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Number of switching state vectors and space vectors in multilevel multiphase converters

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Expressions to calculate the number of switching state vectors and the number of space vectors in multilevel multiphase voltage-source converters are given.

Introduction: Multiphase motor drives have recently attracted much interest in high power applications owing to the recent developments in ship propulsion, traction and aerospace fields [1]. Power levels can go even further if the current multilevel technology [2], also devoted to high power systems, is combined with multiphase technology [3]. Multilevel multiphase converters have thousands of switching states, even with a moderate number of phases and levels. Therefore, the degree of freedom in the modulation process of such converters is very high when compared with classical two level three phase counter parts, which have only eight switching states.

In [4] the modulation problem of a P phase converter was solved in a multidimensional space, in which every switching state vector $v_s = [v_s^1, v_s^2, \ldots, v_s^P]^T$ represents the switching states of all legs. In converters with no connected neutral there are different switching state vectors that provide the same line to line output. All such redundant switching state vectors can be described by a space vector $\omega_s = [\omega_s^1, \omega_s^2, \ldots, \omega_s^{P \; 1}]^T$, where $\omega_s^k = v_s^k \;\; v_s^P$ [4]. All switching state vectors that correspond to a particular space vector can be back calculated as

$$v_s \;\; [\omega_s^1 + n, \omega_s^2 + n, \ldots, \omega_s^{P-1} + n, n]^T \tag{1}$$

where n is an integer number, which can be positive or negative [4].

This Letter provides the expressions to calculate the number of switching state vectors and the number of space vectors in multilevel multiphase converters, including the theoretical case when the converter legs have different numbers of levels.

Two leg converter example: Before considering the general case, the solution for a two leg converter, where the switching state vectors are 2D, is outlined. Fig. 1 shows the switching vectors of a two leg converter with six levels in leg a and five levels in leg b, which have been taken completely arbitrarily. The number of the switching state vectors, which belong to the $\mathcal{U}$, is the number of combinations of all levels of leg a with all levels of leg b: in this case it is $6 \times 5 = 30$ switching state vectors. The space vectors ω_s of this converter are the projections of the switching vectors v_s along the vector $u = [1, 1]^T$ onto the line $b = 0$ [4]. To calculate the number of space vectors of the converter it is necessary to count the number of elements in the set $\mathcal{W}$, or what is the same, to count the elements in the set $\mathcal{L}$. The number of elements in $\mathcal{L}$ is equal to the number of elements in $\mathcal{U}$ minus the number of elements in $\mathcal{M}$, that is, $30 \;\; (5 \times 4) = 10$ space vectors.

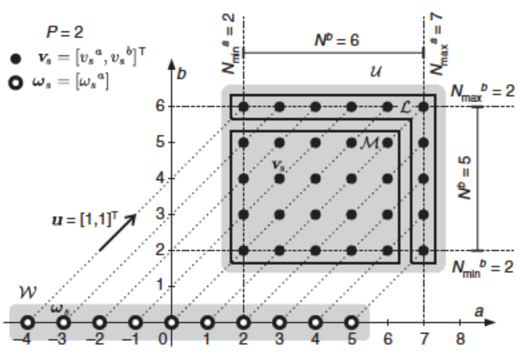

Fig. 1 *Two-dimensional example*

Multiphase converter: The switching state vectors v_s of a generic multi level converter are the elements of the set

$$\mathcal{U} \;\; \{v_s | N_{min}^k \le v_s^k \le N_{max}^k \text{ for all } k \in [1, P]\} \tag{2}$$

where N_{min}^k and N_{max}^k are the minimum and maximum levels that can be reached by the leg k, respectively. The number of switching state vectors, which are in $\mathcal{U}$, is the number of combinations of the levels of all legs:

$$E_{\mathcal{U}} \;\; \prod_{k \; 1}^{P} N^k \tag{3}$$

where $N^k \;\; N_{max}^k \;\; N_{min}^k + 1$ is the number of levels of the leg k. The set $\mathcal{U}$ can be decomposed into two complementary subsets $\mathcal{U} \;\; \mathcal{L} \cup \mathcal{M}$, where the subset $\mathcal{L}$ gathers the switching vectors in which at least one leg (j) reaches the maximum level:

$$\mathcal{L} \;\; \{v_s | v_s \in \mathcal{U}, v_s^j \;\; N_{max}^j \text{ for some } j \in [1, P]\} \tag{4}$$

The remaining switching state vectors, in which no leg reaches the maximum level, are in the subset $\mathcal{M}$:

$$\mathcal{M} \;\; \{v_s | N_{min}^k \le v_s^k \le N_{max}^k \;\; 1 \text{ for all } k \in [1, P]\} \tag{5}$$

The definition of $\mathcal{M}$ is similar to the definition of $\mathcal{U}$, therefore the number of switching vectors in this set is

$$E_{\mathcal{M}} \;\; \prod_{k \; 1}^{P} (N^k \;\; 1) \tag{6}$$

The number of vectors in $\mathcal{U}$, (3), minus the number of vectors in $\mathcal{M}$, (6), gives the number of switching state vectors in $\mathcal{L}$. Since $\mathcal{W}$ and $\mathcal{L}$ have the same number of elements, the number of space vectors ω_s of the con verter is

$$E_{\mathcal{W}} \;\; \prod_{k \; 1}^{P} N^k \;\; \prod_{k \; 1}^{P} (N^k \;\; 1) \tag{7}$$

Proving that the space vectors of the set $\mathcal{W}$ can be counted by counting the switching vectors in $\mathcal{L}$ requires to demonstrate that

- All space vectors are counted.
- Each space vector is counted once.

All space vectors will be counted if for every switching state vector in $\mathcal{M}$ there is another one in $\mathcal{L}$ yielding the same space vector. For a given switching state vector $v_1 \in \mathcal{M}$, the redundant switching state vector $v_2 \;\; v_1 + [n, n, \ldots, n]^T$ shares the same space vector. Additionally, if

$$n \;\; \min_{k \; [1,P]} (N_{max}^k \;\; v_1{}^k) \tag{8}$$

then the switching vector v_2 belongs to the subset $\mathcal{L}$, as is demonstrated in the following:

- From (5) and (8), the value of n is always greater than zero, and $v_2{}^k \;\; v_1{}^k + n > N_{min}^k$.
- From (8), $n \le N_{max}^k \;\; v_1{}^k$. As $v_2{}^k \;\; v_1{}^k + n + (N_{max}^k \;\; N_{max}^k) \;\; [n \;\; (N_{max}^k \;\; v_1{}^k)] + N_{max}^k$, then $v_2{}^k \le N_{max}^k$.

Thus, $N_{min}^k \le v_2{}^k \le N_{max}^k$, so v_2 belongs to $\mathcal{U}$. From (8), $n \;\; N_{max}^j \;\; v_1{}^j$ for some $j \in [1, P]$. The component j of vector $v_2, v_2{}^j \;\; v_1{}^j + n \;\; N_{max}^j$, reaches the maximum level, and consequently v_2 belongs to the subset $\mathcal{L}$.

Proving that every space vector is counted only once requires demonstrating that there are not two different switching state vectors v_1 and v_2 in $\mathcal{L}$ that yield the same space vector ω_s. From (1) and (4) those vectors can be written as:

$$v_1 \;\; [\omega_s{}^1 + n_1, \ldots, \omega_s^{P-1} + n_1, n_1]^T, v_1{}^i \;\; N_{max}^i \tag{9}$$

$$v_2 \;\; [\omega_s{}^1 + n_2, \ldots, \omega_s^{P-1} + n_2, n_2]^T, v_2{}^j \;\; N_{max}^j \tag{10}$$

The value of v_1^j can be calculated from the vector v_2 as $v_1^j = v_2^j + (n_1 \;\; n_2) = N_{max}^j + (n_1 \;\; n_2)$, therefore n_1 must be less or equal to n_2 to fulfil $v_1^j \le N_{max}^j$. In the same way, $v_2^i = v_1^i + (n_2 \;\; n_1) = N_{max}^i + (n_2 \;\; n_1)$, which requires that $n_2 \le n_1$ to fulfil $v_2^i \le N_{max}^i$. Hence, $n_1 = n_2$ and $v_1 = v_2$. As a consequence there are not two different switching vectors in $\mathcal{L}$ that give the same space vector.

Table 1 gives the number of switching state vectors and the number of space vectors for several converters with the same number of levels N in every leg. It shows that the number of switching state vectors increases dramatically with the number of levels and legs of the converter, which makes it impractical to carry out a modulation based on predefined switching sequences stored in a memory, even for converters with a moderate number of phases and levels.

Table 1: Number of switching state vectors and space vectors

v_s/ω_s	$N=2$	$N=3$	$N=5$	$N=7$	$N=9$
$P=3$	8/7	27/19	125/61	343/127	729/217
$P=5$	32/31	243/211	3125/2101	1.68×10^4/ 9031	5.91×10^4/ 2.63×10^4
$P=7$	128/127	2187/2059	7.81×10^4/ 6.17×10^4	8.24×10^5/ $5/4 \times 10^5$	4.78×10^6/ 2.69×10^6
$P=9$	512/511	1.97×10^4/ 1.92×10^4	1.95×10^6/ 1.69×10^6	4.04×10^7/ 3.03×10^7	3.87×10^8/ 2.53×10^8
$P=15$	3.28×10^4/ 3.28×10^4	1.44×10^7/ 1.43×10^7	3.05×10^{10}/ 2.94×10^{10}	4.75×10^{12}/ 4.28×10^{12}	2.06×10^{14}/ 1.71×10^{14}

Conclusions: General expressions to calculate the number of switching state vectors and the number of space vectors in multilevel multiphase converters are provided. These expressions, which are valid even in the theoretical case of converters with different numbers of levels in each leg, show that the number of switching states and space vectors of a converter grow exponentially.

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