PROJECT-BASED LEARNING AND ITS IMPACT ON HIGH SCHOOL STUDENTS' ATTITUDES TOWARDS MATHEMATICS: A QUANTITATIVE STUDY

by

David E. Postlethwait

Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirements for the Degree

Doctor of Philosophy

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ABSTRACT

This quantitative, quasi experimental, nonequivalent control group study analyzed the relationship between students engaging in project-based learning and its impact on their attitudes towards mathematics. Sixty-six high school students taking a non-entry level mathematics class participated in the study. This study compared students taking the same high school mathematics course with one group taught in the traditional way of lecture notes and tests and the second group being taught using projects. These students' responses from the Attitudes Towards Mathematics Inventory were compared using one-way analysis of covariance to determine a difference in student attitudes. The study determined that there was no significant statistical difference in student attitudes toward mathematics between the project-based group and the traditional instruction group when controlling for pretest scores. Future research recommendations include a longer timeframe, focusing on special education students, and making accommodations for English Language Learners to help ameliorate language barriers.

Keywords: project-based learning, attitudes towards mathematics, non-equivalent control group, ATMI

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Dedication

To Grandma and Pap. We did not have much time together on this earth, but it was profound. I know you are watching from Heaven, and I hope I've made you proud.

To Nanny and Pappy. Whether we lived across the yard or four states away you have loved me unconditionally. Thank you for always supporting me.

To my Lord and Savior Jesus Christ, without whom I would not be able to do a small fraction of what I complete. "Now to him who is able to do *exceedingly*, *abundantly*, *immeasurably* more than all we could ask or imagine, according to his power that is at work within us, to him be **glory** in the church and in Christ Jesus throughout all generations, for ever and ever! Amen." (Ephesians 3:20-21).

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List of Abbreviations

Algebra, Functions, and Data Analysis (AFDA)

Attitudes Towards Mathematics Inventory (ATMI)

Project-Based Learning (PBL)

Science, Technology, Engineering, and Mathematics (STEM)

Virginia Department of Education (VDOE)

CHAPTER ONE: INTRODUCTION

Overview

Project-based learning (PBL) is nothing new within the world of education. Mathematics teachers often strive to make connections between the classroom and the world around them with applications, otherwise known as word problems. Unfortunately, students still do not see the connection to their lives in these fictitious scenarios. Students want to know that what they are learning in class will be useful in their lives and the world around them. Shifting the curriculum focus from the traditional memorization of algorithms to perform on exams to a PBL environment can intrigue students and improve their problem-solving skills. It can also help shift how they think about the mathematics studies in their classes. Rote memorization will turn to critical thinking as students actively work to solve real scenarios rather than worksheets of practice problems. This chapter will look at background information on project-based learning, specifically in the mathematics classroom, the problem that exists today, and the significance of this study.

Background

Students want classwork that relates to them as individuals and want to know what they learn has meaning in the world around them (Almarode, 2019; Cukurbasi & Kiyici, 2018; Guo et al., 2020; Jansen & Berry, 2020; Liljedalh, 2020; Serin, 2019). Project-based learning (PBL) is a type of performance assessment "that requires students to demonstrate achievement by producing an extended written or spoken answer, by engaging in group or individual activities, or by creating a specific product" (Lalor, 2017, p. 88) and shows students how the material taught in class applies to the world around them. PBL is becoming increasingly popular in classrooms as educators look for alternative ways to assess students beyond the traditional exam

(Bender, 2012; Hattie, 2017). Kokotsaki et al. (2016) composed a review of quasi-experimental, pretest-post-test designed studies. The authors found a causal link between project-based learning and positive student outcomes in their courses. The authors offer six recommendations—student support, teacher support, effective group work, balancing didactic instruction with independent inquiry method, assessment emphasis on reflection, self and peer evaluation, and student choice and autonomy. These recommendations can help teachers and school leaders implement project-based learning into general school settings (Kokotsaki et al., 2016). These positive-student outcomes continue to drive PBL as a possible alternative assessment. Additional research offers quantitative arguments for integrating PBL into courses not only as a way for students to do well on in-class assessments, but also to increase student performance on state assessments (Craig & Marshall, 2019).

Historical Context

Project-based learning has longstanding roots in education. The core of PBL can be originated in differentiating instruction which has roots back to philosopher Johann Amos Comenius in the 1600s. Comenius worked tirelessly to ensure that everyone was educated, not just the children of the wealthy and elite—and that their education was meaningful (Glenn, 2018). This idea of educating everyone and meeting the needs of individual students continued for centuries and has evolved into differentiated instruction, which is more prominent in education today than ever. Carol Ann Tomlinson (2014, 2017) discussed the importance of meeting the needs of every student through various means. The idea is not to have multiple different tasks for students to complete just for the sake of completing different tasks. The idea, rather, is to make slight modifications to what the students need to complete to reach a common goal. Differentiated instruction easily lends itself to PBL, which has become more prominent in the last few decades (Bender, 2012; Tomlinson, 2017).

In the early 2000s, MathScape (2005) would pose questions to students that would get them thinking about a question. An introductory question, such as "What is the volume of the sinkhole in Guatemala?", could open a lesson. The idea is to intrigue students immediately and get them to want to solve the problem. This leads to students learning mathematics without necessarily thinking about it, but simply solving an interesting problem. A group of middle school students is much more interested in finding the volume of a sinkhole rather than in finding volumes of arbitrary three-dimensional objects (K. Gushue, personal communication, September 15, 2020).

More recently, Robert Kaplinsky (2020) has taken this idea from MathScape (2005) and developed power problem solving for students. It takes the exact concept and rebrands it for the modern classroom. Students are given a problem, and they need to know the answer. They are given something interesting such as how to maximize the profit of a small business or how to create an interesting logo for a company and apply the mathematical concepts they have learned (K. Gushue, personal communication, September 15, 2020). The idea of taking and applying the knowledge is not a new concept in mathematics, but it has become cumbersome. When word problems are mentioned in class, they are frequently accompanied by groans and complaints, possibly because of the farfetched nature of many textbook problems. Oftentimes, application problems have pointless, confusing, or dangerous contexts that distract from what the application problem is trying to achieve, or scenarios are artificially added to mathematical context to make the problem more relatable. When students are presented with an applicable, genuine real-world problem that they want to solve, the process becomes less like a grade requirement and more of an adventure to find the solution. This solution just happens to use mathematics along the way (Barton, 2018).

Social Context

Students perform better academically in a PBL classroom. Across various disciplines, including mathematics, students showed an increase when in a project-based environment. (Holmes & Hwang, 2016; La Prad & Hyde, 2017; Wurdinger et al., 2020). When making mathematical connections in a PBL course, students perform better in class and learn the material at a better rate than in a more traditional class where rote memorization and repetitive algorithms are the primary way of teaching and learning the subject. When connections are made and students enjoy the work they are doing, the mathematics concepts come more naturally, and students want to continue learning. This easily translates to the grades students receive in their mathematics courses. Additionally, students are much more likely to want to do the work when the material is relatable; thus, connections are made, and academic achievement increases (Almarode, 2019; Hattie, 2017).

The impact of PBL can reach farther than simply a student's report card and grade point average. When students have an appreciation and an understanding of mathematics, they are more likely to pursue a career in Science, Technology, Engineering, or Mathematics (STEM) (LaForce et al., 2017). Furthermore, the students will no longer simply be looking to get a good grade, but they will actively search for understanding and want to know why something works. Students will work harder to achieve an understanding of mathematics rather than simply trying to pass the course. Students regularly struggle with mathematics but are given the tools and the desire to want to learn and do well. This productive struggle then translates beyond the mathematics classroom to other areas and is a critical skill for students beyond graduation (San Giovanni, et al., 2020).

Potentially the most important benefit to PBL in the mathematics classroom is the enjoyment that both students and teachers experience daily. Since learning is the top priority of schools, making learning exciting and relatable should be the top priority of teachers (Geurin, 2017; K. Gushue, personal communication, September 15, 2020). Not all students need to have deep admiration for every subject they study in school, but an appreciation for the topics helps students realize that, while they might not like the subject, it holds some significance in some aspects of the world around them (Barton, 2018).

Theoretical Framework

Bloom's taxonomy gives six tiers of questioning. From bottom to top, the tiers are comprised of knowledge, comprehension, application, analysis, synthesis, and creation. Often these levels are broken into two subgroups: low-level, which consists of knowledge, comprehension, and application, and higher-level, which consists of analysis, synthesis, and creation questions. Students are more likely to retain information assessed at higher levels. Furthermore, the higher the level of assessment, the more challenging the assessment—and thus, the more effectively it assesses a student's knowledge. Building students to the higher levels allows them to effectively show more comprehension of the material taught (Bloom et al., 1956). Knowing and understanding the six levels of Bloom et al.'s (1956) taxonomy allows teachers to prepare students for assessments of various levels. Teachers know where their students should be by the end of a lesson, unit, or course. Bloom's taxonomy allows them to plan accordingly, potentially backward, to appropriately prepare students for the final project-based assessment without teaching to a specific test (Bloom et al., 1956; Morton & Colbert-Getz, 2017).

Understanding Bloom et al.'s (1956) taxonomy allows teachers to assess students on a deeper level. Analyzing, synthesizing, and creating are the top of the framework, and that is where project-based learning thrives. Using the information outlined in this framework helps in the proper planning and implementation of project-based learning.

When preparing either entire units or individual lessons, teachers look to see where their material can be differentiated to meet the needs of the various students within their class. Tomlinson (2014, 2017) offers ways to differentiate instruction that lend themselves to project-based learning. When students are offered an open-ended question to answer, they are not restricted to doing work in the same way as their classmates. This is a way to differentiate instruction. If students are answering the same questions and producing a similar product, then the journey is theirs to choose. In a PBL classroom, there does not need to be a different project for each student, but various potential pathways for students to achieve a similar end result (Lalor, 2017).

Bloom's taxonomy and Tomlinson's differentiated instruction both directly relate to PBL in any classroom. Having students work up the levels of Bloom's taxonomy using differentiated instruction will ensure that teachers are meeting the student at the student's level while still having the students meet the goals and objectives of the lessons. PBL also directly lends itself to the higher level of Bloom's taxonomy that will yield more in-depth learning and have higher effect sizes in the classroom (Almarode, 2019; Hattie, 2017).

Problem Statement

Mathematics is often an area of struggle. Many students loathe mathematics before even entering the classroom on the first day of school because they do not consider themselves mathematically inclined. Jo Boaler (2015, 2016) discussed the idea of Carol Dweck's (2006) growth mindset in the mathematics classroom. Growth mindset is the idea that while students have not mastered a topic, they are still able to work towards mastering the material (Dweck, 2006). Additionally, how teachers approach giving feedback to students can impact a student's mindset and how they approach learning and approaching challenging tasks (Boaler & Dweck, 2016). Students are often discouraged if they do not comprehend a topic immediately, but in some cases, their peers seem to understand with ease. The constant judgment that students have compared between themselves and their classmates can shape the students' attitudes towards mathematics as negative (Boaler, 2015). This negative attitude could also stem from traditional assessments and learning that happens in the mathematics classroom, where students watch their teacher complete problems of arbitrary numbers before being released to complete their own (Barton, 2018; Boaler, 2015). Students often do not see the benefit of learning mathematics through rote memorization, reiteration on homework, or an exam. Even with the addition of instructing practical, real-world problems, students see these as fictitious scenarios that would not happen in their daily lives (Boaler & Dweck, 2016).

High school students want to know that what they are learning could be useful to them beyond the classroom. Within the classroom, students will master more topics depending on the learning activities. Activities with a high effect size include summarization, strong classroom cohesion, and classroom discussion (Almarode, 2019). Of course, any classroom will have a mixture of various learning activities, but as more activities with a high effect size are introduced, students can achieve deeper and more meaningful learning (Hattie, 2017). PBL has a positive impact on a student's ability to learn mathematics and perform well academically, which can be beneficial beyond the classroom (Holmes & Hwang, 2016; Wurdinger et al., 2020).

Students' negative attitudes towards mathematics are impacting their ability and achievement in mathematics classes; however, PBL could help address that negative attitude by making mathematics more relatable for students (Hanham et al., 2020). Research has shown there is an impact on student attitudes towards classes when exposed to PBL. Specifically, there is a significant difference in student attitudes towards English classes when engaged in PBL (Duman & Yavuz, 2018); however, more research needs to be conducted to determine the specific impact of PBL on student attitudes towards mathematics. The problem is that the literature has not fully addressed the impact of student attitudes towards mathematics in a PBL classroom.

Purpose Statement

The purpose of this quantitative, quasi-experimental, nonequivalent control group study is to determine the impact project-based learning has on students' attitudes towards mathematics. Quasi-experimental, pre-test/post-test design is used to determine causality between the experimental treatment and the outcome. Within a quasi-experimental nonequivalent control group study, the participants cannot be randomly assigned, and the participants are similar (Gall et al., 2003). The categorical independent variable, type of instruction, is defined as how information is shared between students and teachers in the classroom (Borich, 2014). In this study, there are two categorical groups for the independent variable: traditional instruction, which will be the control group, and project-based learning, which will be the treatment group. Traditional instruction is generally more teacher centered with students working on close-ended problems and taking exams rather than completing projects (Tomlinson, 2014). Conversely, project-based learning is the use of open-ended questions to inspire practical scenarios with a hands-on approach (Boaler, 2015).

The continuous dependent variable, student attitudes towards mathematics, is the students' reaction to the subject as measured by the Attitudes Towards Mathematics Inventory (ATMI) (Tapia & Marsh, 2004). Student attitudes will be compared based on how they are grouped by the categorical independent variable of instructional methods. Students will or will not have been exposed to project-based learning and will be accustomed to traditional instruction and assessments in the mathematics classroom. The researcher will administer the ATMI to all participants in both the experiment and control groups at the beginning and end of the study. Pre-test scores gathered from the first administration of the ATMI will be used as a covariate. Student attitudes towards mathematics will be measured as the dependent variable on a continuous scale using the ATMI at the beginning of the study. After pre-test and post-test inventories are complete, the data will be used to determine a difference in the means after manipulating the independent variable of instructional delivery via project-based learning or traditional instruction. All groups will take identical pre-tests and post-tests at the same time, and then the experiment group will participate in PBL for the duration of the study, but not the control group.

The population for this study will be students in two Northern Virginia high schools—school A and school B—taking Algebra, Functions, and Data Analysis for the first time. A total of at least 66 students will be used and equally divided between the two categories of receiving traditional instruction or receiving PBL. These schools can be considered suburban, with at least 37 percent of students being economically disadvantaged (Virginia Department of Education, 2020).

Significance of the Study

This study will add to the growing body of knowledge on the impacts of PBL in the classroom, especially looking at high school mathematics classrooms. Researchers already know the benefits of PBL in the classroom (Almarode, 2019; Boaler & Dweck, 2016; Hattie, 2017; Kokotsaki et al., 2016; Serin, 2019). This study adds a layer of not only the academic impacts of PBL, but the impacts that PBL could have on the attitudes of students in high school mathematics classes. When students see mathematics as a challenging and often unattainable subject to master, teachers need tools and strategies to combat this attitude (Ganal & Guiab, 2014). This study will add PBL to the list of effective instructional strategies that teachers can use not only for students to improve academically, but also to change their reaction to the subject. Students generally perform poorly in mathematics courses because of their fear and anxiety towards the subject (Tapia & Marsh, 2004). Easing the fear and anxiety by making the approach more meaningful to the students will help them in multiple ways. Additionally, this study will address student attitudes towards mathematics in a project-based environment. Improvement of student attitudes towards mathematics can not only impact how students do within their classes but can also birth or renew a desire for the student to study within the area of science, technology, engineering, or mathematics. This, in turn, can inspire them to pursue those areas as a potential career options lending PBL to extend its effectiveness far beyond the wall of the classroom (LaForce et al., 2017). Similar studies have shown the positive impacts of project-based learning in other disciplines such as English classes (Duman & Yavuz, 2018), but this study extends that into mathematics courses.

By showing PBL improves students' attitudes towards mathematics, the benefits are twofold. First, students will naturally benefit. Having a way to help students understand and consume a traditionally challenging subject will show the students that even though a subject can be challenging, it is not impossible to learn. In addition, teachers will benefit. Often high school mathematics teachers are burdened with trying to make their subject applicable to students. With research backing a PBL approach in the classroom, teachers can shift how they work to meet the mathematical needs of their students. Teachers can use PBL to involve students in mathematics and learn without the fear of failing a traditional exam (K. Gushue, personal communication, September 16, 2020; Liljedalh, 2020).

Research Question

RQ1: Is there a difference between high school students' attitudes towards mathematics in a PBL classroom versus a traditional classroom when controlling for pretest scores?

Definitions

1. *AFDA* – AFDA is an acronym by the Virginia Department of Education (VDOE) referring to the high school Algebra, Functions, and Data Analysis mathematics course students can take as one of the required mathematics courses for graduation (Virginia Department of Education, 2016).
2. *Attitude* – A person's attitude is their reaction towards something, whether positive, negative, or neutral (Tapia & Marsh, 2004).
3. *Attitudes Towards Mathematics Inventory* – The Attitudes Towards Mathematics Inventory (ATMI) is a 40-item instrument developed by Martha Tapia to determine a student's attitude towards mathematics considering multiple variables using a Likert scale (Tapia & Marsh, 2004).
4. *Differentiated instruction* – When teachers differentiate instruction, they offer modifications to the content, assessment, process, or other areas of the instructional process (Tomlinson, 2014).

5. *Project-based learning* – Project-based learning, or PBL, is the use of open-ended questions and tasks and questions to propel classroom topics into practical scenarios with a hands-on approach (Boaler, 2015).
6. *STEM* – STEM is an acronym referring to the integration of science, technology, engineering, and mathematics (Navy & Kaya, 2020).
7. *Traditional instruction* – Traditional instruction is generally more teacher centered with students working on close-ended problems and taking exams rather than completing projects (Tomlinson, 2014).

CHAPTER TWO: LITERATURE REVIEW

Overview

A systematic review of the literature was conducted to explore project-based learning in the high school classroom. This chapter will present a review of the current literature related to the topic of study. In the first section, the theories relevant to project-based learning will be discussed, followed by a synthesis of recent literature regarding project-based learning at all levels and across various disciplines. Literature surrounding project-based learning, its impact, and its effect level in the classroom will be discussed. Additionally, literature reviewing student attitudes towards mathematics will be addressed, including grit and mindsets in the classroom. In the end, a gap in the literature will be identified, presenting a viable need for additional research to be conducted.

Theoretical Framework

Multiple Intelligences

Howard Gardner's theory of multiple intelligences originally stated that people fall into a subset of seven various modes of intelligence, including the following: linguistic, mathematical, musical, bodily-kinesthetic, spatial, interpersonal, and intrapersonal. Students who recognize which intelligence best describes them can use that to their advantage in the classroom. Additionally, teachers who understand these various intelligences can teach to multiple intelligences to best serve their students. Just because a student does not fall under a certain intelligence does not mean they cannot learn or do not understand (Gardner, 1983). More recently, Gardner added naturalistic and existential as additional intelligence that meet his original eight criteria (Gardner, 2011; Tamilselvi & Geetha, 2015). Additionally, it helps teachers to prepare lessons that are varied for students within their classes. Knowing, embracing, and understanding the theory of multiple intelligences permits classroom success from both students and teachers (Gardner, 2011).

Gardner's theory of multiple intelligences lends itself nicely to differentiated instruction and PBL. When teachers are aware of their students' various intelligences, the teacher can then offer students various ways to show mastery in class, such as using different resources, permitting multiple attempts to show the student has learned a skill, and permitting various ways for students to showcase their knowledge, such as an oral presentation or a written report. Teachers need to take these differences among their students seriously and take the time to teach and assess in multiple ways to ensure they are teaching all students, and PBL is an effective way to accomplish that (Borich, 2014; Woolfolk, 2013).

Constructivism

Constructivism is the idea that new knowledge is constructed based on previous experiences. What the learner already knows and understands becomes the foundation of the new topics that are discussed. The constructivism theory is comprised of multiple principles that discuss how the theory is applied to students. The principles of constructivism include seven key components, people learn to learn as they learn, learning is an active process, learning is social, learning is contextual, knowledge is personal, learning exists in the mind, and motivation is key to learning. Each individual principle of constructivism supports what happens within a project-based learning classroom. When teachers employ a constructivist-centered classroom, students genuinely build their knowledge upon prior learning as they continue to learn. Constructivism also discusses cognitive and social learning, which are ever-present in a project-based environment and allow students to grow in areas beyond the content curriculum (Borich, 2014; Woolfolk, 2013).

Zone of Proximal Development

Vygotsky's zone of proximal development outlines the psychological development of children. The first and core zone is the zone of material that the learner can do on their own. The second and middle zone is the zone of proximal development, where a learner can do tasks with some guidance. The third and outer zone include the tasks that the learners cannot do. This theory justifies scaffolding in schools, with teachers giving students work slightly harder than they can do to ultimately get them to the desired level (San Giovanni et al., 2020; Vygotsky, 1934/1986).

Vygotsky's zone of proximal development helps teachers prepare psychologically appropriate lessons and projects for use in their classrooms. Having students work together with others of similar ability, and psychological levels as themselves allows students to apply topics learned in class through brainstorming, conversations, and problem-solving with their peers. Student engagement in verbal communication regarding a topic is a primary tool for transmitting information between people. Students are more likely to recall information that was discussed with their peers as opposed to information that was only heard (Dale, 1946; Lewis, 2019; Vygotsky, 1934/1986).

Theory of Cognitive Development

The basis of Piaget's theory of cogitative development is that children use previous experiences as a basis for new ones. This can be applied to learning new topics in education. Knowing these stages allows teachers to plan lessons that are age-appropriate for students. Furthermore, knowing the developmental stages shows what students should, and should not, be able to do in the classroom at certain ages. Piaget's four stages of cognitive development include sensorimotor from birth to two years, preoperational from two years to seven years, concrete operational from seven years to eleven years, and formal operational from eleven years to adulthood (Piaget, 1968).

Working with high school students, educators familiar with Piaget's theory are aware that their students are in the formal operation stage where abstract thought emerges. Students are more able to make connections without needing physical manipulation to understand the topics discussed in class (Piaget, 1968). Teachers can use this to their advantage as they prepare lessons and assessments for their students who can actively use deductive skills and logical reasoning to complete tasks in a PBL classroom (Borich, 2014; Piaget, 1968; Woolfolk, 2013).

Behaviorism

Behaviorism discusses the idea of learning as a response to an event (Schunk, 2020). Events that occur in the classroom can prompt multiple responses from students depending on their experiences. Teachers can teach students ways to react to certain stimuli within the classroom. For example, an elementary teacher may instruct students to become silent whenever they hear a bell ringing. High school teachers could have the student pick up all handouts on the table near the door as they walk into the classroom. These different stimuli cause the students to behave a certain way. This can also be applied to learning in the classroom. As students are exposed to the material, they can be taught to react a certain way. If in a mathematics classroom, a student encounters a problem that they are unfamiliar with, they can be taught to try different strategies to solve the problem, collaborate with a peer, or consult their notes before asking the teacher. Furthermore, students could also be taught how to react to finding a mistake in their work. If students are taught that mistakes are a part of the learning process and they can use those mistakes to help them achieve mastery, then the students will react appropriately when those situations occur in class (Jansen & Berry, 2020; Schunk, 2020; Woolfolk, 2013).

Bloom's Taxonomy

As discussed in Chapter One, Bloom's taxonomy gives six tiers of questioning divided into lower and higher-level thinking questions. The lower levels include knowledge, comprehension, and application, and these are often included in traditional classrooms where students are sought to return facts taught to them in class on an exam or other assessment. The higher levels include analysis, synthesis, and creation, and these lend themselves to PBL and have a higher effect size on students. The students are much more likely to retain information assessed at the higher levels and can transfer that learning to areas outside of the classroom (Almarode, 2019; Bloom et al., 1956; Hattie, 2017).

Knowing and understanding the six levels of Bloom's taxonomy—as well as behaviorism in the classroom—permits teachers to prepare lessons and assessments for their students at appropriate levels. Being able to scaffold students from the lower level to higher levels of Bloom's taxonomy lends itself to a project-based classroom and prevents teachers from simply teaching to a single exam. Considering what the students need to know by the end of the lesson and using daily learning objectives, teachers can integrate more meaningful lessons and assessments that engage students; thus, maintaining a positive impact on the students' attitudes towards the lessons and assignments conducted in class each day. Understanding how students will respond to certain behaviors in the classroom and connecting those responses to how students learn and are assessed help in reaching all students (Borich, 2014; Schunk, 2020; Woolfolk, 2013).

Related Literature

Project-Based Learning

Project-based learning (PBL) is a type of performance evaluation. Lalor (2017) defines PBL as an assessment "that requires students to demonstrate achievement by producing an extended written or spoken answer, by engaging in group or individual activities, or by creating a specific product" (p. 88). PBL uses open-ended tasks and questions to move classroom topics into practical scenarios. This shift is growing in popularity, as teachers continue to look for additional and alternative ways to assess their students beyond traditional testing methods (Hattie, 2017; Lalor, 2017). Furthermore, PBL offers various outlets for students to demonstrate mastery of content learned in class, such as oral or written presentations rather than simply traditional exams (Boaler, 2015; Hattie, 2017). PBL is becoming increasingly more popular in classrooms as educators look for alternative ways to assess students beyond the traditional test (Hattie, 2017). Kokotsaki et al. (2016) composed a review of quasi-experimental, pretest-post-test designed studies. The authors found a causal link between PBL and positive student outcomes in their courses. The authors ultimately offer six recommendations that can help teachers and school leaders implement project-based learning into general school settings. These recommendations include the following: student support, teacher support, effective group work, balancing didactic instruction with independent inquiry method, assessment emphasis on reflection, oneself and peer evaluation, and student choice and autonomy, which all offer high effect sizes in the classroom (Almarode, 2019; Hattie, 2017; Kokotsaki et al., 2016).

Teachers ultimately want their students to learn, and PBL offers a way for learning to occur while keeping students interested and maintaining content relevance. A PBL classroom motivates students to want to do more than simply score well on an exam (Cukurbasi & Kiyici, 2018). An active project and problem-solving-centered classroom show the students that there is more to learning the content than simply scoring well on an exam or passing a course. This type of learning exemplifies how the knowledge acquired in class relates to the world around them. PBL cannot only offer students various ways to engage with the material, but PBL also offers teachers a different way to assess student knowledge and understanding of the material. Furthermore, PBL brings to the forefront of class some of the unofficial curricula that can help students succeed beyond the classroom (Almarode, 2019; Lalor, 2017; Tomlinson, 2014, 2017).

Advantages

Taking the time as educators outside of class to compose and prepare project-based assessments allows teachers to spend more time during class time facilitating student work. Rather than spending class time within a teacher-centered lecture, instructors can have students actively collaborating, conversing, brainstorming, and problem-solving with their peers in a student-centered environment (Jackson, 2018). Project-based learning allows teachers to assess student content mastery on a learning target or objective by having them produce something that requires more than just a basic exam. When working on a summative exam, students are not able to ask for assistance, are concerned about failing, and can experience test anxiety because that is the final opportunity that the students have to show their understanding of the material. With student-centered projects, active conversations allow students to share ideas and receive and use constant teacher feedback to ensure that they understand the material and are putting that knowledge to use (Hattie, 2017; Jackson, 2018).

Additionally, the implementation of project-based learning yields a larger effect size on student learning than traditional assessments (Almarode, 2019; Hattie, 2017). Having students produce something rather than answer questions on an assessment shows the student has a greater understanding of the material presented in class and is ultimately assessed. Furthermore, analyzing, synthesizing, and creating something yields a greater effect size in student learning than the traditional assessment regularly given in mathematics classes (Hattie, 2017). A student reciting a fact or solving a problem shows a low level of understanding. However, a student analyzing a situation, discussing the situation with their peers, determining an appropriate course of action, implanting that action, and arriving at a solution is not only more meaningful, but also a much higher level of thinking (Bloom et al., 1956; Jackson, 2018).

Interestingly, students also seem to appreciate the hard work that goes into PBL. They no longer see mathematics as a requirement of something they are strictly trying to finish, but rather it is a means to an end for solving their problem. Students would much rather put in work for something meaningful rather than simply completing obligatory practice. Students can more easily see how mathematics works in the world around them in a project-based environment, and that gives merit to the tasks at hand (Hanham et al., 2020). When students see the underlying reason for studying the material, they are inclined to want to learn and do well (Cukurbasi & Kiyici, 2018; Jackson, 2018).

Disadvantages

The greatest disadvantage of project-based learning comes from the teaching perspective. A classroom teacher from any discipline at any level will be required to do significant planning for their project-based learning classroom. For a teacher who regularly uses PBL in their classroom, the planning period will be the busiest time of the day. During class time, however, the teacher will guide students rather than deliver direct instruction; thus, the teacher will be doing much less work while the students are present since students are leading their own instruction during most of the scheduled class time. With proper planning, a teacher using PBL will be a facilitator of learning as students actively work to find the answers to the problems they are presented with (Jackson, 2018). If teachers have a strong collaborative learning team that works together to help in planning lessons and projects, the work becomes less cumbersome. It is important to remember when planning any classroom lesson or activity that those who are doing the most talking are doing the most learning (Geurin, 2016). When students engage with the material and discuss it with their classmates, they will retain much more than a traditional classroom lecture (Treichler, 1967).

Student buy-in can also be a surprising disadvantage to project-based learning. Depending on the discipline, students may not be accustomed to working collaboratively in a problem-solving environment and will voice their discomfort at the beginning of class. This is much more prevalent in secondary students since they have more experience with how various classes traditionally operate. With appropriate teacher planning and preparation, as well as conditioning students how to function in a PBL setting, even a reluctant student can engage and learn in a PBL classroom. Not everybody will initially be comfortable with a nontraditional classroom setup. Both students and some teachers might see the changes as unnecessary at first, but when students and colleagues realize, whether intentionally or not, that student engagement increases as students interact in this way with the material, they are likely to accept and embrace the new learning environment in a PBL classroom (Barton, 2018; K. Gushue, personal communication, September 15, 2020).

Project-Based Learning within the Classroom

John Hattie (2009) conducted a meta-analysis of research done on learning and determined that a hinge point of 0.4 equates to approximately one year of learning in the classroom. Per this meta-analysis, everything done in a classroom is assigned an effect level from negative values that harm student learning to positive values that offer nearly four times that of the 0.4 hinge point. The higher the effect level, the more powerful that aspect of learning is for a student. For example, self-reported grades have an effect size of 1.44. Based on the hinge point of 0.4, this aspect of learning equates to learning more than three times as much in a school year. On the other end of the spectrum, television has an effect size of -0.18. This shows that, ultimately, television hurts a student's education. Obviously, not everything that Hattie researched will always happen in the classroom. Still achieving that 0.4 hinge point will ensure that students are learning what they need to learn as they move forward through their education. Cooperative learning has an effect size of 0.4—right at that hinge point to ensure students are learning a year's worth of material throughout the school year. One teacher cannot be expected to use only one strategy throughout the entire school year, but this research shows that a variety of effective classroom strategies can be included in a teacher's planning, ensuring that students maximize their learning (Hattie, 2012, 2017).

In addition to Hattie's work, Jo Boaler (2016) has completed much research following the work on mindsets done by Carol Dweck (2006). Boaler notes that students grow synapses whenever they make a mistake in class. Students need to make mistakes to learn. Mistakes are an important part of everyday life, and students should not fear making mistakes, but rather embrace mistakes as learning opportunities. Educators have an important responsibility to show students that mistakes are proof that they are trying, and when they make a mistake and correct it, learning is occurring. When mistakes happen, the brain physically grows, and they continue to work towards mastery; thus, a mistake is not something to cause concern, but an opportunity to revisit, reevaluate, and reattempt to find the correct answer. Boaler's research on mindset, along with Hattie's work on effect size in the classroom shows us how PBL can really make students think and realize what they are doing and how it connects to the world around them (Boaler & Dweck, 2016; Dweck, 2006).

Robyn Jackson (2018) also offers insight into project-based learning. When faced with the question of how teachers can cover so much material in just two hours a day, twice a week after school, her words resonated with educators. Jackson comments, “Instead, focus on quality versus quantity and remember, it is not how much we cover that counts; it is how strategic we are about what we cover that will ultimately determine how well our students master the goals of our curriculum,” (2018, p. 178). Teachers do not need to cover every single topic in the curriculum in a project-based setting after school, but if educators covered less material in a much higher quality and more effective way, then students would greatly benefit. When students are presented with a problem that they do not know how to solve, the lesson and strategies from their PBL classes will aid them in finding a solution. This is because the students are accustomed to not knowing how to solve something immediately when they are introduced to it, but rather needing to read and understand before knowing exactly what needs to be done. With these problem-solving tools available, students from a PBL environment thrive on assessments even if the school year did not include the entire curriculum (Holmes & Hwang, 2017; Jackson, 2018; Wurdinger, et al., 2020).

Elementary School

Navy and Kaya (2020) conducted mixed methods research on PBL with their pre-service teachers in elementary schools. The authors conducted semi-structured interviews with participants at the end of the course and analyzed project-based learning unit plans. There were 47 participants in the study—43 females and 4 males. The authors conducted interviews with participants at the end of their course, and they asked questions about project-based learning.

Additionally, project-based learning unit plans were collected and analyzed for their integration of project-based learning. Both the qualitative and quantitative data were collected simultaneously, but they were analyzed independently. This study adds to the body of literature supporting project-based learning in the classroom, specifically the elementary classroom. Teachers can introduce and use project-based learning in their classes as a way to integrate science, technology, engineering, and mathematics (STEM) into courses. In addition, curriculum developers can add project-based learning to assist with the integration of STEM, literacy, and other subjects at the elementary level (Navy & Kaya, 2020).

Elementary school students and teachers are not necessarily strangers to a PBL environment. Very often, students are taught from a young age to work in stations and collaborate with their peers. By the time students are in upper elementary grades and preparing to transition to secondary school, these skills of communication, collaboration, and teamwork learned in a PBL environment are natural to them. In a PBL environment, these young students thrive academically as well as socially. Since students are taught from a young age to inquire, communicate, and ask questions, it is not a new concept to them as they advance from elementary to secondary schools (K. Gushue, personal communication, September 15, 2020; Tsybulsky, 2019).

Secondary School

As students transition from elementary to secondary school, there is a shift in focus from engaging students in an active PBL environment to getting students to do well on assessments and score well enough to pass their respective courses. The focus is less on collaboration—which the students are familiar and comfortable with from elementary school—and more on preparing students for their future of either higher education or employment. Interestingly, teachers of secondary students often struggle with having a PBL environment in their classroom because many of their colleagues, coworkers, and mentors are accustomed to the traditional way of teaching students with paper assessments at the conclusion of the unit. This is where the dissonance in education lies. Students come to secondary school aware of and prepared for PBL classes. Multiple studies (Craig & Marshall, 2019; Holmes & Hwang, 2017; La Prad & Hyde, 2017; LaForce et al., 2017) have analyzed PBL and its impact on the secondary classroom, as well as how students learn.

Students, regardless of the educational setting, can have meaningful exposure to PBL (Craig & Marshall, 2019; La Prad & Hyde, 2017). Students in PBL specialty schools, STEM specialty schools, and traditional schools have impactful PBL lessons. Teacher planning and implantation are vital to the success of PBL in any classroom (Borich, 2014). Students, for example, at the Innovation through Design, Engineering, Arts, and Sciences Academy successfully used PBL in their classes and, in addition to the curriculum content, had successfully covered four important principles of the Coalition of Essential Schools. These included: learning to use one’s mind well; less is more, depth over coverage; personalization; and demonstration of mastery (La Prad & Hyde, 2017). When looking at standardized end-of-year assessment scores, however, PBL does not always yield significantly higher results than students in a traditional classroom setting. When looking strictly at content knowledge, these high-stakes assessments measured similar results from both traditional and PBL students (Craig & Marshall, 2019). PBL in secondary school seeks to teach students the formal curriculum, as well as additional skills. Having similar end-of-course assessment scores with students who learned beyond the mathematics content is successful (Cooper & Murphey, 2021; Murphy & Cooper, 2016).

Beyond academic success students experience from PBL, there are other benefits that extend beyond the report card (Holmes & Hwang, 2016; LaForce et al., 2017). Students who experience PBL can find an increased interest in the STEM fields beyond high school. When PBL is effectively implemented in their coursework students will experience an array of benefits, including: (1) positive student perceptions of project-based learning at their school would predict student interest in STEM careers, (2) higher project-based learning ratings would be positively correlated with science intrinsic motivation and ability beliefs, (3) higher project-based learning ratings would be positively correlated with mathematics intrinsic motivation and ability beliefs, (4) science intrinsic motivation and ability levels would be positively associated with interest in STEM careers, (5) mathematics intrinsic motivation and ability levels would be positively associated with interest in STEM careers, (6) science intrinsic motivation and ability beliefs would mediate the effect of project-based learning on interest in STEM careers, and (7) mathematics intrinsic motivation and ability beliefs would mediate the effect of project-based learning on interest in STEM careers. When looking beyond high school, the experiences that students have impacted their post-high school decisions regardless of gender or race (LaForce et al., 2017).

PBL does have an impact regarding the race and socioeconomic status of students. When analyzing the impacts of PBL on certain student groups, Holmes and Hwang (2016) noted that poverty level was a statistically significant factor for all students regarding achievement in their mathematics class. Again, as consistent with other studies (LaForce et al., 2017), the difference in end-of-course standardized test scores was not statistically significant, but students found themselves immersed in a learning environment that featured more than content (Holmes & Hwang, 2016; LaForce et al., 2017). PBL was observed to close the achievement gap among all students, but specifically students of low socioeconomic status. Overall, PBL did not impact secondary students' academic skills, but PBL did enhance internal cognitive skills and motivational factors. It also had a positive impact on minority students and low socioeconomic students (Holmes & Hwang, 2016; Jensen, 2009, 2013).

As outlined in the next section, higher education students also thrive in a PBL environment, but secondary education seems to have a strong grip on traditional methods of teaching and learning even though research shows students at the secondary level thrive in PBL classes as much as their younger and older peers (Craig & Marshall, 2019; K. Gushue, personal communication, September 15, 2020; Homes & Hwang, 2016; La Prad & Hyde, 2017; LaForce et al., 2017).

Higher Education

Research has been conducted regarding flipped classrooms, flipped learning, blended learning, and other approaches, and teachers as well as students all have varied experiences (Kristensen et al., 2020). Everything from learning outcomes, resources, time, and structure change depending on the content, the learner, and the educator. Full integration of PBL and a flipped classroom worked well and satisfied many students' motivational and learning needs. Students can then see the immediate application of the material discussed in their classes (Kristensen et al., 2020).

As with elementary students, higher education students do well academically in a PBL environment. These students, however, need to be reintroduced to the idea of working collaboratively with their colleagues because they were not exposed to collaboration in secondary school as much as they were in elementary school (K. Gushue, personal communication, September 15, 2020). Students in higher education benefit in various ways from the effective use of project-based learning in their classes across disciplines, including increased content knowledge, learning strategies, skills, motivation, and product quality (Guo et al., 2020). These areas help higher education prepare for the workforce after graduation; thus, exposure to PBL in collegiate courses helps makes graduates more marketable to potential employers (Hart, 2019).

Effects of Project-Based Learning

With project-based learning as a popular alternative to traditional lecture-based teaching models, Chen and Yang (2019) noticed a lack of meta-analyses and conducted their own study to determine the effectiveness of project-based learning. The authors compared 46 effect sizes from 30 different journals between 1998 and 2017. This covers 12,585 students from 189 schools in nine different countries. Calculating Hedges' *d*, the authors computed an effect size of 0.71, which has a medium to large impact on students (Chen & Yang, 2019). When looking at a hinge point of 0.4 being the measure of approximately one year of growth in students, project-based learning yields almost twice as much learning over the course of a school year, particularly when compared to other traditional methods used within the classroom (Chen & Yang, 2019; Hattie, 2017). This translates to students in a PBL centered class learning nearly twice as much as their peers taking the same course in a traditional setting. This calculation, of course, only accounts for using PBL without considering other strategies and techniques of effective classroom instruction. As stated earlier, a teacher is not going to only use one instructional method in their class for the entire school year. Knowing PBL has a high effect size incentivizes teachers to use it more in conjunction with other highly effective strategies. Furthermore, PBL naturally lends itself to additional classroom strategies such as collaboration, student discourse, and metacognitive strategies, which also yield higher effect sizes for students (Almarode, 2019; Hattie, 2017).

Across Disciplines

Balemen and Ozer Keskin (2018) conducted a meta-analysis to demonstrate the effectiveness of project-based learning. The authors analyzed 48 studies in the areas of physics, chemistry, biology, and science. The overall effect size of the studies was very large at 1.063, which showed project-based learning to be 86 percent more effective than traditional teaching methods (Balemen & Ozer Keskin, 2018).

Project-based learning has a place outside of the natural sciences as well. Duman and Yavuz (2018) conducted research on PBL in English classes and its impact on student attitudes towards the subject. Their research found that students exposed to the PBL classroom had better attitudes towards English class than their peers in the traditional course. Additionally, PBL was more likely to have a positive impact on students' overall attention in the class (Duman & Yavuz, 2018).

Project-based learning is more than just another strategy to use in the classroom. PBL offers students a new outlet to showcase what they know, and it allows teachers to guide, adapt, differentiate, and redirect along the way to ensure students are getting the most out of the class and assignments. Teachers can engage students and meet the students at their levels by using PBL in any course they teach. Students, regardless of age or ability level, will benefit from hands-on learning experiences (Lalor, 2017; Tomlinson, 2017).

Within Mathematics

Project-based learning has a positive impact on students within the mathematics classroom (Boaler & Dweck, 2016; Hattie, 2017). Cooperative learning has an effect size of 0.4, which yields approximately one year's worth of student learning. This is in addition to higher effect sizes that come with project-based learning, such as metacognitive strategies (0.6), feedback from teachers (0.75), classroom discussion (0.82), and high teacher expectations of students (1.57). Using Cohen's *d* with a hinge point of 0.4, project-based learning far exceeds the traditional one year of learning that students would receive in a more traditional math classroom (Almarode, 2019; Hattie, 2017).

In addition to the benefits in the classroom, student engagement in project-based learning can be seen outside of school as well. Students know that solving lists of problems are not how mathematics works in the world around them. Even students who do not desire to study mathematics beyond high school know that there is some degree of mathematics in their daily lives. However, they do not see what is studied in high school as applicable to those examples (Liljedalh, 2020). Students may argue that arithmetic is the most common topic of mathematics that they will need to learn, and other areas of mathematics are not as practical. What is often overlooked is how mathematics is embedded into daily activities. When people determine how much money they will have left after a purchase is a form of algebra. Playing the nightly lottery is a form of probability and statistics. Determining how much potting soil to purchase is a form of geometry. People often see and recognize these as important skills to have but do not necessarily understand their mathematical significance. Embedding these various practical scenarios into lessons and classroom experiences can help students realize their significance beyond the need to learn the topics for a test (Boaler, 2015). Additionally, making those connections in practical scenarios helps students retain much more information than facts, skills, or algorithms for a test (Almarode, 2019; Boaler, 2015; Bloom et al., 1956; Hattie, 2012, 2017).

Teachers have a responsibility to ensure that their students learn the material in the curriculum, and there are various ways to do that. Having a classroom that is rich in problem-solving tasks and assessment will not only engage the students more, but also help to make them mathematical thinkers and know how the skills they are learning, both from the official and unofficial curriculums, apply to the world (K. Gushue, personal communication, September 15, 2020; Jackson, 2018). When PBL brings the important skills of problem-solving and critical thinking class lessons, teachers can show students how math is applicable to their lives in realistic and tangible ways. These concrete examples and scenarios remove the ambiguity of why students need to be studying mathematics (Almarode, 2019; Barton, 2018; Boaler, 2015; Hattie, 2017; Smith & Stein, 2018; Smith et al., 2020; Willis, 2010).

PBL is not a novel idea in the world of education. Many elementary and post-secondary classrooms have and continue to use projects as an active part of the learning environment. However, many secondary schools miss the opportunity to seize on what elementary students come into the classroom knowing how to do. Too often, the focus for secondary teachers is to get through the curriculum, teach to the test, and prepare for students for college, but teaching students in a PBL classroom will do all of those and more (K. Gushue, personal communication, September 15, 2020). Students can learn the curriculum by interacting with their peers in addition to the material. In addition to the grade benefits PBL has for students, it also offers the subtle parts of the mathematics curriculum that teachers often hope students walk away with, such as problem-solving and critical thinking skills (Jackson, 2018). Notably, PBL also offers a place for all learners to thrive as teachers can more easily differentiate their instruction.

Frequently school administrators and parents want to know how teachers are reaching all students. While this can be challenging when multiple students all bring multiple learning strengths and weaknesses to the classroom, having a space where students can choose how they show mastery on material allows students and teachers to play to those different strengths while still covering the curriculum (Tomlinson, 2014, 2017).

Student Attitudes Towards School

Students have certain attitudes towards attending school depending on their upbringing, personal experience, and academic experiences (Vernon et al., 2019). For example, a student from a higher socioeconomic family with well-educated parents might see school as a necessity but understand the importance of being educated and putting effort into their studies. These students may plan to attend college and know that how they act and perform in their compulsory education will have a direct impact on their collegiate opportunities (Jensen, 2009). On the other hand, students from lower socioeconomic families may view school in an entirely different light. Students in this group may need to work to help support their families and cannot see school as anything more than an unnecessary requirement that takes up time where the student could rather be working. Additionally, these students often get frustrated when school seems like a requirement where they study topics that do not relate to life. These students often do not see the value in a high school diploma because they are not seeking any additional education beyond high school, and they need to work to help support their families (Jensen, 2013; Vernon et al., 2019).

Teachers have a responsibility to their students to meet the students at their level and help students reach their expectations, both academically and personally (Borich, 2014). While some students will be intrinsically motivated by this, others may need motivation and a connection to real-world scenarios to understand why their time needs to be spent studying (Jensen, 2009). Knowing the different attitudes that students have before entering a course is helpful, but also understanding ways to positively impact student attitudes while they are in class will help students more than passing the course (Vernon et al., 2019).

Elementary School

Teachers of elementary-aged students have a great responsibility for how they impact their students during their primary years. These teachers have a direct impact on how students view education and their attitudes towards school. Many young learners are excited to attend school and have minds open to learning. Furthermore, these young learners have had minimal experience with school and are still shaping their ideas and attitudes towards learning. When students enjoy attending school as well as enjoy how they are learning, their attitude towards school will be more positive. Teachers of the youngest learners can take advantage of the unique possibility they have to positively shape their students' attitudes towards school by making lessons more engaging and interactive for their students as they continue to not only learn about and understand the world around them, but also engage with the academic topics taught to them at that young age (Hacieminoglu, 2019; Hunker & Bill, 2017; Russo & Minas, 2020; Smith et al., 2020; Willis, 2010).

Secondary School

When students get to secondary school, they have had years of academic experience and have already started forming their opinions and attitude towards education. As with elementary teachers, secondary teachers also have an important job of continuing to shape the lives and attitudes of their students. Students, by the time of their adolescence, begin to make their own judgements and decisions about school and their attitude towards it that is impacted by far more than how the teachers present material. Friends also have an impact, possibly greater, on their peers regarding their attitude towards school. Many students, especially by the time they reach high school, have a firm attitude about school as well as specific subjects and think there is no way to change how they feel. This can make the job of the teacher more difficult, but it also makes it much more important that teachers plan how to present material. For example, a mathematics teacher might understand that many students do not have a good attitude towards or appreciate their subject. As a result, the teacher needs to be much more cognizant about how the material is presented, practiced, and assessed, so students, despite their lack of interest in the subject, can still understand the material and succeed in the class (Alistir & Irez, 2020; Ozedmir, 2012).

Higher Education

College students are not immune to how their attitudes impact their mathematics performance. Students who have a negative attitude towards mathematics courses are more likely to have poorer grades in those courses. Thereafter, students who fail university mathematics courses are more likely to have both course and mathematics anxiety. It is imperative for teachers at all levels, including higher education, to find a way to engage their students. College students can be seen as more motivated and as having positive attitudes towards school because, at that point, the students are choosing and paying to attend school. However, while a student might have a desire to earn a degree, that does not mean the student will be interested in all the required courses. Most, if not all, universities require at least one mathematics course to graduate with an undergraduate degree, yet many non-mathematics majors in colleges have poor attitudes towards the subject; thus, the required course is viewed as a stumbling block to achieving the goal. Because of this deeply rooted attitude towards mathematics that some students have prior to attending university, the professors of survey-level courses have a responsibility to engage students in the content and potentially find ways that the courses can relate to other degree-seeking students (Hegeman, 2015; Nunez-Pena et al., 2013).

Within Mathematics

As with any subject, students have varying attitudes towards mathematics. Students who do well in the class may be more inclined to have a positive attitude towards mathematics as opposed to students who struggle with the concepts. Additionally, students who plan to further their education beyond high school have a desire to do well in all courses. Even if students do not like the particular class they are taking, they will still strive to do well, since they see good grades as a necessity to achieve their future goals.

Mathematics teachers seem more pressured than teachers of other subjects to make the topics discussed in their class applicable. Students may not see the direct purpose of learning certain typical mathematics topics such as solving quadratic equations or graphing a line. The other core subjects of English, social studies, and science are more naturally applicable to students. English is the standard language, and reading is an important skill for students to be able to navigate through everyday life. Social studies can be an interesting look at historical events or the study of current global governments and politics, which students know are important to understand as global citizens. Science is the study of living things and directly impacts how students can live their lives. Mathematics, on the other hand, seems less initiative for students to understand why it needs to be studied (Russo & Minas, 2020; Smith & Stein, 2018).

Many experienced mathematics teachers will explain that mathematics is much more about the learning of problem-solving skills and critical thinking rather than rote memorization and the ability to know how to solve a quadratic equation or graph a line. However, students still have a hesitancy about learning the unofficial curriculum and can only see mathematics as the often-difficult topics discussed in class on a regular basis. Teaching mathematics in the traditional way will only fuel students' already preconceived notions and attitudes towards mathematics. To help impact student attitudes towards mathematics, teachers need to put the unofficial curriculum at the forefront of the lesson. This way, students can see they are doing far more than solving fictitious quadratic equations; rather, they are learning about tools that could help in solving a larger problems later. Teachers have a responsibility to ensure they are positively impacting the attitudes towards students by carefully and meticulously crafting everything that happens in the classroom while the students are present (Smith et al., 2020).

Every student enters a classroom with a different background than their peers. Teachers need to be able to guide all of those students regardless of their attitude towards school or mathematics. By building a classroom that is open to the various strengths of the students, teachers are giving students an opportunity to direct their own learning. This newfound confidence can help students to realize that they can, in fact, be successful in all courses, including mathematics. This could also have a positive impact on the students' overall attitude towards school and help students to thrive in an environment where they previously thought they could not (Boaler, 2015; Duman & Yavuz, 2018).

Attitude towards mathematics has an impact on overall mathematics achievement. When students have a positive attitude towards the subject, they are much more likely to be successful in studying mathematics. Conversely, students with a negative attitude towards mathematics will actively work to avoid work in class that directly impacts student grades in the course. Because of this correlation, it is imperative that teachers understand the role they play in student attitudes towards mathematics. When teachers intentionally plan class to be engaging, they are influencing student attitudes towards mathematics, which can result in students actively working to understand and make meaning of the material. This results in higher mathematical achievement (Demir-Lira et al., 2020).

Grit and Mindset

Grit can be defined as maintained perseverance with extreme passion (Duckworth, 2016). On the surface, this sounds more like educators in the classroom rather than students, but with proper planning and execution, teachers can help their students become grittier regarding their classwork in mathematics. When teachers allow students to engage in a productive struggle with the material presented in class, students are more likely to establish grit when working. Students often do not enter the classroom with an extreme passion for a subject. Especially when it comes to mathematics, students often, as will be discussed in a future section, do not have a positive attitude towards mathematics. Effective planning and the use of additional strategies that lend themselves to PBL can also establish a student's grit towards mathematics, which can improve a student's attitude. Student autonomy, for example, and self-pacing are ways that students can find themselves developing a passion for mathematics while engaging in the material in a PBL classroom. This develops students as independent learners and shows they can actively be a part of the mathematical community, while still showing that the students can, in fact, learn and apply mathematics to their daily lives (Duckworth, 2016; San Giovanni et al., 2020; Sanguras, 2018; Yoon et al., 2018).

Carol Dweck (2006) notes that people have either a fixed mindset or a growth mindset. While the idea of a growth mindset is applicable to numerous areas, there is a growing niche of study on growth mindsets in education. Students often incorrectly assume that they are either born intelligent or not, and if they struggle with a concept or skill, then they cannot successfully learn it. What students do not realize is that speed is not always better. Furthermore, students also need to understand that just because they do not understand a topic the first time it is presented does not mean that they cannot grasp the concept later. As mentioned earlier, students fall into multiple intelligences (Gardner, 1983). When teachers work to teach their students of various intelligence, the learners can experience success at different times, but still grasp the concepts discussed in class. Many times, especially as content becomes more difficult, students need more exposure to become proficient in the topic. It is also helpful when students understand that just because they do not understand something when it is first presented, or they struggle with some of the beginning stages of the material, it does not mean the student cannot master the material. It only means that the student has not mastered the material yet (Dweck, 2006; Seeley & Burns, 2015).

A growth mindset is almost imperative to have in a mathematics classroom. Many times, students are frustrated in mathematics courses because they do not grasp the material as quickly as they do in other subject areas. While other subject areas such as English and social studies are more intuitive for students to understand, mathematics has many nuances that students find annoying, overwhelming, and cumbersome. These thoughts cause frustration among students when they do not immediately grasp the topic discussed in class and immediately dismiss themselves as not being able to understand mathematics. What students often do not recognize is how making mistakes helps in the learning process by showing them ways that do not work in solving a problem. What is also often frustrating to students is the seemingly one correct way to solve a mathematics problem rather than multiple methods to arrive at the same correct answer (Boaler, 2002, 2015; Boaler & Dweck, 2016; Dweck, 2006).

Suppose students engage in PBL and can see that they do not need to simply employ one method or strategy to arrive at a correct response. Struggling with and collaborating on problems in class can help students realize that not everybody will understand all topics when they are first presented. Working together through a productive struggle with classmates allows students to see that they are not the only ones who might be struggling with problems. Moreover, it is important for students to realize and understand that simply because they do not master a topic right away does not mean they will never master that topic. Rather, it means that the student has not mastered the topic yet and could use more practice, exposure, and explanation before being able to fully understand the topic. Mistakes, when taken as learning opportunities, can be much more valuable to students than simply understanding a topic immediately. Students, especially when studying mathematics, need to understand that mistakes are not a stopping point where frustration ensues, and they stop learning. Rather mistakes are a time to realize that they might not understand the material yet, but in time with more work, the student can still grasp the content and master the skill covered in class (Barton, 2018; Brock & Hundley, 2016; Jansen & Berry, 2020).

When implemented appropriately, PBL integrates the concepts of grit and mindset in the classroom (Cooper & Murphey, 2021). Teachers can ensure they give appropriate feedback to students, so they continue to learn (Jackson, 2018). Additionally, teachers can explicitly remind students of appropriate ways to think about learning, so students know they are not simply born smart, but rather with the capacity to grow their knowledge and understanding (Cooper &

Murphey, 2021; Duckworth, 2016; Dweck, 2006). When teachers emphasize effort, show students how they can learn even if they have struggled in the past, and exemplify grit and a growth mindset, students can be more motivated to work in class even if they traditionally do not engage in lessons (Mendler, 2021).

Summary

Project-based learning has long been around in the world of education and will likely not be leaving any time soon. Having students complete a project either as part of the instruction, as the assessment, or both bring new depth to the material that more traditional worksheets and assessments do not. Oftentimes, students think they are not good at mathematics because they cannot complete the practice in class or score poorly on their tests. Additionally, students do not easily see where the material they are learning applies to their everyday lives and the world around them. Not only does a project-based lesson or assessment incorporate these real-world ideas, but a project-based classroom also allows students to start thinking on a higher level. Being able to take and analyze a situation, then synthesize that information into a coherent response, forces students to have a thorough understanding of the concepts presented. While there is extensive research on project-based classroom, there are some gaps in the literature. A current gap in the research of the project-based classroom is how the attitudes of high school students who engage in PBL differ from the attitudes of their peers in a traditional high school classroom.

PBL, just as with any classroom pedagogy, varies at different levels and within different disciplines. Differences between elementary, secondary, and higher education classrooms drastically change how a project-based should be implemented and will be perceived by students. Furthermore, a project-based English classroom will look different than a project-based mathematics classroom. A standardized way to introduce projects into a classroom will not work across every discipline. Some courses, such as the arts, have been pursuing project-based learning regularly for some time, but their model will not work the same way in social studies or science classrooms. Appropriately integrating PBL into courses across disciplines, specifically, mathematics, can show students how and why it is important to learn the subject and could shift their overall attitudes towards the course.

Student attitudes towards mathematics vary greatly depending on their mathematical experiences in school. Some students have good experiences and learn to like and appreciate mathematics during their time studying the subject. Other students have less-than-ideal experiences with mathematics. They find the courses challenging, cumbersome, and a roadblock to advancing in school. Classroom teachers have the responsibility to teach students at their level, regardless of what that is when the students enter the teacher's classroom. Every individual instructor that a student has helps in shaping that student's attitude towards the subject of mathematics. Project-based learning and its subsequent, effective classroom strategies can be one tool that teachers use to help positively impact a student's attitude towards mathematics. Not every student sitting in a high school math class is going to become a mathematician, but teachers can use the tools that they have at their disposal to help students learn to appreciate the subject more and potentially alter the student's attitude towards mathematics. Knowing and understanding a potential relationship between PBL and student attitudes towards mathematics can help shape how a teacher conducts their classes. The need for mathematically literate citizens is likely not going to diminish, so teachers can help students find an appreciation for mathematics, so students do not fear or misunderstand the mathematics that regularly occurs in the world around them. This can prepare students to be mathematically literate citizens once they finish school.

When teachers take the job of making mathematics relevant to students seriously, they are offering the potential for students to not only understand and appreciate the subject, but also become problem solvers, thinkers, and inquirers. Effective teachers look for ways to connect to students and ensure that students are maximizing their learning. Furthermore, students can develop a growth mindset where they understand that learning mathematics takes time and is a process. Teachers can also make grittier students who want to put in more effort to understand a confusing topic of study. When teachers effectively plan and implement PBL in their classes, students can see a myriad of results, including the potential to have a more positive attitude towards the subject of mathematics (Jackson, 2018). The question unanswered by the literature is the impact the PBL can have on student attitudes towards mathematics. The literature gives multiple examples of the benefits of PBL (Craig & Marshall, 2019; Holmes & Hwang, 2017; LaForce et al., 2017; La Prad & Hyde, 2017; Wurdinger et al., 2020), as well as the attitudes students have towards studying the subject (Alistir & Irez, 2020; Hacieminoglu, 2019; Hegeman, 2015; Hunker & Bill, 2017; Nunez-Pena et al., 2013; Ozedmir, 2012; Smith et al., 2020; Willis, 2010), but the connection between the two, if one exists, is not clear. Students can experience a myriad of benefits from being in a project-centered classroom (Almarode, 2019; Barton, 2018; Bender, 2012; Boston, 2017; Flores & Malloy, 2009; Hattie, 2017; Jackson, 2018; La Prad & Hyde, 2017; Seeley & Burns, 2015; Willis, 2010), but attitude could be an additional advantage to convince traditional teachers and students that PBL is worth the time and effort for the student’s benefit to not only learn the mathematical content, but also understand and apply the material to other settings which remain at the highest level of learning (Bloom et al., 1956) as well as reaching multiple types of intelligence within the classroom (Gardner, 1983).

CHAPTER THREE: METHODS

Overview

The purpose of this quantitative, quasi-experimental, nonequivalent control group study was to determine if there is a difference between high school students' attitudes towards mathematics after exposure to PBL as opposed to exposure to traditional instruction when controlling for pretest scores. Chapter three begins by introducing the design of the study, including full definitions of all variables. The research questions and null hypotheses follow. The participants, setting, instrumentation, procedures, and data analysis plans are presented.

Design

This research study employed a quasi-experimental nonequivalent control group design. This research design tested the effects of the independent categorical variable of the type of instruction on the dependent continuous variable of student attitudes towards math while controlling for the covariant of pretest scores. This was best served by an analysis of covariance (ANCOVA) (Gall et al., 2003).

A quasi-experimental nonequivalent control group research design assigns the participants to either the control group or experimental group where only the latter receives treatment (Cresswell, 2014; Gall et al., 2003; Johnson & Christensen, 2017). Students were not randomly placed into groups; rather, the groups come from predestined placement, such as a course the students are taking. Thus, convenience sampling was the best way to gather data from these groups. Furthermore, participants in nonequivalent control groups took both a pretest and a post-test to collect data. This required data to be collected from the participants twice—once at the beginning of the study and once after the experimental group has received the treatment of exposure to PBL. Afterward, the ANCOVA was used for comparison of means to determine if any difference exists in the attitudes towards mathematics between the students in the control group and the students in the treatment group.

The researcher examined two key variables, which included type of instruction and student attitudes toward mathematics. According to Borich (2014), the type of instruction is how information is shared between students and teachers during lessons. For example, traditional instruction is where the teacher imparts information to the students (Tomlinson, 2014); whereas PBL is a hands-on approach for students with tangible deliverables (Boaler, 2015). Student attitudes towards mathematics, the continuous dependent variable, was the student's reaction to the subject as measured by the Attitudes Towards Mathematics Inventory (ATMI) (Tapia & Marsh, 2004).

The covariate in this study was the student pretest score measured on the same instrument. The treatment in this study was project-based learning, and it was used throughout the duration of the semester only for the treatment group between the pretest and post-test.

Research Question

The research question for this study asked the following:

RQ: Is there a difference between high school students' attitudes towards mathematics in a PBL classroom versus a traditional classroom when controlling for pretest scores?

Hypothesis

The null hypothesis for this study was as follows:

H_0: There is no difference between high school students' attitudes towards mathematics scores in a PBL classroom versus a traditional classroom when controlling for pretest scores.

Participants and Setting

The population for this study consisted of a convenience sample of high school students in two suburban high schools in northern Virginia during the 2021-2022 school year. The students represented a diverse population of various backgrounds. The school was in a diverse area with a high population of low socioeconomic students. Participants were selected from students enrolled in Algebra, Functions, and Data Analysis (AFDA) which was a sophomore/junior-level mathematics class during the 2021-22 school year. It was a mix of general education students, special education students, and English Language Learners. These students were grouped based on their experience with project-based learning to measure what, if any, difference appeared in their attitudes towards mathematics.

Participants were selected from a convenience sample of students based on whether they have been exposed to project-based learning or have only been exposed to traditional learning. Traditional learning practices could include lectures, practice problems, and traditional exams that do not include a project component within their mathematics classes. At the beginning of the 2021-22 school year, the students were introduced to the study offered an opportunity to opt out of the study should they choose. Students were identified as eligible for the study by their enrollment in an AFDA course.

The sample size for this study exceeded 66 students, which, according to Warner (2013), exceeds the required minimum number of participants for an ANCOVA with a statistical power of .7 at the .05 alpha assuming a medium effect size. The sample came from students taking non-entry level mathematics classes at two high schools in northern Virginia. The students were either in classes that actively use project-based learning during regular instruction and assessment or in more traditional classes with standard end-of-unit assessments. The participants consisted of 28 males and 38 females in AFDA 56% of the students identified as Hispanic, 23% White, 15% Black, 4% Asian, and 2% mixed or other.

The treatment group of students exposed to project-based learning contained 33 students taking AFDA of whom 18 are male, and 15 are female. The students were 52% Hispanic, 21% While, 18% Black, 6% Asian, and 3% mixed or other. The control group of students exposed to traditional instruction and assessment will contain 33 students taking AFDA of whom 19 are male, and 14 are female with 76% Hispanic, 6% While, 12% Black, 3% Asian, and 3% mixed or other.

Instrumentation

To measure students' attitudes towards mathematics, Martha Tapia's (2004) Attitudes Towards Mathematics Inventory (ATMI) is a valid and reliable instrument. Furthermore, a quasi-experimental nonequivalent control group research design allowed determination between the independent variable of project-based learning and the dependent variable of student attitudes towards mathematics. The ATMI was administered to high school mathematics students from two large suburban high schools in northern Virginia. One group featured students who are exposed to PBL, and the other included students who are used to traditional instruction and assessment. The purpose of this instrument was to gather data on student attitudes towards mathematics. Students were administered the ATMI at the beginning and the end of the research study. Prior to taking AFDA, students in neither group will have been exposed to PBL. Baseline pre-test scores were used as a covariate to control for initial group differences on attitudes towards mathematics which were compared with post-test scores gathered from the ATMI. This determined if there was a significant difference in the means of the two groups. See Appendix D for a copy of the instrument and the author's permission to use the instrument.

Tapia and Marsh (2004) developed the ATMI to determine underlying issues with students' attitudes towards mathematics. The items focus on assessing six areas regarding students' attitudes towards mathematics: confidence, anxiety, value, enjoyment, motivation, and parent/teacher expectations. Originally having 49 items, once the individual factor analyses were conducted on each item, the nine lowest scoring items were removed to increase the overall Cronbach alpha level of the instrument.

The ATMI contains 40 items and uses a 5-point Likert scale from strongly disagree to strongly agree. Responses are strongly disagree = A, disagree = B, neutral = C, agree = D, and strongly agree = E. Scores range from 40 to 200, with an average completion time of 10-20 minutes. A full score of 200 indicates a student strongly agrees with each statement, including several reversal statements, such as item 25, indicating mathematics is boring. Numerous additional studies (Khine & Afrai, 2014; Majeed et al., 2013; Yee Lim & Chapman, 2013) verified the reliability of the instrument yielding a Cronbach alpha level of 0.97 by conducting individual factor analyses on the items in the ATMI using the instrument with all age levels in addition to translating the tool into Arabic and having it still yield similar results. Content validity was established in Tapia's (2004) original research by relating the inventory items to five variables: confidence, anxiety, value, enjoyment, and motivation. Each item on the ATMI was subject to factor analysis to ensure the instrument's validity. Additionally, the items showed good internal and test-retest reliability (Tapia & Marsh, 2004).

Procedures

First, the researcher contacted the school division for permission to conduct the research. Once the school division approved, Institutional Review Board (IRB) approval through Liberty University followed. Then, the researcher met with the building principal to discuss the research plan and seek permission, as well as developed a plan for communicating the research proposal with relevant parties. After principal approval, communication to potential students' parents were mailed home to get their permission for students to participate in the study or opt out. Finally, student assent was sought. Documentation for all permissions can be found in Appendix A.

Once IRB, district, parent, and student approval were granted, the researcher distributed the ATMI to students to complete in both groups. Students in both the control and treatment groups were then taught the same standards which can be found in the unit plans for both groups in Appendix B. Students in the treatment group, however, were taught using project-based lessons that can be found in Appendix C. The ATMI was then completed again at the conclusion of one semester of instruction after the control group has been taught using PBL methods in the mathematics class for 18 weeks (one semester of instruction). Once completed, inventory information was analyzed using the procedures outlined in the next section. Students remained anonymous by using student identification numbers on inventory results. To ensure anonymity, surveys were conducted using an online Microsoft Form using student identification numbers rather than student names.

Data Security

During all stages of data collection, any information that could identify participants was protected. Data was stored securely, and only the researcher had access to those data records. Data was stored on a password-protected computer. When not being used, the computer was stored in a locked cabinet. These data will be retained for a period of five years after the completion of this research study.

Data Analysis

For data analysis, SPSS version 28 was used. SPSS is the most used software in education research, used to store, display, and analyze data (Gall et al., 2003). Data was analyzed using a one-way analysis of covariance (ANCOVA). For this study, the dependent variable was students' attitudes towards mathematics, while the independent variable was the type of instruction they received—project-based learning or traditional—with a covariate of pre-test scores on the ATMI. Next will be to "test the statistical significance of the observed differences in the mean scores of the treatment and control group" (Gall et al., 2003, p. 408). Since this study has one categorical independent variable with multiple groups (type of instruction received), one continuous dependent variable (attitudes towards mathematics from the post-test), and one control variable (attitudes towards mathematics from the pre-test), an ANCOVA best fit this study. An ANCOVA analyzed any statistical significance between the two groups and determined if there is a significance in scores between students exposed to project-based learning and those only exposed to traditional learning and assessment while controlling for pretest scores (Gall et al., 2003).

A visual inspection of the data was conducted for missing or inaccurate entries. Box and whisker plots for each group will be used to look for extreme outliers. Kolmogorov-Smirnov was used to confirm the assumption of normality for an anticipated sample size exceeding 66, while Levene's Test of Equality of Error Variance was used to test for equal variance. Additionally, a series of scatterplots between pretest and post-test variables for each group tested for the assumption of linearity and assumption of bivariate normal distribution. Furthermore, noting parallels among the relationship between the dependent variable and the covariate test for an assumption of homogeneity of regression slopes. Partial eta squared was used to calculate the effect size. The null hypothesis will be rejected at the 95% confidence level (Gall et al., 2003; Warner, 2013).

CHAPTER FOUR: FINDINGS

Overview

In Chapter Four, descriptive statistics, data screening procedures, and assumptions for the analysis of covariance (ANCOVA) will be discussed. The results for the null hypothesis will be discussed including the results of the ANCOVA controlling for pretest scores conducting using SPSS Version 28.

Research Question

RQ: Is there a difference between high school students' attitudes towards mathematics in a PBL classroom versus a traditional classroom when controlling for pretest scores?

Null Hypothesis

H_0: There is no significant difference between high school students' attitudes towards mathematics scores in a PBL classroom versus a traditional classroom when controlling for pretest scores.

Descriptive Statistics

The final participants in this study consisted of 66 high school mathematics students taking Algebra, Functions, and Data Analysis (AFDA). The participants were evenly distributed among the control and experiment groups. Of the 66 participants, 42% were male and 58% were female. Fifty-six percent of participants were Hispanic, 23% White, 15% Black, 5% Asian, and 1% mixed or other. Fifty percent of the participants were identified as English Language Learners. Mean posttest scores, standard deviations, and participants for each group can be seen in the following tables. These include the covariate, dependent variable, and adjusted means of the dependent variable.

Table 1

Descriptive Statistics: Covariate: PreTest

Group	*n*	*M*	*SD*
1 - Control	33	10.69	0.81
2 - PBL	33	11.00	0.60

Table 2

Descriptive Statistics: Dependent Variable: PostTest

Group	*n*	*M*	*SD*
1 - Control	33	10.79	0.96
2 – PBL	33	10.79	0.69

Table 3

Descriptive Statistics: Dependent Variable (Adjusted Means): PostTest

Group	*n*	*M*	*SE*
1 - Control	33	10.84	0.14
2 - PBL	33	10.74	0.14

Results

Data Screening

Data screening was conducted on each group's covariate and dependent variable. The researchers sorted the data on each variable and scanned for inconsistencies. No data errors or inconsistencies were identified. Box and whisker plots were used to detect extreme outliers on each dependent variable. These data did show two points within the control group to be outliers greater than three standard deviations from the mean when converted to a z-score, but the researcher retained those data points as they were determined not to greatly impact the overall results of the analysis when comparing the results of the analysis that included those data points with an analysis where those data points were removed. Figures 1 and 2 shows box and whisker plots for the covariate and dependent variables.

Figure 1

Box and whisker plots (covariate).

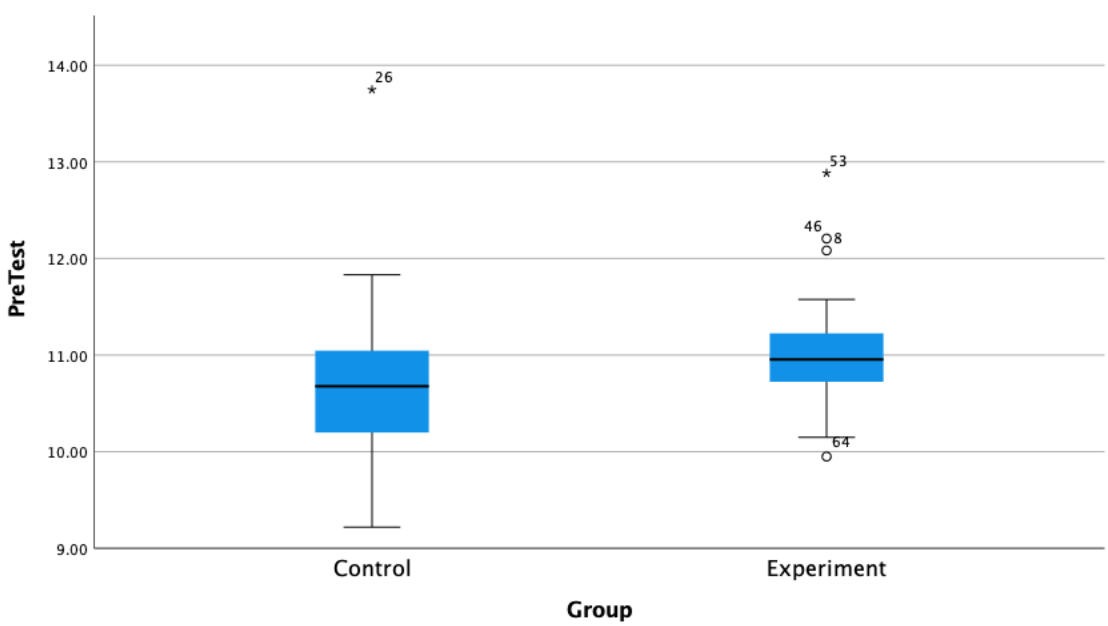
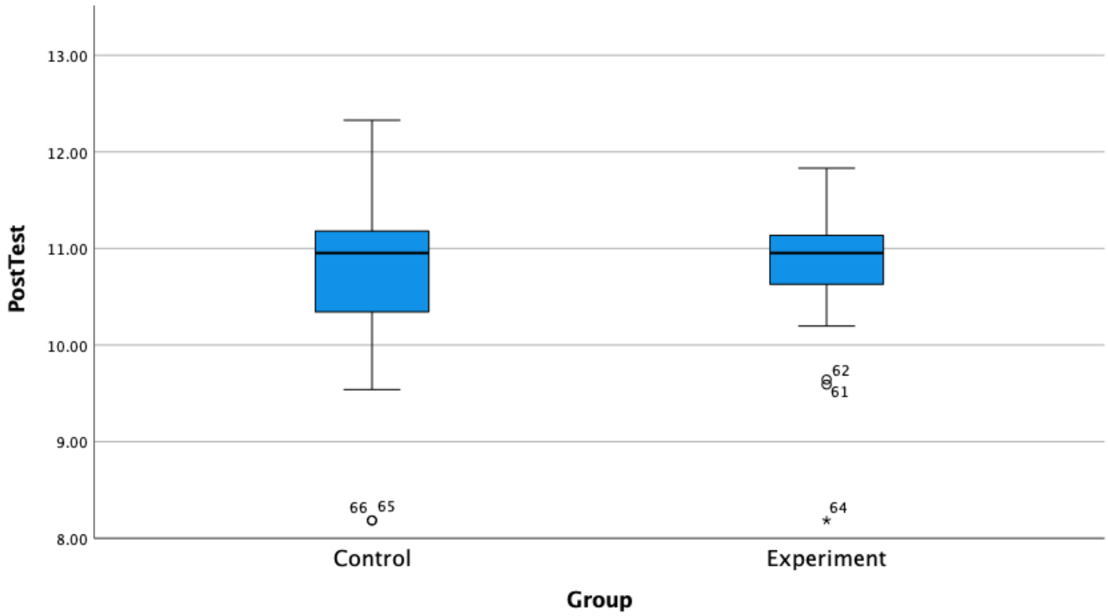


Figure 2

Box and whisker plots (dependent).



Assumptions

An Analysis of Covariance (ANCOVA) was used to test the null hypothesis. The ANCOVA required that the following criteria be met: assumptions of normality, assumption of linearity and bivariate normal distribution, assumptions of homogeneity of slopes, and the homogeneity of variance.

Normality was examined using a Kolmogorov-Smirnov test. Kolmogorov-Smirnov was used because the sample size was greater than 50. After transforming the posttest data by applying the square root function to account for moderate positive skewness, no violations of normality were found. See Table 4 for Tests of Normality.

Table 4

Tests of Normality

	Kolmogorov-Smirnov			
	Groups	Statistic	*df*	Sig.
PreTest	1 - Control	.171	33	.015
	2 – PBL	.126	33	.200
PostTest	1 - Control	.128	33	.183
	2 - PBL	.165	33	.023

The assumptions of linearity and bivariate normal distribution were tested using scatter plots for each group. Linearity was met and bivariate normal distribution was tenable as the shapes of the distributions were not extreme. Figure 3 and Figure 4 include the scatter plots for each group.

Figure 3

Scatter Plot for Control Group

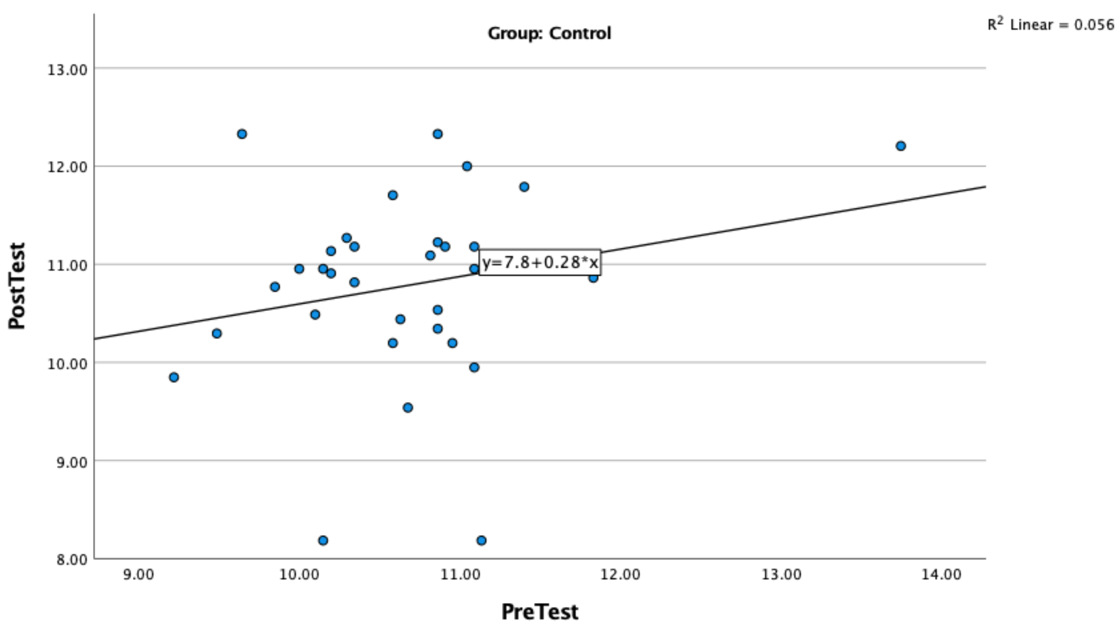
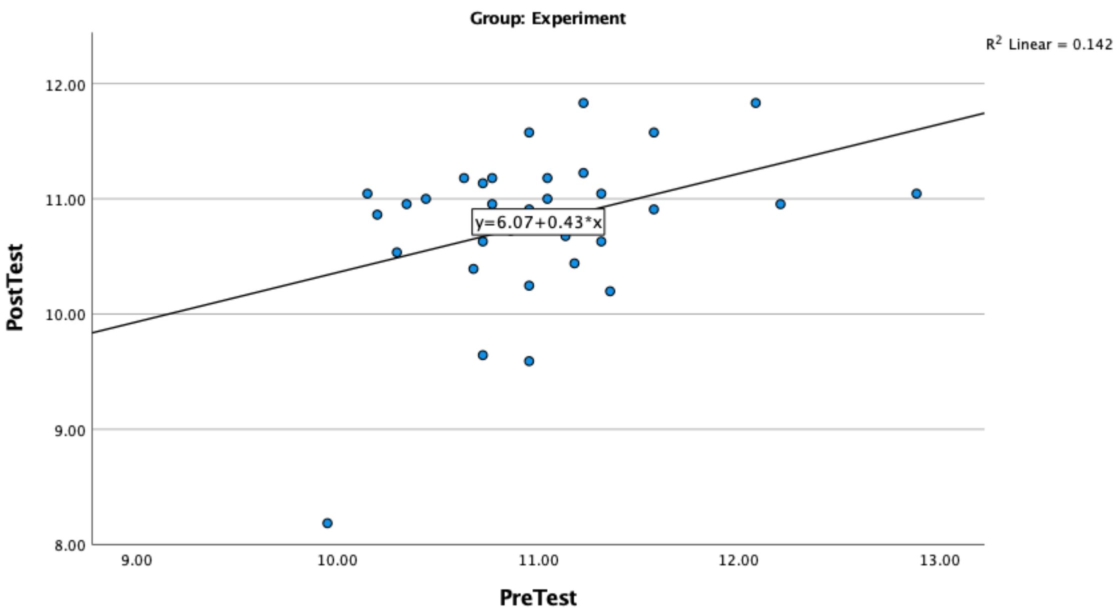


Figure 4

Scatter Plot for PBL Group



The assumptions of homogeneity of slopes was tested and no interaction was found where p = .669. Therefore, the assumption of homogeneity of slopes was met. The assumption of homogeneity of variance was examined using the Levene's test. No violation was found where p = .160. The assumption of homogeneity of variance was met.

Results for Null Hypothesis

An ANCOVA was used to test the null hypothesis regarding student's attitudes towards mathematics in a project-based classroom and a traditional classroom. The null hypothesis was failed to be rejected at a 95% confidence level were $F(1,63) = .216$, p - .664, $\eta_p^2 = .003$. The effect size was very small. Because the null failed to be rejected, a post-hoc analysis was not conducted.

Table 5

Multiple Comparisons of Groups

Pairwise Comparisons

Dependent Variable: days

(I) group	(J) group	Mean Difference (I-J)	*SE*	Sig.[a]	95% Confidence Interval for Difference	
					Lower Bound	Upper Bound
Control	PBL	.094	.202	.644	-.310	.498
PBL	Control	-.094	.202	.644	-.498	.310

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

CHAPTER FIVE: CONCLUSIONS

Overview

The purpose of this quantitative, quasi-experimental, nonequivalent control group study was to determine the impact project-based learning has on students' attitudes towards mathematics. Quasi-experimental, pre-test/post-test design was used to determine causality between the experimental treatment and the outcome since participants could not be randomly assigned to groups. Chapter 5 provides a summary and discussion of the research findings, followed by the study's results considering relevant literature and theory. Methodological and practical implications will be described as well as the study's limitations and recommendations for future research.

Discussion

The purpose of this study was to see if high school students' attitudes towards mathematics would increase when being taught in a project-based environment. This study was inconclusive as to whether being in a project-based environment helped improve students' attitudes towards mathematics when controlling for pretest scores. The literature states that students benefit from engaging in projects and areas that promote critical thinking. While it has been shown that student scores increase when learning in this way, more research needs to be conducted to determine whether PBL has a direct impact on student attitudes towards the subject. While students have a general distaste towards mathematics, it seems that simple changing how the subject is taught might only be the first step in assisting students in gaining an appreciation for mathematics. As the research presently shows, there seems to not be a connection between learning a subject and having a positive attitude towards that subject.

This study attempted to show a difference between high school students' attitudes towards mathematics scores in a PBL classroom versus a traditional classroom when controlling for pretest scores. While the results were inconclusive to determine a direct correlation, there are several connections that can be made between PBL and student attitudes towards mathematics.

Considering the theoretical framework (Bloom et al., 1956; Gardner, 1983; Piaget, 1968; Schunk, 2020; Vygotsky, 1934/1986; Woolfolk, 2013), project-based learning is an ideal classroom environment. When students can use their prior knowledge, life experience, and specific area of intelligence, they can successfully generate new knowledge and understanding of topics discussed in class. Students' attitudes towards mathematics did not decrease when taught in a PBL setting. This research does not support the impact PBL has on students and how they learn in mathematics class, and, as shown in chapter 4, it certainly does not reveal a negative impact on student attitude towards the subject. While actively engaging in mathematics, students were able to learn the material even if there was not significant impact on their attitude towards the subject.

As the literature states, PBL has a positive effect size on students (Almarode, 2019; Hattie, 2009, 2012, 2017), but simply because a certain type of instruction helps students learn mathematics more deeply, more research needs to be conducted to solidify if there is a link between PBL and student attitudes towards mathematics. Knowing there is a possible connection between PBL and student attitudes towards mathematics could help teachers in planning and preparing lessons for students to show what they have learned in class.

Overall, the conclusions from this study support the literature, even though results were inconclusive. Project-based learning still has a positive impact on students even if it does not directly impact how the students feel about the subject.

Implications

Chapter four discussed the hypothesis and what the results for this study add to the body of research. The study did not yield the results anticipated considering the literature. Engaging in PBL allows students to see applications of mathematics in real world scenarios and helps students to know mathematical topics do have a place outside of the classroom. PBL has definite benefits in the class for students of all ages, but while those benefits exist this research does not offer conclusive evidence that PBL impacts a student's attitude towards the subject. Topics in class may help students understand mathematics better, but they do not necessarily improve their attitude towards the subject. Students may still not have the best opinion of mathematics even if they engage in teaching practices that are beneficial to their learning and help them in being more successful in a mathematics course. While this study may not be conclusive enough to associate PBL with student attitudes towards mathematics, it does offer insight into how teachers might be able to continue to offer students ways to apply class knowledge to a practical setting. Additionally, further research could help in determining a relationship between PBL and student attitudes towards mathematics.

While working with students, teachers must reach the needs of all learners which can be done through differentiating, and this lends itself to a PBL classroom (Boaler, 2015; Liljedalh, 2020; Tomlinson, 2014). This research does not diminish the importance of PBL. Still, it does not fully connect the idea of PBL positively impacting student attitudes towards mathematics. Students will still benefit from a hands-on experience that uses their prior knowledge and a practical approach to the subject (Piaget, 1968) which may even foster an appreciation for the subject without completely shifting their attitudes.

Additionally, replicating this research in different, more narrow environments could allow researchers to further look for correlations in PBL and student attitudes. This study attempted to look at the correlation from a general perspective, but more specifically research in general education, special education, and English Language learning groups could benefit could show a relationship between PBL and student attitudes. Looking at these specific subgroups could further the general research attempted in this study and help determine PBL's impact on students' attitudes towards mathematics. Focusing on subgroups rather than the general student population could exemplify Simpson's Paradox arguing that subgroups behave a certain way, but the collective group does not follow the same trend.

Limitations

A larger-than-anticipated percentage of students in this study had a native language that was not English; thus, this made it more challenging for them to complete the ATMI. The inventory, while valid and reliable in English, has been tested in few other languages and could not be translated and tested before this study. While many English Language Learners could have completed the inventory sans issue, it still bares noting their limited English could have impacted results. Translating the ATMI into Spanish, testing for validity and reliability, and then using the translated version could help to ensure that students comprehend the statements.

This study took place over the limited timeframe of one semester and while it did produce results, the researcher wondered what difference and entire year could make in the results of these data. Having students work an entire school year with PBL could yield more desirable results after they experience an entire course with that type of instruction. Additionally, there were a limited number of students available for the study since the study focused on one non-entry level mathematics course. More student participation could have yielded differences and more generalizable results.

An additional limitation could include student apathy towards completing the survey. Some students answered consistently the same for all 40 items in the inventory and this likely swayed the data. While this did not cause any outliers or abnormalities to be scrubbed from the data, it is still a limitation worth mentioning as some students were "neutral" for the entire inventory.

Recommendations for Future Research

Recommendations for future research are as follows:

1. Try to have the ATMI tested and available in the student's native language. The removed the language barrier for students and can give a closer look at how PBL impacts English Language Learners.
2. Have students participate in the study on a longer timeframe. Give students the ATMI at the beginning of the school year and have them participate in PBL for the entire course before taking the inventory again and comparing results. Additionally, longer term studies could also be useful by following students for multiple years in PBL environments.
3. Analyze the impacts of PBL on special education students. This study was void of special education students, but they are an important part of the school population and teachers need to know how to best serve them.
4. Conduct the study in a course that includes a state test. This study was conducted in a course that was not state tests. It would benefit the body of knowledge to see how PBL impacts student attitudes towards mathematics in a state-tested course. Furthermore,

research could be conducted to see if student attitudes towards math are in any way connected with scores on state assessments.

5. Study the relationship between PBL and student attitudes towards mathematics in other levels. This study was conducted with high school students, but future research could see if a connection between PBL and student attitudes towards mathematics exists in elementary, middle school, or college students.

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APPENDIX A

Dear AFDA Parents or Guardians and Students:

As a graduate student in the School of Education at Liberty University, I am conducting research as part of the requirements for a doctoral degree. The purpose of my research is to determine a connection between project-based learning and how students feel about math, and I am writing to invite eligible participants to join my study.

Participants must be high school students enrolled in Algebra, Functions, and Data Analysis (AFDA) for the first time. Participants, if willing, will be assigned to either a control group or a treatment group based on the 2022-2023 school year student placement. Students in the treatment group will do project-based learning (PBL) for three units (approximately three months). Students in the control group will do the three units with normal teaching instructions. Participants will also be asked to participate in a survey at the beginning of the study and again at the end of the study. It should take approximately 20 minutes each to complete the surveys during their regularly scheduled AFDA class. Names and other identifying information will be requested from your student as part of this study, but the information will remain confidential.

For your student to participate, they will complete the survey and during his/her AFDA class.

An opt-out consent document is attached to this letter. The opt-out consent document contains additional information about my research. If you choose to allow your student to participate and your student is willing to participate, then no further action is needed from you. If you or your child DO NOT want to participate, then please sign the attached opt-out form and return it to your student's teacher by September 16. Students who do not participate will still receive the PBL or normal teaching instruction, but they will not complete the surveys.

Sincerely,

David Postlethwait
Graduate Student
[redacted]

APPENDIX B

[redacted] **Mathematics** **Algebra, Functions and Data Analysis**

Unit 1: Linear Programming

Unit Focus

In this unit the focus is on using real world applications to identify optimal solutions. Students will review solving systems of equations and inequalities to determine feasible region solution areas. Students will then use vertices from the feasible region to determine maximum and minimum values in an optimization function.

Suggested Duration: 5 Blocks (Plus 2-3 Blocks max for Activities)

Stage 1 – Desired Outcomes

Established Goals

Virginia Standards of Learning

Students will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

WIDA English Language Development Standards

English language learners communicate for Social and Instructional purposes within the school setting.

English language learners communicate information, ideas, and concepts necessary for academic success in the content area of Mathematics.

Enduring Understandings	Essential Questions
Students will understand that…	***Students will keep considering:***
U1 some practical problems involve multiple linear relationships. Linear programming accounts for all these linear relationships and provides the solution to the problem.	**Q1** What do the various regions mean, including the feasibility regions, in the context of the practical situation being illustrated graphically?
U2 decision makers in many organizations formulate linear programming problems as part of their management activities so the best solutions can be utilized.	**Q2** How does the objective function relate to the practical situation?
	Q3 How do linear programming problems help decision makers make quality decisions for their businesses?

Unit 2: Characteristics of Functions

Unit Focus

The focus of this unit will be to review and extend knowledge of characteristics of various functions to include linear, quadratic, exponential, and logarithmic function families. Characteristics will include domain and range, intervals on which a function is increasing or decreasing, end behaviors, absolute maxima and minima, zeros, intercepts, and vertical and horizontal asymptotes. Students will also determine the values of a function and make connections between multiple representations of functions to include verbal descriptions, tables, equations, and graphs.

Suggested Duration: 6 Blocks

Stage 1 – Desired Outcomes

Established Goals

Virginia Standards of Learning

Students will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

AFDA.1 The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families and their characteristics. Key concepts include

- **a)** domain and range;
- **b)** intervals on which a function is increasing or decreasing;
- **c)** absolute maxima and minima;
- **d)** zeros;
- **e)** intercepts;
- **f)** values of a function for elements in the domain;
- **g)** connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs.
- **h)** end behavior; and
- **i)** vertical and horizontal asymptotes

WIDA English Language Development Standards

English language learners communicate for Social and Instructional purposes within the school setting.

English language learners communicate information, ideas, and concepts necessary for academic success in the content area of Mathematics.

APPENDIX C

Name _________________________________ Period ___ Date____________

Unit 1 Inequalities – Summative Project v2

Three friends work at a restaurant together. Apply your knowledge from our unit on inequalities to model and analyze each of their scenarios below. Be sure to read carefully and answer each question.

Carin is trying to save up for a car and needs to earn at least $550 a week to buy the car they want. As a server, they earn $8 an hour plus tips. The manager has advised them that they can expect to make $225 in tips. Carin's situation can be modeled by the inequality:
$8x+225\geq550$

Solve to determine the minimum number of hours Carin will need to work to achieve their goal. Remember to show your work and interpret your solution in context to the situation. [T1]

Due to a staff shortage, the manager is offering to pay $12 an hour this week plus $225 in tips. Carin is taking advantage of the extra pay and has changed their goal to earn at least $600 this week.
Write the inequality (use one of these: <, >, <, >) Carin would use to model this situation. [T4]

Solve to determine the minimum number of hours Carin will need to work to achieve their new goal. Remember to show your work and interpret your solution in context to the situation. [T1]

D’Andre is working as a host at the restaurant for $12 an hour and as a server for $15 an hour. His goal is to earn at least $500 a week.

Write the inequality D’Andre would use to model his situation. [T4]

Use Desmos to graph the system. Based upon the Desmos graph, identify two possible examples of the number of hours he could be scheduled to work in each position. [T2]

D’Andre decided he wanted to earn tips and therefore accepted a pay reduction to match Carin’s hourly wage. His new pay will be $12 an hour as host and $8 an hour plus tips when serving. His goal is still to earn at least $500 a week.

What is his new inequality assuming he makes $225 in tips? [T4]

Revising (changing to match the new inequality) his Desmos graph, give an example of the number of hours (hostess and serving) he could be scheduled for that allows him to achieve his goal while working less time than previously. [T2]

Eva works at the restaurant and has some cooking experience. The manager has offered to pay her $20 an hour when scheduled as a cook and $15 an hour when scheduled as a server. Eva wants to earn at least $600 a week but the manager will only schedule her for no more than 40 hours in a week.

Write the system of inequalities that would model Eva’s situation. [T4]

Use Desmos to graph the system. Based upon the Desmos graph, identify two possible schedule combinations that fulfill both requirements. [T3]

Eva got called to cover a shift for someone who called out sick and wound up working over 40 hours this week.

In one to two sentences, explain how does this affect(change) the inequalities? [T4]

In one to two sentences, explain how does this affect(change) the graph? [T3]

Name: ____

AFDA Linear Programming Comprehensive D

Solve the linear programming application problem and analyze the situation by answering the given questions. Make sure you complete all the questions to earn all the points.

An automotive shop specializes in oil changes and tire rotations. Each oil change nets a profit of $10 and each tire rotation nets a profit of $15. Each oil change takes 45 minutes while each tire rotation takes 30 minutes. The shop is available for 585 minutes for service work. The shop has at most 16 service bays to use throughout the day. How many oil changes and tire rotations should the automotive shop schedule to maximize profits?

Scenario:
Your boss has come to you and needs you to analyze the profit margin of the operation based on the given constraints, so she can pitch the idea of a pay raise to her boss IN AN HOUR! She is expecting a professional report, including calculations, and answers to all the questions that are specific to your conclusions (do not generalize; Example, do NOT say “I found the number” or “I used x and y”).

Section 1: Define the variables
What do your variables represent? (hint: think of the two things the shop does) [T3]
X =
Y =

Section 2: The Objective Function
What is the company’s goal? (hint: key words are minimize, maximize, profit, or cost) [T1]

Based on your previous answer, write your objective function. [T3]

Section 3: The Constraints
Using words, not numbers, describe the limitations, or the constraints, that the company must consider in this problem. What things do they have to think about when deciding how many of each service they should offer? (hint: You are looking for information that goes together like time, processes, space, costs, etc.) [T1]

Write a constraint representing the amount of time the services require to be completed. (hint: look for information about time and put it together appropriately) [T3]

Write a constraint representing the amount of space available to work in. (hint: look for information about service bays and put it together appropriately) [T3]

Can the company complete negative service? Write two constraints that represent the idea that there cannot be negative oil changes or negative tire rotations. (hint: each constraint will contain one variable and one number with an inequality in the middle) [T3]

Section 4: The Graph and Vertices

Using Desmos, graph the 4 constraints that you created in section 3. Then record (write) the vertices of the feasible region of your graph below. [T2]

Section 5: Evaluating the Vertices and Objective Function

Using your objective function from section 2 and the vertices from section 4, evaluate the value at each vertex. [T2]

Vertices

Objective function

Totals

Section 6: Interpreting the Answer
Based on the answers from section 5, interpret your results. What is the profit at the optimal solution?

What is the optimal solution? (hint: you are telling me what the point represents) [T3]

Section 7: Extension
Your profit margins just changed! The profit on an oil change is now $15 and the profit on a tire rotation is $12. Write the new objective function and solution. [T3]

Explain how your new objective function changed the solution using appropriate vocabulary from this module. [T1]

Your boss just came to you with a change because of a safety issue, only 15 service bays can be used. Explain how this change affects your solution. (hint: you will have to write a new constraint for shop work space and look at the new graph) [T3]

Module 3 Characteristics of Functions PROJECT – Day 1

Team Member Names:

____

____

____

<u>Congratulations!</u> You and your group have been chosen to design the new roller coaster at Math Land! All the rides at Math Land have mathematical themes and you are designing a function coaster.

Part A - Plan

1. Come up with a **name** for your roller coaster
2. You want this roller coaster to have a little bit for every type of guest, so think about what it is going to look like. Your coaster **must go underground at least once**.
3. On a piece of **lined paper** or **on the back of this paper**, **sketch** an idea for your coaster. It must have **at least 5 functions** built into it and **contain at least one of each kind** – linear, quadratic, exponential, and logarithmic. [T1]
4. **Go to Linerider.com** and make sure your roller coaster is safe!! Sketch your roller coaster and press play to see if your customers will make it to the end of your coaster. We don't want any lawsuits! [T0]
5. Capture a **video** and **upload** it to Canvas.

Submitted	Coaster Sketch w/ 5+ functions	LineRider Video
Approved		

Part B - Graph it out! [T0]

1. Take your sketch and **transfer** it to a piece of **graph paper**. Make sure that important parts of your coaster hit easy to find coordinate points.
2. You must **draw and label your axes** and put a **title on your drawing**.
3. Your **x-axis is the length of time** for the coaster and the **y-axis is the height of the coaster**.
4. Your coaster **must go underground at least once**.
5. **Color each function a different color** on your coaster. [T1]

Module 3 Characteristics of Functions PROJECT – Day 2

Part C - Function Characteristics

Coaster Name: ________________________________

Fill in the chart.

Domain & Range [T2]	D:	R:
Intercepts [T3]	x-intercepts	y-intercepts
Zeros [T3]		
Absolute minimum and maximum [T3]	Max:	Min:
Increasing and decreasing intervals [T4]	Increasing:	Decreasing:
End behavior [T4]	As $x \to -\infty$, $y \to$ __________	As $x \to \infty$, $y \to$ __________

Asymptotes [T4]		

Use the information in your chart to fill in the blanks on the next page.

Part D - Fill in the blanks. *(An editable version is in Canvas if you need to adjust to fit your coaster graph*).

Let's go for a ride on the ________________________ roller coaster, which is __________ seconds long. The roller coaster begins at _____ feet from the ground.[T2] After _______ seconds we will reach the maximum height of ________ feet. [T3] Therefore, the values for the domain of this roller coaster are _________ and the range is __________. [T2] We will be moving slow during the intervals of _____________________________________ because we will be going ________ hill, which means these are the (select 1: **increasing**, **decreasing**) intervals. We will be moving very fast during the intervals of ___________________________________ because we will be going ________ hill, which means these are the (select 1: **increasing**, **decreasing**) intervals. [T4] We go underground at the __________ intercept which is ________ seconds from the start and come back above ground at the __________ intercept which is ________ seconds from the start. [T3] Our coaster gets closer and closer to the ground at the end but never touches which means that y = _________. [T4]

Module 3 Characteristics of Functions PROJECT – Day 3

Part E - Poster [T0]

Glue/tape your finished roller coaster graph onto poster paper.

Your poster MUST include the following:

- title
- roller coaster graph
- all of the info from your function characteristic chart

Your poster should be neat, colorful, and free of spelling errors.

Part F - Presentation [T0]

Your group will present your roller coaster and all the information to the class. Each person in your group must speak, so decide ahead of time who is going to say what. A good way to split up speaking is to have each member read a section of Part D.

APPENDIX D

Dear David,

I am attaching the file of the Attitudes Toward Mathematics Inventory (ATMI). Hope you find it useful. If you decide to use it, please let me know. If you have any questions, do not hesitate to ask me.

Sincerely,

Martha Tapia

Martha Tapia, Ph.D.
Retired
[redacted]