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A Computational Model for Biological Tissues Considering the Influence of Injury on Growth and Remodelling

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Biological tissues adapt to changed loading conditions through growth and remodelling (G&R) to reestablish a so-called homeostatic state. On the other hand, loading conditions above their physiological limits, as during trauma or surgical procedures, cause injury and can initiate pathological G&R. Herein, a modelling approach for G&R influenced by injury is presented combining the theories of plasticity and homogenised constrained mixtures. The results show that injury has a significant impact on the G&R behaviour and thus on the accomplishment of homeostasis.

1 Introduction

Soft biological tissues adapt continuously to their mechanical state by the addition of new mass (growth) to the tissue and mass turnover (remodelling) within the tissue. In addition, tissues can also be damaged when unphysiologically loaded. Damage in collagen fibres is considered the most important since elastin deforms elastically up to high strains. Of several damage mechanisms in the fibres, the interstrand delamination, i.e. the permanent elongation of collagen molecular nanostructure, seems to play a crucial role [1]. The presented computational model considers the influence of injury on G&R. The interstrand delamination is modelled using the theory of elastoplasticity for fibre-reinforced matrices as presented by Gasser and Holzapfel in [2]. Growth and remodelling is added following the approach by Cyron et al. in [3] which is based on the homogenized constrained mixture theory.

2 Methods

The deformation gradient $\mathbf{F}$ is multiplicatively decomposed into elastic $\mathbf{F}_\mathrm{e}^i$, plastic $\mathbf{F}_\mathrm{p}^i$, remodelling $\mathbf{F}_\mathrm{r}^i$ and growth $\mathbf{F}_\mathrm{g}^i$ deformation gradients. Two constituents, collagen fibres ($i = \mathrm{c}$) and the elastin matrix ($i = \mathrm{m}$) are considered. Note that both constituents experience the same total deformation $\mathbf{F}$ but different elastic and inelastic deformations, see Fig. 1. Injury and

Table 1: Material and simulation parameters

μ	80 J/kg	$\rho_0^\mathrm{c}(0)$	300 kg/m^3
k_1	160 J/kg	ρ_0^m	650 kg/m^3
k_2	3.5	$\tilde{\mathbf{a}}_0$	$\{0.3, 0.5, 1.0\}$
		$\mathbf{a}_0$	$\tilde{\mathbf{a}}_0/\|\|\tilde{\mathbf{a}}_0\|\|$
	Injury [2]		G&R [4]
τ_0	80 kPa	σ_hom	60 kPa
h	50 kPa	T_r	1.5 s
τ_∞	200 kPa	k_σ	0.02
a_0	0.1		

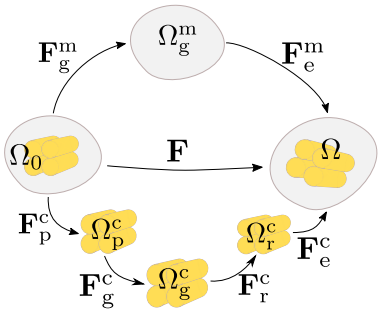

Fig. 1: Decomposition of the deformation gradient

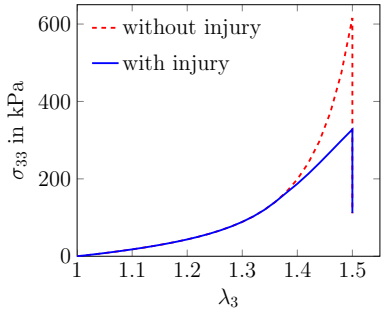

Fig. 2: Cauchy stresses with and without injury over stretch

remodelling are assumed to occur only in the collagen fibres. Growth is the same for both constituents, $\mathbf{F}_\mathrm{g}^\mathrm{c} = \mathbf{F}_\mathrm{g}^\mathrm{m} = \mathbf{F}_\mathrm{g}$. An anisotropic, invariant-based Helmholtz free energy function

$$\Psi(\mathbf{F}_\mathrm{e}^\mathrm{c}, \mathbf{F}_\mathrm{e}^\mathrm{m}, \mathbf{a}_0, A, p, \theta) = \rho_0^\mathrm{c}\left(\psi_\sigma^\mathrm{c}(\mathbf{F}_\mathrm{e}^\mathrm{c}, \mathbf{a}_0) + \psi_A^\mathrm{c}(A)\right) + \rho_0^\mathrm{m}\psi^\mathrm{m}(\bar{\mathbf{F}}_\mathrm{e}^\mathrm{m}) + \rho_0\psi_\mathrm{L}(p, \theta) \tag{1}$$

is defined as a function of the elastic deformations, the fibre direction $\mathbf{a}_0$ in the reference configuration, an injury-related variable A, a pressure p and strain variable θ. The free energy is additively decomposed into a free energy for collagen (superscript c), elastin (m) and a term accounting for the nearly incompressible behaviour of tissues (Ψ_L). A selective Hu-Washizu functional with the constraint for the volumetric deformation $\det(\mathbf{F})/\det(\mathbf{F}_\mathrm{g}) = 1$, which restricts the volume change to the growth deformation, is employed. Since the reference mass density changes due to G&R, the free energies are defined per unit mass and multiplied by the reference mass densities per unit volume of the whole tissue. The total reference mass density is $\rho_0 = \rho_0^\mathrm{c} + \rho_0^\mathrm{m}$. To account for injury, the collagen free energy is again additively decomposed into an elastic

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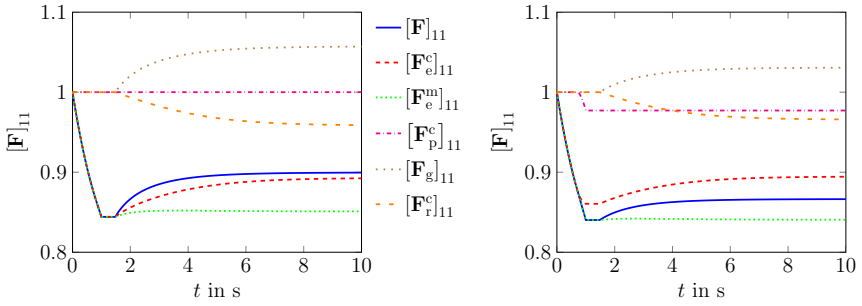

Fig. 3: Decomposed deformation gradient components $[\mathbf{F}]_{11}$ without (left) and with (right) injury over time

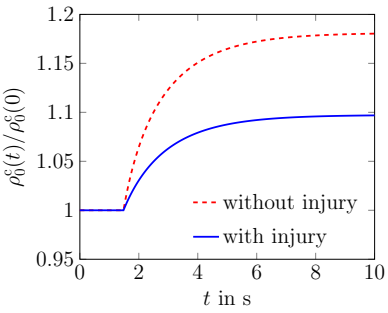

Fig. 4: Reference collagen densities relative to initial reference density over time

term ψ_σ^{c} and a plastic term ψ_A^{c} associated with the permanent elongation due to interstrand delamination. Plastic flow is governed by a combined linear-exponential hardening [2] with the internal variable A as a hardening variable. G&R governs the change of the reference total mass density $\dot{\rho}_0 = \dot{\rho}_0^{\mathrm{c}} + \dot{\rho}_0^{\mathrm{m}}$, where the collagen density rate $\dot{\rho}_0^{\mathrm{c}}$ is governed by the difference between the current Cauchy stress and a homeostatic stress value, whereas the elastin density is assumed to be constant in time (i.e., $\dot{\rho}_0^{\mathrm{m}} = 0$). The detailed approach is presented by Braeu et al. in [4]. Following the slow-growth assumption, the time-scales of growth and elastoplastic deformations can be clearly separated. As a consequence, the plastic deformation and G&R are implemented in a staggered way.

3 Numerical Examples

The features of the model are investigated by a uniaxial tension test on an H1P0 one-element cube with edge length $1\ \mathrm{cm}$. The maximum stretch $\lambda_3 = 1.5$ of the top surface is achieved after $1\ \mathrm{s}$ and held constant afterwards. G&R is activated after $1.5\ \mathrm{s}$ and isotropic growth is considered. Two simulations are compared: in the first simulation, the material deforms only elastically before G&R whereas in the second, it is damaged during loading. The stresses are derived using an exponential function for collagen and a Neo-Hookean approach for the matrix

$$\psi_\sigma^{\mathrm{c}} = \frac{k_1}{2k_2}\left(\mathrm{e}^{k_2(I_4-1)^2} - 1\right), \qquad \psi^{\mathrm{m}} = \frac{\mu}{2}\left(\bar{I}_1 - 3\right), \tag{2}$$

where $k_1,\ k_2,\ \mu$ are material parameters, I_4 the quadratic stretch of collagen in fibre direction and $\bar{I}_1$ the first invariant of the isochoric part of the elastic matrix deformation gradient. The material parameters are listed in Tab. 1.

For both simulations, the Cauchy stresses progressively decrease at $\lambda = 1.5$ due to G&R, converging towards the homeostatic state, see Fig. 2. However, before G&R, the evolution of the stresses differ between both simulations. This difference causes different evolutions of growth and remodelling and thus total deformations, see Fig. 3. The evolution of the reference densities of collagen relative to the initial reference density is depicted in Fig. 4. The density changes with injury $9.7\ \%$ and without $18.0\ \%$. The results show that the impact of injury on G&R is evident and hence should be considered in further research. The approach has also been applied to idealized axisymmetric artery models. These results and modifications of the presented approach will be discussed in future publications. In particular, the biochemical and cellular reaction to injury driving G&R is an essential ingredient to be modelled and incorporated.

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