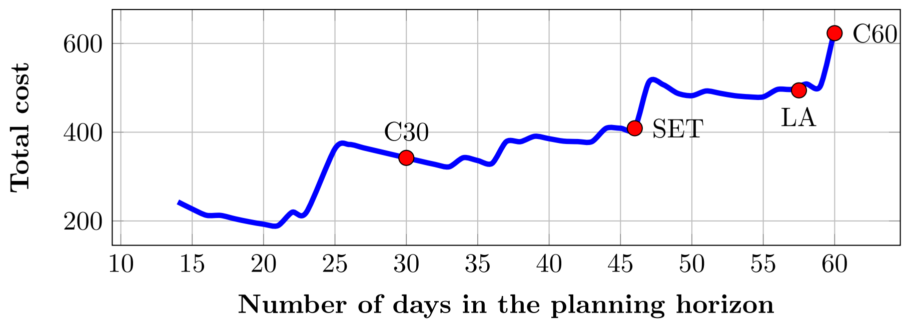

Figure 9: Total cost dataset B: C30, C60, C-SET and C-LA. [Made by the author]

7.4.3 Cost analysis of data set C

Figures 10 and 11 represent the average daily and total cost with data set C, respectively. Note that SET and LA models have similar solutions and fall on top of each other on the blue cost graphs. The SET solution is furthest to the left, and LA solution is furthest to the right.

The SET- and LA models find a lower average daily cost than C30 and C60. Even though the SET model did not prove optimally with CPLEX, it finds the lowest point corresponding to an average daily cost of 7.25 (1000 USD) with a 48-day time horizon. The average daily cost of LA is 7.20 (1000 USD) with a time horizon of 48.33 and was proven optimally. In contrast to Chapter 7.4.1, the LA model finds a feasible solution with a lower average daily cost than SET. When allowing only integer time horizons, the lowest possible average daily cost corresponds with a time horizon of 48 days. However, allowing non-integer time horizons allows the LA model to find a feasible solution with an even lower average daily cost. With that in mind, the cost curve of Figure 10 shows that the average daily cost increases after 48 days on the x-axis. Despite that, the LA model finds a lower average daily cost because, in practice, the average daily decreases between 48 and 48.33 days time horizons.

Figure 11 shows that the total cost change very little between time horizons 33 and 48. When solving the original model with fixed time horizons, the total cost remains 350 (1000 USD) between 33 and 39 day time horizons and 348 (1000 USD) between 40 and 48 day time horizons. Also, recall from Table 10 that the ORG instance represents the sum of time used for 60 individual computations with a fixed time horizon. The time needed to compute all 60 computations with ORG and dataset C was 85120 seconds. However, computations with time horizons between 33 and 48 days only required 3096 seconds, which corresponds to only 3.6 % of the total time of ORG. These characteristics of the total cost and solving time explain that the minimum average cost is in an area of low computational complexity on the blue cost curve of Figure 10. Likewise, the characteristics also explain why the SET and LA models efficiently manage to find the minimum average daily cost.

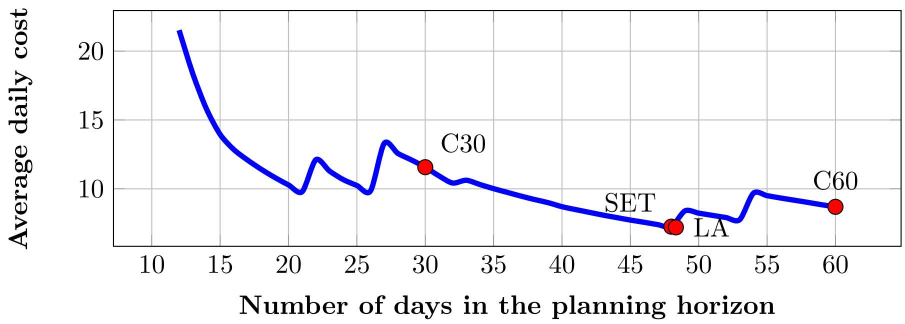

Figure 10: Average daily costs of dataset C: C30, C60, C-SET and C-LA. [Made by the author]

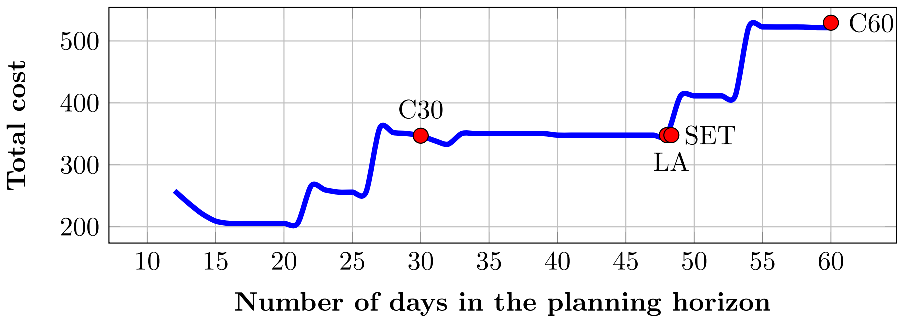

Figure 11: Total cost dataset C: C30, C60, C-SET and C-LA. [Made by the author]

7.4.4 Cost analysis of data set D

Figures 12 and 13 present the average daily cost and total cost with data set D, respectively. These figures need an explanation. Solutions with 28, 29, and 56 days in the time horizon were infeasible. For this reason, solutions with these time horizons have no relevance.

As mentioned in Chapter 7.3, when using the SET model on dataset D, AMPL failed to produce any solution file. For this reason, no results with the SET model are shown in Figures 12 and 13. On the other hand, the LA model obtained a feasible solution and corresponded to an average daily cost of 10.9 with 50 days time horizon. The LA solution also corresponds with the minimum average daily cost on the blue cost curve in Figure 12 but was not proven optimally.

With data set D, proving any optimal solution with either variable or fixed time horizon was impossible. When applying the original model (ORG-instance), non of the solutions were proven optimally. However, using linear approximation with the LA model proved much more efficient than the original model. The grounds for this are that the LA model found the lowest average daily cost in 6 hours, while the original model needed 4 days.

Although data set D was challenging to work with, there are some typical similarities with solutions from the other data sets. For example, the blue cost curve of Figure 13 displays that total costs with fixed time horizons 43-50 are equal. Therefore, the average daily cost decreases when the total cost remains unchanged. As a result, the minimum average daily cost is often found where the same total cost is repeated over many different time horizons. This characteristic of the minimum average daily cost location on the curve is also shared with solutions with data sets A and C.

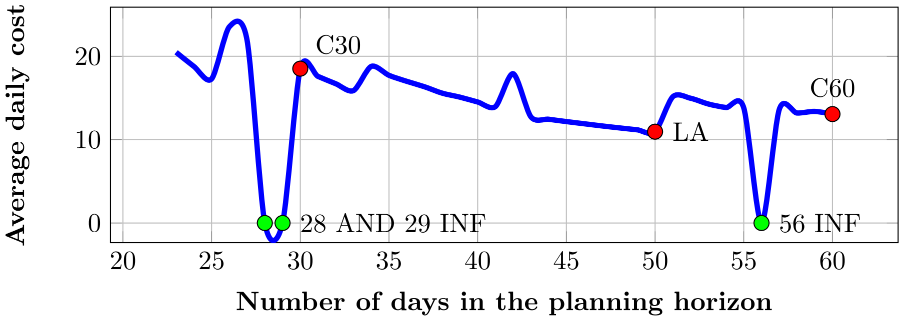

Figure 12: Average daily costs of dataset D: C30, C60, C-SET and C-LA. [Made by the author]

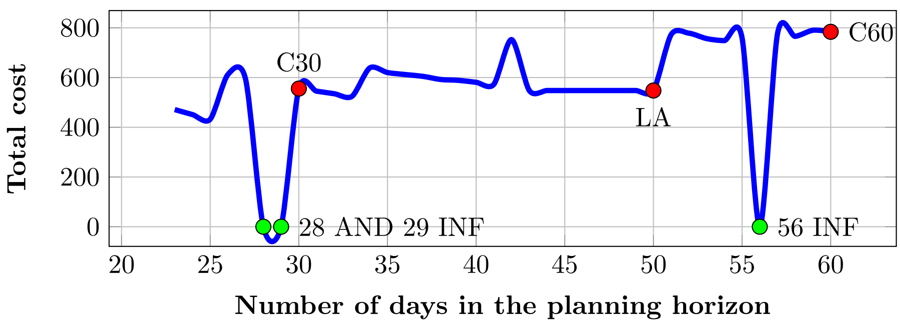

Figure 13: Total cost dataset D: C30, C60, C-SET and C-LA. [Made by the author]

7.4.5 Cost analysis of data set F

Figures 14 and 15 display the average daily cost and total cost of computations with data set F, respectively. Again, the solutions of SET and LA fall on top of each other. The SET solution is furthest to the left, and LA is furthest to the right.

The SET and LA model obtained more cost efficient solutions than C30 and C60. Also, both SET and LA models were solved optimally. The solution of SET corresponds with an average daily cost of 6.75 (1000 USD) with a time horizon of 51 days. The solution of LA corresponds with an average daily cost of 6.66 with a time horizon of 51.67 days.

The optimal average daily costs are also located where the total costs are equal for many time horizons and rise sharply after. This characteristic is clearly shown in Figures 14 and 15. This proves that the SET and LA models work well with data set F. However, it is worth mentioning that when solving data set F with the original model, time horizons over 51 days were not proven optimally. Due to this, it is difficult to say if there are more feasible solutions to find after 51.67 days, which also corresponds to the time horizon of LA.

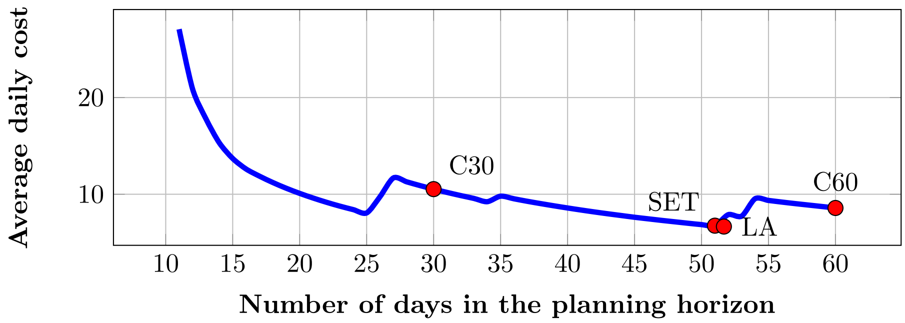

Figure 14: Average daily costs of dataset F: C30, C60, C-SET and C-LA. [Made by the author]

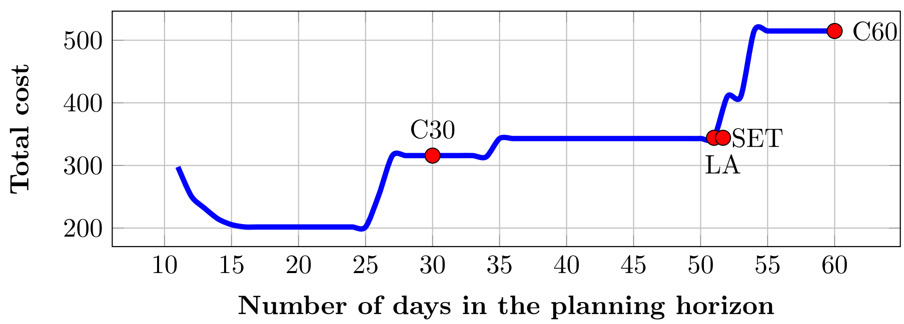

Figure 15: Total cost dataset F: C30, C60, C-SET and C-LA. [Made by the author]

7.5 Assessment of maximum number of visits to ports

The maximum number of visits is an essential parameter of the models which must be sufficiently high enough to obtain a feasible solution. When optimizing AMPL with a given data set, using one specific strategy to set the maximum number of visits is essential to let the 3 models compete on the same terms. In order to have the correct maximum number of visits for the LA and SET instances, the corresponding ORG must first be finished. The ideal strategy is to set the maximum number of visits to be 1 greater than the highest visit number to any customer. For example, imagine an arbitrary instance with a maximum number of visits of 4. The instance is solved, and the highest visit number to any customer shows 4. In this situation, the model should be resolved with 5 as the maximum number of trips. This strategy should be repeated until all 60 computations within ORG are completed. The maximum number of visits of the 60th computation in ORG will then apply to the SET and LA instances. Regrettably, looking back at the AMPL data files, different strategies were used for the various data sets. Due to this sloppy misstep of the author, it is critical to explain the strategy used for the various data sets. Even though different strategies were used, the SET and LA instances consistently applied the maximum number of visits from the 60th computation in ORG.

Here are the strategies for choosing the maximum number of visits for the ORG instances:

- **Data set A:** The chosen strategy was to allow the maximum number of visits to be 1 greater than the highest visit number to any customer.
- **Data set B:** The chosen strategy was to allow the maximum number of visits to equal the highest visit number to any customer.
- **Data set C:** The chosen strategy was to let the maximum number of visits equal 4 until 4 became the highest visit number for any customer. The remaining computations allowed the maximum number of visits to be 1 greater than the highest visit number.
- **Dataset D:** The chosen strategy was to let the maximum number of visits equal 4 until 4 became the highest visit number for any customer. The remaining computations allowed the maximum number of visits to be 1 greater than the highest visit number.
- **Dataset F:** The chosen strategy was to let the maximum number of visits equal 4 until 4 became the highest visit number for any customer. The remaining computations allowed the maximum number of visits to be 1 greater than the highest visit number.

The strategy used with data set B is an issue in letting the models compete on equal terms. For example, instance B-3-2 with 30 day fixed time horizon applied 3 as the maximum number of visits, and the highest number of visits to any customer was 3 in the solution file. Secondly, B-3-2 with 60 days fixed time horizon applied 5 as the maximum number of visits. The problem arises when solving the B-3-2 with SET and LA with variable time horizons between 1 and 60. So when SET and LA apply

5 as the maximum number of visits, they can theoretically obtain a 30-day time horizon with 4 as the highest visit number to any customer. So in practice, the SET and LA models have more variables to consider 30 as the optimal time horizon. To conclude, when completing an ORG instance, the individual computations with fixed time horizons should always apply a maximum number of visits which is at least 1 greater than the highest visit number to any customer.

7.6 Comparison of structure

This sub-chapter goes deeper into the developed models by comparing key numbers from the solutions. Table 12 presents the distance traveled, number of visits, and time horizon of the models. The numbers are presented for the ORG, SET, LA, and the original model with 30 and 60-day time horizons. The 30 and 60-day time horizons were the instances Zojaji and Soltaniani [4] used in their thesis. In order to fit the content of 12 to the page, it was split into two parts. The number of visits from Table 12 represents the total number of visits for a given solution, whereas Chapter 7.5 discussed the maximum number of visits to any port.

Moving on, Table 13 presents the speeds used on the routes of all vessels in each instance. The numbers summarise the amount a specific speed is applied to an instance. Finally, the last column of Table 13 presents the average speed.

Chapter 7.4 was a cost comparison, and the results presented a great potential for cost savings by choosing a time horizon with lower average daily cost rather than using fixed time horizons with 30 and 60 days. However, with large cost savings, the structure of the solutions is affected significantly. The best way to explain new and old structures is by referring to a specific instance. For example, with instance F-4-3, SET, and LA choose time horizons of 51 and 51.67, respectively. The distance traveled from SET and LA solution is 13990 nautical miles. The distance traveled with ORG (30 days) and ORG (60 days) is 12825 and 20986 nautical miles, respectively. Since all models' solutions are cyclic, comparing the average daily distance becomes interesting. With a 51-day time horizon, the average daily distance is 272.5 nautical miles. The average daily distance with a 30-day time horizon is 427.5 nautical miles. This comparison expresses a considerable potential for reduction of the total mileage. Table 13 shows that all models use the same speed with the F-4-3 instance, which signifies that reducing costs without changing the speed is possible.

Moving on to B-3-2, the SET- and LA solutions have very different time horizons, which seems strange. On the other hand, the average daily costs of SET and LA are not far from each other along the y-axis of Figure 8. Since SET was not solved optimally, it is still expected that CPLEX will search for an average daily cost close to the LA solution. So due to the non-optimal solution with SET and proximity to the average daily cost of LA, it is not unreasonable that SET obtained a time horizon of 46 days.

The results of C-4-2 are exciting because they show how minimizing the average daily cost can affect the speed and number of visits. When optimizing with 30 days fixed time horizon, the solution has an average speed of 14.78 knots. When optimizing with variable time horizons, the models can reduce the average speed to a minimum of 13.5 knots. Moving on, the number of visits affects the port operating costs. So with a 48-day time horizon, the solution requires only one extra port visit compared to a fixed 30 days time horizon. Also, when comparing 60 and 48-day time horizons, the 48-day time horizons require four fewer port visits. To put it in perspective: By subtracting 12 days from the 60-day time horizon, it is possible to remove 1/3 of all port visits.

Another important aspect of the C-4-2 is that ORG, SET, and LA show equal mileage, but LA has 0,33 days longer time horizon. This inequality time horizon happens because the vessels in LA start the journey by waiting in the origin port.

	ORG			**SET**			**LA**		
Instance ID	**Distance traveled in nautical miles**	**Number of visits**	**Time horizon**	**Distance traveled in nautical miles**	**Number of visits**	**Time horizon**	**Distance traveled in nautical miles**	**Number of visits**	**Time horizon**
A-4-1	8161	5	53	8161	5	53	10104	6	53.33
B-3-2	26431	11	59	21767	9	46	25654	10	57.50
C-4-2	13990	8	48	13990	8	48	13990	8	48.33
D-5-2	23318	10	50	-	-	No solution	23318	10	50
F-4-3	13990	8	51	13990	8	51	13990	8	51.67

	ORG (30 days)			**ORG (60 days)**		
Instance ID	**Distance traveled in nautical miles**	**Number of visits**	**Time horizon**	**Distance traveled in nautical miles**	**Number of visits**	**Time horizon**
A-4-1	6218	4	30	12825	7	60
B-3-2	16325	7	30	25654	10	60
C-4-2	12825	7	30	20986	12	60
D-5-2	19432	8	30	31091	14	60
F-4-3	12825	7	30	20986	12	60

Table 12: Presents the traveled distance in nautical miles, total number of visits and time horizon. [Made by the author]

		Speed (knots)						
Model	**Instance ID**	**13,5**	**15**	**16,20**	**18**	**19**	**21**	**Average speed**
A-4-1	ORG (30 days)	-	-	4	-	-	-	16,2
	ORG (60 days)	-	-	7	-	-	-	16,2
	ORG	-	-	5	-	-	-	16,2
	SET	-	-	5	-	-	-	16,2
	LA	-	-	3,93	-	-	1,07	17,22
B-3-2	ORG (30 days)	3	3,07	-	-	0,93	-	14,89
	ORG (60 days)	2	0,18	-	-	7,82	-	17,83
	ORG	10,82	1,18	-	-	-	-	13,65
	SET	9	-	-	-	-	-	13,5
	LA	7,59	2,41	-	-	-	-	13,86
C-4-2	ORG (30 days)	2	4,64	-	-	0,36	-	14,78
	ORG (60 days)	11	-	-	-	-	-	13,5
	ORG	8	-	-	-	-	-	13,5
	SET	8	-	-	-	-	-	13,5
	LA	8	-	-	-	-	-	13,5
D-5-2	ORG (30 days)	4	0,73	-	-	3,27	-	15,89
	ORG (60 days)	12,74	1,26	-	-	-	-	13,64
	ORG	9	-	-	-	-	-	13,5
	SET	-	-	-	-	-	-	-
	LA	9	-	-	-	-	-	13,5
F-4-3	ORG (30 days)	7	-	-	-	-	-	13,5
	ORG (60 days)	12	-	-	-	-	-	13,5
	ORG	8	-	-	-	-	-	13,5
	SET	8	-	-	-	-	-	13,5
	LA	8	-	-	-	-	-	13,5

Table 13: Overview of the speeds used by the vessels. [Made by the author]

8 Conclusion

The introduction of this thesis displayed the increase in the volumes of maritime transport since 2010 [1]. As a result, maritime transport has significantly impacted world trade. Due to this impact, the requirements for logistics management have become more complex. Therefore, this thesis focuses on routing and inventory control, which are essential logistics management functions.

Three models have been presented, and two were developed to find the optimal average daily cost and time horizon. The results in the computational study confirm that variable time horizons can result in a considerable cost reduction. Moreover, it is also important to reflect on how the findings of this thesis are applicable in the real world. For example, if the outputs of factories are reduced due to the corona pandemic, it can significantly affect how shipping companies should organize the routing of vessels. At this moment, the shipping company obtains a more complex routing problem. To conclude, under such circumstances, the shipping company should consider the time horizon of the routing plan. The argument for this conclusion is the significant cost reduction obtained with the developed models.

9 Suggestion for future research

The mathematical model from Chapter 6.3 uses linear approximation and is called LA in the computational study. Studying how the LA model works when only allowing integer time horizons would be interesting. As mentioned, the LA model does not require declaring all the time horizons between 1 and 60. Instead, the LA model requires breakpoints of the time horizon. So it would be necessary to compare the non-integer and integer variable time horizons. The challenge is to find a feasible solution where the integer variable time horizon is between two breakpoints.

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A Solution of C-4-2-ORG

```
Total_Cost = 348.013

_total_solve_time = 127.141

_solve_elapsed_time = 12.812

xo :=
D2 P1 2 S1    1
P1 P1 1 S2    1
;

x :=
D1 1 P2 2 S1    1
D2 2 P2 1 S1    1
P1 1 D2 1 S2    1
P1 2 D2 2 S1    1
P2 1 D1 1 S1    1
P2 2 D2 3 S1    1
;

xd :=
D2 1 P1 S2    1
D2 3 D2 S1    1
;

go :=
D2 P1 2 S1 W1    1
P1 P1 1 S2 W1    1
;

g :=
D1 1 P2 2 S1 W1    1
D2 2 P2 1 S1 W1    1
P1 1 D2 1 S2 W1    1
P1 2 D2 2 S1 W1    1
P2 1 D1 1 S1 W1    1
```

```
P2  2  D2  3  S1  W1      1
;

gd  :=
D2  1  P1  S2  W1      1
D2  3  D2  S1  W1      1
;

w  :=
D2  S1      1
P1  S2      1
;

o  :=
D1  1  S1      1
D2  1  S2      1
D2  2  S1      1
D2  3  S1      1
P1  1  S2      1
P1  2  S1      1
P2  1  S1      1
P2  2  S1      1
;

y  :=
D1  1      1
D2  1      1
D2  2      1
D2  3      1
P1  1      1
P1  2      1
P2  1      1
P2  2      1
;

q  :=
D1  1  S1      144
```

```
D2  1  S2      128
D2  2  S1      160
D2  3  S1      144
P1  1  S2      128
P1  2  S1      160
P2  1  S1      160
P2  2  S1      128
;

f  :=
D1  1  P2  2  S1       16
P1  1  D2  1  S2      130
P1  2  D2  2  S1      160
P2  1  D1  1  S1      160
P2  2  D2  3  S1      144
;

fo  :=
P1  P1  1  S2      2
;

fd  :=
D2  1  P1  S2      2
;

t  :=
D1  1      27.5895
D1  2      29.3895
D1  3      29.3895
D1  4      29.3895
D2  1      10.6543
D2  2      13.9938
D2  3      42.6543
D2  4      48
P1  2       5.99691
P1  3       7.99691
P1  4       7.99691
```

```
P2 1     21.9907
P2 2     35.0574
P2 3     36.6574
P2 4     36.6574
;

ts [*] :=
;

s :=
D1 2     138.6
D1 3     138.6
D1 4     138.6
D2 2      97.9444
D2 4      95.8889
P1 1     240.019
P1 2     148
P2 1     200
P2 2     118.4
;

so [*] :=
D1    82.7685
D2    95.8889
P1   240.019
P2    68.0556
;
```

B Solution of C-4-2-SET

```
AverageDailyCost = 7.25026

_total_solve_time = 185449

_solve_elapsed_time = 21621

b = 48

th [*] :=
T48  1
;

xo :=
P1 D2 1 S1    1
P1 D2 2 S2    1
;

x :=
D1 1 P2 2 S1    1
D2 1 P2 1 S1    1
D2 2 P1 2 S2    1
D2 3 P1 1 S1    1
P2 1 D1 1 S1    1
P2 2 D2 3 S1    1
;

xd :=
P1 1 P1 S1    1
P1 2 P1 S2    1
;

go :=
P1 D2 1 S1 W1 T48    1
P1 D2 2 S2 W1 T48    1
;
```

```
g :=
D1 1 P2 2 S1 W1 T48    1
D2 1 P2 1 S1 W1 T48    1
D2 2 P1 2 S2 W1 T48    1
D2 3 P1 1 S1 W1 T48    1
P2 1 D1 1 S1 W1 T48    1
P2 2 D2 3 S1 W1 T48    1
;

gd :=
P1 1 P1 S1 W1 T48    1
P1 2 P1 S2 W1 T48    1
;

w :=
P1 S1    1
P1 S2    1
;

o :=
D1 1 S1 T48    1
D2 1 S1 T48    1
D2 2 S2 T48    1
D2 3 S1 T48    1
P1 1 S1 T48    1
P1 2 S2 T48    1
P2 1 S1 T48    1
P2 2 S1 T48    1
;

y :=
D1 1    1
D2 1    1
D2 2    1
D2 3    1
P1 1    1
P1 2    1
```

```
P2 1     1
P2 2     1
;

q :=
D1 1 S1     144
D2 1 S1     144
D2 2 S2     130
D2 3 S1     158
P1 1 S1     158
P1 2 S2     130
P2 1 S1     128
P2 2 S1     160
;

f :=
D2 1 P2 1 S1      16
D2 3 P1 1 S1       2
P2 1 D1 1 S1     144
P2 2 D2 3 S1     160
;

fo :=
P1 D2 1 S1     160
P1 D2 2 S2     130
;

fd :=
P1 1 P1 S1     160
P1 2 P1 S2     130
;

t :=
D1 1     18.9926
D1 2     20.7926
D1 3     20.7926
D1 4     20.7926
```

```
D1 5     20.7926
D2 1      5.99691
D2 2     21.9969
D2 3     34.4574
D2 4     36.4324
D2 5     36.4324
P1 1     42.4293
P1 2     46
P1 3     48
P1 4     48
P1 5     48
P2 1     13.7938
P2 2     26.4605
P2 3     28.4605
P2 4     28.4605
P2 5     28.4605
;

ts [*] :=
;

s :=
D1 2     138.6
D1 3     138.6
D1 4     138.6
D1 5     138.6
D2 3      17.8556
D2 4     158.081
D2 5     158.081
P1 1     254.576
P1 2     118
P2 1     200
P2 2     148
;

so [*] :=
D1    56.9778
```

```
D2    53.9722
P2   117.237
;

th [*] :=
T48   1
;

b = 48
```

C Solution of C-4-2-LA

```
AverageDailyCost = 7.11672

_total_solve_time = 3022.33

_solve_elapsed_time = 266.781

th = 48.3333

ct = 348.013

ct/th = 7.20026

p :=
16 1    0.869316
16 2    0.0473502
17 2    0.0833333
;

b [*] :=
17  1
;

xo :=
P1 P1 1 S2    1
P1 P1 2 S1    1
;

x :=
D1 1 P2 2 S1    1
D2 1 P2 1 S1    1
P1 1 D2 2 S2    1
P1 2 D2 1 S1    1
P2 1 D1 1 S1    1
P2 2 D2 3 S1    1
;
```

```
xd :=
D2 2 P1 S2     1
D2 3 P1 S1     1
;

go :=
P1 P1 1 S2 W1     1
P1 P1 2 S1 W1     1
;

g :=
D1 1 P2 2 S1 W1     1
D2 1 P2 1 S1 W1     1
P1 1 D2 2 S2 W1     1
P1 2 D2 1 S1 W1     1
P2 1 D1 1 S1 W1     1
P2 2 D2 3 S1 W1     1
;

gd :=
D2 2 P1 S2 W1     1
D2 3 P1 S1 W1     1
;

w :=
P1 S1     1
P1 S2     1
;

o :=
D1 1 S1     1
D2 1 S1     1
D2 2 S2     1
D2 3 S1     1
P1 1 S2     1
P1 2 S1     1
P2 1 S1     1
```

```
P2  2  S1     1
;

y  :=
D1  1     1
D2  1     1
D2  2     1
D2  3     1
P1  1     1
P1  2     1
P2  1     1
P2  2     1
;

q  :=
D1  1  S1     145
D2  1  S1     160
D2  2  S2     130
D2  3  S1     145
P1  1  S2     130
P1  2  S1     160
P2  1  S1     145
P2  2  S1     145
;

f  :=
P1  1  D2  2  S2     130
P1  2  D2  1  S1     160
P2  1  D1  1  S1     145
P2  2  D2  3  S1     145
;

fo  :=
;

fd  :=
;
```

```
t :=
D1 1    23.4051
D1 2    25.2176
D1 3    48.3333
D1 4    48.3333
D1 5    48.3333
D2 1     9.99691
D2 2    27.7747
D2 3    38.9907
D2 4    40.8032
D2 5    48.3333
P1 2     2
P1 3     4
P1 4     4
P1 5     4
P2 1    17.9938
P2 2    31.1813
P2 3    32.9938
P2 4    32.9938
P2 5    32.9938
;

ts [*] :=
;

s :=
D1 2    139.562
D1 3     70.2153
D1 4     70.2153
D1 5     70.2153
D2 3     29.0556
D2 4    157.743
D2 5     89.9722
P1 1    266
P1 2    148
P2 1    200
```

```
P2 2     134.125
;

so [*] :=
D1    70.2153
D2    89.9722
P1  266
P2    92.037
;
```