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Modeling Diminishing Marginal Returns: An Application to the Aircraft Availability Model

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THESIS

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Abstract

The Aircraft Availability Model (AAM) provides the Air Force with a worldwide peacetime requirement for reparable spare parts. Reparable spares are a critical cost area for the Air Force with an average annual budget of one billion dollars. This research models AAM methodology as it relates to the concept of diminishing marginal returns in resource application. Three separate modeling techniques are investigated with the goal of reformulating the AAM as a mathematical programming model that provides a comparable solution and a capable tool for the conduct of sensitivity analysis.

The general formulations presented here are continuous non-linear, continuous linear, and piecewise linear discrete/continuous models. Two formulations of the piecewise linear discrete/continuous model are presented. The piecewise linear model based on AAM sort values shows the dominance of an optimization routine relative to the AAM shopping list greedy heuristic. Research shows that the optimization algorithm provides the greatest improvement to mission design series (MDS) availability per dollar spent. The piecewise linear model based on availability rates provides the capability to maximize the MDS availability level. It has the potential to obtain the highest possible MDS availability relative to reparable spares inventory levels. This mathematical model is discussed in complete detail as a robust platform for conducting extensive post-optimality analysis.

MODELING DIMINISHING MARGINAL RETURNS: AN APPLICATION TO THE AIRCRAFT AVAILABILITY MODEL

I. Introduction

Background

The Air Force Materiel Command implemented the Aircraft Availability Model (AAM) to determine its safety level for reparable items in the Spring of 1988. Reparable spares are a critical cost area for the Air Force with an average annual budget of one billion dollars. The AAM provides the Air Force with a predetermined level of support at the lowest cost (Rexroad, 1992:1). The goal of the model is to minimize procurement costs while achieving a required end item availability target. The model determines system availability based on an established stock level of line-replaceable-units (LRUs). LRUs are defined as those items that can be removed and replaced directly on the aircraft while it remains on the flight line. If an LRU is "down" and there is no spare to replace it, the aircraft is not available. Therefore, these LRUs are a primary contributor to aircraft availability.

The basic mathematical approach of the AAM involves the computation of each item's contribution to system availability. Factors included in the model are the quantity of the item used on each aircraft, the proportion of aircraft in the inventory that contain the item, the number of aircraft being modeled, and the expected back order (EBO) value for the LRU. Each item's number of spares, or stock level, determines its EBO value.

System availability is simply the product of the computed item availability of those parts on the aircraft (Rexroad, 1982:5,6).

The AAM determines which parts to purchase and the order of these purchases based on a sort value. Each part is ranked according to the amount of improvement to aircraft availability the next unit of inventory will provide per unit price. The concept behind this marginal approach is to spend the next dollar on that part which provides the most improvement to system availability. This method provides the decision maker with a shopping list of those items which provide the most improvement to availability per dollar spent. Parts can be procured until funding is depleted or until the target availability value is reached.

The AAM provides a somewhat limited capability to conduct sensitivity, or "what-if" analysis. The model's current output only allows a decision maker to determine the impact of his or her procurement policies on availability goals and funding levels, since other issues are not represented in the model. An alternate formulation that captures the intent of the AAM while providing the capability for in-depth sensitivity analysis would prove valuable to a decision maker.

Statement of the Problem

The purpose of this research is to develop a reformulation of the AAM that provides a comparable solution to the current model while adding the capability to conduct relevant sensitivity analysis.

Research Approach

Each item's availability function demonstrates the principle of diminishing marginal returns. As the number of parts in inventory increases, a point is reached where the next part procured does not provide as much improvement in availability as the last. It is possible to determine the stock level required to realize this condition. This level represents the lower bound stock level for each unit. At the same time, it is possible to determine the level where the next item procured provides a negligible increase to system availability per dollar spent, a legitimate upper bound on the stock level.

Using AAM output data, these bounds can be explicitly defined. The output data provides the capability to construct a mathematical program that achieves an optimal solution comparable to that obtained by the AAM. Enrichment of this mathematical program to include additional constraints provides insight to the sensitivity of the model's optimal procurement plan to outside influences. The research will explore the ability of various mathematical modeling techniques such as continuous nonlinear programming, continuous linear programming, and piece-wise linear discrete/continuous programming to capture the impact of the diminishing marginal returns in a well defined region.

Scope

This study provides a general modeling approach that can be applied to any weapon system under consideration. Research focuses on a specific aircraft (the F-15E) based on AAM output relative to that aircraft. The result is a model representing those factors considered important to the modeled weapon system's availability. This basic framework provides the methodology required to apply equivalent techniques to other systems modeled by the AAM.

II. Literature Review

Overview

A reformulation of the AAM requires a comprehensive understanding of the model. This literature review provides the historical aspects of the model, including its relation to the Multi-Echelon-Technique-for-Recoverable-Item-Control (METRIC) family of spares models and the AAM's development. A discussion of the conceptual framework and mathematical basis of the AAM is included in this section. A review of current literature provides insight into inventory modeling based on its application to model building and constraints, including the use of reparable part models in industry. Finally, the concept of diminishing marginal returns is reviewed and applied to the alternate form of the AAM.

The METRIC Family of Models

The AAM is part of a family of Reparable Item Inventory Models based on the METRIC philosophy used by the United States Air Force. Other related models include the METRIC model, Mod-METRIC, Vari-METRIC, Dyna-METRIC, and the Aircraft Sustainability Model (ASM). In general, each of these models is constructed to minimize expected backorders or maximize aircraft availability. Each of these models assumes independent demand for parts; that is, the failure of any given part is assumed to have no influence on the status of any other parts. The METRIC family of models is then divided into two major categories: those that model stationary demand and those that model dynamic demand. Stationary demand models assume that the distribution of demand for parts over some future time period is constant. The dynamic demand models provide the capability of modeling varying distributions of demand that may occur during contingency operations. Figure 2-1 illustrates the development of the METRIC family of reparable item inventory models.

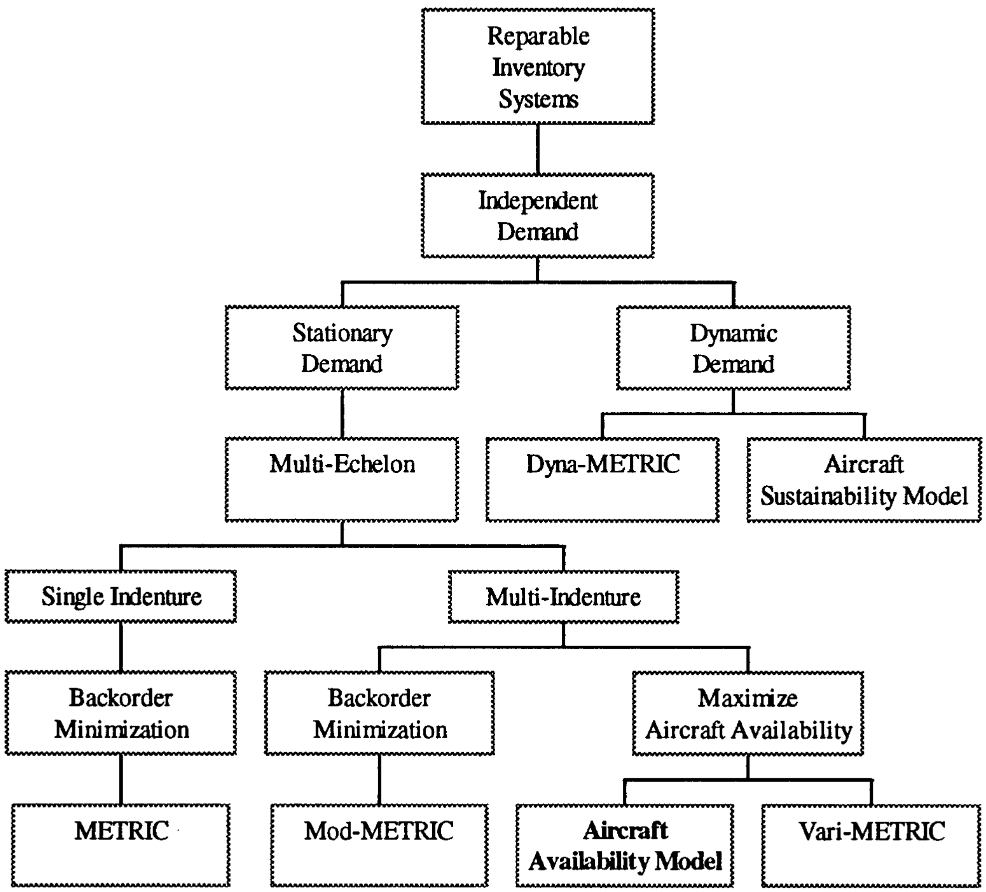


Figure 2-1. Development of Reparable Item Inventory Models (Klinger, 1994:15)

The AAM falls into the stationary demand class of METRIC models. This study is therefore focused in that area.

METRIC Development

A cornerstone of reparable item inventory theory is Palm's Theorem. This theorem enables the estimation of the steady state probability distribution of the number of units in repair from the probability distribution of the demand process and the mean of the repair time distribution (Sherbrooke, 1992:21).

> **PALM'S THEOREM:** If demand for an item is a Poisson process with annual mean *m* and if the repair time for each failed unit is independently and identically distributed according to any distribution with mean *T* years, then the steady-state probability distribution for the number of units in repair has a Poisson distribution with mean *mT* (Sherbrooke, 1992:21).

Palm's Theorem makes it unnecessary to measure the shape of the repair distribution. For any specified mean time *T*, regardless of the distribution, the steady state probability distribution for the number of units in repair is Poisson with mean *mT* (Sherbrooke, 1992:21).

The basis for the METRIC family of models is a mathematical theory that considers the multi-echelon and multi-indenture supply system of reparable parts used by the U.S. Air Force. The multi-echelon element of the supply system represents multiple levels of supply and repair. METRIC and the AAM model the base and depot levels of the hierarchy. Base level supply maintains those items that are immediately available upon a given demand. Base level repair handles those items that can be repaired without the need for extensive overhaul or rework. The depot maintains a supply of reparable units for any number of subordinate bases, thus it is an upper echelon element of the supply system. At the same time depot level repair operates at an upper echelon, conducting major repair and overhaul for those units received from its subordinate bases. The concept of multi-indentured parts breaks down units based on their relationship to the operational end item. The AAM defines operational end items as mission designs (MD) or mission design systems (MDS). (These are discussed in detail later.) First indenture items, or line replaceable units (LRU), are those units that are essential to the operation of the MD. If a first indenture item is down the MD is unavailable. Second indenture items are subunits of first indenture items, third indenture items are subunits of second indenture items, and so on. The status of units at each level of indenture effects the availability of the next higher assembly in the chain of indentured items (O'Malley, 1983:6-3).

The problem environment at its most basic level involves the diagnosis of a malfunction on an aircraft. Once determined, the malfunctioning item is removed from the aircraft and brought into base supply. If a spare is available, it is issued and installed on the aircraft; otherwise a backorder is established for that aircraft. When a base backorder for a first indenture item is issued, the corresponding aircraft has a "hole" and is non-mission capable; it is unavailable for service. This is the extent of the first-indenture stock item scenario. The METRIC model extends only to second indenture items called shop replaceable units (SRU) and two echelons of supply and repair at the base and depot levels as described above (Sherbrooke, 1992:6).

METRIC theory utilizes a system approach aimed at confronting issues on a larger than item level. "In the system approach, questions are asked such as: How can we insure that 95% of our scheduled aircraft flights will not be delayed for lack of spare parts? How much more money do we need to move from 95% to something higher?" (Sherbrooke, 1992:2). The major purpose of the METRIC model is to determine optimal stock levels for each item, subject to a constrained budget or a specified level of system performance such as availability of mission capable aircraft (Sherbrooke, 1967:123).

The theory supporting this purpose involves the development of item and system performance measures. There are two principle measures of item performance: fill rate, the expected percentage of orders filled at the time of placement, and backorders, the expected number of unfilled requests for demand (Sherbrooke, 1992:24). Reparable spares provide the capability to fill a backorder with an item that has been repaired if there is not an item in resupply. The primary system performance measure used in the METRIC model is aircraft availability, the expected percentage of the fleet not down for spares at a random point in time (Sherbrooke, 1992:27).

<u>Application of METRIC Theory in Literature</u>

METRIC theory provides a robust framework upon which several reparable spares models have been based. The basic development tree was provided earlier in this review. A brief description of each model in it is provided here. (For a more in depth discussion of reparable spares modeling see Nahmias, 1981.) Nahmias discusses the significance of METRIC in the field of reparable spares modeling. He notes that METRIC "is important in that it appears to capture many of the significant features of the problem of determining suitable spares levels in a large-scale reparable item inventory system, and, as a result, it is one of the few multi-echelon inventory models to be implemented" (Nahmias, 1981:258). Some weaknesses with the underlying assumptions used in the development of METRIC resulted in the development of other models aimed at addressing some of these weaknesses.

One important element not explicitly accounted for in the METRIC formulation is the relationship of parts in the hierarchical structure, that is, the relationship of an end item to its components or subassemblies (Nahmias, 1981:2). Muckstadt developed the MOD-METRIC model for the explicit consideration of the hierarchical parts structure and it was subsequently implemented by the Air Force Materiel Command (Muckstadt, 1973:472).

METRIC assumes an infinite number of servers at the repair level. This results in the application of the model under steady state conditions (Nahmias, 1981:261). Contingency or war-time operations are not considered under the steady state assumption, leading to the development of the real time METRIC model. This model does not assume first-in-first-out repairs at the depot. The order of depot repairs is determined by base demands generated by simple Poisson processes and repair times are assumed to be independent exponential random variables (Nahmias, 1981:265). The Aircraft Sustainability model is an extension of the AAM and considers contingency operations in its formulation. (For an in-depth discussion of the entire METRIC family of models see Klinger, 1994.)

This brief discussion of METRIC and related models illustrates the importance of the underlying theory and the interest in developing extensions to it with the goal of a deeper understanding of the development of an optimal inventory of reparable items.

AAM Background

In the early 1970s the Air Force established the need for an inventory parts procurement process that could provide a specified level of availability in a time of shrinking budgets. This requirement remains at the forefront of current planning in today's environment of decreased defense spending. The AAM computes the Air Force's worldwide peacetime requirement for reparable spare parts. "It permits the Air Force to set availability goals by aircraft type and compute a worldwide requirement for reparable spare parts which will satisfy these goals at a minimum cost" (Rexroad, July 1992:1).

The AAM Environment

The AAM is focused on those weapon systems containing items with a forecasted level of demand. This allows the computation of availability based on the percentage of aircraft that contain a full set of such parts. Issues such as scheduled or unscheduled on-aircraft maintenance, shortages of nonreparable items, shortages of other types of reparable parts, and the consolidation of reparable item shortages (aircraft cannibalization) are not considered in the model formulation (Rexroad, July 1992:2). As mentioned previously, demand activity is assumed to be steady for reparable spare parts. This is not to say demand is constant but rather it is determined in a "steady state" peacetime environment without surge levels required during wartime operations (Rexroad, July 1992:2).

The model considers a number of different types of aircraft stationed at bases in locations throughout the world. Each Mission Design (MD) or aircraft (F-15 or KC-135, for example) or its associated Mission Design Series (MDS) or subtype (F-15E or KC-135R for example) is considered the end item of concern. These aircraft are supported by an inventory of reparable spare parts that are maintained at a specified stock level. In addition, bases have a limited repair capability that is augmented by depots, which are industrial facilities with extensive repair and overhaul capabilities (Rexroad, July 1992:2).

When a failure occurs on an aircraft, the cause of the failure is isolated to a single component which is then removed and replaced with a serviceable spare from the base inventory as soon as one is available. If base supply has a serviceable unit in the current inventory of spares, aircraft downtime is limited to the time for removal and replacement of the designated LRU. The faulty unit is then either repaired at the local maintenance facility or it is shipped to a depot for extensive repair and/or overhaul. If the part is shipped to a depot, a requisition is placed with the depot for the shipment of a serviceable unit to the base (Rexroad, July 1992:2). This scenario illustrates the first level of indenture in the supply system and is identical to that modeled by the METRIC model.

Base level repair of the failed LRU may identify a failed subassembly within the unit itself. As noted earlier, these subassemblies are designated Shop Replaceable Units (SRU) and are second indenture level items. The faulty SRU is removed from the LRU and replaced by a serviceable item in base supply when possible. SRUs that are unrepairable at the base level are sent to the higher echelon for repair. The AAM definition of a down aircraft is one where no spare is available for a failed LRU; the unfilled demand (back order) causes a "hole" on the aircraft. Lack of a spare SRU delays the repair of the LRU; however, if serviceable spares of the next higher assembly are available, there is no direct SRU effect on aircraft availability (Rexroad, July 1992:3).

Calculation of Expected Back Orders

The AAM utilizes a measure of an aircraft's expected back orders (EBO) to calculate its availability. An EBO is defined as the time weighted unfilled user demands and is found using the following formula (Rexroad, July 1992:5).

$$EBO = \sum_{x>s} (x - s)\, p(x)$$

where,

s is the stock level

x is the number of units in resupply for that site

p(x) is the probability of having x units in resupply given by a negative binomial probability distribution.

The AAM derives the mean of the negative binomial probability distribution, p(x), from the following component data elements:

a. The base pipeline: the expected number of units in base repair or in transit from the depot to the base at a random point in time.

b. The depot pipeline: the expected number of units in depot repair or in transit from the base to the depot at a random point in time.

c. The number of bases at which demands for the component occur.

d. The ratio of depot demands generated at the bases to total depot demands.

e. The expected number of units condemned over the procurement lead time (Rexroad, July 1992:6).

These factors result in the calculation of the worldwide system EBO. This value includes EBOs as a result of both echelons of repair and resupply including stock levels and pipeline values at each level (Rexroad, June 1992:5).

The Negative Binomial Probability Distribution

Sherbrooke provides an in-depth analysis of the use of the negative binomial probability distribution (Sherbrooke, 1992: 60-62). He notes some important characteristics of the distribution which make it a powerful tool in inventory modeling. The negative binomial distribution is a discrete distribution defined by two non-negative arguments which specify the number of trials and the probability of success. This distribution is a generalization of the Poisson distribution, thus Palm's Theorem is applicable. In addition, like the Poisson distribution, the negative binomial distribution maintains independent increments, meaning the number of demands in two nonoverlapping time periods is independent. The two parameters that define the negative binomial distribution can be estimated for a specified mean and a variance to mean ratio greater than one. These characteristics make the negative binomial distribution applicable to modeling the probability of x units in resupply as demonstrated by the AAM.

Calculation of Item Availability

The EBO for each part is dependent on the number of spares for that item and the number of that item in resupply. This value is then translated to an item availability, $q_{h(k),i,n}$, by the following formula:

$$q_{h(k),i,n} = (1 - b(h(k),i)) + b(h(k),i) \cdot \left[1 - \frac{U_{h(k),i} EBO_{i,n}}{T(h(k),i)}\right]^{a(h(k),i)}$$

where,

$q_{h(k),i,n}$ = the probability that an aircraft of MDS h(k) is not missing a unit of component i with n spare units of component i in the system.

a(h(k),i) = the quantity per application (QPA) of component i and is the number of that item present on each aircraft.

b(h(k),i) = the future application percentage (FAP) and is the proportion of aircraft containing the item.

$T(h(k),i)$ = the number of units of component i installed on aircraft of MDS $h(k)$.

$U_{h(k),i}$ = the prorating factor for $MDS_{h(k)}$ and is given by

$$U_{h(k),i} = \frac{a(h(k),i) \cdot b(h(k),i) \cdot F_{h(k)}}{IP}$$

$$\text{where } IP = \sum_{h=1}^{H} \sum_{k=1}^{K(h)} a(h(k),i) \cdot b(h(k),i) \cdot F_{h(k)}$$

$F_{h(k)}$ is the flying hour program for MDS h(k) and IP is the total item program which is the number of flying hours accumulated by units of component i over all MDs (Rexroad, July 1992:26,27).

This formulation of item availability takes into consideration the sharing of components by the various MDs in the inventory by prorating the worldwide EBO for each part.

Calculation of Aircraft Availability

Aircraft availability, A_h, is defined, under the independence assumption, as the product of the LRU item availabilities:

$$A_h = \prod_i q_{h,i,n(i)} \cdot$$

This value provides a system level availability rate and is dependent on the stock level of the various parts (Rexroad, July 1992:13). As stock levels increase so does availability, thus the AAM determines which parts to purchase to achieve a predetermined availability rate based on a sort value, defined in the next section.

Sort Values

Optimal purchasing decisions required to meet the predetermined availability rate are constrained by the MDS budget level and are chosen from the "shopping list." The "shopping list" is generated based on those parts which provide the greatest improvement to availability per dollar spent. This type of marginal analysis is achieved through the formulation of the AAM sort value. Sort values correspond to each potential stock level of an LRU. Sort values are found using the following formula:

$$\text{sort value} = \frac{\ln(q_{h,i,n+1}) - \ln(q_{h,i,n})}{V_{h,i}C_i}$$

where C_i is the cost of a unit of component i and $V_{h,i}$ is a cost prorating factor that adjusts the full cost of common components to account for the benefit of sharing the spare among several aircraft types that use the same item. $V_{h,i}$ is found using the following formula which contains values defined previously.

$$V_{h,i} = \frac{\sum_{k=1}^{K(h)} a(h(k),i) \cdot b(h(k),i) \cdot F_{h(k)}}{IP}$$

This value is the same as the prorating factor used to prorate expected back orders, $U_{h(k),i}$, except in this case it is computed for the MD rather than the MDS. Thus, a spare unit of a common component has a sort value for each aircraft type. The AAM then prorates initialization costs to each of the using aircraft types and accumulates the component's contribution to each aircraft's starting availability rate (Rexroad, July 1992:27,28).

The availability curves are not independent of each other due to the common components across the several MDs. Buy decisions for one aircraft affect the availability of those aircraft that share common components. The AAM accounts for this situation by calculating separate sort value arrays for each shared component relative to each aircraft type. Purchasing decisions are based on the distinct sort value arrays and the final stock level for a particular item may differ for the sharing aircraft. In this case an average buy quantity is determined and the resulting cumulative cost is divided among the sharing aircraft (Rexroad, July 1992:28,29).

<u>Levels of Indenture</u>

The AAM is a multi-echelon (base and depot levels are taken into consideration in the calculation of EBOs), multi-indenture model. To this point SRU modeling has been only briefly mentioned. The AAM definition of availability considers only the status of LRUs, so the impact of SRU stock levels must be determined to model the multi-indentured inventory system. SRU backorders lengthen repair times for LRUs, lowering the probability of a spare LRU being serviceable when demanded (Rexroad, July 1992:32). Figure 2-2 illustrates the effect of SRU backorders.

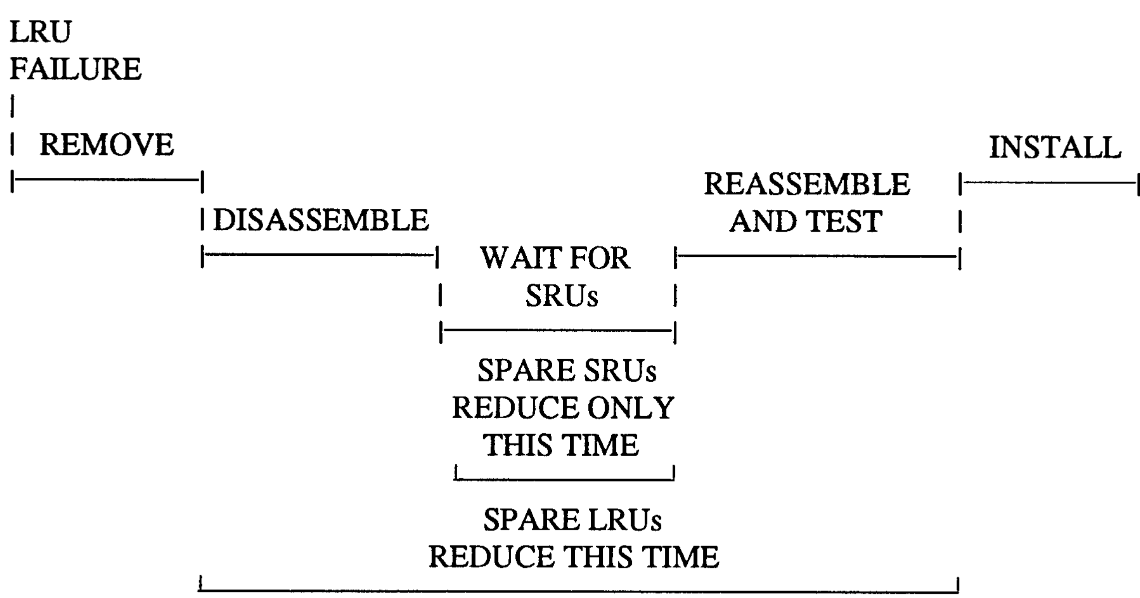


FIGURE 2-2. Levels of Indenture Repair Concept (Rexroad, July 1992:32)

Note the fact that spare LRUs have a direct impact on aircraft availability, while spare SRUs have an indirect impact measured as increased LRU repair time. SRUs are generally lower cost items so the purchase of a number of SRUs has an impact on the LRU EBO value. The impact of lower level items is measured with a recursive technique that begins at the lowest level of indenture. At each level the next higher assembly can be related in the same manner as the first indenture example between the aircraft and LRUs. The concept is to determine a number of subindentured items that are equivalent in price to the next higher assembly. Such an equivalent package of parts is termed an "Lsworth" (LRU's worth) of the component's subassemblies. It receives a sort value and is appropriately included on the optimal shopping list (Rexroad, July 1992:33). Just as the purchase of the next LRU increases aircraft availability, the purchase of an equivalent dollars worth of SRUs provides an improvement due to shorter repair time. If the SRU purchase shows a greater improvement in availability than the purchase of the next LRU, then the SRU package is purchased first.

Reparable Spares Modeling in the United States Navy and Army

The United States Navy (USN) designates items as either reparable or consumable. Consumable items are discarded when deemed no longer useful. Based on the economic value and the feasibility of repair some items are designated as reparable. Although reparable items account for only a small percentage of the total items stocked they account for more than half of the USN budget invested in spare parts. Considering this, research has been conducted to determine operating rules which will minimize the cost per unit time of operating the inventory portion of the reparable system (Schrady, 1967:392).

The USN "reparable system is comprised of three major organizations which maintain the reporting stock point, the user (non-reporting stock point), and the overhaul and repair (O&R) facility" (Schrady, 1967:391). Losses occur when failed items are discarded by the user (not returned to the O&R) and when the O&R cannot economically repair an item. These losses are replaced through the procurement of new items. An inventory of ready-for-issue (RFI) items, supplied by repair and procurement, as well as an inventory of non-ready-for-issue (NRFI) items awaiting for repair at the O&R must be managed. The system then is defined by two inventories, the primary having two input sources with different lead times and costs (Schrady, 1967:392).

The Navy model is deterministic and obtains economic order quantities for procurement and repair. Backorders are not permitted, reflecting the military objective of minimizing disservice as measured by shortages. The formulation treats the two inventories as interdependent parts of a total system, and jointly determines the optimal procurement and repair quantities (Schrady, 1967:392).

Two basic policies are investigated. One maintains the RFI inventory with items repaired in batches thus minimizing procurement activity and minimizing the NRFI inventory. O&R inducts a batch of failed items only when the NRFI inventory level reaches a defined "repair trigger." This type of deterministic system insures regularly spaced O&R inductions of fixed batch sizes resulting in an easy to implement procedure (Schrady, 1967:392). The second policy, the "substitution policy, supplies 100 percent of demand from repaired items until the supply of NRFI items decreases to a point where there are insufficient carcasses on hand to induct another batch. At this time, a procurement quantity is received, and inductions are suspended. This second policy minimizes the RFI inventory (Schrady, 1967:393).

The United States Army (USA) approaches reparable spare inventory modeling in a manner similar to that of the USN. Hoekstra proposes two approaches to handling the problem. The first is similar to the first USN model described above. Two inventories, one of repaired items and the other of newly procured items, are managed separate from one another. Items are repaired in batches when an economic repair quantity is reached and an economic procurement quantity is purchased when the established reorder level is reached. This model is deterministic and calculates the optimal reorder level, procurement quantity, and repair quantity using traditional inventory equations (Hoekstra, 1966:378).

The second policy modeled is a look ahead policy. This model coordinates the actions of each stock manager. Each inventory position and update policy is based on a minimum repair quantity that is established to prevent a stock out position (Hoekstra, 1966:379). The proposed mathematics are not as straightforward as those for the two stock policy but the aim of this model is to more accurately portray the real system. Ultimately the USA built a computer simulation in the SIMSCRIPT simulation language to compare the performance of the proposed systems (Hoekstra, 1966:382).

Reparable Spares Modeling in Industry

The United States military is not alone in the modeling of reparable spares inventory levels. The concept of maintaining an optimal level of non-disposable, high value, repairable units is shared by other elements of industry. Extensive work in this area has been conducted by the American Airlines Decision Technologies division. American manages its inventory of over 5,000 different types of repairable, or rotable, units supporting the operation of a fleet of over 400 aircraft with the use of a PC-based decision support system called Rotables Allocation and Planning System (RAPS). This system provides forecasts of rotable part demand, recommends least cost allocations of parts to airport locations, and calculates the availability level associated with the optimal allocation of each part (Tedone, 1989:61).

The American Airlines problem environment is very similar to that modeled by the family of METRIC models. Each operational aircraft is expected to have a complete complement of fully functional parts before departure. If a rotable part is found to be defective, the part is removed and replaced by a part in the local station stock room if possible. When a spare is issued from the local inventory, a message is sent to the repair facility or exchange base. The base maintains a supply of serviceable units available for use by the field stations. Upon receipt of the request the base ships an identical unit to the station while the station sends the faulty unit to the base for repair. Once repaired, the exchanged unit is placed in the inventory of the exchange base to begin the cycle again (Tedone, 1989:62). This scenario corresponds directly to the METRIC model's first level of supply indenture disregarding local repair capability.

American had been relying on a system developed in the late 1960s. After extensive research by the Decision Technologies and Materials Management divisions it was determined that the system needed to be updated. Their work culminated in the creation of RAPS.

> The purpose of RAPS is not only to recommend allocations of spare parts, but to assist inventory managers and analysts in performing many other analyses relating to rotable part control, such as

- Determining the impact of new fleets on part allocations;
- Making surplus and purchase decisions;
- Evaluating vendor proposals and terms;
- Forecasting future demand; and
- Anticipating allocation behavior through sensitivity analyses.

The ultimate result of a RAPS run is a least-cost allocation of parts. The cost function consists of the sum of inventory ownership costs and expected shortage costs. The solution (allocation) that yields the minimum value of this function is very sensitive to many other quantities (Tedone, 1989:63,64).

The allocation of parts to the stations to satisfy expected demand at a minimum cost is the problem assigned to the American Airlines Materials Management department. One of their roles is to "distribute parts to stations in a cost-effective manner, balancing the cost of part ownership against the cost of part shortage while maintaining an acceptable level of availability. The problem is to find the allocation of least total cost" (Tedone, 1989:62).

RAPS requires a significant amount of data which is managed by a real-time computer system called the Rotable Control System (RCS). Each time a part is removed a tracking record is created that maintains the history of the broken unit from removal to repair as well as that of the serviceable part sent to replace it. Tracking data includes: transit and repair times, part removal histories, and current allocation levels. In addition, American maintains other corporate data bases containing part specific information (Tedone, 1989:62).

Calculation of part demand is critical to the model's formulation. Demand for all parts analyzed in the system is forecasted by a two step process. The first step is the calculation of total system demand across all stations. The distribution of this total demand among the stations is the second step of the process (Tedone, 1989:64).

Total expected system demand is calculated using a linear regression to establish a relationship between monthly part removals and various functions of monthly flying hours. A rolling 18-month history of removals and flying hours is updated on a monthly basis and a best fit regression is used to calculate the coefficients of the relationship. The second phase of the forecasting process distributes the system demand forecast among the individual stations. RAPS assigns a weight to each station that reflects the amount of expected station activity. Station activity is based on the numbers of departing and overnighting aircraft, the scheduled volume of maintenance checks of various levels, and each station's history of part removals (Tedone, 1989:64). At the same time a pattern of demand is established for the stations. Both the volume and this station pattern determine the level of availability, which is a major driver for the total cost of an allocation (Tedone, 1989:65).

The total cost of an allocation is defined as the sum of the inventory ownership cost and the expected shortage cost. Shortage costs are driven by the availability of parts since rotable parts are essential to the operation of the aircraft (much like LRUs are essential to the operation of an MDS). These costs are equivalent to the cost of a flight cancellation. "The cost of a flight cancellation is a highly volatile quantity which varies by fleet, time of day, passenger load, alternative flight availability, and many other factors. In extreme cases the intangible loss of goodwill is a large component of cost" (Tedone, 1989:65). The model uses a network optimization algorithm to select the least costly cycle of cancellations. Shortage costs for each fleet of aircraft are evaluated in that manner. RAPS, like the AAM, then determines the optimal stock level of reparable parts that minimizes total system cost.

The American Airlines system illustrates a high level of capability that is not captured by current Air Force analyses. American's analysts are able to conduct extensive sensitivity analysis on a monthly basis. It is noted here that the data required for each RAPS run provides the capability to formulate those relationships that allow forecasts of the impact of current decisions. The effort and cost required for the Air Force to collect and maintain a data base similar to that maintained by American could outweigh the potential advantages of such a system. In addition, the Air Force has a larger system and serves more sights than American Airlines. A reformulation of current models such as the AAM, which is well established and understood, appears to provide a much more cost effective alternative for the conduct of effective sensitivity analyses.

Modeling Diminishing Marginal Returns

Current literature presents a series of modeling approaches for situations that demonstrate diminishing marginal returns. Diminishing marginal returns are defined as follows:

> As the amount of a variable input utilized by an activity increases, with other inputs held constant, a point is reached where increases in that particular input results in a decreasing marginal increase in output. Output increases, but at an ever decreasing rate per additional unit of resource. Ultimately, the allocation of additional resources to the activity is not justified (Henderson and Quandt, 1971:57).

The AAM can be modeled to fit in this framework. The current formulation demonstrates the concept of diminishing marginal returns (as defined above) where LRUs are considered to be resources applied to the objective of maximizing aircraft availability subject to budgetary limitations. As the number of LRUs in inventory increases, the marginal increase to aircraft availability decreases. Once viable availability functions are determined, a mathematical model can be constructed that provides the optimal level of availability and the corresponding stock level. Deckro and Hebert suggest three modeling approaches for diminishing returns in a project resource setting (1995:9).

The first, a Linear Approximation Model (LADRM), formulates a linear approximation of the nonlinear availability functions (Deckro and Hebert, 1995:9). Such a formulation could be modified so LRU availability is approximated over the possible range prescribed by the inventory upper and lower bounds. This approach requires the introduction of variables which represent the value of the availability function evaluated at its lower and upper bounds. These values indicate the maximum and minimum contributions of a given LRU to aircraft availability.

The slope of the linear approximation is defined as the value of the availability function evaluated at the upper bound (the highest availability level) minus the value of the availability function evaluated at the lower bound (the lowest availability level) divided by the difference between the lower and upper bounds. Considering the shape of a curve that demonstrates diminishing marginal increases as levels of input increase, the value of the approximated slope will be nonpositive (Deckro and Hebert, 1995:10). Alternatively, linear approximations of the LRU availability functions can be developed as a linear objective function that minimizes aircraft unavailability. The model will contain at a minimum a budget constraint. The number of parts to purchase are variables in this model. They are continuous between the specified upper and lower bounds.

This formulation has the advantage of being a traditional linear program. Furthermore, the model's variables are bounded. Many solvers are available to effectively solve large problems of this type. In addition, the solution lends itself to complete sensitivity analysis. Inventory levels and aircraft availability are a direct output of the model formulation and are available relative to the constrained procurement budget. This technique has drawbacks due to the linearization of the nonlinear availability functions which demonstrate diminishing returns. A level of accuracy and the impact of the effects of varying inventory levels may be lost (Deckro and Hebert, 1995:12-14).

Considering the nonlinearity of the availability functions, a viable solution technique is presented with a second approach, the Nonlinear Diminishing Returns Model (DRM). This model directly applies the LRU availability functions. If these functions are Karush-Kuhn-Tucker (KKT) regular and of the correct convexity, any number of available nonlinear programming codes can provide a solution. This model provides increased accuracy and captures the nonlinear nature of the diminishing returns thus providing a more accurate representation of the real world problem (Deckro and Hebert, 1995:14).

Deckro and Hebert point out two primary drawbacks to this solution technique outside the computational advantage of linear versus nonlinear models. First, the KKT restrictions can not always be met when generating the specific underlying functions. An example is a discontinuity within the range of variation for a resource availability for a particular activity. A second drawback is the requirement to assume a continuous resource application (Deckro and Hebert, 1995:15,16). These drawbacks apply to the formulation of the availability curves for each LRU. The curves generated need to be strictly concave so if discontinuities exist they will need to be removed. Additionally, parts are purchased as discrete increments to the stock level so the continuity issue needs to be addressed in an AAM reformulation. (The purchase of one third of an LRU is not feasible.) This leads to a third alternative solution technique which incorporates discrete stock levels.

The Discrete Diminishing Returns Model (DDRM) discretizes the nonlinear availability functions. The intent of this model is to purchase parts in a more realistic discrete fashion while capturing the diminishing returns effect (Deckro and Hebert, 1995:16). This formulation requires the development of zero-one variables which correspond to the possible discrete stock levels of each LRU. Each variable of this type that holds a value of one in the optimal solution indicates the stock level to maintain for the corresponding LRU. Note that for each LRU included in the model only one stock level can be maintained so the sum of its corresponding binary variables will be equal to one, commonly referred to as a special ordered set of type 1 (Deckro and Hebert, 1995:20).

This model is an integer program (IP) that can be solved with available commercial solvers. One limitation of IP solution techniques is that they generally require a large number of iterations and a substantial amount of computer time based on the number of integer variables modeled. This formulation provides an element that can speed the solution process due to the limitation of only one stock level per LRU. Special ordered set constraints limit the number of feasible solutions and can speed computer solution time. In addition, tight bounds on the possible discrete stock levels can aid in problem entry and solution time. The AAM items may exhibit such tight bounds in specific cases.

III. Methodology

Overview of Methodology

The purpose of this research is to suggest a reformulation of the AAM that provides a comparable solution while adding the capability to conduct relevant sensitivity analysis. The method of achieving this purpose involves the development of a viable mathematical programming model that can be modified to investigate the influence of various factors on the optimal solution while providing essentially the same or improved solution as the AAM. Post-optimality analysis incorporates the addition of necessary constraints as well as the analysis of those factors already in the model. This includes traditional sensitivity analysis of the right hand side (RHS) and objective function coefficients. In each case, a range of values is determined over which the respective values can vary and retain the current optimal basis. Simple sensitivity analysis varies each value independent of the other factors in the model. A parametric analysis of multiple factors provides potential for post-optimality excursions aimed at investigating the model's sensitivity to various inputs and potential trade-offs. Each element of the model is defined below.

Data

An AAM run provides the data required to formulate the mathematical model. (Data are from the March 1995 run of the AAM.) The data include the sort values and availability rates for relevant stock levels of those LRUs used on the specified MDS. The model also requires the prorated purchase cost for each component. This study models the AAM output data for the F-15E based on its current buy status, parts requirement, number of aircraft in the Air Force inventory, active funding level, and input received from Air Force Materiel Command Headquarters as of 7 November 1995.

Decision Variables

The decision variables are defined as the various stock levels of those LRUs used on the MDS being modeled (in this case the F-15E). Each variable name corresponds to the stock level and national stock number (NSN) associated with the LRUs. The following assumptions specify an acceptable range of NSN stock levels, as determined relative to output from an execution of the AAM. This allows the formulation of a bounded variable mathematical model.

Assumptions

The AAM is based on a marginal analysis of the improvement to availability per increment of inventory. A reformulation to a mathematical programming model requires the statement of its assumptions. First, the model assumes availability rates increase at a decreasing rate as inventory increases. This ensures the availability functions will be strictly monotonic and concave. (This will prove to be essential for the development of a tractable objective function.) Additionally, it is assumed the sort values are strictly decreasing at a decreasing rate (a monotonic and convex function). Both availability rates and sort values are bounded in the open interval from zero to one.

Objective Function

Formulation of the objective function is the most critical element of the proposed model. The ability to find an optimal solution comparable to that found by the AAM is governed by the shape of the functions represented in the overall objective function. The following factors are taken into consideration in the formulation of a workable objective function:

1. Accuracy: The optimal objective function value should provide a level of availability equal to or greater than that achieved by the AAM. Given that the greedy heuristic approach used in the AAM does not guarantee optimality, an optimization approach should meet or exceed its performance. (This subject will be discussed in further detail later.)

2. Solution time: The approximating function should be continuous and well behaved within the inventory item's upper and lower bounds to reduce the effort of the search algorithm. A continuous formulation is preferred over an integer formulation. While items are purchased in discrete increments, integer programming models often require much more time to reach a final solution than continuous models. It has been shown that the resources required to find an optimal solution to integer models increase exponentially with problem size (Jensen, 1990:9-1).

3. Sensitivity analysis: Integer problems do not readily provide a sensitivity analysis capability. A continuous model, be it non-linear, linear, or piecewise linear, allows the analyst to conduct extensive sensitivity analysis much more directly and readily than an integer formulation.

This research investigates three general objective function formulations: a continuous non-linear model, a continuous linear model, and a piecewise linear discrete/continuous model.

As discussed in Chapter II, the sort value associated with each LRU's stock level contains the natural logarithm of the improvement factor due to the n(j)+1 component divided by the prorated cost of that component. The objective is to maximize the improvement to aircraft availability as a function of the LRU stock levels. The initial objective function formulation considers the exponential nature of the availability values associated to each inventory level. As inventory increases, availability approaches an upper bound of one at a decreasing rate (decreasing marginal returns). A plot of availability data shows that the following functional form approximates the shape of the data (see Appendix A, page A-1 for a plot of the test case data):

$$
\begin{aligned}
1 - a \cdot e^{b \cdot x_{NSN}} &= q_{NSN} \\
a \cdot e^{b \cdot x_{NSN}} &= 1 - q_{NSN} \\
\ln(a) + b \cdot x_{NSN} &= \ln(1 - q_{NSN})
\end{aligned}
$$

where x_{NSN} is the stock level for the part designated by NSN and q_{NSN} is that part's availability contribution at that stock level. Manipulation of the approximating function as shown above allows a linear least squares regression to determine the "best fit" values of *a* and *b*. Table 3-1 contains the estimates for *a* and *b* used in the continuous nonlinear and linear models. Additionally, Table 3-1 provides regression R-squared and F-statistic values to demonstrate the quality of the approximated functional form. The data tabulated here show that the exponential approximation provides an extremely high level of accuracy and confidence. These conditions are indicated by R-squared values near one and F-statistics that in all cases indicate the approximated coefficients are significant at an alpha level of 0.05 in the one tail F-test.

Table 3-1. Regression Analysis of Availability Data

Var. #	NSN	Fitted a	ln(Fitted a)	Fitted b	R-squared	F-Statistic
NSN-1	1005012982522	0.00041062	-7.79785021	-0.90914223	0.99943109	10540.4208
NSN-2	1270013562584FX	0.00010236	-9.18703138	-0.44313548	0.85371114	23.3432
NSN-3	1560012912590FX	0.00168836	-6.38399507	-0.58961422	0.99888776	10777.0052
NSN-4	1620012409687	0.00119007	-6.73374072	-0.32992158	0.93945539	341.3685
NSN-5	1630012251893	0.12384435	-2.08872971	-0.05228081	0.99065109	22146.5369
NSN-6	1650012511153	0.00001616	-11.03318686	-0.32485814	0.91843142	90.077
NSN-7	1650013969218	0.00011345	-9.08413158	-0.74676460	0.99419713	1027.9708
NSN-8	1650013980036	0.00111488	-6.79900823	-0.55967646	0.99899287	11903.0741
NSN-9	1660013387046BO	0.00060903	-7.40363622	-0.62058812	0.95853028	208.0258
NSN-10	1660013389649BO	0.00668540	-5.00782883	-0.60584181	0.99662356	4132.3794
NSN-11	1680012283821	0.00001393	-11.18151106	-0.76880881	0.99111472	334.6371
NSN-12	1680012283822	0.00000316	-12.66652254	-0.80471896	0.96814382	30.3911
NSN-13	2835012180143	0.00036743	-7.90897545	-0.24335330	0.99958457	57747.8156
NSN-14	2835012188080	0.00000746	-11.80569868	-0.20339569	0.95036423	210.6144
NSN-15	4810012518480HS	0.00000254	-12.86772116	-1.58266767	1.00000000	undefined
NSN-16	4810012537542	0.00614650	-5.09187294	-0.67215120	0.99757433	4935.0789
NSN-17	4810013377136TP	0.00006043	-9.71409968	-0.48985035	0.97100772	234.4436
NSN-18	5821012483022FX	0.00024518	-8.31353708	-0.53032270	0.93312518	125.5798
NSN-19	5895012913073FX	0.00005469	-9.81377565	-0.64388566	0.85437785	35.2025
NSN-20	5895013640160LN	0.00270954	-5.91097487	-1.38879930	0.99688605	1600.6788
NSN-21	5895013823225FX	0.00083508	-7.08798666	-0.39531865	0.99932578	23715.0661
NSN-22	5985013902368EW	0.00357285	-5.63439104	-0.39528303	0.99829414	11704.3072
NSN-23	5985013934820EW	0.00051209	-7.57700415	-0.70941160	0.99940425	13420.5129
NSN-24	5998013410403FX	0.00003264	-10.33011201	-0.46259325	0.87528490	42.1097
NSN-25	6110012305147	0.00000170	-13.28597119	-0.10819770	0.86785439	26.2696
NSN-26	6605012400136FX	0.00435235	-5.43703911	-0.15738695	0.99446786	9347.6202
NSN-27	6610013195039	0.02439354	-3.71343697	-0.24717362	0.98516365	2523.2766
NSN-28	6615012444251	0.00001338	-11.22168162	-1.02304883	0.89836208	17.6777
NSN-29	6615013462155	0.00002089	-10.77608277	-0.38083218	0.99964274	22384.3496
NSN-30	6615013527003	0.00097102	-6.93716278	-0.27275089	0.98771809	2010.5137

The continuous non-linear objective function uses the following derived availability function for each LRU.

$$1-\hat{a}\cdot e^{\hat{b}\cdot x_{NSN}} = q_{NSN}$$

Where x_{NSN} is defined as the stock level for the part associated with NSN and q_{NSN} is that part's availability level at stock level x_{NSN}. The first objective function value is then the sum of the natural logarithm of each LRU's inventory level contribution to overall aircraft availability. (The sum of the natural logarithms maintains a linear model as opposed to the product form of aircraft availability as discussed in Chapter II. The exponential of the optimal objective function value is MDS availability.)

A continuous linear objective function is derived using the same values of *a* and *b* found by regressing the AAM data. The functional form for each LRU is as follows:

$$\ln(a) + b \cdot x_{NSN} = \ln(1-q_{NSN})$$

(All equation elements are as defined above.) The objective in this case is to minimize the sum of the natural logarithm of one minus the availability contribution of the LRU stock levels. Note that in this case the functional form results in a minimization of "unavailability." MDS availability is simply one minus the exponential of the optimal solution.

The final objective function formulation investigated here is a piecewise linear discrete formulation. The general form is applied to both availability data and LRU sort values. The concept for this formulation is to capture the discrete nature of the problem while maintaining a continuous and linear model. In general, piecewise linear formulations of non-linear equations require a mixed integer format with binary variables used to define each breakpoint. In the case of a concave function applied to a maximization formulation (convex in the minimization case), where the piecewise slope becomes less favorable as the value of the decision variable increases (as the stock level increases), a mixed integer formulation is unnecessary (Winston, 1994:485). Within the explicitly defined region where the function meets the conditions listed above, the formulation behaves as a piecewise mixed integer problem while maintaining continuous variables. Maintaining continuous variables allows a much more extensive sensitivity analysis capability. (Sensitivity analysis of an integer program is much more limited than that of a continuous linear program and generally requires multiple runs of various scenarios (Nauss, 1979:24-26). See Nauss for a comprehensive discussion of parametric integer programming.) The piecewise linear formulation also requires constraints which equate the sum over each LRU to one. This type of constraint in a continuous formulation ensures a valid convex combination of the feasible stock levels. In addition, only one stock level is possible for each LRU modeled.

Constraints

The initial formulation includes only a budget constraint. The budget level is determined by current Air Force allocations relative to the MDS modeled (the F-15E in this study). Additional constraints are added once the initial formulation is shown to achieve a comparable optimal solution to that of the AAM.

Several opportunities for additional constraints fit within the framework of the basic models explained above. Some specific examples include introducing sub-budgets at the LRU level, constraining the MDS availability, limiting the number of LRUs purchased, fixing the availability contribution of a suite of related LRUs to determine the impact on a part by part basis, and modeling subindentured items. These points will be expanded upon later in this study.

Base Model Formulation

The base model must demonstrate the capability to reproduce purchases comparable to those found using AAM methodology on a shopping list derived for a subset of data. These purchases ultimately result in the optimal stock levels for each LRU. Each model discussed above is presented in mathematical form below.

Continuous Non-linear Program (CNLP)

$$\begin{aligned} &\text{Max} \sum_{NSN} \ln(1 - \hat{a} \cdot e^{\hat{b} \cdot x_{NSN}}) \\ &\text{s.t.} \sum_{NSN} PC_{NSN} \cdot x_{NSN} \leq \text{MDS (F-15E) Budget Allocation} \\ &\quad LB_{NSN} \leq x_{NSN} \leq UB_{NSN} \qquad \forall\ NSN \end{aligned}$$

where x_{NSN} is defined as the stock level for the corresponding NSN, PC_{NSN} is the prorated cost of the NSN, and LB_{NSN} and UB_{NSN} are the NSN inventory lower and upper bounds respectively. The objective function maximizes the sum of the natural logarithms of the approximated availability rates at stock levels x_{NSN} which has the effect of maximizing MDS availability as a function of the NSN stock levels. The budget constraint allows parts to be purchased up to the MDS budget allocation.

Continuous Linear Program (CLP)

$$\text{Min} \sum_{NSN} (\ln(\hat{a}) + \hat{b} \cdot x_{NSN})$$
$$\text{s.t.} \sum_{NSN} PC_{NSN} \cdot x_{NSN} \leq \text{MDS (F-15E) Budget Allocation}$$
$$LB_{NSN} \leq x_{NSN} \leq UB_{NSN} \qquad \forall\ NSN$$

The model elements are defined as in the CNLP. The objective function minimizes the sum of the natural logarithms of the approximated aircraft unavailability at stock levels x_{NSN} which has the affect of minimizing aircraft unavailability as a function of the LRU stock levels.

Piecewise Linear Discrete/Continuous Model (using availability, q) (PLDMq)

$$\text{Max} \sum_{NSN} \ln(q_{NSN_i}) \cdot NSN_i$$
$$\text{s.t.} \sum_{NSN} PC_{NSN} \cdot i \cdot NSN_i \leq \text{MDS (F-15E) Budget Allocation}$$
$$\sum_{i=LB}^{UB} NSN_i = 1 \qquad \forall\ NSN$$
$$0 \leq NSN_i \leq 1 \qquad \forall\ NSN_i$$

where NSN_i is defined as a continuous variable in the region bound by zero and one for the corresponding NSN at stock level i. When NSN_i is equal to one, the stock level for NSN is i. The sum over i for each NSN is equal to one to ensure one feasible inventory position per part modeled. PC_{NSN} is the NSN prorated cost and is multiplied by the stock level, i, to determine the budget increment at that stock level. $q_{NSN,i}$ is the AAM availability rate for the corresponding NSN at stock level i.

The objective function is a monotonic and concave piecewise linear function defined by the discrete LRU availability rates. The objective function maximizes the sum of the natural logarithms of the LRU availability rates at each stock level i. This model provides the maximum MDS availability level based on the discrete LRU availability rates provided by the AAM as constrained by the MDS budget allocation.

Piecewise Linear Discrete/Continuous Model (using Sort Values) (PLDMsv)

$$
\begin{aligned}
&\text{Min} \sum_{NSN} SV_{NSN} \cdot NSN_i \\
&\text{s.t.} \quad \sum_{NSN} PC_{NSN} \cdot i \cdot NSN_i \leq \text{MDS (F-15E) Budget Allocation} \\
&\qquad \sum_{i=LB}^{UB} NSN_i = 1 \qquad\qquad \forall\ NSN \\
&\qquad 0 \leq NSN_i \leq 1 \qquad\qquad \forall\ NSN_i
\end{aligned}
$$

The model elements are defined as in the PLDMq except in this case the piecewise linear objective function is monotonic and convex as defined by the AAM sort values. The objective function minimizes the sum of the LRU sort values at stock level i. This model finds the optimal budget allocation that allows each LRU the potential to move down its vector of sort values which has the affect of providing the maximum improvement to MDS availability per dollar spent. The PLDMsv provides an objective that gives each LRU stock level the same weight as that used to determine its position on the AAM shopping list.

Model Size

The AAM includes 447 different LRUs for the F-15E with 10,260 sort values and their corresponding inventory levels. A reformulation based on the entire data set is a major endeavor and was not necessary to demonstrate the viability of the approaches investigated here. Of the 447 different LRUs, 73 (1,318 sort values) are unique to the F-

15E. This is indicated by a cost proration factor of one, meaning the part is not shared. Furthermore, those LRUs in the subset of unique items that contain three or less data points (32 LRUs), are in a full stock position (indicated by a negative buy position), and/or do not meet the assumptions outlined above are not modeled. This refined subset of LRU data (30 in the non-linear and linear models and 20 in the linear piecewise models) is used in the models presented here.

IV. Results and Analysis

Formulation of Availability Function Approximations

The initial objective function formulation of the mathematical programming model focuses on determining the best functional fit to the LRU availability data. Theoretically, a quadratic function provides a simple form which non-linear solvers can easily handle. The following techniques provide insight to the determination of the correct approximating function:

1. Three point quadratic approximation.
2. Quadratic least squares approximation.
3. Cubic least squares approximation.
4. Three point exponential approximation.

Each technique was applied to data for NSN 1005012982522 as a test case. Full Mathcad software package output for each technique is provided in Appendix A.

An analysis of the results in the test case shows that polynomials of order two and three do not accurately capture the functional characteristics of AAM output data. Most notably, the polynomial functions do not maintain the monotonic assumption within the required inventory bounds. The exponential function meets the model's assumptions and provides the best fit of those techniques investigated in this study (see Appendix A, page A-6). Specifically, one minus the exponential of the given stock levels provides the most accurate approximation of the LRU availabilities. The sum of square error between the fitted and actual values for the three point exponential approximation in the test case is

2.241E-11. This value is three orders of magnitude smaller than the comparable quadratic approximation formula.

Each series of LRU availability terms, when converted to a linear form and regressed, provides the least squares estimates of the exponential parameters shown in Table 3-1 (page 3-5). Based on these tests, each LRU's availability function in the non-linear model as defined in Chapter III has the following form:

$$1-\hat{a}\cdot e^{\hat{b}\cdot x_{NSN}}=q_{NSN}.$$

<u>Analysis of the Continuous Non-Linear Program (CNLP) Performance</u>

The individual LRU availability functions provide the basis for determining the optimal number of additional LRUs to purchase in the CNLP. The model determines the maximum sum of the natural logarithms of the availability functions evaluated at the optimal inventory levels for each LRU. Each variable has an upper and lower bound determined by the AAM availability values that meet the model assumptions. The LRU bounds are the same for all of the models discussed in this study so the models can be readily contrasted and compared.

The optimal solution to this model, as determined by the Microsoft Excel solver on a 486 DX2 personal computer, is similar to that found by the AAM. However, for several reasons the results are not satisfactory for the purpose of this research. (See Appendix B for the Excel results for this model.) Most notably the optimal stock levels are not provided in discrete increments. Furthermore, the approximation functions do not provide an acceptable level of accuracy for the following reasons. The continuous approximations are based on least squares estimates but ultimately model accuracy is limited due to the small marginal differences between data points. In many cases the marginal difference between availability values is on the order of 10^{-6}. Additionally, this formulation requires the sum of the natural logarithms of the approximated availability rates. These computational steps also contribute to the model's inaccuracy. This continuous model does provide a fully capable tool to conduct sensitivity analysis but does not provide the desired discrete solution.

Analysis of the Continuous Linear Program (CLP) Performance

Linear models provide fast solution times as well as a strong sensitivity analysis capability. The LP formulation uses availability data to develop linear functions based on the parameters of the exponential functions in the CNLP formulation. The solution to this model, as determined by the Microsoft Excel Solver, is not comparable to the AAM solution. (See Appendix C for the Excel results for this model.) All but one variable is at either its upper or lower bound. The solution technique chooses an NSN based on its objective function coefficient which is simply the slope of the linear approximating function. The most favorable variables are purchased to the upper bound and the least favorable are set to the lower bound. The final LRU chosen is purchased up to the budget limit. Given the sensitivity of the data and the magnitude of the coefficients, this approximation does not meet the objectives of this research. While the CLP is quick to solve, its accuracy is not acceptable as an alternative for the AAM.

Analysis of the Discrete/Continuous Model Performance

The discrete/continuous model is ideal for the problem environment presented by the AAM. The monotonic and concave (maximization) availability piecewise linear functions allow integer answers in a continuous formulation. (The same is true for the monotonic and convex (minimization) sort value piecewise linear functions.) The strictly increasing or decreasing at a decreasing rate assumption is necessary for this formulation to perform correctly. This requirement uncovered a problem with some of the AAM availability and sort value data.

The sort value data in all cases are strictly decreasing (this is required in the AAM formulation). Unfortunately, these same values do not decrease at a decreasing rate in all cases (they do not demonstrate diminishing marginal returns). The strictly concave assumption is violated. A like condition exists with availability data. These data inconsistencies are attributed to the AAM equal base assumption. This assumption requires additional parts to be held as part of the depot inventory until enough parts are purchased for each base to receive an additional item. The equal base assumption subsequently results in nondiminishing returns in those cases when the AAM redistributes those parts assumed to be held in the depot inventory (Rexroad, 1996).

The PLDMsv and PLDMq require monotonically decreasing and monotonically increasing at a decreasing rate break points, respectively, in order to maintain continuous variables and still offer a discrete solution. This study handles cases of nondiminishing returns by establishing a region about the original AAM optimal inventory purchases that comply with the model assumptions. Those values that do not meet the modeling assumptions are omitted from this demonstration of the approach.

This process explicitly defines the LRU upper and lower bounds that are used in both the PLDMq and PLDMsv. Acceptable ranges are determined based on sort value data as opposed to derived AAM availability rates. Sort values are used by the AAM to rank order the proposed LRU purchases and are therefore considered reliable. When availability data violates the monotonic and strictly concave assumption within the specified sort value ranges, acceptable data points are approximated and included to maintain equivalent bounds. (Thirteen of the 102 availability values modeled are approximated.) In practice, the proposed models should be based on actual AAM output data so that there is no accuracy loss due to approximation or data smoothing.

Data preprocessing for the purpose of this study includes filtering the full set of data to obtain the subset defined in Chapter III. The remaining data are screened as discussed above using an Excel spreadsheet to obtain the final model data set. Those data are then converted to MPS format (an industry standard format for optimization programs (CPLEX, 1994:81)) and loaded into the CPLEX optimization solver. CPLEX is used due to its capability to handle very large models (unlimited variables and constraints (Sharda, 1995:52)) and because it is an efficient and effective solver with sensitivity analysis capabilities. (CPLEX provides sensitivity ranges for the objective function coefficients and constraint right hand sides. These can be expanded through the use of the CPLEX callable routine library and FORTRAN programming.)

The concepts discussed above were applied to the formulation of the PLDMq and PLDMsv. For illustrative purposes, three arbitrary budget levels are implemented in each model. The first budget level is a maximum budget allotment that allows each LRU to be purchased at its upper inventory bound. This model is used to verify the model and data accuracy. Two intermediate funding levels show the model's performance in varying budgetary scenarios. Application of AAM methodology to the explicitly defined subset of data determines the shopping list optimal buy position and is shown in Appendix D. This solution is used as a base case to compare to the optimal solution achieved by the mathematical programming models. Model comparisons are shown in Tables 4-1 and 4-3. Appendix E contains the CPLEX solution output information.

Table 4-1. Comparison of the Shopping List and PLDMsv Optimal Solutions

	Budget=3851653.46		Budget=3185774.84		Budget=2700221.70	
	Shopping List	PLDMsv	Shopping List	PLDMsv	Shopping List	PLDMsv
1005012982522	5	5	4	5	3	4
1270013562584FX	2	2	1	1	0	0
1560012912590FX	5	5	5	5	2	5
1630012251893	131	131	131	131	131	125
1650013969218	2	2	0	0	0	0
1660013387046BO	6	6	4	5	3	3
1660013389649BO	13	13	11	13	9	10
2835012180143	4	4	4	1.529	1	1
2835012188080	4	4	0	0	0	0
4810012537542	13	13	12	13	10	13
4810013377136TP	5	5	5	5	5	4
5821012483022FX	6	6	6	6	4	5
5895012913073FX	11	11	11	11	9	9
5895013640160LN	6	6	4	6	4	4.401
5895013823225FX	4	4	4	3	4	3
5985013902368EW	14	14	14	14	12	13
5985013934820EW	8	8	7	8	5	8
5998013410403FX	3	3	2	3	0	0
6605012400136FX	11	11	9	9	9	8
6610013195039	42	42	42	42	42	39
Optimal Solution	1.38223E-08	1.38223E-08	1.65542E-08	1.41259E-08	2.53607E-08	1.88853E-08
System Availability	0.99831671	0.99831671	0.99804701	0.99810042	0.99763562	0.99793697

This research includes the PLDMsv to determine the impact of implementing an optimization approach as opposed to the greedy heuristic used by the AAM. The objective function coefficients provide each variable the same weight as the AAM shopping list. Opposing solution techniques based on the same objective function vectors can easily be compared and contrasted on a decision variable level. Table 4-1 shows that the PLDMsv clearly outperforms the AAM methodology on the bottom line. The PLDMsv achieves a higher level of system availability per funds applied. The increased improvement in availability achieved by the PLDMsv at a budget level of $2,700,221.70 results in an MDS availability of Q=0.99793697 as compared to the shopping list Q of 0.99763562. (It should be noted that while the numerical differences in the objective functions is small, in this model and in those that follow, literally hundreds of thousands of dollars may be reallocated on the basis of a difference of this magnitude.) A case by case analysis of those items that have different values in the optimal solutions provides insight to the trade-offs being made. Table 4-2 provides a summary of the trade-offs made between the shopping list and PLDMsv optimal solutions.

The PLDMsv purchases items that provide a greater improvement to availability than the AAM purchases for the same budget allocation. This is achieved through the elimination of some items and the addition of others. The additions in Table 4-2 are those items that the PLDMsv purchased below the AAM cut-off sort value. These are items the greedy heuristic could not fit under the budget limitation. The optimization approach selected these items and gave up other items above the AAM cut-off sort value to achieve a higher increase in MDS availability at the same budget level.

Table 4 -2. Trade-Off Analysis of the Shopping List and the PLDMsv Optimal Solutions

Additions

Traded NSNs	Variable	Sort Value	Stk lvl	Cost	Cost @ SV
1005012982522	1	0.0000000005730797	4	29716.86	118867.44
1005012982522	1	0.0000000013760097	3	29716.86	89150.58
	Net Improvement	-0.0000000008029300		Net Increase	29716.86
1560012912590FX	1	0.0000000003652814	5	3231.73	16158.65
1560012912590FX	1	0.0000000017456548	2	3231.73	6463.46
	Net Improvement	-0.0000000013803734		Net Increase	9695.19
1660013389649BO	1	0.0000000009054756	10	17408.46	174084.60
1660013389649BO	1	0.0000000016763722	9	17408.46	156676.14
	Net Improvement	-0.0000000007708966		Net Increase	17408.46
4810012537542	1	0.0000000002067420	13	4377.40	56906.20
4810012537542	1	0.0000000018321009	10	4377.40	43774.00
	Net Improvement	-0.0000000016253589		Net Increase	13132.20
5821012483022FX	1	0.0000000009481328	5	17058.08	85290.40
5821012483022FX	1	0.0000000018820221	4	17058.08	68232.32
	Net Improvement	-0.0000000009338893		Net Increase	17058.08
5895013640160LN	0.5991434	0.0000000006813538	4	33661.84	80673.08
5895013640160LN	0.4008566	0.0000000001091371	5	33661.84	67467.85
5895013640160LN	1	0.0000000011372132	4	33661.84	134647.36
	Net Improvement	-0.0000000003467224		Net Increase	13493.57
5985013902368EW	1	0.0000000008677973	13	13342.04	173446.52
5985013902368EW	1	0.0000000012878343	12	13342.04	160104.48
	Net Improvement	-0.0000000004200370		Net Increase	13342.04
5985013934820EW	1	0.0000000002232926	8	8484.08	67872.64
5985013934820EW	1	0.0000000019062088	5	8484.08	42420.40
	Net Improvement	-0.0000000016829162		Net Increase	25452.24
	Total Improvement	-0.0000000079631238		Total Increase	139298.64

Subtractions

Traded NSNs	Variable	Sort Value	Stk lvl	Cost	Cost @ SV
1630012251893	1	0.0000000019016047	131	8795.12	1152160.72
1630012251893	1	0.0000000026855104	125	8795.12	1099390.00
	Net Loss	0.0000000007839057		Net Decrease	52770.72
4810013377136TP	1	0.0000000012087739	5	8050.00	40250.00
4810013377136TP	1	0.0000000013728717	4	8050.00	32200.00
	Net Loss	0.0000000001640978		Net Decrease	8050.00
5895013823225FX	1	0.0000000017741379	4	53030.98	212123.92
5895013823225FX	1	0.0000000017959398	3	53030.98	159092.94
	Net Loss	0.0000000000218019		Net Decrease	53030.98
6605012400136FX	1	0.0000000019291702	9	15505.66	139550.94
6605012400136FX	1	0.0000000022839261	8	15505.66	124045.28
	Net Loss	0.0000000003547559		Net Decrease	15505.66
6610013195039	1	0.0000000018841562	42	3313.76	139177.92
6610013195039	1	0.0000000020472842	39	3313.76	129236.64
	Net Loss	0.0000000001631280		Net Decrease	9941.28
	Total Loss	0.0000000014876893		Total Decrease	139298.64
	Total Change	-0.0000000064754345		Total Change	0.00

Table 4-3 shows that the optimal buy positions and the resultant optimal MDS availability levels are not the same using AAM methodology and the PLDMq. Like the PLDMsv, the PLDMq finds an optimal solution based on the objective function coefficients. A trade-off analysis in this case differs somewhat from that between the shopping list and the PLDMsv. The objective function coefficients do not apply the same weights to each variable modeled. The objective of the PLDMq is not to find the greatest *per item improvement to availability* but to find the *maximum MDS availability level.*

Table 4-4 provides an item by item analysis of those differences in the shopping list and PLDMq optimal solutions.

Table 4-3. Comparison of the Shopping List and PLDMq Optimal Solutions

	Budget=3851653.46		Budget=3185774.84		Budget=2700221.70	
	Shopping List	PLDMq	Shopping List	PLDMq	Shopping List	PLDMq
1005012982522	5	5	4	4.065	3	3
1270013562584FX	2	2	1	2	0	0
1560012912590FX	5	5	5	5	2	5
1630012251893	131	131	131	131	131	130
1650013969218	2	2	0	0	0	0
1660013387046BO	6	6	4	4	3	3
1660013389649BO	13	13	11	12	9	10
2835012180143	4	4	4	4	1	1
2835012188080	4	4	0	0	0	0
4810012537542	13	13	12	12	10	11
4810013377136TP	5	5	5	4	5	3
5821012483022FX	6	6	6	5	4	5
5895012913073FX	11	11	11	7	9	7
5895013640160LN	6	6	4	5	4	4
5895013823225FX	4	4	4	4	4	4
5985013902368EW	14	14	14	14	12	12.084
5985013934820EW	8	8	7	7	5	6
5998013410403FX	3	3	2	2	0	0
6605012400136FX	11	11	9	9	9	9
6610013195039	42	42	42	37	42	37
Optimal Solution	-1.68471E-03	-1.68471E-03	-1.95490E-03	-1.69693E-03	-2.36718E-03	-1.88526E-03
System Availability	0.99831671	0.99831671	0.99804701	0.99830451	0.99763562	0.99811651

Table 4-4. Trade-Off Analysis of the Shopping List and PLDMq Optimal Solutions

Additions

Traded NSNs	Variable	ln(q)	Stk lvl	Cost	Cost @ SV
1560012912590FX	1	-0.0000962446313659	5	3231.73	16158.65
1560012912590FX	1	-0.0005127214191654	2	3231.73	6463.46
	Net increase	0.0004164767877995		Net increase	9695.19
1660013389649BO	1	-0.0000173701508602	10	17408.46	174084.60
1660013389649BO	1	-0.0000331305488106	9	17408.46	156676.14
	Net increase	0.0000157603979504		Net increase	17408.46
4810012537542	1	-0.0000036000064800	11	4377.40	48151.40
4810012537542	1	-0.0000075200282754	10	4377.40	43774.00
	Net increase	0.0000039200217954		Net increase	4377.40
5821012483022FX	1	-0.0000112700635069	5	17058.08	85290.40
5821012483022FX	1	-0.0000629319801755	4	17058.08	68232.32
	Net increase	0.0000516619166686		Net increase	17058.08
5985013902368EW	0.9161545	-0.0000315529043241	12	13342.04	146680.44
5985013902368EW	0.0838455	-0.0000019175685125	13	13342.04	14542.71
5985013902368EW	1	-0.0000344405930704	12	13342.04	160104.48
	Net increase	0.0000009701202338		Net increase	1118.67
5985013934820EW	1	-0.0000075200282754	6	8484.08	50904.48
5985013934820EW	1	-0.0000155401207471	5	8484.08	42420.40
	Net increase	0.0000080200924717		Net increase	8484.08
	Total Increase	0.0004968093369194		Total Increase	58141.88

Subtractions

Traded NSNs	Variable	ln(q)	Stk lvl	Cost	Cost @ SV
1630012251893	1	-0.0002135728050480	131	8795.12	1152160.72
1630012251893	1	-0.0002207743688676	130	8795.12	1143365.60
	Net Decrease	0.0000072015638196		Net Decrease	8795.12
4810013377136TP	1	-0.0000049000120050	5	8050.00	40250.00
4810013377136TP	1	-0.0000111500621617	3	8050.00	24150.00
	Net Decrease	0.0000062500501567		Net Decrease	16100.00
5895012913073FX	1	-0.0000001400000098	9	8338.98	75050.82
5895012913073FX	1	-0.0000006300001984	7	8338.98	58372.86
	Net Decrease	0.0000004900001886		Net Decrease	16677.96
6610013195039	1	-0.0000003900000760	42	3313.76	139177.92
6610013195039	1	-0.0000013400008978	37	3313.76	122609.12
	Net Decrease	0.0000009500008218		Net Decrease	16568.80
	Total Decrease	0.0000148916149867		Total Decrease	58141.88
	Total Difference	0.0004819177219327		Total Difference	0.00

This analysis shows that the PLDMq purchases items that provide a higher level of MDS availability than the AAM optimal purchases for the same budget allocation. This is achieved through the elimination of some items and the addition of others. The additions in Table 4-4 are those items that the PLDMq purchased below the AAM cut-off sort value. These are items that the greedy heuristic could not fit under the budget limitation. The optimization routine selected these items and gave up other items above the AAM cut-off sort value to achieve a higher level of MDS availability. The greatest advantage provided by the PLDMq is that it does not search for specific items that provide the greatest improvement for each dollar spent. Rather, the PLDMq provides the highest achievable MDS availability level for a given budget allocation.

The above analysis shows that the objective of the PLDMsv is equivalent to that of the AAM. Each technique provides the greatest improvement in availability per dollar spent (constrained by the MDS budget allocation) toward the goal of achieving the highest level of MDS availability. The objective of the PLDMq formulation is to maximize MDS availability subject to the same MDS budget allocation. A comparison of the PLDMsv and PLDMq optimal solutions and resultant MDS availability rates provides insight to which model provides the highest level of MDS availability. The PLDMsv optimal inventory position results in an MDS availability of Q=0.99793697 (exp(-0.002065158)). The PLDMq provides a Q of 0.99811651 (exp(-0.001885264)). It is noted here that thirteen of those availability rates included in the PLDMq are estimated to ensure a monotonic piecewise function, but the success of this formulation remains clear.

Analysis of the Model Solution Techniques

The solution differences noted here are attributed to the nature of the separate solution techniques and objectives. As discussed previously, AAM methodology is a greedy heuristic that selects those items that provide the greatest improvement to availability in descending order until the budget limit is reached. The AAM shopping list approach is similar to a greedy heuristic used to provide an initial solution to Binary Knapsack problems. Like the AAM shopping procedure, the greedy heuristic considers potential items that have been sorted by "increasing indices and insert[s] each new item into the Knapsack if it fits" (Martello and Toth, 1990:28). Martello and Toth show that the greedy heuristic obtains a feasible solution but provides a worst case performance ratio (defined as the optimal continuous solution divided by the heuristic solution) of one half

(Martello and Toth, 1990:29). The greedy heuristic can not guarantee an optimal solution, illustrating a common characteristic of heuristic problem solving known as satisficing. “Heuristic problem-solving methods are strongly associated with satisficing, as opposed to optimizing, behavior. Heuristic methods do not normally guarantee optimal solutions” (Daellenbach, et. al., 1978:650). Optimality is traded for speed.

CPLEX implements the dual simplex algorithm to achieve an optimal solution. This type of algorithm systematically searches the region of feasible solutions to determine the most profitable tradeoffs between budget levels and objective function improvement. Each model's output is the “true” continuous optimal solution, to the degree of accuracy of the data, for the given set of data.

Data Scaling

The CPLEX data found in Appendix E differs from the values in Tables 4-1 and 4-3 by a factor of 1E+16. CPLEX operates with a maximum optimality tolerance of nine decimal places. AAM sort value data is provided in sixteen decimal digit format. Model formulation using this type of raw data results in an extremely sensitive model that is affected by round off error. The natural logarithm of availability rates experiences the same condition as the LRU availability rates approach one. Therefore, the PLDMq is also scaled by a factor of 1E+16. Scaling is required to maintain the AAM's implied significant digits. Given the incremental difference in the AAM's data, the CPLEX routine does not always converge to the same optimal solution when solving a problem that has not been scaled to account for values beyond the ninth decimal place. All post optimality analysis is conducted on output data that has been re-scaled to the original AAM decimal accuracy.

Explanation of the Optimal Basic Feasible Solution

The model output data in Tables 4-1 and 4-3 and Appendix E shows that all model variables are set to discrete inventory positions except for one NSN in all cases. This condition exists due to the model formulation and solution algorithm. The simplex solution algorithm defines each feasible solution as a linear combination of the linearly independent columns of the problem matrix. By definition every feasible solution is a basis of m variables, where the problem matrix contains m rows (constraints) and n columns (variables) (Hillier and Lieberman, 1986:108). The models discussed here have 21 constraints (20 constraints summing to one to ensure a valid convex combination of feasible stock levels, one for each NSN, and the budget constraint) and 102 variables (the 102 data points which define the concave/convex piecewise linear functions). This combination of model parameters results in one more basic variable than NSN modeled and results in one non-integer inventory level.

An acceptable all integer solution is achieved by rounding the non-discrete variable down. The resulting solution may be integer sub-optimal but still exceeds the availability level provided by the AAM optimal buy position. For example, the PLDMq base case requires rounding NSN 5985013902368EW down to a stock level of 12. This results in an objective function decrease of 9.701171634E-7 and a decrease in MDS availability from 0.99811651 to 0.99811555. This example is provided for illustrative purposes only but it shows a negligible decrease in MDS availability to obtain an all integer solution. The resulting solution, however, still exceeds the AAM's results for the same case.

Traditional Sensitivity Analysis

Linear programming provides sensitivity data for the right hand side of the constraints and the objective function coefficients as a post-optimality analysis tool. Data for the PLDMq at a budget level of 2,700,221.70 dollars is provided in Appendix F. This data shows that the budget can decrease to a level of \$2,699,103.03 and increase to a level of \$2,712,445.07 and retain the current optimal basis. (The current mix and values of purchases remains the same except that the fractional amount of the non-discrete stock level varies until a less or more favorable purchasing option is reached.) The availability rate provided by NSN 1005012982522 at a stock level of four can be decreased from the current value of 0.99998862 to 0.99998224 or increased to a value of 0.99999736 and still remain in the optimal basis. This type of analysis is valuable in assessing the impact of such factors as technological improvements or part degradation due to extended shelf life, for example.

Modeling Additional Constraints

Those models proposed here provide an analysis capability beyond the traditional sensitivity tools discussed above with the addition of properly formulated constraints. The additional constraints offered here are examples of the potential of these models. Each constraint is formulated and then applied in the PLDMq to indicate possible impacts on the current optimal solution.

The first proposed constraint models a suite of items that are required to obtain a minimum level of availability. A constraint of this type can be applied in the example of subindentured items that contribute to the availability of a major component such as an engine. Those items that make up the end item of concern are included in a single constraint with the natural logarithm of the availability rate at the associated stock level as a constraint coefficient. The sum of these availability rates is greater than or equal to the natural logarithm of the desired major component availability level. This constraint has the following form:

$$\sum_{NSN \in S} \sum_{i} \ln(q_{NSN_i}) \cdot NSN_i \geq \ln(\mathit{availability})$$

where S is defined as the set of parts that make up the next higher assembly and *availability* is the required availability rate of the next higher assembly.

For illustrative purposes, four arbitrary NSNs were chosen to formulate a constraint as defined above and applied in the PLDMq. The current optimal availability level provided by these NSNs is 0.99972854 (exp(-0.002715)). The additional constraint requires an availability level of no less than 0.999777 (ln(availability) = -0.00223025). The CPLEX output of the PLDMq with this additional constraint is in Appendix G, page G-1. The additional availability requirement for this small subset of NSNs results in a reallocation of the MDS budget resource. The optimal inventory position in this case results in an MDS availability slightly lower than the original optimal value (from 0.99811651 to 0.99798229). This example shows that by requiring a higher level of availability for a major subassembly on the aircraft a portion of MDS availability is sacrificed. While the current AAM formulation would require multiple runs to analyze such a scenario, the proposed model allows these types of trade-offs to be considered directly.

Other possible constraints are based on the concept of sub-budgets within an MDS budget allocation. Those LRUs that are included as part of a sub-budget are summed as in the MDS budget constraint and are further limited at the sub-budget level. The formulation of this type of constraint is as follows:

$$\sum_{\mathrm{NSN} \in S} \sum_{\mathrm{i}} \mathrm{PC}_{\mathrm{NSN}} \cdot \mathrm{i} \cdot \mathrm{NSN}_{\mathrm{i}} \leq \mathrm{Sbudget}$$

where S is defined as the set of parts that make up the specified sub-budget, Sbudget.

Four arbitrary NSNs were chosen to illustrate the implementation of a sub-budget constraint in the PLDMq. The NSNs chosen require $422,032.54 of the MDS budget allocation in the current optimal solution. The additional constraint limits these parts to an allocation of $350,000.00. (CPLEX output of the PLDMq with the addition of this constraint is in Appendix G, page G-4.) The sub-budget restriction results in a redistribution of the MDS budget and a subsequent decrease in the optimal MDS availability rate from 0.9981165 to 0.99809438.

A similar constraint requires a subset of parts to meet a minimum budget allocation.

$$\sum_{\mathrm{NSN} \in S} \sum_{\mathrm{i}} \mathrm{PC}_{\mathrm{NSN}} \cdot \mathrm{i} \cdot \mathrm{NSN}_{\mathrm{i}} \geq \mathrm{Sbudget}$$

where S is defined as the set of parts that make up the specified sub-budget, Sbudget.

The same NSNs modeled in the first sub-budget constraint are modeled here. In this case the budget allocation to these parts is required to meet or exceed $500,000.00. (CPLEX output for this model is found in Appendix G, page G-8.) Once again the optimal budget allocation is adjusted to meet the conditions of the new constraint. The impact is a decrease in MDS availability from 0.99811651 to 0.99806890.

The number of parts purchased can be managed beyond the LRU upper and lower bounds through the formulation of constraints of the following type.

$$\sum_{NSN \in S} \sum_{i} i \cdot NSN_i \leq \text{Stock Level}$$

where S is defined as the set of parts subject to the specified stock level. Constraints of this type apply in various scenarios including the modeling of storage limitations and inventory control programs.

Those NSNs used to demonstrate the sub-budget constraints are used in the formulation of an example inventory control constraint. The total number of parts purchased for this subset of NSNs in the original optimal solution is 20. The additional constraint restricts the total inventory of these part to an upper bound of 15. The optimal MDS availability for this model is 0.99807595 a decrease of 0.00004056 from the basic model (see Appendix G, page G-11 for the CPLEX output for this model).

The above constraint examples demonstrate the versatility and strength of the post-optimality analysis capability of the PLDMq formulation. Note that with the inclusion of new constraints, the optimal basis requires an additional basic variable. This results in one more non-integer stock level. (The conditions of the new constraint are met at equality and the budget is expended to its upper bound.) This condition can be successfully handled by rounding those variables that are non-integer to the nearest integer that satisfies the corresponding constraint. The resulting solution may be integer suboptimal but the additional insight provided outweighs the small decrease in MDS availability.

Parametric Analysis of the RHS and Objective Function

Parametric analysis allows a model parameter to vary over a wide range of values and computes changes to the optimal basis and the resultant effect on the optimal objective function value. Advanced parametric analysis allows the specification of a direction to vary the right hand side or objective function vector and then computes changes to the optimal basis and objective function value over a specified range. In the case of multiple changes to the right hand side, a parametric analysis also determines when constraints become binding, non-binding or infeasible. The CPLEX optimization package does not provide a parametric analysis routine as part of the basic package. The Linear, INteractive, and Discrete Optimizer (LINDO) system does offer the capability to conduct a parametric analysis of both the objective function coefficients and the values of the constraint right hand sides (Schrage, 41-46:1991). The MPS format of the model data developed for use with CPLEX was translated to an acceptable format for the LINDO package and loaded there.

Parametric analysis of the budget constraint provides the decision maker with a powerful tool to investigate the impact of budget changes. For example, the Department of Defense regularly conducts budget "what-if" drills where Air Force funding is subject to a certain level of cutbacks. (There are not many drills to determine how to spend excess defense funding.) Decision makers are then tasked with a determination of what impact a certain percentage cut in funds will have on current programs. Figure 4-1 provides a graph of output from a parametric analysis of the budget constraint in the PLDMq (for parametric output data see Appendix H, page H-1). This graph simulates a thirty percent decrease in the original $2,700,221.70 budget.

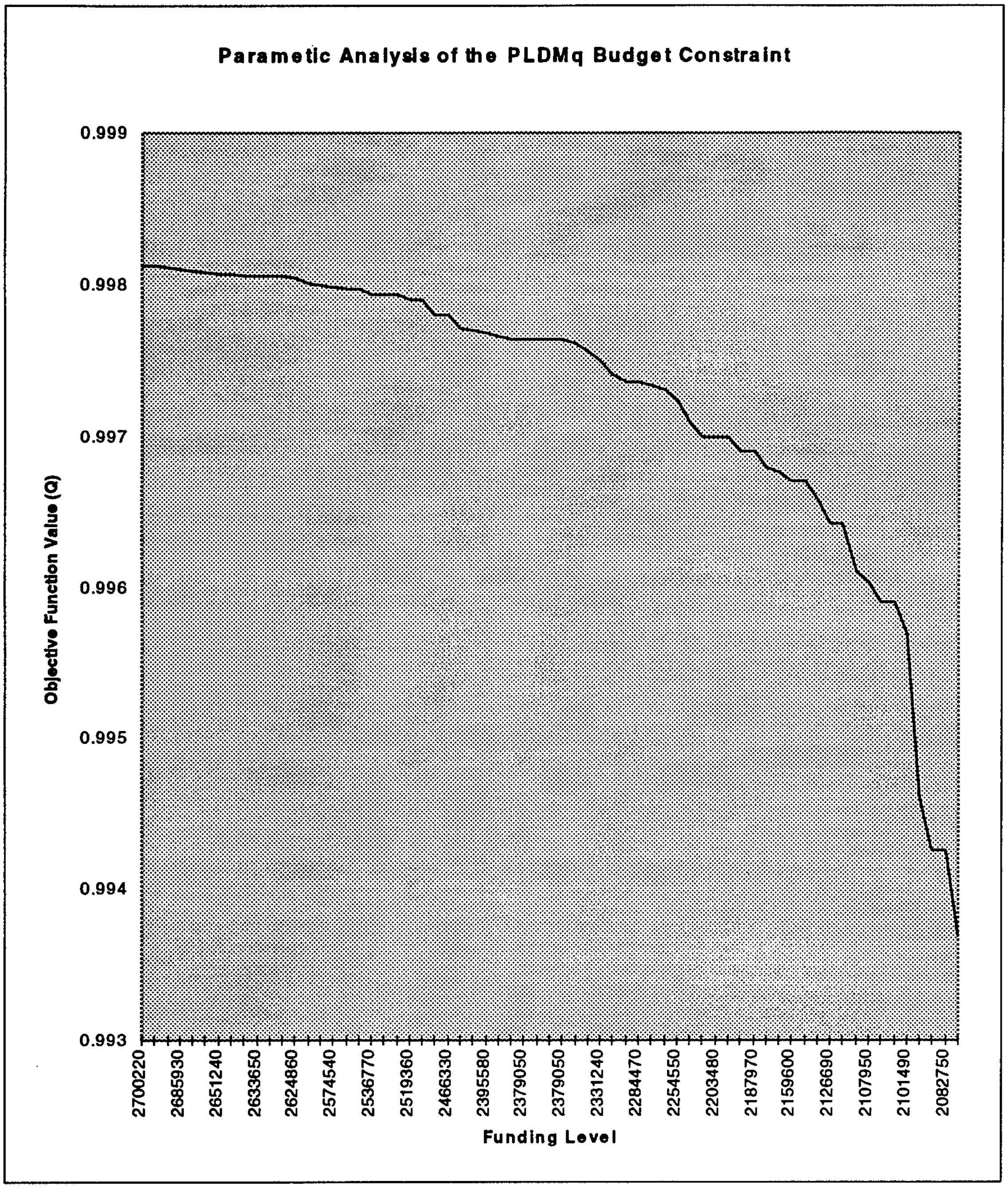


Figure 4-1. Graph of the Parametric Analysis of the PLDMq Budget Constraint

The range of budget values specified in LINDO is from the current funding level to a lower bound of $1,900,200.00 (an approximate thirty percent decrease). The parametric output data and Figure 4-1 show that the model becomes infeasible below a funding level of $2,079,520.00. The current formulation forces the purchase of at least the current lower bound stock level of those parts that have sort values greater than the cutoff sort value determined by the original funding allocation and the AAM shopping list. This example parametric analysis shows that a thirty percent cut results in an unacceptable level of availability. To reach this level of reduction, a decrease in the acceptable minimum availability will be required. (If such drastic cuts were to be considered the bounds could be changed to determine the impact.) A review of Figure 4-1 clearly indicates where the drop off in availability accelerates as funding levels decrease, a critical piece of information in a down-sizing agency or company. Additionally, it is possible to determine the cost of achieving a specified level of MDS availability from that data provided by a parametric analysis. The optimal purchases for each LRU at the specified MDS availability are obtained by noting those variables that have left and entered the original optimal basis.

A parametric analysis of more than one element of the RHS vector indicates the impact of potential resource trade offs. For example, the impact of reallocating funds among those certain groups of LRUs that are constrained to a specified sub-budget level can be investigated with a parametric analysis. The direction vector is defined in such a manner that each dollar decrease to one LRU sub-budget is added to another LRU sub-budget.

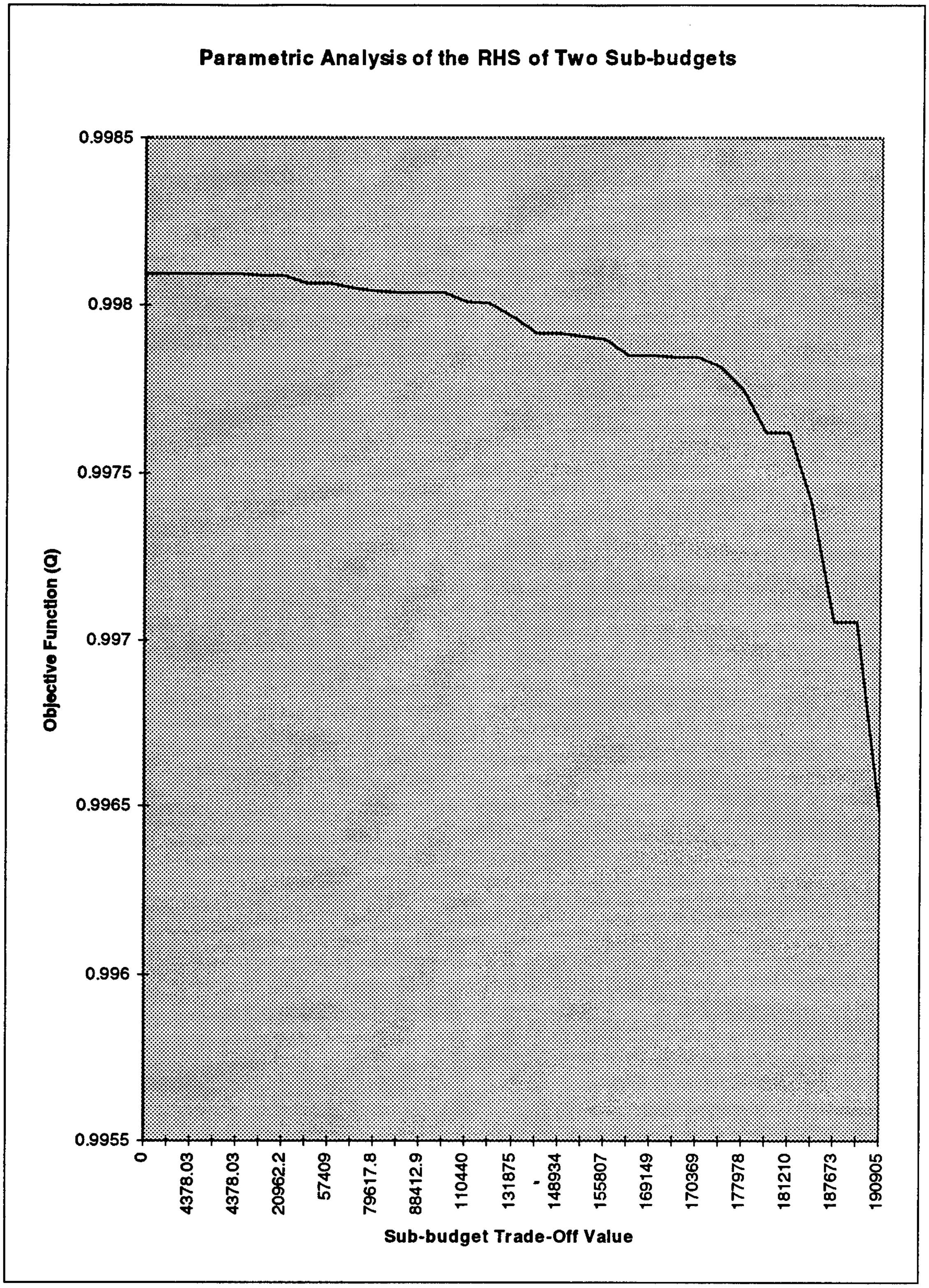


Figure 4-2. Graph of the Parametric Analysis of the RHS of Two Sub-budgets

LINDO output for the parametric analysis of the RHS of two sub-budget constraints is used to construct the graph in Figure 4-2 and can be found in appendix H, page H-2. This type of analysis provides insight to the impact of the allocation of resources. A parametric analysis of multiple constraint right hand sides determines the optimal allocation of program resources. Figure 4-3 is the graph of a specific region of parametric data for the trade-offs in sub-budget allocations. This figure focuses on the region of data that indicates the optimal resource allocation. A reallocation of resources

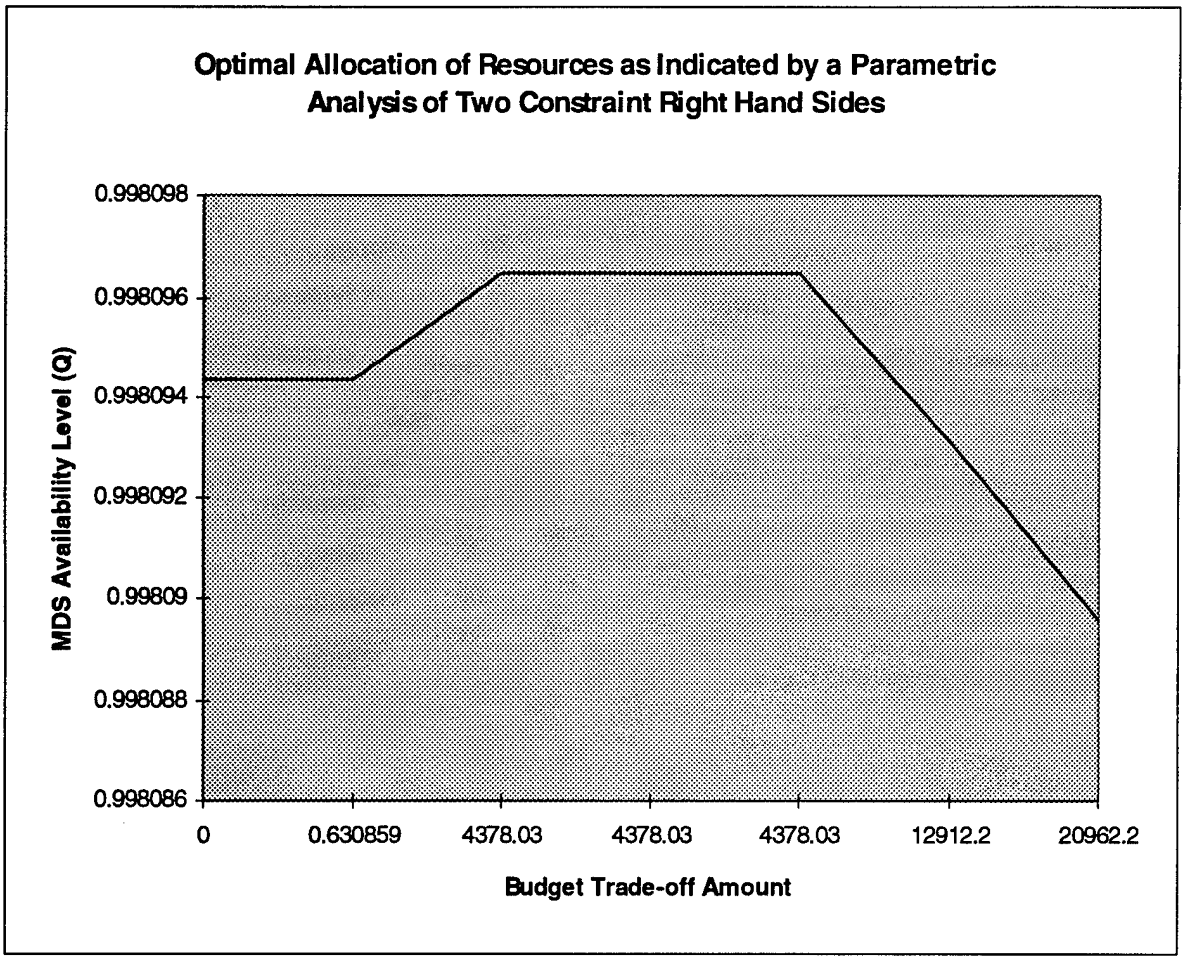


Figure 4-3. The Optimal Allocation of Resources as Indicated by a Parametric Analysis of Two Constraint Right Hand Sides.

between sub-budgets from the initial proposed levels results in an improved overall availability level. The optimal MDS availability level of 0.99809648 is obtained at a budget reallocation of $4,378.03. This budget reallocation actually results in three alternate optimal inventory positions. A practical analysis similar to this provides the decision maker with a great deal of flexibility and insight that does not currently exist in the AAM.

In addition to budgetary trade-offs, multiple inventory control constraints can be compared to determine the optimal allocation of warehouse storage space. Applications in the parametric modeling arena are limited only by the operational setting and the ability of the analyst to model that setting. The output data includes those variables that have left and entered the optimal basis at each step along the specified direction. Noting the changes at each step provides the analyst with the optimal basic solution and objective function value. A well designed parametric analysis provides the optimal allocation of resources over a specified range of values and eliminates the need for several model runs with various constraint right hand side values.

This reformulation of the AAM, subject to its limiting assumptions, provides the ability to extend the analysis of aircraft availability beyond what is currently available to the Air Force. Given the large funding levels necessary to maintain a mission ready airfleet, building upon the already tested models in METRIC and the AAM appears to be a fruitful area for further research.

V. Summary, Conclusions and Recommendations

Summary of Research Effort

The Aircraft Availability Model provides the Air Force with a worldwide peacetime requirement for reparable spare parts. The Air Force reparable spares program operates on an average annual budget of one billion dollars. This research investigates the environment for reparable spares modeling through an in-depth review of METRIC theory and its applications. A review of modeling techniques used in the U. S. Armed forces and related commercial applications provides insight to the methodology being used in the reparable spares arena.

AAM methodology is modeled as it relates to the concept of diminishing marginal returns in resource application. This research applies three separate modeling techniques toward the goal of a reformulation of the AAM as a mathematical programming model that is a capable tool for the conduct of relevant sensitivity analysis.

The general formulations presented here are continuous non-linear, continuous linear, and piecewise linear discrete/continuous models. The continuous non-linear and linear formulations do not meet the objectives of this research effort for reasons discussed in Chapter IV. Two formulations of the piecewise linear discrete/continuous model are presented. The piecewise linear model based on AAM sort values shows the dominance of an optimization routine relative to the AAM shopping list, a greedy heuristic. Research shows that the optimization approach provides the greatest improvement to MDS availability per dollar spent. The piecewise linear model based on LRU availability rates provides the capability to maximize not the improvement to availability but the actual MDS availability rate. It has the potential to obtain the highest possible MDS availability from the reparable spares inventory. This mathematical model is discussed in complete detail as a robust platform for conducting extensive post-optimality analysis.

Conclusions

Current AAM methodology provides the Air Force with a good technique to establish reparable spares inventory levels. The AAM and the METRIC family of models are based on sound principles and have provided excellent results since introduced in the 1970s. This research demonstrates the capabilities of mathematical programming models that are based on valid AAM output data. The goal of this study was to provide a mathematical programming model that provides an optimization based inventory position comparable to that found by the AAM that is capable of a robust sensitivity analysis. Research shows that the proposed reformulation of the AAM provides a higher level of MDS availability while adding a full complement of post-optimality analysis tools.

The suggested reformulation is based entirely on AAM output data. The implementation of the proposed mathematical model requires valid LRU availability rates as calculated by the AAM. Considering the sensitive nature of the model and the solution technique, data accuracy is of the utmost importance. In addition, the data must provide monotonic and concave breakpoints in the piecewise linear model. The strongest attribute of the piecewise linear models investigated here is the capability to provide a discrete solution with continuous variables. Large continuous variable models can be solved with relative speed and ease and provide the capability for post-optimality analysis.

Implementation of a piecewise linear discrete/continuous reformulation of the AAM will provide the Air Force with the highest level of MDS availability per dollar spent on reparable spares inventory levels while adding the capability to conduct extensive sensitivity analysis.

Recommendations for Future Research

Data preprocessing is the main area of concern with respect to implementing the proposed model. Generation of accurate data that meet the model assumptions is a critical requirement for successful results with the piecewise linear formulation. An automated system that can handle the sorting, filtering, and marginal analysis of the AAM output data would be a valuable asset in the implementation of a full scale model. Furthermore, an automated tool that translates the resultant data set to MPS format would be highly advantageous. This translation can be achieved either directly from a text file of data or from a spreadsheet application.

Recall from Chapter IV that the AAM assumes equal bases. Air Force Materiel Command has expressed interest in developing a formulation that models unequal bases. The modeling techniques presented in this study provide a sound framework upon which a model of that nature can be based.

The CPLEX optimization package is a powerful tool that solves extremely large models with relative speed and ease. The current package does not provide a parametric programming capability. The CPLEX callable library and FORTRAN or C programming languages can support a parametric analysis routine. (The CPLEX Optimization, Inc. manual, Using the CPLEX Callable Library, provides a tutorial on the use of CPLEX (pp.3-23) and shows examples of using the CPLEX Callable Library with FORTRAN and C (pp.287-309).) CPLEX is a widely accepted optimization package so research in this area would be widely applicable.

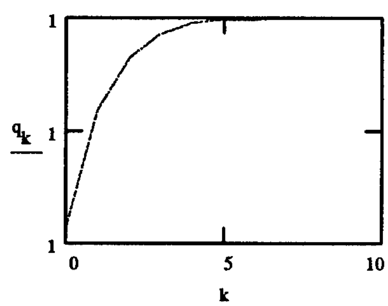
Appendix A. Functional Approximation of A AM LRU Availability data .

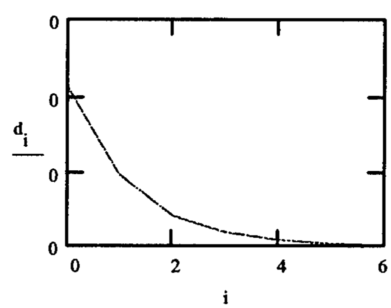
Technique 1. Three point quadratic approximation.
All analysis is based on AAM output data for NSN 1005012982522.
The three points used for the quadratic approximation technique are the LRU lower bound, the AAM optimal buy position, and the LRU upper bound.

$$q := \begin{bmatrix} .99962185 \\ .99983549 \\ .99993070 \\ .99997159 \\ .99998862 \\ .99999553 \\ .99999827 \\ .99999934 \end{bmatrix} \qquad k := 0..7 \qquad i := 0..6 \qquad d_i := q_{i+1} - q_i$$

$$d = \begin{bmatrix} 2.136400000000149\cdot 10^{-4} \\ 9.520999999990121\cdot 10^{-5} \\ 4.089000000007115\cdot 10^{-5} \\ 1.702999999997346\cdot 10^{-5} \\ 6.909999999971106\cdot 10^{-6} \\ 2.740000000001075\cdot 10^{-6} \\ 1.070000000047422\cdot 10^{-6} \end{bmatrix}$$

d is a vector indicating the region of test data is increasing at a decreasing rate.





These graphs confirm the fact that the values are strictly decreasing at a decreasing rate and smooth (monotonic and concave).

$x_k := k$

$q_0 = 0.99962185$

$q_3 = 0.99997159$

$q_7 = 0.99999934$

$$Q := \begin{bmatrix} (x_0)^2 & x_0 & 1 \\ (x_3)^2 & x_3 & 1 \\ (x_7)^2 & x_7 & 1 \end{bmatrix} \qquad q2 := \begin{bmatrix} q_0 \\ q_3 \\ q_7 \end{bmatrix} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} := Q^{-1}\cdot q2 \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{bmatrix} -1.566321428572715\cdot 10^{-5} \\ 1.635696428571865\cdot 10^{-4} \\ 0.99962185 \end{bmatrix}$$

The matrix manipulation shown here determines the parameters of the quadratic: a, b, and c.

$qfit_k := a \cdot (x_k)^2 + b \cdot x_k + c$ $\qquad error_k := qfit_k - q_k$ $\qquad sqerr_k := (error_k)^2$

$qfit_k$
0.99962185
0.999769756428571
0.999886336428572
0.99997159
1.000025517142857
1.000048117857143
1.000039392142857
0.99999934

q_k
0.99962185
0.99983549
0.9999307
0.99997159
0.99998862
0.99999553
0.99999827
0.99999934

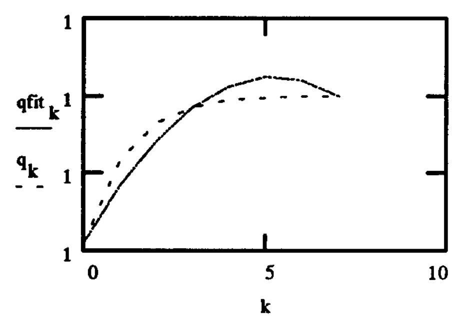
$error_k$
0
$-6.573357142858338 \cdot 10^{-5}$
$-4.436357142845182 \cdot 10^{-5}$
0
$3.689714285715073 \cdot 10^{-5}$
$5.258785714279401 \cdot 10^{-5}$
$4.112214285700855 \cdot 10^{-5}$
0

$sqerr_k$
0
0.000000004320902
0.000000001968126
0
0.000000001361399
0.000000002765483
0.000000001691031
0

$$sse := \sum_{k=1}^{6} sqerr_k$$

$sse = 1.210694138568814 \cdot 10^{-8}$

The error analysis of the quadratic fit indicates a strong relationship between the derived function and the actual data. Notice the error of the fitted points is zero and intermediate points are within 10^{-5} accuracy.



This graph of the fitted and actual data shows a violation of the strictly increasing assumption by the the three point quadratic fit.

Technique 2. Quadratic least squares approximation.
The three point quadratic is not monotonic so a least squares technique is applied in an attempt to remedy this condition.

$$q := \begin{bmatrix} .99962185 \\ .99983549 \\ .99993070 \\ .99997159 \\ .99998862 \\ .99999553 \\ .99999827 \\ .99999934 \end{bmatrix}$$

$k := 0..7$

$x_k := k$

$$x = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{bmatrix} \qquad X := \begin{bmatrix} 1 & x_0 & (x_0)^2 \\ 1 & x_1 & (x_1)^2 \\ 1 & x_2 & (x_2)^2 \\ 1 & x_3 & (x_3)^2 \\ 1 & x_4 & (x_4)^2 \\ 1 & x_5 & (x_5)^2 \\ 1 & x_6 & (x_6)^2 \\ 1 & x_7 & (x_7)^2 \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \\ 1 & 6 & 36 \\ 1 & 7 & 49 \end{bmatrix}$$

$$a := \left(X^T \cdot X\right)^{-1} \cdot X^T \cdot q \qquad a = \begin{bmatrix} 0.9996649445833336 \\ 1.435669642855242 \cdot 10^{-4} \\ -1.427172619052064 \cdot 10^{-5} \end{bmatrix}$$

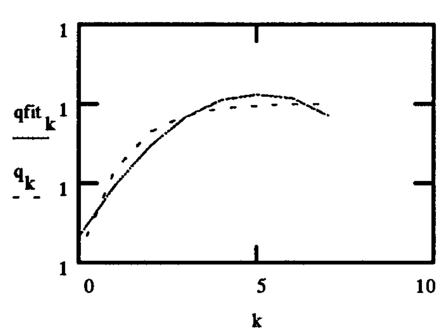
a is a vector of the least squares estimates of the quadratic parameters.

$$qfit_k := a_0 + a_1 \cdot x_k + a_2 \cdot (x_k)^2 \qquad error_k := qfit_k - q_k \qquad sqerr_k := (error_k)^2$$

$qfit_k$	q_k	$error_k$	$sqerr_k$
0.999664944583336	0.99962185	$4.309458333606742 \cdot 10^{-5}$	0.000000001857143
0.999794239821431	0.99983549	$-4.12501785689301 \cdot 10^{-5}$	0.000000001701577
0.999894991607145	0.9999307	$-3.570839285482741 \cdot 10^{-5}$	0.000000001275089
0.999967199940478	0.99997159	$-4.390059522019207 \cdot 10^{-6}$	0.000000000019273
1.00001086482143	0.99998862	$2.224482142987316 \cdot 10^{-5}$	0.000000000494832
1.000025986250001	0.99999553	$3.045625000075436 \cdot 10^{-5}$	0.000000000927583
1.000012564226191	0.99999827	$1.429422619059206 \cdot 10^{-5}$	0.000000000204325
0.999970598749999	0.99999934	$-2.874125000074113 \cdot 10^{-5}$	0.000000000826059

$$sse := \sum_{k=0}^{5} sqerr_k$$

$$sse = 6.275497532314834 \cdot 10^{-9}$$



The quadratic fit in this case provides a lower sum of squared errors than the three point approximation but is still not monotonic in the modeling region.

Technique 3. Cubic least squares approximation.

$$x = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{bmatrix} \qquad X := \begin{bmatrix} 1 & x_0 & (x_0)^2 & (x_0)^3 \\ 1 & x_1 & (x_1)^2 & (x_1)^3 \\ 1 & x_2 & (x_2)^2 & (x_2)^3 \\ 1 & x_3 & (x_3)^2 & (x_3)^3 \\ 1 & x_4 & (x_4)^2 & (x_4)^3 \\ 1 & x_5 & (x_5)^2 & (x_5)^3 \\ 1 & x_6 & (x_6)^2 & (x_6)^3 \\ 1 & x_7 & (x_7)^2 & (x_7)^3 \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \\ 1 & 6 & 36 & 216 \\ 1 & 7 & 49 & 343 \end{bmatrix}$$

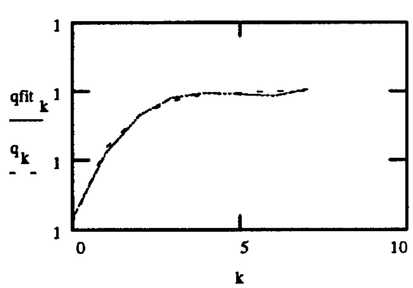
$$a := \left(X^T \cdot X\right)^{-1} \cdot X^T \cdot q \qquad a = \begin{bmatrix} 0.999629848333403 \\ 2.354857142551903 \cdot 10^{-4} \\ -4.936797618843025 \cdot 10^{-5} \\ 3.342499999899051 \cdot 10^{-6} \end{bmatrix}$$

$$qfit_k := a_0 + a_1 \cdot x_k + a_2 \cdot \left(x_k\right)^2 + a_3 \cdot \left(x_k\right)^3 \qquad error_k := qfit_k - q_k \qquad sqerr_k := \left(error_k\right)^2$$

$qfit_k$	q_k	$error_k$	$sqerr_k$
0.999629848333403	0.99962185	$7.998333402703395 \cdot 10^{-6}$	0.000000000063973
0.99981930857147	0.99983549	$-1.618142853054838 \cdot 10^{-5}$	0.000000000261839
0.999930087857159	0.9999307	$-6.121428413052854 \cdot 10^{-7}$	0.000000000000375
0.99998224119047	0.99997159	$1.065119046972374 \cdot 10^{-5}$	0.000000000113448
0.999995823571402	0.99998862	$7.203571402158993 \cdot 10^{-6}$	0.000000000051891
0.999990889999955	0.99999553	$-4.640000044520143 \cdot 10^{-6}$	0.00000000002153
0.999987495476129	0.99999827	$-1.07745238714374 \cdot 10^{-5}$	0.00000000011609
1.000005694999921	0.99999934	$6.354999921409643 \cdot 10^{-6}$	0.000000000040386

$$sse := \sum_{k=0}^{5} sqerr_k$$

$$sse = 5.130555851496908 \cdot 10^{-10}$$



The cubic approximation provides a lower sum of squared errors than the quadratic. This graph shows that the cubic is neither concave nor monotonic.

Technique 4. Three point exponential approximation.
The exponential approximation is derived using the same points as the initial quadratic fit.
The form of the exponential is $c\text{-}ae^{bx_i}\text{=}q_i$.

$$q := \begin{bmatrix} .99962185 \\ .99983549 \\ .99993070 \\ .99997159 \\ .99998862 \\ .99999553 \\ .99999827 \\ .99999934 \end{bmatrix} \qquad k := 0..7 \qquad x_k := 5 + k$$

Note: The current Air Force inventory of 5 parts is included here to scale the problem to an acceptable level for the Mathcad symbolic processor.

$$\frac{e^{x_6 \cdot b} - e^{x_0 \cdot b}}{e^{x_6 \cdot b} - e^{x_3 \cdot b}} = \frac{q_0 - q_6}{q_3 - q_6} \qquad \frac{q_0 - q_6}{q_3 - q_6} = 14.10869565220015$$

$$\frac{e^{11 \cdot b} - e^{5 \cdot b}}{e^{11 \cdot b} - e^{8 \cdot b}} = 14.10869565220015$$

The Mathcad symbolic processor solves the equality shown here for the values of b which satisfy the equation.

$$\begin{pmatrix} -.85775860008003064492 \\ -.857758600080030645 + 2.094395102393195492 3i \\ -.857758600080030645 - 2.0943951023931954923i \end{pmatrix}$$

$$b := -.85775860008003064492$$

The values for a and c are found here. The value for c is assumed to be one for all exponential approximations used in this study.

$$a := \frac{q_3 - q_6}{e^{11 \cdot b} - e^{8 \cdot b}} \qquad a = 0.027593485148153$$

$$c := q_6 + a \cdot e^{b \cdot x_6} \qquad c = 1.000000473375224$$

$$qfit_k := c - a \cdot e^{b \cdot x_k} \qquad error_k := qfit_k - q_k \qquad sqerr_k := \left(error_k\right)^2$$

$qfit_k$
0.99962185
0.99983989480161
0.999932370135986
0.99997159
0.999988223600123
0.999995278103243
0.99999827
0.999999538898185

q_k
0.99962185
0.99983549
0.9999307
0.99997159
0.99998862
0.99999553
0.99999827
0.99999934

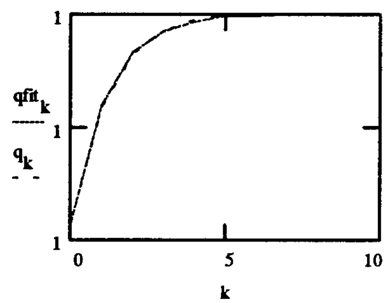
$error_k$
0
$4.404801610236042 \cdot 10^{-6}$
$1.670135986509003 \cdot 10^{-6}$
0
$-3.96399877145015 \cdot 10^{-7}$
$-2.518967572306963 \cdot 10^{-7}$
0
$1.988981850331584 \cdot 10^{-7}$

$sqerr_k$
0
0.000000000019402
0.000000000002789
0
0.000000000000157
0.000000000000063
0
0.00000000000004

$$sse := \sum_{k=1}^{6} sqerr_k$$

$$sse = 2.241221627787435 \cdot 10^{-11}$$

The fit for the exponential function is the best of those attempted in this study. The sum of squared error is three orders of magnitude better than that of the quadratic fit. The least squares exponential fit is determined using Microsoft EXCEL for each LRU included in those models in this study.



Appendix B. The Continuous Non-Linear Program

This Microsoft EXCEL workbook provides a nonlinear reformulation of the Aircraft Availability Model.
The functions for each LRU are based on output data for the F-15E MDS and are of the form $1-a*\exp(b*x)=q$.
The parameters of the exponential (a and b) are derived using a linear least squares regression technique
of the form $\ln(a) + b*x=\ln(1-q)$ and translated to the non-linear form. The objective function is the maximum of the sum
of the natural logarithm of q for each LRU, constrained by the budget and each LRU's acceptable range of stock levels.

		a	b	Variables	Fitted expon.	Price	s.t.: Budget	Lower bnd	Upper bnd	Obj. Fcn.
NSN-1	1005012982522	0.00041062	-0.90914223	2.5651544	0.999960132	29716.86	76228.3333	1	5	-4E-05
NSN-2	1270013562584FX	0.00010236	-0.44313548	2.2087201	0.999961536	13974.63	30866.0466	1	3	-3.8E-05
NSN-3	1560012912590FX	0.00168836	-0.58961422	5	0.999911461	3231.73	16158.65	0	5	-8.9E-05
NSN-4	1620012409687	0.00119007	-0.32992158	0	0.998809927	75403.35	0	0	0	-0.00119
NSN-5	1630012251893	0.12384435	-0.05228081	125	0.99982023	8795.12	1099390	125	131	-0.00018
NSN-6	1650012511153	0.00001616	-0.32485814	0	0.999983844	82240	0	0	0	-1.6E-05
NSN-7	1650013969218	0.00011345	-0.74676460	1	0.999946235	46000	46000	1	2	-5.4E-05
NSN-8	1650013980036	0.00111488	-0.55967646	0	0.99888512	36000	0	0	0	-0.00112
NSN-9	1660013387046BO	0.00060903	-0.62058812	3	0.999905357	57681.59	173044.77	3	6	-9.5E-05
NSN-10	1660013389649BO	0.00668540	-0.60584181	8.6671774	0.999964951	17408.46	150882.211	5	13	-3.5E-05
NSN-11	1680012283821	0.00001393	-0.76880881	0	0.999986071	19672.91	0	0	0	-1.4E-05
NSN-12	1680012283822	0.00000316	-0.80471896	0	0.999996845	20131.4	0	0	0	-3.2E-06
NSN-13	2835012180143	0.00036743	-0.24335330	1	0.999711936	67393.87	67393.87	1	4	-0.00029
NSN-14	2835012188080	0.00000746	-0.20339569	1	0.999993911	70669.82	70669.82	1	4	-6.1E-06
NSN-15	4810012518480HS	0.00000254	-1.58266767	0	0.99999746	13986	0	0	0	-2.5E-06
NSN-16	4810012537542	0.00614650	-0.67215120	9.8954387	0.999992056	4377.4	43316.2936	7	13	-7.9E-06
NSN-17	4810013377136TP	0.00006043	-0.48985035	2.2529203	0.999979958	8050	18136.0084	1	5	-2E-05
NSN-18	5821012483022FX	0.00024518	-0.53032270	3.4553707	0.999960767	17058.08	58941.9907	2	6	-3.9E-05
NSN-19	5895012913073FX	0.00005469	-0.64388566	7	0.999999397	8338.98	58372.86	7	11	-6E-07
NSN-20	5895013640160LN	0.00270954	-1.38879930	3.2531185	0.999970435	33661.84	109505.956	2	6	-3E-05
NSN-21	5895013823225FX	0.00083508	-0.39531865	4	0.999828214	53030.98	212123.92	2	4	-0.00017
NSN-22	5985013902368EW	0.00357285	-0.39528303	11.291135	0.999958821	13342.04	150646.78	9	14	-4.1E-05
NSN-23	5985013934820EW	0.00051209	-0.70941160	5.0159004	0.999985412	8484.08	42555.3003	2	8	-1.5E-05
NSN-24	5998013410403FX	0.00003264	-0.46259325	1	0.999979451	17142.59	17142.59	1	3	-2.1E-05
NSN-25	6110012305147	0.00000170	-0.10819770	0	0.999998302	11429.91	0	0	0	-1.7E-06
NSN-26	6605012400136FX	0.00435235	-0.15738695	9	0.998944269	15505.66	139550.94	4	9	-0.00106
NSN-27	6610013195039	0.02439354	-0.24717362	36	0.999996667	3313.76	119295.36	36	42	-3.3E-06
NSN-28	6615012444251	0.00001338	-1.02304883	0	0.999986619	44301.56	0	0	0	-1.3E-05
NSN-29	6615013462155	0.00002089	-0.38083218	0	0.999979107	13861	0	0	0	-2.1E-05
NSN-30	6615013527003	0.00097102	-0.27275089	0	0.999028979	32725.3	0	0	0	-0.00097
					0.994436557		2700221.7			-0.00558

Appendix C. The Continuous Linear Program

This Microsoft EXCEL workbook provides a linear reformulation of the Aircraft Availability Model.
The functions for each LRU are based on output data for the F-15E MDS and are of the form ln(a)+bx=ln(1-q).
The parameters of the linear approximation were derived using a linear least squares regression technique of the form ln(a) + b*x=ln(1-q). The objective function here is the minimum of the sum of natural log of 1-q for each LRU, constrained by the budget and each LRU's acceptable range of stock levels.

		ln(a)	b	Variables	Objective Fcn	Price	s.t.: Budget	Lower bnd	Upper bnd
NSN-1	1005012982522	-7.79785021	-0.90914223	1	-8.706992446	29716.86	29716.86	1	5
NSN-2	1270013562584FX	-9.18703138	-0.44313548	2.447969	-10.27181326	13974.63	34209.46	1	3
NSN-3	1560012912590FX	-6.38399507	-0.58961422	5	-9.332066153	3231.73	16158.65	0	5
NSN-4	1620012409687	-6.73374072	-0.32992158	0	-6.733740716	75403.35	0	0	0
NSN-5	1630012251893	-2.08872971	-0.05228081	125	-8.623830471	8795.12	1099390	125	131
NSN-6	1650012511153	-11.03318686	-0.32485814	0	-11.03318686	82240	0	0	0
NSN-7	1650013969218	-9.08413158	-0.74676460	1	-9.830896176	46000	46000	1	2
NSN-8	1650013980036	-6.79900823	-0.55967646	0	-6.799008228	36000	0	0	0
NSN-9	1660013387046BO	-7.40363622	-0.62058812	3	-9.265400582	57681.59	173044.77	3	6
NSN-10	1660013389649BO	-5.00782883	-0.60584181	13	-12.88377239	17408.46	226309.98	5	13
NSN-11	1680012283821	-11.18151106	-0.76880881	0	-11.18151106	19672.91	0	0	0
NSN-12	1680012283822	-12.66652254	-0.80471896	0	-12.66652254	20131.4	0	0	0
NSN-13	2835012180143	-7.90897545	-0.24335330	1	-8.152328748	67393.87	67393.87	1	4
NSN-14	2835012188080	-11.80569868	-0.20339569	1	-12.00909437	70669.82	70669.82	1	4
NSN-15	4810012518480HS	-12.86772116	-1.58266767	0	-12.86772116	13986	0	0	0
NSN-16	4810012537542	-5.09187294	-0.67215120	13	-13.82983848	4377.4	56906.2	7	13
NSN-17	4810013377136TP	-9.71409968	-0.48985035	5	-12.16335141	8050	40250	1	5
NSN-18	5821012483022FX	-8.31353708	-0.53032270	2	-9.374182471	17058.08	34116.16	2	6
NSN-19	5895012913073FX	-9.81377565	-0.64388566	11	-16.89651786	8338.98	91728.78	7	11
NSN-20	5895013640160LN	-5.91097487	-1.38879930	6	-14.24377065	33661.84	201971.04	2	6
NSN-21	5895013823225FX	-7.08798666	-0.39531865	2	-7.878623958	53030.98	106061.96	2	4
NSN-22	5985013902368EW	-5.63439104	-0.39528303	9	-9.191938305	13342.04	120078.36	9	14
NSN-23	5985013934820EW	-7.57700415	-0.70941160	8	-13.25229698	8484.08	67872.64	2	8
NSN-24	5998013410403FX	-10.33011201	-0.46259325	1	-10.79270527	17142.59	17142.59	1	3
NSN-25	6110012305147	-13.28597119	-0.10819770	0	-13.28597119	11429.91	0	0	0
NSN-26	6605012400136FX	-5.43703911	-0.15738695	4	-6.066586908	15505.66	62022.64	4	9
NSN-27	6610013195039	-3.71343697	-0.24717362	42	-14.0947289	3313.76	139177.92	36	42
NSN-28	6615012444251	-11.22168162	-1.02304883	0	-11.22168162	44301.56	0	0	0
NSN-29	6615013462155	-10.77608277	-0.38083218	0	-10.77608277	13861	0	0	0
NSN-30	6615013527003	-6.93716278	-0.27275089	0	-6.937162776	32725.3	0	0	0
					-320.3633247		2700221.7		

Appendix D. Application of AAM Methodology to a Subset of Data-The Shopping List

This workbook uses an explicitly defined subset of F-15E AAM output data
and applies AAM methodology to determine the optimal buy postition.
An alternate formulation of the model is then compared to those items
selected from the "shopping list" of this subset of data.

<u>The AAM Shopping List</u>

The cut-off sort value for the first arbitrary budget level is indicated by this shading:

The cut-off sort value for the 2nd arbitrary budget level is indicated by this shading:

NSN	Pro. Cost	Sort Value	ln(q)	stk lvl	Budget at SV	Accum. Bud.
1660013389649BO	17408.46	0.0000000161834380	-0.0003755004914860	5	87042.3	87042.3
5895013640160LN	33661.84	0.0000000147218310	-0.0001960792222744	2	67323.68	154365.98
4810012537542	4377.4	0.0000000142968630	-0.0000638120359447	7	30641.8	185007.78
5985013934820EW	8484.08	0.0000000140849990	-0.0001263979878891	2	16968.16	201975.94
1660013389649BO	17408.46	0.0000000094993569	-0.0002101320761984	6	17408.46	219384.4
5895012913073FX	8338.98	0.0000000086909139	-0.0000006300001984	7	58372.86	277757.26
5985013934820EW	8484.08	0.0000000073765321	-0.0000638120359447	3	8484.08	286241.34
4810012537542	4377.4	0.0000000073335550	-0.0000317105027727	8	4377.4	290618.74
1005012982522	29716.86	0.0000000071909388	-0.0001645235332543	1	29716.86	320335.6
5821012483022FX	17058.08	0.0000000070057637	-0.0001869974829395	2	34116.16	354451.76
1660013389649BO	17408.46	0.0000000054413007	-0.0001154066590923	7	17408.46	371860.22
4810013377136TP	8050	0.0000000052874417	-0.0000393507742316	1	8050	379910.22
6610013195039	3313.76	0.0000000048854066	-0.0000052200136243	36	119295.36	499205.58
5895012913073FX	8338.98	0.0000000045907435	-0.0000002500000313	8	8338.98	507544.56
1560012912590FX	3231.73	0.0000000044178821	-0.0014396958649220	0	0	507544.56
6605012400136FX	15505.66	0.0000000043742659	-0.0026404930354171	4	62022.64	569567.2
5895013640160LN	33661.84	0.0000000043388930	-0.0000500212510419	3	33661.84	603229.04
5985013902368EW	13342.04	0.0000000040514446	-0.0001141165110413	9	120078.36	723307.4
5985013934820EW	8484.08	0.0000000037837814	-0.0000317105027727	4	8484.08	731791.48
6605012400136FX	15505.66	0.0000000037567421	-0.0015530153042349	5	15505.66	747297.14
4810012537542	4377.4	0.0000000036945283	-0.0000155401207471	9	4377.4	751674.54
6610013195039	3313.76	0.0000000036715338	-0.0000013400008978	37	3313.76	754988.3
5821012483022FX	17058.08	0.0000000036690270	-0.0001187470501519	3	17058.08	772046.38
5895013823225FX	53030.98	0.0000000032711363	-0.0003607950787175	2	106061.96	878108.34
6605012400136FX	15505.66	0.0000000032124378	-0.0012395479222008	6	15505.66	893614
1005012982522	29716.86	0.0000000032043008	-0.0000693024013560	2	29716.86	923330.86
1660013389649BO	17408.46	0.0000000030497431	-0.0000623219419718	8	17408.46	940739.32
4810013377136TP	8050	0.0000000029897004	-0.0000219302404659	2	8050	948789.32
1560012912590FX	3231.73	0.0000000028155242	-0.0008674961660170	1	3231.73	952021.05
5985013902368EW	13342.04	0.0000000027838629	-0.0000769729623424	10	13342.04	965363.09

6610013195039	3313.76	0.0000000027473313	-0.0000010500005512	38	3313.76	968676.85
6605012400136FX	15505.66	0.0000000026931256	-0.0011200670409569	7	15505.66	984182.51
1630012251893	8795.12	0.0000000026855104	-0.0002642249043258	125	1099390	2083572.51
1630012251893	8795.12	0.0000000025371144	-0.0002544423677140	126	8795.12	2092367.63
1630012251893	8795.12	0.0000000023962592	-0.0002452400688876	127	8795.12	2101162.75
6605012400136FX	15505.66	0.0000000022839261	-0.0010140840094248	8	15505.66	2116668.41
1630012251893	8795.12	0.0000000022626032	-0.0002365779823642	128	8795.12	2125463.53
1630012251893	8795.12	0.0000000021358174	-0.0002284260872523	129	8795.12	2134258.65
6610013195039	3313.76	0.0000000020472842	-0.0000008400003528	39	3313.76	2137572.41
1630012251893	8795.12	0.0000000020155859	-0.0002207743688676	130	8795.12	2146367.53
6610013195039	3313.76	0.0000000019755079	-0.0000006600002178	40	3313.76	2149681.29
6610013195039	3313.76	0.0000000019298320	-0.0000005100001301	41	3313.76	2152995.05
6605012400136FX	15505.66	0.0000000019291702	-0.0009153788313963	9	15505.66	2168500.71
5985013934820EW	8484.08	0.0000000019062088	-0.0000155401207471	5	8484.08	2176984.79
1630012251893	8795.12	0.0000000019016047	-0.0002135728050480	131	8795.12	2185779.91
5985013902368EW	13342.04	0.0000000018996085	-0.0000516213323581	11	13342.04	2199121.95
6610013195039	3313.76	0.0000000018841562	-0.0000003900000760	42	3313.76	2202435.71
5821012483022FX	17058.08	0.0000000018820221	-0.0000629319801755	4	17058.08	2219493.79
4810012537542	4377.4	0.0000000018321009	-0.0000075200282754	10	4377.4	2223871.19
5895013823225FX	53030.98	0.0000000017959398	-0.0002655552566763	3	53030.98	2276902.17
5895013823225FX	53030.98	0.0000000017741379	-0.0001714747009463	4	53030.98	2329933.15
1560012912590FX	3231.73	0.0000000017456548	-0.0005127214191654	2	3231.73	2333164.88
1660013389649BO	17408.46	0.0000000016763722	-0.0000331305488106	9	17408.46	2350573.34
4810013377136TP	8050	0.0000000016693150	-0.0000111500621617	3	8050	2358623.34
1660013387046BO	57681.59	0.0000000016509086	-0.0000838235130925	3	173044.77	2531668.11
1005012982522	29716.86	0.0000000013760097	-0.0000284104035717	3	29716.86	2561384.97
4810013377136TP	8050	0.0000000013728717	-0.0000070900251342	4	8050	2569434.97
5985013902368EW	13342.04	0.0000000012878343	-0.0000344405930704	12	13342.04	2582777.01
4810013377136TP	8050	0.0000000012087739	-0.0000049000120050	5	8050	2590827.01
5895013640160LN	33661.84	0.0000000011372132	-0.0000117400689144	4	33661.84	2624488.85
5895012913073FX	8338.98	0.0000000010990673	-0.0000001400000098	9	8338.98	2632827.83
2835012180143	67393.87	0.0000000010694991	-0.0002710667352681	1	67393.87	2700221.7
1560012912590FX	3231.73	0.0000000010569970	-0.0002979043691005	3	3231.73	2703453.43
5895012913073FX	8338.98	0.0000000009866293	-0.0000000600000018	10	8338.98	2711792.41
5821012483022FX	17058.08	0.0000000009481328	-0.0000112700635069	5	17058.08	2728850.49
5985013934820EW	8484.08	0.0000000009452809	-0.0000075200282754	6	8484.08	2737334.57
5895012913073FX	8338.98	0.0000000009163556	-0.0000000100000001	11	8338.98	2745673.55
1660013389649BO	17408.46	0.0000000009054756	-0.0000173701508602	10	17408.46	2763082.01
4810012537542	4377.4	0.0000000008959681	-0.0000036000064800	11	4377.4	2767459.41
5985013902368EW	13342.04	0.0000000008677973	-0.0000228702615224	13	13342.04	2780801.45
5821012483022FX	17058.08	0.0000000008488365	-0.0000073500270113	6	17058.08	2797859.53
2835012180143	67393.87	0.0000000008328215	-0.0002149330964632	2	67393.87	2865253.4
5998013410403FX	17142.59	0.0000000007534311	-0.0000192901860544	1	17142.59	2882395.99

1660013387046BO	57681.59	0.0000000007089429	-0.0000667922305514	4	57681.59	2940077.58
1560012912590FX	3231.73	0.0000000006269877	-0.0001704845316620	4	3231.73	2943309.31
2835012180143	67393.87	0.0000000006059478	-0.0001740951536818	3	67393.87	3010703.18
5985013902368EW	13342.04	0.0000000005814403	-0.0000151101141572	14	13342.04	3024045.22
1005012982522	29716.86	0.0000000005730797	-0.0000113800647527	4	29716.86	3053762.08
2835012180143	67393.87	0.0000000005450212	-0.0001373694347488	4	67393.87	3121155.95
1660013389649BO	17408.46	0.0000000004814117	-0.0000089900404103	11	17408.46	3138564.41
5998013410403FX	17142.59	0.0000000004678330	-0.0000112700635069	2	17142.59	3155707
5985013934820EW	8484.08	0.0000000004622789	-0.0000036000064800	7	8484.08	3164191.08
4810012537542	4377.4	0.0000000004327756	-0.0000017100014621	12	4377.4	3168568.48
1270013562584FX	13974.63	0.0000000003722626	-0.0000664422072346	1	13974.63	3182543.11
1560012912590FX	3231.73	0.0000000003652814	-0.0000962446313659	5	3231.73	3185774.84
1270013562584FX	13974.63	0.0000000003590310	-0.0000281603965003	2	13974.63	3199749.47
1660013387046BO	57681.59	0.0000000002952601	-0.0000598817928788	5	57681.59	3257431.06
5895013640160LN	33661.84	0.0000000002722596	-0.0000025800033282	5	33661.84	3291092.9
1660013387046BO	57681.59	0.0000000002581623	-0.0000510113010543	6	57681.59	3348774.49
1660013389649BO	17408.46	0.0000000002523052	-0.0000046000105801	12	17408.46	3366182.95
1005012982522	29716.86	0.0000000002326117	-0.0000044700099905	5	29716.86	3395899.81
5998013410403FX	17142.59	0.0000000002287884	-0.0000073500270113	3	17142.59	3413042.4
5985013934820EW	8484.08	0.0000000002232926	-0.0000017100014621	8	8484.08	3421526.48
4810012537542	4377.4	0.0000000002067420	-0.0000008000003200	13	4377.4	3425903.88
1650013969218	46000	0.0000000001556203	-0.0000617619072273	1	46000	3471903.88
1650013969218	46000	0.0000000001556144	-0.0000234802756595	2	46000	3517903.88
1660013389649BO	17408.46	0.0000000001305148	-0.0000023300027145	13	17408.46	3535312.34
5895013640160LN	33661.84	0.0000000000606521	-0.0000005300001404	6	33661.84	3568974.18
2835012188080	70669.82	0.0000000000357204	-0.0000069300240125	1	70669.82	3639644
2835012188080	70669.82	0.0000000000250328	-0.0000051600133128	2	70669.82	3710313.82
2835012188080	70669.82	0.0000000000174683	-0.0000039300077225	3	70669.82	3780983.64
2835012188080	70669.82	0.0000000000121404	-0.0000030700047125	4	70669.82	3851653.46

Appendix E. CPLEX Solution Output for the PLDMsv and PLDMq

CPLEX Solution Output for the PLDMsv at the Maximum Budget Level: B=$3,851,653

```
PROBLEM NAME     tminsv.mps
DATA    NAME     scaledsv.mps
OBJECTIVE VALUE  1.382233E+08
STATUS           OPTIMAL SOLN
ITERATION        0

OBJECTIVE        obj                  (MIN)
RHS              rhs
RANGES
BOUNDS           BOUND
```

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	1.382233E+08	-1.382233E+08	NONE	NONE	1
2	budget	BS	3851653	-3.197888E-10	NONE	3851653	-0
3	order1	EQ	1	0	1	1	-2326117
4	order2	EQ	1	0	1	1	-3590310
5	order3	EQ	1	0	1	1	-3652814
6	order4	EQ	1	0	1	1	-1.901605E+07
7	order5	EQ	1	0	1	1	-1556144
8	order6	EQ	1	0	1	1	-2581623
9	order7	EQ	1	0	1	1	-1305148
10	order8	EQ	1	0	1	1	-5450212
11	order9	EQ	1	0	1	1	-121404.3
12	order10	EQ	1	0	1	1	-2067420
13	order11	EQ	1	0	1	1	-1.208774E+07
14	order12	EQ	1	0	1	1	-8488365
15	order13	EQ	1	0	1	1	-9163556
16	order14	EQ	1	0	1	1	-606521.1
17	order15	EQ	1	0	1	1	-1.774138E+07
18	order16	EQ	1	0	1	1	-5814403
19	order17	EQ	1	0	1	1	-2232926
20	order18	EQ	1	0	1	1	-2287884
21	order19	EQ	1	0	1	1	-1.92917E+07
22	order20	EQ	1	0	1	1	-1.884156E+07

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	...INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
23	1005012982522-1	LL	0	7.190939E+07	0	1	6.958327E+07
24	1005012982522-2	LL	0	3.204301E+07	0	1	2.971689E+07
25	1005012982522-3	LL	0	1.37601E+07	0	1	1.143398E+07
26	1005012982522-4	LL	0	5730797	0	1	3404680
27	1005012982522-5	BS	1	2326117	0	1	0
28	1270013562584FX1	LL	0	3722626	0	1	132316
29	1270013562584FX2	BS	1	3590310	0	1	0
30	1560012912590FX0	LL	0	4.417882E+07	0	1	4.052601E+07
31	1560012912590FX1	LL	0	2.815524E+07	0	1	2.450243E+07
32	1560012912590FX2	LL	0	1.745655E+07	0	1	1.380373E+07
33	1560012912590FX3	LL	0	1.056997E+07	0	1	6917156
34	1560012912590FX4	LL	0	6269877	0	1	2617063
35	1560012912590FX5	BS	1	3652814	0	1	0
36	1630012251893125	LL	0	2.68551E+07	0	1	7839057
37	1630012251893126	LL	0	2.537114E+07	0	1	6355097
38	1630012251893127	LL	0	2.396259E+07	0	1	4946545
39	1630012251893128	LL	0	2.262603E+07	0	1	3609985
40	1630012251893129	LL	0	2.135817E+07	0	1	2342127
41	1630012251893130	LL	0	2.015586E+07	0	1	1139812
42	1630012251893131	BS	1	1.901605E+07	0	1	0
43	1650013969218-1	LL	0	1556203	0	1	59
44	1650013969218-2	BS	1	1556144	0	1	0
45	1660013387046BO3	LL	0	1.650909E+07	0	1	1.392746E+07
46	1660013387046BO4	LL	0	7089429	0	1	4507806
47	1660013387046BO5	LL	0	2952601	0	1	370978
48	1660013387046BO6	BS	1	2581623	0	1	0
49	1660013389649B5	LL	0	1.618344E+08	0	1	1.605292E+08
50	1660013389649B6	LL	0	9.499357E+07	0	1	9.368842E+07
51	1660013389649B7	LL	0	5.441301E+07	0	1	5.310786E+07
52	1660013389649B8	LL	0	3.049743E+07	0	1	2.919228E+07
53	1660013389649B9	LL	0	1.676372E+07	0	1	1.545857E+07
54	1660013389649B10	LL	0	9054756	0	1	7749608
55	1660013389649B11	LL	0	4814117	0	1	3508969
56	1660013389649B12	LL	0	2523052	0	1	1217904
57	1660013389649B13	BS	1	1305148	0	1	0
58	2835012180143-1	LL	0	1.069499E+07	0	1	5244779
59	2835012180143-2	LL	0	8328215	0	1	2878003
60	2835012180143-3	LL	0	6059478	0	1	609266
61	2835012180143-4	BS	1	5450212	0	1	0
62	2835012188080-1	LL	0	357204	0	1	235799.7

63	2835012188080-2	LL	0	250328.2	0	1	128923.9
64	2835012188080-3	LL	0	174683.1	0	1	53278.8
65	2835012188080-4	BS	1	121404.3	0	1	0
66	4810012537542-7	LL	0	1.429686E+08	0	1	1.409012E+08
67	4810012537542-8	LL	0	7.333555E+07	0	1	7.126813E+07
68	4810012537542-9	LL	0	3.694528E+07	0	1	3.487786E+07
69	4810012537542-10	LL	0	1.832101E+07	0	1	1.625359E+07
70	4810012537542-11	LL	0	8959681	0	1	6892261
71	4810012537542-12	LL	0	4327756	0	1	2260336
72	4810012537542-13	BS	1	2067420	0	1	0
73	4810013377136TP1	LL	0	5.287442E+07	0	1	4.078668E+07
74	4810013377136TP2	LL	0	2.9897E+07	0	1	1.780926E+07
75	4810013377136TP3	LL	0	1.669315E+07	0	1	4605411
76	4810013377136TP4	LL	0	1.372872E+07	0	1	1640978
77	4810013377136TP5	BS	1	1.208774E+07	0	1	0
78	5821012483022FX2	LL	0	7.005764E+07	0	1	6.156927E+07
79	5821012483022FX3	LL	0	3.669027E+07	0	1	2.82019E+07
80	5821012483022FX4	LL	0	1.882022E+07	0	1	1.033186E+07
81	5821012483022FX5	LL	0	9481328	0	1	992963
82	5821012483022FX6	BS	1	8488365	0	1	0
83	5895012913073F7	LL	0	8.690914E+07	0	1	7.774558E+07
84	5895012913073F8	LL	0	4.590744E+07	0	1	3.674388E+07
85	5895012913073F9	LL	0	1.099067E+07	0	1	1827117
86	5895012913073F10	LL	0	9866293	0	1	702737
87	5895012913073F11	BS	1	9163556	0	1	0
88	5895013640160LN2	LL	0	1.472183E+08	0	1	1.466118E+08
89	5895013640160LN3	LL	0	4.338893E+07	0	1	4.278241E+07
90	5895013640160LN4	LL	0	1.137213E+07	0	1	1.076561E+07
91	5895013640160LN5	LL	0	2722596	0	1	2116075
92	5895013640160LN6	BS	1	606521.1	0	1	0
93	5895013823225FX2	LL	0	3.271136E+07	0	1	1.496998E+07
94	5895013823225FX3	LL	0	1.79594E+07	0	1	218019
95	5895013823225FX4	BS	1	1.774138E+07	0	1	0
96	5985013902368E9	LL	0	4.051445E+07	0	1	3.470004E+07
97	5985013902368E10	LL	0	2.783863E+07	0	1	2.202423E+07
98	5985013902368E11	LL	0	1.899608E+07	0	1	1.318168E+07
99	5985013902368E12	LL	0	1.287834E+07	0	1	7063940
100	5985013902368E13	LL	0	8677973	0	1	2863570
101	5985013902368E14	BS	1	5814403	0	1	0
102	5985013934820EW2	LL	0	1.4085E+08	0	1	1.386171E+08
103	5985013934820EW3	LL	0	7.376532E+07	0	1	7.15324E+07
104	5985013934820EW4	LL	0	3.783781E+07	0	1	3.560489E+07
105	5985013934820EW5	LL	0	1.906209E+07	0	1	1.682916E+07
106	5985013934820EW6	LL	0	9452809	0	1	7219883
107	5985013934820EW7	LL	0	4622789	0	1	2389863

108	5985013934820EW8	BS	1	2232926	0	1	0
109	5998013410403FX1	LL	0	7534311	0	1	5246427
110	5998013410403FX2	LL	0	4678330	0	1	2390446
111	5998013410403FX3	BS	1	2287884	0	1	0
112	6605012400136FX4	LL	0	4.374266E+07	0	1	2.445096E+07
113	6605012400136FX5	LL	0	3.756742E+07	0	1	1.827572E+07
114	6605012400136FX6	LL	0	3.212438E+07	0	1	1.283268E+07
115	6605012400136FX7	LL	0	2.693126E+07	0	1	7639554
116	6605012400136FX8	LL	0	2.283926E+07	0	1	3547559
117	6605012400136FX9	BS	1	1.92917E+07	0	1	0
118	6610013195039-36	LL	0	4.885407E+07	0	1	3.00125E+07
119	6610013195039-37	LL	0	3.671534E+07	0	1	1.787378E+07
120	6610013195039-38	LL	0	2.747331E+07	0	1	8631751
121	6610013195039-39	LL	0	2.047284E+07	0	1	1631280
122	6610013195039-40	LL	0	1.975508E+07	0	1	913517
123	6610013195039-41	LL	0	1.929832E+07	0	1	456758
124	6610013195039-42	BS	1	1.884156E+07	0	1	0

CPLEX Solution Output for the PLDMsv at a budget level of $3,185,775

```
PROBLEM NAME      tminsv1.mps
DATA    NAME      scaledsv.mps
OBJECTIVE VALUE   1.412592E+08
STATUS            OPTIMAL SOLN
ITERATION         17

OBJECTIVE         obj                (MIN)
RHS               rhs
RANGES
BOUNDS            BOUND
```

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	1.412592E+08	-1.412592E+08	NONE	NONE	1
2	budget	UL	3185775	0	NONE	3185775	35.11856
3	order1	EQ	1	0	1	1	-7544184
4	order2	EQ	1	0	1	1	-4571848
5	order3	EQ	1	0	1	1	-4220283
6	order4	EQ	1	0	1	1	-5.947828E+07
7	order5	BS	0	1	NONE	1	-0
8	order6	EQ	1	0	1	1	-1.473579E+07
9	order7	EQ	1	0	1	1	-9252830

10	order8	EQ	1	0	1	1	-1.306177E+07
11	order9	BS	0	1	NONE	1	-0
12	order10	EQ	1	0	1	1	-4065884
13	order11	EQ	1	0	1	1	-1.350126E+07
14	order12	EQ	1	0	1	1	-1.20827E+07
15	order13	EQ	1	0	1	1	-1.238494E+07
16	order14	EQ	1	0	1	1	-7699454
17	order15	EQ	1	0	1	1	-2.519087E+07
18	order16	EQ	1	0	1	1	-1.237415E+07
19	order17	EQ	1	0	1	1	-4616516
20	order18	EQ	1	0	1	1	-4093953
21	order19	EQ	1	0	1	1	-2.419253E+07
22	order20	EQ	1	0	1	1	-2.372929E+07

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
23	1005012982522-1	LL	0	7.190939E+07	0	1	6.540882E+07
24	1005012982522-2	LL	0	3.204301E+07	0	1	2.658605E+07
25	1005012982522-3	LL	0	1.37601E+07	0	1	9346753
26	1005012982522-4	LL	0	5730797	0	1	2361067
27	1005012982522-5	BS	1	2326117	0	1	0
28	1270013562584FX1	UL	1	3722626	0	1	-358453
29	1270013562584FX2	BS	0	3590310	0	1	0
30	1560012912590FX0	LL	0	4.417882E+07	0	1	3.995854E+07
31	1560012912590FX1	LL	0	2.815524E+07	0	1	2.404845E+07
32	1560012912590FX2	LL	0	1.745655E+07	0	1	1.346325E+07
33	1560012912590FX3	LL	0	1.056997E+07	0	1	6690169
34	1560012912590FX4	LL	0	6269877	0	1	2503569
35	1560012912590FX5	BS	1	3652814	0	1	0
36	1630012251893125	LL	0	2.68551E+07	0	1	5985825
37	1630012251893126	LL	0	2.537114E+07	0	1	4810737
38	1630012251893127	LL	0	2.396259E+07	0	1	3711057
39	1630012251893128	LL	0	2.262603E+07	0	1	2683369
40	1630012251893129	LL	0	2.135817E+07	0	1	1724383
41	1630012251893130	LL	0	2.015586E+07	0	1	830940
42	1630012251893131	BS	1	1.901605E+07	0	1	0
43	1650013969218-1	LL	0	1556203	0	1	3171657
44	1650013969218-2	LL	0	1556144	0	1	4787052
45	1660013387046BO3	LL	0	1.650909E+07	0	1	7850379
46	1660013387046BO4	LL	0	7089429	0	1	456416.7
47	1660013387046BO5	UL	1	2952601	0	1	-1654717
48	1660013387046BO6	BS	0	2581623	0	1	0
49	1660013389649B5	LL	0	1.618344E+08	0	1	1.556384E+08

50	1660013389649B6	LL	0	9.499357E+07	0	1	8.94089E+07
51	1660013389649B7	LL	0	5.441301E+07	0	1	4.94397E+07
52	1660013389649B8	LL	0	3.049743E+07	0	1	2.613548E+07
53	1660013389649B9	LL	0	1.676372E+07	0	1	1.301313E+07
54	1660013389649B10	LL	0	9054756	0	1	5915528
55	1660013389649B11	LL	0	4814117	0	1	2286249
56	1660013389649B12	LL	0	2523052	0	1	606543.9
57	1660013389649B13	BS	1	1305148	0	1	0
58	2835012180143-1	BS	0.4707312	1.069499E+07	0	1	0
59	2835012180143-2	BS	0.5292688	8328215	0	1	0
60	2835012180143-3	LL	0	6059478	0	1	98039
61	2835012180143-4	LL	0	5450212	0	1	1855549
62	2835012188080-1	LL	0	357204	0	1	2839027
63	2835012188080-2	LL	0	250328.2	0	1	5213974
64	2835012188080-3	LL	0	174683.1	0	1	7620151
65	2835012188080-4	LL	0	121404.3	0	1	1.004869E+07
66	4810012537542-7	LL	0	1.429686E+08	0	1	1.399788E+08
67	4810012537542-8	LL	0	7.333555E+07	0	1	7.049949E+07
68	4810012537542-9	LL	0	3.694528E+07	0	1	3.426295E+07
69	4810012537542-10	LL	0	1.832101E+07	0	1	1.57924E+07
70	4810012537542-11	LL	0	8959681	0	1	6584805
71	4810012537542-12	LL	0	4327756	0	1	2106608
72	4810012537542-13	BS	1	2067420	0	1	0
73	4810013377136TP1	LL	0	5.287442E+07	0	1	3.965586E+07
74	4810013377136TP2	LL	0	2.9897E+07	0	1	1.696115E+07
75	4810013377136TP3	LL	0	1.669315E+07	0	1	4040002
76	4810013377136TP4	LL	0	1.372872E+07	0	1	1358274
77	4810013377136TP5	BS	1	1.208774E+07	0	1	0
78	5821012483022FX2	LL	0	7.005764E+07	0	1	5.917305E+07
79	5821012483022FX3	LL	0	3.669027E+07	0	1	2.640474E+07
80	5821012483022FX4	LL	0	1.882022E+07	0	1	9133745
81	5821012483022FX5	LL	0	9481328	0	1	393907.7
82	5821012483022FX6	BS	1	8488365	0	1	0
83	5895012913073F7	LL	0	8.690914E+07	0	1	7.657417E+07
84	5895012913073F8	LL	0	4.590744E+07	0	1	3.586532E+07
85	5895012913073F9	LL	0	1.099067E+07	0	1	1241411
86	5895012913073F10	LL	0	9866293	0	1	409884
87	5895012913073F11	BS	1	9163556	0	1	0
88	5895013640160LN2	LL	0	1.472183E+08	0	1	1.418832E+08
89	5895013640160LN3	LL	0	4.338893E+07	0	1	3.923594E+07
90	5895013640160LN4	LL	0	1.137213E+07	0	1	8401300
91	5895013640160LN5	LL	0	2722596	0	1	933919.4
92	5895013640160LN6	BS	1	606521.1	0	1	0
93	5895013823225FX2	LL	0	3.271136E+07	0	1	1.124524E+07
94	5895013823225FX3	UL	1	1.79594E+07	0	1	-1644353

95	5895013823225FX4	BS	0	1.774138E+07	0	1	0
96	5985013902368E9	LL	0	4.051445E+07	0	1	3.235728E+07
97	5985013902368E10	LL	0	2.783863E+07	0	1	2.015001E+07
98	5985013902368E11	LL	0	1.899608E+07	0	1	1.177602E+07
99	5985013902368E12	LL	0	1.287834E+07	0	1	6126833
100	5985013902368E13	LL	0	8677973	0	1	2395017
101	5985013902368E14	BS	1	5814403	0	1	0
102	5985013934820EW2	LL	0	1.4085E+08	0	1	1.368294E+08
103	5985013934820EW3	LL	0	7.376532E+07	0	1	7.004265E+07
104	5985013934820EW4	LL	0	3.783781E+07	0	1	3.441309E+07
105	5985013934820EW5	LL	0	1.906209E+07	0	1	1.593532E+07
106	5985013934820EW6	LL	0	9452809	0	1	6623986
107	5985013934820EW7	LL	0	4622789	0	1	2091914
108	5985013934820EW8	BS	1	2232926	0	1	0
109	5998013410403FX1	LL	0	7534311	0	1	4042381
110	5998013410403FX2	LL	0	4678330	0	1	1788423
111	5998013410403FX3	BS	1	2287884	0	1	0
112	6605012400136FX4	LL	0	4.374266E+07	0	1	2.172827E+07
113	6605012400136FX5	LL	0	3.756742E+07	0	1	1.609757E+07
114	6605012400136FX6	LL	0	3.212438E+07	0	1	1.119907E+07
115	6605012400136FX7	LL	0	2.693126E+07	0	1	6550481
116	6605012400136FX8	LL	0	2.283926E+07	0	1	3003022
117	6605012400136FX9	BS	1	1.92917E+07	0	1	0
118	6610013195039-36	LL	0	4.885407E+07	0	1	2.931426E+07
119	6610013195039-37	LL	0	3.671534E+07	0	1	1.72919E+07
120	6610013195039-38	LL	0	2.747331E+07	0	1	8166253
121	6610013195039-39	LL	0	2.047284E+07	0	1	1282157
122	6610013195039-40	LL	0	1.975508E+07	0	1	680768
123	6610013195039-41	LL	0	1.929832E+07	0	1	340383.5
124	6610013195039-42	BS	1	1.884156E+07	0	1	0

CPLEX Solution Output for the PLDMsv at a Budget Level of 2,700,022

```
PROBLEM NAME     tsvscaln.mps
DATA    NAME     scaledsv.mps
OBJECTIVE VALUE  -1.88853E+08
STATUS           OPTIMAL SOLN
ITERATION        49

OBJECTIVE        obj              (MAX)
RHS              rhs
RANGES
BOUNDS           BOUND
```

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	-1.88853E+08	1.88853E+08	NONE	NONE	1
2	budget	UL	2700222	0	NONE	2700222	-256.9537
3	order1	EQ	1	0	1	1	3.627423E+07
4	order2	BS	0	1	NONE	1	0
5	order3	EQ	1	0	1	1	7804840
6	order4	EQ	1	0	1	1	3.101235E+08
7	order5	BS	0	1	NONE	1	0
8	order6	EQ	1	0	1	1	6.637543E+07
9	order7	EQ	1	0	1	1	5.401898E+07
10	order8	EQ	1	0	1	1	4.296243E+07
11	order9	BS	0	1	NONE	1	0
12	order10	EQ	1	0	1	1	1.668968E+07
13	order11	EQ	1	0	1	1	2.243013E+07
14	order12	EQ	1	0	1	1	3.478719E+07
15	order13	EQ	1	0	1	1	3.129361E+07
16	order14	EQ	1	0	1	1	4.597028E+07
17	order15	EQ	1	0	1	1	5.883893E+07
18	order16	EQ	1	0	1	1	5.381042E+07
19	order17	EQ	1	0	1	1	1.967306E+07
20	order18	BS	0	1	NONE	1	0
21	order19	EQ	1	0	1	1	5.482092E+07
22	order20	EQ	1	0	1	1	5.38144E+07

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
23	1005012982522-1	LL	0	-7.190939E+07	0	1	-4.327102E+07
24	1005012982522-2	LL	0	-3.204301E+07	0	1	-1.104049E+07
25	1005012982522-3	LL	0	-1.37601E+07	0	1	-393441.4
26	1005012982522-4	BS	1	-5730797	0	1	-0
27	1005012982522-5	LL	0	-2326117	0	1	-4231179
28	1270013562584FX1	LL	0	-3722626	0	1	-7313460
29	1270013562584FX2	LL	0	-3590310	0	1	-1.077198E+07
30	1560012912590FX0	LL	0	-4.417882E+07	0	1	-3.637398E+07
31	1560012912590FX1	LL	0	-2.815524E+07	0	1	-2.118081E+07
32	1560012912590FX2	LL	0	-1.745655E+07	0	1	-1.131252E+07
33	1560012912590FX3	LL	0	-1.056997E+07	0	1	-5256346
34	1560012912590FX4	LL	0	-6269877	0	1	-1786658
35	1560012912590FX5	BS	1	-3652814	0	1	-0
36	1630012251893125	UL	1	-2.68551E+07	0	1	775979.1

37	1630012251893126	BS	0	-2.537114E+07	0	1	-0
38	1630012251893127	LL	0	-2.396259E+07	0	1	-851387.1
39	1630012251893128	LL	0	-2.262603E+07	0	1	-1774766
40	1630012251893129	LL	0	-2.135817E+07	0	1	-2766847
41	1630012251893130	LL	0	-2.015586E+07	0	1	-3824471
42	1630012251893131	LL	0	-1.901605E+07	0	1	-4944598
43	1650013969218-1	LL	0	-1556203	0	1	-1.337608E+07
44	1650013969218-2	LL	0	-1556144	0	1	-2.519589E+07
45	1660013387046BO3	UL	1	-1.650909E+07	0	1	5401844
46	1660013387046BO4	BS	0	-7089429	0	1	-0
47	1660013387046BO5	LL	0	-2952601	0	1	-1.068467E+07
48	1660013387046BO6	LL	0	-2581623	0	1	-2.51352E+07
49	1660013389649B5	LL	0	-1.618344E+08	0	1	-1.301812E+08
50	1660013389649B6	LL	0	-9.499357E+07	0	1	-6.781361E+07
51	1660013389649B7	LL	0	-5.441301E+07	0	1	-3.170621E+07
52	1660013389649B8	LL	0	-3.049743E+07	0	1	-1.226381E+07
53	1660013389649B9	LL	0	-1.676372E+07	0	1	-3003267
54	1660013389649B10	UL	1	-9054756	0	1	232530.1
55	1660013389649B11	BS	0	-4814117	0	1	-0
56	1660013389649B12	LL	0	-2523052	0	1	-2182104
57	1660013389649B13	LL	0	-1305148	0	1	-5437369
58	2835012180143-1	UL	1	-1.069499E+07	0	1	1.495033E+07
59	2835012180143-2	BS	0	-8328215	0	1	-0
60	2835012180143-3	LL	0	-6059478	0	1	-1.504837E+07
61	2835012180143-4	LL	0	-5450212	0	1	-3.175621E+07
62	2835012188080-1	LL	0	-357204	0	1	-1.851608E+07
63	2835012188080-2	LL	0	-250328.2	0	1	-3.656808E+07
64	2835012188080-3	LL	0	-174683.1	0	1	-5.465131E+07
65	2835012188080-4	LL	0	-121404.3	0	1	-7.275691E+07
66	4810012537542-7	LL	0	-1.429686E+08	0	1	-1.341525E+08
67	4810012537542-8	LL	0	-7.333555E+07	0	1	-6.564418E+07
68	4810012537542-9	LL	0	-3.694528E+07	0	1	-3.037871E+07
69	4810012537542-10	LL	0	-1.832101E+07	0	1	-1.287922E+07
70	4810012537542-11	LL	0	-8959681	0	1	-4642682
71	4810012537542-12	LL	0	-4327756	0	1	-1135547
72	4810012537542-13	BS	1	-2067420	0	1	-0
73	4810013377136TP1	LL	0	-5.287442E+07	0	1	-3.251277E+07
74	4810013377136TP2	LL	0	-2.9897E+07	0	1	-1.160383E+07
75	4810013377136TP3	LL	0	-1.669315E+07	0	1	-468455.6
76	4810013377136TP4	UL	1	-1.372872E+07	0	1	427499.7
77	4810013377136TP5	BS	0	-1.208774E+07	0	1	-0
78	5821012483022FX2	LL	0	-7.005764E+07	0	1	-4.403672E+07
79	5821012483022FX3	LL	0	-3.669027E+07	0	1	-1.505249E+07
80	5821012483022FX4	LL	0	-1.882022E+07	0	1	-1565581
81	5821012483022FX5	UL	1	-9481328	0	1	3390175

82	5821012483022FX6	BS	0	-8488365	0	1	-0
83	5895012913073F7	LL	0	-8.690914E+07	0	1	-7.061465E+07
84	5895012913073F8	LL	0	-4.590744E+07	0	1	-3.175568E+07
85	5895012913073F9	UL	1	-1.099067E+07	0	1	1018352
86	5895012913073F10	BS	0	-9866293	0	1	-0
87	5895012913073F11	LL	0	-9163556	0	1	-1439995
88	5895013640160LN2	LL	0	-1.472183E+08	0	1	-1.185471E+08
89	5895013640160LN3	LL	0	-4.338893E+07	0	1	-2.336726E+07
90	5895013640160LN4	BS	0.5991434	-1.137213E+07	0	1	-0
91	5895013640160LN5	BS	0.4008566	-2722596	0	1	-0
92	5895013640160LN6	LL	0	-606521.1	0	1	-6533461
93	5895013823225FX2	LL	0	-3.271136E+07	0	1	-1125456
94	5895013823225FX3	BS	1	-1.79594E+07	0	1	-0
95	5895013823225FX4	LL	0	-1.774138E+07	0	1	-1.340849E+07
96	5985013902368E9	LL	0	-4.051445E+07	0	1	-1.755861E+07
97	5985013902368E10	LL	0	-2.783863E+07	0	1	-8311077
98	5985013902368E11	LL	0	-1.899608E+07	0	1	-2896820
99	5985013902368E12	LL	0	-1.287834E+07	0	1	-207365.6
100	5985013902368E13	UL	1	-8677973	0	1	564717.2
101	5985013902368E14	BS	0	-5814403	0	1	-0
102	5985013934820EW2	LL	0	-1.4085E+08	0	1	-1.25537E+08
103	5985013934820EW3	LL	0	-7.376532E+07	0	1	-6.063231E+07
104	5985013934820EW4	LL	0	-3.783781E+07	0	1	-2.688482E+07
105	5985013934820EW5	LL	0	-1.906209E+07	0	1	-1.028911E+07
106	5985013934820EW6	LL	0	-9452809	0	1	-2859851
107	5985013934820EW7	LL	0	-4622789	0	1	-209846.8
108	5985013934820EW8	BS	1	-2232926	0	1	-0
109	5998013410403FX1	LL	0	-7534311	0	1	-1.193916E+07
110	5998013410403FX2	LL	0	-4678330	0	1	-1.348804E+07
111	5998013410403FX3	LL	0	-2287884	0	1	-1.550244E+07
112	6605012400136FX4	LL	0	-4.374266E+07	0	1	-4858691
113	6605012400136FX5	LL	0	-3.756742E+07	0	1	-2667690
114	6605012400136FX6	LL	0	-3.212438E+07	0	1	-1208885
115	6605012400136FX7	BS	0	-2.693126E+07	0	1	-0
116	6605012400136FX8	UL	1	-2.283926E+07	0	1	107757.5
117	6605012400136FX9	LL	0	-1.92917E+07	0	1	-328920.9
118	6610013195039-36	LL	0	-4.885407E+07	0	1	-2.569305E+07
119	6610013195039-37	LL	0	-3.671534E+07	0	1	-1.440581E+07
120	6610013195039-38	LL	0	-2.747331E+07	0	1	-6015268
121	6610013195039-39	UL	1	-2.047284E+07	0	1	133720.1
122	6610013195039-40	BS	0	-1.975508E+07	0	1	-0
123	6610013195039-41	LL	0	-1.929832E+07	0	1	-394724.1
124	6610013195039-42	LL	0	-1.884156E+07	0	1	-789449.1

CPLEX Solution Output for the PLDMq at the Maximum Budget Level: B=$3,851,653

```
PROBLEM NAME     tqscale.mps
DATA    NAME     scaledq.mps
OBJECTIVE VALUE  -1.684713E+13
STATUS           OPTIMAL SOLN
ITERATION        0

OBJECTIVE        obj                 (MAX)
RHS              rhs
RANGES
BOUNDS           BOUND
```

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	-1.684713E+13	1.684713E+13	NONE	NONE	1
2	budget	BS	3851653	-5.116621E-10	NONE	3851653	0
3	order1	EQ	1	0	1	1	4.47001E+10
4	order2	EQ	1	0	1	1	2.81604E+11
5	order3	EQ	1	0	1	1	9.624463E+11
6	order4	EQ	1	0	1	1	2.135728E+12
7	order5	EQ	1	0	1	1	2.348028E+11
8	order6	EQ	1	0	1	1	5.10113E+11
9	order7	EQ	1	0	1	1	2.330003E+10
10	order8	EQ	1	0	1	1	1.373694E+12
11	order9	EQ	1	0	1	1	3.070005E+10
12	order10	EQ	1	0	1	1	8.000003E+09
13	order11	EQ	1	0	1	1	4.900012E+10
14	order12	EQ	1	0	1	1	7.350027E+10
15	order13	EQ	1	0	1	1	1E+08
16	order14	EQ	1	0	1	1	5.300001E+09
17	order15	EQ	1	0	1	1	1.714747E+12
18	order16	EQ	1	0	1	1	1.511011E+11
19	order17	EQ	1	0	1	1	1.710001E+10
20	order18	EQ	1	0	1	1	7.350027E+10
21	order19	EQ	1	0	1	1	9.153788E+12
22	order20	EQ	1	0	1	1	3.900001E+09

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.

23	1005012982522-1	LL	0	-1.645235E+12	0	1	-1.600535E+12
24	1005012982522-2	LL	0	-6.93024E+11	0	1	-6.483239E+11
25	1005012982522-3	LL	0	-2.84104E+11	0	1	-2.394039E+11
26	1005012982522-4	LL	0	-1.138006E+11	0	1	-6.910055E+10
27	1005012982522-5	BS	1	-4.47001E+10	0	1	-0
28	1270013562584FX1	LL	0	-6.644221E+11	0	1	-3.828181E+11
29	1270013562584FX2	BS	1	-2.81604E+11	0	1	-0
30	1560012912590FX0	LL	0	-1.439696E+13	0	1	-1.343451E+13
31	1560012912590FX1	LL	0	-8.674962E+12	0	1	-7.712515E+12
32	1560012912590FX2	LL	0	-5.127214E+12	0	1	-4.164768E+12
33	1560012912590FX3	LL	0	-2.979044E+12	0	1	-2.016597E+12
34	1560012912590FX4	LL	0	-1.704845E+12	0	1	-7.42399E+11
35	1560012912590FX5	BS	1	-9.624463E+11	0	1	-0
36	1630012251893125	LL	0	-2.642249E+12	0	1	-5.06521E+11
37	1630012251893126	LL	0	-2.544424E+12	0	1	-4.086956E+11
38	1630012251893127	LL	0	-2.452401E+12	0	1	-3.166726E+11
39	1630012251893128	LL	0	-2.36578E+12	0	1	-2.300518E+11
40	1630012251893129	LL	0	-2.284261E+12	0	1	-1.485328E+11
41	1630012251893130	LL	0	-2.207744E+12	0	1	-7.201564E+10
42	1630012251893131	BS	1	-2.135728E+12	0	1	-0
43	1650013969218-1	LL	0	-6.176191E+11	0	1	-3.828163E+11
44	1650013969218-2	BS	1	-2.348028E+11	0	1	-0
45	1660013387046BO3	LL	0	-8.382351E+11	0	1	-3.281221E+11
46	1660013387046BO4	LL	0	-6.679223E+11	0	1	-1.578093E+11
47	1660013387046BO5	LL	0	-5.788168E+11	0	1	-6.870374E+10
48	1660013387046BO6	BS	1	-5.10113E+11	0	1	-0
49	1660013389649B5	LL	0	-3.755005E+12	0	1	-3.731705E+12
50	1660013389649B6	LL	0	-2.101321E+12	0	1	-2.078021E+12
51	1660013389649B7	LL	0	-1.154067E+12	0	1	-1.130767E+12
52	1660013389649B8	LL	0	-6.232194E+11	0	1	-5.999194E+11
53	1660013389649B9	LL	0	-3.313055E+11	0	1	-3.080055E+11
54	1660013389649B10	LL	0	-1.737015E+11	0	1	-1.504015E+11
55	1660013389649B11	LL	0	-8.99004E+10	0	1	-6.660038E+10
56	1660013389649B12	LL	0	-4.600011E+10	0	1	-2.270008E+10
57	1660013389649B13	BS	1	-2.330003E+10	0	1	-0
58	2835012180143-1	LL	0	-2.710667E+12	0	1	-1.336973E+12
59	2835012180143-2	LL	0	-2.149331E+12	0	1	-7.756366E+11
60	2835012180143-3	LL	0	-1.740952E+12	0	1	-3.672572E+11
61	2835012180143-4	BS	1	-1.373694E+12	0	1	-0
62	2835012188080-1	LL	0	-6.930024E+10	0	1	-3.860019E+10
63	2835012188080-2	LL	0	-5.160013E+10	0	1	-2.090009E+10
64	2835012188080-3	LL	0	-3.930008E+10	0	1	-8.60003E+09
65	2835012188080-4	BS	1	-3.070005E+10	0	1	-0
66	4810012537542-7	LL	0	-6.381204E+11	0	1	-6.301204E+11
67	4810012537542-8	LL	0	-3.17105E+11	0	1	-3.09105E+11

68	4810012537542-9	LL	0	-1.554012E+11	0	1	-1.474012E+11
69	4810012537542-10	LL	0	-7.520028E+10	0	1	-6.720028E+10
70	4810012537542-11	LL	0	-3.600006E+10	0	1	-2.800006E+10
71	4810012537542-12	LL	0	-1.710001E+10	0	1	-9.100011E+09
72	4810012537542-13	BS	1	-8.000003E+09	0	1	-0
73	4810013377136TP1	LL	0	-3.935077E+11	0	1	-3.445076E+11
74	4810013377136TP2	LL	0	-2.193024E+11	0	1	-1.703023E+11
75	4810013377136TP3	LL	0	-1.115006E+11	0	1	-6.25005E+10
76	4810013377136TP4	LL	0	-7.090025E+10	0	1	-2.190013E+10
77	4810013377136TP5	BS	1	-4.900012E+10	0	1	-0
78	5821012483022FX2	LL	0	-1.869975E+12	0	1	-1.796475E+12
79	5821012483022FX3	LL	0	-1.187471E+12	0	1	-1.11397E+12
80	5821012483022FX4	LL	0	-6.293198E+11	0	1	-5.558195E+11
81	5821012483022FX5	LL	0	-1.127006E+11	0	1	-3.920036E+10
82	5821012483022FX6	BS	1	-7.350027E+10	0	1	-0
83	5895012913073F7	LL	0	-6.300002E+09	0	1	-6.200002E+09
84	5895012913073F8	LL	0	-2.5E+09	0	1	-2.4E+09
85	5895012913073F9	LL	0	-1.4E+09	0	1	-1.3E+09
86	5895012913073F10	LL	0	-6E+08	0	1	-5E+08
87	5895012913073F11	BS	1	-1E+08	0	1	-0
88	5895013640160LN2	LL	0	-1.960792E+12	0	1	-1.955492E+12
89	5895013640160LN3	LL	0	-5.002125E+11	0	1	-4.949125E+11
90	5895013640160LN4	LL	0	-1.174007E+11	0	1	-1.121007E+11
91	5895013640160LN5	LL	0	-2.580003E+10	0	1	-2.050003E+10
92	5895013640160LN6	BS	1	-5.300001E+09	0	1	-0
93	5895013823225FX2	LL	0	-3.607951E+12	0	1	-1.893204E+12
94	5895013823225FX3	LL	0	-2.655553E+12	0	1	-9.408056E+11
95	5895013823225FX4	BS	1	-1.714747E+12	0	1	-0
96	5985013902368E9	LL	0	-1.141165E+12	0	1	-9.90064E+11
97	5985013902368E10	LL	0	-7.697296E+11	0	1	-6.186285E+11
98	5985013902368E11	LL	0	-5.162133E+11	0	1	-3.651122E+11
99	5985013902368E12	LL	0	-3.444059E+11	0	1	-1.933048E+11
100	5985013902368E13	LL	0	-2.287026E+11	0	1	-7.760147E+10
101	5985013902368E14	BS	1	-1.511011E+11	0	1	-0
102	5985013934820EW2	LL	0	-1.26398E+12	0	1	-1.24688E+12
103	5985013934820EW3	LL	0	-6.381204E+11	0	1	-6.210203E+11
104	5985013934820EW4	LL	0	-3.17105E+11	0	1	-3.00005E+11
105	5985013934820EW5	LL	0	-1.554012E+11	0	1	-1.383012E+11
106	5985013934820EW6	LL	0	-7.520028E+10	0	1	-5.810027E+10
107	5985013934820EW7	LL	0	-3.600006E+10	0	1	-1.890005E+10
108	5985013934820EW8	BS	1	-1.710001E+10	0	1	-0
109	5998013410403FX1	LL	0	-1.929019E+11	0	1	-1.194016E+11
110	5998013410403FX2	LL	0	-1.127006E+11	0	1	-3.920036E+10
111	5998013410403FX3	BS	1	-7.350027E+10	0	1	-0
112	6605012400136FX4	LL	0	-2.640493E+13	0	1	-1.725114E+13

113	6605012400136FX5	LL	0	-1.553015E+13	0	1	-6.376365E+12
114	6605012400136FX6	LL	0	-1.239548E+13	0	1	-3.241691E+12
115	6605012400136FX7	LL	0	-1.120067E+13	0	1	-2.046882E+12
116	6605012400136FX8	LL	0	-1.014084E+13	0	1	-9.870518E+11
117	6605012400136FX9	BS	1	-9.153788E+12	0	1	-0
118	6610013195039-36	LL	0	-5.220014E+10	0	1	-4.830014E+10
119	6610013195039-37	LL	0	-1.340001E+10	0	1	-9.500008E+09
120	6610013195039-38	LL	0	-1.050001E+10	0	1	-6.600005E+09
121	6610013195039-39	LL	0	-8.400004E+09	0	1	-4.500003E+09
122	6610013195039-40	LL	0	-6.600002E+09	0	1	-2.700001E+09
123	6610013195039-41	LL	0	-5.100001E+09	0	1	-1.200001E+09
124	6610013195039-42	BS	1	-3.900001E+09	0	1	-0

CPLEX Solution Output for the PLDMq at a Budget Level of $3,185,775

PROBLEM NAME	tqscale1.mps	
DATA NAME	scaledq.mps	
OBJECTIVE VALUE	-1.696933E+13	
STATUS	OPTIMAL SOLN	
ITERATION	26	
OBJECTIVE	obj	(MAX)
RHS	rhs	
RANGES		
BOUNDS	BOUND	

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	-1.696933E+13	1.696933E+13	NONE	NONE	1
2	budget	UL	3185775	0	NONE	3185775	-2325298
3	order1	EQ	1	0	1	1	3.902028E+11
4	order2	EQ	1	0	1	1	3.465943E+11
5	order3	EQ	1	0	1	1	1.00002E+12
6	order4	EQ	1	0	1	1	4.814845E+12
7	order5	BS	0	1	NONE	1	0
8	order6	EQ	1	0	1	1	1.20443E+12
9	order7	EQ	1	0	1	1	5.351788E+11
10	order8	EQ	1	0	1	1	2.000538E+12
11	order9	BS	0	1	NONE	1	0
12	order10	EQ	1	0	1	1	1.403239E+11
13	order11	EQ	1	0	1	1	1.425934E+11
14	order12	EQ	1	0	1	1	3.11491E+11

15	order13	EQ	1	0	1	1	1.576249E+11
16	order14	EQ	1	0	1	1	4.17169E+11
17	order15	EQ	1	0	1	1	2.207998E+12
18	order16	EQ	1	0	1	1	5.854402E+11
19	order17	EQ	1	0	1	1	1.749241E+11
20	order18	EQ	1	0	1	1	1.930851E+11
21	order19	EQ	1	0	1	1	9.478286E+12
22	order20	EQ	1	0	1	1	3.033082E+11

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
23	1005012982522-1	LL	0	-1.645235E+12	0	1	-1.324133E+12
24	1005012982522-2	LL	0	-6.93024E+11	0	1	-4.410223E+11
25	1005012982522-3	LL	0	-2.84104E+11	0	1	-1.012028E+11
26	1005012982522-4	BS	0.9347889	-1.138006E+11	0	1	-0
27	1005012982522-5	BS	0.06521113	-4.47001E+10	0	1	-0
28	127001356258 4FX1	LL	0	-6.644221E+11	0	1	-3.503229E+11
29	127001356258 4FX2	BS	1	-2.81604E+11	0	1	-0
30	1560012912590FX0	LL	0	-1.439696E+13	0	1	-1.339694E+13
31	1560012912590FX1	LL	0	-8.674962E+12	0	1	-7.682456E+12
32	1560012912590FX2	LL	0	-5.127214E+12	0	1	-4.142224E+12
33	1560012912590FX3	LL	0	-2.979044E+12	0	1	-2.001568E+12
34	1560012912590FX4	LL	0	-1.704845E+12	0	1	-7.348843E+11
35	1560012912590FX5	BS	1	-9.624463E+11	0	1	-0
36	1630012251893125	LL	0	-2.642249E+12	0	1	-3.838134E+11
37	1630012251893126	LL	0	-2.544424E+12	0	1	-3.064393E+11
38	1630012251893127	LL	0	-2.452401E+12	0	1	-2.348675E+11
39	1630012251893128	LL	0	-2.36578E+12	0	1	-1.68698E+11
40	1630012251893129	LL	0	-2.284261E+12	0	1	-1.076303E+11
41	1630012251893130	LL	0	-2.207744E+12	0	1	-5.156437E+10
42	1630012251893131	BS	1	-2.135728E+12	0	1	-0
43	1650013969218-1	LL	0	-6.176191E+11	0	1	-7.245828E+11
44	1650013969218-2	LL	0	-2.348028E+11	0	1	-4.487301E+11
45	1660013387046BO3	LL	0	-8.382351E+11	0	1	-3.618595E+10
46	1660013387046BO4	BS	1	-6.679223E+11	0	1	-0
47	1660013387046BO5	LL	0	-5.788168E+11	0	1	-4.502132E+10
48	1660013387046BO6	LL	0	-5.10113E+11	0	1	-1.104444E+11
49	1660013389649B5	LL	0	-3.755005E+12	0	1	-3.422225E+12
50	1660013389649B6	LL	0	-2.101321E+12	0	1	-1.809021E+12
51	1660013389649B7	LL	0	-1.154067E+12	0	1	-9.022468E+11
52	1660013389649B8	LL	0	-6.232194E+11	0	1	-4.118795E+11
53	1660013389649B9	LL	0	-3.313055E+11	0	1	-1.604454E+11
54	1660013389649B10	LL	0	-1.737015E+11	0	1	-4.332125E+10

55	1660013389649B11	BS	0	-8.99004E+10	0	1	-0
56	1660013389649B12	UL	1	-4.600011E+10	0	1	3.420445E+09
57	1660013389649B13	LL	0	-2.330003E+10	0	1	-1.435933E+10
58	2835012180143-1	LL	0	-2.710667E+12	0	1	-8.668406E+11
59	2835012180143-2	LL	0	-2.149331E+12	0	1	-4.62215E+11
60	2835012180143-3	LL	0	-1.740952E+12	0	1	-2.105464E+11
61	2835012180143-4	BS	1	-1.373694E+12	0	1	-0
62	2835012188080-1	LL	0	-6.930024E+10	0	1	-2.336286E+11
63	2835012188080-2	LL	0	-5.160013E+10	0	1	-3.802569E+11
64	2835012188080-3	LL	0	-3.930008E+10	0	1	-5.322852E+11
65	2835012188080-4	LL	0	-3.070005E+10	0	1	-6.880135E+11
66	4810012537542-7	LL	0	-6.381204E+11	0	1	-5.690478E+11
67	4810012537542-8	LL	0	-3.17105E+11	0	1	-2.582112E+11
68	4810012537542-9	LL	0	-1.554012E+11	0	1	-1.066862E+11
69	4810012537542-10	LL	0	-7.520028E+10	0	1	-3.6664E+10
70	4810012537542-11	LL	0	-3.600006E+10	0	1	-7.642545E+09
71	4810012537542-12	UL	1	-1.710001E+10	0	1	1.078747E+09
72	4810012537542-13	BS	0	-8.000003E+09	0	1	-0
73	4810013377136TP1	LL	0	-3.935077E+11	0	1	-2.69633E+11
74	4810013377136TP2	LL	0	-2.193024E+11	0	1	-1.141463E+11
75	4810013377136TP3	LL	0	-1.115006E+11	0	1	-2.506321E+10
76	4810013377136TP4	LL	0	-7.090025E+10	0	1	-3.181484E+09
77	4810013377136TP5	BS	1	-4.900012E+10	0	1	-0
78	5821012483022FX2	LL	0	-1.869975E+12	0	1	-1.637814E+12
79	5821012483022FX3	LL	0	-1.187471E+12	0	1	-9.949749E+11
80	5821012483022FX4	LL	0	-6.293198E+11	0	1	-4.764893E+11
81	5821012483022FX5	UL	1	-1.127006E+11	0	1	4.6475E+08
82	5821012483022FX6	BS	0	-7.350027E+10	0	1	-0
83	5895012913073F7	UL	1	-6.300002E+09	0	1	1.559061E+10
84	5895012913073F8	BS	0	-2.5E+09	0	1	-0
85	5895012913073F9	LL	0	-1.4E+09	0	1	-1.829061E+10
86	5895012913073F10	LL	0	-6E+08	0	1	-3.688122E+10
87	5895012913073F11	LL	0	-1E+08	0	1	-5.577183E+10
88	5895013640160LN2	LL	0	-1.960792E+12	0	1	-1.700171E+12
89	5895013640160LN3	LL	0	-5.002125E+11	0	1	-3.178649E+11
90	5895013640160LN4	LL	0	-1.174007E+11	0	1	-1.332686E+10
91	5895013640160LN5	BS	1	-2.580003E+10	0	1	-0
92	5895013640160LN6	LL	0	-5.300001E+09	0	1	-5.777377E+10
93	5895013823225FX2	LL	0	-3.607951E+12	0	1	-1.646578E+12
94	5895013823225FX3	LL	0	-2.655553E+12	0	1	-8.174927E+11
95	5895013823225FX4	BS	1	-1.714747E+12	0	1	-0
96	5985013902368E9	LL	0	-1.141165E+12	0	1	-8.349429E+11
97	5985013902368E10	LL	0	-7.697296E+11	0	1	-4.945316E+11
98	5985013902368E11	LL	0	-5.162133E+11	0	1	-2.720395E+11
99	5985013902368E12	LL	0	-3.444059E+11	0	1	-1.312564E+11

100	5985013902368E13	LL	0	-2.287026E+11	0	1	-4.657726E+10
101	5985013902368E14	BS	1	-1.511011E+11	0	1	-0
102	5985013934820EW2	LL	0	-1.26398E+12	0	1	-1.128512E+12
103	5985013934820EW3	LL	0	-6.381204E+11	0	1	-5.223803E+11
104	5985013934820EW4	LL	0	-3.17105E+11	0	1	-2.21093E+11
105	5985013934820EW5	LL	0	-1.554012E+11	0	1	-7.911716E+10
106	5985013934820EW6	LL	0	-7.520028E+10	0	1	-1.864424E+10
107	5985013934820EW7	UL	1	-3.600006E+10	0	1	8.279619E+08
108	5985013934820EW8	BS	0	-1.710001E+10	0	1	-0
109	5998013410403FX1	LL	0	-1.929019E+11	0	1	-3.967834E+10
110	5998013410403FX2	UL	1	-1.127006E+11	0	1	6.61261E+08
111	5998013410403FX3	BS	0	-7.350027E+10	0	1	-0
112	6605012400136FX4	LL	0	-2.640493E+13	0	1	-1.707087E+13
113	6605012400136FX5	LL	0	-1.553015E+13	0	1	-6.232144E+12
114	6605012400136FX6	LL	0	-1.239548E+13	0	1	-3.133525E+12
115	6605012400136FX7	LL	0	-1.120067E+13	0	1	-1.974772E+12
116	6605012400136FX8	LL	0	-1.014084E+13	0	1	-9.509965E+11
117	6605012400136FX9	BS	1	-9.153788E+12	0	1	-0
118	6610013195039-36	LL	0	-5.220014E+10	0	1	-2.628917E+10
119	6610013195039-37	UL	1	-1.340001E+10	0	1	4.805475E+09
120	6610013195039-38	BS	0	-1.050001E+10	0	1	-0
121	6610013195039-39	LL	0	-8.400004E+09	0	1	-5.605477E+09
122	6610013195039-40	LL	0	-6.600002E+09	0	1	-1.151095E+10
123	6610013195039-41	LL	0	-5.100001E+09	0	1	-1.771643E+10
124	6610013195039-42	LL	0	-3.900001E+09	0	1	-2.422191E+10

CPLEX Solution Output for the PLDMq at a Budget Level of $2,700,222

```
PROBLEM NAME     tqscalen.mps
DATA    NAME     scaledq.mps
OBJECTIVE VALUE  -1.885264E+13
STATUS           OPTIMAL SOLN
ITERATION        54

OBJECTIVE        obj                    (MAX)
RHS              rhs
RANGES
BOUNDS           BOUND

SECTION 1 - ROWS
```

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	-1.885264E+13	1.885264E+13	NONE	NONE	1

2	budget	UL	2700222	0	NONE	2700222	-8672086
3	order1	EQ	1	0	1	1	1.144629E+12
4	order2	BS	0	1	NONE	1	0
5	order3	EQ	1	0	1	1	1.102576E+12
6	order4	EQ	1	0	1	1	1.212335E+13
7	order5	BS	0	1	NONE	1	0
8	order6	EQ	1	0	1	1	2.668801E+12
9	order7	EQ	1	0	1	1	1.690014E+12
10	order8	EQ	1	0	1	1	3.318222E+12
11	order9	BS	0	1	NONE	1	0
12	order10	EQ	1	0	1	1	4.535731E+11
13	order11	EQ	1	0	1	1	3.209315E+11
14	order12	EQ	1	0	1	1	9.610751E+11
15	order13	EQ	1	0	1	1	5.125145E+11
16	order14	EQ	1	0	1	1	1.375968E+12
17	order15	EQ	1	0	1	1	3.554304E+12
18	order16	EQ	1	0	1	1	1.732846E+12
19	order17	EQ	1	0	1	1	5.232746E+11
20	order18	BS	0	1	NONE	1	0
21	order19	EQ	1	0	1	1	1.036399E+13
22	order20	EQ	1	0	1	1	1.08674E+12

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
23	1005012982522-1	LL	0	-1.645235E+12	0	1	-7.583132E+11
24	1005012982522-2	LL	0	-6.93024E+11	0	1	-6.380905E+10
25	1005012982522-3	UL	1	-2.84104E+11	0	1	8.740377E+10
26	1005012982522-4	BS	0	-1.138006E+11	0	1	-0
27	1005012982522-5	LL	0	-4.47001E+10	0	1	-1.886066E+11
28	1270013562584FX1	LL	0	-6.644221E+11	0	1	-7.856113E+11
29	1270013562584FX2	LL	0	-2.81604E+11	0	1	-5.239823E+11
30	1560012912590FX0	LL	0	-1.439696E+13	0	1	-1.329438E+13
31	1560012912590FX1	LL	0	-8.674962E+12	0	1	-7.600412E+12
32	1560012912590FX2	LL	0	-5.127214E+12	0	1	-4.08069E+12
33	1560012912590FX3	LL	0	-2.979044E+12	0	1	-1.960546E+12
34	1560012912590FX4	LL	0	-1.704845E+12	0	1	-7.143732E+11
35	1560012912590FX5	BS	1	-9.624463E+11	0	1	-0
36	1630012251893125	LL	0	-2.642249E+12	0	1	-5.290003E+10
37	1630012251893126	LL	0	-2.544424E+12	0	1	-3.13467E+10
38	1630012251893127	LL	0	-2.452401E+12	0	1	-1.559575E+10
39	1630012251893128	LL	0	-2.36578E+12	0	1	-5.246916E+09
40	1630012251893129	BS	0	-2.284261E+12	0	1	-0
41	1630012251893130	UL	1	-2.207744E+12	0	1	2.451487E+08

42	1630012251893131	LL	0	-2.135728E+12	0	1	-4.011248E+09
43	1650013969218-1	LL	0	-6.176191E+11	0	1	-1.016535E+12
44	1650013969218-2	LL	0	-2.348028E+11	0	1	-1.032635E+12
45	1660013387046BO3	UL	1	-8.382351E+11	0	1	3.299069E+11
46	1660013387046BO4	BS	0	-6.679223E+11	0	1	-0
47	1660013387046BO5	LL	0	-5.788168E+11	0	1	-4.111141E+11
48	1660013387046BO6	LL	0	-5.10113E+11	0	1	-8.426301E+11
49	1660013389649B5	LL	0	-3.755005E+12	0	1	-2.819829E+12
50	1660013389649B6	LL	0	-2.101321E+12	0	1	-1.317112E+12
51	1660013389649B7	LL	0	-1.154067E+12	0	1	-5.208258E+11
52	1660013389649B8	LL	0	-6.232194E+11	0	1	-1.409463E+11
53	1660013389649B9	BS	0	-3.313055E+11	0	1	-0
54	1660013389649B10	UL	1	-1.737015E+11	0	1	6.636321E+09
55	1660013389649B11	LL	0	-8.99004E+10	0	1	-6.053023E+10
56	1660013389649B12	LL	0	-4.600011E+10	0	1	-1.675976E+11
57	1660013389649B13	LL	0	-2.330003E+10	0	1	-2.958652E+11
58	2835012180143-1	UL	1	-2.710667E+12	0	1	2.310903E+10
59	2835012180143-2	BS	0	-2.149331E+12	0	1	-0
60	2835012180143-3	LL	0	-1.740952E+12	0	1	-1.76066E+11
61	2835012180143-4	LL	0	-1.373694E+12	0	1	-3.932542E+11
62	2835012188080-1	LL	0	-6.930024E+10	0	1	-6.82155E+11
63	2835012188080-2	LL	0	-5.160013E+10	0	1	-1.27731E+12
64	2835012188080-3	LL	0	-3.930008E+10	0	1	-1.877864E+12
65	2835012188080-4	LL	0	-3.070005E+10	0	1	-2.482119E+12
66	4810012537542-7	LL	0	-6.381204E+11	0	1	-4.502755E+11
67	4810012537542-8	LL	0	-3.17105E+11	0	1	-1.672214E+11
68	4810012537542-9	LL	0	-1.554012E+11	0	1	-4.347877E+10
69	4810012537542-10	LL	0	-7.520028E+10	0	1	-1.23903E+09
70	4810012537542-11	BS	1	-3.600006E+10	0	1	-0
71	4810012537542-12	LL	0	-1.710001E+10	0	1	-1.906114E+10
72	4810012537542-13	LL	0	-8.000003E+09	0	1	-4.792232E+10
73	4810013377136TP1	LL	0	-3.935077E+11	0	1	-1.423865E+11
74	4810013377136TP2	LL	0	-2.193024E+11	0	1	-3.799149E+10
75	4810013377136TP3	BS	1	-1.115006E+11	0	1	-0
76	4810013377136TP4	LL	0	-7.090025E+10	0	1	-2.920992E+10
77	4810013377136TP5	LL	0	-4.900012E+10	0	1	-7.712008E+10
78	5821012483022FX2	LL	0	-1.869975E+12	0	1	-1.204758E+12
79	5821012483022FX3	LL	0	-1.187471E+12	0	1	-6.701828E+11
80	5821012483022FX4	LL	0	-6.293198E+11	0	1	-2.599613E+11
81	5821012483022FX5	UL	1	-1.127006E+11	0	1	1.087288E+11
82	5821012483022FX6	BS	0	-7.350027E+10	0	1	-0
83	5895012913073F7	BS	1	-6.300002E+09	0	1	-0
84	5895012913073F8	LL	0	-2.5E+09	0	1	-6.851635E+10
85	5895012913073F9	LL	0	-1.4E+09	0	1	-1.397327E+11
86	5895012913073F10	LL	0	-6E+08	0	1	-2.11249E+11

87	5895012913073F11	LL	0	-1E+08	0	1	-2.830654E+11
88	5895013640160LN2	LL	0	-1.960792E+12	0	1	-1.168661E+12
89	5895013640160LN3	BS	0	-5.002125E+11	0	1	-0
90	5895013640160LN4	UL	1	-1.174007E+11	0	1	9.089346E+10
91	5895013640160LN5	LL	0	-2.580003E+10	0	1	-1.094243E+11
92	5895013640160LN6	LL	0	-5.300001E+09	0	1	-3.808426E+11
93	5895013823225FX2	LL	0	-3.607951E+12	0	1	-9.734254E+11
94	5895013823225FX3	LL	0	-2.655553E+12	0	1	-4.809163E+11
95	5895013823225FX4	BS	1	-1.714747E+12	0	1	-0
96	598501390236 8E9	LL	0	-1.141165E+12	0	1	-4.496492E+11
97	5985013902368E10	LL	0	-7.697296E+11	0	1	-1.939171E+11
98	5985013902368E11	LL	0	-5.162133E+11	0	1	-5.610408E+10
99	5985013902368E12	BS	0.9161545	-3.444059E+11	0	1	-0
100	5985013902368E13	BS	0.0838455	-2.287026E+11	0	1	-0
101	5985013902368E14	LL	0	-1.511011E+11	0	1	-3.810184E+10
102	5985013934820EW2	LL	0	-1.26398E+12	0	1	-8.878547E+11
103	5985013934820EW3	LL	0	-6.381204E+11	0	1	-3.355698E+11
104	5985013934820EW4	LL	0	-3.17105E+11	0	1	-8.812915E+10
105	5985013934820EW5	BS	0	-1.554012E+11	0	1	-0
106	5985013934820EW6	UL	1	-7.520028E+10	0	1	6.626255E+09
107	5985013934820EW7	LL	0	-3.600006E+10	0	1	-2.77482E+10
108	5985013934820EW8	LL	0	-1.710001E+10	0	1	-8.242282E+10
109	5998013410403FX1	LL	0	-1.929019E+11	0	1	-3.415639E+11
110	5998013410403FX2	LL	0	-1.127006E+11	0	1	-4.100247E+11
111	5998013410403FX3	LL	0	-7.350027E+10	0	1	-5.194863E+11
112	6605012400136FX4	LL	0	-2.640493E+13	0	1	-1.657881E+13
113	6605012400136FX5	LL	0	-1.553015E+13	0	1	-5.838499E+12
114	6605012400136FX6	LL	0	-1.239548E+13	0	1	-2.838292E+12
115	6605012400136FX7	LL	0	-1.120067E+13	0	1	-1.777949E+12
116	6605012400136FX8	LL	0	-1.014084E+13	0	1	-8.525854E+11
117	6605012400136FX9	BS	1	-9.153788E+12	0	1	-0
118	6610013195039-36	BS	0	-5.220014E+10	0	1	-0
119	6610013195039-37	UL	1	-1.340001E+10	0	1	1.006292E+10
120	6610013195039-38	LL	0	-1.050001E+10	0	1	-1.577429E+10
121	6610013195039-39	LL	0	-8.400004E+09	0	1	-4.24115E+10
122	6610013195039-40	LL	0	-6.600002E+09	0	1	-6.934871E+10
123	6610013195039-41	LL	0	-5.100001E+09	0	1	-9.658592E+10
124	6610013195039-42	LL	0	-3.900001E+09	0	1	-1.241231E+11

Appendix F: CPLEX RHS and Objective Function Coefficient Sensitivity Analysis				
		RHS Sensitivity Ranges		
Constraint Name	Dual Price	Down	Current	Up
budget	8672085.79	2699103.03	2700221.70	2712445.07
		OBJ Sensitivity Ranges		
Variable Name	Reduced Cost	Down	Current	Up
1005012982522-1	0.99992417	-infinity	0.99983549	0.99991131
1005012982522-2	0.99999362	-infinity	0.99993070	0.99993708
1005012982522-3	1.00000874	0.99996285	0.99997159	+infinity
1005012982522-4	1.00000000	0.99998224	0.99998862	0.99999736
1005012982522-5	0.99998114	-infinity	0.99999553	1.00001439
1270013562584FX1	0.99992144	-infinity	0.99993356	1.00001212
1270013562584FX2	0.99994760	-infinity	0.99997184	1.00002424
1560012912590FX0	0.99867144	-infinity	0.99856134	0.99988975
1560012912590FX1	0.99924025	-infinity	0.99913288	0.99989255
1560012912590FX2	0.99959201	-infinity	0.99948741	0.99989535
1560012912590FX3	0.99980396	-infinity	0.99970214	0.99989816
1560012912590FX4	0.99992857	-infinity	0.99982953	0.99990096
1560012912590FX5	1.00000000	0.99983233	0.99990376	+infinity
1630012251893125	0.99999471	-infinity	0.99973581	0.99974110
1630012251893126	0.99999687	-infinity	0.99974559	0.99974872
1630012251893127	0.99999844	-infinity	0.99975479	0.99975635
1630012251893128	0.99999948	-infinity	0.99976345	0.99976397
1630012251893129	1.00000000	0.99977120	0.99977160	0.99977162
1630012251893130	1.00000002	0.99977923	0.99977925	+infinity
1630012251893131	0.99999960	-infinity	0.99978645	0.99978685
1650013969218-1	0.99989835	-infinity	0.99993824	1.00003989
1650013969218-2	0.99989674	-infinity	0.99997652	1.00007979
1660013387046BO3	1.00003299	0.99988319	0.99991618	+infinity
1660013387046BO4	1.00000000	0.99989210	0.99993321	0.99996620
1660013387046BO5	0.99995889	-infinity	0.99994212	0.99998323
1660013387046BO6	0.99991574	-infinity	0.99994899	1.00003325
1660013389649B5	0.99971806	-infinity	0.99962457	0.99990649
1660013389649B6	0.99986830	-infinity	0.99978989	0.99992158
1660013389649B7	0.99994792	-infinity	0.99988460	0.99993668
1660013389649B8	0.99998591	-infinity	0.99993768	0.99995177
1660013389649B9	1.00000000	0.99996082	0.99996687	0.99996753
1660013389649B10	1.00000066	0.99998197	0.99998263	+infinity
1660013389649B11	0.99999395	-infinity	0.99999101	0.99999706
1660013389649B12	0.99998324	-infinity	0.99999540	1.00001216
1660013389649B13	0.99997041	-infinity	0.99999767	1.00002726

2835012180143-1	1.00000231	0.99972666	0.99972897	+infinity
2835012180143-2	1.00000000	0.99976749	0.99978509	0.99978740
2835012180143-3	0.99998239	-infinity	0.99982592	0.99984352
2835012180143-4	0.99996068	-infinity	0.99986264	0.99990196
2835012188080-1	0.99993179	-infinity	0.99999307	1.00006129
2835012188080-2	0.99987228	-infinity	0.99999484	1.00012258
2835012188080-3	0.99981223	-infinity	0.99999607	1.00018387
2835012188080-4	0.99975182	-infinity	0.99999693	1.00024517
4810012537542-7	0.99995497	-infinity	0.99993619	0.99998122
4810012537542-8	0.99998328	-infinity	0.99996829	0.99998501
4810012537542-9	0.99999565	-infinity	0.99998446	0.99998881
4810012537542-10	0.99999988	-infinity	0.99999248	0.99999260
4810012537542-11	1.00000000	0.99999628	0.99999640	+infinity
4810012537542-12	0.99999809	-infinity	0.99999829	1.00000020
4810012537542-13	0.99999521	-infinity	0.99999920	1.00000399
4810013377136TP1	0.99998576	-infinity	0.99996065	0.99997489
4810013377136TP2	0.99999620	-infinity	0.99997807	0.99998187
4810013377136TP3	1.00000000	0.99998593	0.99998885	+infinity
4810013377136TP4	0.99999708	-infinity	0.99999291	0.99999583
4810013377136TP5	0.99999229	-infinity	0.99999510	1.00000281
5821012483022FX2	0.99987953	-infinity	0.99981302	0.99993348
5821012483022FX3	0.99993298	-infinity	0.99988126	0.99994827
5821012483022FX4	0.99997400	-infinity	0.99993707	0.99996306
5821012483022FX5	1.00001087	0.99997786	0.99998873	+infinity
5821012483022FX6	1.00000000	0.99996665	0.99999265	1.00000352
5895012913073F7	1.00000000	0.99999252	0.99999937	+infinity
5895012913073F8	0.99999315	-infinity	0.99999975	1.00000660
5895012913073F9	0.99998603	-infinity	0.99999986	1.00001383
5895012913073F10	0.99997888	-infinity	0.99999994	1.00002107
5895012913073F11	0.99997169	-infinity	0.99999999	1.00002830
5895013640160LN2	0.99988314	-infinity	0.99980394	0.99992079
5895013640160LN3	1.00000000	0.99993904	0.99994998	0.99995907
5895013640160LN4	1.00000909	0.99997917	0.99998826	+infinity
5895013640160LN5	0.99998906	-infinity	0.99999742	1.00000836
5895013640160LN6	0.99996192	-infinity	0.99999947	1.00003755
5895013823225FX2	0.99990266	-infinity	0.99963927	0.99973658
5895013823225FX3	0.99995191	-infinity	0.99973448	0.99978256
5895013823225FX4	1.00000000	0.99978046	0.99982854	+infinity
5985013902368E9	0.99995504	-infinity	0.99988589	0.99993085
5985013902368E10	0.99998061	-infinity	0.99992303	0.99994242
5985013902368E11	0.99999439	-infinity	0.99994838	0.99995399
5985013902368E12	1.00000000	0.99996552	0.99996556	0.99996586

5985013902368E13	1.00000000	0.99997683	0.99997713	0.99997717
5985013902368E14	0.99999619	-infinity	0.99998489	0.99998870
5985013934820EW2	0.99991122	-infinity	0.99987361	0.99996239
5985013934820EW3	0.99996644	-infinity	0.99993619	0.99996975
5985013934820EW4	0.99999119	-infinity	0.99996829	0.99997710
5985013934820EW5	1.00000000	0.99998169	0.99998446	0.99998512
5985013934820EW6	1.00000066	0.99999182	0.99999248	+infinity
5985013934820EW7	0.99999723	-infinity	0.99999640	0.99999917
5985013934820EW8	0.99999176	-infinity	0.99999829	1.00000653
5998013410403FX1	0.99996584	-infinity	0.99998071	1.00001487
5998013410403FX2	0.99995900	-infinity	0.99998873	1.00002973
5998013410403FX3	0.99994805	-infinity	0.99999265	1.00004460
6605012400136FX4	0.99834349	-infinity	0.99736299	0.99901787
6605012400136FX5	0.99941632	-infinity	0.99844819	0.99903130
6605012400136FX6	0.99971621	-infinity	0.99876122	0.99904474
6605012400136FX7	0.99982222	-infinity	0.99888056	0.99905817
6605012400136FX8	0.99991475	-infinity	0.99898643	0.99907161
6605012400136FX9	1.00000000	0.99899986	0.99908504	+infinity
6610013195039-36	1.00000000	0.99999320	0.99999478	0.99999579
6610013195039-37	1.00000101	0.99999765	0.99999866	+infinity
6610013195039-38	0.99999842	-infinity	0.99999895	1.00000053
6610013195039-39	0.99999576	-infinity	0.99999916	1.00000340
6610013195039-40	0.99999307	-infinity	0.99999934	1.00000627
6610013195039-41	0.99999034	-infinity	0.99999949	1.00000915
6610013195039-42	0.99998759	-infinity	0.99999961	1.00001202

Appendix G. CPLEX Solution Output for Models with Additional Constraints

Additional Constraint Modeled: Minimum Availability Requirement for a Suite of Items

```
PROBLEM NAME     tqavail.mps
DATA    NAME     scaledq.mps
OBJECTIVE VALUE  -2.019744E+13
STATUS           OPTIMAL SOLN
ITERATION        49

OBJECTIVE        obj                 (MAX)
RHS              rhs
RANGES
BOUNDS           BOUND
```

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	-2.019744E+13	2.019744E+13	NONE	NONE	1
2	budget	UL	2700222	0	NONE	2700222	-1.795928E+07
3	order1	EQ	1	0	1	1	3.702126E+12
4	order2	BS	0	1	NONE	1	0
5	order3	EQ	1	0	1	1	1.252644E+12
6	order4	EQ	1	0	1	1	5.010691E+13
7	order5	BS	0	1	NONE	1	0
8	order6	EQ	1	0	1	1	4.811602E+12
9	order7	EQ	1	0	1	1	4.385271E+12
10	order8	EQ	1	0	1	1	3.921013E+12
11	order9	BS	0	1	NONE	1	0
12	order10	EQ	1	0	1	1	1.132177E+12
13	order11	EQ	1	0	1	1	5.084468E+11
14	order12	EQ	1	0	1	1	1.854723E+12
15	order13	EQ	1	0	1	1	1.200597E+12
16	order14	EQ	1	0	1	1	2.31384E+12
17	order15	EQ	1	0	1	1	5.512747E+12
18	order16	EQ	1	0	1	1	3.165864E+12
19	order17	EQ	1	0	1	1	9.265769E+11
20	order18	BS	0	1	NONE	1	0
21	order19	EQ	1	0	1	1	1.166002E+13
22	order20	EQ	1	0	1	1	2.406099E+12
23	avail	LL	-2.23025E+12	0	-2.23025E+12	NONE	12.77279

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
24	1005012982522-1	LL	0	-1.645235E+12	0	1	-1.949105E+13
25	1005012982522-2	LL	0	-6.93024E+11	0	1	-6.910135E+12
26	1005012982522-3	LL	0	-2.84104E+11	0	1	-1.811859E+12
27	1005012982522-4	BS	0	-1.138006E+11	0	1	-0
28	1005012982522-5	UL	1	-4.47001E+10	0	1	4.180139E+11
29	1270013562584FX1	LL	0	-6.644221E+11	0	1	-9.153964E+11
30	1270013562584FX2	LL	0	-2.81604E+11	0	1	-7.835526E+11
31	1560012912590FX0	LL	0	-1.439696E+13	0	1	-1.314431E+13
32	1560012912590FX1	LL	0	-8.674962E+12	0	1	-7.480357E+12
33	1560012912590FX2	LL	0	-5.127214E+12	0	1	-3.990649E+12
34	1560012912590FX3	LL	0	-2.979044E+12	0	1	-1.900518E+12
35	1560012912590FX4	LL	0	-1.704845E+12	0	1	-6.843595E+11
36	1560012912590FX5	BS	1	-9.624463E+11	0	1	-0
37	1630012251893125	LL	0	-2.642249E+12	0	1	-6.028483E+12
38	1630012251893126	LL	0	-2.544424E+12	0	1	-4.839109E+12
39	1630012251893127	LL	0	-2.452401E+12	0	1	-3.72965E+12
40	1630012251893128	LL	0	-2.36578E+12	0	1	-2.694593E+12
41	1630012251893129	LL	0	-2.284261E+12	0	1	-1.729803E+12
42	1630012251893130	LL	0	-2.207744E+12	0	1	-8.339022E+11
43	1630012251893131	BS	1	-2.135728E+12	0	1	-0
44	1650013969218-1	LL	0	-6.176191E+11	0	1	-1.443746E+12
45	1650013969218-2	LL	0	-2.348028E+11	0	1	-1.887057E+12
46	1660013387046BO3	UL	1	-8.382351E+11	0	1	8.65607E+11
47	1660013387046BO4	BS	0	-6.679223E+11	0	1	-0
48	1660013387046BO5	LL	0	-5.788168E+11	0	1	-9.468143E+11
49	1660013387046BO6	LL	0	-5.10113E+11	0	1	-1.91403E+12
50	1660013389649B5	LL	0	-3.755005E+12	0	1	-4.889484E+13
51	1660013389649B6	LL	0	-2.101321E+12	0	1	-2.643164E+13
52	1660013389649B7	LL	0	-1.154067E+12	0	1	-1.369795E+13
53	1660013389649B8	LL	0	-6.232194E+11	0	1	-6.699347E+12
54	1660013389649B9	LL	0	-3.313055E+11	0	1	-2.991521E+12
55	1660013389649B10	LL	0	-1.737015E+11	0	1	-1.133518E+12
56	1660013389649B11	LL	0	-8.99004E+10	0	1	-2.919862E+11
57	1660013389649B12	BS	0.2256296	-4.600011E+10	0	1	-0
58	1660013389649B13	BS	0.7743704	-2.330003E+10	0	1	-0
59	2835012180143-1	BS	1	-2.710667E+12	0	1	-0
60	2835012180143-2	LL	0	-2.149331E+12	0	1	-6.49009E+11
61	2835012180143-3	LL	0	-1.740952E+12	0	1	-1.450975E+12
62	2835012180143-4	LL	0	-1.373694E+12	0	1	-2.294063E+12
63	2835012188080-1	LL	0	-6.930024E+10	0	1	-1.338479E+12
64	2835012188080-2	LL	0	-5.160013E+10	0	1	-2.589958E+12

65	2835012188080-3	LL	0	-3.930008E+10	0	1	-3.846837E+12
66	2835012188080-4	LL	0	-3.070005E+10	0	1	-5.107416E+12
67	4810012537542-7	LL	0	-6.381204E+11	0	1	-8.206825E+12
68	4810012537542-8	LL	0	-3.17105E+11	0	1	-3.864164E+12
69	4810012537542-9	LL	0	-1.554012E+11	0	1	-1.715666E+12
70	4810012537542-10	LL	0	-7.520028E+10	0	1	-6.896905E+11
71	4810012537542-11	LL	0	-3.600006E+10	0	1	-2.284091E+11
72	4810012537542-12	LL	0	-1.710001E+10	0	1	-4.671759E+10
73	4810012537542-13	BS	1	-8.000003E+09	0	1	-0
74	4810013377136TP1	LL	0	-3.935077E+11	0	1	-2.963313E+10
75	4810013377136TP2	BS	1	-2.193024E+11	0	1	-0
76	4810013377136TP3	LL	0	-1.115006E+11	0	1	-3.677042E+10
77	4810013377136TP4	LL	0	-7.090025E+10	0	1	-1.407423E+11
78	4810013377136TP5	LL	0	-4.900012E+10	0	1	-2.634143E+11
79	5821012483022FX2	LL	0	-1.869975E+12	0	1	-6.279534E+11
80	5821012483022FX3	LL	0	-1.187471E+12	0	1	-2.517999E+11
81	5821012483022FX4	BS	0	-6.293198E+11	0	1	-0
82	5821012483022FX5	UL	1	-1.127006E+11	0	1	2.102683E+11
83	5821012483022FX6	LL	0	-7.350027E+10	0	1	-5.688214E+10
84	5895012913073F7	UL	1	-6.300002E+09	0	1	1.459621E+11
85	5895012913073F8	BS	0	-2.5E+09	0	1	-0
86	5895012913073F9	LL	0	-1.4E+09	0	1	-1.486621E+11
87	5895012913073F10	LL	0	-6E+08	0	1	-2.976242E+11
88	5895012913073F11	LL	0	-1E+08	0	1	-4.468862E+11
89	5895013640160LN2	LL	0	-1.960792E+12	0	1	-8.560373E+11
90	5895013640160LN3	BS	1	-5.002125E+11	0	1	-0
91	5895013640160LN4	LL	0	-1.174007E+11	0	1	-2.217306E+11
92	5895013640160LN5	LL	0	-2.580003E+10	0	1	-7.346723E+11
93	5895013640160LN6	LL	0	-5.300001E+09	0	1	-1.318715E+12
94	5895013823225FX2	BS	0.1431907	-3.607951E+12	0	1	-0
95	5895013823225FX3	BS	0.8568093	-2.655553E+12	0	1	-0
96	5895013823225FX4	LL	0	-1.714747E+12	0	1	-1.159266E+10
97	5985013902368E9	LL	0	-1.141165E+12	0	1	-1.318221E+11
98	5985013902368E10	BS	0	-7.697296E+11	0	1	-0
99	5985013902368E11	UL	1	-5.162133E+11	0	1	1.390287E+10
100	5985013902368E12	LL	0	-3.444059E+11	0	1	-5.390317E+10
101	5985013902368E13	LL	0	-2.287026E+11	0	1	-1.778133E+11
102	5985013902368E14	LL	0	-1.511011E+11	0	1	-3.398252E+11
103	5985013934820EW2	LL	0	-1.26398E+12	0	1	-6.421389E+11
104	5985013934820EW3	LL	0	-6.381204E+11	0	1	-1.686474E+11
105	5985013934820EW4	BS	0	-3.17105E+11	0	1	-0
106	5985013934820EW5	UL	1	-1.554012E+11	0	1	9.335852E+09
107	5985013934820EW6	LL	0	-7.520028E+10	0	1	-6.283119E+10
108	5985013934820EW7	LL	0	-3.600006E+10	0	1	-1.759989E+11
109	5985013934820EW8	LL	0	-1.710001E+10	0	1	-3.094669E+11

110	5998013410403FX1	LL	0	-1.929019E+11	0	1	-5.007704E+11
111	5998013410403FX2	LL	0	-1.127006E+11	0	1	-7.284378E+11
112	5998013410403FX3	LL	0	-7.350027E+10	0	1	-9.97106E+11
113	6605012400136FX4	LL	0	-2.640493E+13	0	1	-1.585879E+13
114	6605012400136FX5	LL	0	-1.553015E+13	0	1	-5.262483E+12
115	6605012400136FX6	LL	0	-1.239548E+13	0	1	-2.406279E+12
116	6605012400136FX7	LL	0	-1.120067E+13	0	1	-1.489941E+12
117	6605012400136FX8	LL	0	-1.014084E+13	0	1	-7.085813E+11
118	6605012400136FX9	BS	1	-9.153788E+12	0	1	-0
119	6610013195039-36	LL	0	-5.220014E+10	0	1	-4.553016E+11
120	6610013195039-37	UL	1	-1.340001E+10	0	1	1.957161E+10
121	6610013195039-38	BS	0	-1.050001E+10	0	1	-0
122	6610013195039-39	LL	0	-8.400004E+09	0	1	-3.058986E+10
123	6610013195039-40	LL	0	-6.600002E+09	0	1	-6.531156E+10
124	6610013195039-41	LL	0	-5.100001E+09	0	1	-1.041651E+11
125	6610013195039-42	LL	0	-3.900001E+09	0	1	-1.471505E+11

Additional Constraint Modeled: LRU Sub-budget Upper Bound

```
PROBLEM NAME     tqsubhi.mps
DATA    NAME     scaledq.mps
OBJECTIVE VALUE  -1.907437E+13
STATUS           OPTIMAL SOLN
ITERATION        47

OBJECTIVE        obj                 (MAX)
RHS              rhs
RANGES
BOUNDS           BOUND
```

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	-1.907437E+13	1.907437E+13	NONE	NONE	1
2	budget	UL	2700222	0	NONE	2700222	-8329191
3	order1	EQ	1	0	1	1	1.510864E+12
4	order2	BS	0	1	NONE	1	0
5	order3	EQ	1	0	1	1	1.097035E+12
6	order4	EQ	1	0	1	1	1.173229E+13
7	order5	BS	0	1	NONE	1	0
8	order6	EQ	1	0	1	1	2.589686E+12
9	order7	EQ	1	0	1	1	2.539618E+12
10	order8	EQ	1	0	1	1	3.272004E+12

11	order9	BS	0	1	NONE	1	0
12	order10	EQ	1	0	1	1	4.398023E+11
13	order11	EQ	1	0	1	1	4.408471E+11
14	order12	EQ	1	0	1	1	9.259803E+11
15	order13	EQ	1	0	1	1	4.924987E+11
16	order14	EQ	1	0	1	1	1.889828E+12
17	order15	EQ	1	0	1	1	3.481568E+12
18	order16	EQ	1	0	1	1	1.677947E+12
19	order17	EQ	1	0	1	1	5.087288E+11
20	order18	BS	0	1	NONE	1	0
21	order19	EQ	1	0	1	1	1.031613E+13
22	order20	EQ	1	0	1	1	1.045834E+12
23	subbud	UL	350000	0	NONE	350000	-5431347

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
24	1005012982522-1	LL	0	-1.645235E+12	0	1	-5.432913E+11
25	1005012982522-2	BS	0.4345089	-6.93024E+11	0	1	-0
26	1005012982522-3	BS	0.5654911	-2.84104E+11	0	1	-0
27	1005012982522-4	LL	0	-1.138006E+11	0	1	-2.386166E+11
28	1005012982522-5	LL	0	-4.47001E+10	0	1	-5.78436E+11
29	1270013562584FX1	LL	0	-6.644221E+11	0	1	-7.808194E+11
30	1270013562584FX2	LL	0	-2.81604E+11	0	1	-5.143987E+11
31	1560012912590FX0	LL	0	-1.439696E+13	0	1	-1.329992E+13
32	1560012912590FX1	LL	0	-8.674962E+12	0	1	-7.604845E+12
33	1560012912590FX2	LL	0	-5.127214E+12	0	1	-4.084015E+12
34	1560012912590FX3	LL	0	-2.979044E+12	0	1	-1.962762E+12
35	1560012912590FX4	LL	0	-1.704845E+12	0	1	-7.154813E+11
36	1560012912590FX5	BS	1	-9.624463E+11	0	1	-0
37	1630012251893125	LL	0	-2.642249E+12	0	1	-6.698361E+10
38	1630012251893126	LL	0	-2.544424E+12	0	1	-4.241447E+10
39	1630012251893127	LL	0	-2.452401E+12	0	1	-2.364772E+10
40	1630012251893128	LL	0	-2.36578E+12	0	1	-1.028308E+10
41	1630012251893129	LL	0	-2.284261E+12	0	1	-2.020361E+09
42	1630012251893130	UL	1	-2.207744E+12	0	1	1.240593E+09
43	1630012251893131	BS	0	-2.135728E+12	0	1	-0
44	1650013969218-1	LL	0	-6.176191E+11	0	1	-1.000762E+12
45	1650013969218-2	LL	0	-2.348028E+11	0	1	-1.001088E+12
46	1660013387046BO3	UL	1	-8.382351E+11	0	1	3.101281E+11
47	1660013387046BO4	BS	0	-6.679223E+11	0	1	-0
48	1660013387046BO5	LL	0	-5.788168E+11	0	1	-3.913354E+11
49	1660013387046BO6	LL	0	-5.10113E+11	0	1	-8.030726E+11
50	1660013389649B5	LL	0	-3.755005E+12	0	1	-2.413136E+12

51	1660013389649B6	LL	0	-2.101321E+12	0	1	-9.990018E+11
52	1660013389649B7	LL	0	-1.154067E+12	0	1	-2.912974E+11
53	1660013389649B8	BS	0	-6.232194E+11	0	1	-0
54	1660013389649B9	UL	1	-3.313055E+11	0	1	5.236416E+10
55	1660013389649B10	LL	0	-1.737015E+11	0	1	-2.958164E+10
56	1660013389649B11	LL	0	-8.99004E+10	0	1	-1.853303E+11
57	1660013389649B12	LL	0	-4.600011E+10	0	1	-3.809798E+11
58	1660013389649B13	LL	0	-2.330003E+10	0	1	-5.978295E+11
59	2835012180143-1	BS	0.1125429	-2.710667E+12	0	1	-0
60	2835012180143-2	BS	0.8874571	-2.149331E+12	0	1	-0
61	2835012180143-3	LL	0	-1.740952E+12	0	1	-1.52957E+11
62	2835012180143-4	LL	0	-1.373694E+12	0	1	-3.470362E+11
63	2835012188080-1	LL	0	-6.930024E+10	0	1	-6.579226E+11
64	2835012188080-2	LL	0	-5.160013E+10	0	1	-1.228845E+12
65	2835012188080-3	LL	0	-3.930008E+10	0	1	-1.805167E+12
66	2835012188080-4	LL	0	-3.070005E+10	0	1	-2.38519E+12
67	4810012537542-7	LL	0	-6.381204E+11	0	1	-4.535395E+11
68	4810012537542-8	LL	0	-3.17105E+11	0	1	-1.689843E+11
69	4810012537542-9	LL	0	-1.554012E+11	0	1	-4.374073E+10
70	4810012537542-10	BS	0	-7.520028E+10	0	1	-0
71	4810012537542-11	UL	1	-3.600006E+10	0	1	2.740019E+09
72	4810012537542-12	LL	0	-1.710001E+10	0	1	-1.482013E+10
73	4810012537542-13	LL	0	-8.000003E+09	0	1	-4.218032E+10
74	4810013377136TP1	LL	0	-3.935077E+11	0	1	-6.343301E+10
75	4810013377136TP2	BS	1	-2.193024E+11	0	1	-0
76	4810013377136TP3	LL	0	-1.115006E+11	0	1	-2.970547E+09
77	4810013377136TP4	LL	0	-7.090025E+10	0	1	-7.314251E+10
78	4810013377136TP5	LL	0	-4.900012E+10	0	1	-1.620147E+11
79	5821012483022FX2	LL	0	-1.869975E+12	0	1	-1.228155E+12
80	5821012483022FX3	LL	0	-1.187471E+12	0	1	-6.877302E+11
81	5821012483022FX4	LL	0	-6.293198E+11	0	1	-2.716595E+11
82	5821012483022FX5	UL	1	-1.127006E+11	0	1	1.028796E+11
83	5821012483022FX6	BS	0	-7.350027E+10	0	1	-0
84	5895012913073F7	BS	1	-6.300002E+09	0	1	-0
85	5895012913073F8	LL	0	-2.5E+09	0	1	-6.565695E+10
86	5895012913073F9	LL	0	-1.4E+09	0	1	-1.340139E+11
87	5895012913073F10	LL	0	-6E+08	0	1	-2.026709E+11
88	5895012913073F11	LL	0	-1E+08	0	1	-2.716278E+11
89	5895013640160LN2	LL	0	-1.960792E+12	0	1	-9.973747E+11
90	5895013640160LN3	BS	1	-5.002125E+11	0	1	-0
91	5895013640160LN4	LL	0	-1.174007E+11	0	1	-8.03932E+10
92	5895013640160LN5	LL	0	-2.580003E+10	0	1	-4.519976E+11
93	5895013640160LN6	LL	0	-5.300001E+09	0	1	-8.947026E+11
94	5895013823225FX2	LL	0	-3.607951E+12	0	1	-1.009793E+12
95	5895013823225FX3	LL	0	-2.655553E+12	0	1	-4.991004E+11

96	5895013823225FX4	BS	1	-1.714747E+12	0	1	-0
97	5985013902368E9	LL	0	-1.141165E+12	0	1	-4.63374E+11
98	5985013902368E10	LL	0	-7.697296E+11	0	1	-2.030669E+11
99	5985013902368E11	LL	0	-5.162133E+11	0	1	-6.0679E+10
100	5985013902368E12	BS	0	-3.444059E+11	0	1	-0
101	5985013902368E13	UL	1	-2.287026E+11	0	1	4.574921E+09
102	5985013902368E14	LL	0	-1.511011E+11	0	1	-2.8952E+10
103	5985013934820EW2	LL	0	-1.26398E+12	0	1	-8.965821E+11
104	5985013934820EW3	LL	0	-6.381204E+11	0	1	-3.413881E+11
105	5985013934820EW4	LL	0	-3.17105E+11	0	1	-9.10383E+10
106	5985013934820EW5	BS	0	-1.554012E+11	0	1	-0
107	5985013934820EW6	UL	1	-7.520028E+10	0	1	9.535405E+09
108	5985013934820EW7	LL	0	-3.600006E+10	0	1	-2.19299E+10
109	5985013934820EW8	LL	0	-1.710001E+10	0	1	-7.369537E+10
110	5998013410403FX1	LL	0	-1.929019E+11	0	1	-3.356858E+11
111	5998013410403FX2	LL	0	-1.127006E+11	0	1	-3.982684E+11
112	5998013410403FX3	LL	0	-7.350027E+10	0	1	-5.01852E+11
113	6605012400136FX4	LL	0	-2.640493E+13	0	1	-1.660539E+13
114	6605012400136FX5	LL	0	-1.553015E+13	0	1	-5.859766E+12
115	6605012400136FX6	LL	0	-1.239548E+13	0	1	-2.854242E+12
116	6605012400136FX7	LL	0	-1.120067E+13	0	1	-1.788583E+12
117	6605012400136FX8	LL	0	-1.014084E+13	0	1	-8.579022E+11
118	6605012400136FX9	BS	1	-9.153788E+12	0	1	-0
119	6610013195039-36	BS	0	-5.220014E+10	0	1	-0
120	6610013195039-37	UL	1	-1.340001E+10	0	1	1.119919E+10
121	6610013195039-38	LL	0	-1.050001E+10	0	1	-1.350175E+10
122	6610013195039-39	LL	0	-8.400004E+09	0	1	-3.900268E+10
123	6610013195039-40	LL	0	-6.600002E+09	0	1	-6.480362E+10
124	6610013195039-41	LL	0	-5.100001E+09	0	1	-9.090456E+10
125	6610013195039-42	LL	0	-3.900001E+09	0	1	-1.173055E+11

Additional Constraint Modeled: LRU Sub-budget Lower Bound

```
PROBLEM NAME      tqsublo.mps
DATA    NAME      scaledq.mps
OBJECTIVE VALUE   -1.932966E+13
STATUS            OPTIMAL SOLN
ITERATION         49

OBJECTIVE         obj                 (MAX)
RHS               rhs
RANGES
BOUNDS            BOUND
```

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	-1.932966E+13	1.932966E+13	NONE	NONE	1
2	budget	UL	2700222	0	NONE	2700222	-1.774068E+07
3	order1	EQ	1	0	1	1	4.490279E+11
4	order2	BS	0	1	NONE	1	0
5	order3	EQ	1	0	1	1	1.249112E+12
6	order4	EQ	1	0	1	1	2.220438E+13
7	order5	BS	0	1	NONE	1	0
8	order6	EQ	1	0	1	1	4.761164E+12
9	order7	EQ	1	0	1	1	6.144633E+11
10	order8	EQ	1	0	1	1	3.90628E+12
11	order9	BS	0	1	NONE	1	0
12	order10	EQ	1	0	1	1	8.543236E+11
13	order11	EQ	1	0	1	1	1.585285E+11
14	order12	EQ	1	0	1	1	1.839807E+12
15	order13	EQ	1	0	1	1	1.186013E+12
16	order14	EQ	1	0	1	1	4.838033E+11
17	order15	EQ	1	0	1	1	5.477969E+12
18	order16	EQ	1	0	1	1	3.136698E+12
19	order17	EQ	1	0	1	1	9.079679E+11
20	order18	BS	0	1	NONE	1	0
21	order19	EQ	1	0	1	1	1.162952E+13
22	order20	EQ	1	0	1	1	2.168581E+12
23	subbud	LL	500000	0	500000	NONE	1.501948E+07

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
24	1005012982522-1	LL	0	-1.645235E+12	0	1	-1.277073E+12
25	1005012982522-2	LL	0	-6.93024E+11	0	1	-4.057272E+11
26	1005012982522-3	LL	0	-2.84104E+11	0	1	-7.76728E+10
27	1005012982522-4	UL	1	-1.138006E+11	0	1	1.176502E+10
28	1005012982522-5	BS	0	-4.47001E+10	0	1	-0
29	1270013562584FX1	LL	0	-6.644221E+11	0	1	-9.123415E+11
30	1270013562584FX2	LL	0	-2.81604E+11	0	1	-7.774428E+11
31	1560012912590FX0	LL	0	-1.439696E+13	0	1	-1.314785E+13
32	1560012912590FX1	LL	0	-8.674962E+12	0	1	-7.483183E+12
33	1560012912590FX2	LL	0	-5.127214E+12	0	1	-3.992769E+12
34	1560012912590FX3	LL	0	-2.979044E+12	0	1	-1.901931E+12
35	1560012912590FX4	LL	0	-1.704845E+12	0	1	-6.850659E+11
36	1560012912590FX5	BS	1	-9.624463E+11	0	1	-0

37	1630012251893125	UL	1	-2.642249E+12	0	1	5.820603E+10
38	1630012251893126	BS	0	-2.544424E+12	0	1	-0
39	1630012251893127	LL	0	-2.452401E+12	0	1	-6.400841E+10
40	1630012251893128	LL	0	-2.36578E+12	0	1	-1.334189E+11
41	1630012251893129	LL	0	-2.284261E+12	0	1	-2.079314E+11
42	1630012251893130	LL	0	-2.207744E+12	0	1	-2.874456E+11
43	1630012251893131	LL	0	-2.135728E+12	0	1	-3.714613E+11
44	1650013969218-1	LL	0	-6.176191E+11	0	1	-1.43369E+12
45	1650013969218-2	LL	0	-2.348028E+11	0	1	-1.866945E+12
46	1660013387046BO3	UL	1	-8.382351E+11	0	1	8.529977E+11
47	1660013387046BO4	BS	0	-6.679223E+11	0	1	-0
48	1660013387046BO5	LL	0	-5.788168E+11	0	1	-9.34205E+11
49	1660013387046BO6	LL	0	-5.10113E+11	0	1	-1.888812E+12
50	1660013389649B5	LL	0	-3.755005E+12	0	1	-3.377401E+12
51	1660013389649B6	LL	0	-2.101321E+12	0	1	-1.771089E+12
52	1660013389649B7	LL	0	-1.154067E+12	0	1	-8.712068E+11
53	1660013389649B8	LL	0	-6.232194E+11	0	1	-3.877316E+11
54	1660013389649B9	LL	0	-3.313055E+11	0	1	-1.431896E+11
55	1660013389649B10	LL	0	-1.737015E+11	0	1	-3.295754E+10
56	1660013389649B11	UL	1	-8.99004E+10	0	1	3.471632E+09
57	1660013389649B12	BS	0	-4.600011E+10	0	1	-0
58	1660013389649B13	LL	0	-2.330003E+10	0	1	-2.467185E+10
59	2835012180143-1	BS	1	-2.710667E+12	0	1	-0
60	2835012180143-2	LL	0	-2.149331E+12	0	1	-6.342766E+11
61	2835012180143-3	LL	0	-1.740952E+12	0	1	-1.42151E+12
62	2835012180143-4	LL	0	-1.373694E+12	0	1	-2.249866E+12
63	2835012188080-1	LL	0	-6.930024E+10	0	1	-1.323031E+12
64	2835012188080-2	LL	0	-5.160013E+10	0	1	-2.559061E+12
65	2835012188080-3	LL	0	-3.930008E+10	0	1	-3.800492E+12
66	2835012188080-4	LL	0	-3.070005E+10	0	1	-5.045622E+12
67	4810012537542-7	LL	0	-6.381204E+11	0	1	-3.274031E+11
68	4810012537542-8	LL	0	-3.17105E+11	0	1	-8.404577E+10
69	4810012537542-9	BS	0	-1.554012E+11	0	1	-0
70	4810012537542-10	UL	1	-7.520028E+10	0	1	2.542879E+09
71	4810012537542-11	LL	0	-3.600006E+10	0	1	-3.591495E+10
72	4810012537542-12	LL	0	-1.710001E+10	0	1	-9.467294E+10
73	4810012537542-13	LL	0	-8.000003E+09	0	1	-1.63231E+11
74	4810013377136TP1	LL	0	-3.935077E+11	0	1	-2.568849E+11
75	4810013377136TP2	LL	0	-2.193024E+11	0	1	-1.045853E+11
76	4810013377136TP3	LL	0	-1.115006E+11	0	1	-1.868915E+10
77	4810013377136TP4	UL	1	-7.090025E+10	0	1	5542304
78	4810013377136TP5	BS	0	-4.900012E+10	0	1	-0
79	5821012483022FX2	LL	0	-1.869975E+12	0	1	-6.354112E+11
80	5821012483022FX3	LL	0	-1.187471E+12	0	1	-2.555288E+11
81	5821012483022FX4	BS	0	-6.293198E+11	0	1	-0

82	5821012483022FX5	UL	1	-1.127006E+11	0	1	2.139973E+11
83	5821012483022FX6	LL	0	-7.350027E+10	0	1	-4.942429E+10
84	5895012913073F7	UL	1	-6.300002E+09	0	1	1.441392E+11
85	5895012913073F8	BS	0	-2.5E+09	0	1	-0
86	5895012913073F9	LL	0	-1.4E+09	0	1	-1.468392E+11
87	5895012913073F10	LL	0	-6E+08	0	1	-2.939783E+11
88	5895012913073F11	LL	0	-1E+08	0	1	-4.414175E+11
89	5895013640160LN2	LL	0	-1.960792E+12	0	1	-1.66019E+12
90	5895013640160LN3	LL	0	-5.002125E+11	0	1	-2.912112E+11
91	5895013640160LN4	BS	0.3229087	-1.174007E+11	0	1	-0
92	5895013640160LN5	BS	0.6770913	-2.580003E+10	0	1	-0
93	5895013640160LN6	LL	0	-5.300001E+09	0	1	-7.110062E+10
94	5895013823225FX2	LL	0	-3.607951E+12	0	1	-1.159266E+10
95	5895013823225FX3	BS	0.06328207	-2.655553E+12	0	1	-0
96	5895013823225FX4	BS	0.9367179	-1.714747E+12	0	1	-0
97	5985013902368E9	LL	0	-1.141165E+12	0	1	-1.347386E+11
98	5985013902368E10	BS	0	-7.697296E+11	0	1	-0
99	5985013902368E11	UL	1	-5.162133E+11	0	1	1.681946E+10
100	5985013902368E12	LL	0	-3.444059E+11	0	1	-4.806999E+10
101	5985013902368E13	LL	0	-2.287026E+11	0	1	-1.690635E+11
102	5985013902368E14	LL	0	-1.511011E+11	0	1	-3.281589E+11
103	5985013934820EW2	LL	0	-1.26398E+12	0	1	-6.570387E+11
104	5985013934820EW3	LL	0	-6.381204E+11	0	1	-1.816925E+11
105	5985013934820EW4	LL	0	-3.17105E+11	0	1	-1.119049E+10
106	5985013934820EW5	BS	1	-1.554012E+11	0	1	-0
107	5985013934820EW6	LL	0	-7.520028E+10	0	1	-7.031241E+10
108	5985013934820EW7	LL	0	-3.600006E+10	0	1	-1.816255E+11
109	5985013934820EW8	LL	0	-1.710001E+10	0	1	-3.132388E+11
110	5998013410403FX1	LL	0	-1.929019E+11	0	1	-4.97023E+11
111	5998013410403FX2	LL	0	-1.127006E+11	0	1	-7.20943E+11
112	5998013410403FX3	LL	0	-7.350027E+10	0	1	-9.858638E+11
113	6605012400136FX4	LL	0	-2.640493E+13	0	1	-1.587574E+13
114	6605012400136FX5	LL	0	-1.553015E+13	0	1	-5.276041E+12
115	6605012400136FX6	LL	0	-1.239548E+13	0	1	-2.416448E+12
116	6605012400136FX7	LL	0	-1.120067E+13	0	1	-1.49672E+12
117	6605012400136FX8	LL	0	-1.014084E+13	0	1	-7.119709E+11
118	6605012400136FX9	BS	1	-9.153788E+12	0	1	-0
119	6610013195039-36	BS	1	-5.220014E+10	0	1	-0
120	6610013195039-37	LL	0	-1.340001E+10	0	1	-1.998822E+10
121	6610013195039-38	LL	0	-1.050001E+10	0	1	-7.587657E+10
122	6610013195039-39	LL	0	-8.400004E+09	0	1	-1.325649E+11
123	6610013195039-40	LL	0	-6.600002E+09	0	1	-1.895533E+11
124	6610013195039-41	LL	0	-5.100001E+09	0	1	-2.468416E+11
125	6610013195039-42	LL	0	-3.900001E+09	0	1	-3.0443E+11

Additional Constraint Modeled: Inventory Control Level

PROBLEM NAME	tqinvc.mps	
DATA NAME	scaledq.mps	
OBJECTIVE VALUE	-1.925901E+13	
STATUS	OPTIMAL SOLN	
ITERATION	59	
OBJECTIVE	obj	(MAX)
RHS	rhs	
RANGES		
BOUNDS	BOUND	

SECTION 1 - ROWS

NUMBER	......ROW.......	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	obj	BS	-1.925901E+13	1.925901E+13	NONE	NONE	1
2	budget	UL	2700222	0	NONE	2700222	-8188136
3	order1	EQ	1	0	1	1	1.478418E+12
4	order2	BS	0	1	NONE	1	0
5	order3	EQ	1	0	1	1	1.094756E+12
6	order4	EQ	1	0	1	1	1.156978E+13
7	order5	BS	0	1	NONE	1	0
8	order6	EQ	1	0	1	1	2.557141E+12
9	order7	EQ	1	0	1	1	2.958531E+12
10	order8	EQ	1	0	1	1	3.262498E+12
11	order9	BS	0	1	NONE	1	0
12	order10	EQ	1	0	1	1	4.302703E+11
13	order11	EQ	1	0	1	1	6.498736E+11
14	order12	EQ	1	0	1	1	9.115435E+11
15	order13	EQ	1	0	1	1	4.842649E+11
16	order14	EQ	1	0	1	1	1.775209E+12
17	order15	EQ	1	0	1	1	3.451647E+12
18	order16	EQ	1	0	1	1	1.655363E+12
19	order17	EQ	1	0	1	1	5.027452E+11
20	order18	BS	0	1	NONE	1	0
21	order19	EQ	1	0	1	1	1.029645E+13
22	order20	EQ	1	0	1	1	1.029007E+12
23	invcont	UL	15	0	NONE	15	-1.493711E+11

SECTION 2 - COLUMNS

NUMBER	.....COLUMN.....	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
24	1005012982522-1	LL	0	-1.645235E+12	0	1	-5.595145E+11
25	1005012982522-2	BS	0	-6.93024E+11	0	1	-0
26	1005012982522-3	UL	1	-2.84104E+11	0	1	1.622319E+10
27	1005012982522-4	LL	0	-1.138006E+11	0	1	-2.061702E+11
28	1005012982522-5	LL	0	-4.47001E+10	0	1	-5.297664E+11
29	1270013562584FX1	LL	0	-6.644221E+11	0	1	-7.788482E+11
30	1270013562584FX2	LL	0	-2.81604E+11	0	1	-5.104563E+11
31	1560012912590FX0	LL	0	-1.439696E+13	0	1	-1.33022E+13
32	1560012912590FX1	LL	0	-8.674962E+12	0	1	-7.606668E+12
33	1560012912590FX2	LL	0	-5.127214E+12	0	1	-4.085382E+12
34	1560012912590FX3	LL	0	-2.979044E+12	0	1	-1.963674E+12
35	1560012912590FX4	LL	0	-1.704845E+12	0	1	-7.159372E+11
36	1560012912590FX5	BS	1	-9.624463E+11	0	1	-0
37	1630012251893125	LL	0	-2.642249E+12	0	1	-7.442716E+10
38	1630012251893126	LL	0	-2.544424E+12	0	1	-4.861744E+10
39	1630012251893127	LL	0	-2.452401E+12	0	1	-2.861009E+10
40	1630012251893128	LL	0	-2.36578E+12	0	1	-1.400486E+10
41	1630012251893129	LL	0	-2.284261E+12	0	1	-4.501546E+09
42	1630012251893130	BS	0.4358781	-2.207744E+12	0	1	-0
43	1630012251893131	BS	0.5641219	-2.135728E+12	0	1	-0
44	1650013969218-1	LL	0	-6.176191E+11	0	1	-9.942733E+11
45	1650013969218-2	LL	0	-2.348028E+11	0	1	-9.881113E+11
46	1660013387046BO3	UL	1	-8.382351E+11	0	1	3.019919E+11
47	1660013387046BO4	BS	0	-6.679223E+11	0	1	-0
48	1660013387046BO5	LL	0	-5.788168E+11	0	1	-3.831991E+11
49	1660013387046BO6	LL	0	-5.10113E+11	0	1	-7.868001E+11
50	1660013389649B5	LL	0	-3.755005E+12	0	1	-2.256044E+12
51	1660013389649B6	LL	0	-2.101321E+12	0	1	-8.942735E+11
52	1660013389649B7	LL	0	-1.154067E+12	0	1	-2.389332E+11
53	1660013389649B8	BS	1	-6.232194E+11	0	1	-0
54	1660013389649B9	BS	-3.108624E-15	-3.313055E+11	0	1	-0
55	1660013389649B10	LL	0	-1.737015E+11	0	1	-1.3431E+11
56	1660013389649B11	LL	0	-8.99004E+10	0	1	-3.424228E+11
57	1660013389649B12	LL	0	-4.600011E+10	0	1	-5.904364E+11
58	1660013389649B13	LL	0	-2.330003E+10	0	1	-8.596503E+11
59	2835012180143-1	BS	0	-2.710667E+12	0	1	-0
60	2835012180143-2	UL	1	-2.149331E+12	0	1	9.506219E+09
61	2835012180143-3	LL	0	-1.740952E+12	0	1	-1.339445E+11
62	2835012180143-4	LL	0	-1.373694E+12	0	1	-3.185175E+11
63	2835012188080-1	LL	0	-6.930024E+10	0	1	-6.479543E+11
64	2835012188080-2	LL	0	-5.160013E+10	0	1	-1.208908E+12

65	2835012188080-3	LL	0	-3.930008E+10	0	1	-1.775262E+12
66	2835012188080-4	LL	0	-3.070005E+10	0	1	-2.345316E+12
67	4810012537542-7	LL	0	-6.381204E+11	0	1	-4.587493E+11
68	4810012537542-8	LL	0	-3.17105E+11	0	1	-1.735767E+11
69	4810012537542-9	LL	0	-1.554012E+11	0	1	-4.771565E+10
70	4810012537542-10	LL	0	-7.520028E+10	0	1	-3.357472E+09
71	4810012537542-11	BS	1	-3.600006E+10	0	1	-0
72	4810012537542-12	LL	0	-1.710001E+10	0	1	-1.69427E+10
73	4810012537542-13	LL	0	-8.000003E+09	0	1	-4.368543E+10
74	4810013377136TP1	UL	1	-3.935077E+11	0	1	4.108025E+10
75	4810013377136TP2	BS	0	-2.193024E+11	0	1	-0
76	4810013377136TP3	LL	0	-1.115006E+11	0	1	-1.074838E+11
77	4810013377136TP4	LL	0	-7.090025E+10	0	1	-2.82169E+11
78	4810013377136TP5	LL	0	-4.900012E+10	0	1	-4.755545E+11
79	5821012483022FX2	LL	0	-1.869975E+12	0	1	-1.237779E+12
80	5821012483022FX3	LL	0	-1.187471E+12	0	1	-6.949486E+11
81	5821012483022FX4	LL	0	-6.293198E+11	0	1	-2.764718E+11
82	5821012483022FX5	UL	1	-1.127006E+11	0	1	1.004735E+11
83	5821012483022FX6	BS	0	-7.350027E+10	0	1	-0
84	5895012913073F7	BS	1	-6.300002E+09	0	1	-0
85	5895012913073F8	LL	0	-2.5E+09	0	1	-6.44807E+10
86	5895012913073F9	LL	0	-1.4E+09	0	1	-1.316614E+11
87	5895012913073F10	LL	0	-6E+08	0	1	-1.991421E+11
88	5895012913073F11	LL	0	-1E+08	0	1	-2.669228E+11
89	5895013640160LN2	LL	0	-1.960792E+12	0	1	-1.035581E+12
90	5895013640160LN3	BS	1	-5.002125E+11	0	1	-0
91	5895013640160LN4	LL	0	-1.174007E+11	0	1	-4.2187E+10
92	5895013640160LN5	LL	0	-2.580003E+10	0	1	-3.755852E+11
93	5895013640160LN6	LL	0	-5.300001E+09	0	1	-7.800839E+11
94	5895013823225FX2	LL	0	-3.607951E+12	0	1	-1.024754E+12
95	5895013823225FX3	LL	0	-2.655553E+12	0	1	-5.065807E+11
96	5895013823225FX4	BS	1	-1.714747E+12	0	1	-0
97	598501390236 8E9	LL	0	-1.141165E+12	0	1	-4.690199E+11
98	5985013902368E10	LL	0	-7.697296E+11	0	1	-2.068308E+11
99	5985013902368E11	LL	0	-5.162133E+11	0	1	-6.256096E+10
100	5985013902368E12	BS	0	-3.444059E+11	0	1	-0
101	5985013902368E13	UL	1	-2.287026E+11	0	1	6.456878E+09
102	5985013902368E14	LL	0	-1.511011E+11	0	1	-2.518809E+10
103	5985013934820EW2	LL	0	-1.26398E+12	0	1	-9.001723E+11
104	5985013934820EW3	LL	0	-6.381204E+11	0	1	-3.437816E+11
105	5985013934820EW4	LL	0	-3.17105E+11	0	1	-9.223502E+10
106	5985013934820EW5	BS	0	-1.554012E+11	0	1	-0
107	5985013934820EW6	UL	1	-7.520028E+10	0	1	1.073212E+10
108	5985013934820EW7	LL	0	-3.600006E+10	0	1	-1.953646E+10
109	5985013934820EW8	LL	0	-1.710001E+10	0	1	-7.010521E+10

110	5998013410403FX1	LL	0	-1.929019E+11	0	1	-3.332677E+11
111	5998013410403FX2	LL	0	-1.127006E+11	0	1	-3.934323E+11
112	5998013410403FX3	LL	0	-7.350027E+10	0	1	-4.945978E+11
113	6605012400136FX4	LL	0	-2.640493E+13	0	1	-1.661633E+13
114	6605012400136FX5	LL	0	-1.553015E+13	0	1	-5.868515E+12
115	6605012400136FX6	LL	0	-1.239548E+13	0	1	-2.860804E+12
116	6605012400136FX7	LL	0	-1.120067E+13	0	1	-1.792957E+12
117	6605012400136FX8	LL	0	-1.014084E+13	0	1	-8.600893E+11
118	6605012400136FX9	BS	1	-9.153788E+12	0	1	-0
119	6610013195039-36	BS	0	-5.220014E+10	0	1	-0
120	6610013195039-37	UL	1	-1.340001E+10	0	1	1.166661E+10
121	6610013195039-38	LL	0	-1.050001E+10	0	1	-1.25669E+10
122	6610013195039-39	LL	0	-8.400004E+09	0	1	-3.760042E+10
123	6610013195039-40	LL	0	-6.600002E+09	0	1	-6.293394E+10
124	6610013195039-41	LL	0	-5.100001E+09	0	1	-8.856745E+10
125	6610013195039-42	LL	0	-3.900001E+09	0	1	-1.14501E+11

Appendix H: Parametric Analysis Output Data

Parametric Output Data for the PLDMq Budget Constraint

Variable OUT	Variable IN	Pivot Row	RHS Value	Dual Price Before Pivot	Obj Value (scaled)	Obj Value (unscaled)	Q
			2700220	8.67209	-18852600	-0.00188526	0.99811652
NSN16-12	NSN4-130	5	2699100	8.67209	-18862300	-0.00188623	0.99811555
NSN4-130	NSN10-10	5	2690310	8.69993	-18938900	-0.00189389	0.9981079
NSN10-10	NSN7-10	5	2685930	8.95514	-18978100	-0.00189781	0.99810399
NSN7-10	NSN4-128	5	2668520	9.0533	-19135700	-0.00191357	0.99808826
NSN4-128	NSN17-5	5	2659730	9.26866	-19217200	-0.00192172	0.99808013
NSN17-5	NSN4-127	5	2651240	9.4531	-19297400	-0.00192974	0.99807212
NSN4-129	NSN4-128	21	2651240	9.5587	-19297400	-0.00192974	0.99807212
NSN4-128	NSN4-126	21	2642450	9.84875	-19384000	-0.00193840	0.99806348
NSN4-126	NSN16-11	21	2633650	10.463	-19476000	-0.00194760	0.9980543
NSN16-13	NSN4-125	3	2633650	10.7746	-19476000	-0.00194760	0.9980543
NSN4-127	NSN4-126	5	2633650	10.7928	-19476000	-0.00194760	0.9980543
NSN4-126	NSN14-3	5	2624860	11.1227	-19573900	-0.00195739	0.99804452
NSN14-3	NSN20-37	5	2591200	11.3723	-19956700	-0.00199567	0.99800632
NSN20-37	NSN16-12	5	2587880	11.7088	-19995500	-0.00199955	0.99800245
NSN16-12	NSN11-2	5	2574540	12.8771	-20167300	-0.00201673	0.9979853
NSN11-2	NSN10-9	5	2566490	13.3915	-20275100	-0.00202751	0.99797454
NSN10-11	NSN1-2	19	2566490	13.6384	-20275100	-0.00202751	0.99797454
NSN1-2	NSN17-4	19	2536770	13.7605	-20684000	-0.00206840	0.99793374
NSN17-6	NSN12-4	18	2536770	14.2564	-20684000	-0.00206840	0.99793374
NSN12-6	NSN7-8	16	2536770	16.292	-20684000	-0.00206840	0.99793374
NSN7-8	NSN11-1	16	2519360	16.7685	-20975900	-0.00209759	0.99790461
NSN11-3	NSN15-3	14	2519360	17.516	-20975900	-0.00209759	0.99790461
NSN15-3	NSN15-2	14	2466330	17.7407	-21916700	-0.00219167	0.99781073
NSN15-4	NSN15-3	13	2466330	17.85	-21916700	-0.00219167	0.99781073
NSN15-3	NSN10-10	13	2413300	17.9593	-22869100	-0.00228691	0.9977157
NSN10-10	NSN16-10	13	2408930	18.3216	-22949300	-0.00229493	0.9977077
NSN16-10	NSN17-5	13	2395580	19.0013	-23202800	-0.00232028	0.99768241
NSN17-5	NSN11-2	13	2387100	19.0597	-23364500	-0.00233645	0.99766628
NSN11-2	NSN1-1	13	2379050	21.6404	-23538700	-0.00235387	0.9976489
NSN1-3	NSN16-9	17	2379050	22.9017	-23538700	-0.00235387	0.9976489
NSN16-11	NSN7-7	21	2379050	23.4204	-23538700	-0.00235387	0.9976489
NSN7-9	NSN14-2	6	2379050	23.6311	-23538700	-0.00235387	0.9976489
NSN14-4	NSN16-10	9	2379050	27.381	-23538700	-0.00235387	0.9976489
NSN16-10	NSN12-5	9	2365710	27.8395	-23910200	-0.00239102	0.99761184
NSN12-5	NSN7-8	9	2348650	30.2859	-24426800	-0.00244268	0.9975603
NSN7-8	NSN1-2	9	2331240	30.4936	-24957600	-0.00249576	0.99750735
NSN1-2	NSN12-3	9	2301520	32.0428	-25909900	-0.00259099	0.99741236
NSN12-3	NSN12-2	9	2284470	32.7206	-26468000	-0.00264680	0.9973567

NSN12-4	NSN10-8	18	2284470	36.3656	-26468000	-0.00264680	0.9973567
NSN10-8	NSN17-3	18	2280090	36.9406	-26629700	-0.00266297	0.99734057
NSN17-3	NSN12-3	18	2271600	37.8374	-26950700	-0.00269507	0.99730856
NSN12-3	NSN14-3	18	2254550	40.0106	-27633200	-0.00276332	0.99724049
NSN14-3	NSN7-6	18	2220880	43.3898	-29093800	-0.00290938	0.99709485
NSN7-6	NSN10-7	18	2203480	54.4134	-30041100	-0.00300411	0.9970004
NSN10-9	NSN17-2	5	2203480	55.1377	-30041100	-0.00300411	0.9970004
NSN17-4	NSN19-8	19	2203480	55.803	-30041100	-0.00300411	0.9970004
NSN19-8	NSN19-7	19	2187970	63.6575	-31028100	-0.00310281	0.996902
NSN19-9	NSN19-8	2	2187970	66.0043	-31028100	-0.00310281	0.996902
NSN19-8	NSN10-8	2	2172460	68.3512	-32087900	-0.00320879	0.99679635
NSN10-8	NSN17-3	2	2168090	73.3347	-32409000	-0.00324090	0.99676435
NSN17-3	NSN7-5	2	2159600	73.7687	-33034800	-0.00330348	0.99670197
NSN7-7	NSN19-6	21	2159600	74.7033	-33034800	-0.00330348	0.99670197
NSN19-6	NSN7-6	21	2144100	77.0564	-34229600	-0.00342296	0.99658289
NSN7-6	NSN19-5	21	2126690	94.9931	-35883300	-0.00358833	0.9964181
NSN19-7	NSN19-6	19	2126690	139.61	-35883300	-0.00358833	0.9964181
NSN19-6	NSN3-5	19	2111180	202.163	-39018000	-0.00390180	0.9961058
NSN3-5	NSN3-3	19	2107950	229.722	-39760400	-0.00397604	0.99603185
NSN3-3	NSN3-2	19	2104720	394.278	-41034600	-0.00410346	0.99590495
NSN3-4	NSN3-3	12	2104720	529.495	-41034600	-0.00410346	0.99590495
NSN3-3	NSN19-4	12	2101490	664.712	-43182800	-0.00431828	0.99569103
NSN19-4	NSN3-1	12	2085980	701.343	-54057500	-0.00540575	0.99460883
NSN3-1	NSN3-0	12	2082750	1097.79	-57605300	-0.00576053	0.99425603
NSN3-2	NSN3-1	19	2082750	1434.18	-57605300	-0.00576053	0.99425603
NSN3-1	ART	19	2079520	1770.57	-63327300	-0.00633273	0.99368728
			1900200	+INFINITY	INFEASIBLE		
Parametric Output Data for the RHS of Two Sub-budget Constraints							
VAR	VAR	Pivot	RHS	Dual Price	Obj Val	Obj Value	Q
OUT	IN	Row	Val	Before Pivot	(scaled)	(unscaled)	
			0	5.43135	-19074400.	-0.00190744	0.99809438
SLK 24	NSN10-10	17	0.630859	5.43135	-19074400.	-0.00190744	0.99809438
NSN10-10	NSN10-9	17	4378.03	4.8054	-19053300.	-0.00190533	0.99809648
NSN10-11	NSN12-4	5	4378.03	0.12217	-19053300.	-0.00190533	0.99809648
NSN12-6	NSN15-4	12	4378.03	-2.53144	-19053300.	-0.00190533	0.99809648
NSN1-2	NSN11-2	3	12912.2	-3.98014	-19087300.	-0.00190873	0.99809309
NSN11-2	NSN14-3	3	20962.2	-4.34915	-19122300.	-0.00191223	0.9980896
NSN14-3	NSN7-10	3	54624.1	-6.3684	-19336700.	-0.00193367	0.9980682
NSN15-4	NSN15-2	12	57409	-8.68737	-19360900.	-0.00193609	0.99806578
NSN7-10	SLK 23	3	72032.5	-8.90598	-19491100.	-0.00194911	0.99805279
NSN8-1	NSN4-130	23	79617.8	-9.63009	-19564200.	-0.00195642	0.99804549
NSN4-130	NSN7-11	23	88412.9	-9.77114	-19650100.	-0.00196501	0.99803692

NSN7-9	NSN20-38	18	88412.9	-11.0257	-19650100.	-0.00196501	0.99803692
NSN20-36	NSN8-3	21	88412.9	-11.6673	-19650100.	-0.00196501	0.99803692
NSN15-2	NSN10-10	12	110440	-11.8997	-19912200.	-0.00199122	0.99801076
NSN10-10	NSN12-5	12	114817	-12.262	-19965900.	-0.00199659	0.9980054
NSN12-5	NSN12-3	12	131875	-24.2263	-20379100.	-0.00203791	0.99796417
NSN12-3	NSN12-2	12	148934	-26.661	-20833900.	-0.00208339	0.99791878
NSN12-4	NSN10-8	5	148934	-30.306	-20833900.	-0.00208339	0.99791878
NSN10-8	NSN12-3	5	153311	-30.881	-20969100.	-0.00209691	0.99790529
NSN8-3	NSN16-14	21	155807	-33.951	-21053800.	-0.00210538	0.99789683
NSN16-14	NSN8-4	21	169149	-34.1943	-21510100.	-0.00215101	0.9978513
NSN8-2	NSN1-4	10	169149	-34.2561	-21510100.	-0.00215101	0.9978513
NSN12-3	NSN10-7	5	170369	-34.2798	-21551900.	-0.00215519	0.99784713
NSN10-9	NSN10-8	17	170369	-49.4068	-21551900.	-0.00215519	0.99784713
NSN10-8	NSN3-5	17	174746	-67.6039	-21847800.	-0.00218478	0.9978176
NSN3-5	NSN3-3	17	177978	-223.991	-22571700.	-0.00225717	0.99774538
NSN3-3	NSN3-2	17	181210	-388.547	-23827400.	-0.00238274	0.9976201
NSN3-4	NSN3-3	14	181210	-523.764	-23827400.	-0.00238274	0.9976201
NSN3-3	NSN3-1	14	184442	-658.981	-25957000.	-0.00259570	0.99740767
NSN3-1	NSN3-0	14	187673	-1092.06	-29486300.	-0.00294863	0.99705571
NSN3-2	NSN3-1	17	187673	-1428.45	-29486300.	-0.00294863	0.99705571
NSN3-1	ART	17	190905	-1764.84	-35189700.	-0.00351897	0.99648721
			200000	-Infinity	Infeasible		

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13. ABSTRACT *(Maximum 200 words)*

The Aircraft Availability Model (AAM) provides the Air Force with a worldwide peacetime requirement for reparable spare parts. This research models AAM methodology as it relates to the concept of diminishing marginal returns in resource application. Three separate modeling techniques are investigated with the goal of reformulating the AAM as a mathematical programming model that provides a comparable solution and a capable tool for the conduct of sensitivity analysis. The general formulations presented here are continuous non-linear, continuous linear, and piecewise linear discrete/continuous models. Two formulations of the piecewise linear discrete/continuous model are presented. The piecewise linear model based on AAM sort values shows the dominance of an optimization routine relative to the AAM shopping list greedy heuristic. The piecewise linear model based on availability rates provides the capability to maximize the mission design series (MDS) availability level. It has the potential to obtain the highest possible MDS availability relative to reparable spares inventory levels. This mathematical model is discussed in complete detail as a robust platform for conducting extensive post-optimality analysis.

14. SUBJECT TERMS

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