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AN ITERATIVE APPROACH FOR FUZZY MULTI OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM

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Abstract. In this paper, we introduce an iterative method for the solution of fully fuzzy single and multi objective linear fractional programming problems with out converting them into equivalent crisp forms. By introducing fuzzy target and the fuzzy tolerance limit for each objective of the given fuzzy multi objective linear fractional programming problem (FMOLFPP), the given FMOLFPP is reduced to an equivalent single objective non-fractional fuzzy linear programming problem (NFLPP). Then the fuzzy optimal solution of the reduced NFLPP is obtained which inturn provides the Pareto optimal solution of the given FMOLFPP. Numerical examples are provided to illustrate the efficiency of the proposed method.

Keywords:Fuzzy number, fuzzy ranking, fuzzy arithmetic, fuzzy fractional programming(FFP), fuzzy target, fuzzy tolerance limit, Pareto optimal.

AMS Subject Classification: 03E72, 90C32, 90C05, 90C70.

1. Introduction

Linear fractional programming problems(LFPP) are very important in solving real world problems. It plays a major role in corporate companies, institutions, hospitals, financial managements and transportation planning. Linear fractional programming has been an important planning tool for the past four decades. Linear fractional programming is used to achieve the highest ratio of outcome to cost, workers/salary, result/students, patients/doctors, vaccinated/ medicine and etc.

Bellman and Zadeh [4] introduced decision making in a fuzzy environment. Bhargava and Sharma [5] obtained the optimal solution of linear programming problem(LPP) using Gauss method. Farhana Akond Pramy[17] proposed an approach for FMOLFPP using graded mean integration method. Guzel and Sivri [18] solved the multi-objective linear fractional programming (MOLFPP) problem with multiple efficient solution. Loganathan and Ganesan [21, 22, 23] presented a solution approach to fully fuzzy LFPP. They also discussed FMOLFPP.

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Rizk Allah [28] discussed two optimization approaches for bi-level MOLFP problems. Sapan Kumar Das et.al [11, 12, 13, 14, 15] discussed FLFP problem using LU- decomposition method. The also proposed an intelligent dual simplex method to solve triangular neutrosophic linear fractional programming problems. Also they discussed the application of linear fractional programming problem with fuzzy nature in industry sector. Payan [27] proposed a linear modeling to solve MOLFP problem with fuzzy parameters. Surapati [31] discussed fuzzy MOLFPP with Taylor series method. Nuran Guzel [19] proposed a solution method for MOLFPP. Arya and Singh [2] developed fuzzy parametric iterative method for solving multi objective linear fractional optimization problems. Bhati and Singh [6] used Branch and Bound computational method for multi-objective linear fractional optimization problems. Caballero and Hernandez [7] developed restoration of efficiency in a goal programming problem with linear fractional criteria. Chakraborty and Gupta [8] proposed a fuzzy mathematical programming approach for MOLFPP. Veeramani and Sumathi [32] have transformed the given FLFP problem to a MOLFP problem and the resultant problem is converted to a LP problem, using fuzzy mathematical programming method and obtained a crisp optimal solution. Costa [9] solved MOLFPP using interactive method. Arya and Singh [3] used fuzzy efficient iterative method for MOLFPP. Srikant Gupta et.al [30] developed an iterative algorithm for LFPP with minimum number of iterations. Alharbi et.al [1] proposed a solution method for fully FLFP problems using close interval approximation with normalized heptagonal fuzzy numbers. Madineh Farnam et.al [24] used hesitant fuzzy decision environment for multi-objective fractional programming problem. Veeramani et.al [33] solved multi objective fractional transportation problem using neutrosophic goal programming approach. Sahoo et.al [29] proposed MOLFPP with pentagonal intuitionistic fuzzy number. Michael Voskoglou [25] developed a new technique for solving LPP with fuzzy coefficients. Ladji Kane et.al [20] solved fully fuzzy transportation problems with triangular fuzzy numbers. Dai et.al [10] used fuzzy fractional programming model for optimizing water footprint of crop planting and trading. In this paper, we introduce an iterative method for solving single and multi objective FLFPP with out transforming to its equivalent crisp form.

This article is organized as follows: Section 2 introduces fuzzy numbers and offers basic principles and results. Section 3 provides the general FLFPP and the solution methods. Section 4 contains solution algorithm. Section 5 provides numerical examples and the final section provides the conclusion.

2. Preliminaries

In this section, we recall the basic concepts of fuzzy numbers and the related results.

Definition 2.1. *A fuzzy set $\tilde{u}$ defined on the set of real numbers R is said to be a fuzzy number, if its membership function $\tilde{u} : R \to [0,1]$ has the following characteristics:*

- *$\tilde{u}$ is convex, (i.e.) $\tilde{u}(\lambda y_1 + (1-\lambda)y_2) \geq \min\{\tilde{u}(y_1), \tilde{u}(y_2)\}, \lambda \in [0,1]$, for all $y_1, y_2 \in R$.*
- *$\tilde{u}$ is normal, (i.e.) there exists an $y \in R$ such that $\tilde{u}(y) = 1$.*
- *$\tilde{u}$ is piecewise continuous.*

We use $F(R)$ to denote the set of all fuzzy numbers defined on R.

Definition 2.2. *A triangular fuzzy number(TFN) $\tilde{u} = (u_1, u_2, u_3) \in F(R)$ can also be represented as a pair $\tilde{u} = (\underline{u}, \overline{u})$ of functions $\underline{u}(e), \overline{u}(e)$ for $0 \leq e \leq 1$, which satisfies the following requirements:*

- *$\underline{u}(e)$ is a bounded monotonic increasing left continuous function.*
- *$\overline{u}(e)$ is a bounded monotonic decreasing left continuous function.*
- *$\underline{u}(e) \leq \overline{u}(e), 0 \leq e \leq 1$.*

An arbitrary triangular fuzzy number $\tilde{u} = (u_1, u_2, u_3)$ can also be written in its parametric form as $\tilde{u} = (u_0, u_*, u^*)$, where $u_0 = \left(\frac{\underline{u}(1) + \overline{u}(1)}{2}\right)$ is the location index number, $u_* = (u_0 - \underline{u})$ and $u^* = (\overline{u} - u_0)$ are the left and the right fuzziness index functions respectively.

2.1. **Ranking of Triangular Fuzzy Numbers.** Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. We define the magnitude of the triangular fuzzy number $\tilde{u} = (u_0, u_*, u^*)$ by

$$\mathcal{R}(\tilde{u}) = \left(\frac{u^* + 4u_0 - u_*}{4}\right) = \left(\frac{\underline{u} + \overline{u} + u_0}{4}\right).$$

For any two triangular fuzzy numbers $\tilde{u} = (u_0, u_*, u^*)$ and $\tilde{v} = (v_0, v_*, v^*)$ in $F(R)$ we have

- $\tilde{u} \succeq \tilde{v}$ if and only if $\mathcal{R}(\tilde{u}) \geq \mathcal{R}(\tilde{v})$
- $\tilde{u} \preceq \tilde{v}$ if and only if $\mathcal{R}(\tilde{u}) \leq \mathcal{R}(\tilde{v})$
- $\tilde{u} \approx \tilde{v}$ if and only if $\mathcal{R}(\tilde{u}) = \mathcal{R}(\tilde{v})$

2.2. **Arithmetic Operations of Triangular Fuzzy Numbers(TFN's).** Ming Ma et. al. [26] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is lub and glb in the lattice L. i.e. for $u, v \in L$, we define $u \vee v = \max\{u, v\}$ and $u \wedge v = \min\{u, v\}$.

The arithmetic operations on any two triangular fuzzy number(TFN) $\tilde{u} = (u_0, u_*, u^*), \tilde{v} = (v_0, v_*, v^*)$ are defined by

$$\tilde{u} \star \tilde{v} = (u_0, u_*, u^*) \star (v_0, v_*, v^*) = ((u_0 \star v_0), \max\{u_*, v_*\}, \max\{u^*, v^*\}),$$

where $\star \in \{+, -, \times, \div\}$.

Note 1: Division is possible only when the location index number of the denominator fuzzy number is non-zero.

Note 2: For any two triangular fuzzy numbers(TFN's) $\tilde{u} = (u_0, u_*, u^*)$, $\tilde{v} = (v_0, v_*, v^*) \in F(R)$, the following results are true.

(i). $\mathcal{R}(\tilde{u} + \tilde{v}) = \mathcal{R}(\tilde{u}) + \mathcal{R}(\tilde{v})$ and

(ii). $\mathcal{R}\left(\frac{\tilde{u}}{\tilde{v}}\right) = \frac{\mathcal{R}(\tilde{u})}{\mathcal{R}(\tilde{v})}$.

3. FUZZY MULTI-OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEMS

The general form of a FMOLFPP is given by

$$\begin{aligned} \max \tilde{z}(\tilde{y}) &= [\tilde{z}_i(\tilde{y})] \text{ for all } i = 1, 2..., n \\ \text{subject to } \tilde{A}\tilde{\mathbf{y}} &\preceq \tilde{\mathbf{b}} \\ \textit{and } \tilde{\mathbf{y}} &\succeq \tilde{\mathbf{0}} \end{aligned} \tag{1}$$

where $\tilde{z}_i(\tilde{y})= \dfrac{f_i(\tilde{y})}{g_i(\tilde{y})} = \dfrac{\tilde{\mathbf{c}}_i\tilde{\mathbf{y}}+\tilde{\alpha}_i}{\tilde{\mathbf{d}}_i\tilde{\mathbf{y}}+\tilde{\beta}_i}$,
$\tilde{\mathbf{c}}_i, \tilde{\mathbf{d}}_i$ are n dimensional fuzzy vectors and $\tilde{\alpha}_i, \tilde{\beta}_i$ are fuzzy scalars.

Definition 3.1. *Let $\tilde{S}$ be the set of all fuzzy feasible solutions of the FMOLFPP* (1). *A feasible solution $\tilde{\mathbf{y}}^*$ is said to be fuzzy Pareto optimum solution of the FMOLFPP* (1)*, if there does not exist another feasible solution $\tilde{\mathbf{y}} \in \tilde{S}$ such that $\tilde{z}_i(\tilde{\mathbf{y}}) \succeq \tilde{z}_i(\tilde{\mathbf{y}}^*)$ for all i and $\tilde{z}_j(\tilde{\mathbf{y}}) \succ \tilde{z}_j(\tilde{\mathbf{y}}^*)$ atleast one j.*

3.1. **Fuzzy target and fuzzy tolerance.** Let us assume that the fuzzy target as $\tilde{k}_i$ and the fuzzy tolerance limit as $\tilde{l}_i$ for each objective of the FMOLFP problem (1). The membership function of the objective function of FMOLFP problem (1) is given by

$$\mu(\tilde{z}_i(\tilde{y})) = \begin{cases} 1, & if\ \mathcal{R}(\tilde{z}_i(\tilde{y})) > \mathcal{R}(\tilde{k}_i) \\ \mathcal{R}\left(\dfrac{(\tilde{z}_i(\tilde{y})) - \tilde{l}_i}{\tilde{k}_i - \tilde{l}_i}\right), & if\ \mathcal{R}(\tilde{l}_i) \le \mathcal{R}(\tilde{z}_i(\tilde{y})) \le \mathcal{R}(\tilde{k}_i) \\ 0, & if\ \mathcal{R}(\tilde{z}_i(\tilde{y})) < \mathcal{R}(\tilde{l}_i). \end{cases} \tag{2}$$

3.2. **Conversion of FMOLFPP to NFLPP.** As suggested by Rubi Arya and Pitam Singh [3], the FMOLFP problem (1) is reduced to

$$\begin{aligned} F(\tilde{y},\lambda) =& \max[\mu(\tilde{z}_i(\tilde{y})) - \lambda_i], \text{ for all } i = 1, 2..., n \\ \textit{subject to } & \lambda_i \le \mu(\tilde{z}_i(\tilde{y})) \\ & \tilde{A}\tilde{\mathbf{y}} \preceq \tilde{\mathbf{b}} \\ \textit{and } & \tilde{\mathbf{y}} \succeq \tilde{\mathbf{0}}. \end{aligned}$$

Let $\tilde{q}_i = \tilde{k}_i\lambda_i + (1-\lambda_i)\tilde{l}_i, \lambda_i \in [0, 1]$, where $\tilde{q}_i$ is the maximum value of the ith objective function in problem (1). After simplification, we get the non fractional form of (1) as

$$\begin{aligned} F(\tilde{y},\tilde{q}_i) =& \max_{\tilde{y}\in S}\{f_i(\tilde{y}) - \tilde{q}_i(g_i(\tilde{y}))\}, \text{ for all } i = 1, 2..., n \\ \textit{subject to } & \tilde{A}\tilde{\mathbf{y}} \preceq \tilde{\mathbf{b}} \\ \textit{and } & \tilde{\mathbf{y}} \succeq \tilde{\mathbf{0}}. \end{aligned} \tag{3}$$

By taking summation, the single objective non-fractional fuzzy linear programming problem corresponding to the problem (1) is given by

$$\begin{aligned} F(\tilde{y},\tilde{q}_i) =& \max\sum_{i=1}^{n}[f_i(\tilde{y}) - \tilde{q}_i g_i(\tilde{y}] \\ \text{subject to } & \tilde{A}\tilde{\mathbf{y}} \preceq \tilde{\mathbf{b}} \\ \text{and } & \tilde{\mathbf{y}} \succeq \tilde{\mathbf{0}}. \end{aligned} \tag{4}$$

Applying the ranking function on $F(\tilde{y},\tilde{q}_i) = \max\sum_{i=1}^{n}[f_i(\tilde{y}) - \tilde{q}_i g_i(\tilde{y}]$ of the problem (4), we have

$$\mathcal{R}(F(\tilde{y},\tilde{q}_i)) = \mathcal{R}(\max\sum_{i=1}^{n}[f_i(\tilde{y}) - \tilde{q}_i g_i(\tilde{y})])$$

$$\Rightarrow F(y, q_i) = \max\sum_{i=1}^{n}[f_i(y) - q_i g_i(y]. \tag{5}$$

Differentiate the non fractional $F(y, q_i)$ with respect to the non basic variables partially and find its value at all these current basic feasible points. If all these derivatives are negative, then this current solution is optimal. That is

$$\text{If} \qquad \frac{\partial F}{\partial y_i} \leq 0 \text{ (for all non basic variable } y_i), \tag{6}$$

then the current solution is optimal.

Theorem 3.1. *For any fully fuzzy linear fractional programming problem* (4)*, if* $\tilde{z}^* \approx \dfrac{f(\tilde{\mathbf{y}^*})}{g(\tilde{\mathbf{y}^*})} \approx \max\left\{\dfrac{f(\tilde{\mathbf{y}})}{g(\tilde{\mathbf{y}})} / \tilde{\mathbf{y}} \in S\right\}$*, then* $F(\tilde{\mathbf{y}^*}, \tilde{q}_i) \approx \max\{f(\tilde{\mathbf{y}}) - \tilde{q}_i g(\tilde{\mathbf{y}}) / \tilde{\mathbf{y}} \in S\} \approx \tilde{0}$.

Theorem 3.2. *If* $\tilde{\mathbf{y}^*}$ *is an optimal solution of problem* (4)*, then* $\tilde{\mathbf{y}^*}$ *is also a Pareto optimal solution of problem* (1).

Proof: Assume that $\tilde{\mathbf{y}^*}$ is an optimal solution of problem (4). Suppose that $\tilde{\mathbf{y}}^*$ is not a Pareto optimal solution for problem (1). Then there exists an $\tilde{\mathbf{y}} \in S$ such that $\tilde{z}_i(\tilde{\mathbf{y}}) \succeq \tilde{z}_i(\tilde{\mathbf{y}^*})$ for all $i = 1, 2, ..., k$ and $\tilde{z}_i(\tilde{\mathbf{y}}) \succ \tilde{z}_i(\tilde{\mathbf{y}^*})$ for at least one $j = 1, 2, ..., k$.

$\Rightarrow \dfrac{f_i(\tilde{\mathbf{y}})}{g_i(\tilde{\mathbf{y}})} \succeq \dfrac{f_j(\tilde{\mathbf{y}^*})}{g_j(\tilde{\mathbf{y}^*})}$ for all i and $\dfrac{f_j(\tilde{\mathbf{y}})}{g_j(\tilde{\mathbf{y}})} \succ \dfrac{f_j(\tilde{\mathbf{y}^*})}{g_j(\tilde{\mathbf{y}^*})}$ for at least one j.

$\Rightarrow \dfrac{f_i(\tilde{\mathbf{y}})}{g_i(\tilde{\mathbf{y}})} - \tilde{l}_i \succeq \dfrac{f_j(\tilde{\mathbf{y}^*})}{g_j(\tilde{\mathbf{y}^*})} - \tilde{l}_i$ for all i and $\dfrac{f_j(\tilde{\mathbf{y}})}{g_j(\tilde{\mathbf{y}})} - \tilde{l}_j \succ \dfrac{f_j(\tilde{\mathbf{y}^*})}{g_j(\tilde{\mathbf{y}^*})} - \tilde{l}_j$ for at least one j.

$\Rightarrow \dfrac{\left(\dfrac{f_i(\tilde{\mathbf{y}})}{g_i(\tilde{\mathbf{y}})} - \tilde{l}_i\right)}{\tilde{k}_i - \tilde{l}_i} \succeq \dfrac{\left(\dfrac{f_j(\tilde{\mathbf{y}^*})}{g_j(\tilde{\mathbf{y}^*})} - \tilde{l}_i\right)}{\tilde{k}_i - \tilde{l}_i}$ for all i and $\dfrac{\left(\dfrac{f_j(\tilde{\mathbf{y}})}{g_j(\tilde{\mathbf{y}})} - \tilde{l}_j\right)}{\tilde{k}_j - \tilde{l}_j} \succ \dfrac{\left(\dfrac{f_j(\tilde{\mathbf{y}^*})}{g_j(\tilde{\mathbf{y}^*})} - \tilde{l}_j\right)}{\tilde{k}_j - \tilde{l}_j}$ for at least one j.

$$\Rightarrow \frac{\left(\dfrac{f_i(\tilde{\mathbf{y}})}{g_i(\tilde{\mathbf{y}})} - \tilde{l}_i\right)}{\tilde{k}_i - \tilde{l}_i} - \lambda_i \succeq \frac{\left(\dfrac{f_j(\tilde{\mathbf{y}^*})}{g_j(\tilde{\mathbf{y}^*})} - \tilde{l}_i\right)}{\tilde{k}_i - \tilde{l}_i} - \lambda_i \quad \text{for all } i \text{ and}$$

$$\frac{\left(\dfrac{f_j(\tilde{\mathbf{y}})}{g_j(\tilde{\mathbf{y}})} - \tilde{l}_j\right)}{\tilde{k}_j - \tilde{l}_j} - \lambda_j \succ \frac{\left(\dfrac{f_j(\tilde{\mathbf{y}^*})}{g_j(\tilde{\mathbf{y}^*})} - \tilde{l}_j\right)}{\tilde{k}_j - \tilde{l}_j} - \lambda_j \quad \text{for atleast one } j.$$

$$\Rightarrow f_i(\tilde{\mathbf{y}}) - [\lambda_i \tilde{k}_i + (1 - \lambda_i)\tilde{l}_i] g_i(\tilde{\mathbf{y}}) \succeq f_i(\tilde{\mathbf{x}^*}) - [\lambda_i \tilde{k}_i + (1 - \lambda_i)\tilde{l}_i] g_i(\tilde{\mathbf{y}^*}) \quad \text{for all } i \text{ and}$$

$$f_j(\tilde{\mathbf{y}}) - [\lambda_j \tilde{k}_j + (1 - \lambda_j)\tilde{l}_j] g_j(\tilde{\mathbf{y}}) \succeq f_j(\tilde{\mathbf{y}^*}) - [\lambda_j \tilde{k}_j + (1 - \lambda_j)\tilde{l}_j] g_j(\tilde{\mathbf{y}^*}) \quad \text{for atleast one } j.$$

Let $\tilde{q}_i = \lambda_i \tilde{k}_i + (1 - \lambda_i)\tilde{l}_i$, then the above inequalities becomes

$$f_i(\tilde{\mathbf{y}}) - \tilde{q}_i g_i(\tilde{\mathbf{y}}) \succeq f_i(\tilde{\mathbf{y}^*}) - \tilde{q}_i g_i(\tilde{\mathbf{y}^*}) \quad \text{for all } i \text{ and}$$

$$f_j(\tilde{\mathbf{y}}) - \tilde{q}_j g_j(\tilde{\mathbf{y}}) \succeq f_j(\tilde{\mathbf{y}^*}) - \tilde{q}_j g_j(\tilde{\mathbf{y}^*}) \quad \text{for atleast one } j.$$

Summing over k, we get

$$\sum_{i=1}^{i=k} [f_i(\tilde{\mathbf{y}}) - \tilde{q}_i g_i(\tilde{\mathbf{y}})] \succeq \sum_{i=1}^{i=k} [f_i(\tilde{\mathbf{y}^*}) - \tilde{q}_i g_i(\tilde{\mathbf{y}^*})] \text{ and}$$

$$\sum_{j=1}^{j=k}[f_j(\tilde{\mathbf{y}}) - \tilde{q}_j g_j(\tilde{\mathbf{y}})] \succ \sum_{j=1}^{j=k}[f_j(\tilde{\mathbf{y}^*}) - \tilde{q}_j g_j(\tilde{\mathbf{y}^*})].$$

Then by theorem (3.1), we have $F_i(\tilde{\mathbf{y}}, \tilde{q}_i) \succeq F_i(\tilde{\mathbf{y}^*}, \tilde{q}_i)$ for all i and $F_j(\tilde{\mathbf{y}}, \tilde{q}_j) \succ F_j(\tilde{\mathbf{y}^*}, \tilde{q}_j)$ for at least one j, which contradicts the fact that $\tilde{\mathbf{y}^*}$ is an optimal solution of problem (4). Hence the optimal solution of problem (4) is also a Pareto optimal solution of problem (1).

4. ALGORITHM

Step 1: Express all the fuzzy numbers in the FMOLFPP in their parametric form.
Step 2: Express this FMOLFPP into its standard form and obtain the initial fuzzy basic feasible solution(IFBFS) with the corresponding value of each objective z_i.
Step 3: Express the constraints in terms of non basic variables(NBV's) and convert the FMOLFPP in to an equivalent non fractional FLPP in terms of NBV's as

$$F(\tilde{y}, \tilde{q}_i) = \max \sum_{i=1}^{n}[f_i(\tilde{y}) - \tilde{q}_i g_i(\tilde{y}], \text{where } \tilde{q}_i \approx \tilde{z}_i. \tag{7}$$

Step 4: Check for optimality of the current solution using equations (5) and (6). If not optimal, go to the next step.
Step 5: The entering variable is the NBV corresponding to the most positive derivative. Express the constraints obtained in step 3 in terms of this NBV and identify the basic variable(BV) with minimum value which is going to become a new NBV.
Step 6: Express the new FMOLFPP in terms of the new NBV's and obtain the IFBFS with the objective values and then go to step 4.

5. NUMERICAL EXAMPLES

Example 1: Consider a fuzzy linear fractional programming problem discussed by Veeramani and Sumathi [32]

$$\begin{aligned} \max \tilde{z} \approx & \frac{\tilde{5}\tilde{y}_1 + \tilde{3}\tilde{y}_2}{\tilde{5}\tilde{y}_1 + \tilde{2}\tilde{y}_2 + \tilde{1}} \\ \text{subject to } & \tilde{3}\tilde{y}_1 + \tilde{5}\tilde{y}_2 \preceq \tilde{15}, \tilde{5}\tilde{y}_1 + \tilde{2}\tilde{y}_2 \preceq \tilde{10} \\ \text{and } & \tilde{y}_1, \tilde{y}_2 \succeq \tilde{0}. \end{aligned} \tag{8}$$

Solution: Let us assume that all the fuzzy numbers are TFN's and are expressed in their parametric form as: $\tilde{1} \approx (0,1,2) \approx (1, 1-e, 1-e), \tilde{2} \approx (1,2,3) \approx (2, 1-e, 1-e), \tilde{3} \approx (0,3,6) \approx (3, 3-3e, 3-3e), \tilde{5} \approx (3,5,7) \approx (5, 2-2e, 2-2e), \tilde{15} \approx (14,15,16) \approx (15, 1-e, 1-e)$ and $\tilde{10} \approx (8,10,12) \approx (10, 2-2e, 2-2e))$.

The given FLFPP (8) in its parametric form as

$$\begin{aligned} \max \tilde{z} \approx & \frac{(5, 2-2e, 2-2e)\tilde{y_1} + (3, 3-3e, 3-3e)\tilde{y_2}}{(5, 2-2e, 2-2e)\tilde{y_1} + (2, 1-e, 1-e)\tilde{y_2} + (1, 1-e, 1-e)} \\ \text{subject to } & (3, 3-3e, 3-3e)\tilde{y_1} + (5, 2-2e, 2-2e)\tilde{y_2} \preceq (15, 1-e, 1-e) \\ & (5, 2-2e, 2-2e)\tilde{y_1} + (2, 1-e, 1-e)\tilde{y_2} \preceq (10, 2-2e, 2-2e) \\ \text{and } & \tilde{y_1}, \tilde{y_2} \succeq \tilde{0}. \end{aligned} \tag{9}$$

Initial Iteration:
By introducing the non negative slack variables $\tilde{y}_3, \tilde{y}_4$, the standard for of the FLFPP (9) is

$$\begin{aligned}
\max \tilde{z} \approx & \frac{(5,2-2e,2-2e)\tilde{y_1}+(3,3-3e,3-3e)\tilde{y_2}}{(5,2-2e,2-2e)\tilde{y_1}+(2,1-e,1-e)\tilde{y_2}+(1,1-e,1-e)} \\
\text{subject to } & (3,3-3e,3-3e)\tilde{y_1}+(5,2-2e,2-2e)\tilde{y_2}+\tilde{y}_3 \approx (15,1-e,1-e) \\
& (5,2-2e,2-2e)\tilde{y_1}+(2,1-e,1-e)\tilde{y_2}+\tilde{y}_4 \approx (10,2-2e,2-2e) \\
\text{and } & \tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \tilde{y}_4 \succeq \tilde{0}.
\end{aligned} \tag{10}$$

The IFBFS is $\mathbf{\tilde{y}} \approx \begin{pmatrix} (0,0,0) \\ (0,0,0) \\ (15,1-e,1-e) \\ (10,2-2e,2-2e) \end{pmatrix}$ with $\tilde{z}^1 \approx (0,0,0)$.

From equation (10), the basic variables are expressed in terms of non basic variables as
$\tilde{y}_3 \approx (15,1-e,1-e)-(3,3-3e,3-3e)\tilde{y_1}-(5,2-2e,2-2e)\tilde{y_2}$
$\tilde{y}_4 \approx (10,2-2e,2-2e)-(5,2-2e,2-2e)\tilde{y_1}-(2,1-e,1-e)\tilde{y_2}$

Now reduce the FMOLFPP (10) in to an equivalent non fractional FLPP as in equation (7) in terms of non basic variables, we have $\tilde{F}^1(\tilde{y}) \approx (5,2-2e,2-2e)\tilde{y_1}+(3,3-3e,3-3e)\tilde{y_2}$ and check the optimality of this by applying step 4, we have $F^1(y) = 5y_1 + 3y_2$

Differentiating partially with respect to y_1 and y_2, we get,

$\dfrac{\partial F^1(y)}{\partial y_1} = 5$ and $\dfrac{\partial F^1(y)}{\partial y_2} = 3$. Hence the current solution is not optimal. Then by step 5, $\max\left(\dfrac{\partial F^1(y)}{\partial y_1}, \dfrac{\partial F^1(y)}{\partial y_2}\right) = \max(5,3) = 5$.

The non-basic variable y_1 becomes the basic variable and the basic variable y_4 becomes non basic.

First Iteration:
Applying step 6, we have
$\tilde{y_1} \approx (2,2-2e,2-2e)-(2/5,2-2e,2-2e)\tilde{y_2}-(1/5,2-2e,2-2e)\tilde{y}_4$
$\tilde{y}_3 \approx (9,3-3e,3-3e)-(19/5,3-3e,3-3e)\tilde{y_2}+(3/5,3-3e,3-3e)\tilde{y}_4$

and $\tilde{z}^2 \approx \dfrac{(10,2-2e,2-2e)+(1,3-3e,3-3e)\tilde{y_2}-(1,2-2e,2-2e)\tilde{y}_4}{(11,2-2e,2-2e)-(1,2-2e,2-2e)\tilde{y}_4}$

with $\mathbf{\tilde{y}} \approx \begin{pmatrix} (2,2-2e,2-2e) \\ (0,0,0) \\ (9,3-3e,3-3e) \\ (0,0,0) \end{pmatrix}$ and $\tilde{z}^2 \approx (10/11,2-2e,2-2e)$.

Applying step 3, the equivalent non fractional FLPP as in equation (7) is given by $\tilde{F}^2(\tilde{y}) \approx (1,3-3e,3-3e)\tilde{y_2}-(1/11,2-2e,2-2e)\tilde{y}_4$ and check the optimality of this by applying step 4, we have $F^2(y) = y_2 - (1/11)y_4$.

Differentiating with respect to y_2 and y_4, we get, $\dfrac{\partial F^2(y)}{\partial y_2} = 1$ and $\dfrac{\partial F^2(y)}{\partial y_4} = -1/11$. Hence this solution is not optimal. Then by step 5, we have $\max\left(\dfrac{\partial F^2(y)}{\partial y_2}, \dfrac{\partial F^2(y)}{\partial y_4}\right) \approx \max(1,-1/11) = 1$.

The non-basic variable y_2 becomes the basic variable and the basic variable y_3 becomes non basic.

Second Iteration:
Applying step 6 , we have
$\tilde{y}_2 \approx (45/19, 3-3e, 3-3e) - (5/19, 3-3e, 3-3e)\tilde{y}_3 + (3/19, 3-3e, 3-3e)\tilde{y}_4$

$\tilde{y}_1 \approx (20/19, 3-3e, 3-3e) + (2/19, 3-3e, 3-3e)\tilde{y}_3 - (5/19, 3-3e, 3-3e)\tilde{y}_4$

Also $\tilde{z}^3 \approx \dfrac{(235/19, 3-3e, 3-3e) - (5/19, 3-3e, 3-3e)\tilde{y}_3 - (16/19, 3-3e, 3-3e)\tilde{y}_4}{(11, 2-2e, 2-2e) - (1, 2-2e, 2-2e)\tilde{y}_4}$

and $\tilde{\mathbf{y}} \approx \begin{pmatrix} (20/19, 3-3e, 3-3e) \\ (45/19, 3-3e, 3-3e) \\ (0,0,0) \\ (0,0,0) \end{pmatrix}$ with $\tilde{z}^3 \approx (235/209, 3-3e, 3-3e)$.

Applying step 3, the equivalent non fractional FLPP as in equation (7) is given by $\tilde{F}^3(\tilde{y}) \approx -(5/19, 3-3e, 3-3e)\tilde{y}_3 + (59/209, 3-3e, 3-3e)\tilde{y}_4$ and check the optimality of this by applying step 4, we have $F^3(y) = -(5/19)y_3 + (59/209)y_4$.

Differentiating with respect to y_3 and y_4, we get, $\dfrac{\partial F^3(y)}{\partial y_3} = -5/19$ and $\dfrac{\partial F^3(y)}{\partial y_4} = 59/209$. Hence the current solution is not optimal.
Then by steps 5, we have $\max\left(\dfrac{\partial F^3(y)}{\partial y_3}, \dfrac{\partial F^3(y)}{\partial y_4}\right) \approx \max(-5/19, 59/209) \approx 59/209$.
The non-basic variable y_4 becomes the basic variable and the basic variable y_1 becomes non basic.

Third Iteration:
Applying step 6, we have
$\tilde{y}_2 \approx (285/95, 3-3e, 3-3e) - (19/95, 3-3e, 3-3e)\tilde{y}_3 - (1, 3-3e, 3-3e)\tilde{y}_1$
$\tilde{y}_4 \approx (20/5, 3-3e, 3-3e) + (2/5, 3-3e, 3-3e)\tilde{y}_3 - (19/5, 3-3e, 3-3e)\tilde{y}_1$
Also $\tilde{z}^4 \approx \dfrac{(855/95, 3-3e, 3-3e) - (57/95, 3-3e, 3-3e)\tilde{y}_3 + (16/5, 3-3e, 3-3e)\tilde{y}_1}{(35/5, 3-3e, 3-3er) - (2/5, 3-3e, 3-3e)\tilde{y}_3 + (19/5, 3-3e, 3-3e)\tilde{y}_1}$

and $\tilde{y} \approx \begin{pmatrix} (0,0,0) \\ (285/95, 3-3e, 3-3e) \\ (0,0,0) \\ (20/5, 3-3e, 3-3e) \end{pmatrix}$ with $\tilde{z}^4 \approx (855/665, 3-3e, 3-3e)$.

Applying step 3, the equivalent non fractional FLPP as in equation (7) is given by $\tilde{F}^4(\tilde{y}) \approx -(285/3325, 3-3e, 3-3e)\tilde{y}_3 - (5605/3325, 3-3e, 3-3e)\tilde{y}_1$ and check the optimality of this by applying step 4, we have $F^4(y) = -(285/3325)y_3 - (5605/3325)y_1$.
Differentiating with respect to y_3 and y_4, we get

$$\frac{\partial F^4(y)}{\partial y_3} = -285/3325 \quad \text{and} \quad \frac{\partial F^4(y)}{\partial y_1} = -5605/3325.$$

That is $\dfrac{\partial F^4(y)}{\partial y_3} < 0$ and $\dfrac{\partial F^4(y)}{\partial y_1} < 0$ and hence this solution is optimal.

Therefore the optimal solution of the given FLFPP (8) is
$\tilde{y}_1 \approx (3, 3-3e, 3-3e), \tilde{y}_2 \approx (0, 3-3e, 3-3e)$ with $\tilde{z} \approx (1.286, 3-3e, 3-3e)$.

TABLE 1. Flexible optimal solutions for some values of e

e	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{Z}_1$
1	(3,3,3)	(0,0,0)	(1.286,1.286,1.286)
0.9	(2.7,3,3.3)	(-0.3,0,0.3)	(0.986,1.286,1.586)
0.8	(2.4,3,3.6)	(-0.6,0,0.6)	(0.686,1.286,1.886)

Veeramani and Sumathi [32] have transformed the given FLFP problem to a MOLFP problem and the resultant problem is converted to a LP problem, using Fuzzy Mathematical programming method and obtained a crisp optimal solution.

Example 2:
Consider a FFMOLFPP discussed by Durga Prasad Dash et al. [16]

$$\begin{aligned} \max \tilde{z_1} \approx & \frac{\tilde{6}\tilde{y_1} + \tilde{5}\tilde{y_2}}{\tilde{2}\tilde{y_1} + \tilde{7}}, \ \max \tilde{z_2} \approx \frac{\tilde{2}\tilde{y_1} + \tilde{3}\tilde{y_2}}{\tilde{1}\tilde{y_1} + \tilde{1}\tilde{y_2} + \tilde{7}} \\ \text{subject to } & \tilde{1}\tilde{y_1} + \tilde{2}\tilde{y_2} \preceq \tilde{3} \\ & \tilde{3}\tilde{y_1} + \tilde{2}\tilde{y_2} \preceq \tilde{6} \\ \text{and } \quad & \tilde{y_1}, \tilde{y_2} \succeq \tilde{0}. \end{aligned} \tag{11}$$

Solution: Let us assume that all the fuzzy numbers are TFN's and are expressed in their parametric form as $\tilde{1} \approx (0,1,2) \approx (1,1-e,1-e), \tilde{2} \approx (1,2,3) \approx (2,1-e,1-e), \tilde{3} \approx (0,3,6) \approx (3,3-3e,3-3e), \tilde{5} \approx (3,5,7) \approx (5,2-2e,2-2e), \tilde{6} \approx (5,6,7) \approx (6,1-e,1-e)$ and $\tilde{7} \approx (5,7,9) \approx (7,2-2e,2-2e)$).

The given FFMOLFPP (11) in its parametric form

$$\begin{aligned} \max \tilde{z_1} &\approx \left\{ \frac{(6,1-e,1-e)\tilde{y_1} + (5,2-2e,2-2e)\tilde{y_2}}{(2,1-e,1-e)\tilde{y_1} + (7,2-2e,2-2e)} \right\} \\ \max \tilde{z_2} &\approx \left\{ \frac{(2,1-e,1-e)\tilde{y_1} + (3,3-3e,3-3e)\tilde{y_2}}{(1,1-e,1-e)\tilde{y_1} + (1,1-e,1-e)\tilde{y_2} + (7,2-2e,2-2e)} \right\} \\ \text{subject to} \quad & (1,1-e,1-e)\tilde{y_1} + (2,1-e,1-e)\tilde{y_2} \preceq (3,3-3e,3-3e) \\ & (3,3-3e,3-3e)\tilde{y_1} + (2,1-e,1-e)\tilde{y_2} \preceq (6,1-e,1-e) \\ \text{and} \quad & \tilde{y_1}, \tilde{y_2} \succeq \tilde{0}. \end{aligned} \tag{12}$$

First Iteration:
By introducing the non negative slack variables $\tilde{y_3}, \tilde{y_4}$, the standard form of the FLFPP (12) is

$$\begin{aligned} \max \tilde{z_1} &\approx \left\{ \frac{(6,1-e,1-e)\tilde{y_1} + (5,2-2e,2-2e)\tilde{y_2}}{(2,1-e,1-e)\tilde{y_1} + (7,2-2e,2-2e)} \right\} \\ \max \tilde{z_2} &\approx \left\{ \frac{(2,1-e,1-e)\tilde{y_1} + (3,3-3e,3-3e)\tilde{y_2}}{(1,1-e,1-e)\tilde{y_1} + (1,1-e,1-e)\tilde{y_2} + (7,2-2e,2-2e)} \right\} \\ \text{subject to} \quad & (1,1-e,1-e)\tilde{y_1} + (2,1-e,1-e)\tilde{y_2} + \tilde{y_3} \approx (3,3-3e,3-3e) \\ & (3,3-3e,3-3e)\tilde{y_1} + (2,1-e,1-e)\tilde{y_2} + \tilde{y_4} \approx (6,1-e,1-e) \\ \text{and} \quad & \tilde{y_1}, \tilde{y_2}, \tilde{y_3}, \tilde{y_4} \succeq \tilde{0}. \end{aligned} \tag{13}$$

The IFBFS is $\tilde{y} \approx \begin{pmatrix} (0,0,0) \\ (0,0,0) \\ (3,3-3e,3-3e) \\ (6,1-e,1-e) \end{pmatrix}$ with $\tilde{z}_1^1 \approx (0,0,0)$, $\tilde{z}_2^1 \approx (0,0,0)$.
From equation (13), the BV's are expressed in terms of NBV's as

$$\tilde{y}_3 \approx (3,3-3e,3-3e) - (1,1-e,1-e)\tilde{y}_1 - (2,1-e,1-e)\tilde{y}_2$$
$$\tilde{y}_4 \approx (6,1-e,1-e) - (3,3-3e,3-3e)\tilde{y}_1 - (2,1-e,1-e)\tilde{y}_2$$

Applying step 3, the equivalent non fractional FLPP as in equation (7) is given by $\tilde{F}^1(y) \approx (6,1-e,1-e)\tilde{y}_1 + (5,2-2e,2-2e)\tilde{y}_2 + (2,1-e,1-e)\tilde{y}_1 + (3,3-3e,3-3e)\tilde{y}_2$ $\approx (8,3-3e,3-3e)\tilde{y}_1 + (8,3-3e,3-3e)\tilde{y}_2$ and check the optimality of this by applying step 4 we have $\tilde{F}^1(y) = 8y_1 + 8y_2$.

Differentiating with respect to y_1 and y_2, we get $\dfrac{\partial F^1(y)}{\partial y_1} \approx 8$ and $\dfrac{\partial F^1(y)}{\partial y_2} \approx 8$. Hence the current solution is not optimal. By steps 5, we have $\max\left(\dfrac{\partial F^1(y)}{\partial y_1}, \dfrac{\partial F^1(y)}{\partial y_2}\right) \approx$ $\max(8,8) = 8$. Therefore y_1 becomes basic and y_4 becomes non basic.

Second Iteration:

Applying step 6, we have
$\tilde{y}_1 \approx (2,3-3e,3-3e) - (2/3,3-3e,3-3e)\tilde{y}_2 - (1/3,3-3e,3-3e)\tilde{y}_4$
$\tilde{y}_3 \approx (1,3-3e,3-3e) - (4/3,3-3e,3-3e)\tilde{y}_2 + (1/3,3-3e,3-3e)\tilde{y}_4$

Also $\tilde{z}_1^2 \approx \dfrac{(36,3-3e,3-3e) + (3,3-3e,3-3e)\tilde{y}_2 - (6,3-3e,3-3e)\tilde{y}_4}{(33,3-3e,3-3e) + (4,3-3e,3-3e)\tilde{y}_2 - (2,3-3e,3-3e)\tilde{y}_4}$
$\tilde{z}_2^2 \approx \dfrac{(12,3-3e,3-3e) + (5,3-3e,3-3e)\tilde{y}_2 - (2,3-3e,3-3e)\tilde{y}_4}{(27,3-3e,3-3e) + (1,3-3e,3-3e)\tilde{y}_2 - (1,3-3e,3-3e)\tilde{y}_4}$
and $\tilde{y} \approx \begin{pmatrix} (2,3-3e,3-3e) \\ (0,0,0) \\ (1,3-3e,3-3e) \\ (0,0,0) \end{pmatrix}$ with $\tilde{z}_1^2 \approx (36/33,3-3e,3-3e)$, $\tilde{z}_2^2 \approx (12/27,3-3e,3-3e)$).
Applying step 3, the equivalent non fractional FLPP as in equation (7) is given by $\tilde{F}^2(y) \approx (11.92,3-3e,3-3e)\tilde{y}_2 - (5.38,3-3e,3-3e)\tilde{y}_4$ and check the optimality of this by applying step 4, we get $F^2(y) = 11.92y_2 - 5.38y_4$.

Differentiating with respect to y_2 and y_4, we get $\dfrac{\partial F^2(y)}{\partial y_2} = 11.92$ and $\dfrac{\partial F^2(y)}{\partial y_4} = 5.38$. Hence the current solution is not optimal. Then by steps 5, we have $\max\left(\dfrac{\partial F^2(y)}{\partial y_2}, \dfrac{\partial F^2(y)}{\partial y_4}\right)$ $\approx \max(11.92, 5.38) = 11.92$. Therefore y_2 becomes basic and y_3 becomes non basic.

Third Iteration:

Applying step 6, we have
$\tilde{y}_2 \approx (0.75,3-3e,3-3e) - (0.75,3-3e,3-3e)\tilde{y}_3 - (0.25,3-3e,3-3e)\tilde{y}_4$
$\tilde{y}_1 \approx (1.5,3-3e,3-3e) + (0.5,3-3e,3-3e)\tilde{y}_3 - (0.17,3-3e,3-3e)\tilde{y}_4$.
Also $\tilde{z}_1^3 \approx \dfrac{(153,3-3e,3-3e) - (9,3-3e,3-3e)\tilde{y}_3 - (27,3-3e,3-3e)\tilde{y}_4}{(120,3-3e,3-3e) + (12,3-3e,3-3e)\tilde{y}_3 - (4,3-3e,3-3e)\tilde{y}_4}$

$\tilde{z}_2^3 \approx \dfrac{(63, 3-3e, 3-3e) - (15, 3-3e, 3-3e)\tilde{y}_3 - (13, 3-3e, 3-3e)\tilde{y}_4}{(111, 3-3e, 3-3e) - (3, 3-3e, 3-3e)\tilde{y}_3 - (5, 3-3e, 3-3e)\tilde{y}_4}$

and $\tilde{y} \approx \begin{pmatrix} (1.5, 3-3e, 3-3e) \\ (0.75, 3-3e, 3-3e) \\ (0,0,0) \\ (0,0,0) \end{pmatrix}$ with $\tilde{z}_1^3 \approx (1.275, 3-3e, 3-3e)$, $\tilde{z}_2^3 \approx (0.568, 3-3e, 3-3e))$.

Applying step 3, the equivalent non fractional FLPP as in equation (7) is given by $\tilde{F}^3(y) \approx -(37.6, 3-3e, 3-3e)\tilde{y}_3 - (32.07, 3-3e, 3-3e)\tilde{y}_4$ and check the optimality of this by applying step 4 we have $F^3(y) = -37.6y_3 - 32.07y_4$.

Differentiating with respect to y_2 and y_4, we get $\dfrac{\partial F^3(y)}{\partial y_3} = -37.6$ and $\dfrac{\partial F^3(y)}{\partial y_4} = -32.07$. That is $\dfrac{\partial F^3(y)}{\partial y_3} < 0$ and $\dfrac{\partial F^3(y)}{\partial y_1} < 0$ and hence this solution is optimal.

The Pareto optimal solution of the given FMOLFP problem is $\tilde{y}_1 \approx (1.5, 3-3e, 3-3e), \tilde{y}_2 \approx (0.75, 3-3e, 3-3e)$ with $\tilde{z}_1 \approx (1.275, 3-3e, 3-3e), \tilde{z}_2 \approx (0.568, 3-3e, 3-3e)$.

Table 2. Flexible Pareto optimal solutions for some values of e

e	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{z}_1$	$\tilde{z}_2$
1	(1.5,1.5,1.5)	(0.75,0.75,0.75)	(1.275,1.275,1.275)	(0.568,0.568,0.568)
0.9	(1.2,1.5,1.8)	(0.45,0.75,1.05)	(0.975,1.275,1.575)	(0.268,0.568,0.868)
0.8	(0.9,1.5,2.1)	(0.15,0.75,1.35)	(0.675,1.275,1.875)	(-0.032,0.568,1.168)

Durga Prasad Dash et al. [16] have reduced the FMOLFPP to that of FMOLPP and then reducing it to crisp multi objective LPP using ranking function. Reducing the crisp multi objective LPP to a single objective LPP and obtained a crisp solution as $y_1 = 1.1266$ and $y_2 = 0.9731$.

6. conclusion

In this paper, an iterative method is introduced to find the solution of fuzzy single and multi objective LFPP's with out converting into equivalent crisp forms. The ranking method and arithmetic operations are applied on the parametric forms of the fuzzy numbers. By introducing fuzzy target and the fuzzy tolerance limit for each objective of the given FMOLFP problem, the FMOLFPP is reduced to an equivalent single objective non-fractional fuzzy LPP and then the fuzzy Pareto optimal solution is obtained. Numerical examples are provided to illustrate the theory developed in this article. The solutions obtained by the proposed method are compared with solutions by other state-of-art methodologies and found to be with less vagueness and easy to compute.

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