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LACEABILITY PROPERTIES IN THE IMAGE GRAPH OF PRISM GRAPHS

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Abstract. A connected graph G is termed Hamiltonian-t-laceable if there exists in it a Hamiltonian path between every pair of vertices u and v with the property $d(u,v) = t$, $1 \leq t \leq diam(G)$, where t is a positive integer. In this paper, we establish laceability properties in the image graph of Prism graph $Im(Y_n)$.

Keywords: Hamiltonian graph, Hamiltonian laceable graph, Hamiltonian-t-laceable graph, Prism graph, Image graph.

AMS Subject Classification: [2010] 05C45, 05C99.

1. Introduction

Let G be a finite, simple, connected and undirected graph. Let u and v be two vertices in G. The distance between u and v denoted by $d(u,v)$ is the length of a shortest path in G. G is Hamiltonian laceable if there exists in it a Hamiltonian path between every pair of vertices at an odd distance. G is Hamiltonian-t-laceable if there exists in G a Hamiltonian path between every pair of vertices u and v with the property $d(u,v) = t$, $1 \leq t \leq diam(G)$, where t is a positive integer. Throughout this paper, P_m and K_n will denote the path graph and complete graph with m and n vertices respectively.

Laceability in the brick products of even cycles was explored by Alspach et.al. in [1]. A characterization for a 1-connected graph to be Hamiltonian-t-laceable for $t = 1, 2$ and 3 is given in [3] and this was extended to $t = 4$ and 5 by Thimmaraju and Murali [4]. Leena Shenoy [5] studied Hamiltonian laceability properties in product graphs involving cycles and paths. More results in the laceability properties of product graphs can be found in [6], [7], [8], and [11].

The image graph of a graph was introduced by Annapoorna and Murali in [9] to study laceability properties. In this section, we explore laceability properties of some image graphs. The following definition is found in [9].

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In this paper, we establish laceability properties in the edge tolerant of image of Prism graph $Im(Y_n)$.

Definition 1. *The vertex set of* Y_n*, the prism graph having* $2n$ *vertices and* $3n$ *edges is defined as* $V(Y_n) = \{u_1, u_2, ..., u_n\} \cup \{v_1, v_2, ..., v_n\}$ *(taken clockwise) &* $E(Y_n) = \{e_i : 1 \leq i \leq n\} \cup \{e'_i : 1 \leq i \leq n\} \cup \{e_{ii} : 1 \leq i \leq n\}$ *where* e_i *is the edge* $v_iv_{i+1}(1 \leq i \leq n-1)$*,* e_n *is the edge* v_nv_1 *and* e'_i *is the edge* u_iu_{i+1} $(1 \leq i \leq n-1)$*;* e'_n *is the edge* u_nu_1 *and* e_{ii} *is the edge* v_iu_i $(1 \leq i \leq n-1)$*. We call* $v_1, v_2, ..., v_n$ *as the outer cycle vertices (taken clockwise) and* $u_1, u_2, ..., u_n$ *as the inner cycle vertices (taken clockwise).*

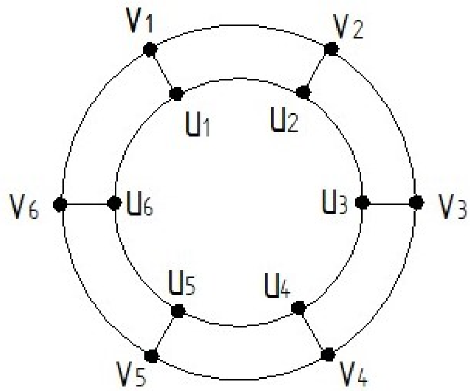

FIGURE 1. Prism graph Y_6

Definition 2. *The image graph of a connected graph* G*, denoted by* $Im(G)$*, is the graph obtained by joining the vertices of the original graph* G *to the corresponding vertices of a copy of* G*.*

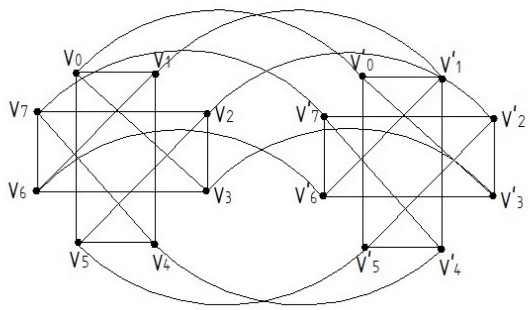

FIGURE 2. Image graph $G(8, 2)$

Definition 3. *A graph* G^* *is k-edge fault tolerant with respect to a graph* G *if the graph obtained by removing any k edges from* G^* *contains* G*, where k is a positive integer.*

Definition 4. *Let P be a path between the vertices v_i and v_j in a graph G and let P' be a path between the vertices v_j and v_k. Then, the path $P \cup P'$ is the path obtained by extending the path P between v_i and v_j to v_k through the common vertex v_j (i.e. if $P : v_i....v_j$ and $P' : v_j....v_k$ then $P \cup P' : v_i....v_j.....v_k$).*

Definition 5. *Let u and v be the two distinct vertices in G. Then u and v are attainable in G if there exists a Hamiltonian path in G between u and v.*

2. Results

Theorem 2.1. *For $n \geq 3$ the graph $H = Im(Y_n)$ is Hamiltonian-1-laceable.*

Proof. Let $V(Y_n)$=$\{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n\}$ (taken clockwise) & $E(Y_n) = \{e_i : 1 \leq i \leq n\} \cup \{e'_i : 1 \leq i \leq n\} \cup \{e_{ii} : 1 \leq i \leq n\}$ where e_i is the edge $v_i v_{i+1}$ $(1 \leq i \leq n-1)$, e_n is the edge $v_n v_1$ and e'_i is the edge $u_i u_{i+1}$ $(1 \leq i \leq n-1)$; e'_n is the edge $u_n u_1$ and e_{ii} is the edge $v_i u_i$ $(1 \leq i \leq n-1)$. We call $v_i : 1 \leq i \leq n$ as the outer cycle vertices (taken clockwise) and $u_i : 1 \leq i \leq n$ as the inner cycle vertices (taken clockwise).

By the definition of image graph, $V(H)$=$v_i \cup v'_i \cup u_i \cup u'_i$, $1 \leq i \leq n$ be the vertex set and $E(H)$=$E_1 \cup E_2 \cup E_3$, where E_1 are the edges in the first copy of Y_n, E_2 are the edges in the second copy of Y_n and E_3 =$\{(v_i, v'_i), (u_i, u'_i), 1 \leq i \leq n\}$.
Clearly, $diam(H)$= $2 + \lfloor \frac{n}{2} \rfloor$.

Since $d(v_i, v_{i+1}) = d(v_i, v'_i) = d(v_i, u_i) = d(u_i, u'_i) = 1$ in H for all $1 \leq i \leq n$, it is enough to prove that there exists a Hamiltonian path in $Im(Y_n)$ between these pairs of vertices.

Claim 2.1.1. *The vertices v_i and v_{i+1} are attainable.*

In H, the path $v_i\Big(P_1^{-1}[n-1]W_{i+2}P_2[n]\Big)\Big(u_{i+1}, u'_{i+1}\Big)\Big(P_2[n]W_i^{-1}P_1^{-1}[n]\Big)\Big(v'_{i+1}, v_{i+1}\Big)$ is a Hamiltonian path between v_i and v_{i+1}.

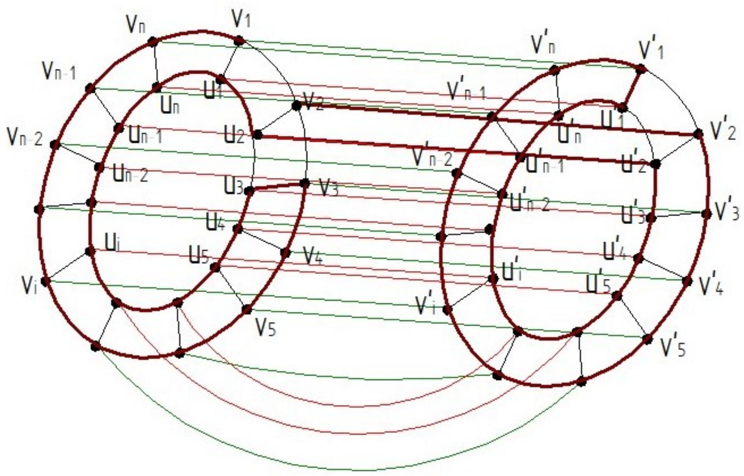

Figure 3. Hamiltonian path between v_1 and v_2 in $Im(Y_n)$

Claim 2.1.2. *The vertices v_i and u_i are attainable.*

In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2[n-1]\Big)\Big(u_{n+i-1},u'_{n+i-1}\Big)\Big(P_2^{-1}[n-1]W_{i+1}^{-1}P_1[n]W_i\Big)$ $\Big(u'_i,u_i\Big)$ is a Hamiltonian path between v_i and u_i.

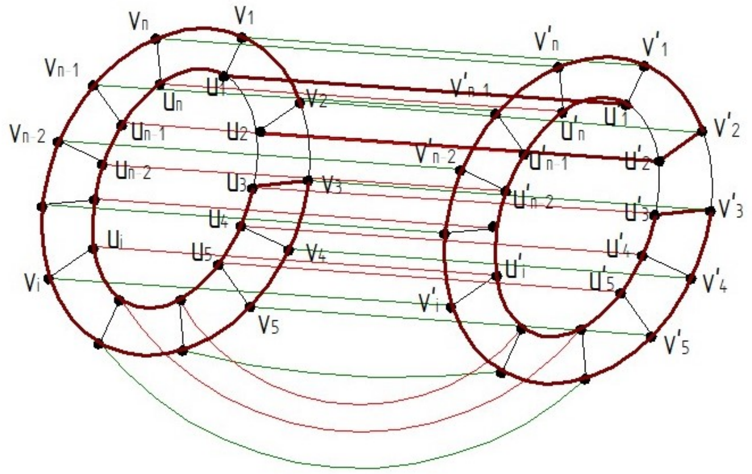

FIGURE 4. Hamiltonian path between v_2 and u_2 in $Im(Y_n)$

Claim 2.1.3. *The vertices v_i and v'_i are attainable.*

In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2[n]\Big)\Big(u_i,u'_i\Big)\Big(P_2^{-1}[n]W_{i+1}^{-1}P_1[n]\Big)v'_i$ is a Hamiltonian path between v_i and v'_i.

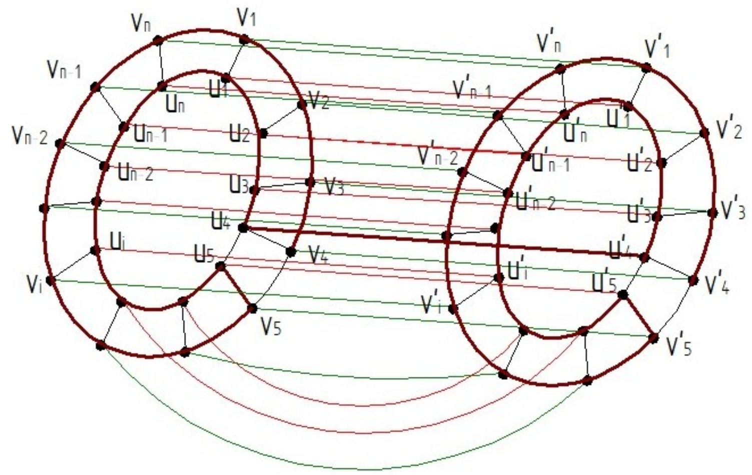

FIGURE 5. Hamiltonian path between v_4 and v'_4 in $Im(Y_n)$

Claim 2.1.4. *The vertices u_i and u'_i are attainable.*

In H, the path $u_i\Big(P_2^{-1}[n]W_{i+1}^{-1}P_1[n]\Big)\Big(v_i,v'_i\Big)\Big(P_1^{-1}[n]W_{i+1}P_2[n]\Big)u'_i$ is a Hamiltonian path between u_i and u'_i. Hence the proof. $\square$

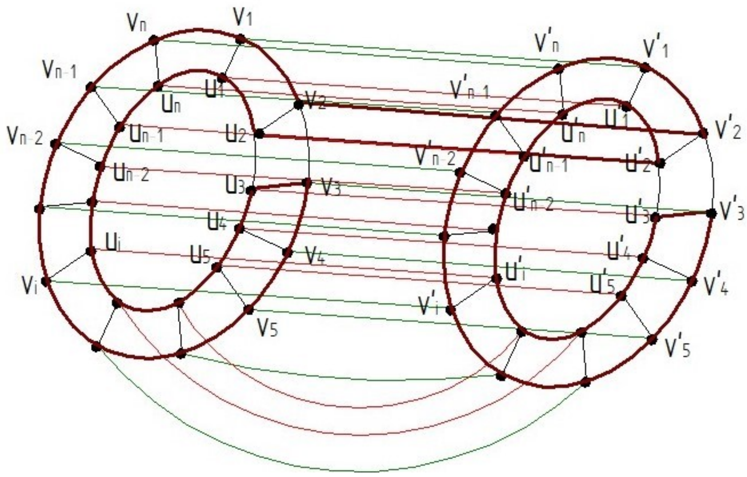

FIGURE 6. Hamiltonian path between u_2 and u'_2 in $Im(Y_n)$

Theorem 2.2. *For $n \geq 3$,the graph $H = Im(Y_n)$ is Hamiltonian-t-laceable for odd $t > 1$.*

Proof. Since $d(v_i, v_{i+t}) = d(v_i, u_{i+t-1}) = d(v_i, v'_{i+t-1}) = d(v_i, u'_{i+t-1}) = d(u_i, u_{i+t}) = t$ in H for all $t > 1$ and $1 \leq i \leq n$, it is enough to prove that there exists a Hamiltonian path in $Im(Y_n)$ between these pairs of vertices.

Claim 2.2.1. *The vertices v_i and v_{i+t} are attainable.*

In H, the path $v_i\Big(P_1^{-1}[n-t]W_{i+t+1}P_2[n-t+1]\Big)\bigcup_{m=0}^{\frac{t-3}{2}}\Big(W^{-1}_{i+2m+1}P_1[2]W_{i+2m+2}P_2[2]\Big)$ $\Big(u_{i+t}, u'_{i+t}\Big)\Big(P_2^{-1}[n]W^{-1}_{i+t-1}P_1[n]\Big)\Big(v'_{i+t}, v_{i+t}\Big)$ is a Hamiltonian path between the vertices v_i and v_{i+t}.

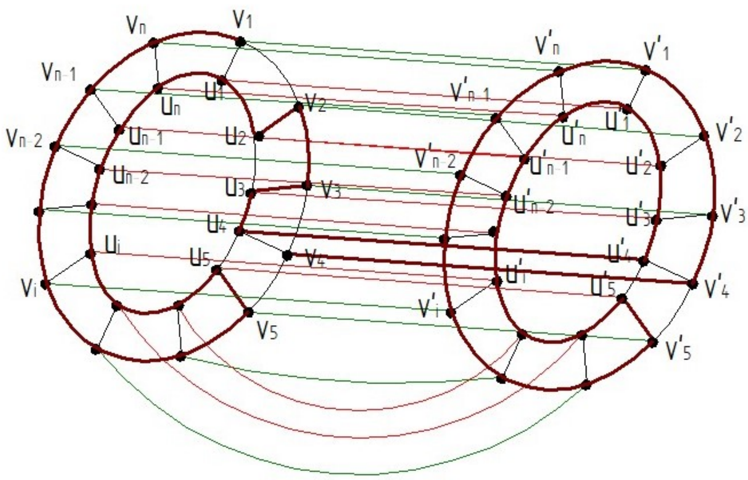

FIGURE 7. Hamiltonian path between v_1 and v_4 in $Im(Y_n)$ ($d(v_1, v_4)$=3)

Claim 2.2.2. *The vertices v_i and u_{i+t-1} are attainable for $t \geq 5$.*

In H, the path $v_i\Big(P_1^{-1}[n-t+1]W_{i+t}P_2[n-t+2]\Big)\bigcup\limits_{m=0}^{\frac{t-5}{2}}\Big(W_{i+2m+1}^{-1}P_1[2]W_{i+2m+2}P_2[2]\Big)$ $W_{i+t-2}^{-1}P_1[2]\Big(v_{i+t-1},u'_{i+t-1}\Big)\Big(P_1[n]W_{i+t-2}P_2^{-1}[n]\Big)\Big(u'_{i+t-1},u_{i+t-1}\Big)$ is a Hamiltonian path between the vertices v_i and u_{i+t-1}.

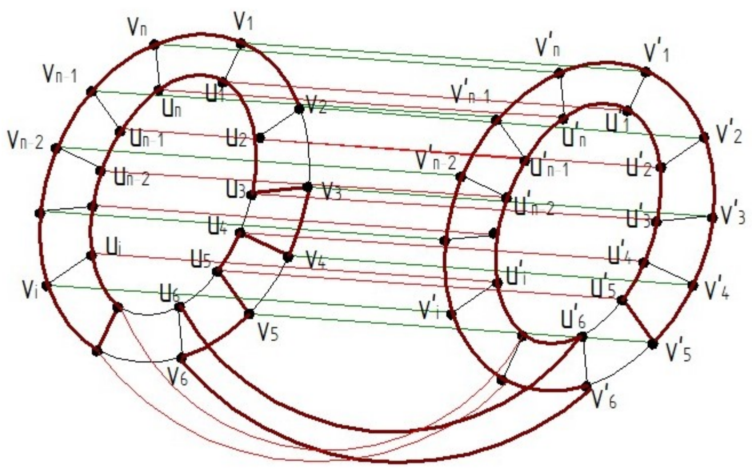

FIGURE 8. Hamiltonian path between v_2 and u_6 in $Im(Y_n)$ ($d(v_2,u_6)$=5)

Remark 1. *In H, the path $v_i\Big(P_1^{-1}[n-3]W_{i+4}P_2[n-2]W_{i+1}^{-1}P_1[3]W_{i+3}\Big)\Big(u_{i+3},u'_{i+3}\Big)$ $\Big(P_2[n-1]w_{i+1}^{-1}P_1^{-1}[n]W_{i+2}\Big)\Big(u'_{i+2},u_{i+2}\Big)$ is a Hamiltonian path between the vertices v_i and u_{i+2} for $t=3$.*

Claim 2.2.3. *The vertices v_i and v'_{i+t-1} are attainable for $t \geq 7$.*

In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2},u_{i+2}^{-1}\Big)\Big(P_2^{-1}[n-t+3]W_{i+t}^{-1}$ $P_1^{-1}[n-t+4]\Big)\bigcup\limits_{m=0}^{\frac{t-7}{2}}\Big(W_{i+2m+3}P_2[2]W_{i+2m+4}^{-1}P_1[2]\Big)\Big(W_{i+t-2}P_2[2]W_{i+t-1}^{-1}\Big)v'_{i+t-1}$ is a Hamiltonian path between the vertices v_i and v'_{i+t-1}.

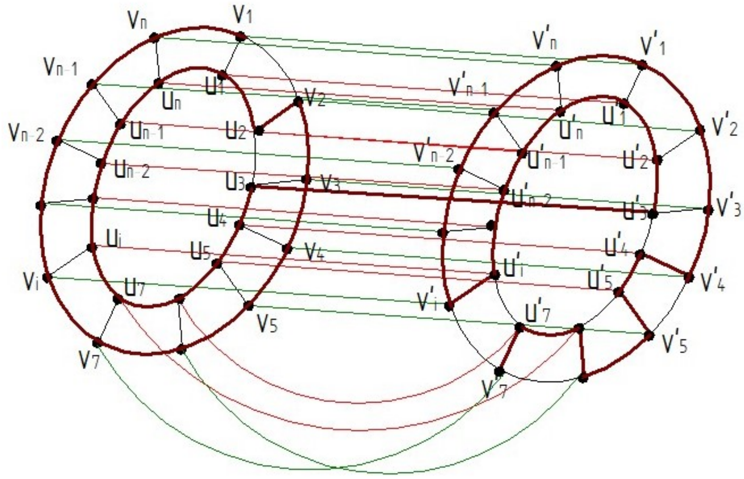

FIGURE 9. Hamiltonian path between v_1 and v_7' in $Im(Y_n)$ ($d(v_1, v_7')$=7)

Remark 2. *In H, the path* $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2}, u_{i+2}^{-1}\Big)\Big[\Big(P_2[n]\Big)^b\Big(P_2^{-1}[n]\Big)^a\Big]$ $\Big(W_{i+t-2}^{-1}P_1^{-1}[n]\Big)v_{i+t-1}'$ *where* $a = \left\{\begin{array}{cc} 1 & t=5 \\ 0 & t\neq 5 \end{array}\right\}$ $b = \left\{\begin{array}{cc} 1 & t=3 \\ 0 & t\neq 3 \end{array}\right\}$ *is a Hamiltonian path between the vertices v_i and v_{i+t-1}' for $t = 3, 5$.*

Claim 2.2.4. *The vertices v_i and u_{i+t-2}' are attainable for $t \geq 7$.*

In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2}, u_{i+2}^{-1}\Big)\Big(P_2^{-1}[n-t+4]W_{i+t-1}^{-1}$ $P_1^{-1}[n-t+5]W_{i+3}\Big)\Big[\bigcup_{m=0}^{\frac{t-7}{2}}\Big(P_2[2]W_{i+2m+4}^{-1}P_1[2]W_{i+2m+5}\Big)\Big]^a u_{i+t-2}'$ where $a = \left\{\begin{array}{cc} 1 & t\geq 7 \\ 0 & t<7 \end{array}\right\}$ is a Hamiltonian path between the vertices v_i and u_{i+t-2}'.

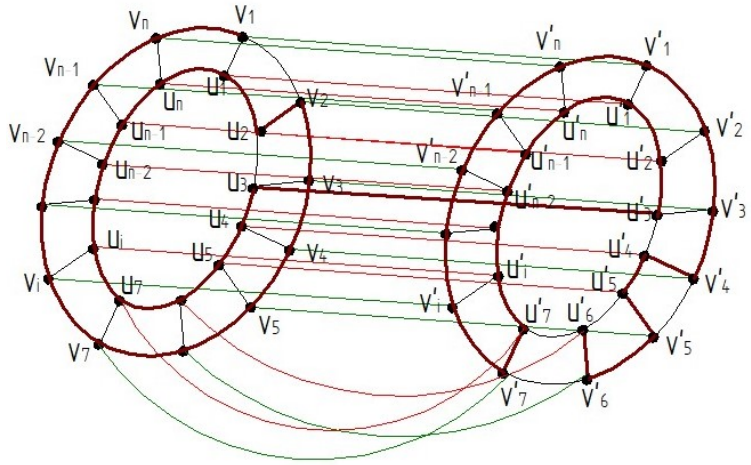

FIGURE 10. Hamiltonian path between v_1 and u_6' in $Im(Y_n)$ ($d(v_1, u_6')$=7)(Claim 5.2.4)

Remark 3. *In H, the path* $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2},u'_{i+2}\Big)\Big(P_2[n-1]W_i^{-1}P_1^{-1}[n]W_{i+1}\Big)$ u'_{i+1} *is a Hamiltonian path between the vertices v_i and u'_{i+1} for $t=3$.*

Claim 2.2.5. *The vertices u_i and u_{i+t} are attainable.*

In H, the path $u_i\Big(P_2^{-1}[n-t]W_{i+t+1}^{-1}P_1[n-t+1]\Big)\bigcup_{m=0}^{\frac{t-3}{2}}\Big(W_{i+2m+1}P_2[2]W'_{i+2m+2}P_1[2]\Big)$ $\Big(v_{i+t},v'_{i+t}\Big)\Big(P_1[n]Wi+t-1P_2^{-1}[n]\Big)\Big(u'_{i+t},u_{i+t}\Big)$ is a Hamiltonian path between the vertices u_i and u_{i+t}.

Hence the proof. □

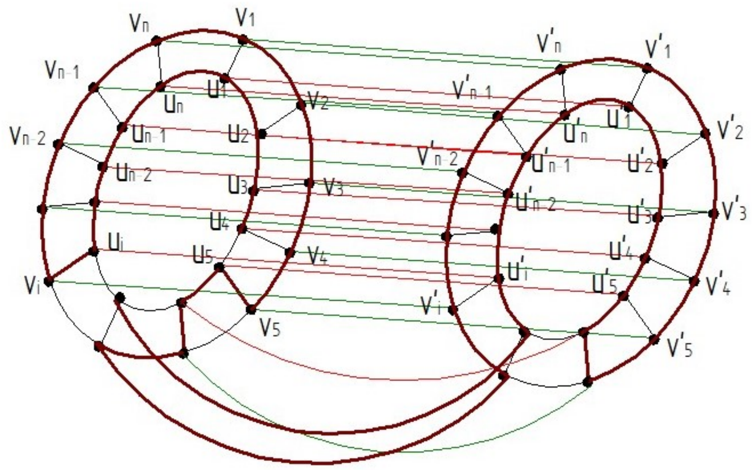

FIGURE 11. Hamiltonian path between u_4 and u_7 in $Im(Y_n)$ ($d(u_4,u_7)$=3)

Combining the above two theorems we observe that there exists a Hamiltonian path in G between every pair of vertices in it at an odd distance. Hence we have the following theorem.

Theorem 2.3. *The graph $H=Im(Y_n)$, $n\geq 3$ is Hamiltonian-laceable.*

Following result establishes the edge tolerant property for the image graph of the prism graph for even t.

Theorem 2.4. *For $n\geq 3$ the 1- edge-fault-tolerant graph $H=Im(Y_n)$ is Hamiltonian-t-laceable for even t, $1\leq t\leq diam(H)$.*

Proof. Since $d(v_i,v_{i+t})=d(v_i,u_{i+t-1})=d(v_i,v'_{i+t-1})=d(v_i,u'_{i+t-1})=d(u_i,u_{i+t})=t$ in H for all $i, 1\leq i\leq n$, it is enough to prove that there exists a Hamiltonian path in $Im(Y_n)$ between these pairs of vertices.

Claim 2.4.1. *The vertices v_i and v_{i+t} are attainable.*

In H, the path $v_i\Big(P_1^{-1}[n-t]W_{i+t+1}P_2[n-t+1]\Big)\Big(W_{i+1}^{-1}\Big)^r\Big[\bigcup_{m=0}^{\frac{t-4}{2}}\Big(W_{i+2m+1}^{-1}P_1[2]W_{i+2m+2}$ $P_2[2]\Big)\Big]^s\Big(v_{i+t-1},u_{i+t}\Big)\Big(u_{i+t},u'_{i+t}\Big)\Big(P_2[n]W^{-1}-i+t-1P_1^{-1}[n]\Big)\Big(v'_{i+t},v_{i+t}\Big)$

where $r = \left\{ \begin{array}{ll} 1 & t = 2 \\ 0 & t \neq 2 \end{array} \right\}$, $s = \left\{ \begin{array}{ll} 1 & t \neq 2 \\ 0 & t = 2 \end{array} \right\}$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices v_i and v_{i+t}.

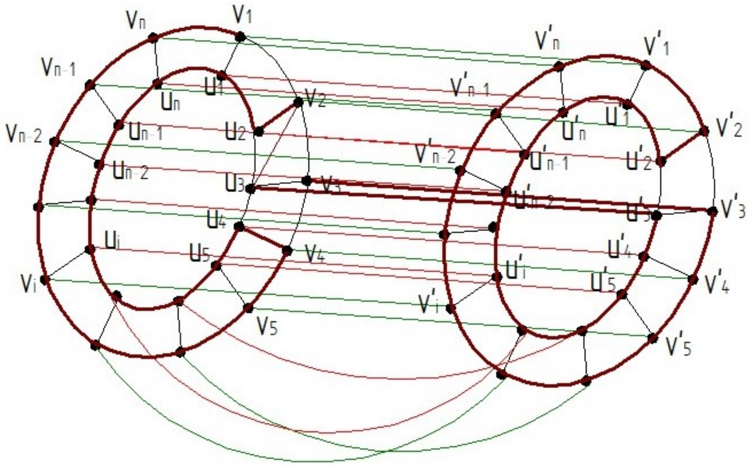

FIGURE 12. Hamiltonian path between v_1 and v_3 in $Im(Y_n)$ ($d(v_1, v_3)$=2)

Claim 2.4.2. *The vertices v_i and u_{i+t-1} are attainable for $t \geq 4$.*

In H, the path $v_i\left(P_1^{-1}[n]\left(v_{i+1}, u_{i+2}\right)P_{n-t+3}^{-1}\right)\left(u_{i+t}, u'_{i+t}\right)\left(P_2[n-1]W_{i+t-2}^{-1}P_1^{-1}[n]W_{i+t-1}\right)$ $\left(u'_{i+t-1}, u_{i+t-1}\right)$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices v_i and u_{i+t-1}.

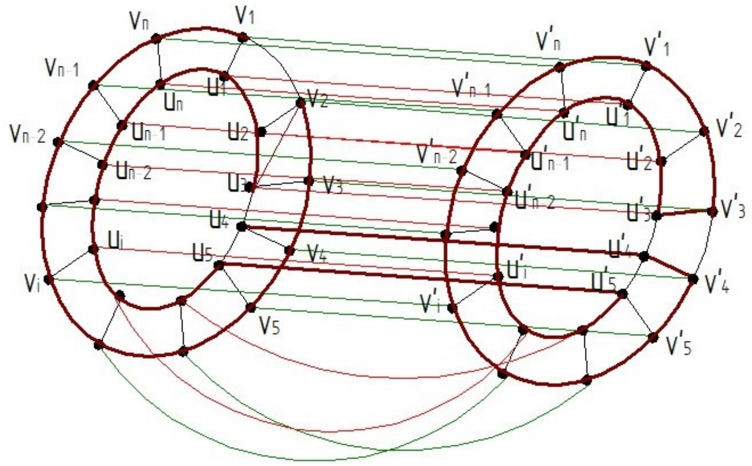

FIGURE 13. Hamiltonian path between v_1 and u_4 in $Im(Y_n)$ ($d(v_1, u_4)$=4)

Remark 4. *In H, the path $v_i\left(P_1^{-1}[n]\left(v_{i+1}, u_{i+2}\right)P_{n-1}^{-1}\right)\left(u_i, u'_i\right)\left(P_2^{-1}[n-1]W_{i+2}^{-1}P_1[n]W_{i+1}\right)$ $\left(u'_{i+1}, u_{i+1}\right)$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices v_i and u_{i+t-1} for $t = 2$.*

Claim 2.4.3. *The vertices v_i and v'_{i+t-1} are attainable for $t \geq 6$.*

In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2}, u'_{i+2}\Big)\big(P_2^{-1}[n-t+3]W_{i+t}^{-1}$ $P_1[n-t+3]\big)\Big(v'_{i+2}, u'_{i+3}\Big)\bigcup_{m=0}^{\frac{t-6}{2}}\Big(W_{i+2m+3}^{-1}P_1[2]W_{i+2m+4}P_2[2]\Big)W_{i+t-1}^{-1}v_{i+t-1}^{-1}$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices v_i and v'_{i+t-1}.

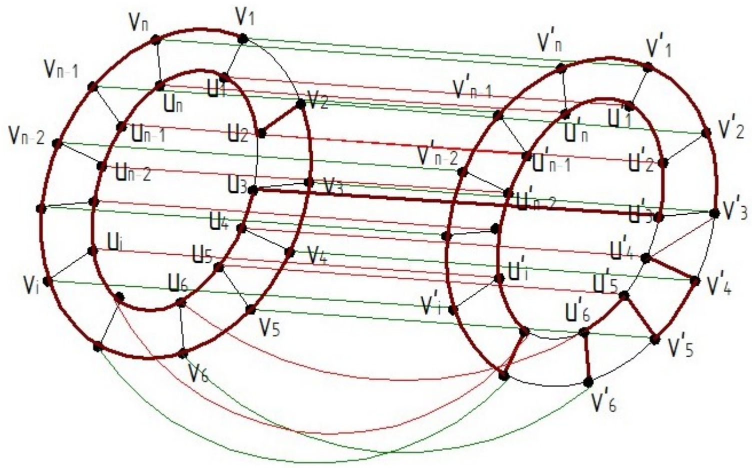

FIGURE 14. Hamiltonian path between v_1 and v'_6 in $Im(Y_n)$ $(d(v_1, v'_6)$=6)(Claim 5.4.3)

Remark 5. *In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2}, u'_{i+2}\Big)\Big(P_2[n]\big(u'_{i+1}, v'_{i+2}\big)$ $\big[P_1[n]\big]^a\big[P_1^{-1}[n]\big]^b\Big)v'_{i+t-1}$ where $a = \left\{\begin{array}{cc} 1 & t=2 \\ 0 & t \neq 2 \end{array}\right\}$, $b = \left\{\begin{array}{cc} 1 & t=4 \\ 0 & t \neq 4 \end{array}\right\}$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices v_i and v'_{i+t-1} for $t = 2, 4$.*

Claim 2.4.4. *The vertices v_i and u'_{i+t-2} are attainable for $t \geq 6$.*

In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2}, u'_{i+2}\Big)\big(P_2^{-1}[n-t+4]W_{i+t-1}^{-1}P_1[n-t+$ $4]\big)\Big(v'_{i+2}, u'_{i+3}\Big)\Big[\bigcup_{m=0}^{\frac{t-8}{2}}\Big(W_{i+2m+3}P_1[2]W_{i+2m+4}P_2[2]\Big)\Big]^a\Big(W_{i+t-3}^{-1}P_1[2]W_{i+t-2}\Big)u_{i+t-2}^{-1}$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices v_i and u'_{i+t-2} where $a = \left\{\begin{array}{cc} 1 & t \geq 8 \\ 0 & t < 8 \end{array}\right\}$.

Remark 6.

1. *In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2}, u'_{i+2}\Big)\Big(P_2[n-2]W_{i-1}^{-1}$ $P_1^{-1}[n-1]W_{i+1}\Big)\Big(u'_{i+1}, v'_i\Big)u'_i$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices v_i and u'_i for $t = 2$.*
2. *In H, the path $v_i\Big(P_1^{-1}[n]W_{i+1}P_2^{-1}[n]\Big)\Big(u_{i+2}, v'_{i+2}\Big)\Big(P_1^{-1}[n]W_{i+3}P_2[n]\Big)u'_{i+2}$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices v_i and u'_{i+2} for $t = 4$.*

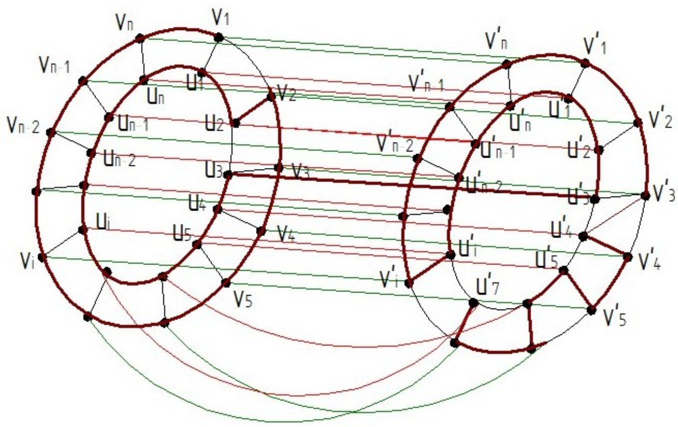

FIGURE 15. Hamiltonian path between v_1 and u'_7 in $Im(Y_n)$ ($d(v_1, u'_7)$=8)(Claim 5.4.4)

Claim 2.4.5. *The vertices u_i and u_{i+t} are attainable.*

In H, the path $u_i\Big(P_2^{-1}[n-t]W_{i+t+1}^{-1}P_1[n-t+1]\Big)\Big(W_{i+1}\Big)^r\Big[\bigcup_{m=0}^{\frac{t-4}{2}}\Big(W_{i+2m+1}P_1[2]W_{i+2m+2}^{-1}P_2[2]\Big)\Big]^s\Big(u_{i+t-1}, v_{i+t}\Big)\Big(v_{i+t}, v'_{i+t}\Big)\Big(P_1[n]W_{i+t-1}P_2^{-1}[n]\Big)\Big(u'_{i+t}, u_{i+t}\Big)$
where $r = \left\{\begin{array}{ll} 1 & t=2 \\ 0 & t\neq 2 \end{array}\right\}$, $s = \left\{\begin{array}{ll} 1 & t\neq 2 \\ 0 & t=2 \end{array}\right\}$ in the 1-edge-fault-tolerant graph H^* is a Hamiltonian path between the vertices u_i and u_{i+t}. Hence the proof. □

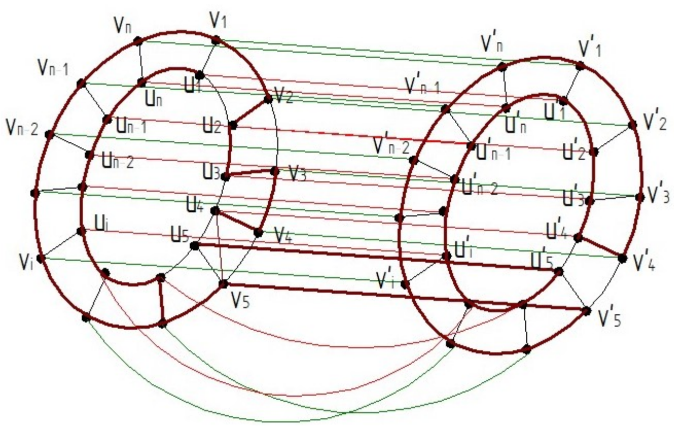

FIGURE 16. Hamiltonian path between u_1 and u_5 in $Im(Y_n)$ ($d(u_1, u_5)$=4)

3. Conclusion

In this paper, the laceabilty properties of the image of prism graph have been explored for all $n \geq 3$. Also, we prove that for all even $t \geq 2$, the 1-edge fault tolerant graph is Hamiltonian-t-laceable. We pose the general problem as follows:

Open Problem: Alspach and Zhang(1989) showed that all cubic cayley graphs over dihedral groups are Hamiltonian by using the concept of brick products. It was proved by Alspach, Chen and McAvaney(1996) that most of the brick products C(2n, m, r) with one cycle (m=1) are Hamiltonian Laceable, in the sense that any two vertices at odd distance apart can be joined by a Hamiltonian path. The problem whether all brick products C(2n, m, r) are Hamiltonian laceable is an open problem. Are the image graphs of brick products graphs C(2n, m, r) with one cycle (m=1) Hamiltonian laceable? For what values of 't' are the image graphs Hamiltonian-t-laceable?

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R. Murali for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.10, N.3.