Received: 10 June 2021 | Accepted: 20 September 2021

DOI: 10.1002/pamm.202100038

An extended Hamilton functional for the thermodynamic topology optimization of hyperelastic structures

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We present our work on a new variational approach for thermodynamic topology optimization of hyperelastic structures: building upon our previous works, we follow a thermodynamic approach for deriving a field equation that describes the evolution of the density. The problem of topology optimization is consequently solved without the need of expensive optimization routines. Furthermore, our new formulation can also be applied to hyperelastic structures which show a remarkable difference to structures optimized for small deformations. Important aspects like tension/compression asymmetry and buckling are inherently included in the topology optimization approach due to the large deformation formulation.

1 Introduction

Topology optimization has become a prominent tool for the design of construction parts and components. In many cases, the topology possessing maximum resistance against deformations is searched, while the boundary conditions, the design space, and the total mass are given. In contrast to other approaches, we make use of a variational method which routes back to thermodynamic extremum principles used for material modeling. Our pioneering works on thermodynamic topology optimization are presented in [1, 2]. A huge benefit of this method is that material non-linearities can be considered within the optimization procedure. Furthermore, the optimal structure is found by solving a partial differential equation instead of a (mathematical) optimization problem. This property allows to implement the thermodynamic topology optimization via the element user routine of standard finite element programs such that external optimization algorithms become obsolete.

In the present work, we recall the fundamental idea of topology optimization via thermodynamic principles. The formulation is based on the recent publication in [4] where an extended Hamilton principle has been discussed for the holistic modeling of thermo-mechanically coupled dissipative processes during microstructure evolution. To this end, we present an extended Hamilton functional whose stationarity conditions yield all governing equations. Then, we apply this strategy to hyperelastic materials as presented in [3]. In this case, the stationarity conditions yield the weak form of the balance of linear momentum and the partial differential equation for the density variable $\rho = \rho(\chi) \in]0, 1]$.

2 An extended Hamilton functional for the thermodynamic topology optimization

The primal variables, for a generalized isothermal material, are provided by the displacements $\boldsymbol{u}$ and the internal variables $\boldsymbol{\alpha}$. The relative density for topology optimization is indicated by $\rho = \rho(\chi)$ with the density variable χ. Physically motivated energetic contributions are stored in the total potential $\mathcal{G} = \mathcal{G}[\boldsymbol{u}, \boldsymbol{\alpha}, \chi]$ and in the dissipation functional $\mathcal{D}_{\boldsymbol{\alpha}}[\boldsymbol{\alpha}]$ in case microstructural evolution is considered. Constraints are included via the functional $\mathcal{C} = \mathcal{C}[\boldsymbol{\alpha}, \chi]$ to account, e.g., for the total mass $\int_\Omega \rho(\chi) \mathrm{d}V = \bar{\rho}\,\Omega$ with the design space Ω and the prescribed averaged density $\bar{\rho} \in]0, 1[$. The process of topology optimization is included by the rearrangement functional $\mathcal{R} = \mathcal{R}[\chi]$. Then, we specify for the extended Hamilton functional

$$\mathcal{H}[\boldsymbol{u}, \boldsymbol{\alpha}, \chi] := \mathcal{G}[\boldsymbol{u}, \boldsymbol{\alpha}, \chi] + \mathcal{D}_{\boldsymbol{\alpha}}[\boldsymbol{\alpha}] - \mathcal{R}[\chi] + \mathcal{C}[\boldsymbol{\alpha}, \chi] \rightarrow \underset{\boldsymbol{u}, \boldsymbol{\alpha}, \chi}{\text{stat}} \tag{1}$$

The negative sign is motivated by the fact that the process of topology optimization evolves in opposite direction to physical processes. For now, we only consider hyperelastic materials with a Neo-Hooke free energy density $\Psi = \rho(\chi)^3 (\frac{1}{2}\mu(I_1 - 3) + \frac{\lambda}{4}(J^2 - 1) - \frac{\lambda}{2}\ln[J] - \mu \ln[J])$ which enters the potential $\mathcal{G} = \int_\Omega \Psi \mathrm{d}V - \boldsymbol{\ell}[\boldsymbol{u}]$. The first invariant of the right Cauchy-Green tensor $\boldsymbol{C}$ is denoted as $I_1 = \mathrm{tr}\boldsymbol{C}$ and the linear terms are collected in $\boldsymbol{\ell}[\boldsymbol{u}]$. Then, the stationarity with respect to the displacements and the density variable need to be evaluated. They read

$$\begin{cases} \delta_u \mathcal{G}[\boldsymbol{u}, \chi](\delta \boldsymbol{u}) = 0 & \forall\, \delta \boldsymbol{u} \\ \delta_\chi \mathcal{G}[\boldsymbol{u}, \chi](\delta\chi) - \delta_\chi \mathcal{R}[\chi](\delta\chi) + \delta_\chi \mathcal{C}[\chi](\delta\chi) = 0 & \forall\, \delta\chi \end{cases} \tag{2}$$

Using the dissipation function Δ_χ for the density variable, we introduce for the rearrangement functional

$$\mathcal{R}[\chi] := \int_\Omega \frac{\partial \Delta_\chi}{\partial \dot{\chi}} \chi \, \mathrm{d}V + \int_\Omega \frac{1}{2} \beta ||\chi||^2 \, \mathrm{d}V \,. \tag{3}$$

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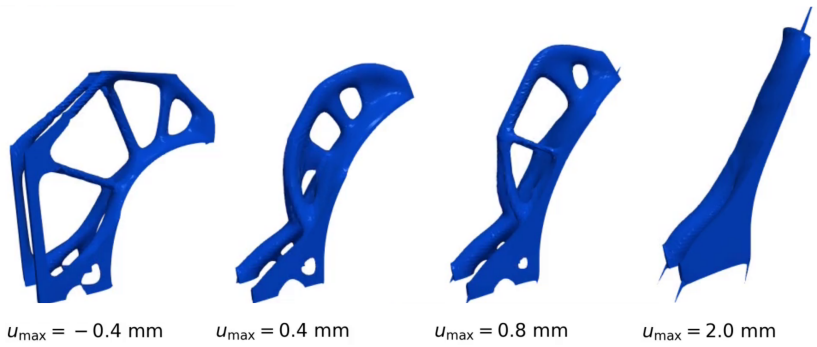

(a) 3D cantilever for different maximum prescribed displacements. Remarkable differences are observed for the different loads. Furthermore, a tension/compression asymmetry is present.

(b) Beam problem using arc-length control for small (top) and large deformations (bottom).

Fig. 1: Numerical results for the thermodynamic topology optimization of hyperelastic structures.

The second term regularizes the non-convex functional and β allows to control the member size. The stationarity with respect to the density variable reads with the Lagrange parameter λ for the mass constraint

$$\begin{cases} \eta\dot{\chi} = -p + \beta\rho' \triangle\rho + \lambda\rho' & \forall\, \boldsymbol{X} \in \Omega \\ \boldsymbol{n} \cdot \nabla\rho = 0 & \forall\, \boldsymbol{X} \in \partial\Omega \end{cases} \tag{4}$$

which is solved via the neighbored element method. For more details, we refer to the original publication [3].

3 Numerical results

The model is applied to two different boundary value problems: a 3D cantilever problem in Fig. 1a and the prominent Messerschmitt–Bölkow–Blohm (MBB) beam in Fig. 1b. The 3D cantilever is optimized for different maximum prescribed displacements. The MBB beam is optimized using arc-length control. For the 3D cantilever, it becomes obvious that tension/compression asymmetry is included by using the given Neo-Hooke energy. Here, the logarithmic term causes a significantly different structure to be optimal for $u_{\max} = -0.4$ mm than for $u_{\max} = 0.4$ mm. Furthermore, the magnitude of the prescribed displacement results in loading states with a varying intensity of the bending moment: the larger the intensity, the smaller the arc structure. This demonstrates the impact of the large deformation setting. This formulation, in principle, also accounts for buckling effects. To investigate this phenomenon, the MBB beam is optimized using an arc-length control scheme in Fig. 1b. For small loads, the almost identical classical result for the MBB is the optimal topology. However, for remarkably larger deformations, the structure is remodeled to adapt to the buckling. Here, different perspectives are presented.

4 Conclusions

Usually, the variational approach provided by an extended Hamilton principle as introduced in [4] is applied for material modeling. However, it can successfully be modified to derive the thermodynamic topology optimization also for hyperelastic structures, cf. [3]. The neighbored element method, cf. [2, 3], provides a fast and robust numerical treatment which, in combination of arc-length control, can be applied for the topology optimization even when buckling is present.

Acknowledgements Open access funding enabled and organized by Projekt DEAL.

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