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Applying Response Surface Methodology to Readiness-Based Leveling of Reparable Items

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APPLYING RESPONSE SURFACE METHODOLOGY
TO READINESS-BASED LEVELING OF REPARABLE ITEMS

THESIS

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DISCLAIMER STATEMENT

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THESIS

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application percentage- percentage of aircraft in a unit that use a particular item

availability- expected percentage of a fleet of aircraft that is not down for spares at a random point in time (Sherbrooke, 1992:27)

backorder- an unfilled demand (Sherbrooke, 1992:24)

base demand rate- rate at which unserviceable LRUs are brought in from the flightline for replacement

base pipeline- number of units of an item either in repair at the base or in resupply back to the base

base processing percentage- percentage of items a base can either repair or condemn

base repair cycle time- days required to repair an item on base assuming test equipment is immediately available

base repair cycle time pipeline quantity- the number of units of an item in repair at the base

base safety level quantity- number of units of an item added to the base stock level to offset variability in the base pipeline

cannibalization- consolidation of item shortages on the smallest number of end items, i.e. aircraft (Sherbrooke, 1992:xxiv)

depot pipeline- number of units of an item either in repair at the depot or in transit to the depot

depot repair cycle time- days required to repair an item at depot assuming test equipment is immediately available

depot repair cycle time pipeline quantity- the number of units of an item in a pipeline that begins at the point when the base declares an item needs depot maintenance and ends at the point when an item is ready for shipment from the depot to the base

essentiality- "This is the relative cost of a backorder on Item i at Base j compared to a backorder on some standard item" (Sherbrooke, 1966:4)

indenture structure- the engineering parts hierarchy (Sherbrooke, 1992:8)

item- type of part (Sherbrooke, 1992:1)

line-replaceable unit (LRU)- a first-indenture item removed from the aircraft while on the flight line; USAF-specific term (Sherbrooke, 1992:8)

multi-echelon problem- tradeoffs between stock at operating locations and supporting depots; the operating location, or base, is the first echelon and the depot is the second echelon; the Air Force is most often considered a two-echelon system (Sherbrooke, 1992:xxi,7)

multi-indenture problem- tradeoffs between stock for an item and its subitems (Sherbrooke, 1992:xxi) order and ship time- time between placing and receiving an item from depot if there was stock on the shelf at depot (Sherbrooke, 1992:6)

order and ship time pipeline quantity- the number of units of an item in resupply to the base from depot

pipeline- the number of units in repair or resupply (Sherbrooke, 1992:14)

reparable- an item that may be repairable depending on the nature of the failure; synonymous with recoverable and repairable, it is a term mostly used in the military services (Sherbrooke, 1992:1)

requirement- for a certain period of time, the number of units of an item determined to be required by all using bases; consists of the base repair cycle, depot repair cycle, and order and ship time pipelines and the base and depot safety levels

special level quantity- number of units of an item added to the base stock level; determined on a case by case basis

shop-replaceable unit (SRU) a second-indenture item removed from the first-indenture item while in the maintenance shop; USAF-specific term (Sherbrooke, 1992:8)

units- quantity of items (Sherbrooke, 1992:1)

Abstract

Reparable items play a large role in determining the readiness of United States Air Force (USAF) weapon systems. Many factors characterizing flying tempo and item repair and transit time influence the level of fleet readiness. Readiness-Based Leveling (RBL) considers these factors as it seeks to maximize aircraft readiness as it allocates spare reparable items between bases and a depot. The purpose of this research was to demonstrate the validity of using response surface methodology (RSM) within the context of RBL in an effort to quantify the influences these factors have on aircraft readiness.

RSM applied designed experiments and least squares regressions in developing a series of empirical models quantifying correlations between one uncontrollable and seven controllable factors and RBL's output. Verification tests indicated the empirical models represented -- to a high degree -- the quantitative relationships present between the inputs and output of RBL. Although valid conclusions cannot be made from the models (a substitute input was used in place of a usual D041 input), the methodology as demonstrated is valid.

APPLYING RESPONSE SURFACE METHODOLOGY

TO READINESS-BASED LEVELING OF REPARABLE ITEMS

I. Introduction

America is committed to defending its interests at home and abroad. The United States Air Force operates many kinds of aircraft as one component of America's defense capability. Today's aircraft are very complex, technological marvels with high price tags and sometimes may not be available for missions. What keeps an aircraft from being available? First, aircraft items alone can be a source of aircraft nonavailability when required replacements are not available. Second, if a replacement is available, the maintenance required to install or test the item may keep an aircraft nonavailable. This research is concerned with the first cause of aircraft nonavailability.

Each aircraft item is one of two types: reparable and nonreparable. A reparable item is capable of being repaired, and a nonreparable item is not capable of being repaired. A line-replaceable unit (LRU) is an example of a reparable item. Oil is an example of a nonreparable item. When comparing these two types of items, reparable items have more impact on an aircraft's availability to fly (Sherbrooke, 1992:5). Reparable items tend to be more expensive and account for the largest portion of the spares budget: $31 billion of spare reparable items in the Air Force's 1990 budget (Sherbrooke, 1992:5). High costs are apparent in the number of reparable items on an aircraft: F-111 with 3758, F-16 with 3306, B-1 with 2650 (Rexroad, 1992: 24). Although reparable items have relatively low demand at the base level, they have longer lead times than nonreparable items. A long lead time means it would take a long period of time to solve a supply problem (Sherbrooke, 1992:5,6).

Managing a reparable item involves large quantities, high cost, low demand, long lead time, and impact on aircraft availability. Managing a set of reparable items is a complex and challenging process. Two of the more significant challenges involve determining the quantity of each item needed and to whom should they be allocated. This research is concerned with the second challenge, also known as central stock leveling (CSL).

Mathematical models are available for CSL. In both the actual CSL system and its model, certain system characteristics may have an influence on the final output or, in this case, how reparable items are allocated. In other words, there may be a relationship between the model's inputs and output(s). Learning about such relationships in a system's model, if they exist, can increase general knowledge about the system's model, if not the actual system under study.

Research Objective

The objective of this research was to develop a method for learning about possible relationships between the inputs and output of Readiness-Based Leveling (RBL). RBL performs CSL via the Multi-Echelon Technique for Recoverable Item Repair (METRIC) model. RBL and METRIC are discussed in more detail in Chapter II.

RBL is designed to allocate the items in a fashion that minimizes the number of expected base backorders for the item. Minimizing backorders is important because it is equivalent to maximizing aircraft availability (Sherbrooke,1992:38,39). A lower number of expected backorders translates to a higher availability of aircraft for conducting military operations.

Air Force Material Command (AFMC) Studies and Analysis (SAO) sponsored this research to investigate which factors or variables affecting RBL are most important. This research developed an empirical model of RBL to determine if every aspect of RBL is equally important or influential in determining the number of expected base backorders.

Scope

This research focused on the aspect of the supply system dealing with allocating reparable items between the bases and the depot. Therefore, it did not focus on the process for determining how many reparable items to allocate. Furthermore, the research only considered two echelons -- base and depot -- and a first-indentured item. One item and one aircraft system (F-16) were considered.

RBL was not critiqued in this study. It is one of several techniques for CSL. Other techniques include the Aircraft Availability Model (AAM), Distribution and Repair in Variable Environments

(DRIVE) model, and fixed safety levels (FSL). An evaluation of the four techniques mentioned here, including METRIC, was performed as part of RAND's Project AIR FORCE (Miller, 1995).

Summary

This chapter described the complex nature and large budget expense surrounding Air Force reparable items. Management has several CSL alternatives for allocating reparable items between the bases and the depot. AFMC has supported the use of RBL with its incorporated METRIC model to solve the allocation problem. In response to inquiries from HQ AFMC/SAO, this research developed an empirical model of RBL to determine which aspects of the multi-echelon, reparable item environment most influence the expected base backorders for a reparable item.

Overview

Chapter II presents background information relevant to this research. It discusses RBL in more detail and introduces an empirical modeling technique referred to as Response Surface Methodology (RSM). Chapter III explains the methodology of this research. It discusses factors to be considered, some preliminary calculations, and the experimentation process. Chapter IV discusses the results of the experiment and resulting empirical model, while Chapter V closes the paper with conclusions and recommendations. For easy reference to most supply-related terms and the usual large population of Air Force acronyms, the reader is referred to the List of Terms included in this document.

II. Literature Review

Introduction

Response Surface Methodology (RSM) was applied to develop an empirical model of Readiness-Based Leveling (RBL). RBL maximizes aircraft readiness as spare parts are allocated between the bases and the depot. After presenting the general purpose and process of RBL, there is an exposition of its underlying model: Multi-Echelon Technique for Recoverable Item Control (METRIC). METRIC considers two supply echelons, base and depot, as it utilizes marginal analysis to distribute spare parts and figure the number of expected backorders resulting from various distribution schemes. A brief section on multi-echelon theory concludes with the computational process of METRIC.

The RSM section incorporates discussion of its purpose and functionality. A section on empirical models discusses how the true output response surface must be approximated because of unknown relationships between the process output response and input variables. Experimental design is an integral part of RSM, and so, along with desirable design characteristics, it is discussed. There is a section on two types of error to be aware of in RSM: random and bias. A section on canonical analysis and its role in analyzing second-order polynomials, along with a chapter summary, complete this chapter.

Readiness-Based Leveling (RBL)

This section provides a background on the purpose and process of RBL. Although *Readiness-Based Leveling* was coined by AFMC/SAO during conference discussions in 1995, its roots date back to the 1960s when its underlying solution procedure, the Multi-Echelon Technique for Recoverable Item Control (METRIC), was developed (Sherbrooke,1966). An exposition of METRIC follows a discussion of the purpose of RBL.

<u>RBL Purpose</u>

The purpose of Readiness-Based Leveling is in its name. Leveling is the activity of allocating spares of reparable items (i.e. stock) between the bases and the depot. Going one step further, RBL is a central stock leveling (CSL) method because the allocation determination is made at a centralized location. *Readiness-Based* means the allocation is done in a manner that maximizes Air Force readiness. In this context, the measure of readiness is aircraft availability. RBL minimizes expected base backorders for a reparable item. As stated in Chapter I, minimizing backorders is important because it is equivalent to maximizing aircraft availability (Sherbrooke,1992:38,39).

Multi-Echelon Technique for Recoverable Item Control (METRIC)

METRIC provides the procedure for meeting the goal of RBL: allocation of reparable (that is, recoverable) items in a fashion that maximizes readiness by minimizing expected backorders. The model's developer describes it as follows:

> METRIC is a mathematical model translated into a computer program, capable of determining base and depot stock levels for a group of recoverable items; its governing purpose is to optimize system performance for specified levels of system investment. METRIC is designed for application at the weapon-system level, where a particular line item may be demanded at several bases and the bases are supported by one central depot (Sherbrooke, 1966:2).

METRIC's Purposes

Three purposes of the METRIC model are

> 1. Optimization. A major purpose of the model is to determine optimal base and depot stock levels for each item, subject to a constraint on system investment or system performance...
> 2. Redistribution. The model can take fixed stock levels on each item and optimally allocate the stock between the bases and depot....
> 3. Evaluation. The model provides an assessment of the performance and investment cost for the system of any allocation of stock between the base and depot (Sherbrooke, 1966:2).

METRIC's Mathematical Assumptions

1. "System Objective of Minimizing the Expected Number of Backorders" (Sherbrooke,1966:6).

The objective is defined in this way: "Take a fixed period of time and add together the number of days on which any unit of any item at any base is backordered. Dividing this number by the length of the period and taking the expected value of the statistic yields a number that is independent of the period length (Sherbrooke 1966: 6). This objective was preferred because "...backorders are a convex function of base stock level when the depot stock level is constant" (Sherbrooke 1966: 6). Sherbrooke showed impracticalities of using other objective functions which were maximizes: fill rate, ready rate, operational rate. This assumption is reasonable for a goal of maximizing aircraft availability because both minimizing backorders and maximizing availability were stated to be equivalent.

2. "Compound Poisson Demand" (Sherbrooke 1966: 8).

This assumption has not followed observation (Sherbrooke, 1992:47). First of all, there are no observations of clusters of demand (Sherbrooke, 1992:62). Second of all, a compound Poisson distribution of demand is characterized by a variance to mean ratio (VTMR) that is constant over the demand's observed time period. However, empirical data has shown the VTMR to increase with time (Sherbrooke, 1992:61,62). Hence, the negative binomial distribution is used to generalize a Poisson distribution (with nonconstant mean) while allowing a dynamic VTMR. The two parameters used to define the negative binomial distribution are an item's mean demand rate and VTMR (Sherbrooke, 1992:61).

3. "Demand is Stationary Over the Prediction Period" (Sherbrooke 1966: 9).

For time intervals of equal length within the prediction period, it seems reasonable that the distribution of demands for an item will be the same. This assumption would appear to be more true for prediction periods with a consistent environment, that is at peace or at war.

4. "Decision on Where Repair Is to Be Accomplished Depends on the Complexity of the Repair Only" (Sherbrooke 1966: 10).

This assumption also means base workload has no impact on where the item will be repaired. A study found .3 percent of 10,965 items were sent to the depot for workload reasons (Weifenbach, 1966: 4).

5. "Lateral Resupply [Between Bases] is Ignored "(Sherbrooke 1966: 10).

This assumption is reasonable since the number of lateral shipments "is typically small" (Sherbrooke 1966: 10).

6. "System is Conservative" [Items Are Not Condemned] (Sherbrooke 1966: 10).

A study found only 4.1 percent of 10,965 items were condemned (Weifenbach, 1966: 4). "The condemnation rate must be considered for procurement purposes, but the procurement process in not considered in the METRIC optimization" (Sherbrooke, 1966:10-11).

7. "The Depot Does Not Batch Units of a Recoverable Item for Repair Unless There is an Ample Supply of Serviceable Assets" (Sherbrooke 1966: 11).

"In those few cases where setup cost is an important factor and demand is reasonably high, so that some batching is indicated, the estimate of depot repair time should include the average waiting time before depot repair is initiated" (Sherbrooke, 1966:11). The METRIC model used in this research factored in depot delay.

8. "Recoverable Items May Have Different Essentialities" (Sherbrooke 1966: 11).

It seems reasonable to assume different types of items may have associated with them degrees of need by the bases. Some items may restrict the capability of an aircraft more than another.

9. "Demand Data from Different Bases Can Be Pooled" (Sherbrooke, 1966:12).

Any averaging technique deemed satisfactory by the user may be used to come up with an initial estimate of demand per flying-program element: per flying hour, or for this research, per 100 flying hours.

Multi-Echelon Theory

Bases and the depot make up the multi-echelon environment. Certain types of characteristics at both locations are of interest. One characteristic is the demand rate for an item at the depot, which is a function of the demand rate for an item at the base.

The mean depot demand rate is

$$\sum_{j=1}^{J} \lambda_{ij} \bar{f}_{ij} \left(1 - r_{ij}\right) = \sum_{j=1}^{J} \theta_{ij} \left(1 - r_{ij}\right) \qquad (2\text{-}1)$$

where

λ_{ij} : mean customer arrival rate with item i at base j
$\bar{f}_{ij}$: mean demand per customer with item i at base j
r_{ij} : probability item i can be repaired at base j
$(1-r_{ij})$: probability item i at base j must be shipped to depot for repair
θ_{ij} : mean demand for item i at base j (Sherbrooke,1966:13).

The expected backorders at a point in time for a given stock level is

$$B(s) = \sum_{x=s+1}^{\infty} (x - s) p\left(x \middle| \lambda T, VTMR\right) \qquad (2\text{-}2)$$

where

s : spare stock for an item defined as the sum of stock on hand plus on order plus in repair minus backorders; a constant value under the one-for-one replacement policy
λ : mean customer arrival rate where demand for an item is negative binomial*
T : mean resupply (repair) time for an arbitrary distribution $\Psi(t)$
$p(x|\lambda T, VTMR)$: negative binomial* probability density for a mean customer rate λT
(Sherbrooke,1966:13,14)

* Recall the discussion in METRIC's second mathematical assumption.

METRIC's Computational Process

There are five stages to computing the solution. Only one type of item is considered through the first four stages. Stage five considers all types of items simultaneously and is included for completeness. The scope of this research considers only one item. "The essential idea is to compute the depot delay in resupplying bases, and this depends on the depot stock level. Then we can compute the backorders at each base which depend on the resupply delay from depot and the base stock level" (Sherbrooke,1992:48). The five stages are briefly explained.

Stage 1: Compute depot's average delay per demand

The depot's average response time to a base's resupply request is a function of the depot spare stock, s_{io}. The bounds on this response time can be seen when the depot has either an infinite or zero amount of spare stock. If the spare stock is infinite, then the response time is just the average order and ship time, O_{ij}. If the spare stock is zero, then the depot's average repair time, D_i, must be added to the average order and ship time. Thus, the delay *at the depot* will be between zero and the depot's average repair time (Sherbrooke, 1966:14).

The depot delay can be computed as a function of the depot spare stock. Let λ_i be the expected number of customers for item i who arrive at the depot in a certain time period; so $\lambda_i = \Sigma\lambda_{ij}(1\text{-}r_{ij})$ for all bases j. A resupply delay will occur at the depot when there are more items in repair than there were in the spare stock. In the case of a resupply delay, the difference between the items in repair and the spare stock quantity denotes how many items are being delayed. Using Equation 2-2, the expected quantity being delayed at any point in time is (Sherbrooke, 1966:14):

$$B(s_{io}|\lambda_i D_i) = \sum_{x = s_{io}+1}^{\infty} \left(x - s_{io}\right)p\left(x|\lambda_i D_i, VTMR_i\right) \quad (2\text{-}3)$$

> The total expected system delay over any time period is simply the expected number of units on which delay is being incurred at a random point in time multiplied by the length of the time period. Since we are interested in the average delay per demand, we must then divide by the expected number of demands over that time period.... Thus, the average delay per demand...is (Sherbrooke, 1966:15):

$$\frac{\sum_{x = s_{io}+1}^{\infty} \left(x - s_{io}\right)p\left(x|\lambda_i D_i, VTMR\right)}{\lambda_i \bar{f}_i} = \delta(s_{io})D_i \quad (2\text{-}4)$$

where

$$\delta(s_{io}) = \frac{\sum_{x=s_{io}+1}^{\infty} (x - s_{io}) p\left(x \middle| \lambda_i D_i, VTMR\right)}{\lambda_i D_i \bar{f}_i} \tag{2-5}$$

Stage 2: Compute Expected Backorders

Expected backorders for each level of depot stock, s_{io}, is a function of the base stock, s_{ij}. Equation 2-2 is used assuming that $s = s_{ij}$, $\lambda = \lambda_{ij}$, and $T = r_{ij}A_{ij} + (1 - r_{ij})(O_j + \delta(s_{io})D_i)$ (Sherbrooke, 1966:15).

Stage 3: Determine Optimal Allocation of Items to the Bases

For each level of depot stock, s_{io}, the marginal allocation procedure determines where to distribute the next unit of item i. The criteria used is which base would experience the largest drop in backorders as a consequence of receiving the additional unit (Sherbrooke, 1966:15).

Stage 4: Select Minimum Expected System Backorders

The minimum expected system backorders is selected for each level of constant total system stock, $s_{io}+s_{ij}$. For example, a total system stock of ten units of item i could be distributed between the depot and bases in various ways: one at depot and nine to the bases, two at depot and eight to the bases, and so on. Associated with each distribution scheme for this constant stock level is an expected number of backorders at the bases. This stage will select for one type of item -- independently of the other types of items -- that distribution scheme corresponding to the least expected backorders. (Sherbrooke, 1966:16).

Stage 5: Determine Next Investment Among the Items

> Now we consider the multi-item problem. Marginal analysis is again employed. Using the item backorder functions computed in Stage 4, the next investment is allocated to that item which produces the maximum decrease in expected backorders divided by unit cost. This [marginal analysis approach] is similar to the procedure we used in

> Stage 3, except that there we were dealing with a single item so that unit cost was not a variable.... After each allocation, the system investment and system backorders are computed. Allocation terminates when the investment target is just exceeded or alternatively, when the expected backorders are just less than a target value (Sherbrooke, 1966: 16).

Since only one item was analyzed in this research, Stage 5 was not included in the analysis but is shown here for completion.

Summary

RBL considers different aspects of the multi-echelon environment as it takes a quantity of reparable items, one type at a time, and distributes them between the bases and depot. RBL uses marginal analysis to determine where the next unit should be distributed. It minimizes expected base backorders which is equivalent to maximizing aircraft availability. Perhaps not all aspects are equally important in affecting RBL's outcome. RSM is a mathematical tool which can be used to determine the degree of influence factors have on a particular system.

Response Surface Methodology (RSM)

This research applied RSM to develop an empirical model, or function, for RBL.

> Response surface methodology comprises a group of statistical techniques for empirical model building and model exploitation. By careful design and analysis of experiments, it seeks to relate a response, or output variable to the levels of a number of predictors, or input variables, that affect it (Box and Draper, 1987:1).

This brief discussion of RSM includes a description of the structure of an empirical model, as applied to RSM. Then, experimental design, which provides the structure for data generation to assist model building, is covered. Models fall short of representing their respective process perfectly, implying the presence of error; two types of error can exist in RSM, random and bias. A discussion of these errors is followed by a description of canonical analysis, useful in providing a straightforward description of the approximated response surface. Three experimental design characteristics helpful in minimizing random and bias error will complete this chapter.

Building An Empirical Model

Process experts understand how a process works; they know what makes it run well or not run well. If something related to the process changes, they know how it will affect the process because they understand underlying relationships governing it. This knowledge is acquired over time and may not be easy to convey to someone inexperienced with the process.

RSM is a technique that enables a person to become an "expert" about a process, so to speak. Before applying RSM to a process, an analyst may not know anything about the process. He or she may not know which factors are meaningful or what, if any, relationships exist among the factors or between the factors and the process output. However, with some time-saving advice from the experts at the beginning of an analysis, RSM can shed light on process characteristics that were not only unknown to the analyst, but also may not have been totally understood by the experts. The time-saving expert advice includes which factors are likely to be meaningful to the process, what is the process output, and where can data be collected.

The product of incorporating expert advice and RSM is an empirical model of the process. A graphical depiction of the relationship between the factors and the process output is called a response surface. A true response surface represents the true process outcome. The empirical model represents the response surface in equation form. There can be many possible models, all representing the process and its true response surface to different degrees of accuracy. Two examples of models representing a process where at least three factors were considered are:

$$g_1(\mathbf{x},\boldsymbol{\beta}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \quad (2\text{-}6)$$

$$g_2(\mathbf{x},\boldsymbol{\beta}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{13} x_1 x_3 \quad (2\text{-}7)$$

I say "at least three factors were considered" because more factors may have been considered originally but were determined to not be meaningful in representing the process. The factors are represented by the vector $\mathbf{x}$. The vector $\boldsymbol{\beta}$ represents how the factors influence the process output. Model $g_2(\mathbf{x},\boldsymbol{\beta})$ contains one more term than $g_1(\mathbf{x},\boldsymbol{\beta})$. Since the models differ, they will have different accuracies in representing the process output. Perhaps the extra term in $g_2(\mathbf{x},\boldsymbol{\beta})$ makes a positive difference, enhancing the model's accuracy.

An RSM empirical model "can be thought of as a Taylor's series expansion of the true underlying theoretical function...truncated after terms of *d*th order" (Box and Draper, 1987:21). Higher degree polynomials usually provide a better approximation of the true response surface. For a polynomial of degree d, a smaller region of interest usually results in a better approximation. "Region of interest" refers to the range of values over which the factors, or input variables, can vary. In practice, a first or second degree polynomial might be adequate for a limited range of the factors (Box and Draper,1987:21,22). Equations 2-6 and 2-7 above represent possible first and second degree polynomials, respectively. A more formal representation of a second degree polynomial approximation of the response, y, has the following form:

$$\hat{y} = b_0 + \sum_{i=1}^{N} b_i x_i + \sum_{i=1}^{N} b_{ii} x_i^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_{ij} x_i x_j \qquad \text{(2-8)}$$

The statistical techniques used in developing RSM's empirical models include experimental design, least squares regression, and, if the result is a second order polynomial, canonical analysis. The following sections briefly describe these techniques, with the exception of least-squares regression; the reader is assumed to be familiar with least squares regression.

Experimental Design

The source of the data used to develop an empirical model can be either prior observation or planned experimentation. In the context of RSM, designed experiments provide virtually all of the data for empirical model building and exploitation. A designed experiment is "a test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes in the output response" (Montgomery, 1991:1). Many experimental designs are efficient in that they provide a great deal of information from relatively few experiments, or design points.

An experimental strategy can involve a series of tests. A screening design might constitute the first experiment. A screening design is a relatively small scale (in terms of the data generated) experiment that assesses the significance of all of the original factors. Those factors exhibiting only a negligible influence on the process output can be eliminated from further consideration and experimentation. Such screening reduces the dimensionality of all the subsequent designs included in the sequential experimental strategy.

"Purposeful changes" to the values, or levels, of the factors can be implemented simultaneously or by one-at-a-time. During one-at-a-time changes, one factor level is varied as all other factor levels remain constant. Purposeful, simultaneous changes to the factor levels, done in context of an experimental design, are more efficient in gathering meaningful data and can also detect interactions among the factors; one-at-a-time changes cannot detect interactions.

As a result of purposeful, simultaneous changes to the factor levels, the analysis to determine reasons for changes in the output response is relatively straightforward. The "reasons" most likely lie in the correlations that may exist between the output and the factors. The resulting form of the empirical model will approximate these relationships.

Desirable Design Characteristics

There are certain characteristics desirable in experimental designs: orthogonality, rotatability, and small number of design points to name a few. A design is orthogonal if all "n-dimensional design vectors...are at right angles," implying the factors are uncorrelated with each other (n denotes number of observations. There is an n-dimensional design vector for each factor included in the experiment) (Box and Draper, 1987:479). Orthogonality is desirable because it allows all the factor effects to be measured independently from each other (Schmidt and Launsby, 1989:3.2). Not all designs, however, are orthogonal. The demands of orthogonality often require more runs than is practical in an experimental setting. Fortunately, orthogonality is not critical; modern computing technologies allow estimation of factor effects even when design are not orthogonal (Box and Draper, 1987:510). In order to simplify the process of developing orthogonal designs, factor levels are usually coded in the following way:

$$x_{iu} = \frac{\left(\xi_{iu} - \bar{\xi}_i\right)}{S_i} \quad (2\text{-}9)$$

where $\bar{\xi}_i$ is the mean of the settings of factor i
S_i is the half-width of the range of settings for factor i

The result of such a coding scheme is the highest setting for each factor takes on the value of one while the lowest setting takes on a value of minus one (Box and Draper, 1987:480).

An experimental design is rotatable if the variance of the empirical model's predicted response is a function of a design point's distance from the design center (Box and Draper, 1987:440). All design points equidistant from the design center are equally important in terms of the information they contain about the response surface. The variance associated with the information is the same for each of the equidistant points. Therefore, a rotatable design provides the same amount of accuracy in every direction from the design center. In other words, none of the equidistant points provides an output response with more or less variance than any other equidistant point. Although rotatability is desirable, it also is not critical to the success of the empirical modeling effort (Box and Draper, 1987:510).

The number of runs required by an experiment depends on the number of factors involved and the design implemented. Using as few runs as possible is desirable because it saves time and money. Ideally, experimental designs would be orthogonal, rotatable, and require a minimum number of runs. In practice, resource limitations, operating restrictions, and formal policy preclude the use of ideal experimental designs. In spite of such obstacles, properly designed experiments can still support empirical model building and exploitation.

Types of Error

Empirical models can contain two types of errors: random and bias (Box and Draper, 1987:208). Random error results when it is possible to observe different output responses for the same factor level inputs; that is, there is some probabilistic distribution associated with the output response for a set of factor level inputs. Random error results from some uncontrollable effect on the process. Bias error is the difference between the output response of the empirical model and the true response function for a given set of factor level inputs. Bias error results from the inability of the empirical model to fully represent the relationships between the factors and output response.

Random error does not apply to RBL because its underlying solution procedure, METRIC, although inherently stochastic, exhibits "deterministic" behavior if viewed in the context of a black box. A given set of inputs always results in the same response, regardless of the number of runs conducted.

Thus, the error present in an empirical model of RBL is completely attributable to bias -- sometimes referred to as lack-of-fit. With no random, or "pure" error, it is not possible to measure the quality of the empirical model in terms of statistical significance. The adequacy of the empirical model must, therefore, be measured in terms associated with the least-squares fitting procedure such as R^2, adjusted R^2, and mean square error.

Canonical Analysis

If the resulting empirical model is a second-order polynomial, the description of the approximated surface can be made more straightforward through applying a canonical analysis. "Canonical analysis is a method of rewriting a fitted second degree equation in a form in which it can be more readily understood" (Box and Draper, 1987:332). Canonical analysis is helpfully applied where factor dependence exists. Factor dependence means "the response function for one factor is not independent of the levels of the other factors" (Box and Draper, 1987:329). A second order empirical model -- before canonical analysis -- exhibits factor dependence by the presence of interaction terms. Factor dependencies can make response surface description difficult, thus it is by elucidating factor dependencies that canonical analysis aids in response surface description.

If the empirical model is more readily understood, then it will be simpler to address several possible concerns. Does the region of interest also happen to contain an extreme point (although searching for such points is not the goal of this research)? If an extreme point exists, is it a maximum or minimum? Are there alternative optima? Is it possible to optimize a second response within the region of interest? Are there directions of insensitivity with respect to the relationship between the output response and factors? Can the output response be improved by simultaneous changes in the factor levels? Perhaps a basic mechanism, or "natural law," is theorized to exist between two or more factors. Canonical analysis could increase knowledge about the existence of such a relationship (Box and Draper, 1987:329,330).

There are two canonical forms: A and B. When canonical analysis is performed in practice, the A canonical form is developed first, but it may not always be appropriate to develop the B canonical form. Briefly, the A canonical form is "achieved by a rotation of axes which removes all cross-product terms" (that is, interactions) from the empirical model (Box and Draper, 1987:332). The A canonical form addresses the factor dependence discussed previously.

The B canonical form results from "a change of origin to remove first-order terms..." (Box and Draper, 1987:332,333). The B canonical form effectively positions the empirical model over the stationary point (i.e. center of the system represented as an extreme point, ridge, or saddle point), if one exists. The B canonical form is appropriate when a stationary point exists and is close (not much greater than one experimental-design unit) to the center of the experimental design region (Box and Draper, 1987:339). That is, the movement of the axes' origin should not go beyond the experimental design region.

Summary

This chapter discussed the process under study - RBL - and the elements of a general methodology - RSM - for developing an empirical model of the process. RBL considers different aspects of the multi-echelon environment as it takes a quantity of reparable items, one type at a time, and allocates them between the bases and depot. RBL uses marginal analysis to determine where the next unit should be allocated. Its underlying model, METRIC, was discussed via its purposes, assumptions, and five-stage computational process. In this research, only the first four stages were used since only one item was analyzed.

RSM is a group of statistical techniques used to relate various process inputs to a process output. The output may have an analytical or theoretical relationship with the inputs. The true relationship is unknown and, therefore, approximated by an empirical model. The empirical model's response surface approximates the true response surface within a limited region. RSM uses experimental designs to generate the data required to develop the corresponding empirical models.

Random and bias errors were discussed, but random error was of no concern to this research because of the "deterministic" nature of the METRIC model. Bias error, the difference between the approximated surface and the true surface, was a concern and something to minimize. Canonical analysis facilitates a straightforward interpretation of a second-order polynomial model. Through axes rotations and translation, canonical analysis can highlight factor dependencies and the principal features of an empirical model.

Chapter III covers the details of applying RSM in order to develop an empirical model for the response surface of RBL. As the details of RSM are covered, the actual analysis of RBL takes place. The analysis in Chapter III should be taken in the following context: this paper is demonstrating the validity of applying RSM to RBL, and due to necessary data adjustments mentioned later, any conclusions about correlations between the output and factors are for the purpose of illustrating interpretations that could be made. Chapter IV discusses the conclusions drawn from the analysis in Chapter III while Chapter V considers studies to either improve or extend the work presented in this paper.

III. Analysis

Introduction

The previous chapter stated that response surface methodology (RSM) could develop an empirical model of the readiness-based leveling (RBL) technique. The goal of developing an empirical model of RBL is to approximate its output, or response, as a function of its input factors. The resulting empirical model can then be analyzed to determine the significance of the relationships between RBL's output response and input variables, or factors. This study required a reparable item to provide a basis for determining factor levels. A signal processor, National Stock Number (NSN) 1270-01-396-6750WF, was chosen from a group of 66 F-16 line-replaceable units (LRU) because of its high activity in the areas of demand rate, worldwide requirement, and expected base backorders (EBO). Appendix A, Table A.1, lists the values for these criteria for all of the 66 LRUs and shows the ranking procedure through which the item was chosen (Figure A.1). The data sets used to make the NSN determination, and referenced later in this chapter, are in Appendix B.

Refer to the List of Terms for a definition of requirement. The requirements data set in Appendix B includes input and output used with the D041 Recoverable Item Requirements System (henceforth referred to as D041). D041 prioritizes each individual item, down to the second indenture level, in the order of its marginal contribution to aircraft availability. Thus D041 determines the optimal quantity of all items needed to maintain the desired availability of a particular weapon system. The comprehensive D041 solution process requires a great deal of computing resources.

RBL uses the D041 output to allocate all required items to either the depot or one of the operational bases. Since this study focused on only one type of item, it did not justify the computer resources typically consumed by D041. Therefore, a method was needed to provide a substitute requirement input for RBL. Appendix C describes the method used in this study to directly estimate the requirement.

There are four major phases to this analysis: selecting factors and factor levels, developing and analyzing a first-order empirical model, developing and analyzing a second-order empirical model if necessary, and verifying the models.

Factors and Factor Levels

Table 3.1 lists the seven controllable factors used in this research. The factors are controlled in the sense that their values were purposely chosen, or controlled, in an experimental setting. Air Force Materiel Command (AFMC) Studies and Analysis (SAO) suggested the seven factors because they cover the main aspects of the logistic system under study, and they are presumed to be influential in RBL's computation of expected base backorders. The List of Terms defines each factor. Table 3.1 also lists the two levels at which the factors were set. Appendix D explains why the levels were chosen. The levels shown as a percentage represent a percentage decrease (low level) or increase (high level) in the current level. The current levels were collected from the data sets described in Appendix B. The term "Vary" in Table 3.1 refers to a current level that depends on the base.

Table 3.1 Controllable Factors and Factor Levels: Model Development

FACTOR	LEVEL		
	Low	Current	High
1. Flying hours/day for each base (FH)	-20%	vary	+20%
2. Application Percentage (AP)	-10%	vary	+10%
3. Demand rate per 100 flying hours (DR)	-30%	1.0026	+30%
4. Base processing Percentage (BP)	0.59	0.77	0.95
5. Base repair cycle time (BRCT)	3	5	9
6. Order and ship time (OST)	9	9	23
7. Depot repair cycle time (DRCT)	20	25	30

An item's requirement quantity (REQ) (as calculated in Appendix C) was also considered as a factor. It is not shown in Table 3.1 because it is not a controllable factor; its value is a function of the seven factors listed above (see Appendix C).

First-Order Strategy

A first-order model is developed through a first-order strategy, and it considers only the main factors as possible terms in the model; interaction among the factors and quadratic effects are not considered. In general, the choice of experimental design determines the types of models that may be estimated.

Every design has associated with it a resolution, which is usually depicted with Roman numerals; that is, IV denotes resolution four. "A design is of resolution R if no p-factor effect is aliased with another effect containing less than R-p factors" (Montgomery, 1991:339). For example, two-factor effects are not aliased with one-factor (or main) effects in a resolution IV design, but they are aliased with two-factor and higher effects. The effects of "aliased" factors cannot be differentiated from one another. Designs of resolution III or IV are generally used to develop first-order models, whereas second-order models require designs of at least resolution V.

In this research, the experimental design used in the first-order strategy was 2^{7-1}_{VII}. This nomenclature represents 64 runs in a resolution VII design. AFMC/SAO suggested three-factor interactions were the highest interactions to be concerned about. With a resolution VII design, three-factor interactions are aliased with four-factor and higher interactions. Thus, assuming four-factor and higher interactions are not significant, there is no confusion attributable to aliasing when interpreting the meaningfulness of three-factor or lower interactions. The 2^{7-1}_{VII} design appears with the RBL outputs in Appendix E. This appendix also has one example of a portion of a design point (in this case the first design point) that served as input to RBL.

RBL's METRIC model is stochastic in the sense that there is a probability distribution associated with the number of backorders that may occur at any point in time. However, this stochastic element is represented as an expected value; namely, expected base backorders. For a given set of levels for each of the seven controllable factors, calculations in Appendix C will produce a requirement quantity that serves as input to METRIC (along with five of the controllable factors). METRIC will output the same EBO value for a given set of inputs. So, although it incorporates probability distributions for backorders, the expected backorder quantity will not vary in cases of repeated inputs. METRIC is not a deterministic model, but if viewed as a "black box," it may seem to exhibit such a characteristic. For this reason, the statistical inferences typically associated with linear regression models are not valid. However, there are other indicators such as R^2, SSE, and MSE that indicate the quality of the model fit.

Appendix F contains information about the experiments' output to include the following: cumulative distribution functions (CDF) of both REQ and expected back orders (EBO) (Figures F.1,F.2), scatter plots of each factor matched against EBO (Figures F.3 through F.10), statistics on the distributions of REQ and EBO (Table F.1), statistics on each of the controllable factors at both the low and high levels (Table F.2), and a correlation matrix (Table F.3). From the CDFs and skewness statistic, both REQ and EBO show evidence of skewness. This information is useful later for considering factor transformations. In the scatter plots and statistics table, the levels of BP and DR have a noticeable effect on the average EBO value, while the levels of BRCT have a noticeable effect on the spread of EBO values. Noteworthy correlations exist between REQ and EBO and BP and EBO. Thus, it appears supply situations calling for a low item requirement are correlated with low EBO values, while a high base repair percentage is also correlated with low EBO values. DRCT is notable for its lack of correlation with any other factor or the response.

After conducting experiments specified in the design, a first-order model was sought. SAS software was used for a portion of the analysis and its statistical procedures are denoted by capital letters. The SAS code is included in Appendix G. The RSQUARE procedure used the R^2 statistic as criteria while seeking the best one-factor model, the best two-factor model, and so on until all eight factors were incorporated into a model. The results of the RSQUARE procedure are included in Appendix H. The relationships between R^2 and MSE and the number of factors in a model are plotted in Figures 3.1 and 3.2, respectively. Based on these two criteria, Figures 3.1 and 3.2 indicate models with three to five factors as being roughly equivalent.

Recall that the seven-factor design is orthogonal. The zero correlation values in Table F.3 support this fact. A useful result of the orthogonal design is the coefficient estimates of a regression (incorporating only orthogonal factors) reflect the effect each factor has on the response. Furthermore, since a regression coefficient reflects the effect on the response of a one unit change in the factor (e.g. from -1 to 0 or 0 to 1), doubling the coefficient's value reflects the effect on the response from changing the factor level between -1 and 1. Figure 3.3 graphs the factor effects on the response. It is the difference between the average EBO value when a factor is at its high level and the average EBO value when the factor is at its low level. The first three factors (BP, DR, FH) have the greatest impact of the seven

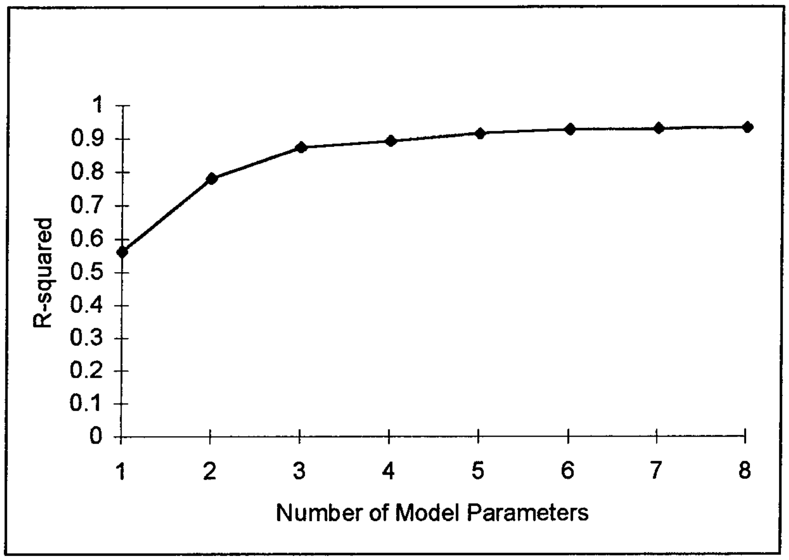

Figure 3.1 R^2 vs. Number of Model Parameters

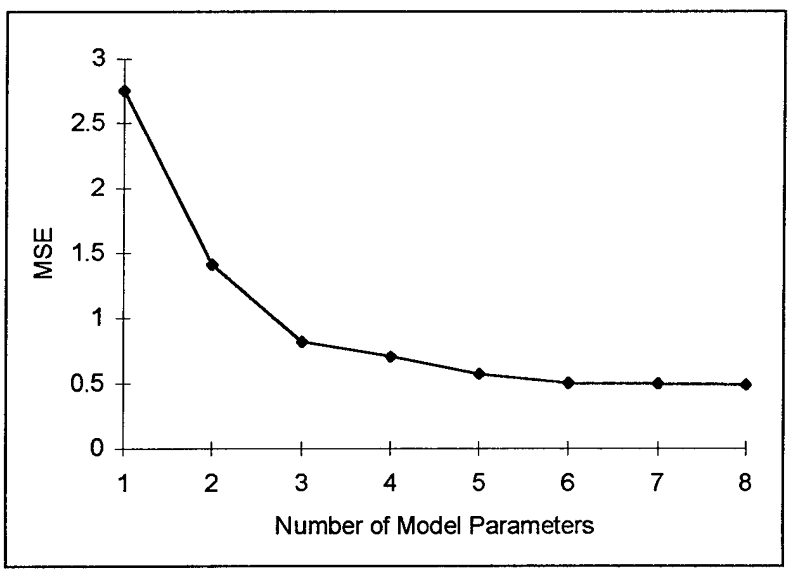

Figure 3.2 MSE vs. Number of Model Parameters

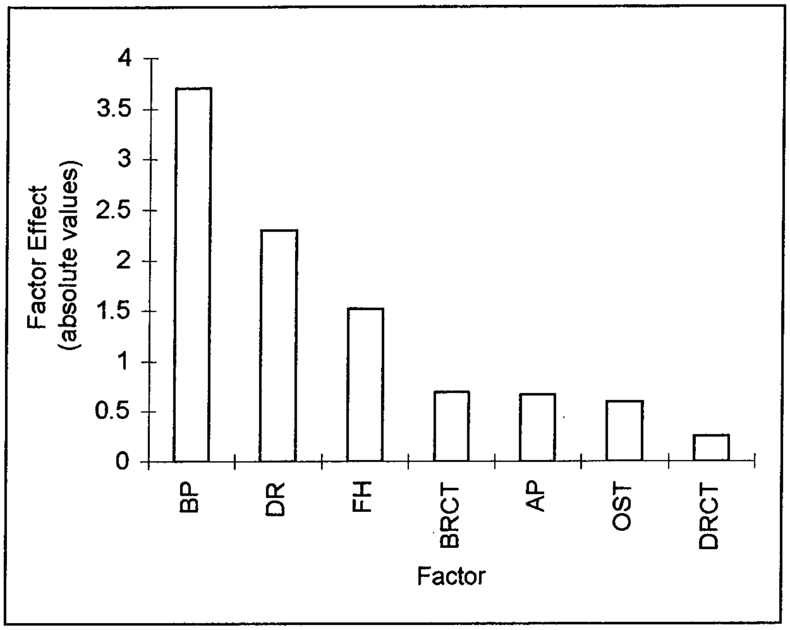

Figure 3.3 Effects of Controllable Factors on EBO

controllable factors. The next three factors (BRCT, AP, OST) have less of an impact and are similar in magnitude. REQ is not included in this graph because such an effect cannot be determined; REQ is not controllable and it did not have an experimental design vector of +1 and -1. Actually, it had 64 different "levels" that were a result of the calculations in Appendix C. Thus, it is not possible to come up with an effect commensurate to those in Figure 3.3. Furthermore, when REQ is added to a regression equation including the other factors, the coefficients and intercept term change due to the correlation exhibited in Table F.3 between REQ and the other factors and the EBO response.

Using the results of RSQUARE as a starting point, five first-order models were developed. The order in which they are discussed basically follows the order in which they were developed. The first model developed was:

$$EBO = 8.114 + 0.7569\ FH + 1.1509\ DR - 1.8519\ BP \qquad (3\text{-}1)$$

With an R^2 of 0.874, it incorporated the three factors in Figure 3.3 with the largest effects on EBO. Figure 3.4 shows that the residuals appear to increase with higher predicted values. In light of the seven controllable factors, Equation 3-1 represents the most parsimonious model that was developed. At the same time, its factors explain the influence of level changes on EBO well.

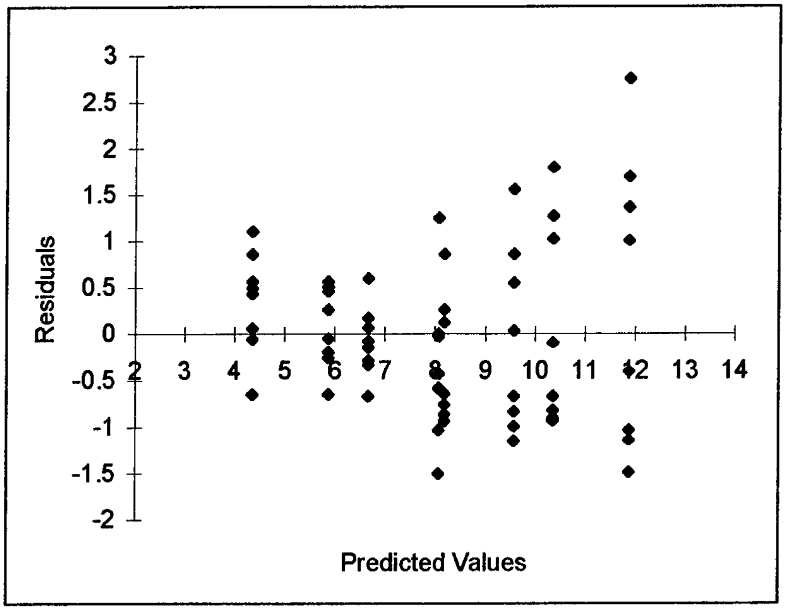

Figure 3.4 Residuals vs. Predicted Values for Equation 3-1

The second model developed was:

$$EBO = 8.114 + 0.7569\ FH + 0.3331\ AP + 1.1509\ DR - 1.8519\ BP - 0.3482\ BRCT + 0.2965\ OST \quad (3\text{-}2)$$

With an R^2 of 0.926, it is the opposite extreme on the "parsimony scale," incorporating six factors. Figure 3.5 definitely shows a pattern.

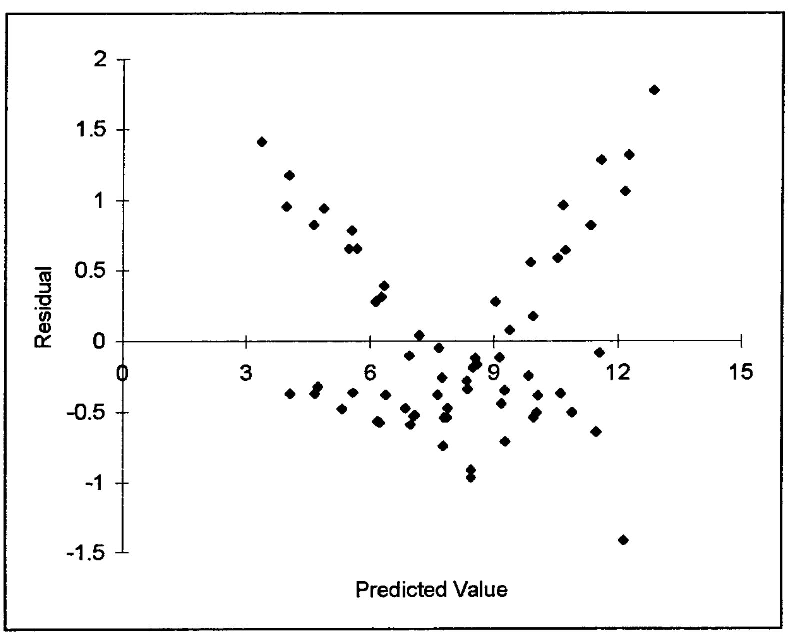

Figure 3.5 Residuals vs. Predicted Values for Equation 3-2

The dispersion of the residuals from zero seems to be related to the magnitude of the predicted values. Such patterns usually indicate a model specification error. In other words, there is a structural pattern in the EBO response that is not modeled by the terms (factors and response) of Equation 3-2.

The graphical and correlation relationships demonstrated in Table F.3 and Figure F.10, between the REQ factor and the EBO response, strongly suggest including REQ in an empirical model of EBO. The RSQUARE procedure first used REQ in building a five-factor model. The factor was excluded from larger models, however, until it reentered in the eight-factor model. The five-factor model including REQ was:

$$6.6010 + 0.4819\ FH + 0.7384\ DR - 1.4758\ BP - 0.6922\ BRCT + 0.0099\ REQ \qquad (3\text{-}3)$$

Its R^2 equaled 0.915 and Figure 3.6 shows a less severe pattern similar in shape to Equation 3-2's. However, Figure 3.7 shows somewhat of a curving pattern between the residuals and REQ. This pattern may call for a transformation on the REQ values.

Recall the small presence of skewness among the EBO values in Figure F.2. Such skewness is a cause for the residual patterns present in Equations 3-2 and 3-3 (Montgomery, 1991:100). Usually a

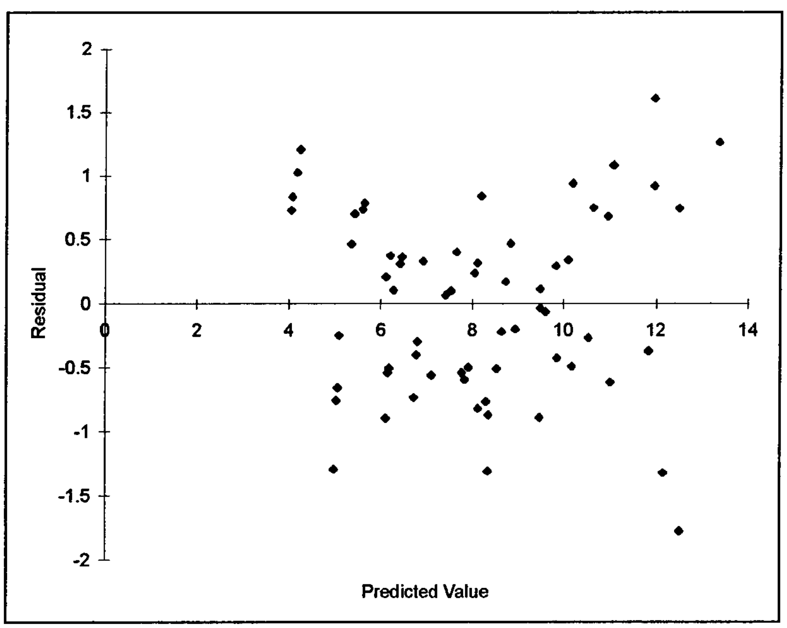

Figure 3.6 Residuals vs. Predicted Values for Equation 3-3

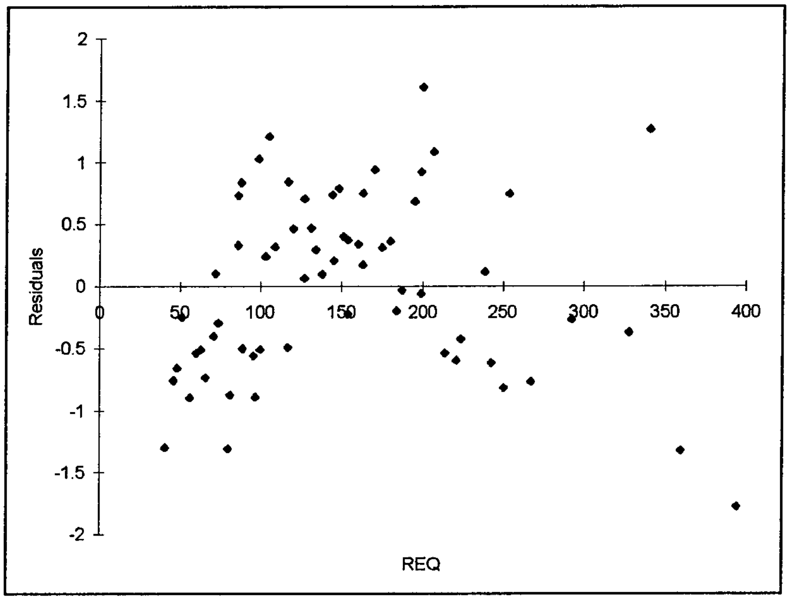

Figure 3.7 Residuals vs. REQ for Equation 3-3

transformation on the response values accommodates the skewness, resulting in a better-fitting model. A Box-Cox transformation analysis transformed the EBO response iteratively, seeking a transformed EBO response that resulted in the best fit for Equation 3-3 (Montgomery, 1991: 105). Figure 3.8 shows a square-root transformation as the best one for the given responses and model.

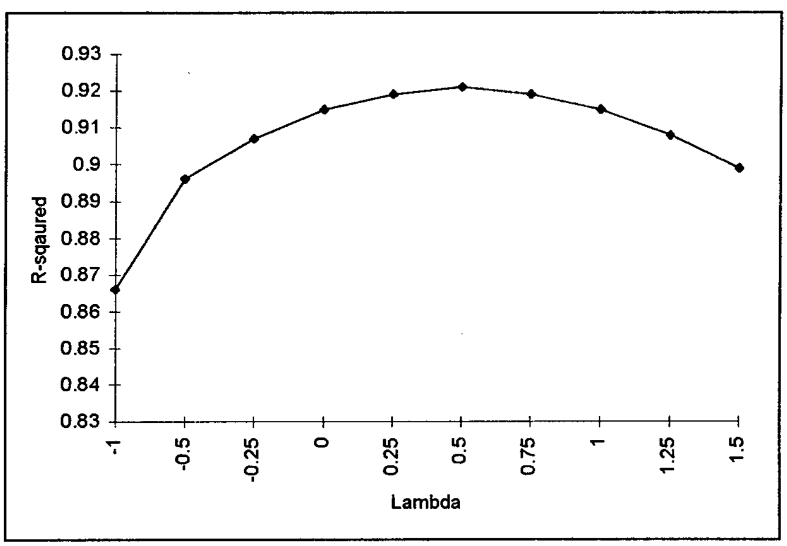

Figure 3.8 Box-Cox Transformation Results

The transformation was incorporated into the fourth model:

$$EBO^{1/2} = 2.6251 + 0.099\ FH + 0.1506\ DR - 0.2783\ BP - 0.092\ BRCT + 0.0013\ REQ \qquad (3\text{-}4)$$

Its R^2 equaled 0.922 and Figure 3.9 shows the transformation has eradicated the pattern. However, Figure 3.10 still shows a pattern similar to the one for Equation 3-3. A natural logarithm transformation was used to compress the scale of the REQ values. With this transformation of REQ, the Box-Cox

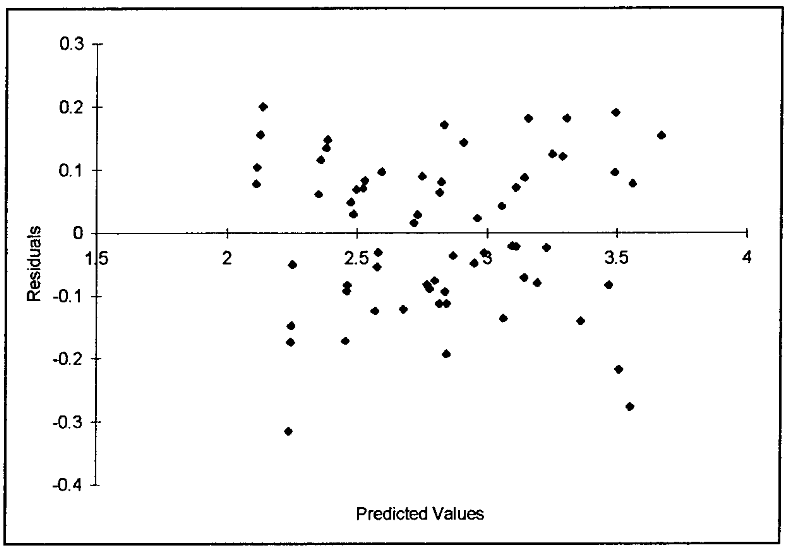

Figure 3.9 Residuals vs. Predicted Values for Equation 3-4

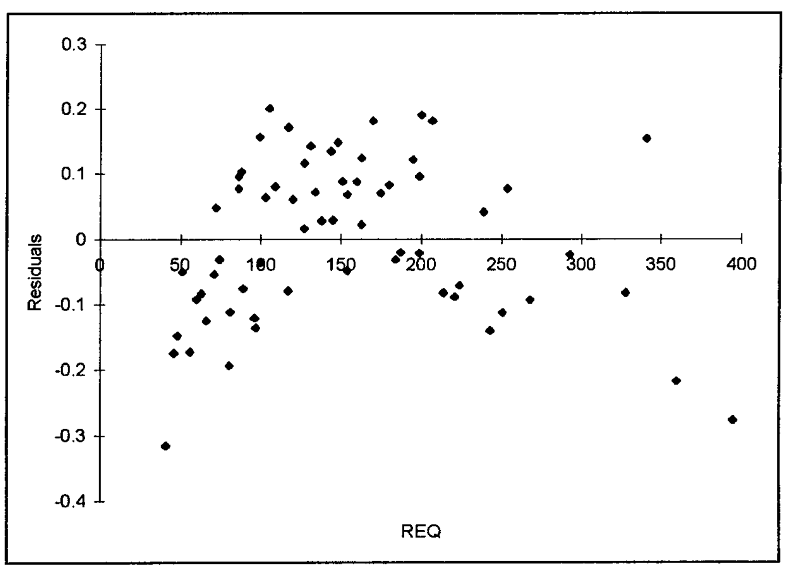

Figure 3.10 Residuals vs. REQ for Equation 3-4

transformation applied in Equation 3-4 is no longer valid. The fifth model was similar to Equation 3-3 except for the REQ transformation:

$$EBO = -7.214 + 0.1978\ FH + 0.2964\ DR - 0.9899\ BP - 1.1777\ BRCT + 3.1341\ \ln REQ \quad (3\text{-}5)$$

Its R^2 equaled 0.956 and Figure 3.11 shows a fair plot of the residuals. Figure 3.12 shows an elimination of the pattern that was present in Figures 3.7 and 3.10.

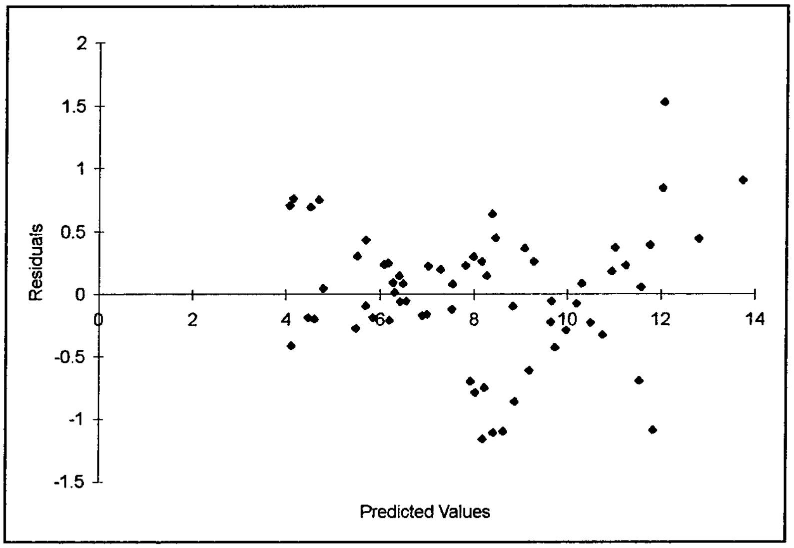

Figure 3.11 Residuals vs. Predicted Values for Equation 3-5

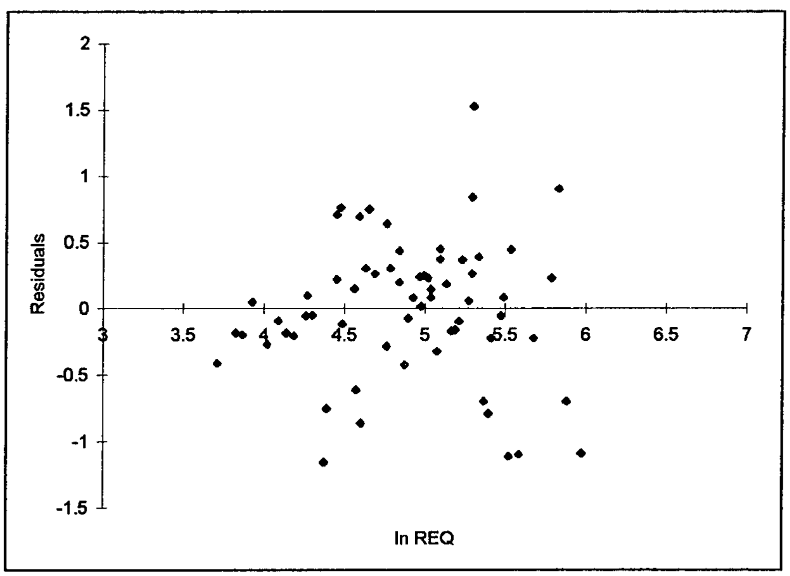

Figure 3.12 Residuals vs. ln REQ for Equation 3-5

Of the five models discussed in this section, Equations 3-1 and 3-5 are noteworthy. Between the two, Equation 3-1 is more parsimonious, offers a straightforward interpretation, and provides good explanatory capability. It shows FH, DR, and BP to be, collectively, the most influential on EBOs of the eight factors considered. FH and DR are positively correlated with EBO, and BP is negatively correlated.

Equation 3-5 offers the trade-off between a slightly more cumbersome model and a higher degree of explanatory capability. FH, DR, and BP have the same influences on EBO. The last factor in Equation 3-5, REQ, is positively correlated with EBO. Interestingly, BRCT is negatively correlated with EBO. Intuitively, it may not make sense that an increase in the amount of an item's repair time is correlated with a lower EBO value. Soon it is shown the interaction effects between BRCT and the other factors cause the negative sign. Interaction effects can be investigated with the first-order design output. Investigating such effects can reveal relationships among factors that the first-order models do not consider.

The presence of interactions means the response surface has curvature. Twenty-one two-way interactions were possible between the seven controllable factors. The ensuing analysis was not applicable to REQ for the same reasons its effect on EBO could not be derived like it was for the other factors. Appendix I presents the interactions. The MSE of the plotted average effects and the factors' scales may influence the degree of interaction present. An interaction effect exists if a factor's effect on the EBO depends on the level of another factor. For example, see Figure 3.12. Initially consider BRCT at the high level (+1). FH's effect on EBO as FH's level changes from -1 to +1 is approximately one. Now consider BRCT at the low level (-1). FH's effect on EBO as FH's level changes from -1 to +1 is approximately two. Therefore, the effect FH has on EBO depends on the level of BRCT: one if BRCT is high, two if BRCT is low. Other noteworthy interactions are depicted between BRCT and DR, BP, and OST in Figures I.12, I.15, and I.18, respectively.

The interactions between BRCT and the other factors in the model (FH, DR, BP) provides an explanation for the counterintuitive negative correlation between EBO and BRCT in Equation 3-5. In Equation 3-5, a low level of FH is correlated with a lower quantity of EBO. Reconsider Figure 3.12 and FH at its low level. With FH at its low level, the high level of BRCT corresponds to the lower EBO value.

Equation 3-5, a low level of FH is correlated with a lower quantity of EBO. Reconsider Figure 3.12 and FH at its low level. With FH at its low level, the high level of BRCT corresponds to the lower EBO value.

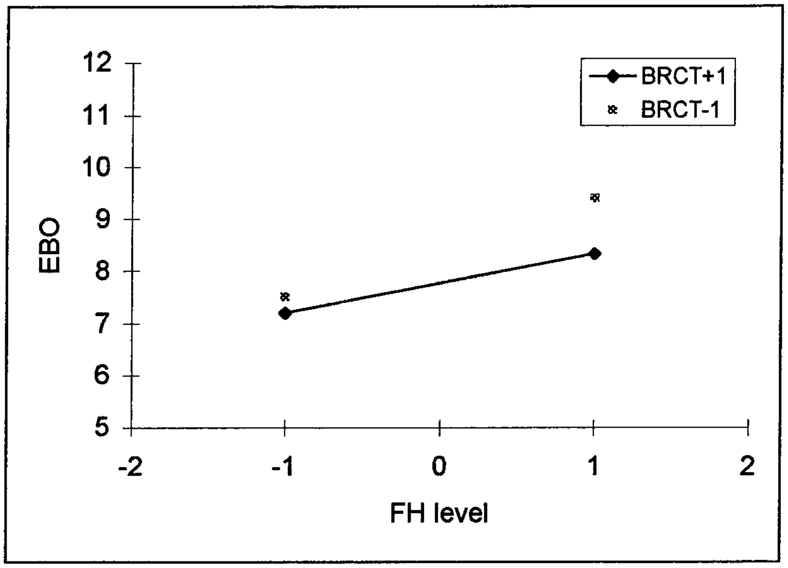

Figure 3.13 Interaction Between FH and BRCT

Since BRCT is negatively correlated with EBO, a high level of BRCT would certainly contribute to lower EBO as far as the regression is concerned. The same reasoning holds true between DR and BRCT in Figure I.12 in Appendix I. The reasoning does not hold up between BP and BRCT in Figure I.15, but the EBO values are so close anyway. Although the interactions illustrate the role of BRCT's negative sign, it does not resolve the counter-intuitiveness.

The presence of interactions suggest that the response surface of EBO exhibits a degree of curvature. Although the fitted models might be suitable empirical models of RBL, the addition of quadratic terms offers the possibility of improving their explanatory capability.

Second-Order Strategy

The original factorial design points were augmented in order to estimate quadratic terms. The result was a face-centered central composite design (FCCCD) (Montgomery, 1991:546). In addition to the original two-level factorial design, the FCCCD included a center point and two axial points for each factor. The three levels for each of the factors in the FCCCD were necessary for developing a quadratic model. One advantage of a FCCCD is that it "fits" within the design space of the original two-level design where the design settings are already at their extremes.

The FCCCD design is not rotatable because the axial points stay within the two-level design space. The number of center points determine the orthogonality of the augmented design. For a 2^{7-1}_{VII} design, 22 center points are required for orthogonality (Montgomery, 1991:546). With the METRIC model, multiple center points do not provide any new information: for a unique set of inputs, the same output occurs. Just one center point provides all the information. Table 3.2 summarizes the difference between a design with 22 center points and a design with one center point; all 14 axial points are included.

Table 3.2: Impact of Center Points on FCCCD

Design	R^2	Sqrt MSE	Response Mean	Coefficient of Variation (%)	R^2 for Linear Terms	R^2 for Quadratic Terms	R^2 for Interaction Terms
22 center pts.	0.9988	0.0955	8.4061	1.14	0.8981	0.0603	0.0404
1 center pt.	0.9988	0.1191	8.2503	1.44	0.9179	0.0396	0.0413

Table 3.2 shows an increase in MSE and variation as the number of center points decreased from 22 to one, but the amounts are minimal. The second-order strategy used a design with one center point because it made sense as far as the information it has to contribute, and the impact of the missing 21 center points is minimal.

Typically, center points are generated before axial points in order to determine whether or not there is curvature in the response surface (that is, to test for a quadratic effect) (Schmidt and Launsby, 1989:322). However, investigation of the interaction effects with the first-order design has already shown the presence of curvature. Therefore, the center and axial points were incorporated into the FCCCD simultaneously in order to estimate all second-order terms. The augmented design points and their corresponding RBL responses are shown in Appendix J.

Appendix K contains information for the 79 experiment's outputs similar to that shown in Appendix F. The mean and median values of REQ and EBO increased slightly, while the skewness increased for REQ and decreased for EBO (Table K.1). Table K.2 contains average response values for the center points. The statistics on each of the controllable factors at both the low and high values were very similar to those for the factorial data set. The correlation matrix in Table K.3 shows two notable relationships that were present in Table F.3 as well: BP and EBO (-0.74), REQ and EBO (0.70). The scatter plots in Figures K.3 through K.10 do not show any evidence of quadratic effect except for REQ in Figure K.10.

After conducting experiments specified in the design, a second-order model was sought. The SAS code is included in Appendix L. As was done for the first-order strategy, procedure RSQUARE was run on the 44 terms and its results are in Appendix M. Figures 3.14 and 3.15 plot the relationships between R^2 and MSE against the number of factors in a model. A three or four-factor model (certainly no more than five) appeared to be adequate in terms of explanatory capability. A plot of factor effects similar to Figure 3.3 cannot be shown because of the presence of REQ in all of the models except the one-factor model. Recall REQ is not orthogonal to the controllable factors, therefore parameter estimates may change as the models incorporating REQ change.

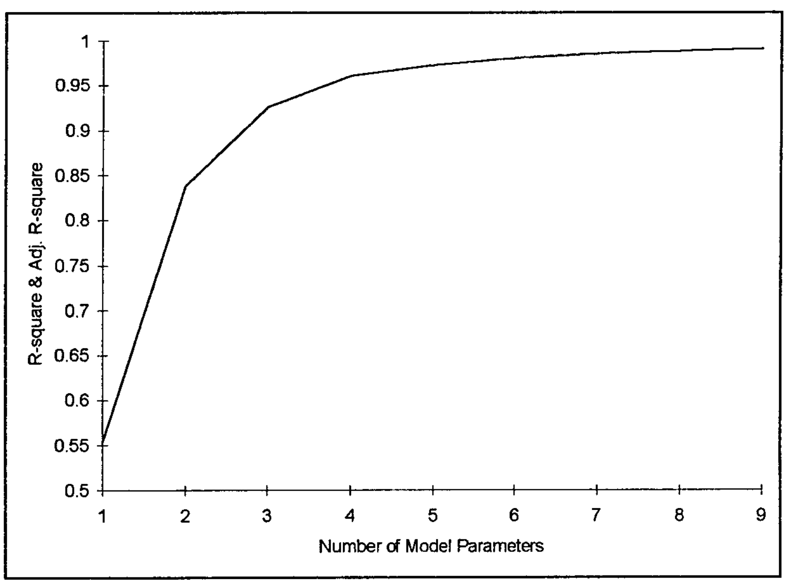

Figure 3.14 R-square vs. Number of Model Parameters

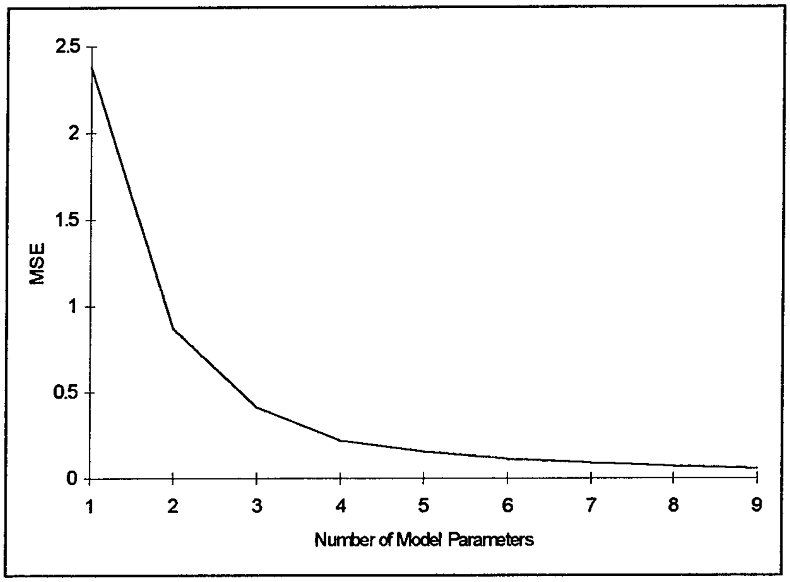

Figure 3.15 MSE vs. Number of Model Parameters

SAS procedure RSREG (response surface regression) generated a model with all linear, interaction, and quadratic terms: 44 terms total, not including the intercept (see Appendix N). The R^2 equaled 0.999. The possibility of meaningful three-way and higher interactions is practically nonexistent because the error sum of squares is almost zero. However, a 44-term model is not parsimonious.

Two second-order models were analyzed with three and four factors. They were generated by the RSQUARE procedure; these two models are the best three-factor and four-factor models in terms of R^2. The three-factor model was:

$$EBO = 4.8183 - 0.8805\ BP + 0.0239\ REQ - 0.008\ BRCT*REQ \qquad (3\text{-}6)$$

Its R^2 equaled 0.925 and Figure 3.16 shows no strong pattern. Figure 3.17 shows a curving pattern similar to the one experienced in the first-order models, suggesting a needed transformation.

The four-factor model was:

$$EBO = 2.535 + 0.0522\ REQ - 0.0053\ BP*REQ - 0.0077\ BRCT*REQ - 0.00008\ REQ^2 \qquad (3\text{-}7)$$

Its R^2 equaled 0.961 and Figures 3.18 and 3.19 show good scatter plots with an eradication of the curving pattern in the REQ plot.

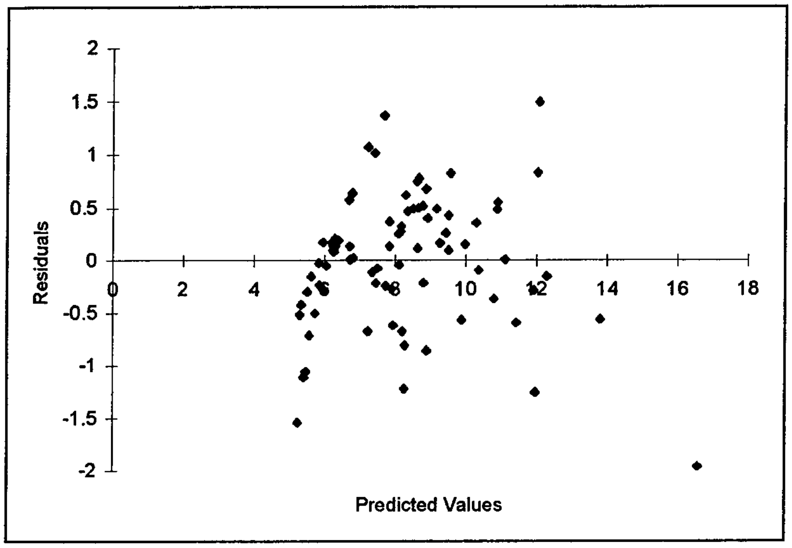

Figure 3.16 Residuals vs. Predicted Values for Equation 3-6

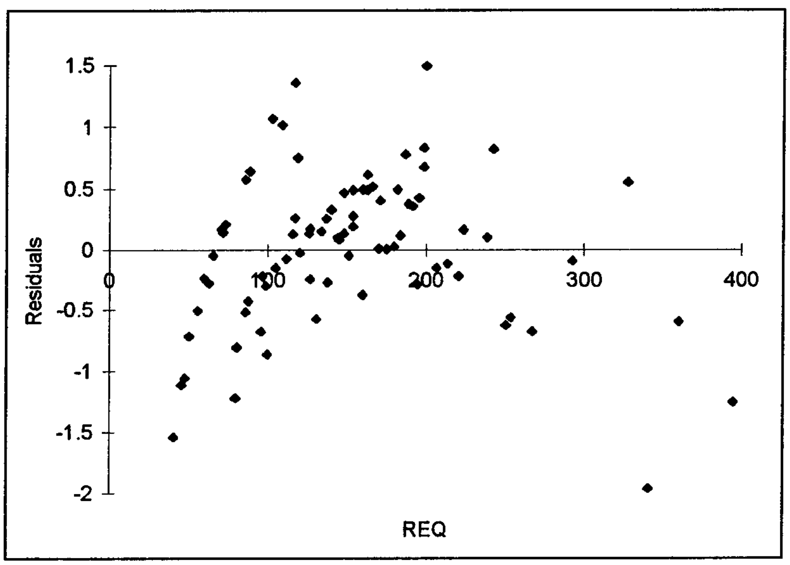

Figure 3.17 Residuals vs. REQ for Model 3-6

In comparing all seven models, a couple stand out. Equation 3-6 is more parsimonious than any other model except for Equation 3-1 however, for the same levels of parsimony, Equation 3-6 has a larger R^2 than Equation 3-1. Equation 3-7 has the highest R^2 plus it has one less factor than the model with the next highest R^2, Equation 3-5. The transformations in Equations 3-4 and 3-5 do not afford factor effect interpretations as straightforward as Equation 3-6, which has no transformations. Furthermore, models

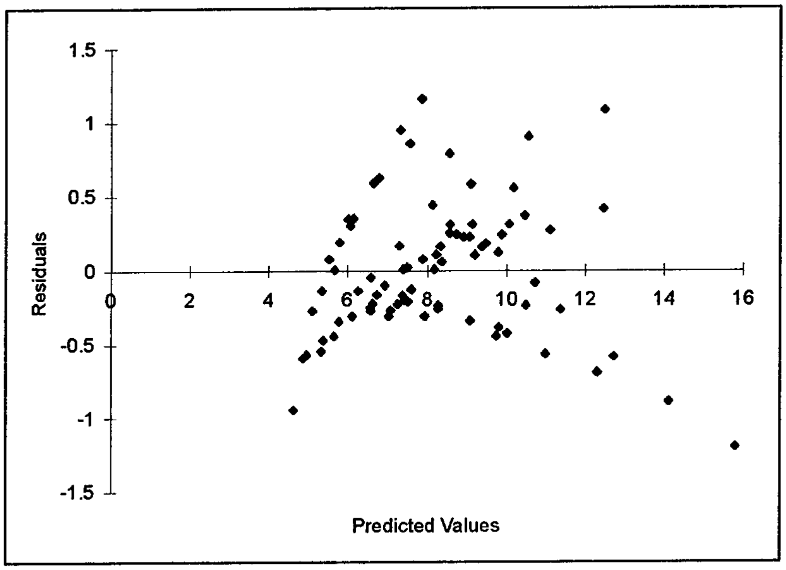

Figure 3.18 Residuals vs. Predicted Values for Equation 3-7

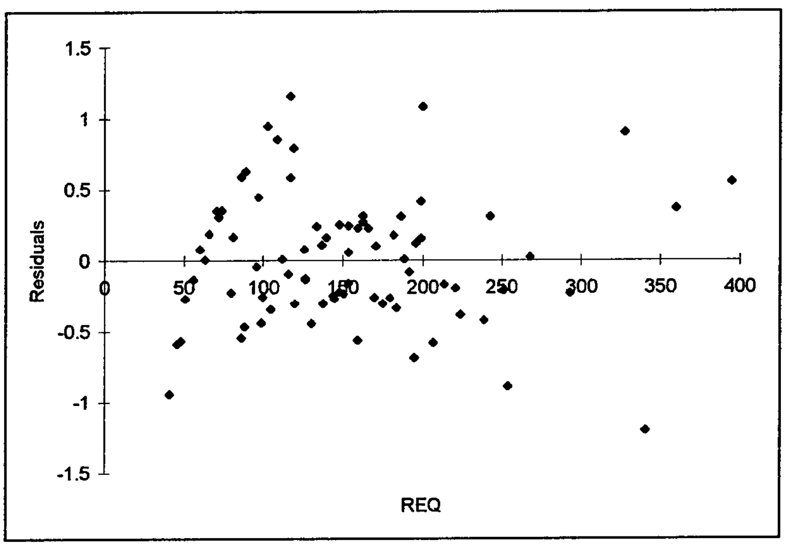

Figure 3.19 Residuals vs. REQ for Equation 3-7

without transformations are simpler to use because they do not require additional computation time to perform transformations. Although Equation 3-7 has a transformation, it can be easily interpreted as the main source of curvature in the RBL response surface. In light of this discussion on parsimony, explanatory capability, and interpretability, Equation 3-6 appears to have the best combination of these three model attributes.

A canonical analysis was performed in an effort to provide additional information on the surface curvature within the design region. Appendix O explains the development of the A canonical form shown here:

$$EBO = 9.8356 + 0.5046\ FH - 0.3908\ AP + 0.7903\ DR - 0.9595\ BP + 0.5559\ BRCT + 2.0464\ OST + 0.6450\ DRCT - 0.0117\ REQ + 0.2839\ FH^2 + 0.0473\ AP^2 - 0.0411\ DR^2 - 0.0592\ BP^2 - 0.1276\ BRCT^2 - 0.2647\ OST^2 - 0.4368\ DRCT^2 - 0.9197\ REQ^2 \quad (3.8)$$

Two points can be made from Equation 3-8. First, by virtue of its coefficient's magnitude, REQ has a large linear correlation with EBO. With an approximate mean of 153 in the second-order design, on average REQ affects EBO by approximately -1.7 backorders. Second and similarly, since REQ^2 's coefficient is the largest of the quadratic terms, requirement plays the largest role in shaping the response surface.

Interaction and quadratic effects have shown to be useful in developing models that are more explanatory than the first-order models while at the same time more parsimonious. However, the explanatory capability of the first and second-order models, as given in the R^2 statistic, only applies to the experimental data set. Using a verification data set provides a means for determining the flexibility of the model to explain behavior when new design points are considered.

Model Verification

The verification process proceeded in three steps:

1. Different factor levels were chosen and coded.
2. The coded levels were ran in RBL according to the second-order design.
3. A comparison was made between RBL's observed EBO responses and the empirical models' predicted responses.

The new factor levels were within the design space but not at levels used in the development of the first and second-order models. These design points tested a model's accuracy in areas of the response surface where no previous information (via RBL runs) existed. Table 3.3 shows the new factor levels for the controllable factors.

Table 3.3 Controllable Factors and Factor Levels: Model Verification

FACTOR	LEVEL Low	Center	High
1. Flying hours/day for each base (FH)	-18%	-5%	+10%
2. Application Percentage (AP)	-5%	current	+5%
3. Demand rate per 100 flying hours (DR)	-15%	+5%	+15%
4. Base processing Percentage (BP)	0.67	0.75	0.85
5. Base repair cycle time (BRCT)	5	6	8
6. Order and ship time (OST)	10	13	19
7. Depot repair cycle time (DRCT)	23	27	29

Three measures of empirical model performance were used. First, for each design point, the difference between the observed RBL output and an empirical model's predicted value was calculated, resulting in a prediction error. The ratio of this difference and the observed value provides a measure of error as a percentage of the observed value. Second, the predicted EBO values were plotted against the observed EBO values. A high correlation pattern indicated good model performance. Third, the stability of the empirical models' parameters were investigated. That is, for an existing model's particular set of factors, the model was refit to the verification data set. If the new model's parameter estimates were very similar to the original model, then confidence in the original model's explanatory capability is justified since basically the same model explained the response surface in different regions. These three measures were applied to all seven models previously developed.

The uncoded design points and RBL output are shown in Appendix P, Table P.1 for the factorial points, and Table P.2 for the center and axial points. Table 3.4 contains information for the verification runs' outputs similar to that shown in Appendices F and K. Compared to the second-order design, the new verification design points resulted in higher EBO and REQ values with almost half the standard deviations. REQ and EBO have less skew which is evident not only in their skew statistics but also in the close mean and median values. The correlation matrix in Table P.3 shows two notable relationships that were present in Tables F.3 and K.3 as well: BP and EBO (-0.657), REQ and EBO (0.736).

Table 3.4 REQ and EBO Distribution Statistics

	REQ	EBO
Mean	152.835	8.573
Median	152.000	8.618
Std. Dev.	37.313	1.174
Skewness	0.650	0.160
Kurtosis	0.608	-0.198

Once the verification runs were complete, the first of the three measures of empirical model performance, percentage of error, was calculated. Table 3.5 shows the average error percentage along with other performance statistics for all of the models. Model 3-7 sustained the least amount of error against the verification data set. The second measure of empirical model performance, correlation of EBO values, is also shown Table 3.5, and Model 3-7 had the highest value for this measure as well. Figure 3.20 shows the high degree of correlation of Model 3-7 graphically.

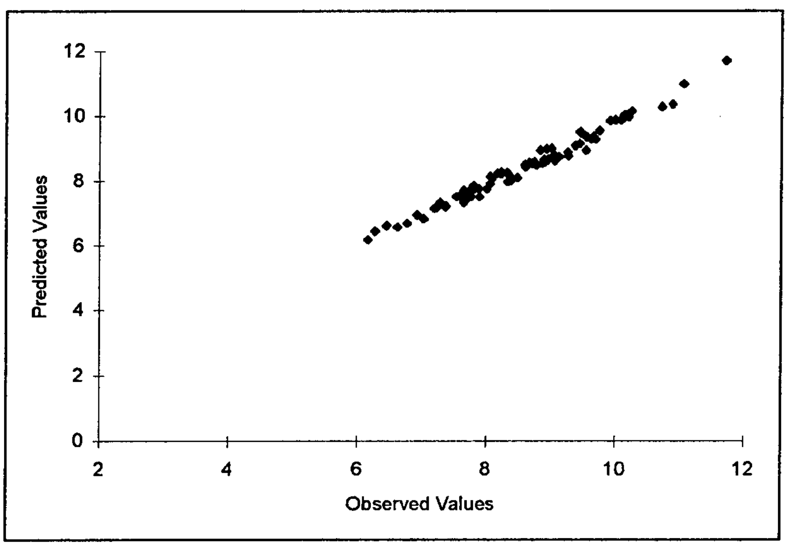

Figure 3.20 Predicted Values vs. Observed Values for Model 3-7

The third measure of empirical model performance was the stability of the models' parameters. Table 3.6 shows the two regression equations for each model: one for the original data set and one after refitting the model to the verification data set. None of the models' parameter signs changed between regressions on both data sets. The amount of change in the parameters' magnitudes was not large; Model 3-7's

Table 3.5 Statistics on All Models' Performances With Verification Data Set

				Verification Information						
							Error (%)		Residual Bounds	
Model	Form for Response	Factors Used	R-square from experimental design	R-square after fitting factors to new data	Correlation Between Observed vs. Predicted	Average error (%)	Minimum	Maximum	Minimum	Maximum
(3-1)	EBO	FH,DR,BP	0.874	0.862	0.925	6.0	5.10E-07	14.90	-6.10E-01	1.62
(3-2)	EBO	FH,BP,DR,AP,BRCT.OST	0.926	0.965	0.975	6.9	0.30	14.10	-0.03	1.21
(3-3)	EBO	FH,BP,DR,BRCT,REQ	0.915	0.952	0.967	6.3	0.10	14.10	-0.23	1.15
(3-4)	sqrt EBO	FH,BP,DR,BRCT,REQ	0.922	0.950	0.960	7.2	0.02	14.20	-0.09	1.31
(3-5)	EBO	FH,BP,DR,BRCT,ln REQ	0.956	0.977	0.988	3.2	0.09	7.00	-0.33	0.63
(3-6)	EBO	BP,REQ,BRCT*REQ	0.925	0.967	0.980	3.8	0.30	8.30	-0.44	0.79
(3-7)	EBO	REQ,BP*REQ, BRCT*REQ, REQ-squared	0.961	0.987	0.990	2.2	0.04	6.30	-0.16	0.60

Table 3.6 Stability of Models' Parameters

Model	Form for Response	Factors Used	Original Model with Experimental Data / Model of Same Factors Fit to Verification Data
(3-1)	EBO	FH,DR,BP	8.114 + 0.7569 FH + 1.1509 DR - 1.8519 BP 8.595 + 0.8185 FH + 1.2133 DR - 1.6704 BP
(3-2)	EBO	FH,BP,DR,AP,BRCT.OST	8.114 + 0.7569 FH + 0.3331 AP + 1.1509 DR -1.8519 BP - 0.3482 BRCT + 0.2965 OST 8.7674 + 0.8186 FH + 0.3688 AP + 1.2126 DR - 1.6702 BP - 0.5339 BRCT + 0.3856 OST
(3-3)	EBO	FH,BP,DR,BRCT,REQ	6.6010 + 0.4819 FH +0.7384 DR - 1.4758 BP - 0.6922 BRCT + 0.0099 REQ 6.0038 + 0.307 FH + 0.4734 DR - 1.0743 BP - 1.1168 BRCT + 0.0177 REQ
(3-4)	sqrt EBO	FH,BP,DR,BRCT,REQ	2.6251 + 0.099 FH + 0.1506 DR - 0.2783 BP - 0.092 BRCT + 0.0013 REQ 2.5106 + 0.0585 FH + 0.0906 DR - 0.1909 BP - 0.1824 BRCT + 0.0028 REQ
(3-5)	EBO	FH,BP,DR,BRCT,ln REQ	- 7.214 + 0.1978 FH + 0.2964 DR - 0.9899 BP - 1.1777 BRCT + 3.1341 ln REQ - 9.7725 + 0.1201 FH + 0.1899 DR - 0.8424 BP - 1.3455 BRCT + 3.6994 ln REQ
(3-6)	EBO	BP,REQ,BRCT*REQ	4.8183 - 0.8805 BP + 0.0239 REQ - 0.008 BRCT*REQ 4.4038 - 0.7369 BP + 0.0287 REQ - 0.0095 BRCT*REQ
(3-7)	EBO	REQ,BP*REQ, BRCT*REQ, REQ-squared	2.535 + 0.0522 REQ - 0.0053 BP*REQ - 0.0077 BRCT*REQ - 0.00008 REQ-squared 2.0049 + 0.0599 REQ - 0.0048 BP*REQ - 0.0092 BRCT*REQ - 0.0001 REQ-squared

parameters are the most stable. Since the parameters' values did not change by a large amount, confidence can be placed in Model 3-7's original parameter estimates. Model 3-7's performance should not unduly overshadow the other models' performances; all models performed well.

Summary

This chapter has shown how response surface methodology can be applied to characterize a response function for the RBL technique. Several empirical representations of the response surface were developed. The confidence in these representations ranged from fairly high to very high. By exploring several models with different levels of parsimony, the influences of different factors on EBO could be investigated. The high explanatory capability (R^2), which all models possessed, substantiated the influences of factors within that model.

The *method* of applying RSM is emphasized since the scope of this research precluded the use of the computationally intensive D041 model. Therefore, any statements made about factor influences must be taken in the context of first, a notional D041 requirement input and, second, the design space defined by the extreme factor levels of the first-order design. Otherwise, the steps taken, analyses conducted, and interpretations made in this study are typical of a classic RSM study of RBL.

IV. Conclusions

The purpose of this research was to demonstrate that response surface methodology (RSM) could be used to explore possible relationships between some of the factors affecting readiness-based leveling (RBL) and its output response, expected backorders (EBO). Empirical models were developed based on statistics of goodness of fit, such as R^2 and MSE, and an analysis of the residuals. By virtue of the models developed and their level of explanatory capability, the demonstration is complete. Throughout this research, two issues related to developing and interpreting the response surface models consistently emerged.

First, the use of the Recoverable Item Requirements System (D041) model to generate the spares requirement was unwarranted for the scope and purpose of this research. Whereas D041 prioritizes each individual item, down to the second-indenture level, in the order of its marginal contribution to aircraft availability, this research focused on one first-indentured item. Unfortunately, in practice D041's output is the RBL input. Since a substitute D041 input was used, any specific conclusions concerning the effect of the factors investigated on EBO are somewhat suspect. Not until the methodology demonstrated in this paper is applied with data used in practice can valid conclusions regarding those relationships be made.

Second, the model at the heart of RBL, Multi-Echelon Technique for Recoverable Item Control (METRIC), exhibits a "deterministic" behavior if it is viewed as a "black box;" a given set of inputs always yields the same output. Because of this behavior, replications of any designs points served no purpose. Usually, replications are performed in order to estimate pure error in a process, but METRIC does not have pure error. Hence, all of the models' prediction error is due to lack of fit.

The behavior of all seven models in this study was such that no new interpretations of factor influences had to be made between one model and another. Provided the models develop in a similar manner using the D041 input, the insight gained will have a degree of commonality among the models. If the models do not develop in such a manner, determining the "best" model to represent the process in question will require more thought. It will be this model which supports conclusions about factor influences in RBL.

V. Considerations for Further Study

As response surface methodology (RSM) is by nature an iterative process, the line of research begun in this paper is by no means complete. This chapter will highlight new perspectives for further RSM analysis on Readiness-Based Leveling (RBL).

The first new perspective would address the fact that a substitute D041 requirement was used as an RBL input. Because the actual D041 requirement values were not incorporated in this study, definitive conclusions concerning the effect of the factors investigated on expected backorders (EBO) were not made. Certain factors were highlighted as having large effects on RBL's output, but only for the sake of demonstrating the kinds of observations that could be made in an RSM study. Thus, RSM should be applied to the RBL technique with the D041 requirement values incorporated. If the factor effects happen to be similar to those shown in this study, then the models developed in this research gain more credibility. Furthermore, a comparison could be made between the requirements generated in this study and the requirements generated by the D041 model; high correlation would tend to validate the requirement calculations performed in this study.

The second new perspective would consider applying RSM to other resource-leveling techniques. There are other ways to allocate reparable items between the bases and the depot; for instance, one thought is to keep a certain or minimal amount of stock at the depot with the remainder allocated between the bases. If this leveling technique was analyzed, then the resulting factor influences could be compared to the analysis described in the preceding paragraph.

The third new perspective would attempt to modify the design region under study by extending the extreme factor level values. This modification should not be done without sufficient rationale. For instance, the item used in this study had a specified demand rate, and a degree of change below and above this demand rate denoted the extreme factor levels. To widen the difference between the extreme values for this item, just to modify the design space, should not be done unless considered realistic. There are other ways to modify the design region. For example, all data in this study applied to bases and units flying the F-16 aircraft. If other weapon systems were considered, each might have a unique response surface. Other examples include the consideration of second-indentured items or items that belong to different weapon subsystems.

The fourth new perspective would consider base and depot safety levels as controllable factors. These two factors would be in addition to the seven controllable factors and one uncontrollable factor included in this study. Thus, the two major components of the requirement calculation, pipelines and safety levels, are directly controllable within the experimental environment. Incorporating safety level factors would allow an analysis of their influence on expected backorders. For example, if it is believed that the depot repair pipeline could be controlled in the field and decreased, then less items would be required in that pipeline. A certain number of items would then become available as safety stock. Theoretically, increasing the safety stock should result in better aircraft availability. In the context of an empirical model incorporating the pipelines and safety levels of interest, the changes in pipelines and safety levels could be imputed into the model and a predicted expected backorder quantity computed.

Whichever new perspective(s) may be taken, the validity of applying RSM to RBL remains and the result is a response surface reflecting how inputs are affecting an output. The "how" is manifested in the resulting empirical model. The "why" behind relationships between inputs and output cannot be garnered from an empirical model. Instead, such a question is answered through a mechanistic model: one that reflects the true relationships and is incapable of error. Natural laws are such models. Even though an empirical model is not the mechanistic model, such modeling presents a structured, systematic approach to learning more about a process; in this case, readiness-based leveling of reparable items.

Appendix A

National Stock Number Selection

Table A.1 shows the ranking by national stock number (NSN) of 66 F-16 line-replaceable units against three criteria: expected base backorders (EBO), requirement (REQ), demand rate (DR). The table continues on the next page. On the third page, Figure A.1 explains the method for choosing an NSN.

Table A.1 NSN Ranking on Three Criteria

Rank	NSN	EBO	NSN	REQ	NSN	DR
1	5985012122950WF	13.3252	5985012122950WF	168	1280012804855WF	1.2464
2	1270013590877WF	10.2277	1270012383662WF	147	1270011022965WF	1.0536
3	5895011420803WF	9.5443	1270013966750WF	135	1270013966750WF	1.0026
4	1270013851879WF	9.481	1270012330011WF	115	1270013590877WF	0.9111
5	6625011938861WF	8.8804	5945011709363WF	115	1270011022966WF	0.8544
6	1270992512706WF	8.5395	1270992512706WF	95	5895012489012WF	0.8
7	1270013333608WF	8.3767	6615013619746WF	84	1270011022962WF	0.7663
8	1270012383662WF	8.0889	5895012489012WF	83	6605010876645WF	0.7405
9	1270013966750WF	7.6478	1260012511150WF	76	1270011022963WF	0.7375
10	1270011022965WF	7.5499	1290012279260WF	67	5895011420803WF	0.6342
11	6605993708249WF	6.6578	1270012352370WF	66	5895013558414WF	0.6255
12	1270011022966WF	6.5061	6605010876645WF	61	1270012223829WF	0.5596
13	1270010932256WF	6.482	6625011938861WF	58	1270992512706WF	0.5528
14	1260013510592WF	6.2798	5895011420803WF	51	1270013093077WF	0.431
15	1270011022963WF	5.823	1270992255327WF	47	1270012122990WF	0.4205
16	1270013093077WF	5.6941	1270013093077WF	43	1290012279260WF	0.4191
17	1290012279260WF	4.9667	5999010803978WF	43	1270012352370WF	0.4186
18	1260012511150WF	4.5987	6610013081859WF	43	1270992255327WF	0.4172
19	5985013083647WF	4.4767	5998013227746WF	40	5985012935451WF	0.4019
20	5998013309073WF	4.2984	1260013510592WF	39	1270010932256WF	0.388
21	1290013406317WF	4.0771	1290013765449WF	39	1290013406317WF	0.3832
22	6610013081859WF	3.8963	6615011273160WF	38	6605993708249WF	0.3733
23	1290013765449WF	3.8878	1270013333608WF	37	5985013083647WF	0.3348
24	5895013558414WF	3.5301	5895011435443WF	37	1270013851879WF	0.3335
25	1270992255327WF	3.4794	6605993708249WF	37	6615013619746WF	0.3311
26	1270012122990WF	3.142	6615013517337WF	36	6605010463533WF	0.3292
27	1270012827914WF	3.0281	1270013590877WF	35	1270012383662WF	0.3178
28	5985012935451WF	3.0071	1290013223711WF	34	6615013566851WF	0.3084
29	1270011022962WF	2.9835	6605010463533WF	34	1290013223711WF	0.3078
30	1270012352370WF	2.8766	5895012301075WF	33	1290013765449WF	0.3078
31	1270012223829WF	2.8253	1270013851879WF	31	1260013510592WF	0.2957
32	5841010963945WF	2.8195	5895013558414WF	31	1270013333608WF	0.2875
33	1290013223711WF	2.8177	1270010932256WF	30	1270011336494WF	0.2873
34	5999010803978WF	2.7098	6615013246374WF	30	1260012511150WF	0.2635
35	6605010463533WF	2.467	5841010963945WF	29	5998011230046WF	0.2627
36	5841010964833WF	2.3508	6615010427834WF	29	1270012827914WF	0.2512
37	6615013246374WF	2.3282	5841010964833WF	28	1270010453976WF	0.2426

Table A.1 (continued)

Rank	NSN	EBO	NSN	REQ	NSN	DR
38	5895012301075WF	2.1919	5998013309073WF	28	6615013528570WF	0.2375
39	5998011230046WF	2.1604	1270012122990WF	26	5985012122950WF	0.2305
40	1280012804855WF	2.0456	5985013164588WF	26	5841010964833WF	0.2167
41	6615011273160WF	2.0357	6615011496398WF	26	5998013309073WF	0.2146
42	5998013227746WF	2.0147	6615011297445WF	24	5841010963945WF	0.2127
43	5895012489012WF	1.9696	1270011022962WF	22	1270012330011WF	0.1721
44	5895011435443WF	1.9245	1290013406317WF	21	6615013246374WF	0.1706
45	6615010427834WF	1.8722	1270012223829WF	19	5895012301075WF	0.1511
46	6605010876645WF	1.7211	1270011022966WF	17	5985013164588WF	0.135
47	1270011336494WF	1.6615	5998011230046WF	17	6615011273160WF	0.1196
48	1270010946872WF	1.5834	1270011022965WF	16	6610013081859WF	0.1175
49	6615013566851WF	1.4984	1270012827914WF	16	6610013728170WF	0.1175
50	1280011216879WF	1.4733	6610010397817WF	16	1290010800203WF	0.0877
51	1270012330011WF	1.3265	5985012935451WF	14	6625011938861WF	0.0816
52	6615013619746WF	1.2779	1290010800203WF	13	5895011435443WF	0.0727
53	1290010800203WF	1.264	1270011022963WF	12	6615013517337WF	0.0723
54	6615011297445WF	1.053	5821010771313WF	12	1280011216879WF	0.0721
55	5821010771313WF	0.9839	6615013566851WF	12	5985013164589WF	0.054
56	6610010397817WF	0.9065	1280011216879WF	11	1270010946872WF	0.0522
57	5985013164588WF	0.8477	5985013083647WF	11	5999010803978WF	0.0502
58	1270010453976WF	0.8396	6610013728170WF	10	5821010771313WF	0.0496
59	6615013528570WF	0.7655	6625011146771WF	10	5945011709363WF	0.0443
60	6615013517337WF	0.7344	1280012804855WF	9	6615011297445WF	0.0342
61	6615011496398WF	0.6773	1290013041615WF	9	5998013227746WF	0.0243
62	5985013164589WF	0.5741	1270011336494WF	8	1290013041615WF	0.0234
63	6625011146771WF	0.2753	5985013164589WF	8	6615011496398WF	0.0214
64	1290013041615WF	0.2074	6615013528570WF	7	6610010397817WF	0.0202
65	6610013728170WF	0.0959	1270010946872WF	6	6615010427834WF	0.0158
66	5945011709363WF	0.059	1270010453976WF	3	6625011146771WF	0.0089

Criteria Choose an item with high activity

- High expected base backorders
- A requirement greater than 50 units
- A demand rate greater than .5

Ranks = (EBO rank) + (REQ rank) + (DR rank)

Choose items with a low total rank

Meet Criteria

NSN	Ranks
1270013966750WF	9+3+3=15
1270992512706WF	7+6+13=26
5895011420803WF	4+14+10=28
5895012489012WF	43+8+6=57
6605010876645WF	46+12+8=66

Do Not Meet Criteria

NSN	Ranks
1270013590877WF	2+27+4=33
1270012383662WF	8+2+27=37
5985012122950WF	1+1+39=41
1270011022965WF	10+48+2=60
1270011022966WF	12+46+5=63
6625011938861WF	5+13+51=69

Figure A.1 Method for Choosing an Active Item

Appendix B

AFMC/SAO Data Set

There are three data sets in this appendix: scenario data, item data, requirements data. AFMC/SAO assembled the data sets in order to conduct an assessment of the effects of depot stocking strategies on expected base backorders ("Setting," 1995:unnumbered). The data sets encompass 66 items (LRUs) on more than 1,400 F-16/A,B,C,D aircraft stationed at 55 installations around the world. They include data pertaining to flying rates, item management, and requirements. Descriptions of the data set terms can be found in the List of Terms. The data categories and specific fields are portrayed in Table B.1.

Table B.1 Data Set Descriptions

Scenario data:	Flying hours per day per base (FH)
Item data:	Demand rate per 100 flying hours (DR)
	Base Processing Percentage (BP)
	Quantity per Application (QPA)
	Application Percentage (AP)
Requirements data:	Order and Ship Time (OST)
	Base Repair and Cycle Time (BRCT)
	Depot Repair Cycle Time (DRCT)
	OST Pipeline Quantity (OSTQ)
	BRCT Pipeline Quantity (BRCTQ)
	Base Safety Level Quantity (BSL)
	DRCT Pipeline Quantity (DRCTQ)
	Requirements (REQ)

The scenario and item data sets come from AFMC databases which collect information on operations. The same holds true for the OST, BRCT, and DRCT fields in the requirements data set. The D041 produced the remaining fields of the requirements data set. The primary purpose of the D041 is similar to RBL. The D041 essentially makes a prioritized "shopping list" of items by which the highest priority item will make the largest marginal increase in aircraft availability. Given a requirement of a certain item, RBL allocates the items between the bases and the depot in such a way that minimizes the number of expected base backorders.

Appendix C

Substitute Method for Providing RBL the D041 Requirement Input

Requirement

The full requirement for an item consists of pipeline quantities and safety levels. The requirement (REQ) is calculated as follows:

$$REQ = BRCTQ + OSTQ + BSL + DRCTQ + DSL$$

Pipeline Equations

The pipeline quantities for an item are computed in a straightforward fashion, summing over all bases that use the item:

The Base Repair Cycle Pipeline Quantity (BRCTQ):

$$BRCTQ = \sum_{i=1}^{m} FH_i * DR_i * QPA_i * AP_i * BP_i * BRCT_i$$

The Order and Ship Time Pipeline Quantity (OSTQ):

$$OSTQ = \sum_{i=1}^{m} FH_i * DR_i * QPA_i * AP_i * (1 - BP_i) * OST_i$$

The Depot Repair Cycle Pipeline Quantity (DRCTQ):

$$DRCTQ = \sum_{i=1}^{m} FH_i * DR_i * QPA_i * AP_i * (1 - BP_i) * DRCT_i$$

These quantities are computed for each item (this study only covered one item). Incorporated within the pipeline quantity equations are the following:

The Item Flying Hours per Day (Item FH):

$$\text{Item FH} = \sum_{i=1}^{m} FH_i * QPA_i * AP_i$$

The Daily Demand Rate (DDR):

$$DDR = \sum_{i=1}^{m} \text{Item FH}_i * DR_i$$

Incorporating base processing percentage into DDR emphasizes items either staying at the base or going to the depot, depending on the pipeline equation. Finally, incorporating the pipeline time results in the number of items that are in the pipeline at any point in time.

<u>Safety Level Regression</u>

The base and depot safety levels are two remaining quantities necessary to compute a requirement. They attempt to account for variability in their respective pipelines (see List of Terms for base pipeline and depot pipeline). A simple way to account for these two quantities was to develop a regression accounting for the total safety level (TSL) as a function of the pipelines:

$$TSL\{BSL+DSL\} = f(OSTQ,BRCTQ,DRCTQ)$$

DRCTQ was not significant in the first regression. The results of the final regression are included in Figure C.1. The regression equation used for determining the total safety level:

$$TSL = 13.572 + 1.023\ OSTQ + 1.866\ BRCTQ \qquad \text{(C-1)}$$

There is another type of stock called the special level quantity (see List of Terms). Due to its heavy dependence on a base's special circumstances, AFMC/SAO suggested it not be included.

The impact of Equation C-1 on the requirement quantity was assessed. If Equation 3-1's prediction error was small relative to the requirement quantity, then adverse effects on the requirement quantity would be minimal. However, the prediction error was large. Calculations were based on D041 output data containing pipeline and safety level information for the 66 items shown in Appendix A. Figure C.2 plots the correlation between the observed total safety level (BSL + DSL) and the predicted safety level (Equation 3-1). Because the plotted points are not tight and following the observed values, the plot graphically substantiates the low R^2 value in Figure C.1. Table C.1 presents statistics on the impact

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.76629
R Square	0.58720
Adjusted R Squar	0.57409
Standard Error	15.42067
Observations	66

ANOVA

	df	SS	MS	F
Regression	2	21310.05851	10655.03	44.80724
Residual	63	14981.21422	237.7971	
Total	65	36291.27273		

	Coefficients	Standard Error	t Stat	P-value
Intercept	13.57229	2.60195	5.21619	2.16E-06
OSTQ	1.02303	0.26860	3.80873	0.000319
BRCTQ	1.86593	0.26250	7.10828	1.29E-09

Figure C.1 Safety Level Regression Results

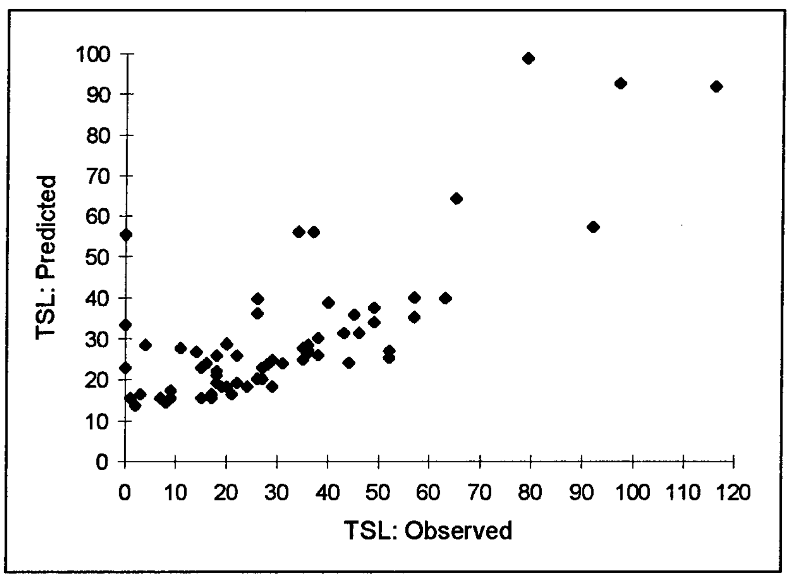

Figure C.2 Predicted TSL vs. Observed TSL

of Equation 3-1's predicted safety level values. The errors are substantial, and because the TSL can make up a large proportion, if not all, of the requirement, the large prediction errors have a large adverse effect on REQ.

Table C.1 Statistics on Impact of TSL Prediction

	TSL as a Percentage of REQ (Observed values)	TSL Prediction Error: Resid. / Observed TSL (Absolute Value: %)	TSL Prediction Error as a Percentage of Observed REQ
Average	57	39.8	43.0
Minimum	0	1.1	0.7
Maximum	100	104	579

Appendix D

Factor Levels

AFMC/SAO suggested all of the factor levels. No analysis per se was responsible for the chosen levels. The ranges over which the factors varied come from a combination of AFMC/SAO logistic analyst experience, Air Staff goals, and existing maximums used in supply models.

The variations for FH, AP, DR, and BP are based on experience. The low levels for BRCT, OST, and DRCT incorporate Air Staff targets, namely: BRCT (3), OST (11), DRCT (80% of current value). For the NSN used in this study, the target had already been surpassed for OST, so the current level served as its low OST level. The high levels for BRCT and OST incorporate Standard Base Supply System (SBSS) caps, or limits. The limits were 10 and 24 for BRCT and OST, respectively. However, since the data set described in Appendix B used integer values for the pipelines, the limits were reduced by one [BRCT (9), OST (23)] in order to provide an integer center point for each of them. The high level for DRCT was chosen to be the same percentage deviation from the current level as the Air Staff target, that is, a 20% increase. The Air Staff targets and SBSS caps apply to all items, not just the one used in this research.

Appendix E

First-Order Design and Example Input

Table E.1 shows the 64 coded design points and RBL output for the first-order strategy.

Table E.1 First-Order Design and RBL Output

FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
-1	-1	-1	-1	-1	-1	1	80	7.0322
1	-1	-1	-1	-1	-1	-1	97	8.5783
-1	1	-1	-1	-1	-1	-1	81	7.4781
1	1	-1	-1	-1	-1	1	134	10.1305
-1	-1	1	-1	-1	-1	-1	117	9.6924
1	-1	1	-1	-1	-1	1	199	12.8787
-1	1	1	-1	-1	-1	1	163	11.3879
1	1	1	-1	-1	-1	-1	200	13.5777
-1	-1	-1	1	-1	-1	-1	41	3.7057
1	-1	-1	1	-1	-1	1	56	5.22
-1	1	-1	1	-1	-1	1	48	4.4142
1	1	-1	1	-1	-1	-1	63	5.6758
-1	-1	1	1	-1	-1	1	66	5.9942
1	-1	1	1	-1	-1	-1	89	7.4172
-1	1	1	1	-1	-1	-1	74	6.5089
1	1	1	1	-1	-1	1	109	8.4329
-1	-1	-1	-1	1	-1	-1	96	6.5563
1	-1	-1	-1	1	-1	1	154	8.4289
-1	1	-1	-1	1	-1	1	127	7.4838
1	1	-1	-1	1	-1	-1	163	8.9119
-1	-1	1	-1	1	-1	1	187	9.4561
1	-1	1	-1	1	-1	-1	243	10.3921
-1	1	1	-1	1	-1	-1	199	9.546
1	1	1	-1	1	-1	1	328	11.4766
-1	-1	-1	1	1	-1	1	86	4.7928
1	-1	-1	1	1	-1	-1	120	5.8261
-1	1	-1	1	1	-1	-1	99	5.2184
1	1	-1	1	1	-1	1	144	6.3359
-1	-1	1	1	1	-1	-1	145	6.3299
1	-1	1	1	1	-1	1	214	7.2313
-1	1	1	1	1	-1	1	175	6.7314
1	1	1	1	1	-1	-1	251	7.3185
-1	-1	-1	-1	-1	1	-1	100	8.0286
1	-1	-1	-1	-1	1	1	160	10.4347
-1	1	-1	-1	-1	1	1	131	9.3121
1	1	-1	-1	-1	1	-1	170	11.1347
-1	-1	1	-1	-1	1	1	195	11.6355
1	-1	1	-1	-1	1	-1	254	13.2452
-1	1	1	-1	-1	1	-1	207	12.1556

Table E.1 (continued)

FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
1	1	1	-1	-1	1	1	341	14.6264
-1	-1	-1	1	-1	1	1	46	4.2952
1	-1	-1	1	-1	1	-1	60	5.6152
-1	1	-1	1	-1	1	-1	51	4.8517
1	1	-1	1	-1	1	1	72	6.3742
-1	-1	1	1	-1	1	-1	71	6.3746
1	-1	1	1	-1	1	1	103	8.2945
-1	1	1	1	-1	1	1	86	7.2552
1	1	1	1	-1	1	-1	117	9.0331
-1	-1	-1	-1	1	1	1	138	7.6271
1	-1	-1	-1	1	1	-1	184	8.7465
-1	1	-1	-1	1	1	-1	151	8.0587
1	1	-1	-1	1	1	1	239	9.6065
-1	-1	1	-1	1	1	-1	224	9.4337
1	-1	1	-1	1	1	1	360	10.8426
-1	1	1	-1	1	1	1	293	10.2733
1	1	1	-1	1	1	-1	395	10.7383
-1	-1	-1	1	1	1	-1	88	4.9195
1	-1	-1	1	1	1	1	127	6.1358
-1	1	-1	1	1	1	1	105	5.4592
1	1	-1	1	1	1	-1	148	6.4285
-1	-1	1	1	1	1	1	154	6.5861
1	-1	1	1	1	1	-1	221	7.2424
-1	1	1	1	1	1	-1	180	6.8322
1	1	1	1	1	1	1	268	7.5371

Table E.2 shows the first design point, uncoded, for six of the 45 bases that use the item. The settings in this table correspond to the first design point in Table E.1. "Users" field is the number of bases that use the item. "NCT" is the condemnation rate which was not considered in this study. "C-factor" denotes whether the base is situated overseas or in the continental United States and also was not studied.

Table E.2 Partial Uncoded Design Point for RBL Input

NSN		Users	DRCT		REQ	
1270013966750		45	30		80	
Base	DDR	BP	BRCT	NCT	OST	C-factor
FB2027	0.21526	0.59	3.00	0.00	9	1.00
FB2805	0.05118	0.59	3.00	0.00	9	1.00
FB2823	0.04002	0.59	3.00	0.00	9	1.00
FB4488	0.04511	0.59	3.00	0.00	9	1.00
FB4803	0.12855	0.59	3.00	0.00	9	1.00
FB4823	0.07495	0.59	3.00	0.00	9	1.00

Appendix F

Experiment Output for First-Order Strategy

Table F.1 REQ and EBO Distribution Statistics

	REQ	EBO
Mean	152.9219	8.1140
Median	141.0000	7.5105
Std. Dev.	81.2109	2.4887
Skewness	0.9690	0.5549
Kurtosis	0.6840	-0.2415

Table F.2 EBO Values for High and Low Factor Levels

	FH+	AP+	DR+	BP +	BRCT+	OST+	DRCT+
Mean	8.8709	8.4470	9.2649	6.2621	7.7657	8.4104	8.2413
Std. Dev.	2.5315	2.5558	2.4482	1.2429	1.8581	2.5554	2.5621
	FH-	AP-	DR-	BP -	BRCT-	OST-	DRCT-
Mean	7.3571	7.7809	6.9630	9.9658	8.4622	7.8175	7.9866
Std. Dev.	2.2351	2.4137	1.9642	1.9905	2.9806	2.4239	2.4472

Table F.3 Correlation Matrix

	FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
FH	1								
AP	0	1							
DR	0	0	1						
BP	0	0	0	1					
BRCT	0	0	0	0	1				
OST	0	0	0	0	0	1			
DRCT	0	0	0	0	0	0	1		
REQ	0.345	0.1623	0.5176	-0.472	0.4315	0.2116	0.0754	1	
EBO	0.3065	0.1349	0.4661	-0.75	-0.141	0.1201	0.0516	0.706	1

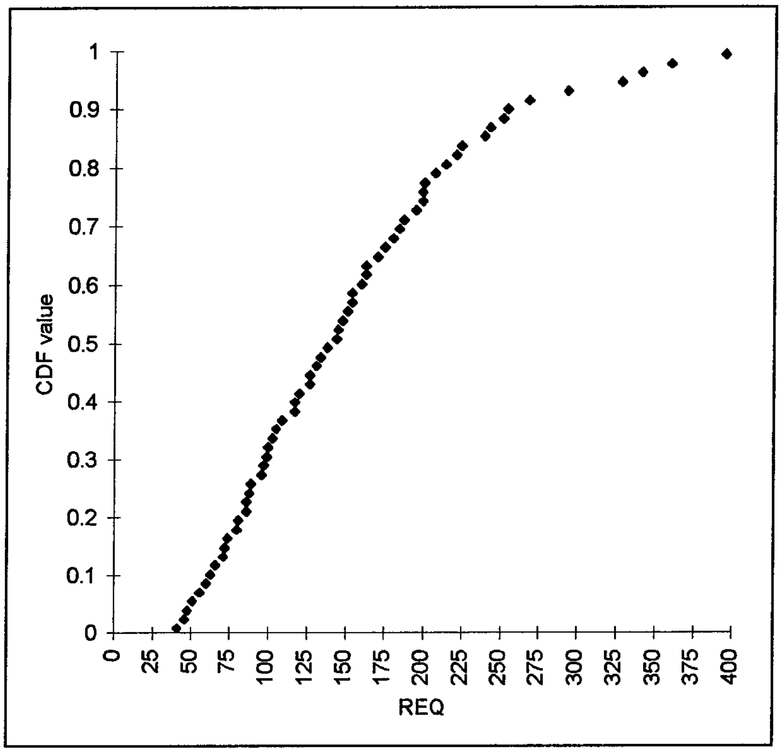

Figure F.1 CDF of REQ

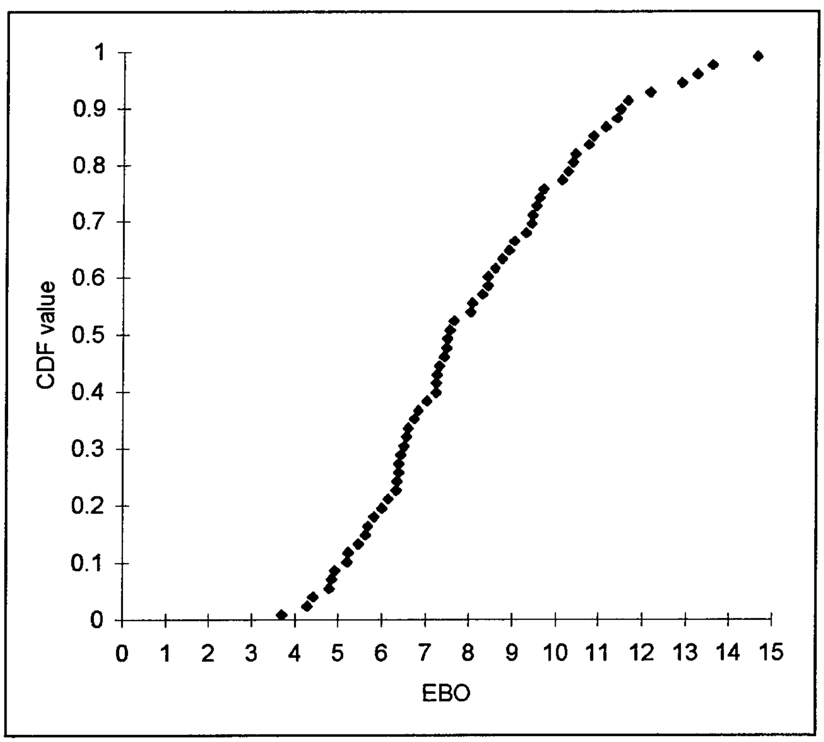

Figure F.2 CDF of EBO

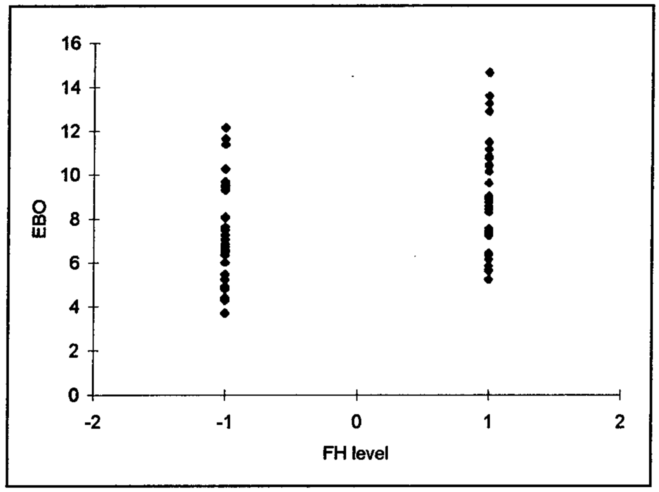

Figure F.3 EBO vs. FH

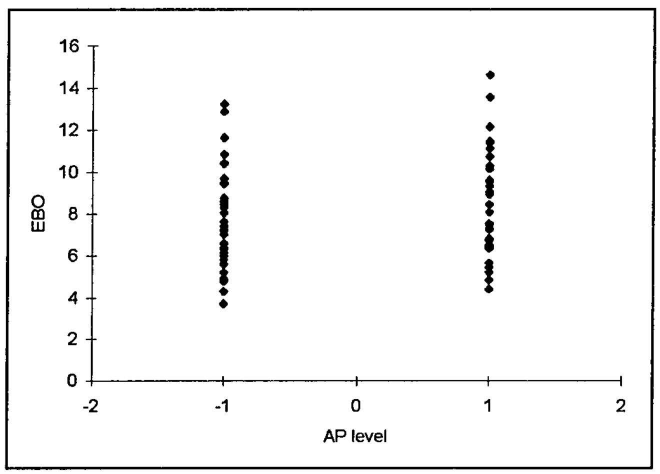

Figure F.4 EBO vs. AP

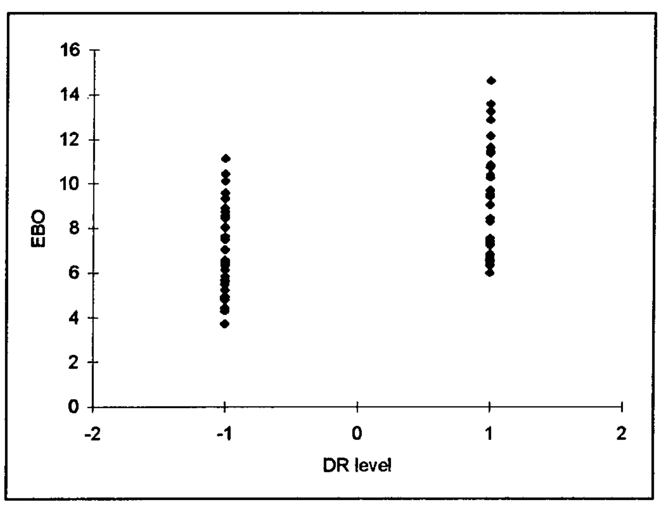

Figure F.5 EBO vs. DR

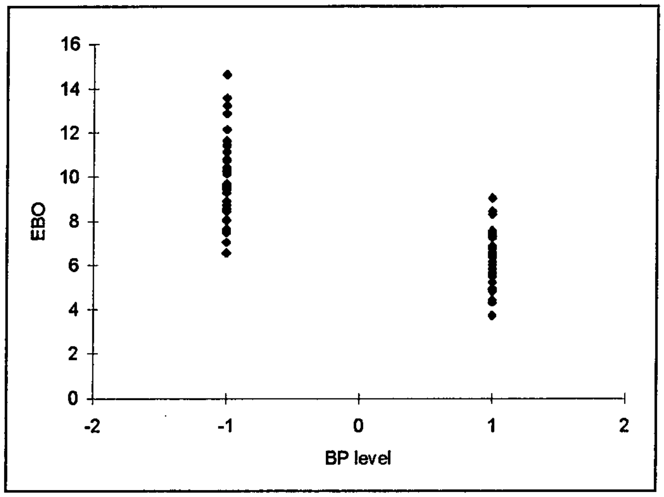

Figure F.6 EBO vs. BP

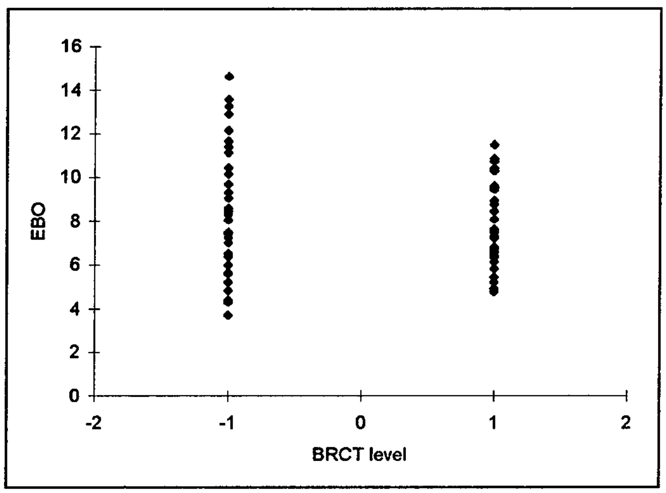

Figure F.7 EBO vs. BRCT

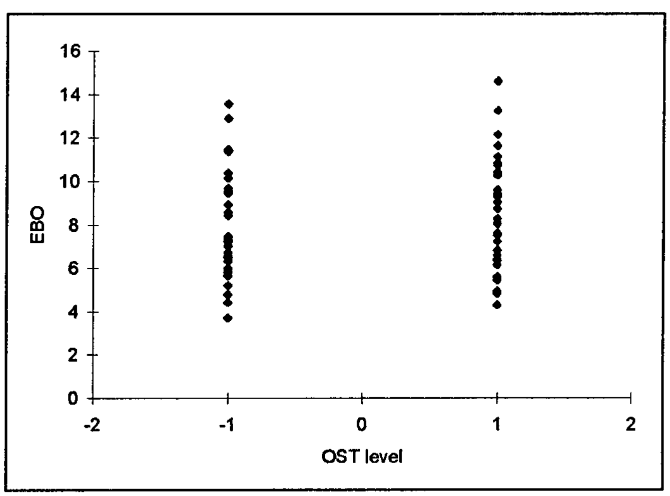

Figure F.8 EBO vs. OST

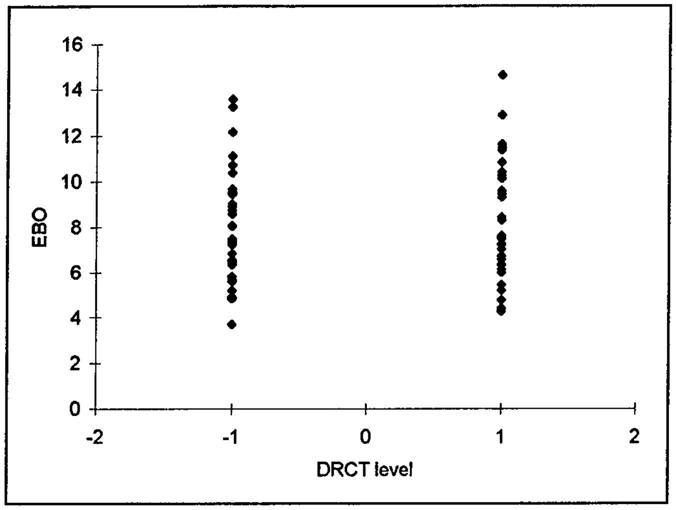

Figure F.9 EBO vs. DRCT

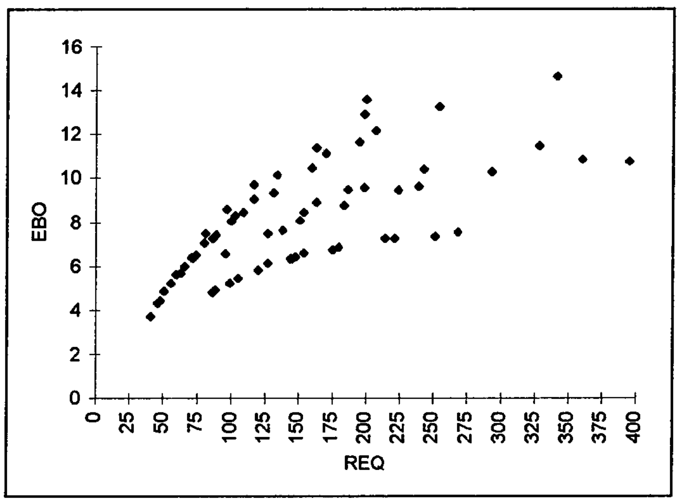

Figure F.10 EBO vs. REQ

Appendix G

SAS Code For First-Order Strategy

```
OPTIONS NODATE NONUMBER LINESIZE= 80;
TITLE 'Thesis:Applying RSM to RBL by Captain Todd May';
TITLE2 'First-Order Strategy';
DATA CUBIC;
 INFILE FACTORAL;
 INPUT FH AP DR BP BRCT OST DRCT REQ EBO;
PROC PLOT DATA=CUBIC;
 PLOT EBO*FH; PLOT EBO*AP; PLOT EBO*DR; PLOT EBO*BP; PLOT EBO*BRCT;
 PLOT EBO*OST; PLOT EBO*DRCT; PLOT EBO*REQ; PLOT REQ*FH; PLOT REQ*AP;
 PLOT REQ*DR; PLOT REQ*BP; PLOT REQ*BRCT; PLOT REQ*OST; PLOT REQ*DRCT;
DATA NORM64              /* Normal prob. points for QQ plots */;
 DO I=1 TO 64;
   J=(I-.5)/64;
   OUTPUT;
 END;
PROC SORT DATA=CUBIC OUT=SORTCUBE;
 BY EBO;
DATA QQEBO               /* Data set of EBO and QQ points */;
 SET NORM64 (KEEP=J);
 SET SORTCUBE (KEEP=EBO);
PROC PLOT DATA=QQEBO                   /* QQ plot of EBO */;
 PLOT J*EBO='*';
 TITLE3 'QQ Plot of EBO';
PROC SORT DATA=CUBIC OUT=SORTREQ;
 BY REQ;
DATA QQREQ               /* Data set of REQ and QQ points */;
 SET NORM64 (KEEP=J);
 SET SORTREQ (KEEP=REQ);
PROC PLOT DATA=QQREQ                   /* QQ plot of REQ */;
 PLOT J*REQ='*';
 TITLE3 'QQ Plot of REQ';
TITLE3;
PROC RSQUARE DATA=CUBIC     /* Find models with highest R-square */
        OUTEST=EST ADJRSQ SSE MSE CP B SELECT=1;
 MODEL EBO=FH AP DR BP BRCT OST DRCT;
PROC PRINT DATA=EST;
PROC PLOT DATA=EST;
 PLOT _CP_*_P_='C' _P_*_P_='P' / OVERLAY;
 TITLE3 'Best First-Order Models Without REQ';
TITLE3;
PROC RSQUARE DATA=CUBIC       /* Same procedure including REQ */
        OUTEST=EST ADJRSQ SSE MSE CP B SELECT=1;
 MODEL EBO=FH AP DR BP BRCT OST DRCT REQ;
PROC PRINT DATA=EST;
PROC PLOT DATA=EST;
 PLOT _CP_*_P_='C' _P_*_P_='P'/OVERLAY;
```

```
 TITLE3 'Best First-Order Models with REQ';
TITLE3;
DATA EFFECT;
 INFILE EFFECTS;
 INPUT LABEL $ VALUE;
 VALUE=VALUE*2;
PROC SORT DATA=EFFECT;
 BY VALUE;
DATA NORM7;
 DO I=1 TO 7;
   J=(I-.5)/7;
   OUTPUT;
 END;
DATA PLOT1          /* Factor effects (except REQ) and QQ points */;
 SET NORM7;
 SET EFFECT;
PROC PLOT DATA=PLOT1    /* QQ plot of factor effects (except REQ) */;
 PLOT J*VALUE=LABEL;
 TITLE3 'QQ Plot of Factor Effects';
TITLE3;
PROC STEPWISE DATA=CUBIC;
 MODEL EBO=FH AP DR BP BRCT OST DRCT REQ/MAXR;
```

Appendix H

SAS RSQUARE Output For First-Order Strategy

Thesis: Applying RSM to RBL by Captain Todd May
First-Order Strategy

N = 64 Regression Models for Dependent Variable: EBO

In	Rsq	Adj Rsq	C(p)	MSE	SSE	Parameter Estimates Intercept	FH	AP	DR	BP	BRCT	OST	DRCT	REQ
1	0.5625	0.5554	286.8	2.753	170.7	8.1140	.	.	.	-1.8519	.	.	.	.
2	0.7798	0.7726	116.6	1.409	85.9	8.1140	.	.	1.1509	-1.8519	.	.	.	.
3	0.8737	0.8674	44.08	0.821	49.3	8.1140	0.7569	.	1.1509	-1.8519	.	.	.	.
4	0.8936	0.8864	30.32	0.703	41.5	8.1140	0.7569	.	1.1509	-1.8519	-0.3482	.	.	.
5	0.9149	0.9076	15.43	0.572	33.2	6.6010	0.4819	.	0.7384	-1.4758	-0.6922	.	.	0.00989
6	0.9262	0.9185	8.467	0.505	28.8	8.1140	0.7569	0.3331	1.1509	-1.8519	-0.3482	0.2965	.	.
7	0.9289	0.9200	8.358	0.495	27.7	8.1140	0.7569	0.3331	1.1509	-1.8519	-0.3482	0.2965	0.1274	.
8	0.9306	0.9205	9.000	0.492	27.1	7.5709	0.6582	0.2866	1.0028	-1.7169	-0.4717	0.2359	0.1058	0.00355

Appendix I

Interaction Effects with First-Order Design Points

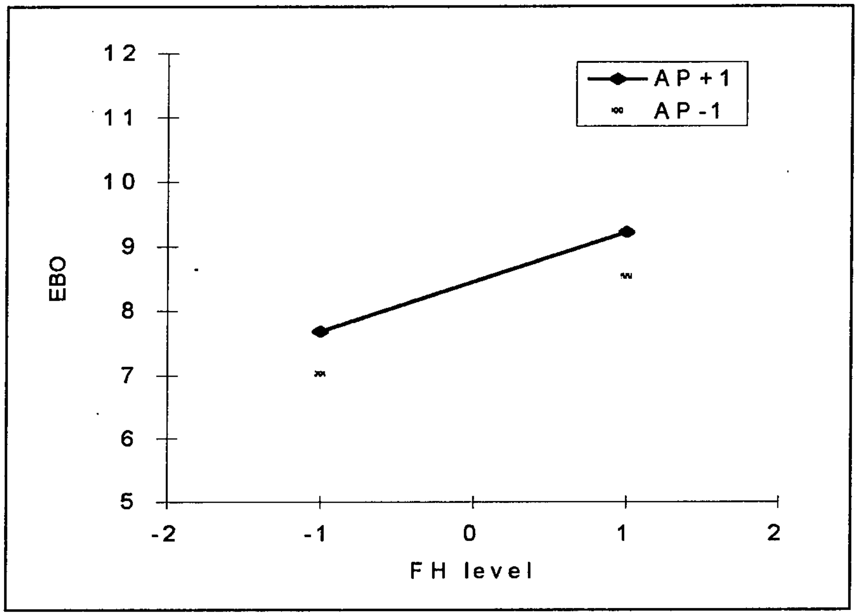

Figure I.1 Interaction Between FH and AP

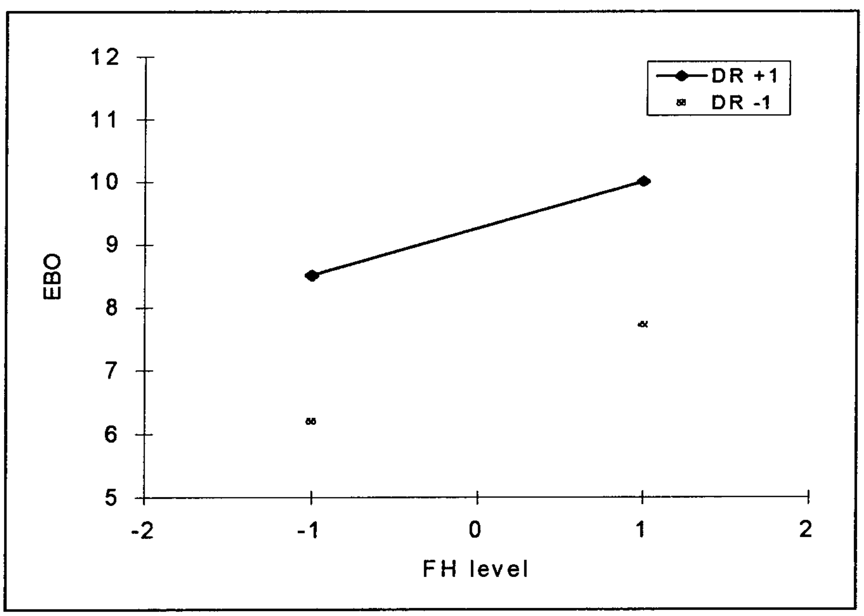

Figure I.2 Interaction Between FH and DR

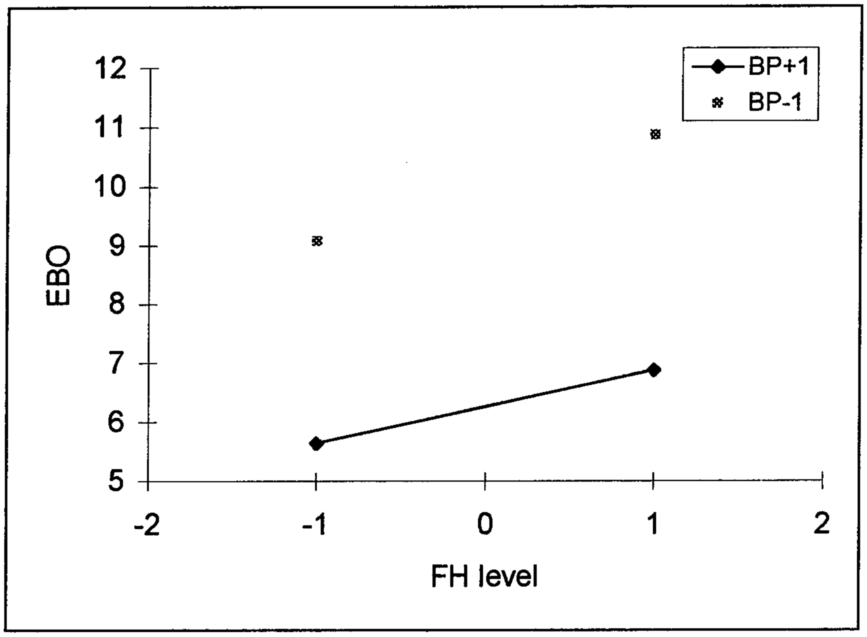

Figure I.3 Interaction Between FH and BP

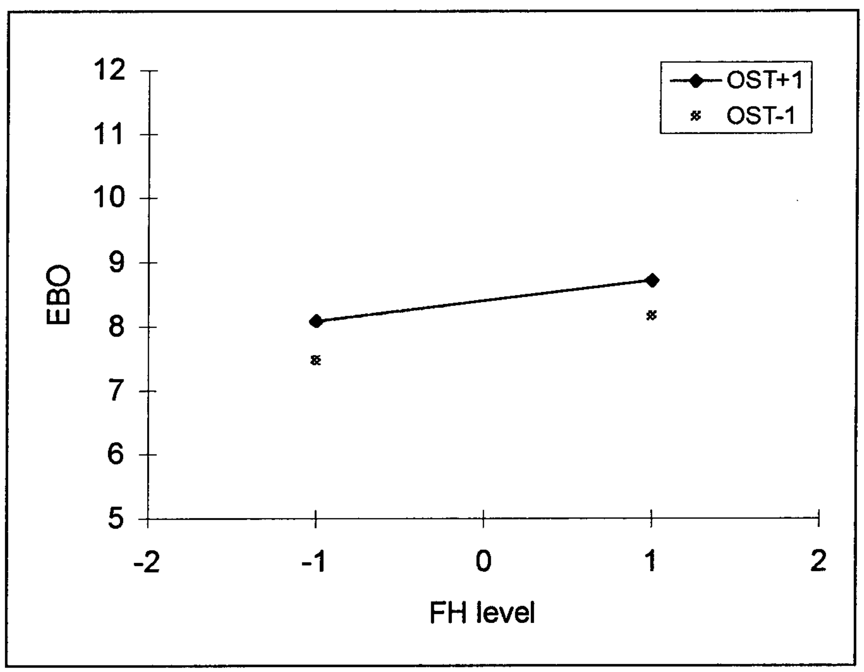

Figure I.4 Interaction Between FH and OST

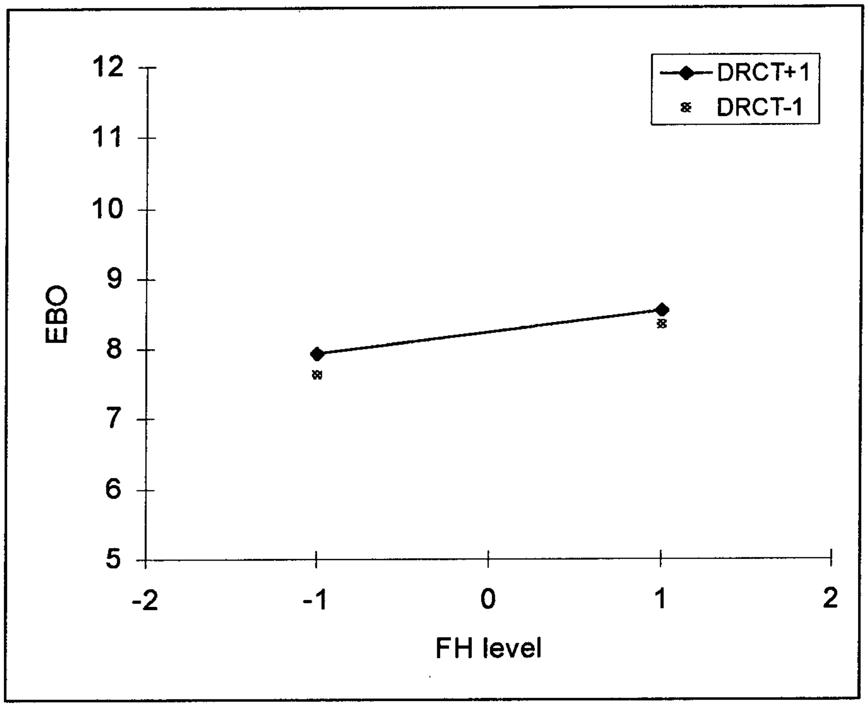

Figure I.5 Interaction Between FH and DRCT

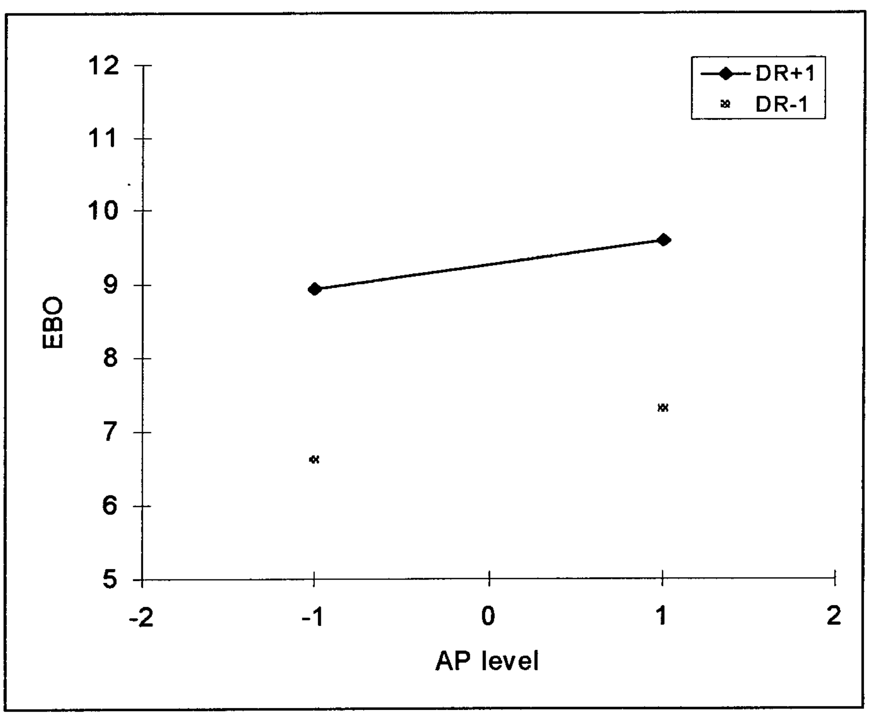

Figure I.6 Interaction Between AP and DR

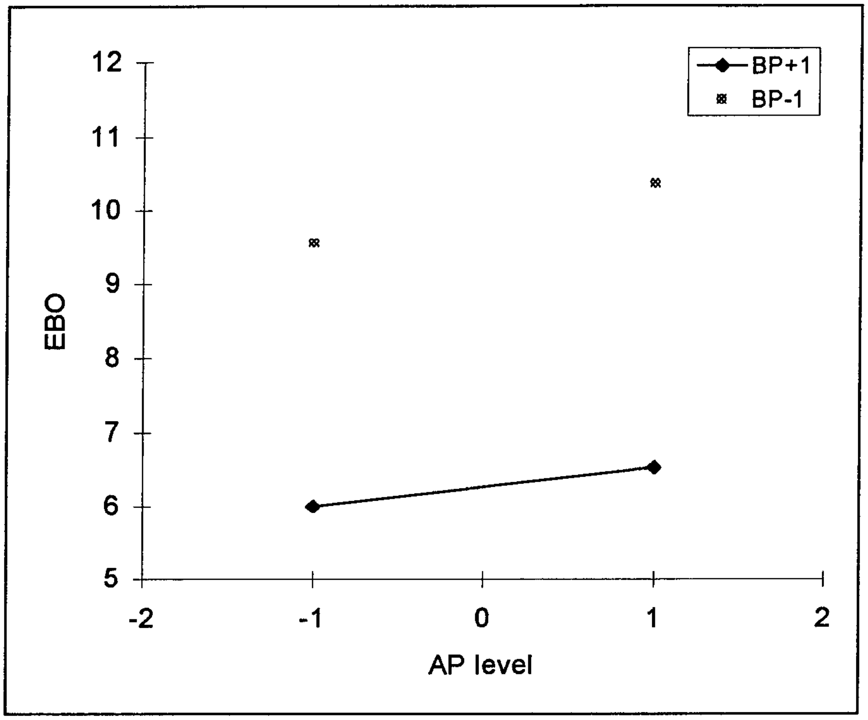

Figure I.7 Interaction Between AP and BP

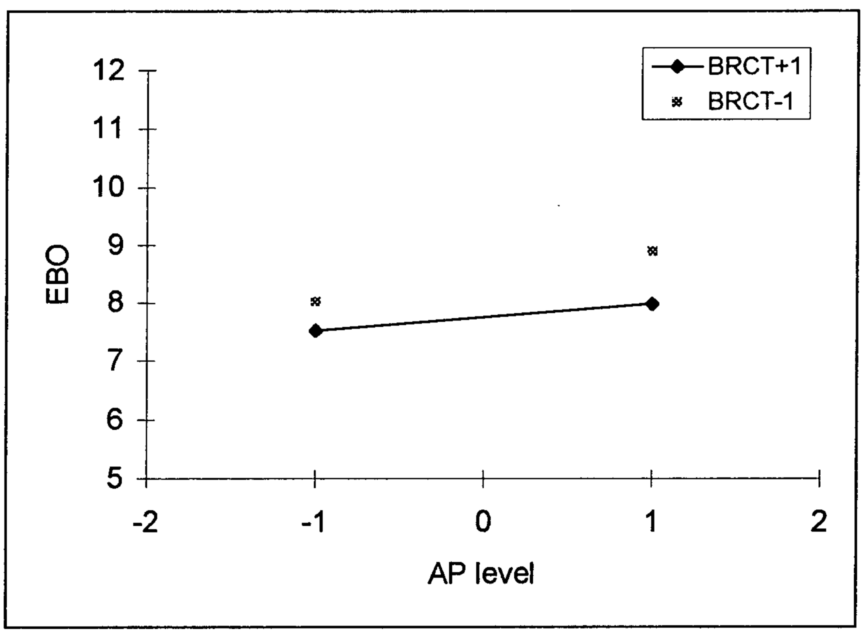

Figure I.8 Interaction Between AP and BRCT

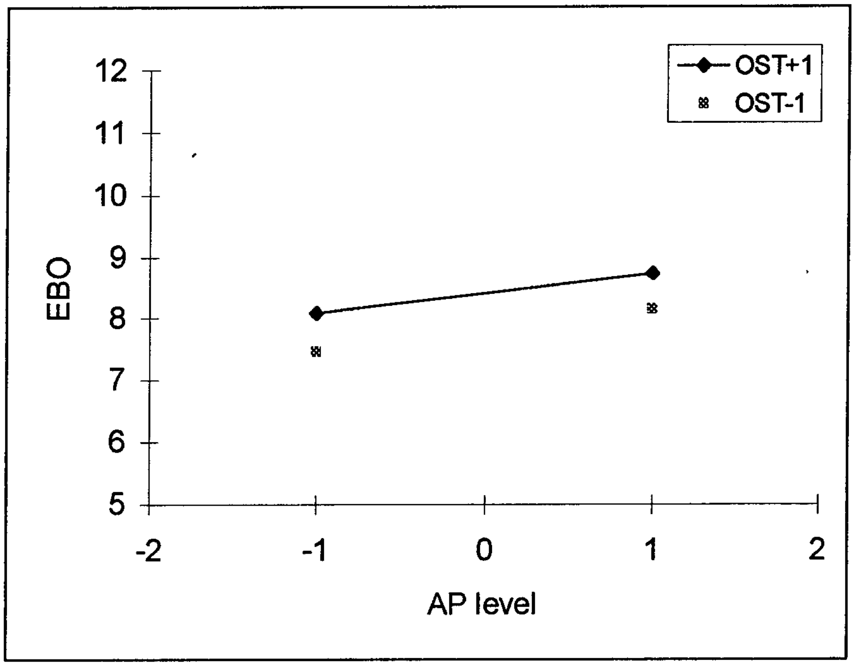

Figure I.9 Interaction Between AP and OST

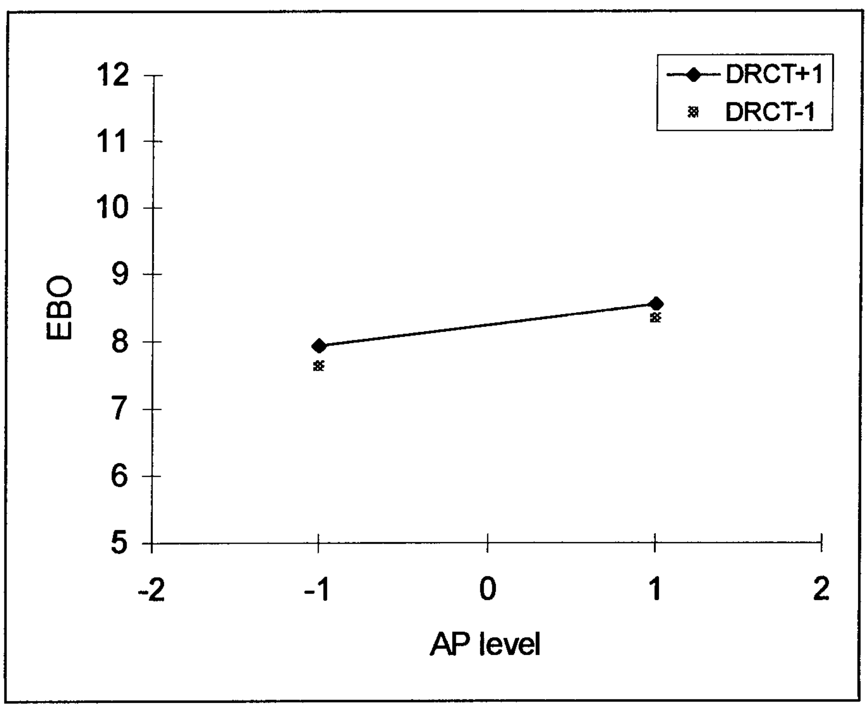

Figure I.10 Interaction Between AP and DRCT

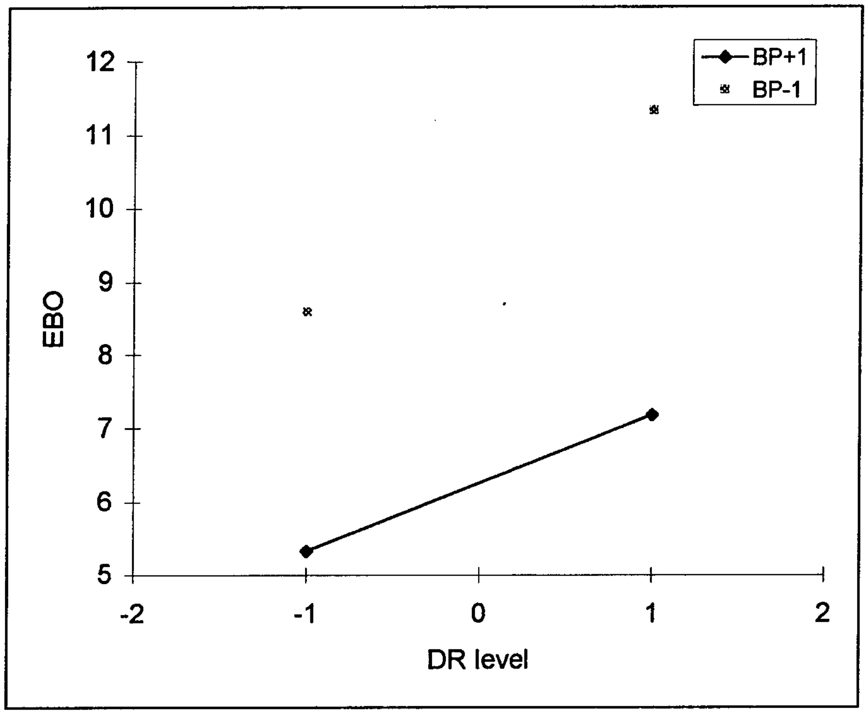

Figure I.11 Interaction Between DR and BP

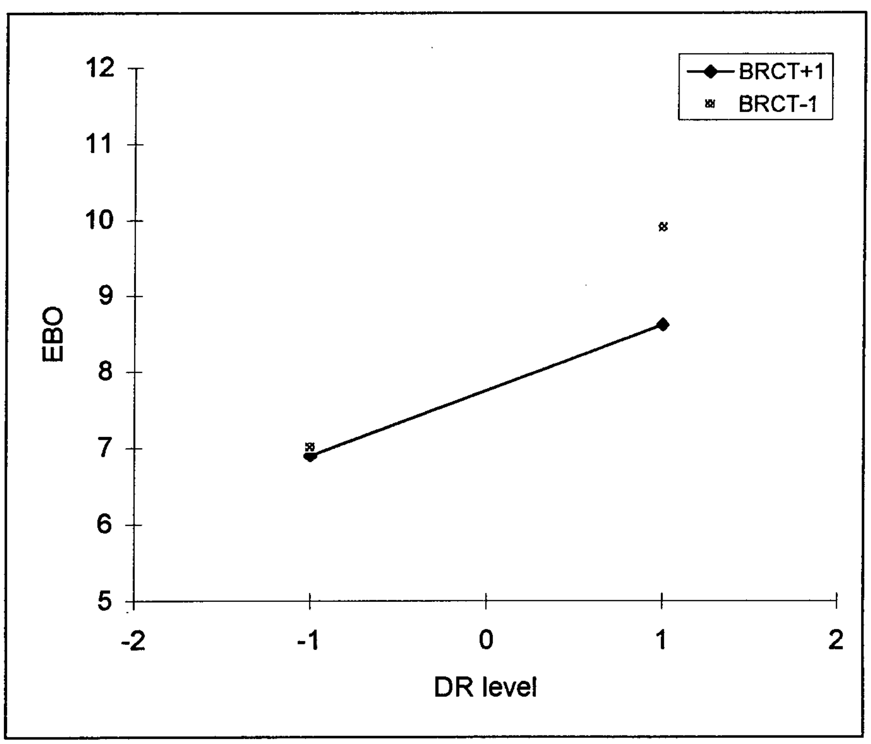

Figure I.12 Interaction Between DR and BRCT

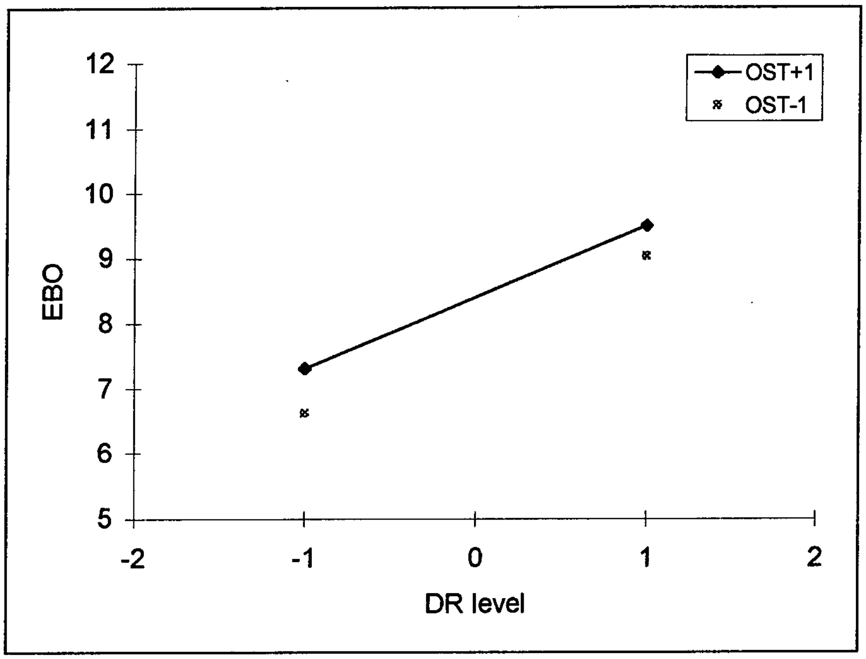

Figure I.13 Interaction Between DR and OST

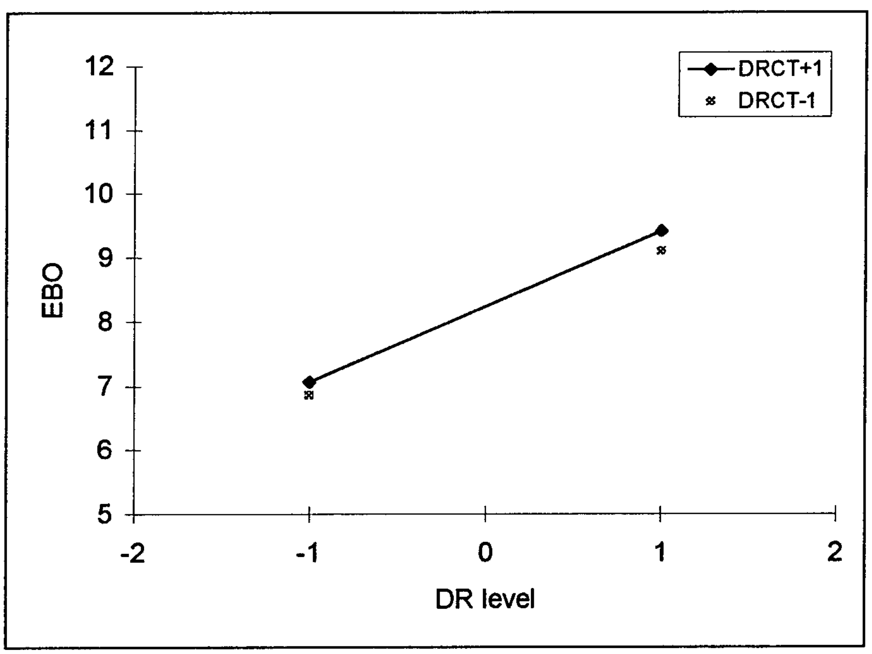

Figure I.14 Interaction Between DR and DRCT

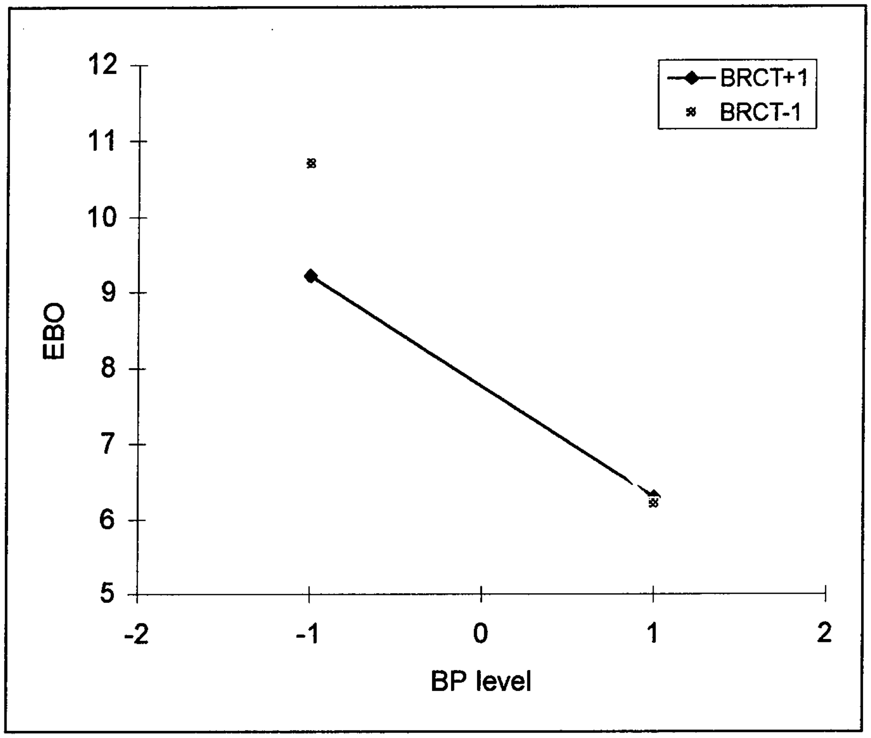

Figure I.15 Interaction Between BP and BRCT

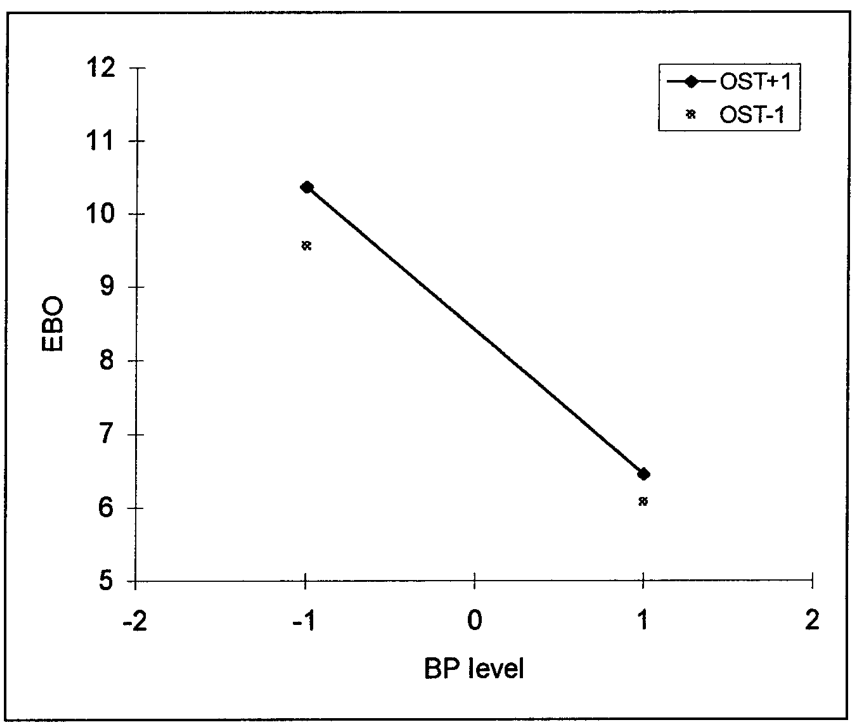

Figure I.16 Interaction Between BP and OST

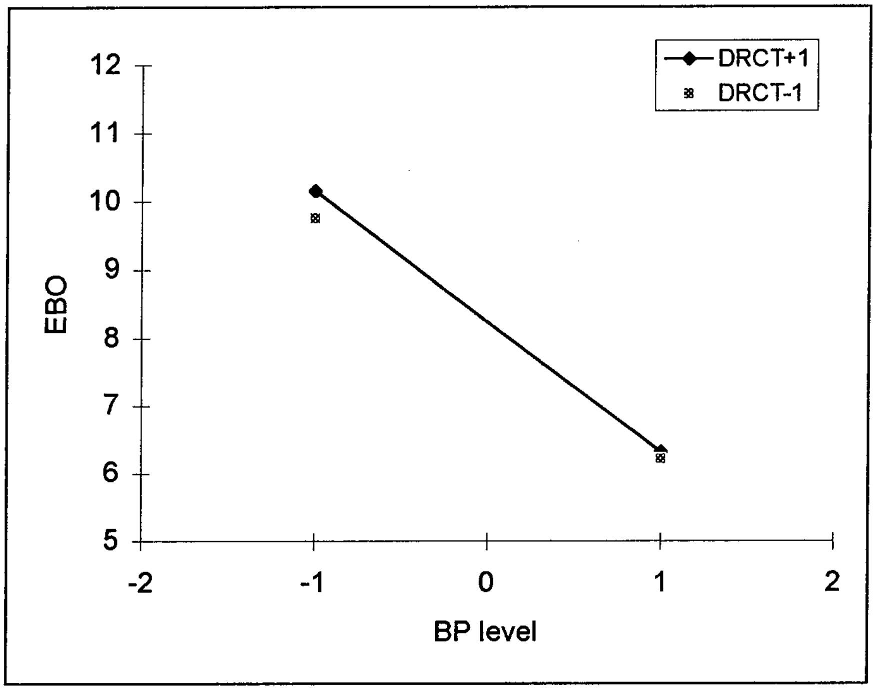

Figure I.17 Interaction Between BP and DRCT

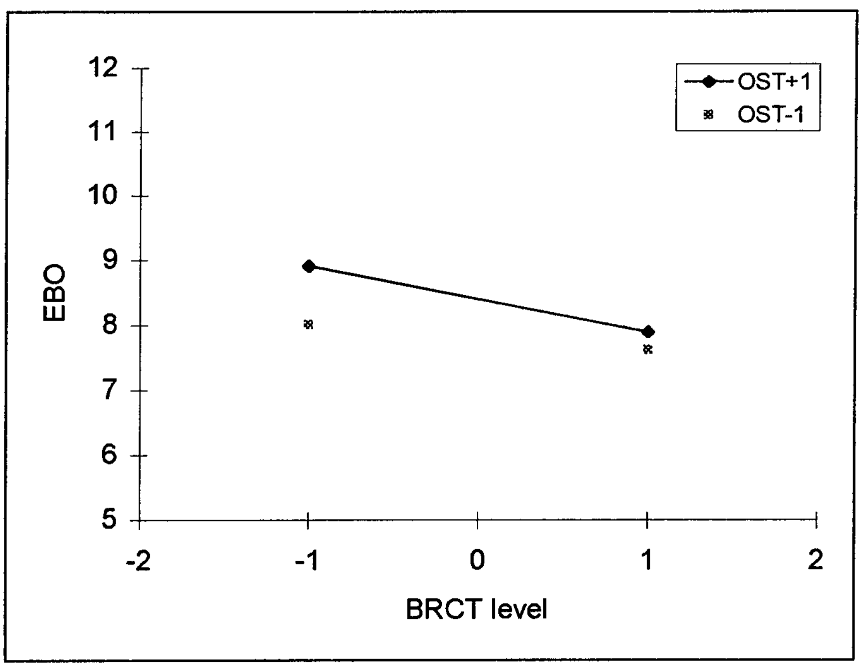

Figure I.18 Interaction Between BRCT and OST

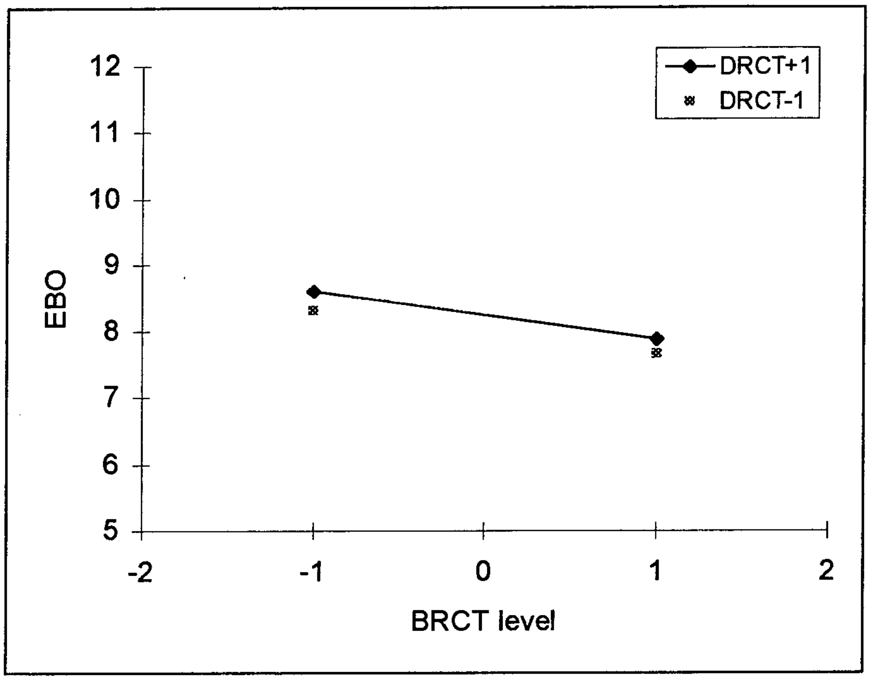

Figure I.19 Interaction Between BRCT and DRCT

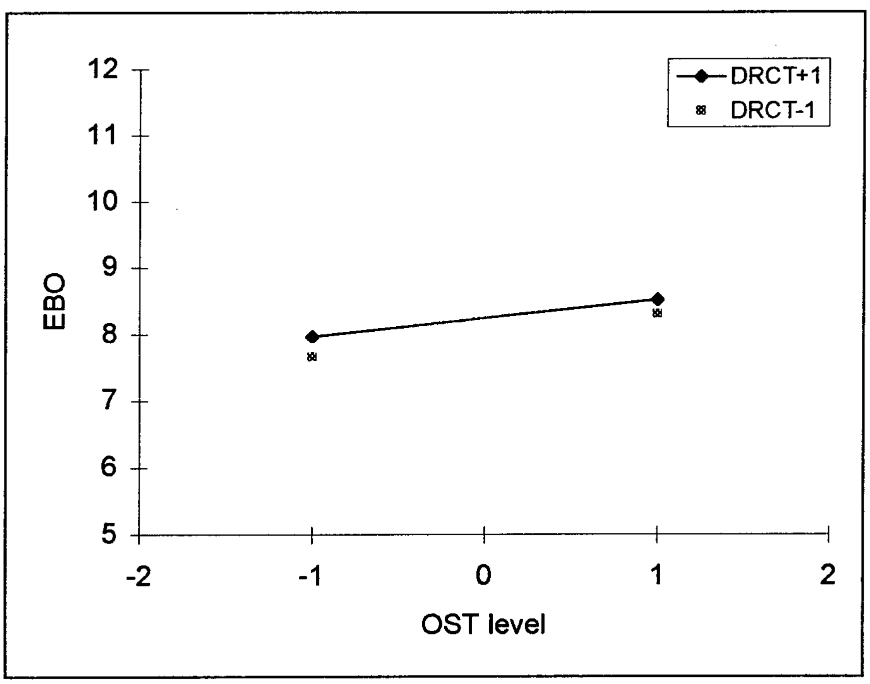

Figure I.20 Interaction Between OST and DRCT

Appendix J

Center and Axial Design Points for Second-Order Design

Table J.1 Coded Center and Axial Points for Second-Order Design

FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
0	0	0	0	0	0	0	154	8.9920
-1	0	0	0	0	0	0	126	7.9682
1	0	0	0	0	0	0	182	9.6661
0	-1	0	0	0	0	0	140	8.4965
0	1	0	0	0	0	0	166	9.3048
0	0	-1	0	0	0	0	112	7.4250
0	0	1	0	0	0	0	196	9.9338
0	0	0	-1	0	0	0	192	10.6515
0	0	0	1	0	0	0	116	6.8452
0	0	0	0	-1	0	0	119	9.3650
0	0	0	0	1	0	0	189	8.2022
0	0	0	0	0	-1	0	137	8.3493
0	0	0	0	0	1	0	171	9.3123
0	0	0	0	0	0	-1	148	8.8269
0	0	0	0	0	0	1	160	9.1426

Appendix K

Experiment Output Information for Second-Order Strategy

Table K.1 REQ and EBO Distribution Statistics

	REQ	EBO
Mean	153.1013	8.2503
Median	145.0000	8.2022
Std. Dev.	73.9831	2.2926
Skewness	1.0226	0.4207
Kurtosis	1.2899	-0.0113

Table K.2 EBO Values for Center Levels

	FH	AP	DR	BP	BRCT	OST	DRCT
Mean	9.5706	9.5567	9.5936	9.5821	9.5762	9.5683	9.5427
Std. dev.	1.0023	1.0472	0.9262	0.7209	1.0334	1.0421	1.0564

Table K.3 Correlation Matrix

	FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
FH	1								
AP	0	1							
DR	0	0	1						
BP	0	0	0	1					
BRCT	0	0	0	0	1				
OST	0	0	0	0	0	1			
DRCT	0	0	0	0	0	0	1		
REQ	0.3457	0.1626	0.5186	-0.4727	0.4323	0.2119	0.0755	1	
EBO	0.3048	0.1345	0.4631	-0.7437	-0.1426	0.1212	0.0515	0.7017	1

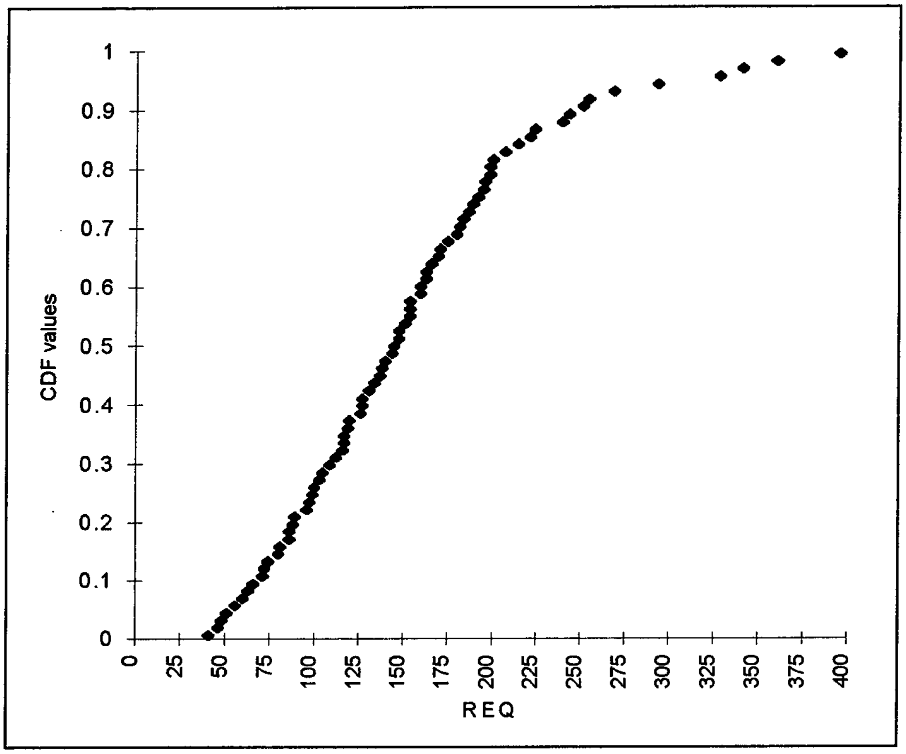

Figure K.1 CDF of REQ

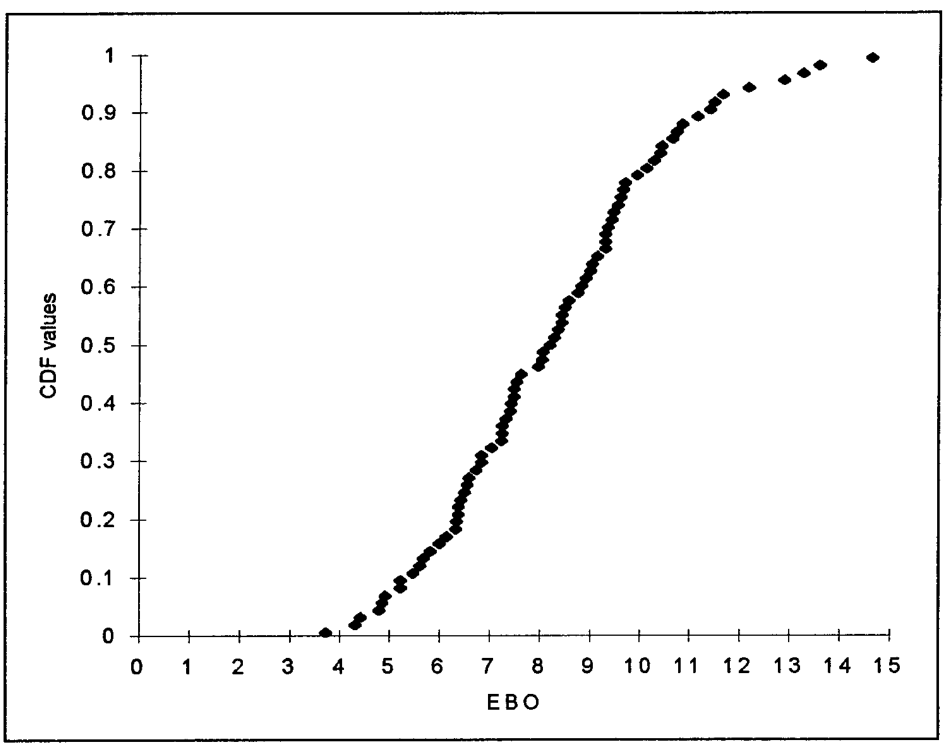

Figure K.2 CDF of EBO

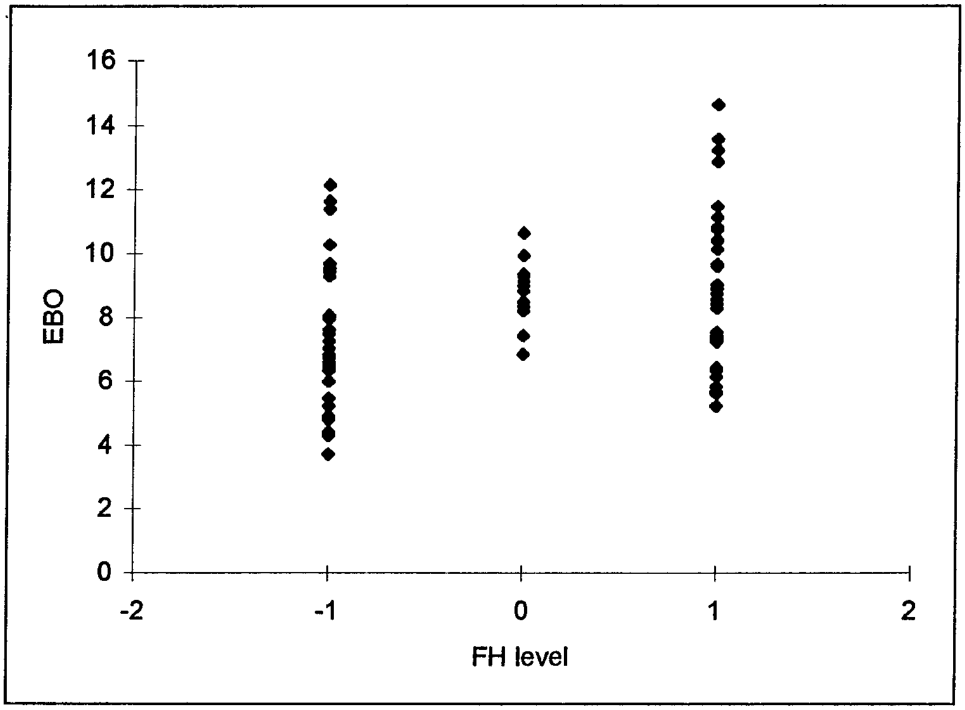

Figure K.3 EBO vs. FH

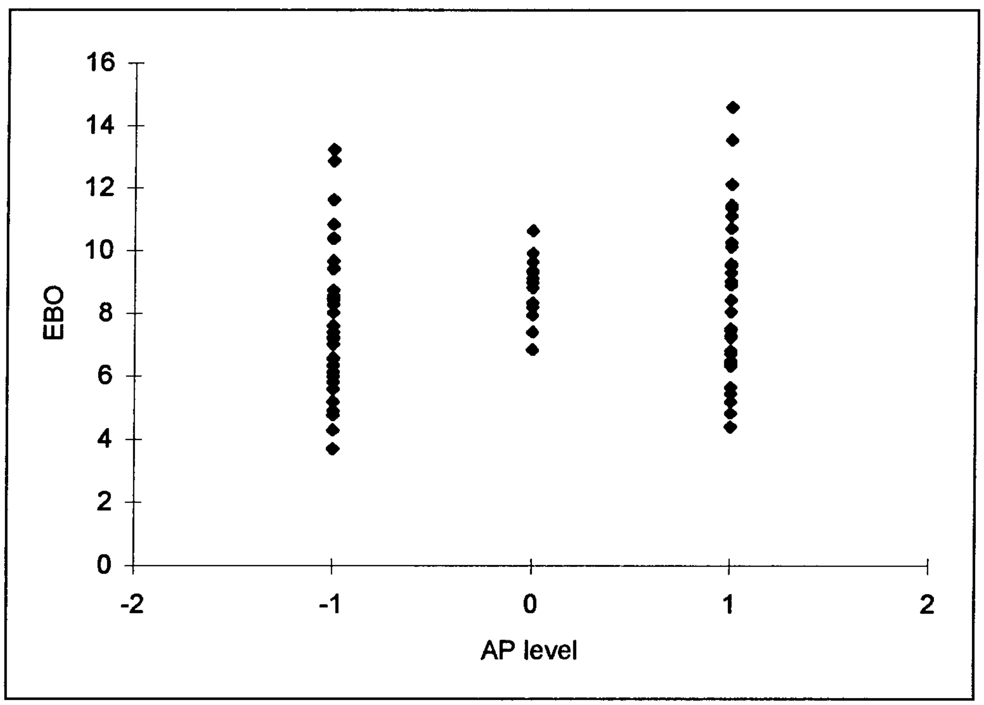

Figure K.4 EBO vs. AP

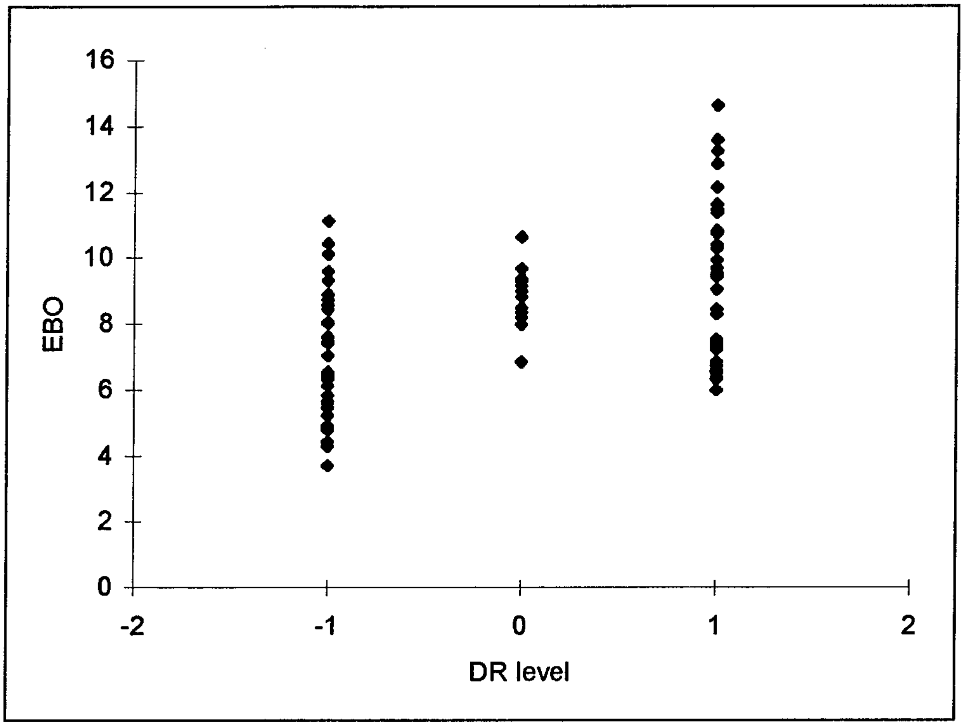

Figure K.5 EBO vs. DR

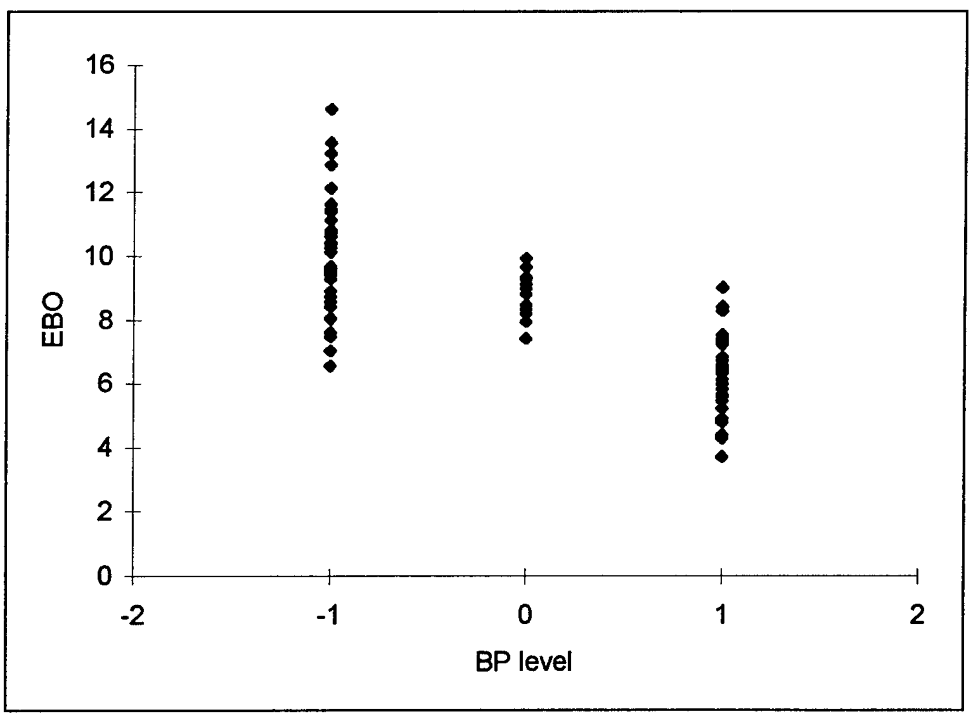

Figure K.6 EBO vs. BP

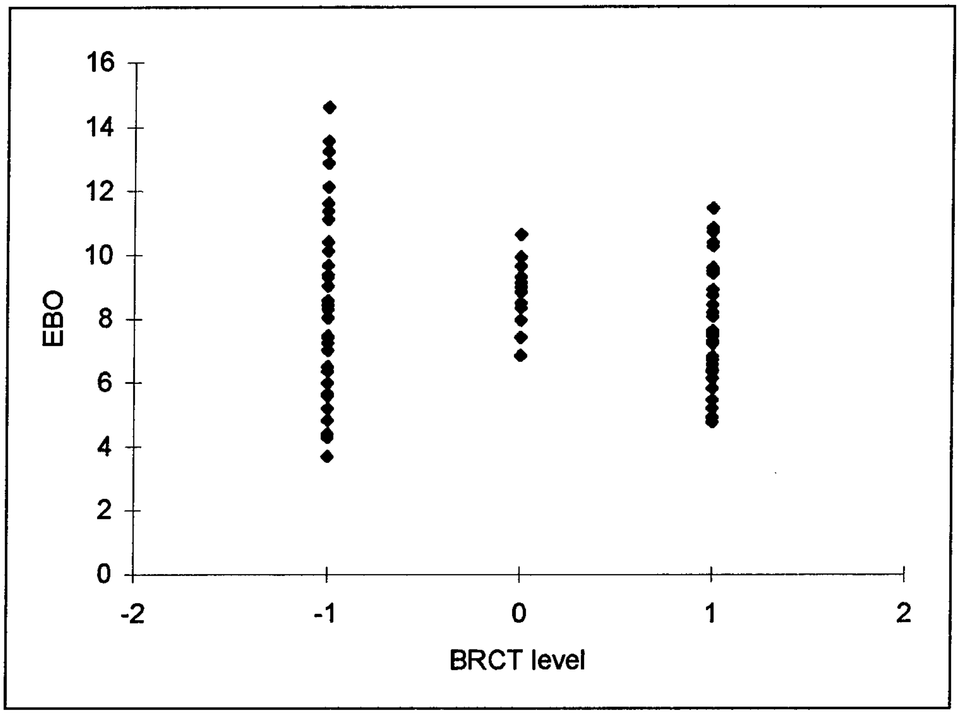

Figure K.7 EBO vs. BRCT

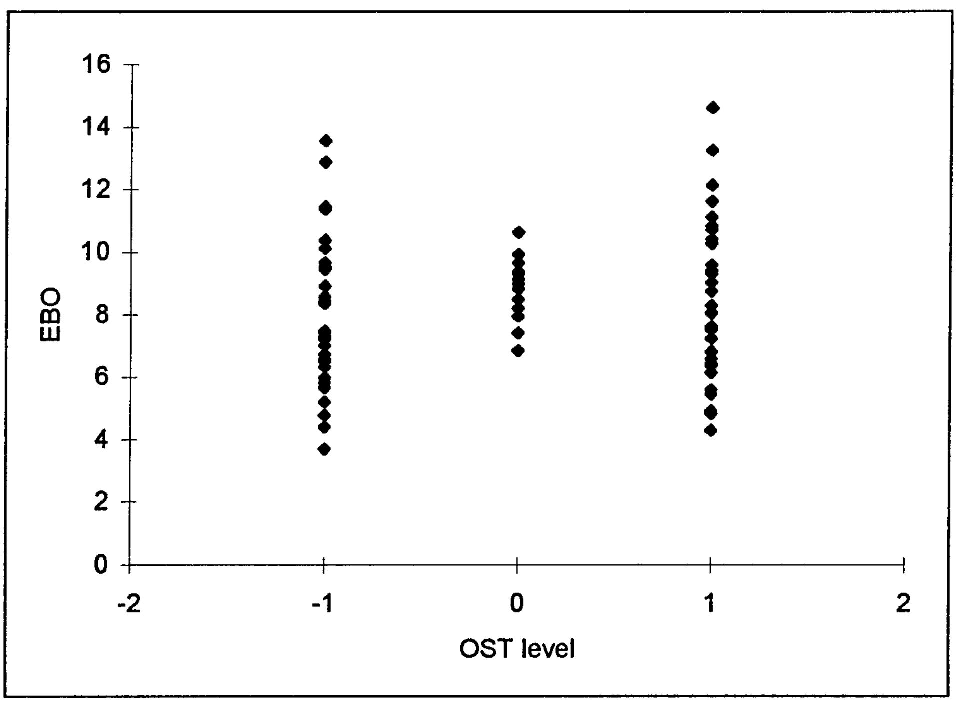

Figure K.8 EBO vs. OST

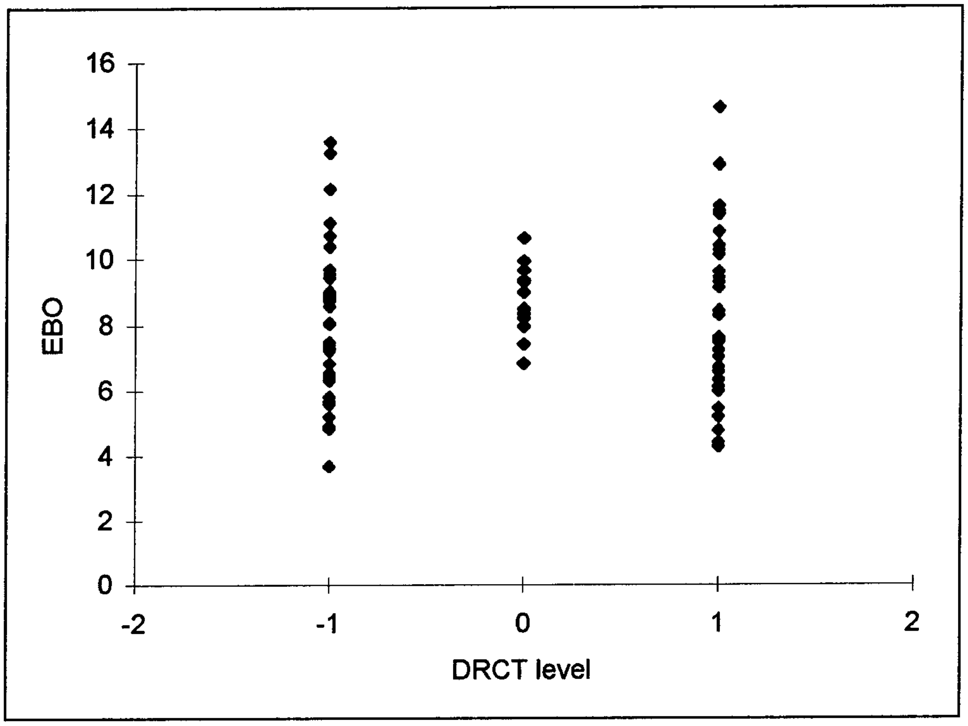

Figure K.9 EBO vs. DRCT

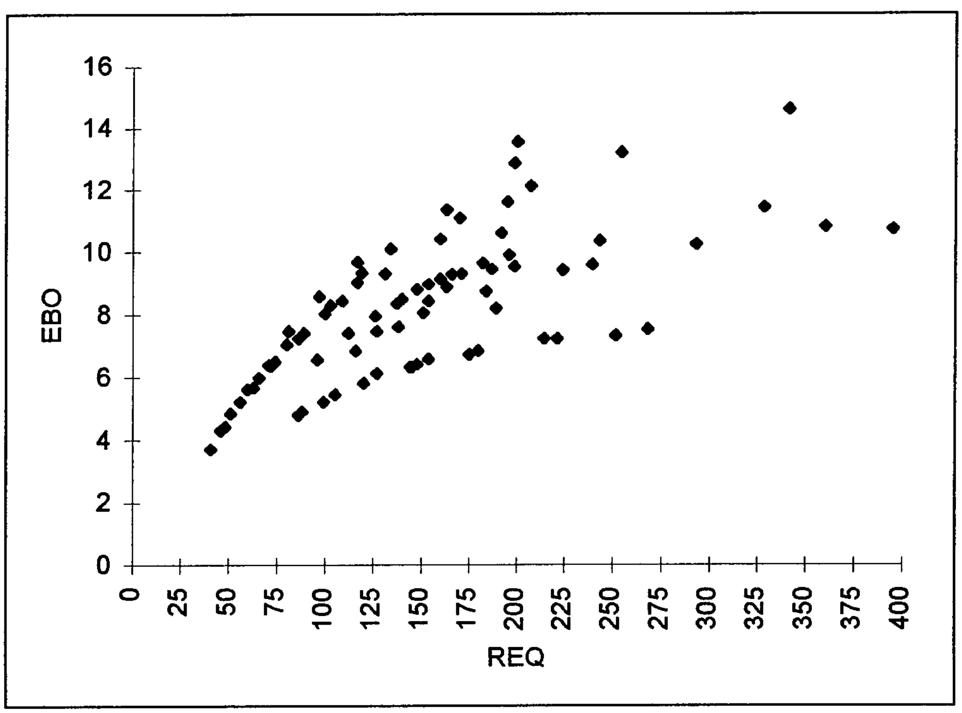

Figure K.10 EBO vs. REQ

Appendix L

SAS Code for Second-Order Strategy

```
OPTIONS NODATE NONUMBER LINESIZE=80;
TITLE 'Thesis: Applying RSM to RBL by Captain Todd May';
TITLE2 'Second-Order Strategy';
DATA CUBIC;
 INFILE FACTORAL; INPUT FH AP DR BP BRCT OST DRCT REQ EBO;
DATA CENTER22;
 INFILE CENTER22; INPUT FH AP DR BP BRCT OST DRCT REQ EBO;
DATA CENTER;
 INFILE CENTER; INPUT FH AP DR BP BRCT OST DRCT REQ EBO;
DATA AXIAL;
 INFILE AXIAL; INPUT FH AP DR BP BRCT OST DRCT REQ EBO;
DATA ALL22;
 SET CUBIC CENTER22 AXIAL;
DATA ALL1;
 SET CUBIC CENTER AXIAL;
PROC RSREG DATA=ALL22;
 TITLE3 'With 22 Center Points | Without REQ';
 MODEL EBO=FH AP DR BP BRCT OST DRCT;
PROC RSREG DATA=ALL1;
 TITLE3 'With 1 Center Point | Without REQ';
 MODEL EBO=FH AP DR BP BRCT OST DRCT;
PROC RSREG DATA=ALL22;
 TITLE3 'With 22 Center Points | With REQ';
 MODEL EBO=FH AP DR BP BRCT OST DRCT REQ;
PROC RSREG DATA=ALL1;
 TITLE3 'With 1 Center Point | With REQ';
 MODEL EBO=FH AP DR BP BRCT OST DRCT REQ;
DATA NORM7;
 DO I=1 TO 7; J=(I-.5)/7; OUTPUT; END;
DATA NORM21;
 DO I=1 TO 21; J=(I-.5)/21; OUTPUT; END;
DATA ONEWAY;
 INFILE ONEWAY; INPUT LABEL $ VALUE;
PROC SORT DATA=ONEWAY;
 BY VALUE;
DATA TWOWAY;
 INFILE TWOWAY; INPUT LABEL $ VALUE;
PROC SORT DATA=TWOWAY;
 BY VALUE;
DATA QQONEWAY;
 SET NORM7 (KEEP=J); SET ONEWAY;
DATA QQTWOWAY;
 SET NORM21 (KEEP=J); SET TWOWAY;
PROC PLOT DATA=QQONEWAY;
 TITLE3 'QQ Plot of One-Way Parameter Estimates w/o REQ';
 PLOT J*VALUE=LABEL;
```

```
PROC PLOT DATA=QQTWOWAY;
  TITLE3 'QQ Plot of Two-Way Parameter Estimates w/o REQ';
  PLOT J*VALUE=LABEL;
PROC MEANS DATA=ALL1 NWAY NOPRINT; CLASS REQ FH; VAR EBO;
  OUTPUT OUT=REQFH MEAN=;
PROC MEANS DATA=ALL1 NWAY NOPRINT; CLASS REQ AP; VAR EBO;
  OUTPUT OUT=REQAP MEAN=;
PROC MEANS DATA=ALL1 NWAY NOPRINT; CLASS REQ DR; VAR EBO;
  OUTPUT OUT=REQDR MEAN=;
PROC MEANS DATA=ALL1 NWAY NOPRINT; CLASS REQ BP; VAR EBO;
  OUTPUT OUT=REQBP MEAN=;
PROC MEANS DATA=ALL1 NWAY NOPRINT; CLASS REQ BRCT; VAR EBO;
  OUTPUT OUT=REQBRCT MEAN=;
PROC MEANS DATA=ALL1 NWAY NOPRINT; CLASS REQ OST; VAR EBO;
  OUTPUT OUT=REQOST MEAN=;
PROC MEANS DATA=ALL1 NWAY NOPRINT; CLASS REQ DRCT; VAR EBO;
  OUTPUT OUT=REQDRCT MEAN=;
TITLE3 'Interaction Plot';
PROC PLOT DATA=REQFH; PLOT EBO*FH=REQ;
PROC PLOT DATA=REQAP; PLOT EBO*AP=REQ;
PROC PLOT DATA=REQDR; PLOT EBO*DR=REQ;
PROC PLOT DATA=REQBP; PLOT EBO*BP=REQ;
PROC PLOT DATA=REQBRCT; PLOT EBO*BRCT=REQ;
PROC PLOT DATA=REQOST; PLOT EBO*OST=REQ;
PROC PLOT DATA=REQDRCT; PLOT EBO*DRCT=REQ;
DATA ALLSTEP;
  SET ALL1; A=FH; B=AP; C=DR; D=BP; E=BRCT; F=OST; G=DRCT; H=REQ;
  AA=A*A; BB=B*B; CC=C*C; DD=D*D; EE=E*E; FF=F*F; GG=G*G; HH=H*H;
  AB=A*B; AC=A*C; AD=A*D; AE=A*E; AF=A*F; AG=A*G; AH=A*H; BC=B*C;
  BD=B*D; BE=B*E; BF=B*F; BG=B*G; BH=B*H; CD=C*D; CE=C*E; CF=C*F;
  CG=C*G; CH=C*H; DE=D*E; DF=D*F; DG=D*G; DH=D*H; EF=E*F; EG=E*G;
  EH=E*H; FG=F*G; FH=F*H; GH=G*H;
TITLE3;
PROC RSQUARE DATA=ALLSTEP OUTEST=EST ADJRSQ SSE MSE CP B SELECT=1;
  MODEL EBO= A B C D E F G H AA BB CC DD EE FF GG HH AB AC AD AE
      AF AG AH BC BD BE BF BG BH CD CE CF CG CH DE DF DG DH EF
      EG EH FG FH GH;
PROC STEPWISE DATA=ALLSTEP;
  MODEL EBO= A B C D E F G H AA BB CC DD EE FF GG HH AB AC AD AE
      AF AG AH BC BD BE BF BG BH CD CE CF CG CH DE DF DG DH EF
      EG EH FG FH GH/MAXR;
```

Appendix M

SAS RSQUARE Output For Second-Order Strategy

Thesis:Applying RSM to RBL by Captain Todd May
Second-Order Strategy

N = 79   Regression Models for Dependent Variable: EBO

```
Parameter
     Rsq     Adj   C(p)   MSE    SSE Estimates
In           Rsq                     Intercept    A     B     C
                   D     E     F     G     H     AA
                   BB    CC    DD    EE    FF    GG
                   HH    AB    AC    AD    AE    AF
                   AG    AH    BC    BD    BE    BF
                   BG    BH    CD    CE    CF    CG
                   CH    DE    DF    DG    DH    EF
                   EG    EH    FG    FH    GH

 1  0.5530  0.5472  12846  2.380  183.2   8.2503    .     .     .
              -1.8534     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .
------------------------------------------------------------------
 2  0.8384  0.8341  4599  0.872  66.3   3.8438    .     .     .
                   .     .     .     .  0.0306     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   . -0.00962     .     .     .
------------------------------------------------------------------
 3  0.9254  0.9224  2085  0.408  30.6   4.8183    .     .     .
              -0.8805     .     .     .  0.0239     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   .     .     .     .     .     .
                   . -0.00799     .     .     .
------------------------------------------------------------------
```

```
Parameter
       Rsq      Adj    C(p)   MSE    SSE Estimates
In              Rsq                        Intercept     A      B      C
                         D      E      F      G      H      AA
                         BB     CC     DD     EE     FF     GG
                         HH     AB     AC     AD     AE     AF
                         AG     AH     BC     BD     BE     BF
                         BG     BH     CD     CE     CF     CG
                         CH     DE     DF     DG     DH     EF
                         EG     EH     FG     FH     GH

 4  0.9607  0.9586  1068  0.218  16.1   2.5350      .      .      .
                         .      .      .      .  0.0522      .
                         .      .      .      .      .      .
                 -0.00008       .      .      .      .      .
                         .      .      .      .      .      .
                         .      .      .      .      .      .
                         .      .      .      . -0.00528     .
                         . -0.00773    .      .      .
--------------------------------------------------------------------------
 5  0.9726  0.9708 724.0  0.154  11.2   1.2633      .      .      .
                         . -1.3488     .      .  0.0651      .
                         .      .      .      .      .      .
                 -0.00011       .      .      .      .      .
                         .      .      .      .      .      .
                         .      .      . -0.3106      .      .
                         .      .      .      . -0.00517     .
                         .      .      .      .      .
--------------------------------------------------------------------------
 6  0.9806  0.9789 496.8  0.111   8.0   3.5063  0.3256      .  0.4502
                         .      .      .      .  0.0457      .
                         .      .      .      .      .      .
                 -0.00008       .      .      .      .      .
                         .      .      .      .      .      .
                         .      .      .      .      .      .
                         .      .      .      . -0.00697     .
                         . -0.00636    .      .      .
--------------------------------------------------------------------------
 7  0.9850  0.9835 370.1  0.087   6.1   1.3297      .      .      .
                         . -1.3725     .      .  0.0633      .
                         .      .      .      .      .      .
                 -0.0001        .      .      . -0.2104      .
                         .      .      .      .      .      .
                         .      .      . -0.3370      .      .
                         .      . 0.2067      . -0.00495      .
                         .      .      .      .      .
--------------------------------------------------------------------------
```

```
Parameter
       Rsq      Adj    C(p)   MSE    SSE Estimates
 In             Rsq                          Intercept     A       B       C
                        D       E      F      G      H      AA
                        BB      CC     DD     EE     FF     GG
                        HH      AB     AC     AD     AE     AF
                        AG      AH     BC     BD     BE     BF
                        BG      BH     CD     CE     CF     CG
                        CH      DE     DF     DG     DH     EF
                        EG      EH     FG     FH     GH

 8  0.9884  0.9871 274.2 0.068   4.8   1.3389      .      .      .
                      . -1.3950      .      .  0.0625     .

                      .      .      .      .      .      .
                  -0.0001      .      .      . -0.2229    .

                      .      .      .      .      .      .
                      .      .      . -0.3548     .      .
                      .      .  0.1851     . -0.00486     .
                      .      .      . -0.00094     .
---------------------------------------------------------------------------
 9  0.9911  0.9899 199.2 0.053   3.7   1.7785      .      .      .
                      . -1.3801      .      .  0.0605     .
                      .      . -0.3329      .      .      .
                 -0.00009      .      .      . -0.2290    .

                      .      .      .      .      .      .
                      .      .      . -0.3628     .      .
                      .      .  0.1888     . -0.00494     .
                      .      .      . -0.00096     .
```

Appendix N

SAS RSREG Output for Second-Order Strategy

Thesis: Applying RSM to RBL by Captain Todd May
Second-Order Strategy
With 1 Center Point | With REQ

Response Surface for Variable EBO

Response Mean	8.250330
Root MSE	0.119082
R-Square	0.9988
Coef. of Variation	1.4434

Regression	Degrees of Freedom	Type I Sum of Squares	R-Square	F-Ratio	Prob > F
Linear	8	376.299742	0.9179	3317.0	0.0000
Quadratic	8	16.234916	0.0396	143.1	0.0000
Crossproduct	28	16.938236	0.0413	42.659	0.0000
Total Regress	44	409.472894	0.9988	656.3	0.0000

Residual	Degrees of Freedom	Sum of Squares	Mean Square
Total Error	34	0.482140	0.014181

Parameter	Degrees of Freedom	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEPT	1	9.835604	1.179608	8.338	0.0000
FH	1	0.913769	0.236621	3.862	0.0005
AP	1	0.432539	0.132931	3.254	0.0026
DR	1	1.304706	0.331180	3.940	0.0004
BP	1	-1.892005	0.443462	-4.266	0.0001
BRCT	1	0.435110	0.335136	1.298	0.2029
OST	1	0.599052	0.172097	3.481	0.0014
DRCT	1	0.116336	0.092599	1.256	0.2176
REQ	1	-0.002073	0.009899	-0.209	0.8354
FH*FH	1	-0.127816	0.084431	-1.514	0.1393
AP*FH	1	0.033818	0.026124	1.295	0.2042
AP*AP	1	-0.044994	0.079160	-0.568	0.5735
DR*FH	1	0.055962	0.058758	0.952	0.3476

Parameter	Degrees of Freedom	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
DR*AP	1	0.027548	0.035098	0.785	0.4380
DR*DR	1	-0.298809	0.111825	-2.672	0.0115
BP*FH	1	-0.167288	0.072223	-2.316	0.0267
BP*AP	1	-0.091889	0.039971	-2.299	0.0278
BP*DR	1	-0.241237	0.104795	-2.302	0.0276
BP*BP	1	-0.230176	0.097093	-2.371	0.0236
BRCT*FH	1	-0.035559	0.057387	-0.620	0.5396
BRCT*AP	1	-0.020199	0.035270	-0.573	0.5706
BRCT*DR	1	-0.070601	0.080126	-0.881	0.3844
BRCT*BP	1	0.356599	0.076028	4.690	0.0000
BRCT*BRCT	1	-0.018368	0.083969	-0.219	0.8282
OST*FH	1	0.029283	0.029782	0.983	0.3324
OST*AP	1	0.020075	0.021154	0.949	0.3493
OST*DR	1	0.039083	0.039069	1.000	0.3242
OST*BP	1	-0.269582	0.088677	-3.040	0.0045
OST*BRCT	1	-0.051798	0.040898	-1.267	0.2139
OST*OST	1	-0.083621	0.079153	-1.056	0.2982
DRCT*FH	1	0.001648	0.021865	0.0754	0.9403
DRCT*AP	1	-0.024601	0.017310	-1.421	0.1644
DRCT*DR	1	0.022658	0.027609	0.821	0.4176
DRCT*BP	1	-0.099056	0.039345	-2.518	0.0167
DRCT*BRCT	1	-0.010651	0.025838	-0.412	0.6828
DRCT*OST	1	-0.013700	0.018284	-0.749	0.4588
DRCT*DRCT	1	0.046734	0.078307	0.597	0.5546
REQ*FH	1	0.000995	0.001650	0.603	0.5504
REQ*AP	1	0.000313	0.000869	0.361	0.7205
REQ*DR	1	0.002021	0.002565	0.788	0.4361
REQ*BP	1	-0.002038	0.002330	-0.875	0.3879
REQ*BRCT	1	-0.002984	0.001568	-1.903	0.0656
REQ*OST	1	-0.000865	0.001013	-0.854	0.3992
REQ*DRCT	1	0.000457	0.000556	0.823	0.4162
REQ*REQ	1	-0.000024	0.000023	-1.049	0.3018

Appendix O

Canonical Analysis

Canonical analysis was performed on the 44-term, second-order model shown in Appendix N, henceforth referred to as the full model. Thus, the analysis begins with a second-order model incorporating all main effects, two-way interactions, and quadratic terms. The canonical form A rotates the factor axes so that they align with the principal axes of the response surface. The two-way interactions are removed as a result of the axis rotation. The canonical form A for the full model is:

$$EBO = 9.8356 + 0.5046\ FH - 0.3908\ AP + 0.7903\ DR - 0.9595\ BP + 0.5559\ BRCT + 2.0464\ OST + 0.6450\ DRCT - 0.0117\ REQ + 0.2839\ FH^2 + 0.0473\ AP^2 - 0.0411\ DR^2 - 0.0592\ BP^2 - 0.1276\ BRCT^2 - 0.2647\ OST^2 - 0.4368\ DRCT^2 - 0.9197\ REQ^2$$

The canonical form B offers a further simplification to the response surface representation. If the response surface has a stationary point within the design region, the rotated factor axes can be translated to coincide with the stationary point. The translation eliminates the linear terms from the model, leaving only the constant and quadratic terms. For the response surface under investigation, the canonical form B was not developed because a stationary point did not exist.

Appendix P

Verification Output

Table P.1 Verification Design and Output for Factorial Points

FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
-0.9	-0.5	-0.5	-0.56	-0.33	-0.86	0.8	105	7.6585
0.5	-0.5	-0.5	-0.56	-0.33	-0.86	-0.4	126	8.8702
-0.9	0.5	-0.5	-0.56	-0.33	-0.86	-0.4	106	7.8944
0.5	0.5	-0.5	-0.56	-0.33	-0.86	0.8	147	9.6303
-0.9	-0.5	0.5	-0.56	-0.33	-0.86	-0.4	127	8.9122
0.5	-0.5	0.5	-0.56	-0.33	-0.86	0.8	178	10.7262
-0.9	0.5	0.5	-0.56	-0.33	-0.86	0.8	149	9.5546
0.5	0.5	0.5	-0.56	-0.33	-0.86	-0.4	181	10.8817
-0.9	-0.5	-0.5	0.44	-0.33	-0.86	-0.4	79	6.1665
0.5	-0.5	-0.5	0.44	-0.33	-0.86	0.8	106	7.3630
-0.9	0.5	-0.5	0.44	-0.33	-0.86	0.8	89	6.6184
0.5	0.5	-0.5	0.44	-0.33	-0.86	-0.4	110	7.6519
-0.9	-0.5	0.5	0.44	-0.33	-0.86	0.8	107	7.3779
0.5	-0.5	0.5	0.44	-0.33	-0.86	-0.4	133	8.4800
-0.9	0.5	0.5	0.44	-0.33	-0.86	-0.4	111	7.6783
0.5	0.5	0.5	0.44	-0.33	-0.86	0.8	151	9.0671
-0.9	-0.5	-0.5	-0.56	0.67	-0.86	-0.4	118	7.2865
0.5	-0.5	-0.5	-0.56	0.67	-0.86	0.8	163	8.6734
-0.9	0.5	-0.5	-0.56	0.67	-0.86	0.8	136	7.8105
0.5	0.5	-0.5	-0.56	0.67	-0.86	-0.4	167	8.9019
-0.9	-0.5	0.5	-0.56	0.67	-0.86	0.8	164	8.7490
0.5	-0.5	0.5	-0.56	0.67	-0.86	-0.4	202	9.6534
-0.9	0.5	0.5	-0.56	0.67	-0.86	-0.4	168	8.9675
0.5	0.5	0.5	-0.56	0.67	-0.86	0.8	235	10.0898
-0.9	-0.5	-0.5	0.44	0.67	-0.86	0.8	108	6.2707
0.5	-0.5	-0.5	0.44	0.67	-0.86	-0.4	136	7.1925
-0.9	0.5	-0.5	0.44	0.67	-0.86	-0.4	114	6.4498
0.5	0.5	-0.5	0.44	0.67	-0.86	0.8	153	7.5389
-0.9	-0.5	0.5	0.44	0.67	-0.86	-0.4	137	7.2248
0.5	-0.5	0.5	0.44	0.67	-0.86	0.8	185	8.0964
-0.9	0.5	0.5	0.44	0.67	-0.86	0.8	154	7.5808
0.5	0.5	0.5	0.44	0.67	-0.86	-0.4	195	8.2407
-0.9	-0.5	-0.5	-0.56	-0.33	0.43	-0.4	119	8.2417
0.5	-0.5	-0.5	-0.56	-0.33	0.43	0.8	164	9.9101
-0.9	0.5	-0.5	-0.56	-0.33	0.43	0.8	137	8.8439
0.5	0.5	-0.5	-0.56	-0.33	0.43	-0.4	168	10.1980
-0.9	-0.5	0.5	-0.56	-0.33	0.43	0.8	165	9.9935
0.5	-0.5	0.5	-0.56	-0.33	0.43	-0.4	204	11.0578
-0.9	0.5	0.5	-0.56	-0.33	0.43	-0.4	170	10.1437
0.5	0.5	0.5	-0.56	-0.33	0.43	0.8	236	11.7272
-0.9	-0.5	-0.5	0.44	-0.33	0.43	0.8	92	6.7732
0.5	-0.5	-0.5	0.44	-0.33	0.43	-0.4	115	7.7659

Table P.1 (continued)

FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
-0.9	0.5	-0.5	0.44	-0.33	0.43	-0.4	96	7.0258
0.5	0.5	-0.5	0.44	-0.33	0.43	0.8	129	8.3431
-0.9	-0.5	0.5	0.44	-0.33	0.43	-0.4	115	7.9012
0.5	-0.5	0.5	0.44	-0.33	0.43	0.8	156	9.2741
-0.9	0.5	0.5	0.44	-0.33	0.43	0.8	130	8.3793
0.5	0.5	0.5	0.44	-0.33	0.43	-0.4	163	9.5498
-0.9	-0.5	-0.5	-0.56	0.67	0.43	0.8	146	8.0685
0.5	-0.5	-0.5	-0.56	0.67	0.43	-0.4	182	8.9332
-0.9	0.5	-0.5	-0.56	0.67	0.43	-0.4	151	8.3340
0.5	0.5	-0.5	-0.56	0.67	0.43	0.8	208	9.4792
-0.9	-0.5	0.5	-0.56	0.67	0.43	-0.4	183	9.0116
0.5	-0.5	0.5	-0.56	0.67	0.43	0.8	253	10.1873
-0.9	0.5	0.5	-0.56	0.67	0.43	0.8	210	9.4565
0.5	0.5	0.5	-0.56	0.67	0.43	-0.4	263	10.2523
-0.9	-0.5	-0.5	0.44	0.67	0.43	-0.4	115	6.4554
0.5	-0.5	-0.5	0.44	0.67	0.43	0.8	153	7.6083
-0.9	0.5	-0.5	0.44	0.67	0.43	0.8	128	6.9296
0.5	0.5	-0.5	0.44	0.67	0.43	-0.4	162	7.7356
-0.9	-0.5	0.5	0.44	0.67	0.43	0.8	154	7.6489
0.5	-0.5	0.5	0.44	0.67	0.43	-0.4	197	8.1896
-0.9	0.5	0.5	0.44	0.67	0.43	-0.4	163	7.7812
0.5	0.5	0.5	0.44	0.67	0.43	0.8	221	8.6070

Table P.2 Verification Design and Output for Center and Axial Points

FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
-0.25	0	0.17	-0.11	0	-0.43	0.4	152	9.0360
-0.9	0	0.17	-0.11	0	-0.43	0.4	133	8.3779
0.5	0	0.17	-0.11	0	-0.43	0.4	174	9.6853
-0.25	-0.5	0.17	-0.11	0	-0.43	0.4	146	8.7100
-0.25	0.5	0.17	-0.11	0	-0.43	0.4	158	9.2584
-0.25	0	-0.5	-0.11	0	-0.43	0.4	126	8.0648
-0.25	0	0.5	-0.11	0	-0.43	0.4	166	9.3829
-0.25	0	0.17	-0.56	0	-0.43	0.4	168	9.7455
-0.25	0	0.17	0.44	0	-0.43	0.4	133	8.0150
-0.25	0	0.17	-0.11	-0.33	-0.43	0.4	141	9.1217
-0.25	0	0.17	-0.11	0.67	-0.43	0.4	175	8.6184
-0.25	0	0.17	-0.11	0	-0.86	0.4	144	8.7755
-0.25	0	0.17	-0.11	0	0.43	0.4	168	9.4411
-0.25	0	0.17	-0.11	0	-0.43	-0.4	147	8.8770
-0.25	0	0.17	-0.11	0	-0.43	0.8	155	9.0550

Table P.3 Correlation Matrix

	FH	AP	DR	BP	BRCT	OST	DRCT	REQ	EBO
FH	1								
AP	0	1							
DR	-0.003	0	1						
BP	0.001	0	-0.005	1					
BRCT	0.003	0	-0.015	0.005	1				
OST	0.003	0	-0.015	0.004	0.015	1			
DRCT	-0.003	0	0.015	-0.004	-0.015	-0.015	1		
REQ	0.499	0.159	0.509	-0.414	0.407	0.273	0.090	1	
EBO	0.447	0.144	0.481	-0.657	-0.217	0.185	0.080	0.736	1

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Reparable items play a large role in determining the readiness of United States Air Force (USAF) weapon systems. Many factors characterizing flying tempo and item repair and transit time influence the level of fleet readiness. Readiness-Based Leveling (RBL) considers these factors as it seeks to maximize aircraft readiness as it allocates spare reparable items between bases and a depot. The purpose of this research was to demonstrate the validity of using response surface methodology (RSM) within the context of RBL in an effort to quantify the influences these factors have on aircraft readiness.

RSM applied designed experiments and least squares regressions in developing a series of empirical models quantifying correlations between one uncontrollable and seven controllable factors and RBL's output. Verification tests indicated the empirical models represented -- to a high degree -- the quantitative relationships present between the inputs and output of RBL. Although valid conclusions cannot be made from the models (a substitute input was used in place of a usual D041 input), the methodology as demonstrated is valid.

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