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EXACT CLOSED FORM SOLUTIONS OF COMPOUND KDV BURGERS' EQUATION BY USING GENERALIZED (G'/G) EXPANSION METHOD

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Abstract. In this investigation, the compound Korteweg-de Vries (Kd-V) Burgers equation with constant coefficients is considered as the model, which is used to describe the properties of ion-acoustic waves in plasma physics, and also applied for long wave propagation in nonlinear media with dispersion and dissipation. The aim of this paper to achieve the closed and dynamic closed form solutions of the compound KdV Burgers equation. We derived the completely new solutions to the considered model using the generalized $\left(\frac{G'}{G}\right)$-expansion method. The newly obtained solutions are in form of hyperbolic and trigonometric functions, and rational function solutions with inverse terms of the trigonometric, hyperbolic functions. The dynamical representations of the obtained solutions are shown as the annihilation of three-dimensional shock waves, periodic waves, and multisoliton through their three dimensional and contour plots. The obtained solutions are also compared with previously exiting solutions with both analytically and numerically, and found that our results are preferable acceptable compared to the previous results.

Keywords: Compound Korteweg-de Vries Burgers equation, Generalized (G'/G) expansion method, Anti-Kink solitons.

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1. Introduction

The nonlinear evolution equations are used as to express the complex physical phenomenon in various fields of mathematical physics, mostly in fluid mechanics, solid state-physics, plasma physics, plasma waves, nonlinear optics, chemical physics, quantum mechanics, optical fibers, electricity, geochemistry, meteorology, protein chemistry, chemical reactive materials, in ecology, population model, heat flow, wave propagation, propagation of shallow water waves, chemical kinematics, quantum field theory, bio physics, elastic media, electromagnetic, acoustics, material sciences, economics, control theory, mathematical physics and so on [1, 15, 55, 25].

Analytical solutions of nonlinear evolution equations play an important role in understanding the inner mechanism of the physical phenomena. There are many analytical methods to study the traveling wave solutions of non-linear evolution equations such as inverse scattering method [15], Backlund transformation method [55], Hirota method [25], expansion method [66], Sine-Cosine function method [67], *Tanh*, *Coth* Function method [18], Exp-function method [24], F-expansion method [68], Lie Symmetry method [33, 34, 35, 36, 37, 38, 39, 40, 41, 52] and so on.

Apart form the above methods M Wang, et al. [66] proposed the $\left(\frac{G'}{G}\right)$-expansion method and applied on Kd-V equation, modified Kd-V equation, Variant Boussinesq equations, Hirota-Satsuma equations. The key idea behind the $\left(\frac{G'}{G}\right)$ expansion method is that the assumed solutions on the nonlinear evolution equations can be written of the form $\sum_{i=0}^{n} a_i \left(\frac{G'}{G}\right)^i$ where $G = G(\xi)$ satisfies the second order linear differential equation

$$G'' + \lambda G' + \mu G = 0, \tag{1}$$

where $a_i(i = 0, 1, 2, \cdots, n)$, $a_n \neq 0$, λ, μ are constants, and n is a positive integer. Many researchers applied $\left(\frac{G'}{G}\right)$-expansion method on different nonlinear equations such as G Ebadi, A Biswas [16] solved Kuramoto-Sivashinsky (KS) equation, B Agheli, R Darzi and A Dabbaghian [2] solved time fractional seventh-order Lax equation, time fractional seventh-order Sawada-Kotera-Ito equation, time fractional seventh-order Kaup-Kupershmidt equation, A Kurt, O Tasbozan and D Baleanu [42] solved Conformable Nizhnik-Novikov-Veselov System, F Kangalgil [26] solved the Perturbed Wadati-Segur-Ablowitz equation, M A Ghabshi, E V Krishna and M Alquran [20] solved Klein-Gordon system.

The (G'/G) expansion method was improved by S Zhu [72] and called as the extended (G'/G) expansion method, and applied on KdV equation, where the assumed solution was of the form $\sum_{i=-n}^{n} a_i \left(\frac{G'}{G}\right)^i$, $a_i(i = 0, \pm 1, \pm 2, \pm 3, \cdots, \pm m)$ are constants but both a_{-n}, and a_n can not be zero at same time, and where $G = G(\xi)$ satisfies (1), the extended (G'/G) expansion method is generalized and improved by H Naher, et al.[47] and applied on $(3+1)$-dimensional modified KdV-Zakharov-Kuznetsev equation, the method is known as generalized and improved (G'/G) expansion method, where the assumed solutions was written in the form $\sum_{i=-n}^{n} a_i \left(d + \frac{G'}{G}\right)^i$, $a_i(i = 0, \pm 1, \pm 2, \pm 3, \cdots, \pm m)$, d, are constants but both a_{-n}, and a_n can not be zero at same time and where $G = G(\xi)$ satisfies (1).

H. Naher and F. A. Abdullah first introduce the (G'/G) expansion and the generalized (G'/G) expansion methods [48] and solve nonlinear differential equations like KdV equation. Further many researcher are used to the mention methods to solved nonlinear partial differential equations like Kadomtsev-Petviashivli-Benjamin-Bona Mahony (KP-BBM) equation , modified KdV Zakharov-Kuznetsov (KdV-ZK) equation , Boussinesq equation , potential Yu-Toda-Sasa-Fukuyama and Gardner KP equation , symmetric-Regularized Long Wave equation, Burgers' and ZK-BBM equations, Zhiber-Shabat and Lioville equations, complex fractional Schrödinger equation, fifth order KdV and KdV-ZK equations [4, 5, 6, 7, 57, 58, 3, 49, 31, 8]. In $\left(\frac{G'}{G}\right)$ expansion method, the assumed solutions of the nonlinear partial differential equations are of the form

$$\sum_{j=0}^{m} a_j \left(\frac{G'}{G}\right)^{j} + \sum_{j=1}^{m} b_j \left(\frac{G'}{G}\right)^{-j}$$

where , $a_j, b_j, j = 0, 1, 2, \cdots, m$, are constants and $G = G(\xi)$ satisfies the ordinary differential equation of the form

$$P_1 G G'' - P_2 G G' - P_3 (G')^2 - P_4 G^2 = 0 \tag{2}$$

where P_1, P_2, P_3 and P_4 are free parameters , and d is a constant. In the generalized $\left(\frac{G'}{G}\right)$ expansion method, the assumed solutions of the nonlinear partial differential equations are of the form

$$\sum_{j=0}^{m} a_j \left(d + \frac{G'}{G}\right)^{j} + \sum_{j=1}^{m} b_j \left(d + \frac{G'}{G}\right)^{-j}$$

, where $a_j, b_j, j = 0, 1, 2, \cdots, m$, and d are constants and satisfies the ordinary differential equation of the form (2).

The compound Kd-V Burgers equation illustrates the propagation of ion acoustic waves in plasma physics [28], and also used to explain the propagation of thermal pulse through single crystal [50, 63] and applied as model for long wave propagation in nonlinear media with dispersion and dissipation [64]. The compound Kd-V Burgers equation was solved analytically by using, special truncated expansion method [22], He's variation iteration method [9] and by combination method [69]. It is also solved by two different form of $\left(\frac{G'}{G}\right)$-expansion methods [70, 46], and also numerically solved by [54], [60].

We have used the generalized (G'/G) expansion method to study the analytical solutions of the compound Kd-V Burgers equation. The paper is arranged as follows: in section 2, details about the (G'/G) expansion method is given. In section 3, generalized (G'/G) expansion method is applied on the compound Kd-V Burgers equation. Results and graphs of the some of the solutions are given in the section 4 and, in section 5 some comparison of the obtained solutions are done by both analytic and numeric way and in section 6 contains some important results and conclusion respectively.

2. Generalized (G'/G) expansion Method

Let us consider the nonlinear partial differential equation of the form

$$W\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial t^2}, \frac{\partial^2 u}{\partial x \partial t}, \frac{\partial^2 u}{\partial x^2}, \cdots\right) = 0 \tag{3}$$

where u is a unknown function of x and t. The method follows the the following steps as:

Step-1. First of all we convert eq. (3) an ordinary differential equation by using wave transformation $X = x - kt$, where k is a constant , as

$$R\left(u, \frac{du}{dX}, \frac{d^2u}{dX^2}, \cdots\right) = 0 \tag{4}$$

where $u = u(X)$

Step-2. Assuming the solutions of the Eq. (4) can be written as follows:[48]

$$u(X) = \sum_{j=0}^{m} a_j \left(d + \frac{G'}{G}\right)^j + \sum_{j=1}^{m} b_j \left(d + \frac{G'}{G}\right)^{-j} \tag{5}$$

where either a_m or b_m can be zero but both a_m and b_m can not be zero at same time, and a_j $(j = 0, 1, 2 \cdots m)$ and b_j $(j = 1, 2 \cdots m)$, and d are arbitrary constants where $G = G(X)$ satisfies Eq. (2).

Step-3. The degree of the polynomial m, which is a positive integer can be calculated by homogeneous balancing between the highest order derivative terms with highest order nonlinear terms appearing in Eq. (4).

Step-4. With the value of m substitute the value of Eq. (5) and Eq. (2) into Eq. (4), then Eq. (4) becomes a polynomial of $\left(d + \frac{G'}{G}\right)^N$ $(N = 0, \pm 1, \pm 2, 3, \cdots)$, then collecting the coefficients of the obtained same degree polynomial is equal to zero, yields a system of algebraic equations for $a_j (j = 0, 1, 2, \cdots, m)$ and $b_j (j = 1, 2, 3, \cdots, m)$ and k.

Step-5. To get the values of $a_j (j = 0, 1, 2, \cdots, m)$, $b_j (j = 1, 2, 3, \cdots, m)$ and k, solve the resulted system of algebraic equations.

The general solutions of (2) are represented by five family with satisfying some conditions. Let $\Omega = (P_1 - P_3)$, $\Delta = P_2^2 + 4P_4\Omega$, C_1 and C_2 are integration constants, we have the following general solutions as follows:

Family-I when $P_2 \neq 0$ and $\Delta > 0$

$$\left(\frac{G'}{G}\right) = \frac{P_2}{2\Omega} + \frac{\sqrt{\Delta}}{2\Omega} \left(\frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{2\Omega} X\right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{2\Omega} X\right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{2\Omega} X\right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{2\Omega} X\right)} \right) \tag{6}$$

Family-II when $P_2 \neq 0$ and $\Delta < 0$

$$\left(\frac{G'}{G}\right) = \frac{P_2}{2\Omega} + \frac{\sqrt{-\Delta}}{2\Omega} \left(\frac{C_1 \sin\left(\frac{\sqrt{-\Delta}}{2\Omega} X\right) + C_2 \cos\left(\frac{\sqrt{-\Delta}}{2\Omega} X\right)}{C_1 \cos\left(\frac{\sqrt{-\Delta}}{2\Omega} X\right) + C_2 \sin\left(\frac{\sqrt{-(\Delta)}}{2\Omega} X\right)} \right) \tag{7}$$

Family-III when $P_2 \neq 0$ and $\Delta = 0$

$$\left(\frac{G'}{G}\right) = \frac{P_2}{2\Omega} + \frac{C_2}{C_1 + C_2 X} \tag{8}$$

Family-IV when $P_2 = 0$ and $\Omega P_4 > 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\Omega P_4}}{\Omega} \left(\frac{C_1 \sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega} X\right) + C_2 \cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega} X\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega} X\right) + C_2 \sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega} X\right)} \right) \tag{9}$$

Family-V when $P_2 = 0$ and $\Omega P_4 < 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{-\Omega P_4}}{\Omega} \left(\frac{C_1 \sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega} X\right) + C_2 \cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega} X\right)}{C_1 \cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega} X\right) + C_2 \sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega} X\right)} \right) \tag{10}$$

Step-6. Substitute the value $a_j (j = 0, 1, 2, \cdots, m)$, $b_j (j = 1, 2, 3, \cdots, m)$ and k in Eq. (5) and using (6), (7), (8), (9), (10), we get the desired solutions of Eq. (3).

3. Solutions of the compound Korteweg-de Vries Burgers Equation

Consider the general form of the CKdV-B equation of the form [9]

$$\frac{\partial u}{\partial t}+Au\frac{\partial u}{\partial x}+Bu^2\frac{\partial u}{\partial x}+C\frac{\partial^3 u}{\partial x^3}=D\frac{\partial^2 u}{\partial x^2} \tag{11}$$

where A, B are nonlinear coefficients, C is the dispersive coefficient, D is the dissipation coefficient and $u=u(x,t)$.
Now using the wave transformation $X=x-kt$ where k is a constant. Then Eq. (11) is converted into an ordinary differential equation of the form

$$-k\frac{du}{dX}+Au\frac{du}{dX}+Bu^2\frac{du}{dX}+C\frac{d^3u}{dX^3}-D\frac{d^2u}{dX^2}=0 \tag{12}$$

Integrating Eq. (12) once, we have

$$-ku+\frac{1}{2}Au^2+\frac{1}{3}Bu^3+C\frac{d^2u}{dX^2}-D\frac{du}{dX}+c=0 \tag{13}$$

where c is the integration constant, and $u=u\,(X)$.
Now balancing the term between u^3 with $\frac{d^2u}{dX^2}$ in Eq. (13), we get $m=1$. So the assumed solution of Eq. (13) is of the form

$$u(X)=a_0+a_1\left(d+\frac{G'}{G}\right)+b_1\left(d+\frac{G'}{G}\right)^{-1} \tag{14}$$

Now substitute the value of Eq. (14) into Eq. (13), we get a function of polynomial of $\left(d+\frac{G'}{G}\right)$, then taking the coefficient of same power of $\left(d+\frac{G'}{G}\right)^i$, $(i=0,\pm1,\pm2,\pm3)$, we get a set of system of algebraic equations , then we solve the system of algebraic equations by using Maple we get the following set of solutions.
Set-1

$$a_0=-\frac{\left(A\sqrt{-\frac{6C}{B}}P_1-12Cd\Omega-6CP_2-2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}},\, b_1=\frac{\sqrt{-\frac{6C}{B}}\left(d^2\Omega+P_2d-P_4\right)}{P_1},$$

$$k=-\frac{\left(3A^2CP_1^2+6BC^2\Delta+2BD^2P_1^2\right)}{12P_1^2BC},\, a_1=0,$$

$$c=\frac{\left(3A^3CP_1^2\sqrt{-\frac{6C}{B}}+18ABC^2\Delta\sqrt{-\frac{6C}{B}}+6ABD^2P_1^2\sqrt{-\frac{6C}{B}}+72BC^2D\Delta-8BD^3P_1^2\right)}{72B^2CP_1^2\sqrt{-\frac{6C}{B}}} \tag{15}$$

Set-2

$$a_0=-\frac{\left(A\sqrt{-\frac{6C}{B}}P_1-12Cd\Omega-6CP_2-2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}}, a_1=\frac{\sqrt{-\frac{6C}{B}}\Omega}{P_1}, b_1=0,$$
$$k=-\frac{\left(3A^2CP_1^2+6BC^2\Delta+2BD^2P_1^2\right)}{12P_1^2BC},$$

$$c=\frac{\left(3A^3CP_1^2\sqrt{-\frac{6C}{B}}+18ABC^2\Delta\sqrt{-\frac{6C}{B}}+6ABD^2P_1^2\sqrt{-\frac{6C}{B}}-72BC^2D\Delta+8BD^3P_1^2\right)}{72B^2CP_1^2\sqrt{-\frac{6C}{B}}} \tag{16}$$

Set-3

$$a_0=-\frac{\left(A\sqrt{-\frac{6C}{B}}+2D\right)}{2B\sqrt{-\frac{6C}{B}}}, a_1=\frac{\sqrt{-\frac{6C}{B}}\Omega}{P_1}, b_1=\frac{\sqrt{-\frac{6C}{B}}\Delta}{4\Omega P_1},$$
$$k=-\frac{\left(3A^2CP_1^2+24BC^2\Delta+2BD^2P_1^2\right)}{12P_1^2BC}, d=-\frac{P_2}{2\Omega},$$
$$c=\frac{\left(3A^3CP_1^2\sqrt{-\frac{6C}{B}}+72ABC^2\Delta\sqrt{-\frac{6C}{B}}+6ABD^2P_1^2\sqrt{-\frac{6C}{B}}-288BC^2D\Delta+8BD^3P_1^2\right)}{72B^2CP_1^2\sqrt{-\frac{6C}{B}}} \tag{17}$$

For Set-1:
Putting the value of Eq. (15) into Eq. (14), using (6) and the following simplified wave solutions are obtained

$$u(x,t)=-\frac{\left(A\sqrt{-\frac{6C}{B}}P_1-12Cd\Omega-6CP_2-2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}}+\frac{\sqrt{-\frac{6C}{B}}\left(d^2\Omega+P_2d-P_4\right)}{P_1}$$
$$\left(d+\frac{P_2}{2\Omega}+\frac{\sqrt{\Delta}}{2\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)}\right)\right)^{-1} \tag{18}$$

where $X=x-\frac{\left(3A^2CP_1^2+6BC^2\Delta+2BD^2P_1^2\right)}{12P_1^2BC}t$.
Putting the value of Eq. (15) into Eq. (14), using (7) and the following simplified wave

solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 6CP_2 - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\left(d^2\Omega + P_2 d - P_4\right)}{P_1}$$
$$\left(d + \frac{P_2}{2\Omega} + \frac{\sqrt{-\Delta}}{2\Omega}\left(\frac{C_1\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right) + C_2\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right)}\right)\right)^{-1} \tag{19}$$

where $X = x - \frac{\left(3A^2CP_1^2 + 6BC^2\Delta + 2BD^2P_1^2\right)}{12P_1^2BC}t$.
Putting the value of Eq. (15) into Eq. (14), using (8) and the following simplified wave solution is obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 6CP_2 - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}}$$
$$+ \frac{\sqrt{-\frac{6C}{B}}\left(d^2\Omega + P_2 d - P_4\right)}{P_1}\left(d + \frac{P_2}{2\Omega} + \frac{C_2}{C_1 + C_2X}\right)^{-1} \tag{20}$$

where $X = x - \frac{\left(3A^2CP_1^2 + 2BD^2P_1^2\right)}{12P_1^2BC}t$.
Putting the value of Eq. (15) into Eq. (14), using (9) and the following simplified wave solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\left(d^2\Omega - P_4\right)}{P_1}$$
$$\left(d + \frac{\sqrt{\Omega P_4}}{\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right) + C_2\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)}{C_1\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right) + C_2\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)}\right)\right)^{-1} \tag{21}$$

where $X = x - \frac{\left(3A^2CP_1^2 + 24BC^2P_4\Omega + 2BD^2P_1^2\right)}{12P_1^2BC}t$.
Putting the value of Eq. (15) into Eq. (14), using (10) and the following simplified wave solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\left(d^2\Omega - P_4\right)}{P_1}$$
$$\left(d + \frac{\sqrt{-\Omega P_4}}{\Omega}\left(\frac{C_1\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right) + C_2\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)}{C_1\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right) + C_2\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)}\right)\right)^{-1} \tag{22}$$

where $X = x - \frac{\left(3A^2CP_1^2 + 24BC^2P_4\Omega + 2BD^2P_1^2\right)}{12P_1^2BC}t$.
For Set-2:
Putting the value of Eq. (16) into Eq. (14), using (6) and the following simplified wave

solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 6CP_2 - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\Omega}{P_1}$$
$$\left(d + \frac{P_2}{2\Omega} + \frac{\sqrt{\Delta}}{2\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)}\right)\right) \tag{23}$$

where $X = x - \frac{\left(3A^2CP_1^2+6BC^2\Delta+2BD^2P_1^2\right)}{12P_1^2BC}t$.
Putting the value of Eq. (16) into Eq. (14), using (7) and the following simplified wave solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 6CP_2 - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\Omega}{P_1}$$
$$\left(d + \frac{P_2}{2\Omega} + \frac{\sqrt{-\Delta}}{2\Omega}\left(\frac{C_1\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right) + C_2\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right)}\right)\right) \tag{24}$$

where $X = x - \frac{\left(3A^2CP_1^2+6BC^2\Delta+2BD^2P_1^2\right)}{12P_1^2BC}t$.
Putting the value of Eq. (16) into Eq. (14), using (8) and the following simplified wave solution is obtained

$$\begin{aligned} u(x,t) &= -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 6CP_2 - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}} \\ &\quad + \frac{\sqrt{-\frac{6C}{B}}\Omega}{P_1}\left(d + \frac{P_2}{2\Omega} + \frac{C_2}{C_1 + C_2X}\right) \end{aligned} \tag{25}$$

where $X = x - \frac{\left(3A^2CP_1^2+2BD^2P_1^2\right)}{12P_1^2BC}t$.
Putting the value of Eq. (16) into Eq. (14), using (9) and the following simplified wave solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\Omega}{P_1}$$
$$\left(d + \frac{\sqrt{\Omega P_4}}{\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right) + C_2\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)}{C_1\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right) + C_2\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)}\right)\right) \tag{26}$$

where $X = x - \frac{\left(3A^2CP_1^2+24BC^2P_4\Omega+2BD^2P_1^2\right)}{12P_1^2BC}t$.
Putting the value of Eq. (16) into Eq. (14), using (10) and the following simplified wave

solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}P_1 - 12Cd\Omega - 2DP_1\right)}{2BP_1\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\Omega}{P_1}$$
$$\left(d + \frac{\sqrt{-\Omega P_4}}{\Omega}\left(\frac{C_1\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right) + C_2\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)}{C_1\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right) + C_2\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)}\right)\right) \tag{27}$$

where $X = x - \frac{\left(3A^2CP_1^2 + 24BC^2P_4\Omega + 2BD^2P_1^2\right)}{12P_1^2BC}t$.

For Set-3:

Putting the value of Eq. (17) into Eq. (14), using (6) and the following simplified wave solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}} + 2D\right)}{2B\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{\Delta}\sqrt{-\frac{6C}{B}}}{2P_1}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)}\right)$$
$$+\frac{\sqrt{-\frac{6C}{B}}\sqrt{\Delta}}{2P_1}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{\Omega}X\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}X\right)}\right)^{-1} \tag{28}$$

where $X = x - \frac{\left(3A^2CP_1^2 + 24BC^2\Delta + 2BD^2P_1^2\right)}{12P_1^2BC}t$.

Putting the value of Eq. (17) into Eq. (14), using (7) and the following simplified wave solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}} + 2D\right)}{2B\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\sqrt{-\Delta}}{2P_1}\left(\frac{C_1\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right) + C_2\sin\left(\frac{\sqrt{-(\Delta)}}{2\Omega}X\right)}\right)$$
$$+\frac{\sqrt{-\frac{6C}{B}}\Delta}{2P_1\sqrt{-\Delta}}\left(\frac{C_1\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}X\right) + C_2\sin\left(\frac{\sqrt{-(\Delta)}}{2\Omega}X\right)}\right)^{-1} \tag{29}$$

where $X = x - \frac{\left(3A^2CP_1^2 + 24BC^2\Delta + 2BD^2P_1^2\right)}{12P_1^2BC}t$.

Putting the value of Eq. (17) into Eq. (14), using (6) and the following simplified wave solution is obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}} + 2D\right)}{2B\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}\Omega}{P_1}\left(\frac{C_2}{C_1 + C_2X}\right) \tag{30}$$

where $X = x - \frac{\left(3A^2CP_1^2 + 2BD^2P_1^2\right)}{12P_1^2BC}t$.

Putting the value of Eq. (17) into Eq. (14), using (9) and the following simplified wave

solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}+2D\right)}{2B\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}}{P_1}\left(-\frac{P_2}{2}+\sqrt{\Omega P_4}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)+C_2\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)}{C_1\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)+C_2\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)}\right)\right) + \frac{\sqrt{-\frac{6C}{B}}P_4}{P_1}\left(-\frac{P_2}{2\Omega}+\frac{\sqrt{\Omega P_4}}{\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)+C_2\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)}{C_1\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)+C_2\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}X\right)}\right)\right)^{-1} \tag{31}$$

where $X = x - \frac{\left(3A^2CP_1^2+96BC^2P_4\Omega+2BD^2P_1^2\right)}{12P_1^2BC}t$. Putting the value of Eq. (17) into Eq. (14), using (10) and the following simplified wave solutions are obtained

$$u(x,t) = -\frac{\left(A\sqrt{-\frac{6C}{B}}+2D\right)}{2B\sqrt{-\frac{6C}{B}}} + \frac{\sqrt{-\frac{6C}{B}}}{P_1}\left(-\frac{P_2}{2}+\sqrt{-\Omega P_4}\left(\frac{C_1\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)+C_2\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)}{C_1\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)+C_2\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)}\right)\right) + \frac{\sqrt{-\frac{6C}{B}}P_4}{P_1}\left(-\frac{P_2}{2\Omega}+\frac{\sqrt{-\Omega P_4}}{\Omega}\left(\frac{C_1\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)+C_2\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)}{C_1\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)+C_2\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}X\right)}\right)\right)^{-1} \tag{32}$$

where $X = x - \frac{\left(3A^2CP_1^2+96BC^2P_4\omega+2BD^2P_1^2\right)}{12P_1^2BC}t$.

4. Results and Discussion

In this section, we show the three-dimensional surface plots and corresponding three-dimensional contour plots, as well as kink and anti-kink waves, multi-soliton structures, periodic solitons , dynamics of the solutions of the compound Kd-V Burgers equation.

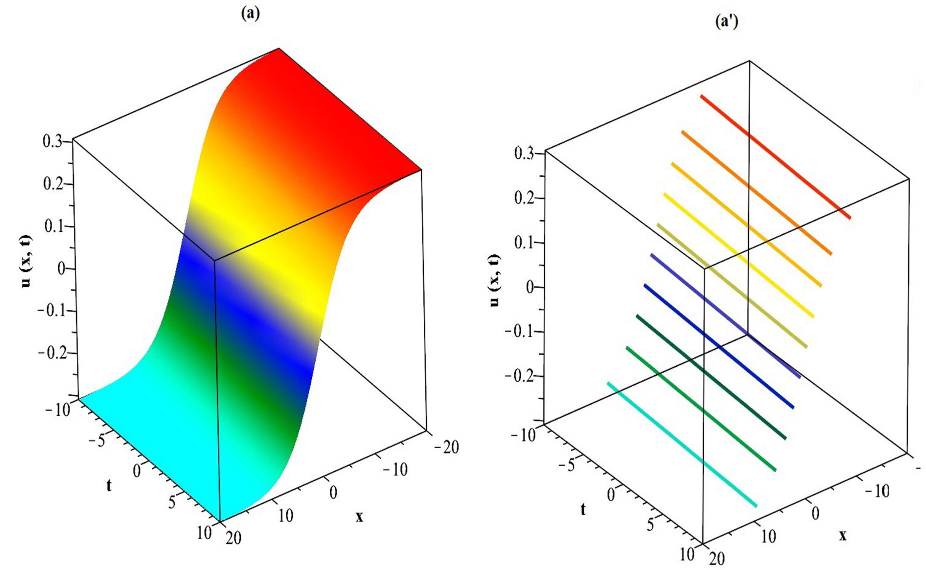

FIGURE 1. (a) and (a') are 3D plot and contour plot of (18) respectively, where $P_1 = 40$, $P_2 = 1$, $P_3 = 2$, $P_4 = 1$, $d = 2$, $C_1 = 1$, $C_2 = 0$. Here the 3D plot of anti-kink solutions and contour plot are drawn within the interval $-20 \le x \le 20$, $-10 \le t \le 10$ and $-10 \le x \le 10$, $-10 \le t \le 10$ respectively.

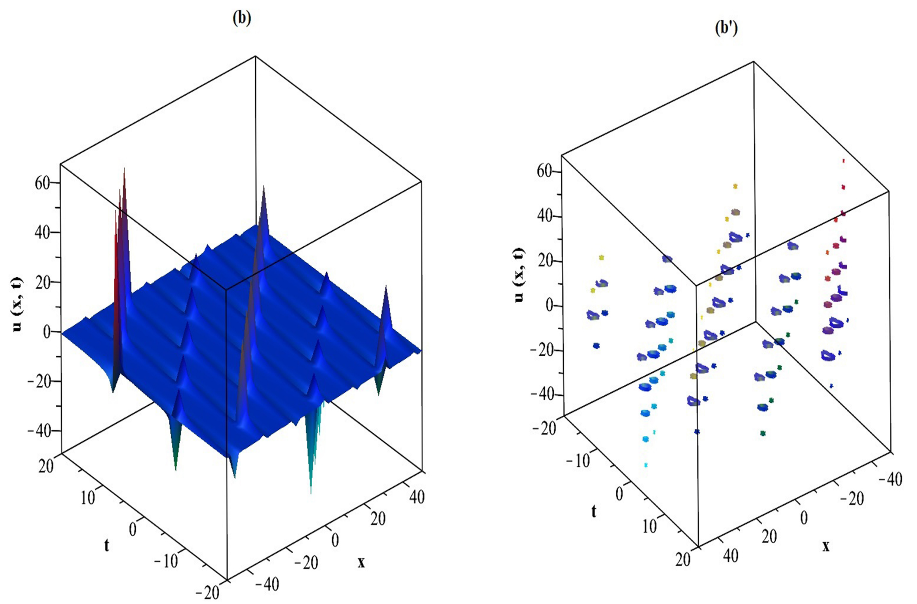

FIGURE 2. (b) and (b') are 3D plot and contour plot of (19) respectively, where $\mathbf{P_1 = 25, P_2 = 1, P_3 = 1, P_4 = -1, d = 2, C_1 = 10, C_2 = 20}$, within the interval $\mathbf{-50 \le x \le 50}$ and $\mathbf{-20 \le t \le 20}$.

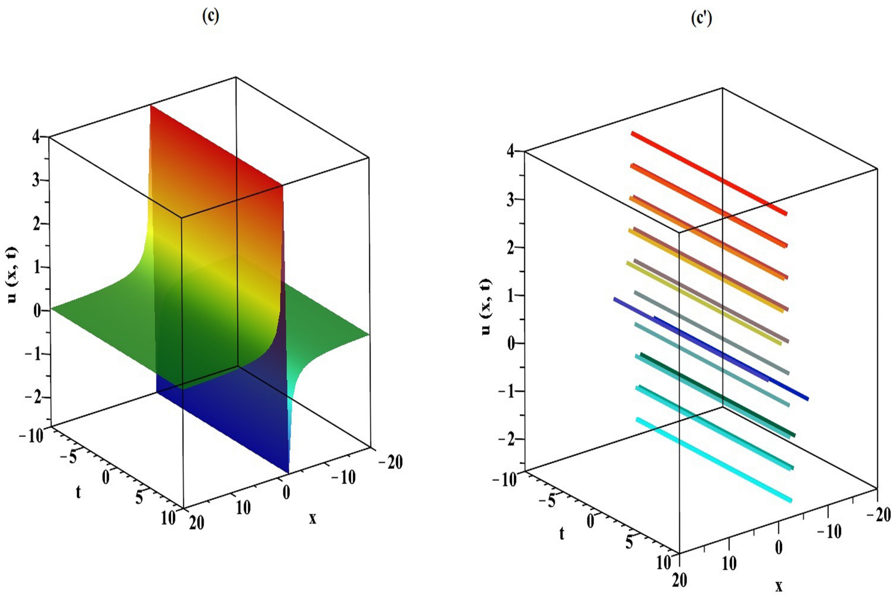

FIGURE 3. (c) and (c') are 3D plot and contour plot of (20) respectively, where $P_1 = 3$, $P_2 = 4$, $P_3 = 5$, $P_4 = 2$, $d = 2$, $C_1 = 1$, $C_2 = 1$.Here the 3D graph and contour plot are drawn within the interval $-20 \leq x \leq 20$, $-10 \leq t \leq 10$ and $-10 \leq x \leq 10$, $-10 \leq t \leq 10$ respectively.

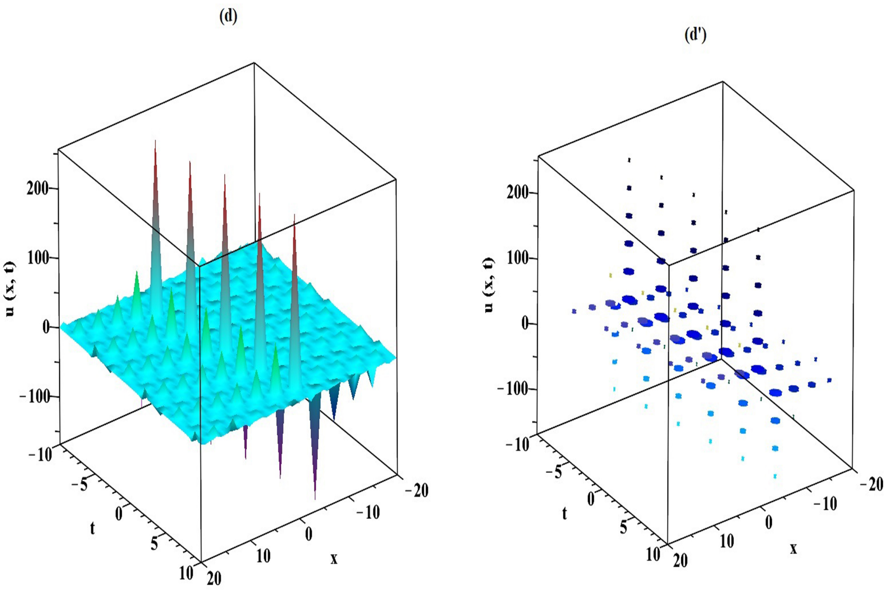

FIGURE 4. (d) and (d') are 3D plot and contour plot of (22) respectively, where $P_1 = 3$, $P_2 = 0$, $P_3 = 2$, $P_4 = -1$, $d = 2$, $C_1 = 1$, $C_2 = 0$, within the interval $-20 \leq x \leq 20$ and $-10 \leq t \leq 10$.

Figure 1-4 show the kink waves or shock waves and traveling wave profiles of the compound Kd-V Burgers equation with constant coefficients by using the generalized expansion method approach through three-dimensional surface plots and corresponding three-dimensional contour plots. The obtained solutions are in trigonometric, hyperbolic , and rational forms. For graphical representation of the solutions of the CKdV-B equation for, Figure 1-4 the arbitrary values of the coefficients, are taken as $A = 10$, $B = -3$, $C = 2$, $D = 10$. In Figure 1 the annihilation of surface plot and contour plot of solution (18) represents the of anti-kink solitons, and similar type of dynamic structures of anti-kink/ kink solitons also exists for the solutions (21), (23), (26), (28). As seen from the figure 1, with time there is a smooth transition of the shock region which clearly resembles to a typical classical shock wave formation in non-linear dynamics. Figure 2 provides the information about the multisoliton structures of (19), similar waves will exists for the solutions of (27), (29). A clear train of soliton propagating with varying amplitude over a well defined span of space and time is seen. Figure 3 shows a typical formation of double layer type structure for such solution. The same double layer structure also exists for the solutions of (20), (25), and the typical double layer structure with a sharp transition from zero to positive potential and negative to zero potential over the specified time interval is seen which is interesting, as the solutions apart from giving soliton or shock profile, a also admit a double layer solution.

Figure 4 shows that the wave profile of the multisoliton with periodic structures of (22) similarly, it is interesting to see that these all admit both compressive as well as rarefactive solitary structure. In all the cases of soliton propagation e.g. Figure 2 and Figure 4, and for their respective solutions as mentioned above, we can see the propagation of both compressive as well as rarefactive soliton with distinctive contour plots.

5. Analytical and numerical comparison of the obtained solutions.

Nahar et. al. [70] investigated that the solutions of the compound KdV-Burger's equation are of the form $u(x,t) = a_0 + a_1\left(\frac{G'}{G}\right)$, where a_0, a_1 are constants, and $G = G(\xi)$ satisfies the ordinary differential equation of the form (1) where $\xi = x - kt$, where k is a constant. On the other hand for the same form of differential equation as mentioned in equation number (1), above, Zayed and Gepreel [13] found that the solutions of the compound KdV-Burgers equation are of the form $u(x,t) = a_0 + a_1\left(\frac{G'}{G}\right)$, and of the form $u(x,t) = a_0 + b_1\left(\frac{G'}{G}\right)^{-1}$ where a_0, a_1, and b_1 are constants, and $G = G(\xi)$ satisfies the ordinary differential equation of the form (1) , where $\xi = x - kt$, where k is a constant. In our results the solutions of compound Kd-V Burgers equation are of the form $u(x,t) = a_0 + a_1\left(d + \frac{G'}{G}\right)$, $u(x,t) = a_0 + b_1\left(d + \frac{G'}{G}\right)^{-1}$, and of the form $u(x,t) = a_0 + a_1\left(d + \frac{G'}{G}\right) + b_1\left(d + \frac{G'}{G}\right)^{-1}$, where a_0, a_1, d, and b_1 are constants, and $G = G(\xi)$ satisfies the ordinary differential equation of the form (2), where $\xi = x - kt$, where k is a constant. It is to be noted that the values of a_0, a_1, b_1 are different for each different set of solutions. It is to be noted that solutions mentioned in eq. no.s (28), (29), (31), (32) in the present Manuscript are of completely new forms.

We have tried to validate our mathematical and analytical findings with the available existing literature. For example, pan et. al [54], investigated the solution of compound Kd-V Burgers equation and they reported (Fig. 2 of Example 4.1) that their numerical solution obtained by using the classical Crank- Nicolson scheme and Alternating Segment Crank Nicolson difference scheme, admitted shock type travelling wave. The analytical solution as mentioned in equation number (18) of the present Manuscript and its graphical output (mentioned in Fig.1) quite resemble to the typical shock wave type solution mentioned by pan et. al. So, we can conclude that the newly introduced methodology of finding solutions of the non-linear partial differential equations like compound Kd-V Burgers equation discussed in the present manuscript should be quite useful and effective to the real-world phenomenon where in this case arise.

6. Conclusion

In this paper, we have successfully obtained the new type of solitary wave solution as mentioned in Eq. No. (18)) and also other completely new forms of solutions of compound Kd-V Burgers with constant coefficients as mentioned in equation numbers (28), (29), (31), and (32). By using the generalized $\left(\frac{G'}{G}\right)$-expansion method,it is found that the newly obtained closed-form solutions are of trigonometric, hyperbolic, rational forms. The obtained solutions are also verified by back substituting it with Maple. By using Maple, the three-dimensional surface plots and corresponding three-dimensional contour plots are provided to understand the importance of the obtained solutions, the solutions of dynamical structures of waves are helpful in numerical and theoretical studies of the governing equations. The methods used and the solutions obtained may be useful to study the propagation characteristics of solitary waves/shock waves especially in degenerate plasma medium which are prevalent and well established/studied in astrophysical compact objects, e.g., non-rotating white dwarf stars, neutron stars, black holes [59, 32, 12, 13, 51, 43] etc. as well as to study the femtosecond dynamics of electron gas confined in metallic and semiconductor plasma, ultra-intense laser plasma interaction [17, 65, 61, 11, 23, 45]. The generalized method can be used further to study the nonlinear partial differential equations, which exists in different domains of classical as well as quantum plasma physics and mathematical physics.

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