Chinese Journal of Polar Science, Vol 20, No. 1, 72-80, June 2009

Preliminary results on relationship between thermal diffusivity and porosity of sea ice in the Antarctic

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Received May 30, 2009

Abstract The in situ sea - ice temperature, salinity and density observed from Chinese Antarctic Zhongshan Station have been applied to calculate the vertical profile of sea ice porosity. Based on numerical method, a number of schemes on sea - ice thermal diffusivity versus porosity have been accessed and one optimized scheme is identified by an optimal control model with an advanced distributing parameter system. For simplicity, the internal heating source item was neglected in the heat conduction equation during the identification procedure. In order to illustrate the applicability of this identified scheme, the vertical ice temperature profiles have been simulated and compared with measurements, respectively by using identified scheme and by classical thermodynamic formulae. The comparisons indicated that the scheme describing sea - ice thermal diffusivity and porosity is reasonable. In spite of a minor improvement of accuracy of results against in situ data, the identified scheme has a more physical meaning and could be used potentially in various applications

Key words sea ice, porosity, optimal identification, numerical simulations, Antarctica

1 Introduction

The polar-regions are mostly covered by ice. Ice growth and melting are controlled by heat conduction. In the view of the composition, sea ice consists of pure ice, brine, solid salt and gases Its physical, optical, mechanical and thermal characteristics largely depend on the originations and compositions of those materials

Sea ice is very sensitive to the temperature, particularly when it closes to the melting point A slightly change of temperature will alter its properties[1,2]. The salinity and density are additional important physical properties of sea ice. The sea ice porosity, which defined as the ratio of brine and gas to its total volume is the function of sea ice temperature, salinity and density[3]. The sea-ice porosity is calculated by determining the gas and brine content in unit volume. The brine and gas content in sea ice are usually obtained from measurements of bulk density, temperature and salinity. The calculation of sea ice porosity began in early 60 s[4] and much more progresses have been made in 80 s. A classical result referred to Cox and Weeks (1983)[5] who analyzed the phase diagram of sea ice and derived equations that determine the gas and brine volumes. Their calculation of ice porosity is suitable for temperature range from -2 ℃ to -30 ℃. Leppäranta and Manninen (1988)[6] discussed the calculation of the ice porosity suitable for the temperature range above -2 ℃. In various sea ice applications[3,7,8], however, the brine volume/pocket concept is commonly used instead of porosity because of a time consuming process to measure the ice density. Nowadays, with the development of new technology to analysis brine and air bubbles in ice core samples[8-10] and the in situ determination of sea ice salinity and density[11,12], much work have been done on evaluation of sea ice properties using ice porosity[13-16]. For example, the maximum compressive strength of sea ice has been estimated using ice density and porosity[14].

In this paper, a relationship between thermal diffusivity and porosity is defined and applied to calculate the ice temperature regime. We first applied mathematical distributing parameter system and the optimal control theory to identify sea ice thermal diffusivity dependent on ice porosity based on measured sea ice temperature, salinity and density from Chinese Antarctic Zhongshan Station in 2006. The scheme is then verified by simulating sea ice temperature profiles and comparing with observations from Zhongshan station in 2005. We further made a standard ice temperature regime calculation with concepts of thermal heat conductivity and volumetric heat capacity[17], the errors of modeled ice temperature are investigated. Our motivation is to find the applicability of sea ice porosity on thermodynamic modeling.

2 Theoretical backgrounds of sea ice thermal diffusivity and porosity

Thermal diffusivity is one of the key components determining sea-ice heat conduction process. It is the most directly observable thermal property, as it is directly related to the rate of temperature change of ice floe and defined as the ratio of thermal conductivity (k_{si}) to volumetric heat capacity ($\rho_{si} c_{si}$)

$$\lambda \equiv \frac{k_{si}}{\rho_{si} c_{si}} \tag{1}$$

where ρ_{si} and c_{si} are ice density and specific heat capacity, respectively. Assuming that sea ice consists of uniformly and randomly distributed spherical air bubbles, Schwerdtfeger (1963) derived formulae to calculate the effective specific heat capacity and thermal conductivity of sea ice on the basis of Maxwell's principle[17]. The formulae read:

$$c_{si,eff}(z) = -\frac{S_{si}(z)}{\alpha T(z)^2} L + \frac{S_{si}(z)}{\alpha T(z)} (c_w - c_{pi}) + c_{pi} \tag{2}$$

$$k_{si,eff}(z) = k_{pi} - (k_{pi} - k_b) \frac{S_{si}(z) \rho_{si}}{\alpha \rho_w T(z)} \tag{3}$$

where $\alpha = -0.0182$ ℃$^{-1}$, z is the depth in sea ice in [cm], $S_{si}(z)$ is sea ice salinity expressed in [‰], $T(z)$ is sea ice temperature in ℃, L is the latent heat of fusion, C_w is the specific heat capacity of water and C_{pi} is the specific heat capacity of pure ice. The latent heat has the value of $L = 334$ J/g, for pure ice and decreases to $L = 264$ J/g if the ice salinity is 8‰. The specific heat capacity for water and pure ice are given by $C_w = 4.19$ J/(g·℃) and $C_{pi} = 2.09$ J/(g·℃).

In equation (3), k_{pi} and k_b are the thermal conductivities of pure ice and brine, $k_{pi} = 2.1$ w/(m·℃), the thermal conductivity for the brine is determined in [w/(m℃)] as: $k_b(T) = 0.52 + 0.023T + 0.000059T^2$, ρ_{si} is sea ice density in [kg/m^3], ρ_w is the water density, $\rho_w = 1000$ kg/m^3.

Sea ice has been regarded as a two-phase porous medium in some idealized cases[18-20], however, the format of existence of brine and gas in sea ice is far more complex. Although sea ice has been treated rarely as three-phase or even four-phase composite materials with respect to its thermal conductivity and diffusivity discussions, others, such as methane ethane hydrate, segregated ground ice, terrestrial porous media have been considered as multiphase composite materials to study their thermal properties in relation to their porosity in many theory studies[21-24].

Cox and Weeks (1983) have given the statistical relationship of the porosity used sea ice phase diagram[5], but it only relates to the proportion of the components rather than the distribution. Their calculation for the porosity depends on the ice phase diagram and is appropriate for a temperature range of -2 ℃ to -30 ℃, which can be described as

$$\upsilon = \upsilon_a + \upsilon_b \tag{4}$$

$$\upsilon_a = 1 - \frac{\rho_{si}}{\rho_{pi}} + (\rho_{si} \cdot S_{si}) \cdot \frac{F_2(T)}{F_1(T)} \tag{5}$$

$$\upsilon_b = \frac{\rho_{si} \cdot S_{si}}{F_1(T)} \tag{6}$$

$$\rho_{pi} = (0.917 - 1.403 \times 10^{-4} \cdot T) \times 10^3 \tag{7}$$

where υ is the porosity of sea ice in [‰], υ_a is the volume of air per unit volume of sea ice in [‰], υ_b is the volume of brine per unit volume of sea ice in [‰], ρ_{si} and ρ_{pi} are the density of the sea ice and the pure ice in [kg/m^3] respectively, $S_{si}(z)$ is sea ice salinity expressed in [‰], $T(z)$ is sea ice temperature in [℃]. Assuming gas bubbles and brine pockets are disconnected and sea ice temperature ranging between -2.0 ℃ and -22.9 ℃, the functions F_1 and F_2 can be parameterized as: $F_1(T) = -4.372 - 22.45T - 0.6397T^2 - 0.01074T^3$, and $F_2(T) = 0.08903 - 0.01763T - 5.33 \times 10^{-4} T^2 - 8.801 \times 10^{-4} T^3$, respectively.

From equations above, it is not difficult to conclude that thermal diffusivity (λ) and the ice porosity (υ) are functions of temperature (T_{si}), salinity (S_{si}) and density (ρ_{si}). Therefore, one may use a relationship between thermal diffusivity and porosity to investigate the thermal regime of sea ice. In this case, the temperature, salinity and density are applied to calculate ice porosity, rather than to parameterize the thermal heat conductivity and volumetric heat capacity. Because we want to argue that ice porosity is a more physical related concept with more directly linked applications. First, we need to identify the scheme between λ and υ. An optimal control model with an advanced distributing parameter system is applied to identify the relationship between thermal diffusivity and porosity.

3 The method of parameter optimal identification

By assuming no internal heat source term, the one-dimensional heat-conduction equation can be simplified as:

$$\frac{\partial}{\partial t}T(z, t) = \lambda \frac{\partial^2}{\partial z^2}T(z, t) \tag{8}$$

where λ is the thermal diffusivity, T is sea ice temperature, $T(z, t)|_{t=0} = T_0(z)$ is the initial temperature, $T(z, t)|_{z=z_1} = T_1(t)$ and $T(z, t)|_{z=z_2} = T_2(t)$ are boundary conditions, z_1 and z_2 are the upper and lower boundaries, $T_1(t)$ and $T_2(t)$ are temperatures accordingly, $(z, t) \in \Omega \times I$, $\Omega = [z_1, z_2]$, $I = [0, t_{ob}]$, $0 < t_{ob} < +\infty$, t_{ob} is the time for observation.

Based on the fact that sea ice temperature varies with depth and time, so does the thermal diffusivity. This variety belongs to the change of thermodynamic property of sea ice and reflects its nature characteristics. For a given depth and a certain moment, however, the physical property and the thermal diffusivity of sea ice can be regarded as constants. By inverse calculation procedure, the thermal diffusivity could be identified and the relationship between the thermal diffusivity and the porosity can be found out[25].

Mathematically, the inverse problem typically has more than one solution. In order to be more precise, a proper function (scheme) of thermal diffusivity is first assumed, the equation (8) is then applied with initial and boundary conditions to calculate the ice temperature. The calculated ice temperature should be the least squared smallest against the measured values. In other word, the objective function can be expressed as:

$$\min \sum (T_{cal}(z, t) - T_{ob}(z, t))^2 \tag{9}$$

where $T_{cal}(z, t)$ is the calculated temperature, $T_{ob}(z, t)$ is the observed temperature.

In order to reduce the ranges of identified parameters, according to the maximum and the minimum data for measured temperature, salinity, density, the thermodynamic formulae by Schwerdtfeger (1963) and the calculation for ice porosity developed by Cox and Weeks (1983), we calculate thermal diffusivity and porosity respectively. Then we try to find the optimized relationship between thermal diffusivity and the porosity (Fig. 1).

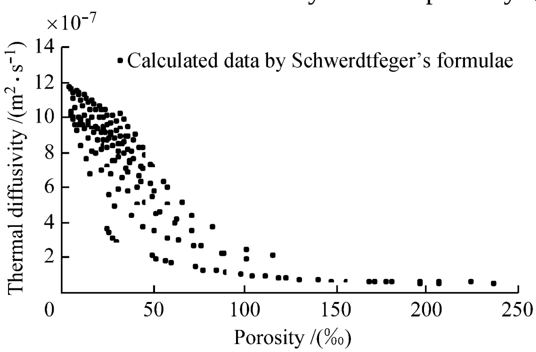

Fig. 1 The thermal diffusivity as a function of the ice porosity given by Schwerdtfeger's formulae and measured data

As extreme cases by definition, we can say that porosity of pure ice (special case of sea ice) is 0 and the thermal diffusivity is the same as thermal diffusivity of pure ice (1.08×10^{-6} m^2/s,); the porosity of pure brine (another special case of sea ice) is 1000 and the thermal diffusivity has the same order of magnitude of pure sea water. Therefore, the threshold relationship values between thermal diffusivity and porosity may be: zero porosity versus pure ice thermal diffusivity and 1000 porosity versus thermal diffusivity of pure sea water.

We have applied seven ice temperature profiles observed from Chinese Antarctic Zhongshan Station, 2006[26] to process our parameter optimal identification. For simplicity, the solar heating effect was neglected (Eq. 8), the ice temperature profiles were obtained from ice floe well below surface in winter season. Table 1 shows the technical details of temperature measurement.

In this case we consider only the heat transfer in the sea ice rather than the mass balance[27]. Thereby, we construct the optimal control system and identify the thermal diffusivity in the range of parameters by using the porosity calculated and the measured data in the ice where there is no phase change (Table 1). The temporal and spatial interval for identifying are 5 s and 5 mm, while the temporal and spatial interval for the ice temperature profiles are 0.5 h and 60 mm respectively. In the process of identifying the initial temperatures for each node is derived through polynomial fitting measurement temperature of 0 moment daily and the boundary conditions are gained through linear interpolating the temperature of the upper and lower boundaries. The half implicit difference scheme and genetic algorithm is applied to solve the optimal control problem[25].

Table 1 Time and depth of seven temperature profiles used for parameter optimal identification. We selected 24 hours measurements for each temperature profile

Date	25 Apr.	21 May	18 June	27 July	2 Oct	1 Nov.	18 Nov.
Range in ice (cm)	6 - 48	6 - 60	6 - 84	6 - 120	42 - 150	36 - 150	42 - 162

We use formulations from literatures and newly modified to identify the relationship between the thermal diffusivity and the porosity (Table 2). By comparing seven identified results[3, 14, 28-30], we choose the optimized formulation $\lambda(\upsilon) = a(1+\upsilon)^{-b}$ that is derived by improving $\lambda(\upsilon) = a\upsilon^{-b}$. $\lambda(\upsilon) = a\upsilon^{-b}$ is applied to evaluate the mechanical property of sea ice in Bohai[6], but in mathematics we need $\upsilon \geq 1$. Some trouble will occur if $\upsilon \to 0$, though the case take place hardly in reality. The optimized relationship (10) between the thermal diffusivity and the porosity of the Antarctic sea ice is determined by identifying and fitting ultimately (Fig. 2), which can be described as

$$\lambda(\upsilon) = 10.8 \times 10^{-7} (1+\upsilon)^{-0.302} \quad (10)$$

Table 2 Formulations of sea ice properties and porosity from literatures and modified in the inverse process

No.	Formulation	Reference and Origin	Error
1	$\lambda(\upsilon) = a(1 - b\sqrt{\upsilon})^{-1}$	[28], used in the relation of sea ice tensile strength and brine volume	1.41%
2	$\lambda(\upsilon) = a(1 - b\upsilon)^{4}$	[29], used in the relation of the elastic modulus and the porosity	1.55%
3	$\lambda(\upsilon) = a(1 - b\sqrt{\upsilon})^{-2}$	[29], used in the relation of the effect stress and the porosity	1.42%
4	$\lambda(\upsilon) = a \cdot e^{-b\upsilon}$	[30], used in the elastic modulus and the porosity	1.52%
5	$\lambda(\upsilon) = a(1+\upsilon)^{b}$	[3], used in the relation of the mechanical indicator and the porosity	1.41%
6	$\lambda(\upsilon) = a(1 - b\sqrt{\upsilon})^{2}$	[14], used in the relation of maximum intensity and the porosity	1.43%
7	$\lambda(\upsilon) = a(1 + b\upsilon)^{-1}$	Newly modified form based on references	1.46%

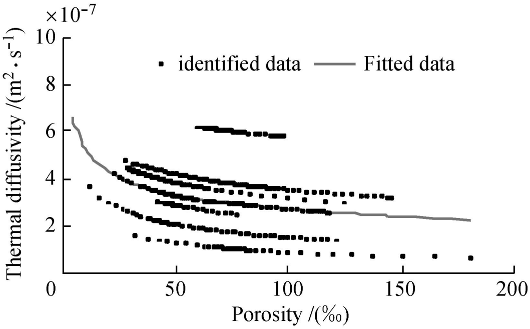

Fig 2 The optimized curve fitted with identified data from Antarctic Zhongshan Station in 2006.

4 The validation of newly derived scheme

In order to validate the identified result, we calculate sea-ice porosity and then derive the vertical temperature profiles by using scheme (10) and the heat-conduction equation (8). The calculated results are then compared with the measured ice temperature from Chinese Antarctic Zhongshan Station, 2005. Figure 3 shows the calculated and measured vertical temperature profiles at 13:00 on various days. The simulated and measured ice temperature profiles agreed well.

For comparison, we also applied Schwerdtfeger's (1963) formulae (2), (3) and the heat-conduction equation (8) to calculate the vertical temperature profiles. Figure 4 shows the results.

From Fig 3 and Fig 4 we can conclude that the trends of temperature deviations are quite the same. The average errors between observed and measured ice temperature are 1.41% (Fig 3) and 1.71% (Fig 4), respectively. This indicates the scheme of thermal diffusivity and porosity is applicable in terms of sea ice thermodynamic calculation.

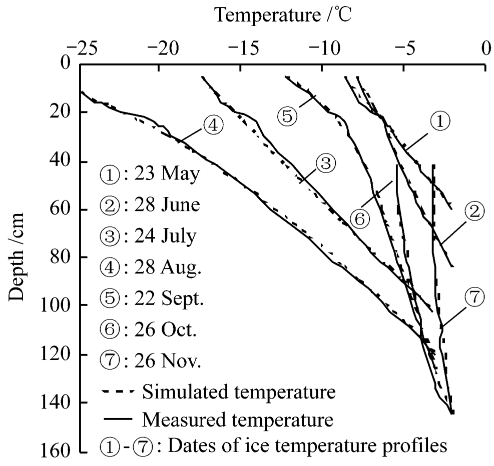

Fig 3 The simulated and measured ice temperature profiles from Antarctic Zhongshan Station in 2005. The thermal diffusivity versus ice porosity was applied in the thermal heat conduction equation.

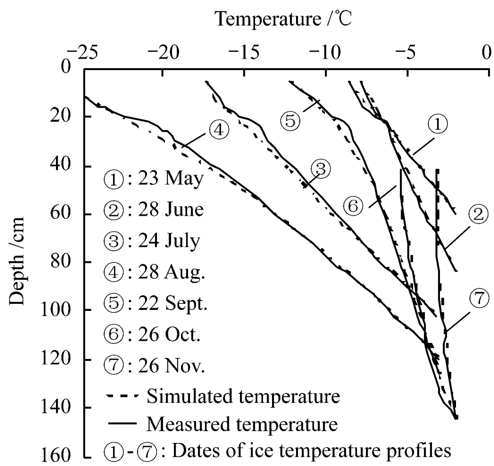

Fig 4 The simulated and measured ice temperature profiles from Antarctic Zhongshan Station in 2005. The thermal heat conductivity and volumetric heat capacity introduced by Schwerdtfeger[17] were applied in the thermal heat conduction equation.

5 Conclusions

The thermal heat conduction equation plays an important role in sea ice thermodynamics. Based on mathematical optimal parameter identification method, we have identified an optimal relationship between thermal diffusivity and the ice porosity. The scheme can directly be applied in heat conduction equation resolving the ice temperature regime. The numerical simulations of ice temperature regime show good agreement with observations. The simulated ice temperature profiles are also not differ far from those calculated by the classical heat conduction equation with thermal heat conductivity and volumetric heat capacity. Although the identified scheme, Eq. (10) was confined only to the polar night condition, i. e. without internal solar heating source term, we see still potential applicable of Eq. (10) for the Antarctic sea ice. This is because the ice layer is normally thick, the internal solar heating will be largely confined in the upper layer of ice or snow, deeper in the ice, the temperature regime is linear, i. e. Eq. (8) is well valid.

Additionally, since no solar radiation available in Antarctic winter, Eq. (8) is valid and can be used to solve the entire ice temperature regime with our identified scheme (10).

The sea ice porosity has more physical meaning rather than temperature and salinity. From the point of view of sea ice mechanism, sea ice engineering and sea ice biology aspects, it is more convenience to use ice porosity concept, for example, for the moment the porosity is applied to the evaluation of mechanical property, dielectric permittivity, sea ice scattering and primary productivity. Therefore in such cases discussion on thermal parameter, especially the thermal diffusivity from the point of view of the porosity is of critical importance.

We do have various simplifications in our optimal parameter identification procedure, there are still problems remain unsolved, for example, one may need to identify the thermal diffusivity versus ice porosity when solar radiation is available. In this case, the full thermal heat conduction equation is to be applied. The extensive field data under all weather conditions is urgently needed.

Acknowledgements We are grateful to Mr. Yinke Dou and Ruibo Lei who have been participated 21st and 22nd Chinese Antarctic scientific expedition to carry out field measurements. We also thank Mr Jiaping Ye and Li Dong, leaders of 21st and 22nd Chinese Antarctic Zhongshan Station to provide excellent logistical support to ensure the high quality in situ data observations.

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