Master’s degree thesis

LOG950 Logistics

Localization of semi-central warehouses and inventory management for Mørenot AS

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Preface

This master's thesis marks the end of our five years as students at Molde University College, where we have completed our master's degree in logistics with a specialization in logistics analytics. The research has been conducted from November 2022 to May 2023. It has been an interesting journey to work with Mørenot AS to analyze a real case.

We would like to express our sincere gratitude to our master thesis supervisor, professor Arild Hoff, at Molde University College for his feedback, support, and guidance. Throughout our writing process he has provided us with insightful discussions and kept a positive and supporting attitude towards our work. His continuous support has unquestionably elevated the quality of our thesis, shaping it into the comprehensive work we think it is today.

We would also want to give a huge thanks to Mørenot for providing an interesting case for us to work with in addition to a good collaboration through meetings. They provided us with necessary and relevant data as well as information about the company's operations.

Abstract

Mørenot is a Norwegian company mainly selling products to the fishing industry. Their current inventory policy is characterized by long lead times and several challenges when deciding the service level for their different warehouse locations. They wanted to investigate the possibility of decentralizing their warehouse operations to solve some of the issues. Some of the improvements might be small, but in today's business environment every small improvement in the business operations matters and can give a competitive advantage over their competitors.

This thesis investigates the potential to decentralize Mørenot's warehouse operations by introducing semi-central warehouses. We have used a quantitative approach to solve the problem where we incorporated location and demand data to identify suitable locations for these warehouses. By creating a facility location model, we obtained possible locations for semi-central warehouses.

The number of semi-central warehouses was determined by analyzing factors such as lead time for uncovered demand, total relevant inventory costs, and service level at the locations. This enabled us to propose a solution to Mørenot that some semi-central warehouses should be established, which could improve their operations.

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1.0 Introduction

Mørenot is a Norwegian company providing different products to fishery, fish farming and the seismic industry (Mørenot, 2022). They have over thirty facilities worldwide, but their core business is in Norway and Europe and most of their business is in the fishing sector. They have over eight hundred employees and in 2019 they had over 1,1 billion NOK in revenue. Some of the products they produce are trawls, longline and pots for offshore fishing and nets and mooring equipment for fish farming to mention a few. They have production facilities in Norway, Poland, China, and Lithuania. Their main warehouse is in Søvik located on the outskirts of Ålesund, while they operate two other semi-central warehouses. Everything they import is transported to one of these three warehouses. In addition to these three they have several smaller warehouses/service locations scattered along the Norwegian coast which are served from the main warehouse.

Figure 1 Mørenot Facility Locations (Mørenot, 2022)

Mørenot operates several locations along the Norwegian coastline in addition to some locations scattered around Europe, US and one in Asia (shown in Figure 1). Their main market is here in Norway serving fishing vessels and fish farms along the whole coast. The products Mørenot produces and imports varies a great deal. In their product catalogue they have more than one hundred thousand different items spanning from gloves to chains, fishing hooks, pots, and nets.

The items produced abroad are transported by ship from Asia and truck from Europe to Søvik outside of Ålesund in Norway. Here the products either go into production where they are made into different products like nets or get transported by truck to the various locations where the product is needed. The various locations or warehouses included in this thesis are all located along the coast and are connected to a port. This allows customer vessels to birth near the warehouses to pick up purchased goods.

Mørenot operates with several different divisions within the fishing industry (Mørenot, 2022). They sell equipment related to fishery where they sell equipment like trawls, fishing lines pots etc. Their aquaculture division produces equipment for fish farming and moorings etc. related to fish farming. Their offshore division produce items related to seismic operations offshore. The last division they operate in is in digital space where they deliver solutions to monitor equipment and service of equipment. Many of the divisions share a lot of the different items they sell, as well as raw materials used in production. One example of this is ropes which are one of their biggest item categories. Regardless of this we will only analyze their fishery division. This is their biggest division, and it has the most items.

They would like for us to investigate the possibility to moving some of their products away from the central warehouse in Ålesund to more decentralized warehouses closer to its customers to reduce its lead times. Therefore, the problem will be a combination of a facility location and inventory management problem and finding an optimal combination of warehouses to use for different products while still having a satisfactory service level.

1.1 Problem description and research questions

We want to investigate the possibility to decentralize their warehouse operations by introducing one or several semi-central warehouses. By introducing semi-central warehouses, it is possible for Mørenot to increase their service level while reducing their inventory costs. A higher service level will lead to higher customer satisfaction which could help customer retention and attract new customers. This can give Mørenot a stronger competitive position in the market and provide a better service compared to their competitors.

Based on this problem description we created this research problem:

How can Mørenot determine the locations for semi-central warehouses to improve service levels and reduce inventory costs?

From the broader research problem, we derived three more specific research questions that we will address throughout the thesis:

How can a facility location model help determine the location of semi-central warehouses for Mørenot?

How many semi-central warehouses should they establish, when focusing on lead time for uncovered demand?

How do inventory management related costs affect the number of semi-central warehouses to establish?

1.2 Structure of the thesis

The first chapter gives a brief overview of Mørenot and how it operates. In the second chapter we have conducted a literature review where relevant research is presented. In the third chapter we have gone into more detail about how we intend to solve the problem. The fourth chapter provides an in-depth case description. We have also described the data we gathered from Mørenot. In chapter five we have applied different models and calculations to the problem to solve it. Chapter six consists of a discussion of the results and how different inputs might affect the results. Lastly, we have written our conclusion to the thesis summarizing the main findings.

2.0 Literature review

In this chapter relevant literature will be presented that forms the theoretical framework for this thesis. There is a lot of relevant research written on the topic of operations research. We have tried to narrow it down to the most important literature relevant to the scope of this problem. One part of the research area is optimization. Within the optimization field there are many sub fields which will be used throughout the thesis.

2.1 Operations research

Operations research is a field within the mathematics aiming to analyze a real-life problem to make an educated decision to solve the problem. There are several “subfields” within operations research. Some of them are statistics, mathematical modeling, and optimization. They are often used to find an optimal or near optimal solution to a problem and aiding the decision makers to make the correct decision.

The term operations research was first introduced at the end of second world war, even though several of the fields within operations research had been practiced for several hundred years (Larnder, 1984). The field have had a significant increase in the size of problems it can solve when the computers were introduced. There have also been several programs developed and the algorithms that previously had to be calculated by hand is now done by computers speeding up the solving process, making it more accessible and easier to use.

Further in the thesis we will focus on the fields mainly within optimization and mathematical modeling.

2.2 Optimization and mathematical modeling

An optimization problem has the goal of finding a minimum or maximum value for a given problem. It can be solved by creating a mathematical model, using algorithms to solve the problem to optimality, or heuristics to find a good solution quickly. All optimization problems consist of an objective function, decision variables and constraints (Winston & Albright, 2016). The objective function, decision variables and constraints can be modelled mathematically to explain a problem in a general way.

A mathematical model is generally built up in the following manner:

Decision Variables

The decision variables in a mathematical model are often represented by the symbols $X_1, X_2, \dots, X_n$. These values can symbolize whatever is necessary depending on what sort of system you wish represent in the model. They can for instance represent how much to transport on a certain link, whether to open a fire station or not, how many units to keep as a safety stock, or how many people you should hire for a certain project, to mention a few examples.

Constraints

The constraints in an optimization problem are usually represented as an equality or inequality which "constraints" the decision variables in the model. These values can restrict the maximum or minimum values a decision variable can have, force them to be binary or integer, and even force them to be exactly equal to a set value. One might want to limit the weight transported on a truck to be no greater than the maximum load allowed, or make sure that a manufacturing company produces at least as much product as their customers demand.

Objective function

The objective function of an optimization problem is usually an equation which states what should be maximized or minimized. You may wish to maximize profit or minimize transport distance or lead time.

2.2.1 Mathematical programming techniques

Mathematical Programming (MP) can be divided into several subgroups or categories. LP, or Linear Programming is part of a larger group of mathematical programming models. This is a method of finding an optimal solution in a mathematical model which has linear objective function and linear constraints (Ragsdale, 2016). The benefit of problems that can be modeled with linearity is that they are relatively easy for a computer to solve. The Simplex algorithm is the most used tool for solving such problems (Nabli, 2009). There will always be at least one optimal solution assuming the problem is feasible. Given enough time, an optimal solution will always be found.

Other problem types that are not as simple to compute solutions for are Integer Programming (IP) and Mixed Integer Programming (MIP) problems. In IP problems, all variables must be integer values, while MIP has a mix of forced integer and continuous values for variables. It makes sense to use integer or binary values for variables that represent values that cannot be divided in smaller units realistically. For instance, deciding how many employees to hire for a certain period should be an integer value, as people are never divided in fractions. One can also use binary values where a decision variable is only allowed to be either 0 or 1.

Non-linear Programming (NLP) is used when objective functions or constraints cannot be stated in a linear form. This type of problems is usually much harder to solve than LP problems and do not always result in obtaining the optimal solution of the model. In NLP problems you may end up with solutions using local maximum or minimum values that are not as good as the global optimum you are looking for. This might also be the case for larger IP/MIP problems where it can be difficult to verify that the solution is optimal.

2.3 Stochastic Optimization

If there is no uncertainty in the data the problem is deterministic, meaning it will produce the same result each time, however most of the time there is uncertainty in the data. In order to capture the uncertainty of the supply chain one might wish to consider stochastic supply and/or demand. Stochastic optimization refers to a collection of methods for maximizing or minimizing an objective function where randomness is included (Lauren, 2014). Randomness is usually applied to the system in the constraints or the objective function. Like with deterministic optimization problems, there is not necessarily one single solution type that works the best for all cases.

Stochastic optimization problems may be divided in two groups of problems: single-period and multi-period. Single-period, or single stage problems try to find the single optimal solution to the problem, while multi-period or multistage problems try to find an optimal sequence of decisions. While single stage problems may be solved by modified versions of deterministic methods, a multistage problem is dependent on future decisions which will alter the final optimal sequence of decisions. Multistage models are therefore much more reliant on statistical assumptions and approximations.

When solving single stage problems, one might assign statistical distributions to the relevant input values of the model. The model can then be run several times resulting in different input values, and therefore different output values as well. By running the model enough times, we may end up with a significant statistical distribution of the objective function value. This can tell us what the performance of the system would be with the stochasticity we are expecting.

2.4 Multi objective optimization

The concept of multi objective optimization is a natural end point when attempting to model complex real-world systems. There will often be more relevant objectives than just one. Normally, one might focus on maximizing profit or minimizing resource utilization, but what if allowing the profit to reduce by 1% could increase employee satisfaction by 50%? Surely, this could be a valid trade-off. Multi objective optimization allows you to focus on more than just one objective.

There are several ways to do this. For instance, one could combine several objectives in one single model and give them weights according to their importance. You may also do several steps, where one solves a model for one objective at the time, and then includes the previous objective solution as a value in a constraint while you solve for the next objective. This is sometimes referred to as Multi-Level Programming and can be a useful approach if the hierarchical order of the objectives is meaningful and the trade-offs from the other objectives need to be considered (Caramia & Dell'Olmo, 2020).

If one has an extensive list of objectives though, this could have a negative effect on the latter ones, as they will become increasingly constrained as you go down the hierarchy of objectives, maybe even ending in a situation of infeasibility. By ordering objectives in this manner, the less important objectives have very little impact on the overall optimal solution.

The mentioned scenario of having an extensive list of objectives with a hierarchical order can have implications on Pareto-optimality. Pareto-optimality, with regards to multi-objective optimization refers to a state where no further improvements can be made in one objective without sacrificing the performance of another objective (Dellnitz et al., 2005). This means that the objective one tries to optimize, may not be improved upon even more without decreasing the performance of the previous objective.

By utilizing Goal-programming, one could also try to even out the deviation of the optimal solution for the individual objectives. By solving a model for one objective at the time, you can store the optimal solutions as constraints in a final collective model which takes all the previous objectives into account at the same time. In this final model you may calculate how much each objective value deviates from its optimal solution, and thereby make sure that all the objectives have a fairly even deviation from their individual optimum and that no one objective is "left out".

2.5 Facility Location:

Facility location problems (FLP) is the science concerned with the optimal placement of facilities (Daskin et al., 2005). Facility location is a critical strategic decision that has far-reaching implications for an organization. It involves selecting the most suitable locations for new facilities, which can impact the company's competitiveness, profitability, and growth potential over an extended period. Since the decision can be costly and complex, it is usually made with a long-term perspective in mind.

There are many books and publications written about the different types and variations of facility location problems. One of the more comprehensive collection of problems is a book called “Facility Location: Concepts, Models, Algorithms and Case Studies” (Zanjirani & Farahani, 2009). Here they have collected different publications by different authors to give a collection of relevant different problem settings. Some of the problems discussed are location allocation problem, covering problem and location-inventory problem. Whereas the last one is most relevant for our thesis.

2.5.1 Different types of facility location problems

There are many different types of facility location problems. Andreas Drexl and Andreas Klose have written a good article “Facility Location Models for Distribution System Design” (Klose & Drexl, 2005) where they go into details of different variants of location problems. The first type of location problem they describe is the most basic form of location problem where the facility can be located anywhere on the plane. These are called continuous location models. In this model as with many of the other location models the objective is to minimize or maximize the distance from the facilities to the customer or demand nodes. The distances are often calculated using the Euclidean distances. This gives a distance as a straight line between two points on the feasible plane. Using the continuous model, you need to have the x and y coordinates for the locations.

A different type of facility location problem is the discrete localization model. Here the nodes are predetermined in a network model. The objective is to find out what nodes of facilities to use in the network. One example of such a problem is a p-median problem where the objective is to minimize the sum of distances between the nodes in the graph and the closest facility. Other variants of this is p-center, UFLP, covering and anti-covering problems (Dantrakul et al., 2014).

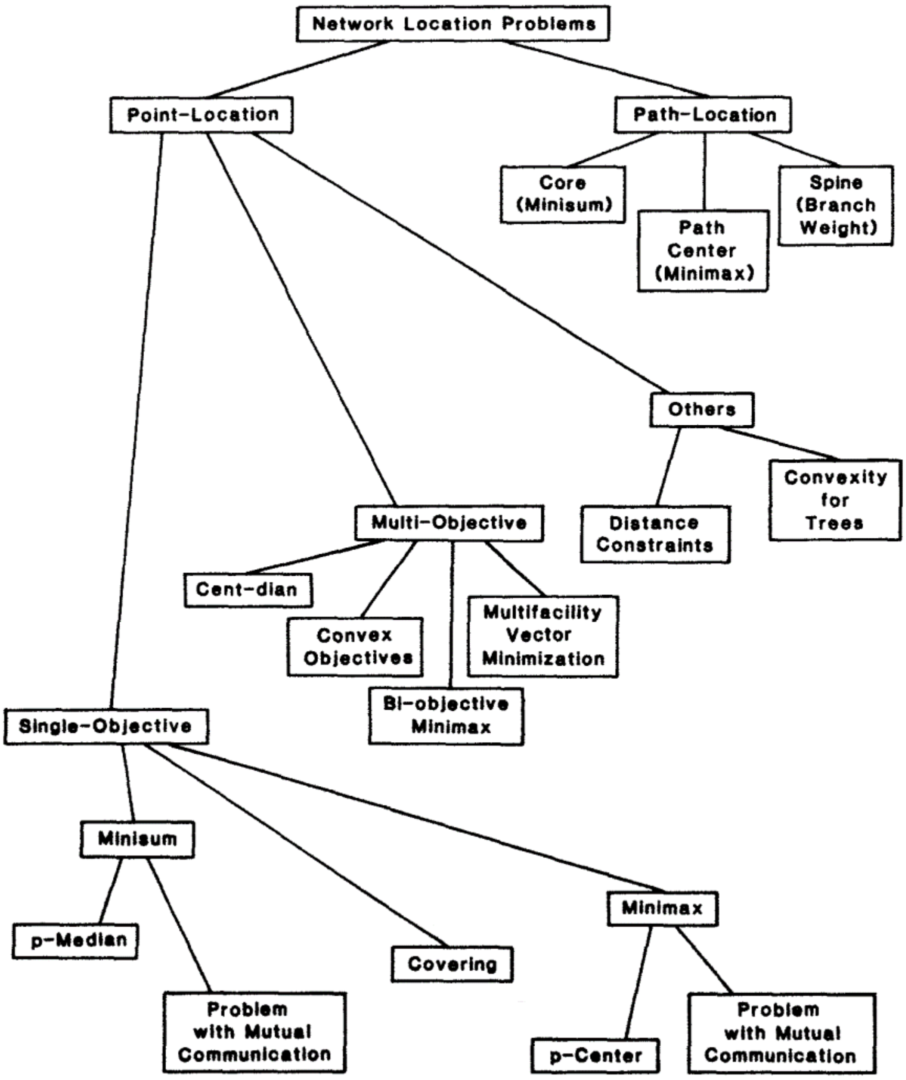

Figure 2 Overview of different network location problems (Tansel et al., 1983)

In Figure 2 we can see an overview of different network location problems and how they relate to each other. Here we see the paths of Point-Location and Single-Objective leading to Minisum, Covering and Minimax problems (Tansel et al., 1983). As our priority is looking at reducing the overall weighted distance in the system for all customers, the most suitable type of model would be made from a single Minisum objective. This means that there is one single objective function which has a goal of minimizing the total sum of all costs in the system.

This will however allow for big fluctuations and variety in how high costs are for each location or customer. The Minimax objective would try to reduce the single largest cost value for any location or customer, while allowing the total sum to be higher than in the

Minisum objective case. This means the model would reduce the largest single cost to make sure it is never extremely large for any location or customer but could mean a somewhat larger cost for everyone in the system.

Covering problems look at cases where there is a maximum or minimum value that the cost values can have for a location to be considered “covered” by another. A common example is that a “customer” may only be considered covered by a hospital if an ambulance can drive to the person in need and provide treatment within a certain maximum time limit.

2.5.2 Single-objective discrete facility location

The book Location Science (Laporte et al., 2019) is a book going in great detail about different location science theory and how to apply different models to a problem. The first concepts described in the book are the ones that are most relevant for us. They go into great detail about different discrete location problems like P- median, P- center and covering problems. These are all problems where there are a set of predetermined possible locations, and the objective is to find a subset of these possible locations to establish.

2.5.2.1 P- Median problem

One type of discrete problem is the p-median problem(P-MP). In this problem the objective is to establish p number of candidate locations which is fixed beforehand (Marin & Pelegrin, 2019). The cost is to be minimized and the costs are all the costs associated with allocating the facilities, often cost between nodes or distance. P-median problems can be used for a wide variety of problems spanning from placemats of cache proxies in a computer network to locations of distributions centers and clustering of different routes.

The classical p median problem can be formulated as:

Parameters:

i	Candidate locations to establish facility	I= {1, 2, …, i}
I	Set of candidate locations	
j	possible location with demand	J= {1, 2, ..., j}
J	Set of possible locations	
d_j	Demand at location *j*	j ∈ J
c_{ij}	Transportation cost from location i to location *j*	i ∈ I, j ∈ J
p	Number of facilities to establish.	

Variables:

X_{ij}	1 if user j is supplied from facility i, 0 otherwise.	∀ i ∈ I, j ∈ J
Y_i	1 if location i is chosen 0, otherwise	∀ i ∈ I

$$min \sum_{i\in I} \sum_{j\in J} d_j c_{ij} x_{ij} \qquad (1)$$

Subject to:

$$\sum_{i\in J} x_{ij} = 1 \ \ \forall\, j \in J \qquad (2)$$

$$\sum_{j\in J} x_{ij} \le n y_i \qquad \forall\ i \in I \qquad (3)$$

$$\sum_{i\in I} y_i = p \qquad (4)$$

$$x_{ij} \in \{1,0\} \ \ \forall i \in I, j \in J \qquad (5)$$

$$y_i \in \{1,0\} \ \ \forall i \in I. \qquad (6)$$

The objective function (1) is to minimize the sum of the total cost. The first constraint (2) ensures that all nodes are supplied from one location. The second constraint (3) ensures that a node can serve up to n locations if it is established. The third constraint (4) ensures that p candidate locations are chosen. The last two constraints ensures that the variables x and y is binary.

2.5.2.2 P- Center problem

A p-center problem (P-CP) is a problem where the objective is to minimize the maximum distance between a demand point and a closest point in that set of nodes (Çalık et al., 2019). A classic example of a problem that can be solved as a p-center problem is allocation of emergency services like fire, police, and ambulance service. A p-center problem ensures that the maximum distance from an established node to the nodes it serves is minimized.

A p-center problem can be formulated as:

Parameters:

i	Candidate locations to establish facility	I= {1, 2, …, i}
I	Set of candidate locations	
j	possible location with demand	J= {1, 2, ..., j}
J	Set of possible locations	
z	Maximum distance between the demand points and facility	
d_{ji}	Distance from location j to location i	j ∈ J, i ∈ I
p	Number of facilities to establish.	

Variables:

X_{ij}	1 if user j is supplied from facility i, 0 otherwise.	∀ i ∈ I, j ∈ J
Y_i	1 if location i is chosen 0, otherwise	∀ i ∈ I

Minimize z (1)

Subject to:

$$\sum_{i \in I} d_{ji} x_{ij} \leq z \quad \forall j \in J \qquad (2)$$

$$\sum_{i \in I} x_{ij} = 1 \ \ \forall j \in J \qquad (3)$$

$$x_{ij} \leq y_i \qquad (4)$$

$$\sum_{i \in I} y_i \leq p \qquad (5)$$

$$x_{ij} \in \{1,0\} \ \ \forall i \in I, j \in J. \qquad (6)$$

$$y_i \in \{1,0\} \ \ \forall i \in I. \qquad (7)$$

The first constraint (2) makes sure that all the distances between the nodes and the facilities they are assigned to are smaller than the objective value z (minimize max distance). The second constraint (3) as in p-median problem ensures that all nodes are served from one facility. The third constraint (4) ensure that we do not assign demand points to locations with no facility. The next constraint (5) restricts number of assign facilities to be less or equal to p. The last two, (6) and (7), are constraint ensuring the variables are binary.

2.5.2.3 Fixed charge facility location problems

Fixed charge facility location problems (FLP) are a type of problem that is similar in nature as the previous discussed but incorporate an extra element in terms of a cost of establishing a facility(Fernandez & Landete, 2019). A FLP can be modeled similar to the previous models, but with some changes:

Parameters:

i	Candidate locations to establish facility	I= {1, 2, …, i}
I	Set of candidate locations	
j	possible location with demand	J= {1, 2, ..., j}
J	Set of possible locations	
f_i	Fixed cost of establishing facility in location i	i ∈ I
d_j	Demand at location j	j ∈ J
c_{ij}	Transportation cost from location i to location j	i ∈ I, j ∈ J
q_i	Capacity at location i	i ∈ I

Variables:

X_{ij}	1 if user j is supplied from facility i, 0 otherwise.	∀ i ∈ I, j ∈ J
Y_i	1 if location i is chosen 0, otherwise	∀ i ∈ I

Objective function:

$$min \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \qquad (1)$$

Subject to:

$$\sum_{i \in J} x_{ij} = 1 \ \ \forall \ j \in J. \qquad (2)$$

$$\sum_{j \in J} d_j x_{ij} \leq q_i y_i \qquad \forall \ i \in I. \qquad (3)$$

$$x_{ij} \in \{1,0\} \ \ \forall i \in I, j \in J. \qquad (4)$$

$$y_i \in \{1,0\} \ \ \forall i \in I. \qquad (5)$$

As one can see it is quite similar to the model formulation for P-MP. The main change is in the objective function (1) where we want to minimize the sum of cost between the chosen (X_{ij}) nodes plus the sum of fixed charge or setup cost of choosing node i. f_i denotes the setup cost. The second (3) constraint is also changed. Previously (P-MP) it said that a facility could serve n locations if chosen. Now it ensures that the capacity at a facility is not exceeded ($\sum_{j \in J} d_j x_{ij}$ denotes the demand for the link while $q_i y_i$ denotes the capacity).

This also ensures that a node cannot be assigned to a node that a facility is not established on (capacity will be 0 as y_i=0). The model also decides how many facilities to open as we want to open as few as possible if setup cost>0.

2.5.2.4 Covering Location Problems

Covering location problems deal with situations where one must locate facilities that provide a service for customers that may only receive the service if they meet a certain criterion (e.g., time, distance, etc.)

An example would be a customer (e.g., a person) that can only receive emergency help from an ambulance if they are under a certain travel time from the closest facility (e.g., the ambulance can arrive in less than 7 minutes to where the person is located.) These problems are referred to as covering problems, and if the conditions that are asked for are met, then the customer is covered (Garcia & Marin, 2019).

A covering location problem can be formulated as follows:

For each pair of potential location and customer there is a known constant $a_{jk} \in \{0,1\}$ representing whether the customer can be served by a facility from location j.
For each $j \in N$ there is also an associated cost c_j that must be paid to open a facility at location j.

Parameters:

n	Number of potential locations	N = {1,2,…,n}
N	Set of potential locations	
k	Number of customers to be served	
K	Set of customers	K = {1, 2,…, k}
G	Set of links in the covering coefficient matrix	G = {(N x M)}
c_j	Fixed cost of establishing a facility in site j	j ∈ N
a_{jk}	Covering coefficient. a_{jk} = 1 if customer k can be covered from location j	$a_{jk} \in \{0,1\}$; $(j,k) \in G$

Variables:

U_j U_j = 1 if a remedy is in location j, otherwise 0 $U_j \in \{0,1\}; j \in N$

Objective function:

$$\min \sum_{j \in N} c_j U_j \qquad (1)$$

Subject to:

$$\sum_{j \in N} a_{jk} U_j \geq 1 \ \ \forall \, k \in K. \qquad (2)$$

$$U_j \in \{0,1\}; j \in N. \qquad (3)$$

The objective function (1) minimizes the sum of fixed costs only for the facilities that are established. The first constraint (2) counts how many facilities serve each customer site j and makes sure each site is covered by at least one facility. The last constraint (3) ensures the variables are binary.

The main difference between this type of problem and the others discussed previously, is that the latter assumes that all nodes may be served by all other nodes, while this is not the case in covering location problems. This is specifically addressed by the a_{jk} parameter, telling us which nodes can cover which.

2.6 Inventory management

Inventory management has become one of the key elements of the supply chain management and can greatly affect the performance of a business (Priniotakis & Argyropoulos, 2018). Inventory management is about deciding how much inventory to store. This is important because a lot of inventory held leads to unnecessary high cost, while too low stock might lead to stockout and no possibility to deliver to the customer on time. Inventory management includes several terms and research areas like service level, safety stock, inventory level and many more. All of which have huge research fields.

2.6.1 Lead time

The definition of lead time is the time it takes to complete a process from start to finish. The lead time in a supply chain is often the time between a customer needs/orders a product until it receives it. Every company tries to reduce the lead time, this will give them a competitive advantage over their competitors if they can produce and deliver the products faster (Tersine & Hummingbird, 1995). A short lead time is highly dependent on what product one is producing. If someone buys a ship, they do not expect to receive it the same day, while in industries like oil and gas the lead time to get spare parts must be as short as possible (hours) due to the high cost of downtime.

2.6.2 Service level and safety stock

Service level is the expected probability of not experiencing a stockout situation during the next cycle of replenishment, or the probability of not having any lost sales (Radasanu, 2016). The safety stock levels are determined by the service level. The safety stock level must be large enough to cover the demand of customers, yet not so high that the company loses a lot of money due to high carrying costs.

Safety stock is inventory that is carried to prevent stockouts and situations where backorders occur. The safety stock serves as a buffer against various deviations such as delays in deliveries, inaccuracies in demand forecast, insufficient or poor quality materials from vendors, and differences between planned and actual inventories.

A common way to calculate the safety stock is: $SS = Z * \sigma_{LT}$ Where Z is the service factor derived from the desired service level in a normal distribution. If you want a service level of 95% the Z value is 1,64. σ_{LT} is the standard deviation of the demand during the lead time.

Uncertainty in demand fluctuations and difficulty in predicting future variability is the reason for having a safety stock and is how you can maintain a certain service level even with stochastic demand patterns. If there is no uncertainty there is no need to hold safety stock.

There are different types of service levels. The two most widely used service levels are P1, cyclic service level and P2, filling degree service level (Schneider, 1981).

P1 service level is where we calculate the probability of not getting a stockout at random time during the cycle, often called cyclic service level. This means that if we have a 90% service level there will be a stockout occurring in 10% of the cycles. P2 service level is defined as the fraction of the demand not being met at a given time, often called fill rate. It focuses on both the probability of a stockout occurring and the size of the stockout. There are also other ways of calculating service level such as time between stockouts (TBS) and ready rate.

A very high service level means that customers will get what they want, when they want it almost all the time. This results in a high satisfaction level among customers which will help maximize sales. As mentioned, this does come with its own costs, mainly carrying costs, which is the cost of always having items in stock and will increase along with higher levels of stock. There are several risks included with having large inventories as well, such as damages to goods, expiration and lowering of prices. The higher the level of stocks, the larger the risks and costs will be as well.

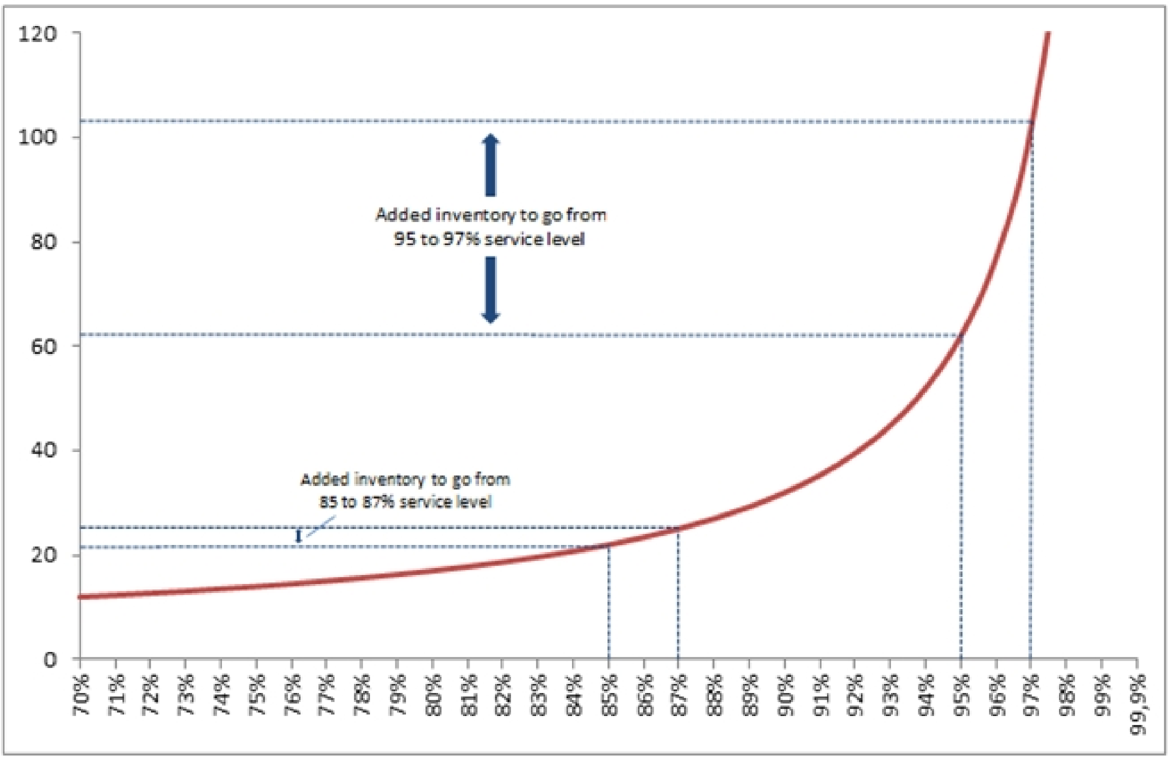

Figure 3 How service level affects inventory levels (Schalit & Vermorel, 2023)

As we can see in Figure 3, the relationship between service level and inventory value is exponential, growing extremely fast as it gets closer to 100%. The graph displays two instances where the service level is increased by only two percentage points. When going from 85% to 87% there is only a slight increase in the value of inventory needed, but when going from 95% to 97% there is a massive increase in inventory for a small improvement in service level. This is exactly why it is quite common to see service levels around 95%.

The service level is the trade-off border between the costs of inventory and the cost of stockouts. It makes sense to find a balance which maximizes the overall return for the company, but this is usually a complex endeavor, due to the analysis of the situation being very sensitive and at the same time, challenging to measure properly. While reducing the safety stock levels will immediately grant you with more money being available, it might take years to know how this impacted the longevity of a satisfaction in a customer's relationship due to an increased number of stockouts.

The sensitivity a customer has for stockouts also varies depending on the type of product in question, meaning that there is a theoretical optimal service level for each product individually. To lower the complexity, a heuristic approach is often used, such as ABC-analysis.

An ABC-analysis is based on the idea that a product is more "important" both for the retailer and the customer if it generates more revenue than other products. Making this assumption lets you categorize the products according to their respective sales volume.

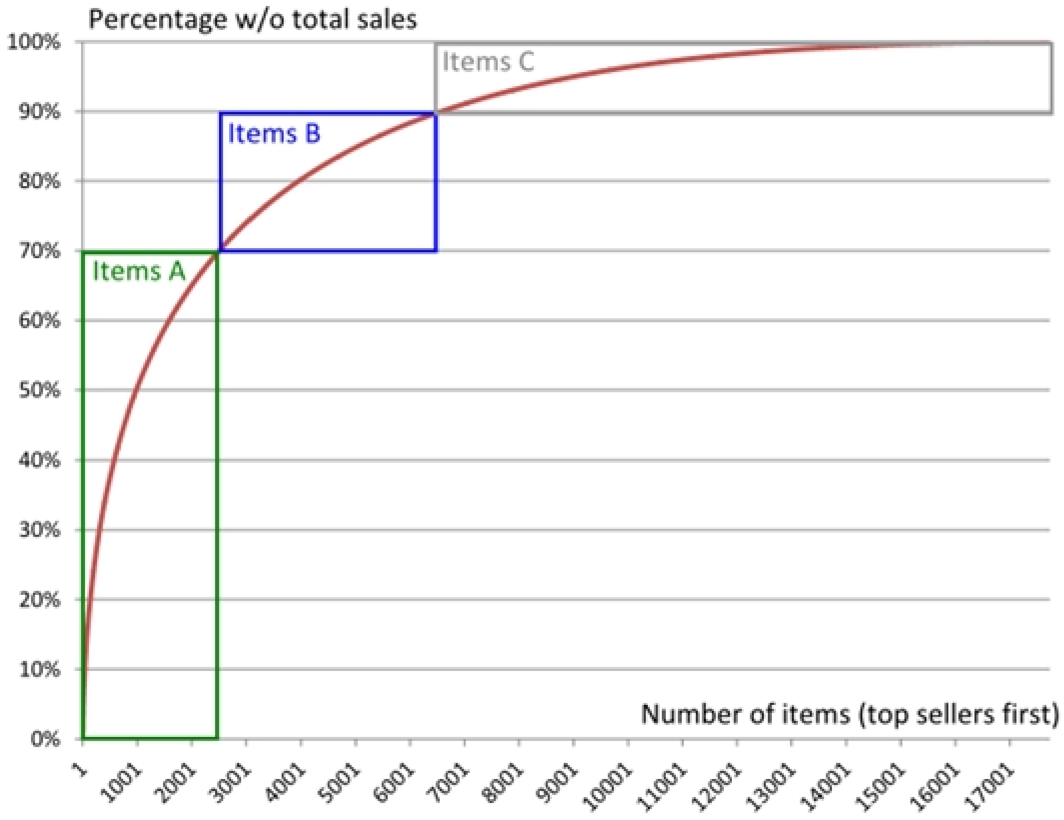

Figure 4 ABC-items and value distribution (Schalit & Vermorel, 2023)

Figure 4 shows an example of the distribution of items in some company based on value of total sales. The items are sorted by the top sellers in descending order going left to right. As we can see the first 2 500 or so items (15%) make up 70% of all the sales revenue for the company. These are classified as the A-items. The next 20% or so of items make up another 20% of the total value. These are the B-items. Lastly, the final 65% of all the items only make up the last 10% of overall revenue. These are the C-items.

From this graph it is evident that it would make more sense to put more effort into making sure A-items have satisfactory service levels before spending too many resources on B- and C-items, as they simply are not worth as much and will not have as big of an impact in terms of lost sales.

Since the service level is a balancing act between the costs of inventory and the costs of stockouts, finding values for these are an essential part of determining an appropriate target service level. In general, inventory costs can be numerous and may not be easily isolated but can usually be identified in a way that allows one to attach a number to its cost. Cost of working capital, cost of storage space, interest rates, etc. are relevant values.

One might also want to look at things like cost of expiration, cost of obsolescence, cost of destroyed inventory, etc. to get the bigger picture.

While numerous and occasionally complex, most of the values used for finding a total cost number for inventory are real, existing numbers that can be extracted in one way or another, but when finding the cost of stockouts things are not as easy.

The most obvious cost of stockout may be the lost sales, which is relatively easy to keep track of, but this is only one part of many. Extensive research has shown that stockouts pose a significant risk to customer satisfaction and may lead to a long-term weakening of a company's client base (Ranjan & Puri, 2012).

A company's effort to improve its service level involve a variety of sub-goals that must be monitored and continuously improved. To meet the desired service level, a company focuses on individual goals such as delivery time, delivery readiness, delivery flexibility, quality, and reliability.

2.6.3 Optimal order size (EOQ)

One important aspect of inventory theory and decisions linked to inventory theory is the optimal order size or economic order quantity (EOQ). The model is developed to find the "balance point" of the inventory holding costs and the order costs (Kumar, 2019). Using this relatively simple model the decision takers in a company can find out how much of an item they should order and how often they should order that amount, just to mention some of the possibilities.

The EOQ model is:

$$Q^* = \sqrt{\frac{2DS}{IC}}$$

Where:

Q* = the optimal order quantity

D = Demand for the product in the time period

S = Order cost or setup cost (cost of making an order)

I = Inventory internal interest rate (percentage rate to store an item in one time period)

C = Item cost per unit

Often the denominator in the fraction is noted as H, inventory holding cost (Cost of holding one unit of inventory for one time period normally one year).

The EOQ model can be visualized by a graph shown below. Here we can observe that the Q* value is at the point where the order and holding cost lines intersect:

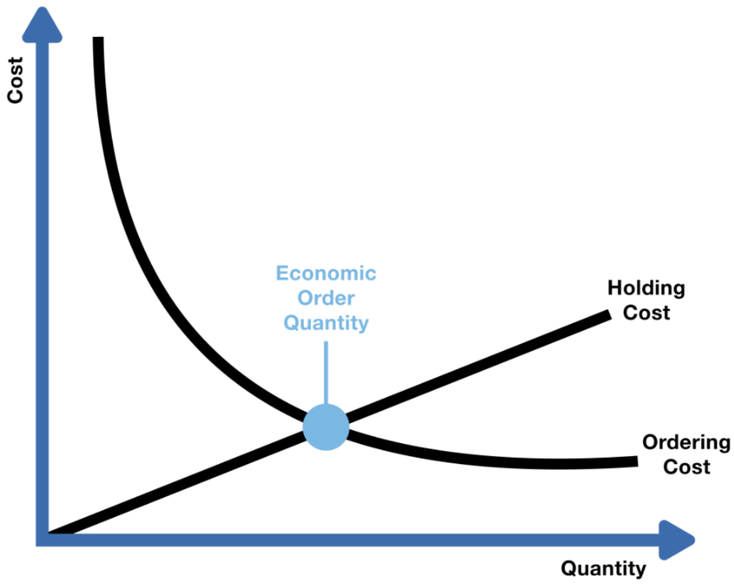

Figure 5 EOQ Model

When we have the EOQ value for an item it is possible to calculate how often is ordered by doing Q*/total demand and end up with T* which represents how many times one must reorder to meet the demand. If T* is 0,2 the product should be ordered five times in the period. If we include lead time (the time it takes from ordering an item to when it arrives) we can also calculate the reorder point or the inventory level at which a new order must be placed to not get into a stockout situation.

This simple EOQ model assumes that the demand is known and deterministic and constant, but in real life it almost never is. Demand for a product often varies from month to month, but also within a month there can be fluctuations in the demand. Trends in the market and seasonality also affect demand. This leads to the order size either being too big and we have too much in stock or too low and the company is not able to meet the demand (stockout). Having too little stock on hand can be solved using safety stock but comes with additional costs of tying up capital in extra inventory.

There are several ways the basic EOQ model can be extended. One way is to include several time periods in the planning horizon and solve the problem using algorithms or heuristics like those developed by Wagner Whitin and Silver Meal (Djunaidi et al., 2019). Using these algorithms, it is possible to account for multiple periods and seasonality. It is also possible to extend the model to include coordinated ordering of several products and discounts for different thresholds for Q.

2.7 Combined Inventory management and facility location

There has been a lot of research done on the problems individually (Location problems and inventory management) but fewer publications where these two have been done together. One of the first publications we could find regarding both problems was written by Vaidyanathan Jayaraman in 1998 in the article "Transportation, facility location and inventory issues in distribution network design: An investigation" (Jayaraman, 1998). In this paper he describes how to minimize the total cost by deciding what warehouse and production sites to open. The different sites have different cost associated to storing the products and operating the locations.

In the article "Location-inventory problem in supply chains: a modelling review" (Farahani et al., 2015) they look at different location inventory problem models and how they have been applied in different research articles. In the objective function for the basic location-inventory (LIP) optimization model they include both fixed cost of locating, transportation costs, inventory costs like cost of safety stock, holding costs and shortage costs. This is quite a versatile model than can relatively easily be customized to add or remove constraints as fitted to a specific case. The article has an extensive literature review categorizing different LIP articles and outlining in a table different characteristic of the model parameters they have used, what type of model it is and what type of solution technique(algorithm) used.

They point out in the article that even though LIP shows the possibility of real-world applications, a surprisingly small amount of research has been done of applying real world data to the model and figure out what modeling approach is the best and most accurate. Looking at the trend of research for LIP for the past thirty years it is only in the last decade articles and research about LIP have become more popular, but then on a more general basis and not applied to real world data.

2.8 Models and theory applied to real world cases.

In this part we will outline some research done both by researchers and other students where facility location theory and models have been used to solve real world cases. Although these cases are not directly related to our case, we can draw some similarities. First of all, all of the cases below utilize facility location to solve a discrete facility location problem. Secondly even though the cases discuss widely different businesses, it is fairly easy to transfer and adapt the theory and models to different business cases.

2.8.1 Amazon drone delivery

In 2017 there was done a study to find the optimal number of locations and charging stations for Amazons drone delivery system in San Francisco (Shavarani et al., 2018). The study was conducted to mitigate delivery times in a congested city using drones. They used the shortest path algorithm and euclidean distances to calculate the different distances between demand nodes and possible locations for take-off and changing locations. Because of a drones limited lift capacity they assume only one customer's demand can be met for each trip. They argue that it is an uncapacitated problem but as technology gets better and the drone capacity increases the model need to be changed to allow for more than one unit of demand making the problem a capacitated problem.

One important constraint in the drones is their range and a drone might need to visit several recharging stations before arriving at the customer. They also included the cost of opening and operating the different launch sites and recharging locations as well as cost of operating and maintain their drone fleet.

They developed a mixed integer non-linear model to solve the problem, but because it is NP-hard an exact solution might not be possible to find. Therefor they used a generic algorithm to solve the problem. Using this algorithm, they proposed a total of two launch sites and 22 recharge stations. When comparing drone delivery investment in drones and locations they find out that they can break even on the investment in just about a year, saving about 49 cents per km transported compared to traditional delivery by truck.

According to a Wall Street Journal article (De Avila, 2022), Amazon started a trial project of their drone delivery service for real customers in a small town called San Jaquin just north east of San Francisco in 2022 just five years after the case study was conducted by Shavarani, Nejad, Rismanchain and Izbirak. If the trial project is a success, they expect Amazon to open for drone delivery in more cities in the coming months and years.

2.8.2 Facility location for cloud computing servers

Another example of a facility location application is facility location in cloud computing. In cloud computing facility location theory and operations research can be used to determine where to locate data centers. A data center is the core of any cloud computing businesses, and the location of the datacenters are influenced by a lot of factors. In the paper "Applying operations management models for facility location problem in cloud computing environments" (Babu & Krishna, 2013) they discuss why the location of the datacenters have such an important role.

First of all, the centers must be as close to the demand as possible to reduce the time it takes for the customer to access the datacenter. Secondly the cost of operating a datacenter varies a lot. Datacenters use a lot of energy to be online and therefore factors like electricity price, climate etc. influences where the optimal location of the centers should be. Other factors as data laws, stability in the area (wars, nature disasters etc.) have an impact as well. In the paper they have given three different solutions (center, median and maximal covering) to locate the data centers in India. And the three different solutions are discussed and concludes that the final decision will be a compromise between the three solutions.

2.8.3 Case study for ATM site selection

The Automated Teller Machine (ATM) site selection is an important decision for banks to make and was studied in a case study (Celik Turkoglu et al., 2018) and various studies in the literature have investigated this issue utilizing different methodologies. The primary goal of effective ATM placement is to reduce costs while maintaining customer satisfaction. This study focuses on Uncapacitated Facility Location Problems (UFLP) and Capacitated Facility Location Problems (CFLP) in relation to ATM placement. The models are implemented for a private bank's ATMs located in a district of Istanbul. The findings demonstrate that ATM capacity is a significant factor.

They present UFLP and CFLP models for the problem for which they then solve a real case study about ATM deployment. For distances between ATM locations and district centers is calculated with the great-circle distance metric, using coordinate data for each ATM. In the case with no limits on capacity, they open 13 ATMs to minimize the distance-related costs, while in the capacitated case they must open five additional ATMs to meet customer demand. There is also an increase in the total cost of the system when the capacity constraint is incorporated to the model due to the extra ATMs needed.

In this paper they utilize fixed costs for establishing facilities, which will not be the case for us. They present both capacitated and uncapacitated situations where we will only look at the uncapacitated case. By including capacities at locations, we would likely also experience an increase in costs.

Although these three cases are not directly related to the fishing industry, some similarities in the models can be drawn to Mørenot's case. They all have locations where they decide where to establish some sort of service to serve a set of customers. All the problems are also constrained by a set of logical constraints that can be changed to other cases like Mørenot's.

2.8.4 Other master's theses

There are written other master's theses for Mørenot before, and our thesis will use some of this work and try to improve or use their findings in our research. The first and most relevant thesis for our research is the thesis "Localizing warehouse facilities for Mørenot AS" (Kwidzynski, 2022). In this thesis he investigates the possible locations to establish warehouses for Mørenot using facility location models. His thesis focuses more on the various locations a warehouse should be located given different inputs of cost for operating and opening warehouses. He then discusses the different solutions' pros and cons.
In this thesis several basic facility location models are used, and different solutions have been given for different P- values and for different input data. What we will try to do compared to Kwidzynski is to find out how many semi-central locations Mørenot should operate based on several factors.

Our thesis will focus more on a combination of the lead time for uncovered demand and inventory management related costs to find one optimal solution given the input data provided by Mørenot. We will utilize similar models in the first part of our thesis regarding finding the optimal distribution of semi-central warehouses, but we will take this a step further by also looking at how costs from inventory management affect the results.

We will also use a better developed dataset. Where his thesis estimated a lot of the data, our dataset is more extensive, opening the possibility for a more accurate model.

Two other theses written for Mørenot are “Multi-criteria inventory classification to improve inventory management practices at Mørenot AS” (Bassore & Natwijuka, 2022) and “An Exploratory Case Study on the Properties of Organizing Master Data with a Product Structure in the Fishing Industry” (Gjervik & Taklo, 2022).

The first of these two theses explore how placing products in different product categories (ABC/XYZ) can be used by Mørenot to find out what products they should focus their capital on to reduce their lead time. In addition to this they show how the various categories can be used with Mørenot’s inventory strategy. The last thesis looks at Mørenot’s challenges with a massive dataset of master data. They investigate how Mørenot can get a better data structure in their ERP system for the company and employees to operate more efficiently.

3.0 Method and data

This is a quantitative thesis aiming to solving a problem for the Norwegian company Mørenot AS. The case is provided to us by them, and they want us to investigate whether they should store some of their products on more “local” warehouses, closer to their customers.

Quantitative research is all about calculating and verifying or discarding a theory. It uses data and mathematical models to calculate results which helps give the author’s ideas credibility and reproducibility. This makes it possible for the reader to verify that the calculations and the results are correct. The data is often collected from surveys, experiments or from public or private databases.

On the other hand, we have qualitative research. This is all about using theory and discussing whether a topic or theory is correct. It often uses relevant theory to give the reader a deeper understanding of the research being conducted. The theory used to conduct qualitative research is often gathered from other research papers from the same research field or from comparable topics. Theory and information about the topic can also be gathered through interviews etc.

Our thesis is mainly a quantitative thesis as we use data and mathematical models to prove the research we are conducting. Of course, we will used available literature during the research, but the results are gathered and verified using mathematical models and a quantitative approach.

By using the theory and literature mentioned in chapter 2 we can create an optimization problem. The problem will be a facility location problem combined with some inventory theory.

We aim at creating a facility location model and then analyze different aspects, such as lead time for uncovered demand and costs. In the analysis we can easily tweak and alter the input values which allows us to see how it affects the outputs, giving a better insight of the variables and constraints and how it changes the solution.

We will also use inventory management theory to see how the costs for the different solutions from the facility location problems compare to each other and using this information try to give a recommendation to Mørenot about what locations they should establish as semi-central warehouses.

3.1 Data

First of all, the data needs to be collected and understood. Then the relevant data needs to be extracted. This process is time consuming, and it is important to get right as it is the foundation of the following model and solutions given. If incorrect data is used the model will not give a satisfactory answer (not grounded in reality). Mørenot extracts data from their ERP systems and sends this over to us in spreadsheet format. This gives us the most up to date data possible, as it is recorded in real time into the ERP system as it happens and is the same data that they operate with to make decisions.

3.1.1 What data do we need?

When we started to gather data, we had some ideas about what we needed, so we had a meeting with representatives from Mørenot where we discussed what data were available. From this meeting we discussed several different variables that we could include in the dataset.

To solve the problem, we need some important pieces of data. The most important piece of data we need is the locations and demand at the different facilities or warehouses in Norway. In addition to this we need to know the categories and price for the different items and know how the demand varies throughout the year. If there is a lot of variation in demand, then we should collect data for several years so we can identify the patterns and include it in the analysis.

Ideally, the data should be on the most granular level since this allows us to aggregate parts of the data if we want to. Disaggregating the data is more challenging and might introduce uncertainty in the dataset if you are required to estimate values.

This data is used for creating the facility location model and these datapoints are important input values in the model we developed. A basic facility location model can utilize locations of facilities, capacities of facilities, costs (travel times), and demand to obtain a solution. Further we can use some descriptive statistics and calculations to look at various aspects for the different facility location solutions to compare them to each other and try to come with some recommendations for how many semi-central warehouses to establish.

In many cases good understanding of the business environment and expertise can give companies close to optimal performance, without developing a model. Due to the complexity of such an interconnected system in the real world, there will most likely be multiple solutions with very similar output values as the final result, even with a large variety of values for the decision variables.

If we manage to make a model that is good at representing the real-world situation, then it will be possible for Mørenot to act and choose to store some of their product on more warehouses than just the three main warehouses. The goal is to create an Excel workbook which can solve a facility location model. It can then be updated in the future to account for changes in demand and new products, etc.

3.2 Solving the problem

Using quantitative methods, we analyzed the solution given by the optimization model. This gave us information about what locations Mørenot should use as semi-central storage locations to minimize the total lead time for uncovered demand and inventory costs. We can conduct analysis for various aspects of the problem. This allows us to compare different inputs and outputs Mørenot can change.

3.2.1 Tools used

There are several different ways of solving optimization problems. These include different programming languages and programs, heuristics, and spreadsheet software like Microsoft Excel. We will use Excel and the OpenSolver plug-in to get a solution and analyze it.

Excel is a well-known piece of software that most people in the industry has at least some experience with and this includes the people working at Mørenot. By using Excel spreadsheets as our interface, it will be easier for others to look at the model and understand what is happening. This also enables Mørenot to use this spreadsheet model as a decision tool in the future by changing the input data. We utilize Excel as it has an easy-to-use visual interface, which we find it easy to work with. If there are errors in the model it is relatively easy to figure out what is wrong thanks to the feedback the software provides to the end user. The visual user interface shows the model setup in a way that is easy to understand for those who work on it.

Even though we may have an easy-to-use interface, developing a mathematical model is not an easy task, and it is essentially performed the same way regardless of which piece of software is being used to solved it. The way we set up the spreadsheet model inside Excel depends on how we have designed the mathematical model in the first place. All formulas and cells must be set up in a way that considers all parameters and variables and must fit so the logic of the model is correct. This can be a challenge in larger models, but a nice feature of using spreadsheets is flexibility. Should we wish to expand or reduce the size of the model, this can be done easily assuming the spreadsheet has been set up properly.

A “Solver” is a piece of software used to solve optimization problems. By interacting with a user-interface which lets you design a model in a normal spreadsheet, the Excel Solver then uses the cells in the spreadsheet as input parameters and variables in order to build a mathematical model in a way that can be solved by a computer. A solver engine then computes the calculations necessary and provides the user with an output based on how the model is set up.

The built-in Excel Solver only allows for a maximum of 200 variables and 200 constraints. There is a decent chance that we end up with developing a model which utilizes more than 200 variables and therefore, another tool must be used instead.

The OpenSolver plug-in for Excel is a free plug-in which does not have artificial limitations on the maximum number of variables and constraints you can use in a model, while also providing support for more solver engines. Apart from this, the software operates in almost the same way as the built-in solver package. We have experimented with larger models containing several hundred variables to test the capabilities and limitations of the OpenSolver Plugin. We have successfully run a model with more than 2500 variables and therefore we see this as a suitable tool for solving our models.

There are other solver software packages which would be suitable to use for this case besides Excel Solver. The thing they have in common is that they all utilize the mathematical model in order to solve the problem. The largest differences are usually the user-interface and which solver engines the software allows you to access and use for running the model.

One example is AMPL (A Mathematical Programming Language). AMPL is a more direct programming approach to solving optimization problems, where one writes programming code in a simplistic interface that is solved by the solver. Such a model can be written in plain text in a text editor which then reads the text files containing the proper code to run the model and output a solution file (Fourer et al., 2003)

While AMPL may be suitable for large scale, complex problems, it is also more challenging to work with, especially for those not familiar with optimization models. A paid license for AMPL does not have any artificial restrictions on the number of variables or constraints you can have in a model.

The recurring downside with most solver software package alternatives outside of Excel is the price. There are very high costs for licenses for many of the other options as these are mainly aimed at being used in larger firms that can handle the cost. They can often cost between $6000 and $14 000 per license depending on what license type is chosen (AMPL Optimization Inc., 2022). Microsoft Excel is relatively cheap in comparison and is already utilized by most companies to some extent anyways, so the barrier of entry is much smaller for the Excel Solver, which is a free plug-in one simply needs to activate in the settings of the software.

There are also several other ways to solve mathematical optimization models. One way is to use other programming languages such as Python, R studio or Mathlab. These use different plugins that run the algorithms (solvers) such as simplex and so on. Another way of solving optimization problems is by using heuristics. These are algorithms that are fast and efficient at finding a solution. The solution may not always be the optimal one, but it is often quite close to the best possible outcome. Regardless of what software one uses to solve the optimization problem the mathematical formulation is the same and one can relatively easily “translate” the problem from Excel to AMPL or Python and vice versa.

4.0 Case description

The purchasing of goods to the various locations is done by as many as forty people at Mørenot. After discussions with them it became clear that this leads to a lot of “small” deliveries to the various locations in addition to overstocking items. Another problem is that due to long lead times and expensive freight from Asia to Norway they try to have full containers when ordering. Therefore, when so many employees are ordering it can be challenging to have full utilization of the containers. The ordering process is currently quite “segregated” between the various locations and with better ordering policies across the organization there can be a significant reduction in lead times and costs.

One possible way to aggregate the purchasing of items and minimize overstocking is to introduce several semi-central warehouses that do the purchasing for the other local warehouses. This can also lead to a higher service level and less backordering (fewer stockouts) due to more than just their main warehouse in Søvik having items on stock. The idea of having one or multiple semi-central warehouses will be explored further and is the focus of this thesis.

When getting insight in their operations, it became clear that their location at Søvik was the most important. Even though they imported some items to the other locations, most of the goods purchased from overseas came by ship to Søvik. Also, most of the stock today is located at their Søvik location (in addition to Gangstøvika which are located just a few kilometers away). Therefore, we decided that it made sense to always use Søvik as the main warehouse in our calculations. This is justified because Søvik already operates as their main warehouse and has well-established infrastructure. If any other locations were to operate as a main warehouse additional investments might be necessary.

4.1 What is the problem?

Mørenot operates within the fishing industry and therefore their customers have different needs of different products throughout the year. Some of their products might have a clear seasonal trend, while other items have a constant demand throughout the year. Other external factors such as fish price, trends in what kind of fish people eat and so on might influence what kind of fishery is popular at different times.

Because of this, if a product is not in stock at the correct time it might lead to a loss of sale as the season for the specific type of fishing can be over within a month or two. With an average shipping time from Asia to Norway of 6 weeks alone, the long lead times means the customer will in most cases go to a competitor to purchase their fishing goods. Therefore, it is important for Mørenot to have a high service level ensuring that the items are at the correct location at the correct time.

The aim of this thesis is to provide suggestions for the number and locations of semi-central warehouses to be established. The decision regarding the number of warehouses will be based on an analysis of the inventory costs and the lead time for uncovered demand.

While changing the lead time (e.g., transportation time) might be difficult, using different methods discussed in this thesis they can ensure that more of the items requested at the various locations are already in place when the customer is there. Thereby from the customers point of view there is zero lead time. And if the items happened to not be in stock, they would be in a location relatively close, in a semi-central warehouse making the lead times shorter.

The traditional definition of lead time will not be used in this thesis. Instead, we will focus on the waiting time a customer experiences when an item they wish to purchase is not available at that point in time. While Mørenot always has lead times for all products they order from all warehouses and locations to meet demand according to their service levels, we will focus specifically on the uncovered demand. That is, the demand customers have for items that there are no more items left of in stock. When a warehouse or location experiences a stockout, that is the uncovered demand we wish to look at the lead time for.

It is this lead time for the uncovered demand that a customer will actually experience the effects of and is what could potentially result in a lost sale or even the loss of a customer long term. In general, throughout this thesis, we will refer to this as the lead time for uncovered demand.

When calculating inventory costs, we will look into setup, holding, safety stock and stockout costs at different service levels and combinations of semi-central warehouses.

4.2 How we solved the problem

After getting the problem described by Mørenot we found out that we had to choose between several already established locations to use as semi-central warehouses or hubs.

To do this we researched, among other things, facility location models and work done by Laporte and Nickel, in particular the book “Location Science” regarding facility location.

This book gave us good insight and a good starting point of which models were suitable and which were not for Mørenot’s problem. In the end we ended up developing a P-MP model where we decide a P number of locations to open at a minimum cost (distance as a cost). This is a discrete model where the location can be selected among a selected number of alternatives and the costs (can be represented as distance, monetary value, travel time etc.,) are known.

We chose to use a P-MP model because we wish to minimize the total sum of lead time for the uncovered demand in the system. We do not necessarily wish to reduce the single largest occurrence of lead time for uncovered demand(P-CP), but rather reduce the overall sum of it for all customers. This meant that a P-MP model as explained in chapter 2.5.2.1 was right for our situation. In our model we will not have fixed costs for establishing a facility, as we will simply expand the inventory use at the chosen locations. As Mørenot has items that can be stored outside and room to spare, we handle the inventory capacity as being infinite. This was confirmed by Mørenot during our discussions. While this makes the model somewhat simpler, there could be other costs related to expanding a facility in this fashion to accommodate for storing more items. Therefore, we do not solve a fixed-charge facility location problem.

After we solve the model, we obtain P number of locations to use as semi-central storage locations along with the distribution of which locations they each serve. Then we can investigate the total lead time for uncovered demand for the different outputs.
When we know this, we can go one level deeper into the inventory management field to calculate the costs aspect of the system and how the distribution of semi-central warehouses and who they serve will affect monetary costs.

For instance, it may be the case that the model suggests one location should have a non-negligible increase in its stock which may surpass the extra capacity that is currently available today. In this case, there would be a need for physical expansion of space and/or buildings which results in added costs.

Other relevant costs that might arise are increased salary costs due to needing more staff to handle the increased amount of stock. Having more inventory in stock also results in higher inventory management related costs (this is explored in detail from chapter 5.3).

4.3 Gathering and description of raw data

After several meetings with representatives from Mørenot they put us in contact with their Item Coordinator from their operations department. He had a lot of knowledge of their data and how to manipulate it. With his help we figured out what data was available and what data we should include in our research.

Mørenot uses an ERP system called Navision which in turn is integrated into Power BI. Power BI is software developed by Microsoft used to analyze large datasets for companies (Microsoft, 2023). It can be used to visualize and share data and is useful to create dashboards for employees to have easy access to up-to-date data to help with decision making. Because of their use and knowledge of how to extract data from their data systems the data we got were up to date and correct representation of their operations. In PowerBI it is possible to send queries to extract data from their databases in a user-friendly format.

From these queries the tables we obtained contained only the data we needed, and a minimal amount of cleaning the data was necessary. Huge thanks to the item coordinator for enabling this and giving us a good explanation of the data, saving us a lot of time digging through huge datasets we have no prior knowledge of. The most important piece of data we gathered was their transaction data for 2022.

4.3.1 Transaction data

The transaction data is a rather large dataset with about 82 thousand observations and contained the following variables:

Company:

This variable is the same for all observations in the dataset, MNF or Mørenot Fishery as we only wanted to analyze their fishery department and not aquaculture or offshore divisions as explained earlier.

Item No, category and description:

This variable is a unique number/string noting the different items and the category they belong to as well as a brief description of the product. The product description might be useful in the later analysis if we want to take a deeper look at some of the items that “stands out” in the dataset and it is easy for us to know if it is a bolt or a big net. There are 11771 unique items divided in 17 categories in the dataset with a total of 82667 observations.

Location Code:

This variable denotes what location the different items were shipped in/out/transferred from. In total there are 19 different locations in the dataset. Some of them are at the same location and will later be merged, see chapter 4.3.2

Qty:

This variable tells us how many of the given items have been sold, purchased, or transferred. It is negative if it has gone out of a location and positive if it comes to a location.

Avg Price:

This variable tells us the average price for the item in the given transaction. The prices are all purchase prices for Mørenot and do not represent the sales value. The price for the items can vary a bit throughout the year and therefore we need to take that into consideration in the later analysis. The prices are in NOK.

Start of week:

These variables state what week the transaction took place. We decided to have the data on a weekly basis. The reason for this is that if we had it daily the dataset would become unnecessarily large, and we would have a far greater “resolution” of the data than what is necessary for this thesis. We still opted to have the data on weekly basis at it easier to aggregate the data to months rather than the opposite way from month to weeks. The data is for the whole of 2022.

Transaction type:

These last variable states whether the observation is going into a location, going out (sold or production) or is transferred (between two locations). The most important transaction type for our thesis is the "out" observations as they represent the demand at the various locations.

4.3.2 Distance and location data

The first thing we did when cleaning the data was that we aggregated some of the locations. This is due to Mørenot acquiring other businesses, which has resulted in some locations having multiple entries in the dataset as Mørenot have not aggregated the new company into the Mørenot locations in their database, even though they are located at the same address. Therefore, we had to combine some of the locations which had the same address. This left us with 11 locations from the 19 in the original dataset.

The next piece of data that we need to create a facility location model is the distance or travel times between the various locations. To get this we simply needed the addresses to the various locations, which meant we could use google maps to get the distances and travel time between the various locations. This was done using the google maps API in google sheets, so we did not have to look up each location individually. The results of this can be seen in Table 1 and Table 2.

Table 1 Distances between all eleven locations (km)

Distances (km)	Søvik	Båtsfjord	Fosnavåg	Harøy	Gangstøvika	Tromsø	Skjervøy	Bekkjarvik	Hildre	Salthella	Avaldsnes
Søvik	0	1907	140	45,2	49,8	1417	1514	502	12,3	507	596
Båtsfjord	1906	0	2003	1895	1912	831	684	2329	1887	2334	2383
Fosnavåg	140	2003	0	184	106	1514	1610	450	151	455	543
Harøy	45,2	1896	184	0	93,8	1407	1503	546	46,8	551	640
Gangstøvika	50,2	1913	106	94,2	0	1424	1520	468	61,3	473	562
Tromsø	1417	831	1514	1407	1424	0	159	1875	1398	1880	1951
Skjervøy	1514	684	1611	1503	1520	159	0	2145	1495	2150	2199
Bekkjarvik	502	2327	450	546	468	1875	2145	0	514	6,4	109
Hildre	12,3	1888	151	46,8	60,9	1398	1495	513	0	518	607
Salthella	508	2332	455	552	473	1880	2151	6,4	519	0	114
Avaldsnes	596	2381	544	640	562	1951	2199	109	607	114	0

Table 2 Travel times between all eleven locations (hours)

Travel times (hours)	Søvik	Båtsfjord	Fosnavåg	Harøy	Gangstøvika	Tromsø	Skjervøy	Bekkjarvik	Hildre	Salthella	Avaldsnes
Søvik	0	26	3	0,9	0,8	22	23	9	0,2	9	11
Båtsfjord	26	0	28	26	26	11	9	31	26	31	31
Fosnavåg	3	28	0	3	2	23	25	8	3	8	10
Harøy	0,8	26	3	0	2	22	23	10	0,9	10	12
Gangstøvika	0,8	26	2	2	0	22	23	9	1	9	10
Tromsø	22	11	23	22	22	0	4	29	22	29	29
Skjervøy	23	9	25	23	23	4	0	29	23	29	29
Bekkjarvik	9	31	8	10	9	28	29	0	10	0,2	2
Hildre	0,2	26	3	0,9	0,9	22	23	10	0	10	11
Salthella	10	32	8	10	9	29	29	0,2	10	0	2
Avaldsnes	11	31	10	12	10	29	29	2	11	2	0

Below in Figure 6 is a visualization where in Norway the eleven warehouses are located (Søvik in red). As one can see they are scattered along the western coast of Norway.

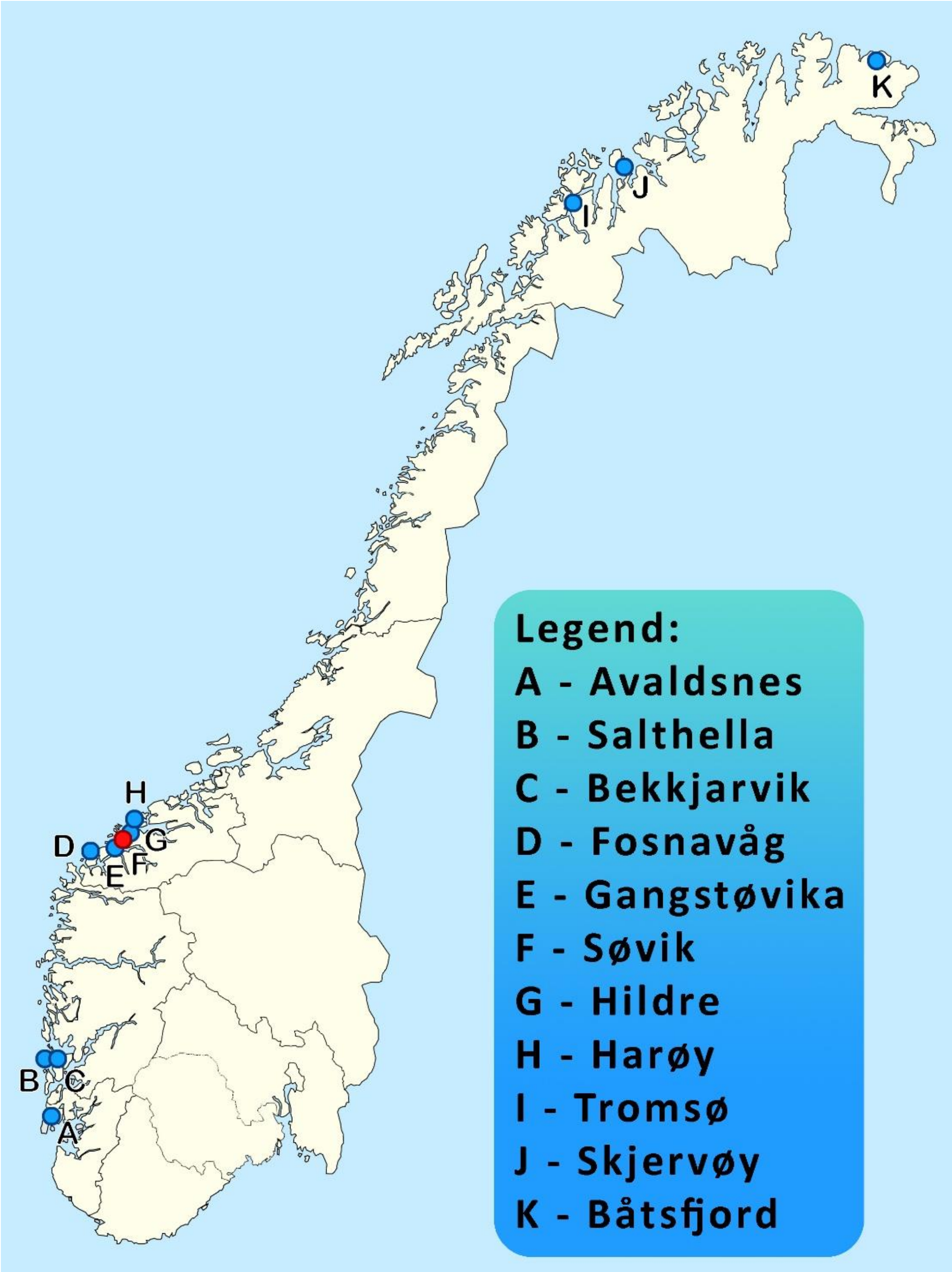

Figure 6 Geographic locations of the eleven warehouses

4.3.3 Other data

In addition to the data mentioned above we also had information regarding their holding costs and ordering cost. They had actual numbers for these values which will help a lot with the inventory management part of our work. Both the holding cost and ordering cost are central parts in most of the calculations done in inventory management and not having to estimate these numbers will give a much better answer.

Their holding costs are calculated using an interest rate of 20% which means that it costs 20% of the item's value to store it for one year. While the ordering cost are 700 NOK meaning it costs Mørenot 700 NOK to make a new order. The ordering cost is assumed to be the same regardless of the order size. These values were calculated and used by Mørenot in their daily operations and were provided to us by them.

4.4 Accuracy of the gathered data

The data we got for the year of 2022 is as accurate as it can be as it is the real collected data by Mørenot. The data could be daily, but the difference would be negligible as there are a few transactions for the same item at the same location in the same week and small price differences for the item within a week. Another variable that might impact the accuracy of the data when used in an analysis is the Covid-19 pandemic. The numbers in the dataset might be inflated/deflated compared to a more "normal" year regarding the pandemic.

One way to check if the data from 2022 is representative is to compare it to previous years before the pandemic, but that is hard because in the years leading up to the pandemic Mørenot acquired several smaller businesses within the same industry. Although the data is hard to compare and verify compared to previous data, during conversations with Mørenot we were assured that 2022 was within their expectations of a "normal" year and we therefore assume the data is representative of their normal operations.

A weakness of the data is that some of the products are similar and can be substitutes for each other but with different item numbers. One example of this is their ropes. The ropes can be made at different factories, hence have different item numbers, while being the same product. This is due to Mørenot purchasing other companies that make the same products resulting in the same product having different item numbers. They are working on gathering information about products that are the same, but with different item numbers. However, this is an ongoing process, and the data is not available for us. Because of this we use the items and item numbers as they are.

4.5 Cleaning and observations of data

After getting the raw data in Power BI, we exported it as a CSV file that can be manipulated using various software. We tried several different methods of extracting and manipulating the data we needed. The first we tried was using Pivot tables in Excel. Pivot tables are user friendly and easy to apply filters and alter the data based on calculations, but due to the large size of the dataset, it was slow to use. Therefore, we did the initial cleaning using R Studio which was a much faster way to clean the data.

4.5.1 Aggregating by year.

After importing the whole dataset into R with the correct locations the dataset had just above 50 thousand observations. What we then did with the dataset was to aggregate so each location has the total quantity for each product (sum for each product at each location) and used the average price for each product (mean for each item price for each location) if a product is sold more than one time during the year. This reduced the dataset to about 13700 observations. We also calculated some values that might be relevant for later analysis like average demand for each item per location per week, standard deviation for demand for each item per location, percentage of total sales (demand*price) for the items at a location.

After cleaning the data, the dataset was reduced the dataset from over eighty thousand observations to just above 13700 observations. This will help speed up further calculations and model in addition to being an accurate dataset. This dataset was then exported from R Studio as an CSV file, which can be used to do the modeling and calculations in Excel.

4.5.2 Seasonal variations

After cleaning all the data, we wanted to see whether any seasonal variations were present in the data as the fishing industry changes a lot during the year for different types of fishing. If there is compelling evidence for seasonal trends for some of the products, it needs to be considered in the analysis. In Figure 7 we can see the sum weekly demand at all locations for each product category.

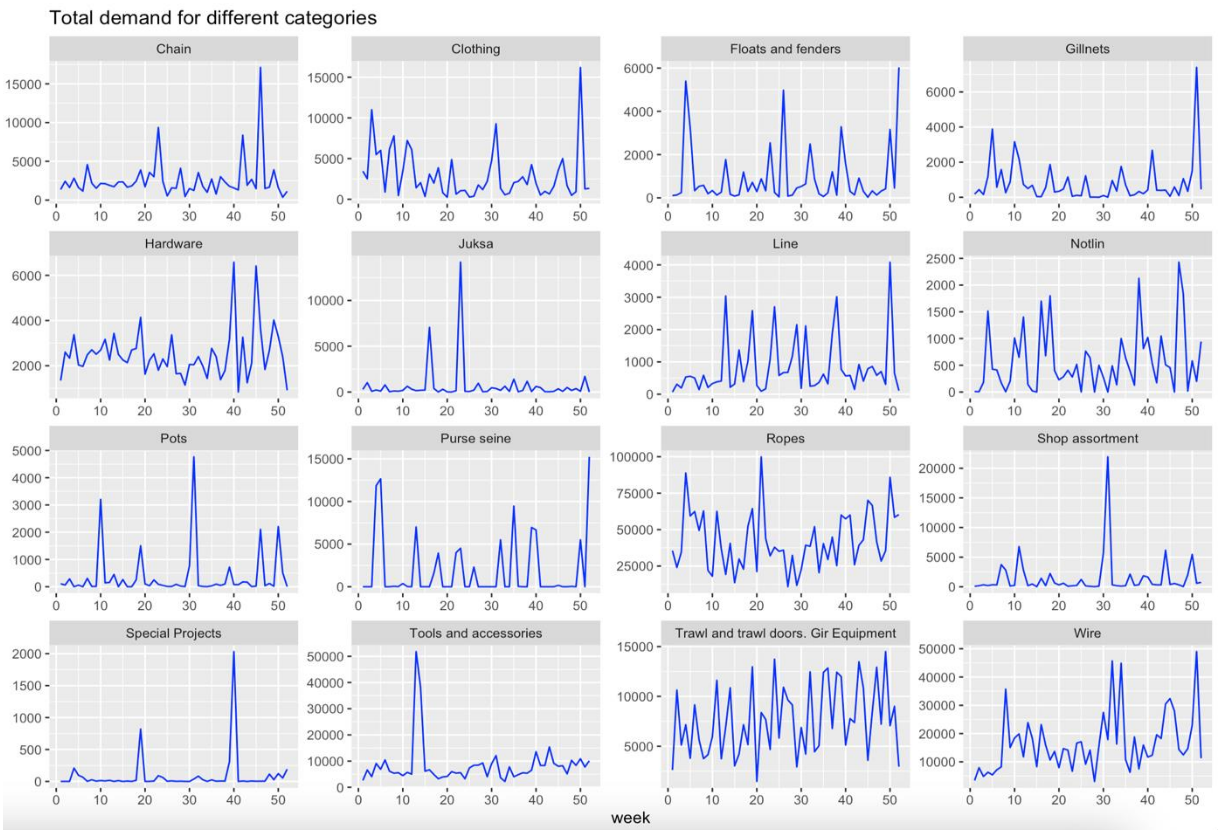

Figure 7 Aggregated weekly demand for all product categories.

In these plots we do not observe any significant evidence of seasonal variations for the year of 2022. Therefore, we choose to not include season variations into our analysis.

5.0 Development of models and analysis

The main objective of this thesis is to provide a suggested number of semi-central locations that might help reduce lead time for uncovered demand as well as inventory costs. The first task is to figure out what locations could serve as a semi-central warehouse. To do this we used a facility location model.

There are several different facility location models that can be used, but the P-MP model is the one fitting this problem the best. We will create a P-MP model for P=1 to P=11 and then investigate several ways of measuring what value of P (number of semi-central warehouses) is best with regards to lead time for uncovered demand and inventory costs.

5.1 Facility location using a P-MP model.

Since we did not consider the costs of establishing or opening new locations, but rather expanding existing locations or even just altering the distribution of items, we did not include fixed establishing costs in the model. This meant it was essentially "free" for the model to open any location it wanted and that the cost was the same for all locations, which in turn meant that this would not impact which location it chose to include in its solution.

The only consideration the model had to take was the weighted demand of the locations. This was calculated as the demand for the product group at one location, multiplied by the distance to all other locations. The combination of P locations which had the lowest sum of total weighted demand for serving all demand for its locations would represent the locations which should act as semi-central warehouses for the smaller locations surrounding it. Specifically which warehouses are served by the individual semi-central warehouses is also shown in the solution from the model.

This model was run once for each of the different product groups. With 16 product groups this meant we ran the model 16 * 11 = 176 times to be able to compare the results and find which value of P gave the best result.

5.1.1 Mathematical model for P-MP model

In this part we will formulate the P-MP model we will use throughout our thesis.

Parameters:

n	Number of locations	
N	Set of locations	$N = \{1,2,3\ldots n\}$
$c_{f,t}$	Distance from location f to t	$f \in N; t \in N$
d_t	Demand at location t	$t \in N$
$C_{f,t}$	Weighted "cost" from location f to t. Calculated as demand at location t*distance from node f to node t ($C_{f,t} = d_t * c_{f,t}$)	
P	Number of semi-central warehouses to establish	

The “cost” of the model is not a monetary cost, but rather a value that shows a total weighted distance between all locations. This allows links with a short distance, but a large demand to have a bigger weight.

The parameter P will be used throughout the thesis where it describes what P-MP solution we are referring to where it represents how many semi-central warehouses are selected in the solution. For instance, we will refer to the parameter both as “P=4” and “P4” when we have selected four semi-central warehouses to be established, as these are used interchangeably.

Variables:

The fist variable states whether a semi-central warehouse is established in location f:

$$U_f \qquad U_f \in \{1,0\}$$

U is a binary variable which takes the value 1 if a semi-central location is established in location f, and 0 otherwise.

The second variable states if a location t is served from the semi-central warehouse at location f.

$$V_{t,f} \qquad V_{t,f} \in \{1,0\}$$

V is also binary 1 if location t is served from semi-central warehouse f otherwise 0.

Objective function:

The objective function is to minimize the total weighted distance for the system:

$$min \sum_{f \in N} \sum_{t \in N} C_{f,t} * V_{f,t} \qquad (1)$$

Here the C represents the weighted cost from node f to node t where the weighted cost is demand in node f multiplied with the distance from node f to t.

Constraints:

A location must be served by exactly one other location:

$$\sum_{f \in N} V_{f,t} = 1 \;\; \forall \; t \in N. \qquad (2)$$

A semi-central warehouse can serve up to n locations if it is established (U_f=1).

$$\sum_{t \in N} V_{f,t} \leq n * U_f \; \forall \, f \in N \; (3)$$

Number of semi-central warehouses to be established should be equal to P:

$$\sum_{f \in N} U_f = P. \qquad (4)$$

The value of U_1 must be equal to 1.

$$U_1 = 1 \qquad (5)$$

Table 3 Index of locations for the set N

i	Location
1	Søvik
2	Båtsfjord
3	Fosnavåg
4	Harøy
5	Gangstøvika
6	Tromsø
7	Skjervøy
8	Bekkjarvik
9	Hildre
10	Salthella
11	Avaldsnes

This constraint forces Søvik (the first location in the list from Table 3) to always be chosen, because it is the main warehouse (explained in chapter 4.0). This essentially means that for P4 Søvik plus three locations are established.

While we have different product groups we look at, these are not specified mathematically in the model. This means that every time we run the model, we do not distinguish between different products but rather change the input values of demand. So, while we have different product groups to analyze, the model runs as if there is only one group.

Problem size

This problem has a total of 132 variables, 11*11 for the $V_{f,t}$ variables and 11 for the U_f variables.

The number of constraints is calculated in the following manner:

We have the four constraints (2), (3), (4) and (5), plus binary constraints:

Constraints (2) applies to each location in the set N which is equal to 11. Constraints (3) also applies to each location in the set N which is also equal to 11. Constraint (4) simply forces one sum to be equal a specific number which is only one constraint. Constraint (5) forces one location to always be used. This is also only one constraint.

Finally, we add the two binary constraints on the variables. The number of constraints is one per variable which means the calculation is the same as the number of variables in the model: 11 for the U_f variables and 11*11 = 121 for the $V_{f,t}$ variables.

This gives us a grand total of (11 + 11 + 1 + 1 + 11 + 121) = 145 constraints in total for the model.

5.1.2 Why did we use a weighted distance matrix?

A weighted distance matrix can provide a more accurate representation of the cost or effort required to service locations, as it takes both the physical distance and the demand for products at each location into account. Using distances alone would assume that all locations are equally important, which may not be the case. For example, a location with higher demand for products may be more valuable, or in our case, more important than other locations with lower demands.

Since our focus is to reduce the lead time for uncovered demand, the specifics of the items and their value (which can vary significantly between customers) are not of interest to us in this scenario. This is explained in detail in chapter 6.2.1. By incorporating demand by multiplying the distances with the respective demands, we assign greater weight to locations with higher demand. This gives us a better representation of the importance of each location.

5.1.3 Spreadsheet Model

To solve the mathematical model by using the Excel Solver we must design a suitable spreadsheet. We need input tables for distances and demand, as well as an intermediate weighted distance matrix and finally an output table.

In Table 4 you can see the distances between all eleven locations. This, together with the demand, is used for making the weighted distance table.

Table 4 Distance table for all eleven locations

Distances (km)	Søvik	Båtsfjord	Fosnavåg	Harøy	Gangstøvika	Tromsø	Skjervøy	Bekkjarvik	Hildre	Salthella	Avaldsnes
Søvik	0	1907	140	45,2	49,8	1417	1514	502	12,3	507	596
Båtsfjord	1907	0	2003	1895	1912	831	684	2329	1887	2334	2383
Fosnavåg	140	2003	0	184	106	1514	1610	450	151	455	543
Harøy	45,2	1896	184	0	93,8	1407	1503	546	46,8	551	640
Gangstøvika	50,2	1913	106	94,2	0	1424	1520	468	61,3	473	562
Tromsø	1417	831	1514	1407	1424	0	159	1875	1398	1880	1951
Skjervøy	1514	684	1611	1503	1520	159	0	2145	1495	2150	2199
Bekkjarvik	502	2327	450	546	468	1875	2145	0	514	6,4	109
Hildre	12,3	1888	151	46,8	60,9	1398	1495	513	0	518	607
Salthella	508	2332	455	552	473	1880	2151	6,4	519	0	114
Avaldsnes	596	2381	544	640	562	1951	2199	109	607	114	0

In Table 5 you can see the demand inputs. These are the demands for each product group for each location.

Table 5 Demand for each product group at each location

Demand	Søvik	Båtsfjord	Fosnavåg	Harøy	Gangstøvika	Tromsø	Skjervøy	Bekkjarvik	Hildre	Salthella	Avaldsnes
Product Group	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11
A	3309	56	0	0	83583	37463	4668	20	0	245	407
B	365	1859	209	0	94725	256	45864	4951	0	4209	0
C	24696	1187	1904	0	5504	169	1564	204	0	13430	60
D	9258	108	15595	14646	44	5	2256	87	282	307	0
E	3259	680	70	0	58853	31467	6885	5912	0	5927	24
F	337	1501	28330	0	13	0	5463	0	0	0	0
G	1765	4143	1668	0	33966	0	1014	4	0	0	0
H	8392	0	20	0	9210	0	2239	0	0	9216	555
I	61	1049	601	0	15542	1	72	712	0	165	0
J	22385	200	0	0	0	0	0	0	0	75277	0
K	975886	39626	71826	17581	541182	161824	211634	9158	4474	134217	364
L	396	421	1235	0	73914	959	710	54	0	48	0
M	706	1	30	12	2128	6	265	1	0	574	812
N	25870	2245	710	50	68634	13448	5058	33269	36	279282	106
O	44714	196	7	0	143089	72465	98293	4	0	202	15
P	159764	0	0	0	25429	204953	26617	1	0	10381	0

By multiplying one row (one product group) from Table 5 with the distances in Table 4 we get the weighted distances in Table 6. In this example we have multiplied the product group labeled "I" in the demand table. This table represents the C_{ft} parameter as explained in chapter 5.1.1. Some values being zero simply means there was no demand for that product group at that location.

Table 6 Example of weighted distance table

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11
L1	0	2000443	84140	0	773992	1417	109008	357424	0	83655	0
L2	116327	0	1203803	0	29716304	831	49248	1658248	0	385110	0
L3	8540	2101147	0	0	1647452	1514	115920	320400	0	75075	0
L4	2757	1988904	110584	0	1457840	1407	108216	388752	0	90915	0
L5	3062	2006737	63706	0	0	1424	109440	333216	0	78045	0
L6	86437	871719	909914	0	22131808	0	11448	1335000	0	310200	0
L7	92354	717516	968211	0	23623840	159	0	1527240	0	354750	0
L8	30622	2441023	270450	0	7273656	1875	154440	0	0	1056	0
L9	750	1980512	90751	0	946508	1398	107640	365256	0	85470	0
L10	30988	2446268	273455	0	7351366	1880	154872	4557	0	0	0
L11	36356	2497669	326944	0	8734604	1951	158328	77608	0	18810	0

Running the solver with these inputs gives us the following output shown in Table 7. The total cost comes from multiplying the binary variables in the table by the weighted distance tables and thereby summing together the ones who are being used.

Table 7 Example output for P=4 and product category = I

								Total distance (cost)					114841	
V	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	Sum		U_f
L1	1	0	0	0	0	0	0	0	1	0	1	3		1
L2	0	1	0	0	0	1	1	0	0	0	0	3		1
L3	0	0	0	0	0	0	0	0	0	0	0	0		0
L4	0	0	0	0	0	0	0	0	0	0	0	0		0
L5	0	0	1	0	1	0	0	0	0	0	0	2		1
L6	0	0	0	0	0	0	0	0	0	0	0	0		0
L7	0	0	0	0	0	0	0	0	0	0	0	0		0
L8	0	0	0	1	0	0	0	1	0	1	0	3		1
L9	0	0	0	0	0	0	0	0	0	0	0	0		0
L10	0	0	0	0	0	0	0	0	0	0	0	0		0
L11	0	0	0	0	0	0	0	0	0	0	0	0		0
Served	1	1	1	1	1	1	1	1	1	1	1		Sum p	4
													Chosen p	4

In the rightmost column in Table 7 are the U_f variables showing which locations should be used as semi-central warehouses for this particular case. The established warehouse L1 serves the locations L1, L9 and L11. The warehouse L2 serves locations L2, L6 and L7. The warehouse L5 serves locations L3 and L5. The warehouse L8 serves locations L4, L8 and L10. These, along with the total cost in the top right are the values we extract for further analysis.

This is just to showcase how we developed the spreadsheet model. There are several different ways to create this within excel and it is up to the user to design it the way that suits them. The problem could of course be modeled in other software like AMPL or Python etc. using the same mathematical model as described in chapter 5.1.1 and the solutions would be the same.

5.1.4 Finding the semi-central warehouse locations

We decided to split the demand based on product groups and run the model separately for each of them. This was done to see if there was a significant variation in the "location" of the demand for each group compared with all demand summed together.

The model described in chapter 5.1.1 was run a total of 16 times for each value of P, one time for each of the 16 different categories of products. This means that for each P we get 16 solutions, one for each of the product categories. One of the outputs for this can be seen in Table 8. Here is the output for P = 4 when the model is run 16 times. A, B, C, …, P represents the 16 different product categories, and the binary numbers represents the U_f variable from the solution which notes whether a location should serve as a semi-central warehouse or not.

The running of the solver software was automated using a simple macro, so we did not have to run the model 16 times for each P and copy the results from each run. This enabled us to simply change the value of P in the model and press a button in the Excel sheet and that macro was run 16 times and collected the 16 results in a table. This was a real time saver as the model had to be run 16 times for each P and then for eleven different values for P which resulted in having to run the model 176 times.

Table 8 Optimal semi-central warehouse locations for the different product groups when P=4.

P = 4																
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
L1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
L2	0	0	1	0	0	1	1	0	1	1	0	1	0	0	0	0
L3	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0
L4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
L5	1	1	0	0	1	0	1	1	1	0	1	1	1	1	1	1
L6	1	0	0	0	1	0	0	0	0	0	0	1	0	1	1	1
L7	1	1	1	1	0	1	1	1	0	0	1	0	1	0	1	0
L8	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0
L9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L10	0	0	1	0	0	0	0	1	0	1	1	0	0	1	0	1
L11	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
TC	392520	1361352	575676	259162	1707766	672	178988	65447	114841	0	61397704	291776	70962	2972859	240654	4232109

Table 8 tells us that for product group A the solution is to establish a semi-central warehouse in locations L1, L5, L6 and L7. For product group B we can see that the solution is to establish a semi-central warehouse at locations L1, L5, L7, L8. Similar solutions can be seen for product group C through P. The total cost for each product group can be seen at the bottom.

The next step then becomes how to decide what four locations to establish a semi-central warehouse in. As we can see in Table 8 there is no consensus from the 16 solutions on where to locate so we need to do some aggregation from the solutions. To do this we looked at different methods:

The first thing we did was to sum up the binary values for each location, this shows how many of the 16 product groups the model established in each location. These calculations are shown in the "Sum served" column in Table 9. We chose the locations with the highest sum until we had chosen as many warehouses as necessary for the given P.

The problem however occurs when there are several warehouses that share the highest sum as seen in Table 9 for warehouse L2, L6 and L10 all have a sum of six product categories. Therefore, we needed a better method to decide which of these to choose.

Table 9 Calculations to determine what 4 locations to serve as a semi-central location for P=4

	Sum served	U_f	cost*variable	U_f
L1	16	1	73 862 487	1
L2	6	?	1 161 953	
L3	3		259 834	
L4	1		259 161	
L5	12	1	73 026 977	1
L6	6	?	9 837 684	
L7	10	1	64 543 136	1
L8	3		3 183 958	
L9	0		0	
L10	6	?	69 243 795	1
L11	1		70 962	

The last calculation we did in Table 9 was to take the sum of the total cost for the solution for each product category multiplied with the binary variable (U_f). The reason we did this was because the "total cost" calculations include both distance, demand and the $V_{f,t}$

variable which describes which semi-central warehouse serves which locations. Then we chose the P highest values from this calculation, and it represents where the semi-central warehouses should be. L1 (Søvik) will always serve 16 item categories and have the highest weighted cost, therefore it is always selected.

Using this method, we found out for P4 the semi-central warehouses should be at location L1, L5, L7 and L10 (Søvik, Gangstøvika, Skjervøy and Salthella). This method was done for all P = 1 to P = 11 and we end up with the following result:

Table 10 Semi-central warehouse locations for P1 to P11

P	Semi-central warehouse locations
1	Søvik
2	Søvik, Skjervøy
3	Søvik, Skjervøy, Salthella
4	Søvik, Gangstøvika, Skjervøy, Salthella
5	Søvik, Båtsfjord, Gangstøvika, Skjervøy, Bekkjarvik
6	Søvik, Båtsfjord, Gangstøvika, Tromsø, Skjervøy, Salthella
7	Søvik, Båtsfjord, Gangstøvika, Tromsø, Skjervøy, Salthella, Fosnavåg
8	Søvik, Båtsfjord, Fosnavåg, Harøy, Gangstøvika, Tromsø, Skjervøy, Bekkjarvik
9	Søvik, Båtsfjord, Fosnavåg, Harøy, Gangstøvika, Tromsø, Skjervøy, Bekkjarvik, Salthella
10	Søvik, Båtsfjord, Fosnavåg, Harøy, Gangstøvika, Tromsø, Skjervøy, Bekkjarvik, Salthella, Hildre
11	Søvik, Båtsfjord, Fosnavåg, Harøy, Gangstøvika, Tromsø, Skjervøy, Bekkjarvik, Salthella, Hildre, Avaldsnes

As one can see Søvik is represented in all the eleven different results as it was the extra constraint we added and in P11 all the warehouses are their own semi-central warehouse. Skjervøy is also represented in all the solutions (P2-P11) indicating that it has a lot of demand, or the adjacent locations close by have a lot of demand in total.

5.1.5 Finding what locations the semi-central locations serve

After figuring out where the semi-central warehouses should be we need to find out what locations they serve. In the previous chapter we ran the model 16 times, one for each category for each value of P and we found the semi-central locations. Now we need to find out what locations they each serve. If we had only run the model one time for each P (using total demand or just focusing on one product group) and not aggregating the solutions we would not have to do this step as the model and $V_{f,t}$ variable tells us what semi-central a "customer" or location is served from.

We use the same P-MP model as in chapter 5.1.1 but with altering one constraint. We already have a constraint forcing Søvik to be one of the selected locations, but now we expand this constraint to force all the 11 U_f variables to be equal to an input. The input is the answer seen in Table 10. For P4 we force the U_1, U_5, U_7 and U_{10} variables to be 1 where U_1 is Søvik, U_5 is Gangstøvika, U_7 is Skjervøy and U_{10} is Salthella.

This constraint can be defined as the amount of set N that is part of the solution:

$$N_a = \{1, 5, 7, 10\}$$

$$U_i = 1 \; \forall \; i \in N_a$$

For each value of P, a set of chosen locations are found and given as set N_a. The U_i variables will be equal to 1 for all locations i that are found in the set N_a, otherwise the value of U_i will be zero.

For example, in the case of P=4, the values of U_i look like this:

Table 11 Example of U_i values for P=4

i	U_i
1	1
2	0
3	0
4	0
5	1
6	0
7	1
8	0
9	0
10	1
11	0

The demand was also set to be the total demand instead of being separated by product group (not run once for each category). This gave us a result which showed which locations the semi-central warehouses should serve, shown in Table 12 is the solution for P=4.

Table 12 Output for P4 with forced locations to see what semi-central location serves what warehouse.

V	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11
L1	1	0	0	1	0	0	0	0	1	0	0
L2	0	0	0	0	0	0	0	0	0	0	0
L3	0	0	0	0	0	0	0	0	0	0	0
L4	0	0	0	0	0	0	0	0	0	0	0
L5	0	0	1	0	1	0	0	0	0	0	0
L6	0	0	0	0	0	0	0	0	0	0	0
L7	0	1	0	0	0	1	1	0	0	0	0
L8	0	0	0	0	0	0	0	0	0	0	0
L9	0	0	0	0	0	0	0	0	0	0	0
L10	0	0	0	0	0	0	0	1	0	1	1
L11	0	0	0	0	0	0	0	0	0	0	0

Here we can see that L1 (Søvik) serves L4 and L9, L5 (Gangstøvika) serves L3, L7 (Skjervøy) serves L2 and L6 and L10 (Salthella) serves L8 and L11.

This meant that we had obtained the optimal combination of semi-central warehouses and locations they serve for every single value of P. In Table 13 and Table 14 below one can see the answer from the complete model. Cells marked with green are the semi-central warehouses while the white cells below it is the warehouses it serves (all semi-central warehouses serve themselves). For P=1 Søvik was chosen and serves all other warehouses, At P=3 Søvik serves Fosnavåg, Harøy, Gangstøvika and Hildre. Skjervøy serves Båtsfjord and Tromsø while Salthella serves Bekkjarvik and Avaldsnes. See Table 13 and Table 14 for all possibilities of P and the various locations.

Table 13 Model answer P1 to P6, semi-central warehouses marked with green.

P1	P2	P3	P4	P5	P6
Søvik	**Søvik**	**Søvik**	**Søvik**	**Søvik**	**Søvik**
Båtsfjord	Fosnavåg	Fosnavåg	Harøy	Harøy	Harøy
Fosnavåg	Harøy	Harøy	Hildre	Hildre	Hildre
Harøy	Gangstøvika	Gangstøvika	**Gangstøvika**	**Båtsfjord**	**Båtsfjord**
Gangstøvika	Bekkjarvik	Hildre	Fosnavåg	**Gangstøvika**	**Gangstøvika**
Tromsø	Hildre	**Skjervøy**	**Skjervøy**	Fosnavåg	Fosnavåg
Skjervøy	Salthella	Båtsfjord	Båtsfjord	**Skjervøy**	**Tromsø**
Bekkjarvik	Avaldsnes	Tromsø	Tromsø	Tromsø	**Skjervøy**
Hildre	**Skjervøy**	**Salthella**	**Salthella**	**Bekkjarvik**	**Salthella**
Salthella	Båtsfjord	Bekkjarvik	Bekkjarvik	Salthella	Bekkjarvik
Avaldsnes	Tromsø	Avaldsnes	Avaldsnes	Avaldsnes	Avaldsnes

Table 14 Model answer P7 to P11, semi-central warehouses marked with green.

P7	P8	P9	P10	P11
Søvik	**Søvik**	**Søvik**	**Søvik**	**Søvik**
Harøy	Hildre	Hildre	**Båtsfjord**	**Båtsfjord**
Hildre	**Båtsfjord**	**Båtsfjord**	**Fosnavåg**	**Fosnavåg**
Båtsfjord	**Fosnavåg**	**Fosnavåg**	**Harøy**	**Harøy**
Fosnavåg	**Harøy**	**Harøy**	**Gangstøvika**	**Gangstøvika**
Gangstøvika	**Gangstøvika**	**Gangstøvika**	**Tromsø**	**Tromsø**
Tromsø	**Tromsø**	**Tromsø**	**Skjervøy**	**Skjervøy**
Skjervøy	**Skjervøy**	**Skjervøy**	**Bekkjarvik**	**Bekkjarvik**
Salthella	**Bekkjarvik**	**Bekkjarvik**	Avaldsnes	**Hildre**
Bekkjarvik	Salthella	Avaldsnes	**Hildre**	**Salthella**
Avaldsnes	Avaldsnes	**Salthella**	**Salthella**	**Avaldsnes**

The problem would be solved here if we knew how many semi-central warehouses they would want to operate, but we have no other indications of how many semi-central warehouses they want, and it is difficult to know how many they wish to establish. There are several factors not included in this analysis that could affect the number of locations. Some locations might need more staff to operate, the cost of operations might increase, insurance etc. We chose not to include these costs in our calculations as these can be hard to estimate. Therefore, we focused on minimizing the lead time for uncovered demand to begin with.

To find out what value (or values) of P have the smallest total lead time for uncovered demand we need to do more calculations. The first thing we did was to use the solutions for the facility location to look at the total lead times of uncovered demand (chapter 5.2), then we looked at the relevant inventory costs in chapter 5.3 to see if any of the eleven solutions stand out by having a low total lead time of uncovered demand and low inventory cost.

5.2 Investigating which value of P has the lowest lead time for uncovered demand

After solving the facility location for P1 to P11 and finding what location to use as semi-central warehouses we do not know how many semi-central locations would be the optimal number to open. Therefore, we created a way to calculate the total lead time corresponding to uncovered demand at a given service level. Using this we can see if a different number of semi-central locations has an impact on the total lead time for the uncovered demand.

We want to investigate the lead time for uncovered demand for the customers, more specifically the total lead time of items that are out of stock. We only consider the items that get stocked out at each location and the items and deliveries that occur on a normal basis are not part of this analysis. Therefore, we only look at the lead time for uncovered demand for the different facility location compositions we found in chapter 5.1.

In chapter 5.1 we used a P-MP model to decide what locations to establish as semi-central warehouses while having Søvik as the main warehouse. Even though our analysis does not consider the day-to-day normal deliveries from the main warehouse at Søvik, it is still considered in the next calculations where we see how much lead time for the uncovered demand is represented in the different solutions. Because Søvik has frequent day-to-day deliveries it also has a safety stock related to these deliveries.

This leads Søvik to have extra stock available as its safety stock is calculated based on the total demand for all of Mørenot. This gives Søvik the possibility to use this safety stock (which is in the same warehouse and therefore has essentially no lead time) related to the normal deliveries when a stockout situation occurs. This gives a considerably smaller lead time compared to if Søvik must order and wait for an express delivery from their suppliers in Asia and China in particular.

We assume that Søvik does not have direct deliveries to local warehouses when a stockout occurs, and that all uncovered demand must go through the previous step in the chain (first through a semi-central warehouse then Søvik main warehouse). The local warehouse that is assigned by the P-MP model to be served by a semi-central warehouse must be covered only by that semi-central warehouse during a stockout and not get deliveries from Søvik directly (unless it is served by Søvik). This is due to the idea of minimizing the waiting time for the customer when a stockout occurs. For normal deliveries for meeting demand, direct deliveries are commonplace.

We have a hypothesis on how the different number of semi-central warehouses will affect the total lead time for uncovered demand in the system. We estimated that this value would be relatively high with P=1 as shown in the left chart in Figure 8 and start to fall as the number of semi-central warehouses increases, but only to a point. It is this point we want to find where the total stockout lead times are the shortest.

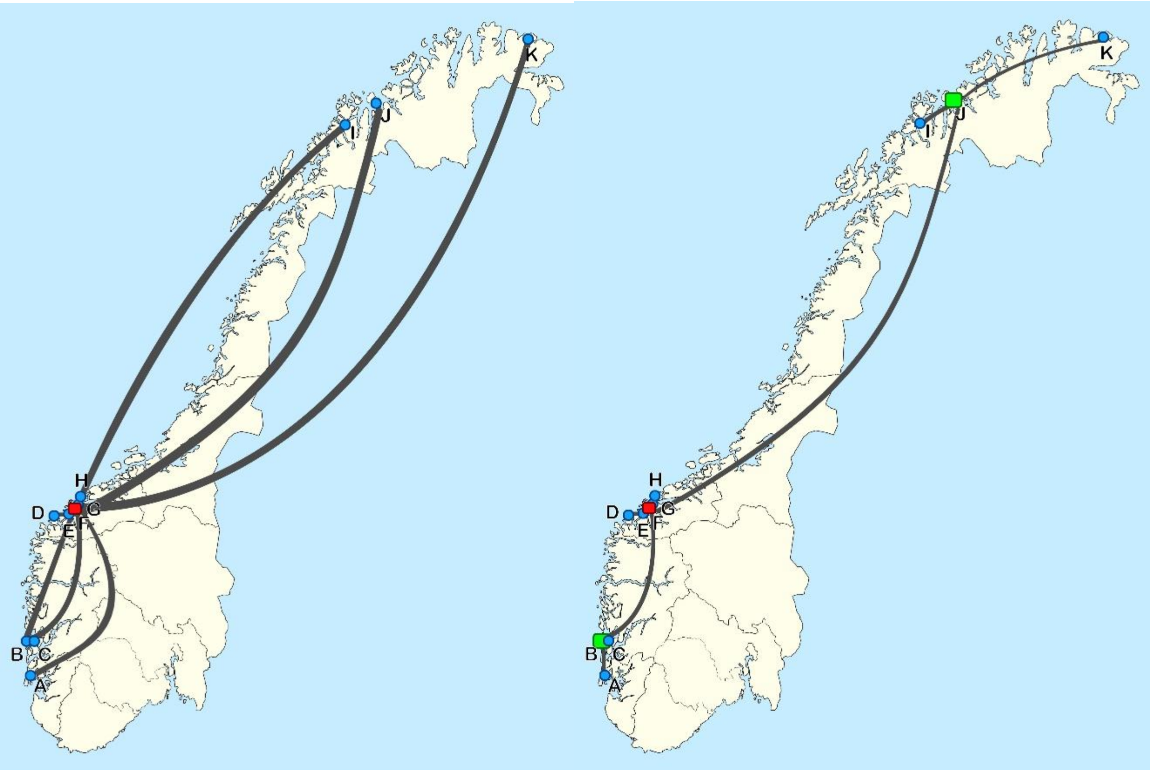

Figure 8 Transportation distances with zero semi-central warehouses vs 2 semi-central warehouses (green) and a main warehouse (red).

The benefits of adding semi-central warehouses can be seen in Figure 8. One can imagine that the gray lines represent transport distances between the locations, and in the right image the sum of the distances the gray lines are covering is quite a lot smaller than in the left image.

While the distance may be longer for an item going from a local warehouse back to Søvik in the right picture (if a stockout occurs at the semi-central warehouse), this scenario rarely happens due to the safety stocks at the semi-central warehouses, which deliver most of the uncovered demand. In the case of the left picture, the long way to Søvik occurs every time there is a stockout at the local warehouse, making it a more common scenario.

At some point we would expect the values to start increasing again, because if we think of the case P=11 where every single location acts as a semi-central warehouse, then we will essentially be back at the start where there were none. Since every single warehouse only serves itself, the benefits of being served by another semi-central warehouse are lost. All the locations get their stock from Søvik, just as in the case of P=1.

We figured out that the total lead time for uncovered demand could be graphed as a type of parabolic curve, and the value would start to fall as the number of semi-central locations increases, up to a point. However, at one value of P we would expect the lead time to begin to increase again. This is natural as if we think logically about the problem there is not really any difference between P1 and P11.

In P1 Søvik serves all warehouses and there is only Søvik that is chosen as a semi-central warehouse. However, in the case of P11 all the locations are semi-central warehouses and gets all its stock from Søvik, just as in the case of P1. The main difference between P1 and P11 would be that in P11 it can be assumed that all warehouses have a higher service level as they all operate as semi-central warehouse. Then our job is to find out at what value of P is the total lead time of uncovered demand is minimized (bottom of this curve). In Figure 11 one can see the output graph of the calculations we did, confirming this theory.

5.2.1 How we calculate the total lead time for uncovered demand given a service level

In a system where customers purchase products from a location with a warehouse, it is expected that the customer’s demand will be covered as much as possible, up to a point. This is referred to as the service level, as described earlier in chapter 2.6.2.

If we use a 95% service level, it means that a warehouse should be able to deliver 95% of a customer’s demand from stock in a P2 service level situation. Or in other words, 5% of items a customer wishes to purchase is not in stock.

As it is a common industry standard, we have continued to work with a service level of 95% in our calculations, but as our spreadsheet setup is made to be as dynamic as possible, we can easily change this value for any other service level we wish to look at for comparison.

In a system where we have the local warehouses, semi-central warehouses, and the main warehouse at Søvik we have three main layers to work with when calculating the total value of the lead times of uncovered demand in the system. From the point of view of a local warehouse that does not serve any other warehouses, they simply have to worry about meeting 95% of the demand of its customers. We can then assume that 5% of the total demand of any local warehouse is what the next level back in the chain will have to cover.

It is important to mention that the only lead time we look at is the total lead time for the uncovered demand. We are looking specifically at the lead time that comes from stockouts at the locations. The normal lead time from normal stock replenishments of goods for all locations is always present but is not part of our calculations. We focus strictly on the extra lead time that will be added if any one item is not in stock at the time a customer wants it, as this is the lead time that a customer will experience directly. We also use just the transportation times to calculate the lead time for the uncovered demand, and do not include extra time such as administrative time spent placing an order.

Calculating the lead time value in this fashion means that items with larger sales volumes will get a much higher cost than others. Something small like fishing hooks or knives that have large sales numbers will get a higher value than something large like a gillnet even though you can fit many units of a smaller item on one truck for transport. So, one might argue that 10 000 fishing hooks will take as much time in transport as a gillnet.

However, it is important to remember that when thinking about lead time from service levels we are looking at the situation that arises when the specific item a customer wants is out of stock. If fishing hooks suddenly are out of stock when a customer needs it, the transportation time for that one single fishing hook will be just as long as for the gillnet.

Let us consider an example where Tromsø is selected as a semi central warehouse and serves two other local warehouses: Skjervøy and Båtsfjord. In this case (95% P2 service level), the total demand for Tromsø is 5% of the demand from Skjervøy and 5% of the demand from Båtsfjord in addition to its own demand from its customers. This is because the location chosen as semi-central warehouses are not new warehouses that get established, but rather an expansion of the already existing ones, so if Tromsø is selected as a semi-central warehouse it would simply be expanded to some degree. This is explained in Figure 9 below.

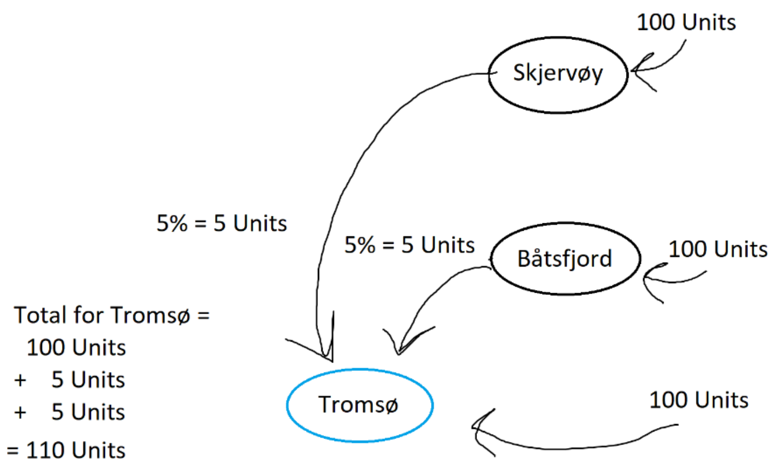

Figure 9 Explanation of quantities for a semi-central warehouse.

Here we see all three locations have a demand of one hundred units of the same item, but since Tromsø acts as a semi-central warehouse which serves the two others at Skjervøy and Båtsfjord, it will get 5% of their uncovered demand as well, for a total of 110 units in this case.

This also goes one step further back (to the main warehouse of Søvik) since all links eventually end back at Søvik as it is the main warehouse that all others are served from. If we take an example of the entire system with P=3 where we have two semi-central warehouses together with Søvik it would look something like Figure 10.

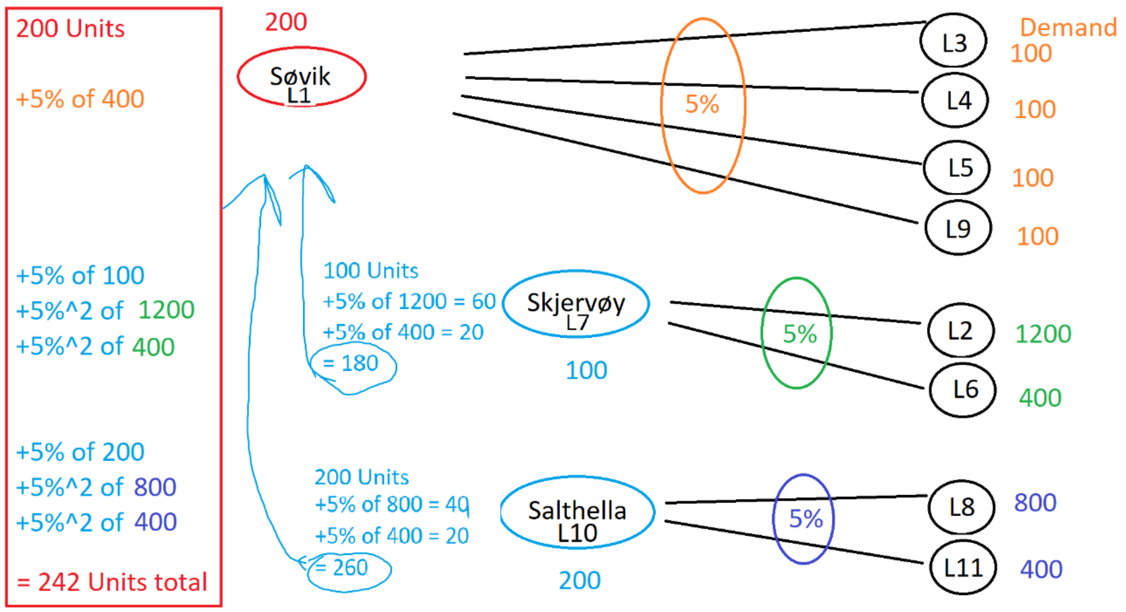

Figure 10 Service level and quantities for the P3 situation where Søvik is the main warehouse and Skjervøy and Salthella are semi-central warehouses.

As we see in Figure 10 it is important to keep track of the uncovered demand that is added to each location as we go through the "layers" as only 5% of the previous amount is passed along. The demand for each location is shown on the far-right side, as well as the demands for Søvik, Skjervøy and Salthella being 200, 100 and 200 respectively.

Moving from the right to the left we can see that 5% of the total demand for the locations on the right side is "passed along" to the three semi-central warehouses. For instance, 5% of the demand from L2 of 1200 units is passed along to Skjervøy (L7) leaving it with 60 units from L2 along with the other demand it must cover from L6 as well as its own demand. We again take 5% of these 60 units from L2 and are left with 3 units of the demand from L2 that Søvik (L1) must cover. This leaves us with Søvik having a total of 42 extra units of demand to cover, compared to their original demand of 200, giving them a total of 242 units.

Most of the stockout items in Søvik are delivered from the main warehouse safety stock, although this stock is also susceptible to being stocked out. These items must either be express delivered from their suppliers in Asia, wait until the next normal delivery or the customer will need to procure the item elsewhere. Despite this we have included in the calculations that 95% of the demand in Søvik is out of stock and is delivered from Asia.

Later calculations show that we get equivalent results regardless of the number of items shipped from Asia.

It is important to keep track of how much total demand each location is responsible for. By starting with the demand for every single location/warehouse we can extract the remainder of the service level from them individually and track them through the system. For instance, a local warehouse that does not serve any other will have 5% of its original demand served by its semi-central warehouse, and 5% of that again will be served by the main warehouse at Søvik. This means Søvik is responsible for 5%^2 = 0,0025 = 0,25% of the original demands at the local warehouses in addition to the 5% of demands at the semi-central warehouses.

5.2.2 Calculating the total lead time for uncovered demand for Mørenot

By adding up the correct amounts of demand at the locations, we can obtain a value for the total lead time of uncovered demand in the system overall. This was done by taking the total quantity of the demand on a link between two locations and multiplying it with the transportation time between them. This would give us a crude value of the lead time for the uncovered demand.

Let's consider an example where the local warehouse Tromsø is to be served by the semi-central warehouse at Skjervøy. The demand that Skjervøy would be responsible for covering would be the 5% of demand from Tromsø which is uncovered, and this would have to be transported on the link between the two.

If Tromsø has a total demand of 700 000 units, Skjervøy would have to cover 0,05 * 700 000 = 35 000 of these. This is then multiplied by the transportation time (as shown in Chapter 4.3.2 above) to give a lead time value of 35 000 units * 3,70 hours = 129 500 lead time hours.

We have also included a last link representing the lead time cost of having empty stock at the main warehouse in Søvik. Assuming imports from Asia we have added a lead time of 1080 hours (45 days), which will make any items covered by Søvik's 95% service level get a relatively high value, even with few items. The lead time of 45 days is the average time it takes to send a shipment by sea from China to Norway (Shipfreight, 2023).

This calculation is done for all links connecting all the locations in the system and a total sum of lead time hours of uncovered demand is obtained. This calculation is then done for the eleven solutions found by the facility location model in chapter 5.1.

We use this total lead time of uncovered demand value as a way to compare the different combinations of semi-central warehouses and locations they serve as the value of P changes. By plotting the values from all eleven possibilities, we were able to find which value of P was the best performer overall.

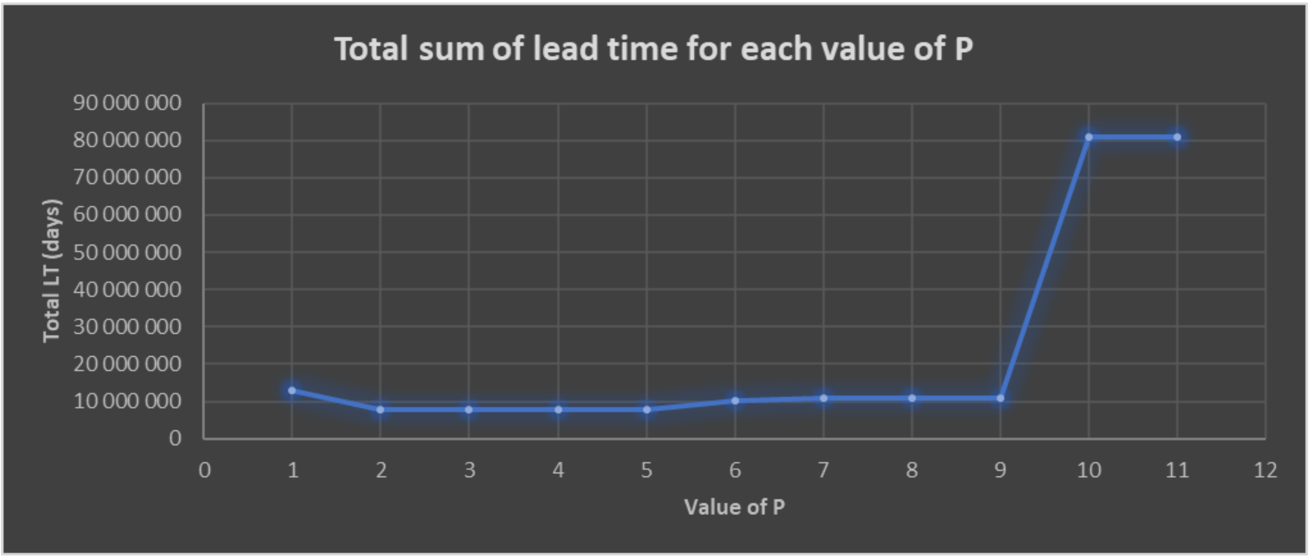

Figure 11 Total sum of lead time of uncovered demand for each value of P

Figure 11 shows how the total values change as P changes. The graph shows a steady pattern from P=1 up to P=9 with P=2 to P=5 being very close to each other. There is a large jump when going from P=9 to P=10 and P=11. This happens because P=10 is the first time the model made Søvik not serve any other locations, only the other semi-central warehouses.

Table 15 shows the exact values which makes it clear that P=2, P=3, P=4 and P=5 is very close to each other with P=4 having the lowest total uncovered lead time by a small margin.

Table 15 Total sum of lead time for uncovered demand for each value of P at 95% service level

P	Total LT (days)
1	12 819 219
2	7 863 102
3	7 949 563
4	**7 689 181**
5	7 898 177
6	10 318 539
7	10 732 160
8	10 734 843
9	10 897 950
10	80 986 676
11	81 042 852

The small lead by P=4 tells us that there may be good reasons to pick another value of P if one takes other factors into account such as inventory management related costs. For instance, choosing P=2 over P=4 will only increase the lead time value by 2,3%, but this may have other benefits due to only having to expand capacity at two locations rather than four. We also tried using other values for the service level ranging from 75% to 99,9%. While the total lead time for the uncovered demand changed, the lowest value was still for the P4 solution and the P2-P5 solutions were still fairly low with values around the same area as P4.

5.2.3 Comments about Søvik as main warehouse with different service levels

In the calculations done up to this point, we used a service level of 95% for all the locations. One interesting thing to change is the service level for Søvik. It is their main warehouse and as mentioned earlier it is assumed they have a relatively large safety stock at their disposal so a service level of 95% might be low. By changing the service level in our calculations for Søvik we can check if a higher service level changes what facility location solutions have smaller lead times for uncovered demand. If we use a service level of 99%, 99,9% and 100% at Søvik and keep a service level of 95% at the other locations, we get the following results:

Table 16 Lead time for uncovered demand at different service levels for Søvik.

P	99 %	99,9 %	100 %
1	2 619 669	**324 770**	**69 782**
2	2 017 047	701 685	555 533
3	**2 220 392**	931 328	788 099
4	2 211 751	979 329	842 393
5	2 307 531	1 049 635	909 869
6	3 292 128	1 711 186	1 535 526
7	3 454 009	1 816 425	1 634 471
8	3 456 693	1 819 109	1 637 155
9	3 507 841	1 845 066	1 660 314
10	17 507 503	3 223 304	1 636 170
11	17 557 522	3 273 322	1 686 189

In Table 16 we can see that with a service level of 99% it is still P1 to P5 that are among the lowest values, the same as using 95% service level. However, if we increase the service level to 99,9% we can see that P1 gets a significantly lower value. It is natural to have fewer semi-central warehouses when the service level at Søvik increases. We also included 100% service level, although this is only possible in theory as the safety stock would be incredibly large. It is included to show how the lead times for the uncovered demand would be if we had no stockouts at their main warehouse in Søvik and no express orders would have to be delivered from Asia.

Table 17 Total lead time of uncovered demand with service level of 99% at all locations

P	Total LT (days)
1	523 934
2	**393 173**
3	433 390
4	431 094
5	450 998
6	657 406
7	690 349
8	690 886
9	701 549
10	14 719 294
11	14 729 563

In Table 17 we have used a service level of 99% at all the nodes to calculate the total lead time of the uncovered demand. Here we can see we end up with the same results as in the previous tests where the P1 to P5 solutions come out with a relatively low total lead time.

From Table 16 and Table 17 we can see that for up to 99% service level it is the same P solutions that have a low value, and these five solutions will be investigated further with respects to inventory costs at different service levels in the next chapter.

In chapter 5.3 we will look at the inventory costs for the different facility location distributions found in chapter 5.1. Here we will include penalty costs in terms of transportation costs to perform an express delivery for the items that are out of stock. We can then compare the different costs for the eleven different solutions and use the three methods (facility location, total lead time of uncovered demand and inventory costs) to give recommendations to Mørenot about how many semi-central locations they should open and what warehouses they serve.

5.3 Inventory management

After finding the minimum lead times of uncovered demand we saw that multiple of the facility location solutions gave quite similar values. Therefore, we wanted to investigate if there are any other possibilities to decide which of the first six options (P1 to P6) are the best.

One way of doing this is by using inventory theory and looking at the different costs associated with holding inventory (Silver et al., 2016). There are four main costs associated with holding stock. The setup cost, holding cost, safety stock cost and stockout costs. These four elements summed together represent the total relevant cost or TRC for short.

We want to investigate the four solutions found in chapter 5.2 where we calculated the lead time for uncovered demand. As these four solutions P2, P3, P4 and P5 gave quite similar total values we want to see if there are any bigger differences in the costs. We also wanted to include the case of P1 and P6 to compare it to our solutions for completeness.

To accomplish this, we could not use each item individually as there were too many items and many of them were not sold in more than one week at a location during the year. Therefore, with lots of periods with no sales and a few periods with large sales, the standard deviations would become quite large, resulting in inaccurate conclusions.

Therefore, we transformed the data into 16 aggregate items representing a typical product for each of the product categories. To get the demand for the aggregated items we took the average demand of the item for each week for each location. If an item is not sold in a location one week the demand for that item is set to zero so it will be included when calculating the standard deviation. The standard deviation is calculated for the 52 weeks of 2022. The item cost is calculated as a weighted average for the items in the category and for each location.

5.3.1 Total relevant cost formula

The cost elements are represented with the following parameters:
D is the demand at the location
Q is the optimal ordering size from the Wilson formula or EOQ formula: $\sqrt{\frac{2*D*A}{r*v}}$
A is the setup costs or the cost associated with doing one order.
r is the holding costs in percentage of the value for storing the item (often for one year)
v is the item value
k is the safety factor or safety level. This comes from normal distribution. If we have a safety level of 0,95 or 95% the k is 1,64. For 99% the k is 2,33 and so on.
σ_L Standard deviation of demand during lead time
B_1 Stockout cost per item short.
$P_{u\geq}(K)$ Probability of a stockout occurring at the given safety level with corresponding K

The setup cost is represented as: $\frac{D}{Q} * A$

The setup cost is how many orders one needs to satisfy the demand ($\frac{D}{Q}$), multiplied with the setup cost per order.

The holding cost is: $\frac{1}{2}Qvr$

The holding cost is the average stock level ($\frac{1}{2}Q$) multiplied with the cost of the item and internal interest rate.

The safety stock cost is: $k\sigma_L vr$

The safety stock cost is the safety factor multiplied with the standard deviation of demand during the lead time multiplied with the cost of holding one item in stock (vr).

The shortage cost or penalty cost of stockout is: $\frac{D}{Q}B_1 P_{u\geq}(k)$

This is the cost of stockout. It is the number of order cycles times the penalty cost times the chance of a stockout occurring.

Then the total relevant cost can be represented as:

$$TRC = \frac{D}{Q}A + \frac{1}{2}Qvr + k\sigma_L vr + \frac{D}{Q}B_1 P_{u\geq}(k)$$

5.3.2 Stockout Cost B_1

To calculate the total relevant costs, we need different penalty costs. We use a B_1 penalty cost which is a fixed cost if a stockout occurs. In our case, this represents the cost of an express delivery for one item that is out of stock. This means that the value of the B_1 cost can vary from within Norway and to Asia. Different values for B_1 will be explored and its effect on the solution.

Important to note that we are working with aggregated products, and therefore we do not have a value for the sizes/weights of the individual products. Therefore, it is challenging to figure out a proper way to calculate the transportation cost per (aggregated) item. To put this into perspective, a small item like a hook can be sent at low cost by mail, while bigger items might require more expensive specialized transportation.

To get the penalty cost we looked around at different Norwegian transport companies and their price lists. They were quite similar priced, and we just ended up choosing Bring and their prices to include. We got the transport cost in Norway from the Bring pricing table

(Bring, 2022). This table showed the cost of transport between zones (areas in Norway) and the weight of the item transported.

From this we simply picked one of the average weight and distance from the table which gave us a value of 603 NOK as the penalty cost for the semi-central and local warehouses. As we do not know the weight of the generalized products, we are using this as an estimate. Later testing showed that although the penalty cost affected the TRC, the solution in terms of which of the six P1, P2, P3, P4, P5 and P6 (amount of semi-central warehouses) were cheapest remained unchanged.

The transport cost from Norway to Asia is much more difficult to find for one item. As most of the freight is transported in shipping containers and not as a single item in one container (if size allows it). Using the Freightos Baltic Index (Freightos, 2023) which is an index showing how much container freight costs to and from different parts of the world based on real life, live transport costs from many different carriers. This gives us the best possible estimate for the cost of transporting a container from Norway to Asia. As of 02.05.23 the cost of transporting a container is 1399 USD for a 40-foot shipping container (About half the price for a standard 20-foot TEU).

We experimented with different costs ranging from 200 to 7500 NOK in penalty cost with no significant changes to which value of P was the cheapest option. Using this information, we settled on a price of 750 NOK for one item in penalty cost as we assume there will be transported multiple items in one container. The results of the changes in penalty cost can be seen in Figure 14.

5.3.3 Optimal order size and service level

One can find the optimal combination of the order size (Q) and the safety factor (k) by iteratively using the formulas below and solving for Q and k until they are no longer changing (Silver et al., 2016).

$$Q = Q_w \sqrt{1 + \frac{B_1}{A} P_{u\geq}(k)}$$

Where Q_w is the original Q value (optimal order size) from the standard EOQ formula, B_1 is the penalty cost per stockout and A is the setup cost. $P_{u\geq}(k)$ is the probability of a stockout occurring in each order cycle (this value is updated with a new k value from the next formula).

$$k = \sqrt{2 \ln \left(\frac{DB_1}{\sqrt{\pi} Q \sigma_L v r}\right)}$$

Where D is the demand, B_1 is the penalty cost for a stockout occurring, Q is the output from the previous formula, σ_L is the standard deviation of the demand during lead time, v is the item cost and r is the interest rate.

This iterative process can also be done automatically in excel using solver and minimizing total cost (TRC) and having the safety factor (k) as a variable in a model. Given this the Q also updates in the spreadsheet, and we are left with the optimal value of Q, k and TRC.

5.3.4 Other calculations and assumptions

We need to take account of the demand that is uncovered in one location and move the uncovered demand to the location that it is served by. This means that uncovered demand in a local warehouse is moved to the semi-central warehouse, and the uncovered demand at the semi-central warehouse is moved to the main warehouse in Søvik.

The number of units that will not be covered directly from stock for each order cycle can be calculated using the formula:

$$\sigma_L G_u(k)$$

This is the number of units short at a location in a cycle (Silver et al., 2016). σ_L is the standard deviation of demand during the lead time, and $G_u(k)$ is the standard normal loss function. Its value is found in the normal distribution table and is defined for each value of k, which in turn comes from the service level.

Since we operate with costs per week (and the order cycles are not fixed to a week), we will have to adjust this value by dividing it by the order cycle. This is obtained by dividing the order size Q by the demand D.

$$\frac{\sigma_L G_u(k)}{\frac{Q}{D}}$$

This leaves us with the number of units short per week. This is the number we add to the demand for the supply location. We update the demands for each aggregated product at every location. This means that by changing the service level (k) the input values such as demand, Q and σ_L changes. These new values will then be used to calculate the total relevant costs for each of the different solutions found in chapter 5.1.

5.3.5 Inventory management results

After setting up the excel sheet to be dynamic such that it would update all affected values automatically, the first thing we did was to force the service levels for all the locations to be 90%, 95% and 95%. We chose these values as these are some of the more common service levels companies try to operate at. The outputs were as follows:

			0,9			
	Setup	Holding	Safety stock	Stockout	Total/week	Total Year
P1	19 793	19 793	31 869	1 792	73 247,0	3 808 842
P2	19 745	19 745	30 129	1 786	71 405,6	3 713 091
P3	19 731	19 731	29 425	1 759	70 645,3	3 673 555
P4	19 748	19 748	29 123	1 785	70 404,8	3 661 049
P5	**19 835**	**19 835**	**28 230**	**1 793**	**69 693,6**	**3 624 068**
P6	19 800	19 800	30 862	1 791	72 253,0	3 757 156
			0,95			
	Setup	Holding	Safety stock	Stockout	Total/week	Total Year
P1	19 611	19 611	38 763	886	78 872,1	4 101 349
P2	19 586	19 586	37 056	885	77 112,6	4 009 855
P3	19 578	19 578	36 333	872	76 360,8	3 970 764
P4	19 591	19 591	36 142	885	76 209,1	3 962 872
P5	**19 648**	**19 648**	**34 981**	**887**	**75 164,3**	**3 908 542**
P6	19 618	19 618	37 849	886	77 971,2	4 054 501
			0,99			
	Setup	Holding	Safety stock	Stockout	Total/week	Total Year
P1	19 465	19 465	52 820	176	91 925,3	4 780 117
P2	19 460	19 460	50 908	176	90 003,4	4 680 177
P3	19 458	19 458	50 055	173	89 145,0	4 635 538
P4	19 463	19 463	49 975	176	89 076,6	4 631 982
P5	**19 481**	**19 481**	**48 313**	**176**	**87 450,3**	**4 547 414**
P6	19 469	19 469	51 886	176	90 998,6	4 731 929

Figure 12 Forced service level of 90%, 95% and 99% for all locations with a B1 penalty cost of 750.

In Figure 12 we can see the solutions. In the first table we can see the setup, holding, safety stock and stockout costs for P1 to P6, with the lowest total cost in bold font.

Note that the setup and holding costs are the same. This is because we used the EOQ value, when in fact the optimal Q value is calculated from the iterative process described in chapter 5.3.3 using the formula: $Q = Q_w\sqrt{1 + \frac{B_1}{A}P_{u\geq}(k)}$. We chose to omit these calculations because when the service level is high (low $P_{u\geq}(k)$ value), and the ratio of the penalty cost B_1 to setup cost A is low, the root expression would have a value close to 1 and the optimal Q value would be close to the value of Q from the EOQ formula.

We can also see the total cost for a week and a year (total cost * 52). Based on this we can see that if we force the service level to be 0,9, 0,95 or 0,99 we see that P5 is the cheapest for all the three service levels tested.

This also shows the exponential growth in costs that occurs as one increases the service level. This is shown in more detail for the P4 case in Figure 13. Note that the increased total cost comes from the significant increase in safety stock that is required to satisfy the high service level.

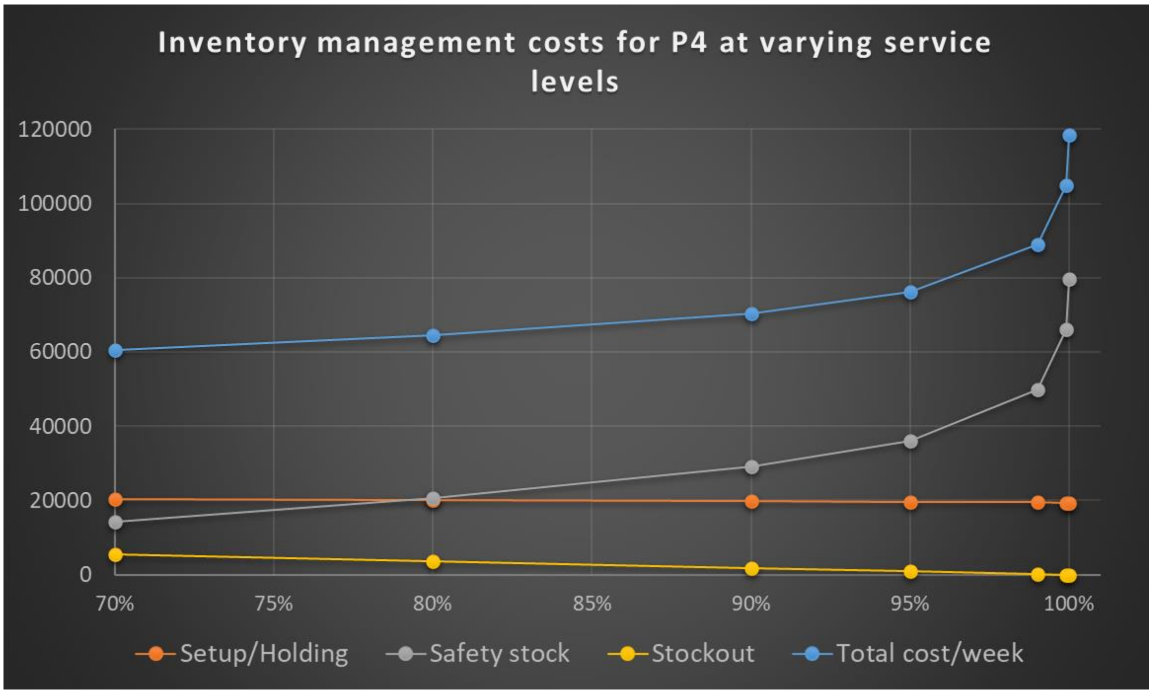

Figure 13 Inventory management costs for P4 at varying service levels

This graph shows the relationship between the different inventory costs and how the service level affects them. Note that as the service level increases towards 100% the stockout costs go towards zero while safety stock goes towards infinity. At 100% service level there would in theory be infinite safety stock and no stockouts.

5.3.6 Optimal service level to minimize cost.

We can take this one step further by using a simple model and solver in excel to choose the optimal service level at each individual location to minimize cost. This can be done for each of the six values of P, and we end up with the minimum cost when only changing the service level. Normally, an iterative process is used to find the optimal values for k and Q as described in chapter 5.3.3. In our case, we let the model choose the optimal service level and k values for each location. This in turn results in new values for Q which are close to optimal for the given k. This means that the model will automatically find the optimal combination of values for service level for all locations with the given input data.

Figure 14 shows the optimal costs for varying B1 penalty costs at Søvik. When comparing this output with Table 15 we can see that we get similar results where the costs are falling from P1 to P5 before starting to increase again.

Optimal values B1=200						
	Setup	Holding	Safety stock	Stockout	Total/week	Total Year
P1	20 044	20 044	4 180	2 756	47 023,4	2 445 219
P2	19 895	19 895	4 040	2 317	46 146,3	2 399 606
P3	19 839	19 839	3 924	2 103	45 705,8	2 376 699
P4	19 835	19 835	3 994	2 081	45 744,7	2 378 723
P5	**19 778**	**19 778**	**3 900**	**1 596**	**45 051,0**	**2 342 653**
P6	19 955	19 955	4 338	2 401	46 649,3	2 425 763

Optimal values B1=750						
	Setup	Holding	Safety stock	Stockout	Total/week	Total Year
P1	19 665	19 665	6 871	3 042	49 242,8	2 560 626
P2	19 842	19 842	4 313	3 797	47 794,3	2 485 304
P3	19 795	19 795	4 149	3 591	47 329,7	2 461 144
P4	19 796	19 796	4 197	3 567	47 354,9	2 462 456
P5	**19 747**	**19 747**	**4 042**	**3 096**	**46 632,2**	**2 424 874**
P6	19 925	19 925	4 465	3 987	48 301,7	2 511 689

Optimal values B1=2000						
	Setup	Holding	Safety stock	Stockout	Total/week	Total Year
P1	19 859	19 859	5 253	7 568	52 538,0	2 731 977
P2	19 760	19 760	4 796	7 161	51 478,2	2 676 866
P3	19 725	19 725	4 551	6 968	50 968,1	2 650 339
P4	19 733	19 733	4 569	6 934	50 967,8	2 650 328
P5	**19 699**	**19 699**	**4 298**	**6 494**	**50 190,3**	**2 609 893**
P6	19 832	19 832	5 032	7 339	52 034,0	2 705 770

Optimal values B1=7500						
	Setup	Holding	Safety stock	Stockout	Total/week	Total Year
P1	19 653	19 653	6 962	22 314	68 581,1	3 566 216
P2	19 611	19 611	6 009	21 956	67 186,8	3 493 715
P3	19 597	19 597	5 572	21 783	66 548,8	3 460 538
P4	19 619	19 619	5 497	21 748	66 482,2	3 457 075
P5	**19 610**	**19 610**	**4 972**	**21 357**	**65 548,3**	**3 408 512**
P6	19 660	19 660	6 460	22 101	67 881,1	3 529 819

Figure 14 Minimum cost for different penalty costs using the service level as a variable.

The difference between the four tables is the B1 stockout cost for the warehouse at Søvik. While we have good estimates for the stockout costs at locations that have supply warehouses in Norway, it is not as clear how this would be calculated for stockouts at Søvik. As it was not clear what this cost should be, we experimented with different values (200, 750, 2000 and 7500 NOK) and found that while the costs increased overall with an increase in this value, the distribution of what facility location system (value of P) is the best one relative to the others, did not.

In Figure 15 we can see the different input values we tried, and the costs of the different solutions summarized in a graph. The first thing that is evident is that it is much cheaper overall to use the optimal service levels (last four clusters) no matter the P1 to P6 facility location. The next thing to notice is that within each cluster the P5 solution comes out best or just a few NOK more expensive than the second best option. We can also see that each cluster acts similarly to the graph in Figure 11 where the value for lead time for uncovered demand is falling to P5 and then starts to climb again in P6 showing a correlation between lead times and costs.

We can also see in the four rightmost clusters that changing the B1 penalty cost for Søvik does not change what facility location distribution is the best (P5).

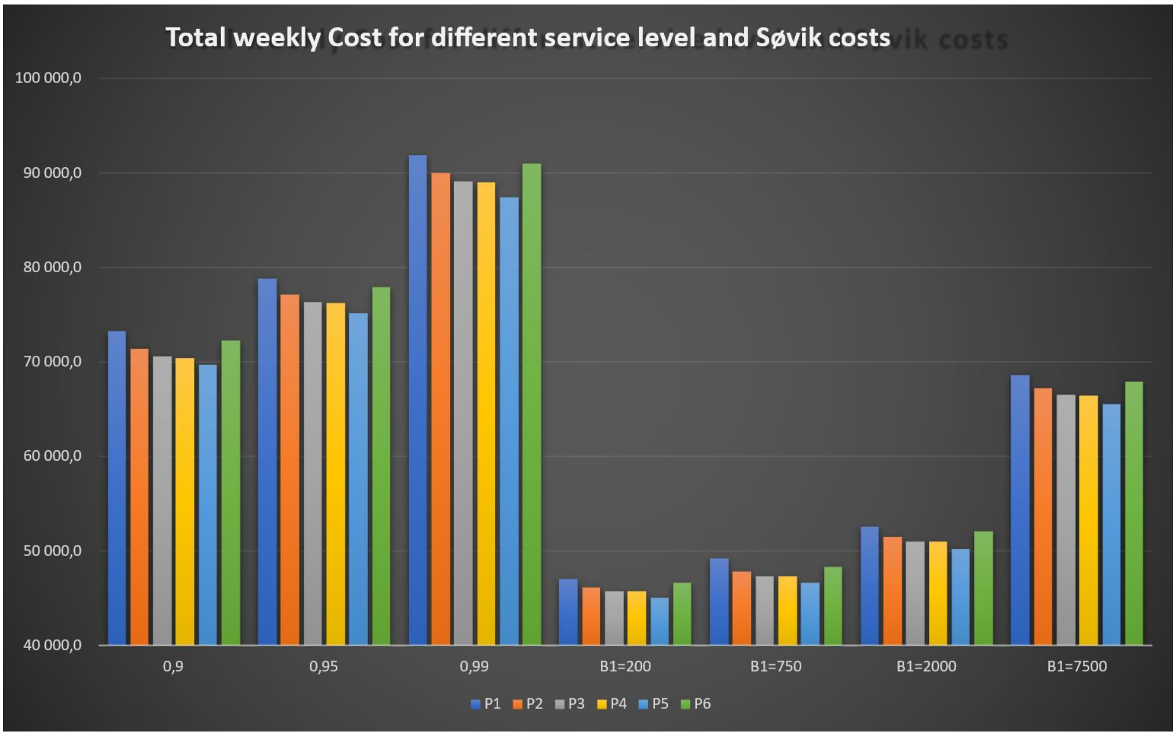

Figure 15 Costs for different service level and penalty costs

Note that the Y-axis starts at a value of 40 000 to emphasize the differences between the different P-values, as the differences are only a few percent.

In Table 18 we can see the differences in cost in detail for when the B_1 penalty cost is 750. While there are differences and P5 is indeed the cheapest option, there is only a 5,6% increase in costs by going with the most expensive option of P1. This suggests that much like with our findings regarding the lead time for uncovered demand, there is not necessarily a large variation in costs between the different alternatives we have investigated, and the optimal value of P is rather sensitive. As a result, introducing additional factors for comparison could shift the optimal value for P quite easily due to its sensitivity.

Table 18 Difference in costs for the P values (B_1=750)

P value	Total/week	Cost difference compared to the optimal value (P5)
P1	49 242,8	5,6 %
P2	47 794,3	2,5 %
P3	47 329,7	1,5 %
P4	47 354,9	1,5 %
P5	**46 632,2**	**0,0 %**
P6	48 301,7	3,6 %

Table 19 the service level for each location in the P5 solution is shown. Note that the service level is quite similar for stockout penalty costs of 750 and 2000 at Søvik. The penalty cost for the other locations is set at 603 as explained in chapter 5.3.2.

Table 19 Service levels for different B1 costs for P5

Location	Stockout cost B_1=750	Stockout cost B_1=2000
Søvik	50,0 %	50,0 %
Harøy	95,6 %	96,0 %
Hildre	99,5 %	99,6 %
Båtsfjord	94,1 %	95,4 %
Gangstøvika	96,2 %	96,7 %
Fosnavåg	94,8 %	94,8 %
Skjervøy	83,7 %	87,5 %
Tromsø	90,0 %	90,1 %
Bekkjarvik	87,8 %	89,7 %
Salthella	98,8 %	98,8 %
Avaldsnes	89,0 %	89,1 %

There is a slight increase in service level for all locations when changing the B1 cost at Søvik from 750 to 2000. The higher B1 cost means it is more expensive to have a stockout at Søvik. This means it is cheaper to have fewer units of stock being transferred back to the main warehouse.

It is also notable that there is such a low service level of 50% at Søvik. This comes from the fact that keeping a safety stock is more expensive than getting a stockout (explained in detail in chapter 6.3).

Finally, we can see that the different service levels do not change much when B1 is changing. This tells us that the individual service levels at each location are not very sensitive to changes in the B_1 stockout cost.

Table 20 Service levels at various locations for P1 to P6. B1=750 at Søvik, 603 elsewhere.

	P1	P2	P3	P4	P5	P6
Søvik	50,0 %	50,0 %	50,0 %	50,0 %	50,0 %	50,0 %
Harøy	93,4 %	93,6 %	95,6 %	93,8 %	95,6 %	95,6 %
Hildre	95,6 %	95,6 %	99,6 %	79,2 %	99,5 %	99,5 %
Gangstøvika	96,1 %	96,2 %	96,3 %	96,3 %	94,1 %	94,1 %
Fosnavåg	86,5 %	87,3 %	94,8 %	99,2 %	96,2 %	96,2 %
Skjervøy	99,7 %	98,8 %	84,0 %	83,5 %	94,8 %	94,8 %
Båtsfjord	75,1 %	75,3 %	94,2 %	94,2 %	83,7 %	75,8 %
Tromsø	70,8 %	68,6 %	90,0 %	90,1 %	90,0 %	83,3 %
Salthella	82,6 %	83,3 %	75,6 %	75,0 %	87,8 %	75,5 %
Bekkjarvik	93,9 %	94,2 %	98,3 %	98,4 %	98,8 %	98,3 %
Avaldsnes	76,0 %	90,0 %	87,7 %	87,3 %	89,0 %	87,4 %

In Table 20 we can see the different optimal service levels for P1 to P6 to minimize the total relevant costs. Here we can see that for all the locations (except Søvik) most of the service levels are in the 75-96% range.

6.0 Results and discussion

Throughout this thesis we first solved a facility location P-MP model to find out what locations should be established as semi-central warehouses. Using this information, we calculated the lead time for the uncovered demand and the total relevant costs for holding inventory. With the results given from these two calculations we could see that a pattern emerged in terms of how many semi-central locations should be established.

6.1 P-MP model results

After solving the P-MP model we were left with eleven different solutions where we had from one to eleven semi-central warehouses. Each of these eleven solutions are solved to optimality for the different values of P.

If we look at the map of the possible geographical locations of the warehouses in Figure 6, we can see that there are some natural clusters. For choosing three semi-central warehouses it intuitively makes sense to have one cluster in the south, one around Søvik, and one up north assuming evenly distributed demand across Norway.

When we solved the P-MP model for P3 the model selected Søvik along with Skjervøy and Salthella as semi central warehouses. If we look at their location in combination with the warehouses they serve from Table 13, we can see that Skjervøy serves the northern warehouses, Salthella the southern while Søvik serves the rest.

If they were to open only one semi-central warehouse it should be opened in Søvik. This is because it is forced to be chosen by the model. Søvik will be used as one of the locations in all the solutions as it is the main warehouse. When we select two warehouses it should be located at Søvik and Skjervøy. The rest of the results is shown in addition to the warehouses they serve in Table 13.

In the solutions for P=4 to P=11 we can see that both Søvik and Gangstøvika is chosen among other locations. Even though Søvik and Gangstøvika are only located 50 km away from each other they are chosen because the demand in the Ålesund area is so large that it needs several semi-central warehouses to minimize cost (weighted distance).

In order to investigate which value of P is the best fit for Mørenot's problem we need to put these eleven combinations in the context of lead time for uncovered demand as well as costs.

6.2 Lead time for uncovered demand

After we created the P-MP model we needed to create a way to calculate the total lead time for uncovered demand for the different solutions. This enabled us to see what combinations of semi-central warehouses had the lowest lead time for uncovered demand.

Initially, we used a service level of 95% for all the locations to calculate the total lead time for uncovered demand. This gave us a solution where P2 to P5 were within 3,5% from the lowest value which were at P4. The total uncovered lead time were significantly higher at P1 compared to P4 and from P6 to P9 it steadily increased indicating that the more semi-central warehouses they establish the more total lead time for uncovered demand the customers experience.

Note that when P = 10 Søvik is no longer serving any other locations except the semi-central location, and the value for lead time for uncovered demand seen in the graph in Figure 11 spikes. For P11 or eleven semi-central warehouses there are no warehouses that are served by someone else and therefore P11 and P1 (no semi-central warehouses) are the same in terms of facility location. Every location is served by the main warehouse at Søvik.

As we tested altering the value of the service level, we found it did not change what combination of locations resulted in the lowest total sum of uncovered demand lead time. We tried values of service level for all locations ranging from 75% to 99,9%. They all resulted in P4 having the lowest value, indicating that the service did not have a significant impact on our results. While the values increased or decreased for all combinations, the relative differences between them stayed almost the same and we ended up with graphs similar to Figure 11 for all the service level values we tried. This gave us confidence that establishing between 2 and 5 semi-central warehouses would result in the lowest value for lead time for uncovered demand in the system.

To have a service level of just 95% at the main warehouse might be low. Therefore, we tried changing the service level for Søvik. While still having a service level of 95% at all other locations, we changed the service level at Søvik from 95% to 99%, 99,9% and 100%. From the previous calculations we saw that most of the lead time was related to the main warehouse at Søvik, so by increasing the service level at their main stock we expect the total lead time of uncovered demand to fall quite a bit.

The results from these calculations, shown in Table 16, indicate that the higher the service level is at the main warehouse, the fewer semi-central locations are needed. However, it is important to note that an increase in service level only tells a part of the story as there might be extra costs associated with such a high service level and these must be balanced to find an optimal solution.

6.2.1 Assumptions for why we use individual items

One might argue that some products will be more important than others and that this should impact the calculations for the lead time values (such as penalty cost for having a stockout), but we have no data on the importance of different products, and whether someone is missing a small, but critical screw can be just as important as someone missing a full net if the result in both cases is that the boat is not able to go fishing at that time.

Finding such a value for the importance of a product would essentially be impossible, as the value of an item is a subjective opinion which will vary greatly depending on the situation of the customer. Each customer knows the value of not having each item (down time/penalty cost etc.) but Mørenot cannot calculate the value of each item they have for every customer. One item may be extremely important for a customer one day and almost worthless the next and different customers valuate products differently. Because of this we will have to assume that all products are as important as any other.

Since most of the transportation within Norway is done by trucks on roads, it would be beneficial to find out how many of an item fit on a truck.

To do this, however, we would need to know how much space every single individual item used, which could then be converted into a number representing truckloads. If they waited until a full truck was ready the total lead times for uncovered demand would also get higher as the item would have to wait for enough back ordered items to accumulate to be able to send a full truck. Therefore, we assume each backordered item would be sent as a single item as an express delivery.

6.3 Inventory costs

After we calculated the lead time for uncovered demand, we wanted to see how the different combinations of semi-central warehouses affected inventory costs. This could be compared to the lead time values calculated previously and would allow us to find a balance between service levels and inventory related costs.

After testing various inputs such as B1 costs and service level (shown in Figure 15) the P5 facility location solution always gave the lowest total relevant costs. We can see in Table 18 that all the solutions P1 to P6 have relatively equal costs with P5 having a slightly lower cost than the rest.

From the optimal service levels for the P5 solution we can see that all of the locations have a service level in the 88%-95% range with Søvik being the "odd one out" with just 50%. This is the lowest possible service level as this means there is a 50/50 chance to be in a stockout situation due to there not being any safety stock present.

The service level of 50% at Søvik can be explained by the term for safety stock in the TRC formula, there are four factors present. The standard deviation of demand during the lead time σ_L, item value v and the interest rate r are fixed, while the k value changes depending on the service level. Due to the large value of σ_L (long lead time from Søvik to Asia), a minor change in the k value can lead to a substantial change in safety stock costs. In the term for the stockout cost, the $\frac{D}{Q}B_1$ part is relatively small compared to $\sigma_L vr$.

When the service level is increased, k is increased and $P_{u\geq}(k)$ is decreased. k is increasing at a faster rate than $P_{u\geq}(k)$ is decreasing. As a result, when the service level is increased, the safety stock term will increase faster than the term for stockout costs will decrease. Because of this, the model will focus on minimizing the cost by decreasing the service level resulting in a low value for k. This explains the 50% service level at the main warehouse in Søvik.

The TRC formula can be used to calculate the cost, but also to figure out much more information about their inventory strategy. One could find information such as the number of units in safety stock, order cycles, reorder points, etc.

We omitted to calculating this as we have made a generalized product so such calculations would not give Mørenot any real use. In addition to this we saw that in their data systems and in power BI they had already set up calculations for such values with their current inventory plan.

6.4 Overall recommendations

Both the uncovered demand lead time and inventory costs showed a trend where the costs and lead times where lowest when establishing 2,3,4 or 5 semi-central warehouses. Four semi-central warehouses had the lowest total lead time for uncovered demand while five had the lowest inventory costs. This led to the conclusion that having two to five semi-central warehouses seems good as the costs and lead times only varied by a few percent between them.

From the warehouses geographical locations, we see that there are three natural clusters. From this information we can argue that they should at least establish three locations to ensure that not one of the transportation links (and the corresponding lead time) becomes long. This is what a P-CP model does where it minimizes the single largest distance.

Table 21 Semi-central locations for P3, P4 and P5

P3	P4	P5
Søvik	**Søvik**	**Søvik**
Fosnavåg	Harøy	Harøy
Harøy	Hildre	Hildre
Gangstøvika	**Gangstøvika**	**Båtsfjord**
Hildre	Fosnavåg	**Gangstøvika**
Skjervøy	**Skjervøy**	Fosnavåg
Båtsfjord	Båtsfjord	**Skjervøy**
Tromsø	Tromsø	Tromsø
Salthella	**Salthella**	**Bekkjarvik**
Bekkjarvik	Bekkjarvik	Salthella
Avaldsnes	Avaldsnes	Avaldsnes

In Table 21 we have our recommendation for where to locate the semi-central warehouses and what locations they serve. All these three solutions provided good results in terms of lead time for uncovered demand and inventory costs. We can see from Table 21 that when selecting three warehouses Søvik (their main warehouse) is selected and serves Fosnavåg, Harøy, Gangstøvika and Hildre. Skjervøy serves Båtsfjord and Tromsø while Salthella serves Bekkjarvik and Avaldsnes. The table also shows the results for four and five selected warehouses.

The locations and connections for P3 is shown on a map in Figure 8. Similar figures are shown below for P4 as well as P5 in Figure 16.

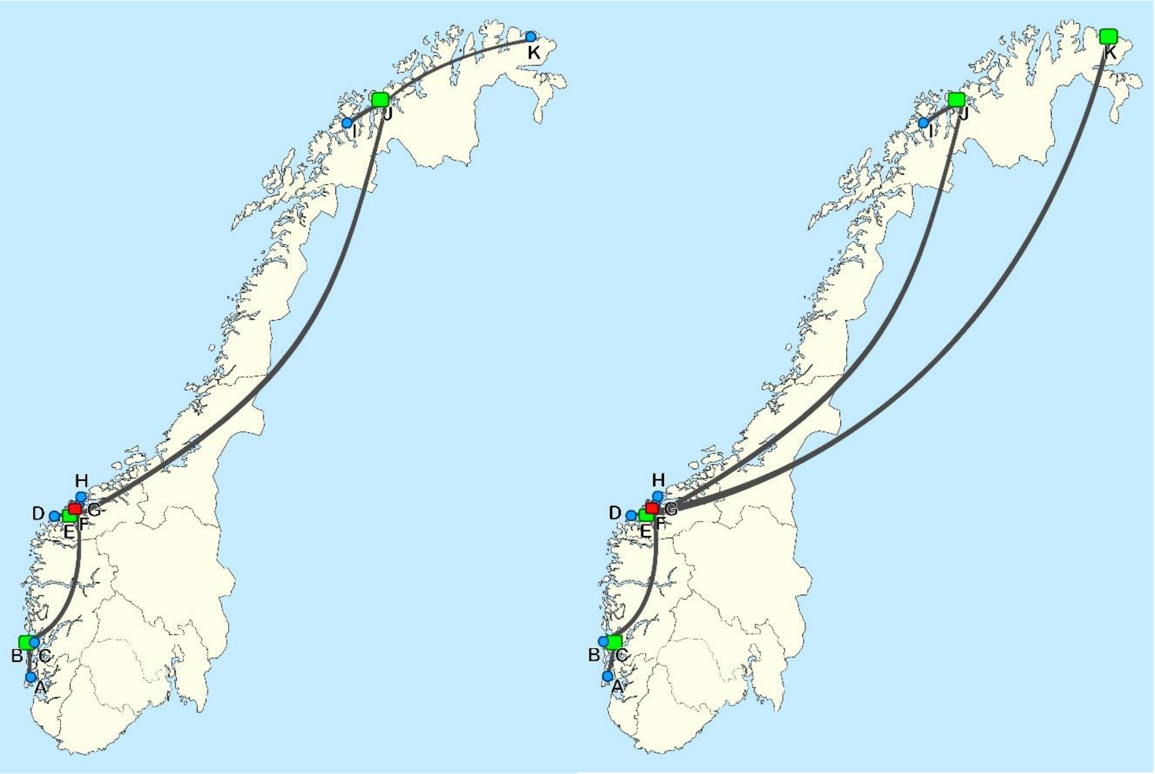

Figure 16 Visualization of transportation distances for P=4 (left) and P=5 (right). Main warehouse at Søvik in red, semi-central warehouses in green.

In Table 22 we have summarized the service level for all the locations for the different number of semi-central locations. The service levels are derived from the model where we minimized total relevant cost where the service level was the variable. We can see that the service levels in most cases are relatively similar for the different number of semi-central locations.

Table 22 Service levels for locations for P3, P4 and P5 with B1=750

	P3	P4	P5
Søvik	50,0 %	50,0 %	50,0 %
Harøy	95,6 %	93,8 %	95,6 %
Hildre	99,6 %	79,2 %	99,5 %
Gangstøvika	96,3 %	96,3 %	94,1 %
Fosnavåg	94,8 %	99,2 %	96,2 %
Skjervøy	84,0 %	83,5 %	94,8 %
Båtsfjord	94,2 %	94,2 %	83,7 %
Tromsø	90,0 %	90,1 %	90,0 %
Salthella	75,6 %	75,0 %	87,8 %
Bekkjarvik	98,3 %	98,4 %	98,8 %
Avaldsnes	87,7 %	87,3 %	89,0 %

To conclude we recommend establishing between three and five locations. If more than five locations are selected, the costs and lead time for uncovered demand will start to increase. All these three solutions will have more or less the same costs and lead time for uncovered demand given the data we have used.

6.5 Additional info

There are other possible calculations to do with their data to further strengthen their inventory management. Such analysis can be beneficial to further strengthening their competitiveness, but because our thesis focused on the facility location and giving Mørenot a recommendation of how many semi-central warehouses they should operate such analysis were not investigated in much detail in this thesis.

6.5.1 ABC inventory classification

In this thesis we used all items, it might be a better approach to solve the problems separately for each of the A, B and C items using a correct ABC analysis. It is safe to assume that with the different categories of ABC items Mørenot would wish to have different inventory strategies. It is normal for the A items to represent most of the total sales value but represent a relatively small amount of the total items. Therefore, it is smart to have more of these items in stock, and therefore have a higher service level for these as well. The A items can be called the “core” items of the business and it is these items that generate the most revenue. The same can be said about B and C items where B is less important to stock at a significant level while C items should have a more reserved safety stock.

We did a basic ABC analysis of the items at each location for Mørenot. In Figure 17 below is a graph of the ABC items for each location from the result of the analysis.

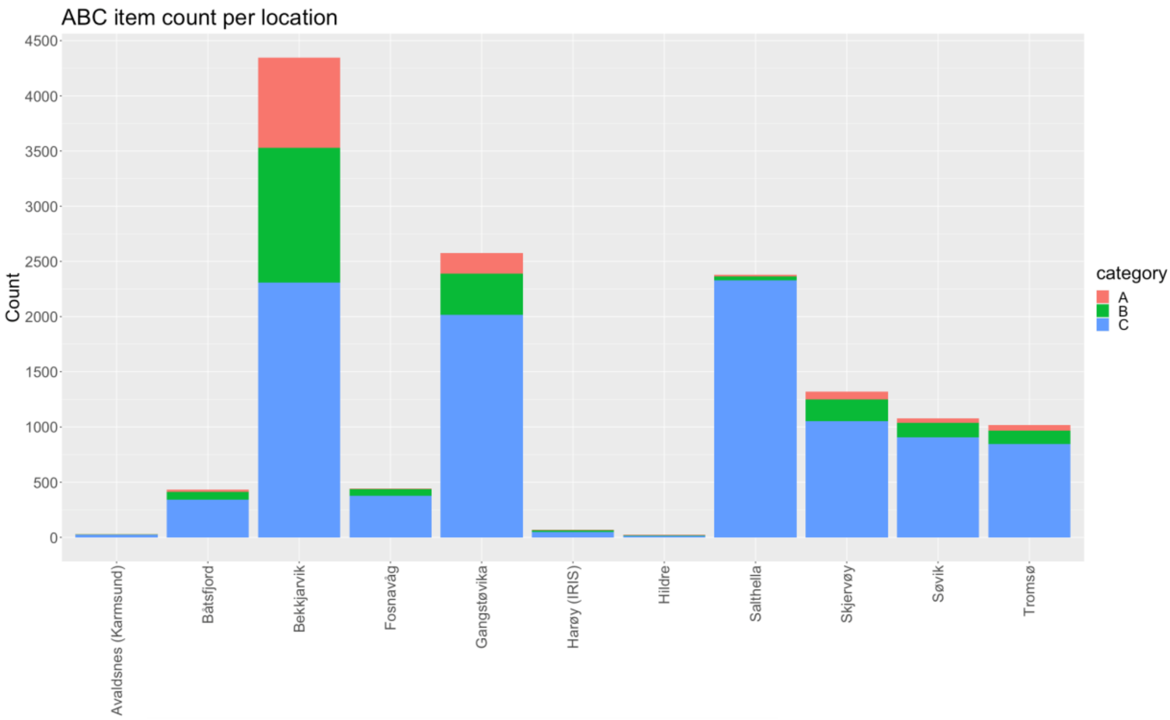

Figure 17 ABC items distribution

We chose that 70% of the total sales value at each location represents A items, the next 20% B items and the last 10% the C items. From this graph it becomes clear that for most of the locations there are just a few A items at the location, meaning that there are a few of the products that represents a large amount of the total sales value.

This can be used to have a different safety stock levels for A, B and C items as well as different ordering cycles and sizes to mention a few. This was also discussed more thoroughly in (Bassore & Natwijuka, 2022) master's thesis with a more in depth inventory classification.

Although ABC analysis is not extensively explored in this thesis it could be interesting for Mørenot to combine the findings done in this thesis with the inventory classification thesis written by Bassore and Natwijuka to see if there are any differences in what semi-central warehouses they should establish when focusing on different classifications of the inventory. Further analysis combining this thesis with the one mentioned can be beneficial for Mørenot and their operations.

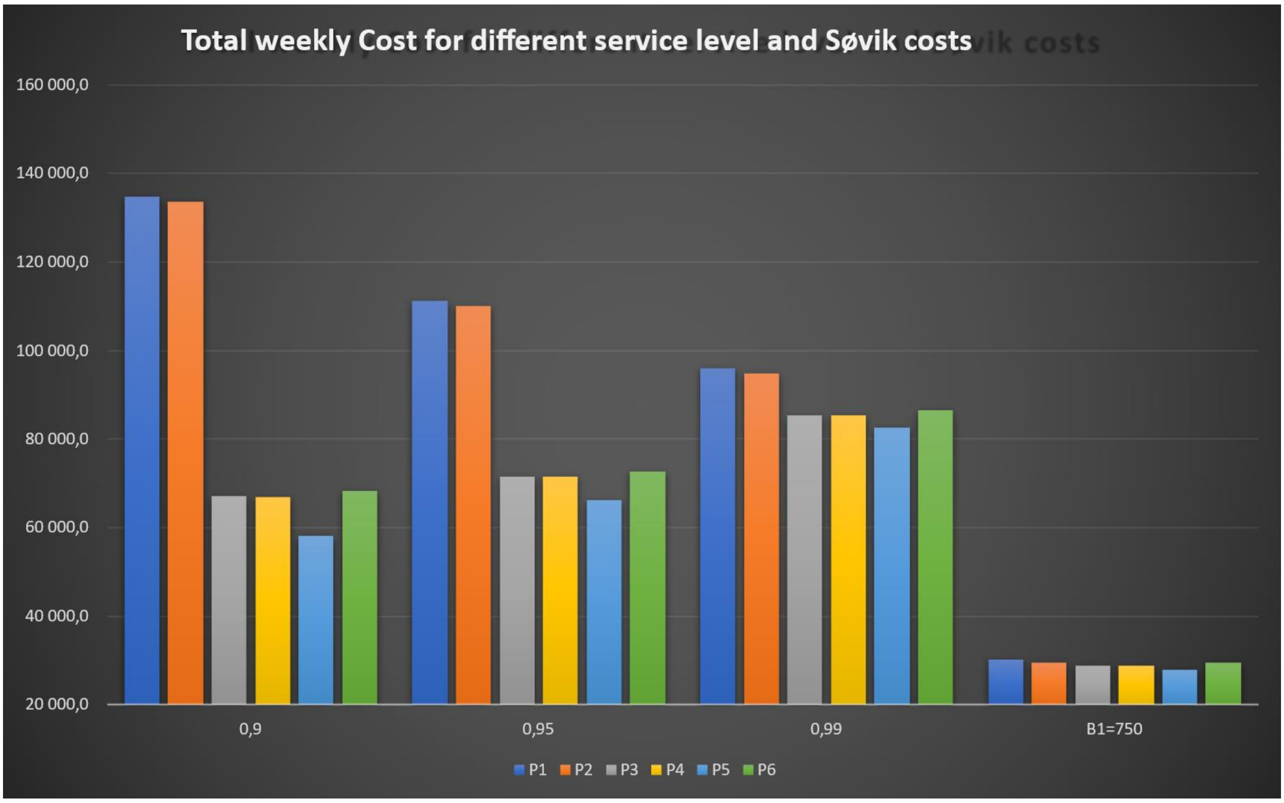

Figure 18 Cost for A items at different service levels

After we conducted the simple ABC analysis, we used the output of this to check the cost at varying degree of service level. In Figure 18 we can see the different costs when only using the A items. The different semi-central locations and the locations they serve stayed the same as we wanted to compare the costs and how many semi-central locations to establish. We can see then that when only using the A items it is still cheapest to establish five semi-central warehouses regardless of the different input values for service level we tested.

In Table 23 we can see the service level for the P5 solution using A items.

Table 23 Difference in service level for only A items for P5 (B1=750)

Location	Service level
Søvik	50,0 %
Harøy	95,4 %
Hildre	99,4 %
Båtsfjord	87,1 %
Gangstøvika	94,3 %
Fosnavåg	50,0 %
Skjervøy	58,6 %
Tromsø	87,8 %
Bekkjarvik	91,0 %
Salthella	92,3 %
Avaldsnes	58,7 %

These are the service levels for only the A items. We would expect the B and C items to have a lower service level than the A items.

6.5.2 Effects of fixed establishing costs

One of the costs we have omitted from the model is the fixed cost to establish a new warehouse. We have omitted this as Mørenot told us that they do not have any capacity constraints on their warehouses and that an increase in demand will not lead to extra space being built. Even so, if we had included fixed costs, the solution would trend towards opening fewer warehouses assuming the fixed cost for the different warehouses are somewhat similar.

Since the savings in inventory management costs only amount to approximately 136 000 NOK per year for P5 vs P1, the return on investment would only be positive after a very long time period, considering the large costs of establishing four additional warehouse locations compared to just one. Comparing P1 and P2, the difference in costs of only 75 000 NOK per year means the establishing costs of the extra warehouse for P2 could be at most 2 250 000 NOK for a positive return on the investment within thirty years. Assuming the same cost for establishing the necessary warehouses for P5, the total cost of 9 000 000 NOK for four extra warehouses would take approximately 66 years to break even.

Other costs we did not include were the cost of hiring additional workers as this cost is constant among all the locations as an extra worker costs the same regardless of location. The number of workers needed would be related to the demand, which remains constant. Other costs that are omitted that are difficult to compute are the extra operational costs such as electricity, equipment costs, etc.

6.5.3 Transferability of the models to other businesses and industries

We have created the models and calculations within a dynamic excel workbook that updates automatically. In addition to this the mathematical models upon which it is built can be used to solve problems for other divisions at Mørenot as well as other companies. The mathematical models are general, and it is possible to add new constraints, variables and objectives to the model, tailor made to the other company's needs.

The biggest difference between Mørenot's case and other companies would be the type, and amount of data they have available. The amount of data provided for the model will affect the accuracy and realism of the model's output. Mathematical models also offer the opportunity to customize the model to fit the company's needs. Therefore, the optimization problem can be relatively easily solved by other companies and industries in a similar way as we have solved the case for Mørenot.

This essentially means that with little effort Mørenot can extract a new dataset from PowerBI and import it to the excel workbook. Then all the calculations and models should be updated and run with the new input data. It is also possible for them to use data from different divisions like aquaculture in a similar manner.

6.5.4 Limitations

It might be unrealistic with express deliveries from Asia for stockout shipments. These shipments from Asia are usually planned a long time in advance. There is also usually a high safety stock level at the main warehouse which reduces the necessity of express shipments in the first place.

We have only looked at Asia (China) as a possibility to get extra items when Søvik is out of stock. In reality, Mørenot has more operations and suppliers in other locations in Europe, etc. It is more feasible to get express deliveries from these suppliers as they are located closer to Norway and are therefore provide faster deliveries. But today, most of their imports come from China, so we only included this in our calculations.

Another limitation to our study is we have not differentiated between the items they produce and the items they purchase. Items they produce might contain several other items (which increases the demand for these). Some of the production items can be categorized as produce to order, or even engineer to order and calculating lead times and stockout for such one-of-a-kind items are not relevant.

We have also not included capacity constraints in our analysis. This mainly comes from discussions from Mørenot, although when not included a nail and a large fishing net is considered to use the same space which is unrealistic. If item size were included, we could say more about safety stock. For example, one hundred extra nails are no big deal, but one more fishing net could take up considerable storage space.

For other companies, the storage capacities at the warehouses may be necessary to take into consideration along with volume usage per item.

6.5.5 Further research

A natural next step for this research is to make a model which includes more aspects of the system at the same time. Values such as lead times and inventory costs could be included together with the facility location aspect to make a more comprehensive model. This would give an answer that balances several different aspects at once providing a more accurate solution that better represents the real-world case.

It would also be interesting to investigate the possibility to have more deliveries from the European suppliers as well as Asia, which could provide better options for the lead time on imports. This could ultimately reduce the number of semi-central warehouses needed.

7.0 Conclusion

The main objective of this thesis was to investigate the possibility to decentralize Mørenot´s warehouse operations by introducing one or several semi-central warehouses. This was done by creating a facility location model in Excel where input data for their locations and demands was used to find suitable locations to establish semi-central warehouses. To determine the number of semi-central warehouses we analyzed several factors like lead time for uncovered demand, total relevant costs related to holding inventory as well as service level at the locations.

Based on the analysis conducted in this thesis, we can answer the research questions outlined at the beginning of our study.

How can a facility location model help determine the location of semi-central warehouses for Mørenot?

To determine the locations of the semi-central warehouses a P-MP facility location model were used to minimize the weighted total cost between the various locations. This gave us the locations for where to establish semi-central warehouses, as well as the warehouses they serve. We obtained eleven different combinations ranging from establishing one warehouse (P=1), up to eleven (P=11).

How many semi-central warehouses should they establish, when focusing on lead time for uncovered demand?

We calculated the total lead time for uncovered demand for each of the eleven solutions obtained from the P-MP model. We found that the values varied quite a bit in the different solutions. The lowest lead times where in the solution where four semi-central warehouses were established in Skjervøy, Salthella, Gangstøvika in addition to the main warehouse in Søvik. Even though establishing four gave the lowest total lead time for uncovered demand, the solution for two to five semi-central warehouses only deviated by 3,4% and is therefore not too far from the best solution.

How do inventory management related costs affect the number of semi-central warehouses to establish?

We calculated the total relevant costs related to inventory management for the first six combinations from the P-MP model. By altering the service level at each location, we obtained the corresponding costs, which allowed us to compare the different combinations. This led to establishing five semi-central warehouses having the lowest cost overall, regardless of the service level. All combinations from establishing one to six warehouses had quite similar costs, with the highest value only being 5,6% larger than the lowest.

From the analysis and discussions conducted in this thesis, when assuming accurate input data and reasonable estimations, our suggestion is for Mørenot to establish between three and five semi-central warehouses. This indicates that it would be beneficial for Mørenot to further explore the feasibility of establishing additional semi-central warehouses.

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Figure list

Tables

Appendix

Short and full name of nodes

Short	Name
L1	Søvik
L2	Båtsfjord
L3	Fosnavåg
L4	Harøy
L5	Gangstøvika
L6	Tromsø
L7	Skjervøy
L8	Bekkjarvik
L9	Hildre
L10	Salthella
L11	Avaldsnes