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A novel approach for the consideration of plastic material behavior in thermodynamic topology optimization

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In order to find optimal structures for realistic applications, it is essential to include the real material behavior in the optimization process. For this purpose, this research focuses on thermodynamic topology optimization accounting for plasticity for which a surrogate material model is developed. Characteristically, the stress/strain diagram resulting from physical loading and unloading shows a hysteresis for classical plasticity models. Our material model takes only the physical loading during the optimization process into account. To this end, during a virtual unloading in the optimization process, the dissipation of energy is suppressed which yields the same elasto/plastic deformation state as for physical loading. By using this novel material model, optimized structures can be computed without resourceful classical path-dependent plasticity computation.

1 Motivation and Idea

During an iterative optimization process an increase of stiffness and therefore a decrease of deformation can be observed. This leads to an overshooting of strains that are evolutionary reduced without any physical unloading. The virtual unloading also applies to plastic material in optimization, so that the dissipation as part of the classical hysteresis in stress/strain diagram (cf. Fig. 1) may not occur. Hence, we present a novel surrogate material model within the thermodynamic topology optimization without dissipation and therefore without the time-consuming path-dependent computation of discrete loading steps.

2 Thermodynamic topology optimization

The thermodynamic topology optimization (TTO) is an established approach for topology and material optimization, cf. [2,3]. Here, we present a novel approach of the thermodynamic topology optimization extended for plastic material behavior. This can be included by means of the extended Hamilton functional [4]

$$\mathcal{H}[\boldsymbol{u},\boldsymbol{\varepsilon}^{\mathrm{p}},\chi] := \bar{\mathcal{H}}[\boldsymbol{u},\boldsymbol{\varepsilon}^{\mathrm{p}},\chi] - \mathcal{R}[\chi] + \mathcal{C}[\boldsymbol{\varepsilon}^{\mathrm{p}},\chi] \qquad \text{with} \qquad \bar{\mathcal{H}}[\boldsymbol{u},\boldsymbol{\varepsilon}^{\mathrm{p}},\chi] := \mathcal{G}[\boldsymbol{u},\boldsymbol{\varepsilon}^{\mathrm{p}},\chi] + \mathcal{D}[\boldsymbol{\varepsilon}^{\mathrm{p}}] \tag{1}$$

which makes use of the plastic strains $\boldsymbol{\varepsilon}^{\mathrm{p}}$ as internal variable. Furthermore, the functional depends on the displacement $\boldsymbol{u}$ and the topology is defined by the continuous density variable $\chi \in [\chi_{\min}, 1]$ with $\chi_{\min} > 0$. For a detailed definition of the Gibbs energy $\mathcal{G}$, the dissipation-related work $\mathcal{D}$, the rearrangement term $\mathcal{R}$, and the constraint functional $\mathcal{C}$ we refer to [1]. By requiring the Hamilton functional to become stationary, its solution results in the optimized structure. The stationarity condition with respect to all independent variables reads

$$\begin{cases} \delta_{\boldsymbol{u}}\mathcal{H} = 0 = \displaystyle\int_\Omega \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}} : \delta\boldsymbol{\varepsilon}\, \mathrm{d}V - \int_\Omega \boldsymbol{b}^\star \cdot \delta\boldsymbol{u}\, \mathrm{d}V - \int_{\Gamma_\sigma} \boldsymbol{t}^\star \cdot \delta\boldsymbol{u}\, \mathrm{d}A & \forall\, \delta\boldsymbol{u} \\ \delta_{\boldsymbol{\varepsilon}^{\mathrm{p}}}\mathcal{H} = 0 = \displaystyle\int_\Omega \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}^{\mathrm{p}}} : \delta\boldsymbol{\varepsilon}^{\mathrm{p}}\, \mathrm{d}V + \int_\Omega \frac{\partial \Delta^{\mathrm{diss}}}{\partial \dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}} : \delta\boldsymbol{\varepsilon}^{\mathrm{p}}\, \mathrm{d}V + \delta_{\boldsymbol{\varepsilon}^{\mathrm{p}}}\mathcal{C} & \forall\, \delta\boldsymbol{\varepsilon}^{\mathrm{p}} \\ \delta_{\chi}\mathcal{H} = 0 = \displaystyle\int_\Omega \frac{\partial \Psi}{\partial \chi}\, \delta\chi\, \mathrm{d}V - \int_\Omega \eta\, \dot{\chi}\, \delta\chi\, \mathrm{d}V - \int_\Omega \beta\, \nabla\chi \cdot \nabla\delta\chi\, \mathrm{d}V + \delta_{\chi}\mathcal{C} & \forall\, \delta\chi \end{cases} . \tag{2}$$

Here, the balance of linear momentum can be detected for the first condition $(2)_1$, and the field equation in the third condition $(2)_3$ needs to be solved to update the topology.

3 Surrogate material model for plasticity

The surrogate material model for plasticity is derived from the second stationarity condition $(2)_2$ with the Helmholtz free energy $\Psi = 1/2\ (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{p}}) : \chi^3\, \mathbb{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{p}})$. To consider that only the physical loading path analogous to the classical plastic stress/strain hysteresis is relevant during the optimization process, we assume a dissipation free material evolution with a vanishing dissipation function $\Delta^{\mathrm{diss}} = 0$. This can be argued because each decrease of strains and also plastic strains during the optimization process is caused by the modified topology and the simultaneously increased stiffness. The yield

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condition $||\boldsymbol{\sigma}^{\mathrm{dev}}|| = r$ with $r = \sqrt{2/3}\,\sigma^{\mathrm{Y}}_{\mathrm{exp}}$ and the volume preservation of plastic strains $\boldsymbol{\varepsilon}^{\mathrm{p}} : \boldsymbol{I} = 0$ are taken into account in the constraint functional $\mathcal{C}$ by using Lagrange parameters. Thus, formulation of the surrogate material model results in the non-linear algebraic equation

$$\mathbb{P} : \left(-\boldsymbol{\sigma} + \frac{r^2}{\boldsymbol{\sigma}^{\mathrm{dev}} : \mathbb{E}_0 : \boldsymbol{\sigma}^{\mathrm{dev}}}\, \boldsymbol{\sigma}^{\mathrm{dev}} : \mathbb{E}_0 \right) = \mathbf{0} \qquad \Leftrightarrow \qquad \mathbb{P} : \boldsymbol{s} = \mathbf{0}\,. \tag{3}$$

A full derivation can be found in [1]. In order to differentiate between the elastic and plastic case, the indicator function

$$\Phi\left(\boldsymbol{\varepsilon}\right) := ||\boldsymbol{\varepsilon}|| - \varepsilon^{\mathrm{Y}}_{\mathrm{exp}} \qquad \text{with} \qquad \varepsilon^{\mathrm{Y}}_{\mathrm{exp}} = \frac{r}{||\mathbb{P} : \chi^3\, \mathbb{E}_0 : \boldsymbol{\varepsilon}_{\mathrm{tens}}||} \qquad \text{so that} \qquad \Phi = \begin{cases} < 0 & \text{elastic} \\ \geq & \text{plastic} \end{cases} \tag{4}$$

is formulated as function of the strains, cf. the derivation in [1].

For updating the plastic strains, Eq. (3) needs to be solved numerically, which is an ill-posed problem due to the projection into the deviator subspace. Therefore, the solution space is defined by the kernel of the deviator operator $\mathbb{P}$. One eigenvector revealed that $\boldsymbol{s} = \begin{pmatrix} \zeta & \zeta & \zeta & 0 & 0 & 0 \end{pmatrix}$ with $\zeta \in \mathbb{R}$ solves the equation completely. This hydrostatic pressure structure suggest the transformation into the eigenspace and then $\boldsymbol{s}$ is solved via a Newton update scheme. Due to the missing impact of the volumetric part, the deviator part of the plastic strains is computed afterwards to ensure the volume preservation constraint.

The update of the plastic strains takes place in the complete process of thermodynamic topology optimization, also including the previously mentioned update of the displacements and density. For more details about the surrogate material model and the complete optimization we refer to [1].

4 Numerical results

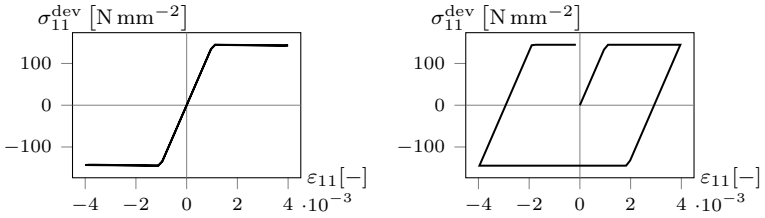

Fig. 1: Stress/strain curves for the proposed surrogate material model (left) and the classical elasto-plastic model resulting in hysteresis (right)

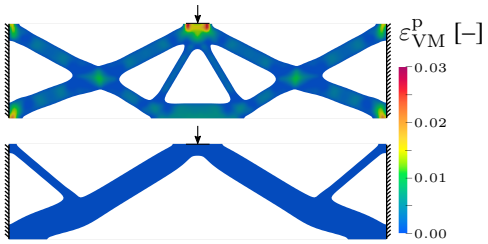

Fig. 2: Comparative results for plastic (above) and elastic (below) TTO for the quasi 2D clamped beam

For validation of the proposed surrogate model, the stress/strain curve related to a prescribed tension and pressure loading and unloading is compared to the hysteresis curve in Fig. 1. No dissipation can be detected and the results are not path-dependent, as intended. This makes it possible to compute the physical loading simultaneously to the optimization procedure without any additional finite element simulation for the physical loading history.

The optimization of different boundary value problems shows different structures if plastic or only elastic material behavior is taken into account, cf. the clamped beam loaded with a prescribed displacement in Fig. 2. It should be noted that in the evolutionary optimization including plasticity, the type of loading, i. e. force or displacement, as well as the magnitude affects the resulting structure. If an external force is selected too large, yielding can occur due to the limits of the ideal plasticity model.

Figure 2 also shows the distribution of plastic strains for the clamped beam. Verifying the values of plastic strains with a classical elasto-plastic computation for a fixed topology shows small differences.

A further analysis of the numerical results can be found in [1].

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