© Universiti Tun Hussein Onn Malaysia Publisher's Office

IJIE

http://penerbit.uthm.edu.my/ojs/index.php/ijie
ISSN : 2229-838X e-ISSN : 2600-7916

The International Journal of Integrated Engineering

Design and Characterization of a Non-Linear Variable Inerter in Vehicle Suspension System

Goh, K.Y.[1], Soong, M.F.[1*], Ramli, R.[1], Saifizul, A.A.[1], Khoo, S.Y.[1]

[1]Department of Mechanical Engineering, Faculty of Engineering,
Universiti Malaya, 50603 Kuala Lumput, MALAYSIA

*Corresponding Author

DOI: https://doi.org/10.30880/ijie.2023.15.03.021
Received 1 August 2023; Accepted 15 August 2023; Available online 19 October 2023

Abstract: Inerter is a two-terminal component in suspension system such that the force at the two terminals is directly proportional to the relative acceleration of these two points. Studies have shown that the inerter can provide satisfactory vibration isolation for a number of suspension applications, including train suspension, building suspension and vehicle suspension. In the context of vehicle suspension, the existing passive inerter has been shown to provide benefits to vehicle dynamics performance measures, such as ride comfort and road holding ability. However, a basic passive inerter has fixed characteristic, and hence its potential is limited. This study overcome this limitation by incorporating variable inertia in inerter flywheel, however its non-linear characteristic needs to be determined. The method of achieving variable inertia in inerter flywheel is through introduction of movable masses or sliders attached with springs into inerter flywheel. The change of moment of inertia is caused by position change of sliders due to centrifugal force when the flywheel is rotating. Results showed that the proposed variable inerter exhibits a non-linear force-acceleration relationship with respect to its operating rotational speed. A vehicle suspension system equipped with a variable inerter is also able to further reduce vertical vehicle body acceleration and vehicle's dynamic tire load when compared with vehicle suspension system without inerter and equipped with a passive inerter, which indirectly relates to a better vehicle ride and handling performance improvements. Hence, it can be proved that the proposed variable inerter is better than a passive inerter and is able to provide better ride comfort and road holding ability to a vehicle.

Keywords: Variable inerter, non-linear inertance, vehicle suspension system, ride comfort, road holding ability.

1. Introduction

A vehicle suspension system is a system of springs, dampers and linkages which are connected between wheel and vehicle body. When designed and tuned accordingly, a vehicle suspension serves the purpose of isolating vehicle body from vibration coming from the ground due to irregularities, as well as maintaining consistent contact between tire and road surface by minimizing normal load vibrations. The former is important for ride comfort or passenger comfort, while the latter is important for tire-road holding ability which indirectly relates to vehicle handling and safety capability [1].

Regardless of the complexity of a modern vehicle suspension system, it can currently be generalized down to two major elements, which are spring and damper element. They are used to support static load of a vehicle, temporarily store and dissipate the undesirable energy, and literally dampen vehicle response, resulting in reduction of wheel-load vibration and firm contact on the ground. A two-terminal element called inerter can also be joined as a suspension component where it has property that the force at its two terminals is directly proportional to their relative acceleration [2]. It provides translational inertia by utilizing rotational inertia of a flywheel and converting it back to translational effect through several feasible motion conversion mechanisms, such as rack-and-pinion, ball- screw and hydraulic mechanisms. In force-current analogy [3], the correspondence between mass and capacitor was omitted due to the fact that one terminal

*Corresponding author: mfsoong@um.edu.my

of mass element is grounded, which meant it was only applicable to grounded capacitor in a circuit. However, a capacitor needs not necessary be grounded and hence an electrical circuit may not have a mechanical equivalent of direct mass-spring-damper system. Therefore, Smith has introduced the concept of ideal inerter and it has no restriction of terminal on the ground [2].

Smith and Wang had made a comparative study of several simple passive suspension struts which includes damper and inerter to obtain their performance improvements [4]. The results showed that the suspension performance measures had improved, namely ride comfort, tire load and suspension's ability to carry load, when these were independently optimized. In 2014, Soong highlighted that the benefit in ride performance brought by inerter in parallel suspension layout exists not just for ordinary passive suspension, but for semi-active suspension as well [5]. From the analysis, it was observed that inerter force and spring force were anti-phase to each other. This cancellation of forces gave better vibration isolation to a sprung mass in parallel layout. The inerter is not only proven to be beneficial to a typical passenger car, but also on heavy vehicles such as truck and bus [6]. It is able to reduce the sprung mass acceleration and dynamic tire load of heavy vehicles with larger optimum inertance than that for passenger cars no matter in series or parallel arrangement layouts [7].

A two-stage inerter-spring-damper suspension system was also established, and showed that it is able to provide better low-frequency vibration damping performance than a classic inerter-spring-damper suspension [8]. The inerter can also be incorporated with an active actuator and formed active tuned inerter damper suspension. The result indicated that the it can improve the vehicle ride comfort without changing the dynamic tire load [9]. The inerter was later being implement in semi-active and active vehicle suspension system, and the result proved that adding an inerter can improve a vehicle's suspension performance. Moreover, Soong also had discovered that a switchable inerter can theoretically give significant better ride and tire load performance compared to a basic passive inerter [10]. From this perspective, it can be concluded that a variable inerter with continuous variability in its characteristics is able to further improve vehicle ride performance.

This study intends to design a variable inerter which would alter its characteristics known as inertance depending on its operating rotational speed. From derivation of mathematical modeling of proposed variable inerter, it showed that the inertance of a variable inerter changes according to its rotational speed. Moreover, the design parameters of variable inertia flywheel would also bring an effect to its performance. These parameters include density or material, length of hollow slot, thickness of inerter flywheel, pitch of ball-screw and spring constant applied in the inerter flywheel. This study also identified the effect of each design parameter of variable inertia flywheel in its suspension performance. This new design of variable inerter is then proved to exhibit non-linear characteristic and to be able to provide performance improvements to vehicle suspension system. Vertical sprung mass acceleration and dynamic tire load are two main criteria to be minimized which correspond to ride comfort and road holding ability.

2. Design and Modeling of a Variable Inerter

Inerter is a two-terminal component with the property that the same and opposite force applied to the terminals is proportional to their relative acceleration, which can be mathematically stated as Equation (1).

$$F_{inerter} = b(a_2 - a_1) \quad (1)$$

in which $F_{inerter}$ is the force at the terminals, b is the property of the inerter known as inertance, while a_1 and a_2 give the relative acceleration of the terminals. In this study, the inertance of variable inerter is a function of its design variables. The type of inerter used is a ball-screw inerter employed with a variable inertia flywheel. The inertance of ball-screw inerter can be identified by Equation (2).

$$b = (\frac{2\pi}{p})^2 J \quad ; \quad J = \frac{1}{2} m r^2 \quad (2)$$

in which p is pitch or lead of ball-screw, J is overall rotary moment of inerter flywheel, m is mass of inerter flywheel, while r is radius of inerter flywheel.

A speed-dependent variable ball-screw inerter flywheel is designed such that it consists of three major parts, which are a cylindrical flywheel, cuboid hollow slots and spherical slotted sliders, as illustrated in Figure 1 and Table 1 displayed the symbol. There will be a number of hollow slots to be introduced into the original cylindrical flywheel and each hollow slot is attached with a spring and a slider in order to change its moment of inertia and hence the inertance of flywheel. The changes on moment of inertia and inertance of flywheel are caused by position change of sliders due to centrifugal force when it is rotating at various rotational speeds. When the flywheel is rotating at a very low speed, the displacement of sliders is almost zero which indicates that the sliders are around the center of flywheel. At this position, the variable inertia flywheel has its smallest moment of inertia. The displacement of sliders is at maximum where it is furthest from center of flywheel when the flywheel is rotating at very high speed. At this position, the flywheel has its largest moment of inertia and inertance.

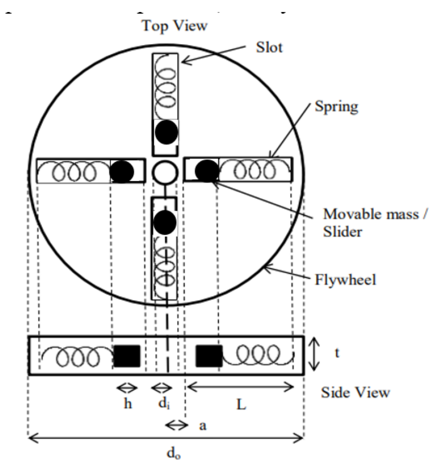

Fig. 1 - Design of variable inertia flywheel

Table 1 - List of symbols in variable inertia flywheel

Parameter	Symbol
Outer diameter	d_o
Inner diameter	d_i
Distance between tangential center hole and hollow slot	a
Length of hollow slot	L
Length of movable mass/ slider	h
Thickness of flywheel	t

The overall rotary inertia of the variable inerter can be identified as in Equation (3).

$$J_{overall} = J_{flywheel} - nJ_{hollow\ slot} + nJ_{slider} \quad (3)$$

in which $J_{flywheel}$ is the original flywheel inertia, $J_{hollow\ slot}$ is rotary inertia of hollow slots in flywheel, J_{slider} is rotary inertia of slotted sliders in flywheel, while n is the number of hollow slots and sliders in inerter flywheel. $J_{flywheel}$ is the moment of inertia of original flywheel and can be derived as shown in Equation (4)

$$J_{flywheel} = \frac{\rho\pi t}{32}({d_o}^4 - {d_i}^4) \quad (4)$$

in which ρ is density of flywheel, t is thickness of flywheel, d_o is outer diameter of flywheel, while d_i is inner diameter of flywheel. $J_{hollow\ slot}$ is the moment of inertia of hollow slots introduced in the flywheel where it is cuboid in shape and can be derived as shown in Equation (5).

$$J_{hollow\ slot} = \rho L t^2[\left(\frac{L^2+t^2}{12}\right) + \left(L_o + \frac{L}{2}\right)^2]; \quad L_o = \frac{d_i}{2} + a; \quad (5)$$

in which ρ is density of hollow slot or flywheel, t is thickness of hollow slot or flywheel, L is length of hollow slots, L_o is distance between center of flywheel and hollow slot, while a is distance between tangential center of flywheel and hollow slot. J_{slider} is the moment of inertia of the sliders to be inserted within hollow slots in the flywheel where it is spherical in shape and can be derived as shown Equation (6),

$$J_{slider} = \frac{\rho\pi t^3}{6}[\frac{1}{10}t^2 + \left(\frac{d_i}{2} + \mathrm{a} + \mathrm{e} + \frac{t}{2}\right)^2] \quad (6)$$

in which ρ is density of slider or flywheel, t is thickness of slider or flywheel, d_i is inner diameter of flywheel, a is distance between tangential center of flywheel and hollow slot, while e is position of slider with respect to rotational speed of flywheel. In this case, the position of sliders can be determined as Equation (7). As in structural diagram of variable inertia flywheel in Figure 1, the centrifugal force of flywheel when rotating is balanced by the spring force attached with slider in the flywheel.

$$e = \frac{\rho\pi t^3\omega^2}{6k-\rho_s\pi t^3\omega^2}\left(\frac{d_i}{2} + a + \frac{t}{2}\right) \tag{7}$$

in which ρ is density of flywheel, t is thickness of flywheel, ω is angular speed of flywheel, k is spring constant, d_i is inner diameter of flywheel, while a is distance between tangential center of flywheel and hollow slot. However, the radius of rotation of sliders, r from centre should be limited by outer and inner diameter, and hence Equation (8) is applied to illustrate the conditions.

$$e = \begin{cases} 0; & F_c < F_k \\ \frac{\rho\pi t^3\omega^2}{6k-\rho\pi t^3\omega^2}\left(\frac{d_i}{2} + a + \frac{t}{2}\right); & F_c = F_k \\ L - \frac{t}{2}; & F_c > F_k \end{cases} \tag{8}$$

while F_c is centrifugal force when the flywheel is rotating and F_k is force of spring attached on the slider.

3. Characterization and Analysis of Variable Inerter

The relationship of each design parameters on the inertance of variable inerter can be determined through a set of data of variable inerter as reference and be used to compare with other datasets. Table 2 shows the reference dataset of design parameters of variable inertia flywheel.

Table 2 - Reference dataset of design parameters of variable inertia flywheel

Parameter	Value	Parameter	Value
Density , ρ	2700 kg/m^3	Thickness, t	0.04 m
Outer diameter, *do*	0.15 m	Ball-screw pitch, p	0.025 m
Inner diameter, *di*	0.02 m	Spring constant, k	1030 N/m
Distance between center hole and slots, a	0.005 m	Number of slots introduced, n	4
Length of slots, L	0.05 m		

In order to verify that the variable inerter can exhibit a non-linear force-acceleration relationship which is indicated by variable inertance as opposed to constant inertance, a graph of inertance against rotational speed of inerter flywheel is plotted based on the formulated mathematical modeling, as shown in Figure 2.

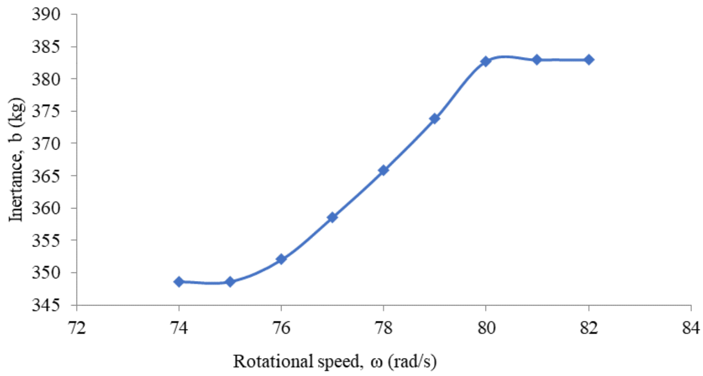

Fig. 2 - Graph of inertance against rotational speed of variable inertia flywheel

Based on Figure 2, it can be observed that inertance is increased with rotational speed of inerter flywheel between 75 rad/s to 80 rad/s. There will be minimum inertance which is about 349 kg at any rotational speed of inerter flywheel below 75 rad/s. When the inerter flywheel is rotating below certain speed, its centrifugal force is insufficient to push the sliders away from the center of inerter flywheel, causing its moment of inertia to be minimum and hence the same goes to its inertance. Meanwhile, for rotational speed of inerter flywheel above 80 rad/s, its inertance is at maximum, approximately 383 kg. When the rotational speed of inerter flywheel goes beyond 75 rad/s, the centrifugal force will push the sliders to move along the hollow slots employed in the flywheel and increase its inertance. However, the maximum distance of sliders can be displaced is limited by the length of the hollow slots, hence the inertance of inerter flywheel reaches its maximum after 80 rad/s.

The characteristics of a variable inertia flywheel is not only dependent on the position of its sliders when rotating, but also on its material and design parameters. In order to study the effect of each design parameter on inertance and suspension performance, parametric analysis on the modeling is needed and plotted in graph. Taking Table 2 as reference data, each parameter is then varied under certain range to study its impact on inertance and also in vehicle suspension performance measures, namely ride comfort and road holding ability. There are five major design parameters that are selected to be studied, namely material or density, length of hollow slots introduced, thickness of inerter flywheel, pitch of ball-screw and spring constant employed in inerter flywheel. Firstly, a variable inerter can be made of materials with different densities. In this study, the materials used for fabrication of variable inertia flywheel can be polylactic acid, PLA (1240 kg/m^3), aluminium (2700 kg/m^3) and steel (8000 kg/m^3). Figure 3 shows the characteristics of inerter with different densities under varies rotational speed.

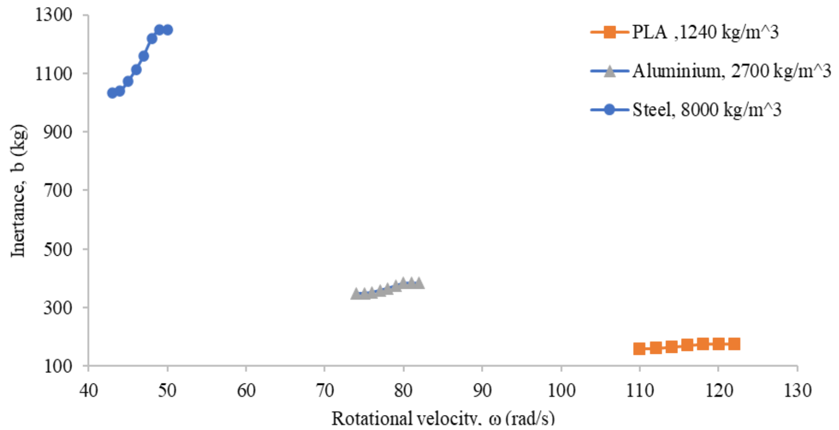

Fig. 3 - Comparisons between graph of inertance against rotational speed of inerter flywheel with different densities (PLA, Aluminium, Steel)

As illustrated in Figure 3, it can be observed that the ranges of inertance with respect to rotational speed of inerter flywheel are quite different for all three materials. Under the same dimensions of variable inertia flywheel design parameters, material density gives a significant effect to its mass where it contributes to its overall inertance as described in Equations 4, 5 and 6. PLA is able to provide a maximum inertance of 176 kg, aluminium gives 382 kg inertance at its maximum, while steel reaches its highest point at 1250 kg of inertance. Moreover, the sliders with higher mass would give rise to higher centrifugal force when it rotates, and hence it can be easily displaced since it has higher force to compress the spring. Therefore, the inerter flywheel with higher mass can accomplish its function at lower rotational speed, such as that from steel, followed by aluminium and PLA. Length of hollow slots introduced in variable inertia flywheel also can affect inertance of a variable inerter. If the only deviation from a set of similar dimensions of variable inerter is the length of hollow slots, the flywheel with longer hollow slots has larger variation of inertance since it can displace further and increase its moment of inertia as displayed in Figure 4. The further the sliders displaced from the center of flywheel, the larger the moment of inertia, and thus higher inertance. However, the inerter with longer hollow slots employed has lower maximum inertance, as it has lower mass in overall.

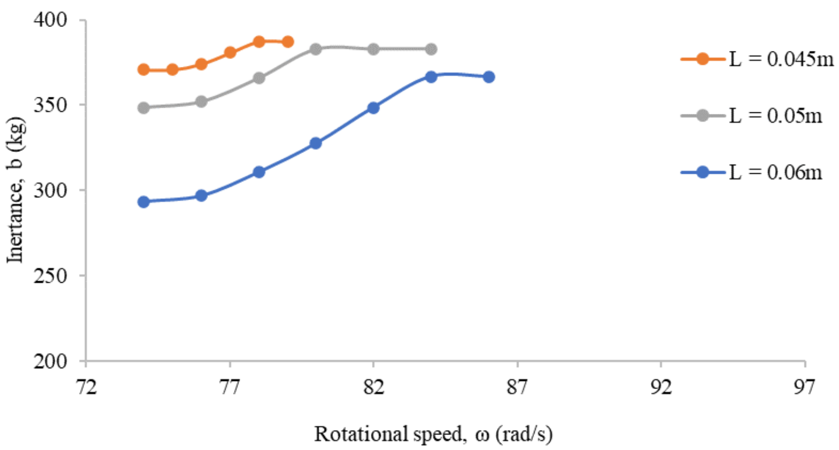

Fig. 4 - Comparisons between graph of inertance against rotational speed of inerter flywheel with length of hollow slots (0.045m, 0.05m, 0.06m)

The following factor that can alter the inertance of inerter flywheel is its thickness. Making the thickness to be varied from 0.03 m to 0.05 m, the overall inertance is increased and it works in lower range of rotational speed. Due to the fact that when the variable inertia flywheel is designed, the thickness of the flywheel is related to the size of the sliders. Hence, increment of flywheel thickness will give rise to mass of sliders. Same as the theory mentioned before in the aspect of density, higher thickness of flywheel and slider provide higher overall inertance to the system and also being displaced easily when it rotates. Therefore, inerter flywheel with 0.05 m thickness has the highest mass which give highest inertance at lowest rotational speed, followed by flywheel with 0.04 m and 0.03 m thickness, as in Figure 5.

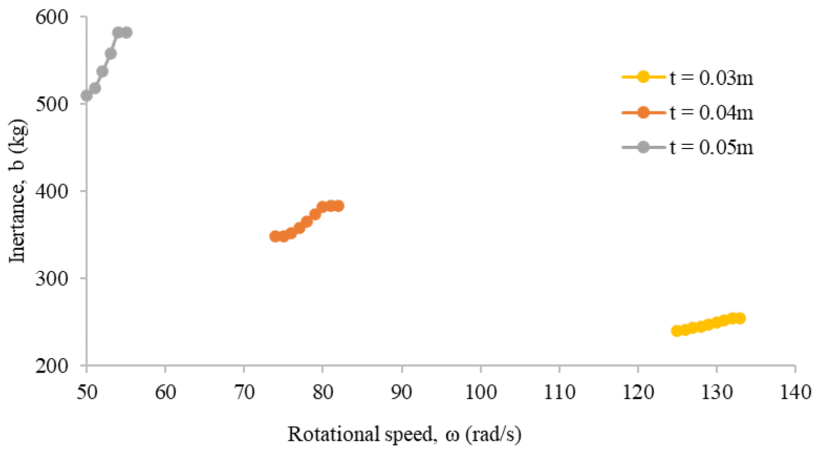

Fig. 5 - Comparisons between graph of inertance against rotational speed of inerter flywheel with varying thickness (0.03m, 0.04m, 0.05m)

Pitch or lead of ball-screw is also an important factor which can affect its characteristics where it relates to the angle of thread and hence rotational speed. Pitch of a ball-screw refers to the distance between screw threads, while lead represents the linear distance travelled for each complete turn of the screw. For single start screws, lead and pitch are equivalent. As shown in Figure 6, ball-screw with small pitch has higher inertance when compared to ball-screw with large pitch due to the fact that it will translate higher number of rotations and speed as compared to large pitch ball-screw for a same linear distance input. In the case of high rotational speed of ball-screw, it has higher centrifugal force that pushes the sliders towards outer casing of flywheel, causing an increase of moment of inertia, and the same goes to its inertance.

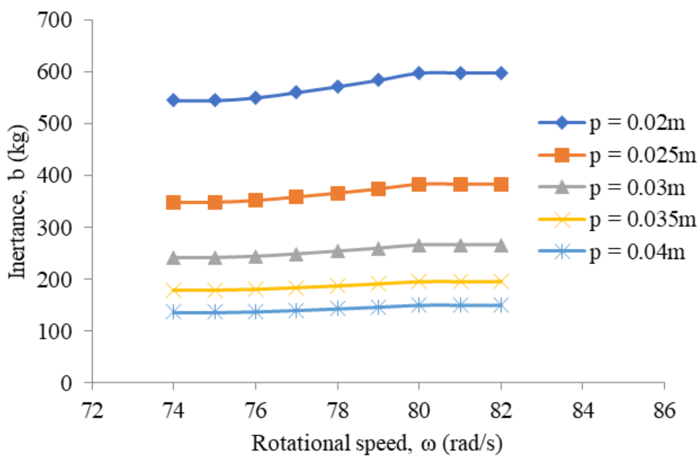

Fig. 6 - Comparisons between graph of inertance against rotational speed of inerter flywheel with varying ball-screw pitch

Lastly, the variable that can alter the characteristic of a variable inerter is the spring constant embedded in inerter flywheel. Spring is an elastic element to store mechanical energy where it will return to its original shape after deformed, that is, after being stretched or compressed. When a force is applied onto a spring, the spring reverts an equal but opposite force to the object. Meanwhile, spring constant is a characteristic of a spring which is used to measure the ratio of force to displacement caused by a spring. In short, it is used to measure the stiffness of a spring. A stiffer spring has higher spring constant and executes smaller displacement than spring with lower spring constant for the same applied force. In variable inertia flywheel, a stiffer spring embedded requires a higher rotational speed of inerter flywheel in order to displace the sliders since it requires higher centrifugal force to compensate spring force as illustrated in Figure 7.

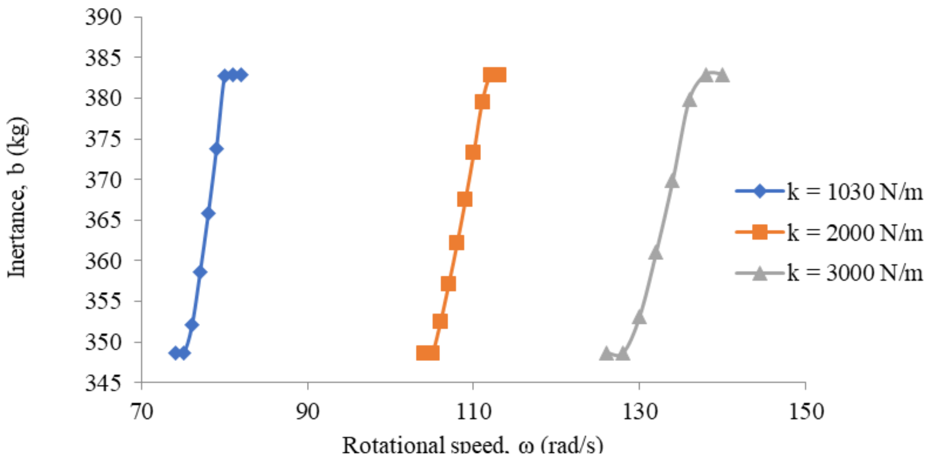

Fig. 7 - Comparisons between graph of inertance against rotational speed of inerter flywheel embedded with varying stiffness of spring

4. Variable Inerter in Vehicle Suspension System

A vehicle suspension system is generally equipped with spring and damper element, and it aims in providing good vibration isolation for passenger comfort and good tire-road contact. A parallel inerter with fixed characteristics added to vehicle suspension system was proved to further isolate a vehicle from vibration where it opposes and eliminates the spring force, giving better vibration isolation [5]. Switchable inerter is also discovered to further improve vehicle ride performance when compared to passive inerter [10]. The superior ride performance in vehicle suspension system can be determined by the reduction of vertical sprung mass acceleration and dynamic tire load. In order to identify the suspension performance of a variable inerter in vehicle suspension, a two degree-of-freedom quarter vehicle model was taken as a representative system in this study. For parallel inerter, equations of motion are displayed as in Equation (9) and (10).

$$m_s \ddot{z}_s = k(z_u - z_s) + c(\dot{z}_u - \dot{z}_s) + b(\ddot{z}_u - \ddot{z}_s) \tag{9}$$

$$m_u \ddot{z}_u = k_t(z_g - z_u) - k(z_u - z_s) - c(\dot{z}_u - \dot{z}_s) - b(\ddot{z}_u - \ddot{z}_s) \tag{10}$$

while m_s, m_u are sprung and unsprung masses, z_s, z_u are sprung and unsprung mass displacements, $\dot{z}_s, \dot{z}_u$ *and* $\ddot{z}_s, \ddot{z}_u$ denote the corresponding velocities and accelerations, z_g is vertical road displacement, k , k_t are suspension stiffness and tire stiffness, c is damping coefficient, while b is inertance. The vehicle model, as illustrated in Equations (9) and (10), was constructed in MATLAB/ Simulink environment and utilized to judge the suspension performance of a variable inerter. Table 3 shows the quarter vehicle parameters of a typical passenger car.

Table 3 - Quarter vehicle parameters of a passenger car [5]

Vehicle parameter	Passenger car
Sprung mass, m_s (kg)	317.5
Unsprung mass, m_u (kg)	45.4
Suspension stiffness, k (N/m)	22000
Suspension damping, c (Ns/m)	1500
Tire stiffness, k_t (N/m)	192000

The vehicle model is then simulated for sprung and unsprung mass responses due to step input road profile of 0.1m. Step input profile is a transient input with fixed step height to emulate a vehicle hitting an obstacle such as a curb or a bump. While vertical sprung mass acceleration and dynamic tire load are two common factors that affect the vehicle passenger comfort, therefore they are chosen to be the performance indicators. The output from vehicle suspension systems without inerter, with passive inerter and with variable inerter were recorded and compared, as in Figures 8 and 9. According to previous studies, the optimum inertance of a passive inerter is approximately 6kg for the passenger car parameters used in this study [5]. Therefore, a variable inerter should also provide similar range of inertance to the suspension system. Following the analysis from previous section, there are some design parameters of the variable inertia flywheel that can be altered in order to determine a set of variables which is able to give a better suspension performance to the vehicle. Table 4 shows a set of design parameters of variable inertia flywheel which can provide better performance to a vehicle suspension system.

Table 4 - Dataset of design parameters of variable inertia flywheel which can give better suspension performance from parametric analysis result

Parameter	Value	Parameter	Value
Density , ρ	1240 kg/m^3	Thickness, t	0.03 m
Outer diameter, do	0.10 m	Ball-screw pitch, p	0.05 m
Inner diameter, di	0.02 m	Spring constant, k	1030 N/m
Distance between center hole and slots, a	0.005 m	Number of slots introduced, n	4
Length of slots, L	0.04 m		

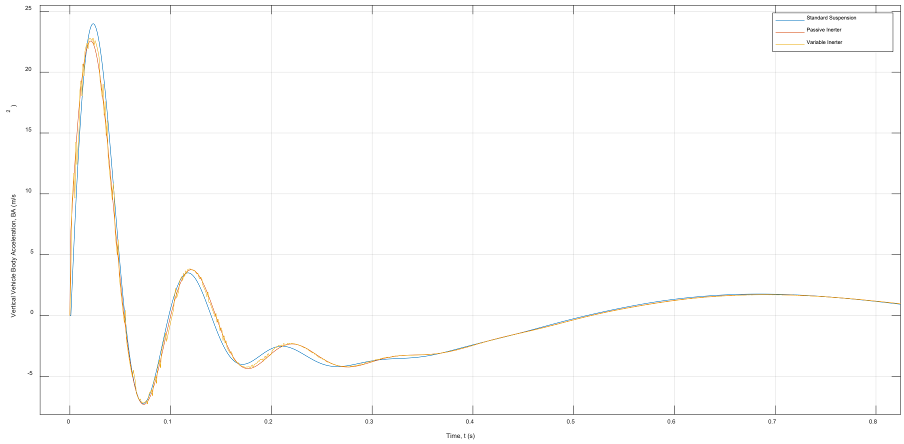

Fig. 8 - Comparison of vertical sprung mass acceleration among vehicle suspension systems without inerter, with passive inerter and with variable inerter

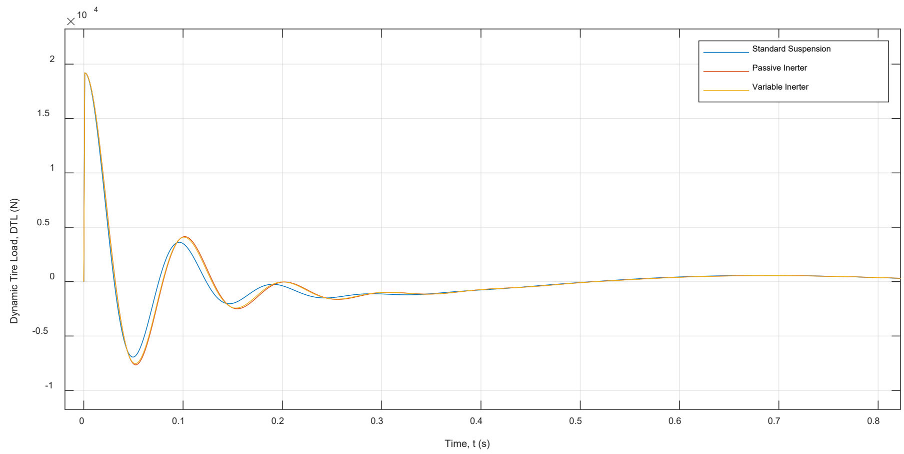

Fig. 9 - Comparison of dynamic tire load among vehicle suspension systems without inerter, with passive inerter and with variable inerter

Table 5 - Performance analysis of suspension systems without inerter, with passive inerter and variable inerter

		Rise Time (ms)	Overshoot (%)	Settling Time (s)
Vertical Vehicle Body Acceleration, BA	Without Inerter	13.721	0.658	2.43
	Passive Inerter	12.714	0.667	2.28
	Variable Inerter	12.222	0.667	2.23
Dynamic Tire Load, DTL	Without Inerter	0.7949	0.658	2.18
	Passive Inerter	0.7943	0.704	2.03
	Variable Inerter	0.7909	0.704	1.84

Referring to Figures 8 and 9 with analysis on the graph as in Table 5, a vehicle suspension system equipped with variable inerter seems to have a better suspension performance when compared to vehicle suspension systems without inerter and with passive inerter. From Table 5, suspension system equipped with variable inerter has shorter rise time and settling time in vertical vehicle body acceleration and dynamic tire load, therefore it was proved that it has the ability to further increase ride comfort to passenger and provide good road holding ability. However, the percentage overshoot of variable inerter increased slightly when compared to conventional suspension system, but is the same as suspension system with passive inerter. Hence, it is proved that a variable inerter has better performance than a passive inerter. To clearly figure out the improvement of a variable inerter when compared to a passive inerter, the root-mean-square (RMS) value of vertical vehicle body acceleration and dynamic tire load are also computed for each suspension systems, as described in Table 6.

Table 6 - Comparison of RMS vertical vehicle body acceleration and dynamic tire load of suspension systems without inerter, with passive inerter and variable inerter

	RMS Vertical Vehicle Body Acceleration, BA (m/s^2)	Improvement in BA(%)	RMS Dynamic Tire Load, DTL (N)	Improvement in DTL (%)
Without Inerter	2.0046	Reference	1178.81	Reference
Passive Inerter	1.9644	2.0	1240.26	-5.2
Variable Inerter	1.9625	2.1	1229.78	-4.3

As described in Table 6, the RMS vertical vehicle body acceleration of suspension system equipped with variable inerter decreased from 2.0046 m/s^2 to 1.9625 m/s^2, which bring an improvement of 2.1% when compared to a passive suspension system. However, it was insignificant when compared to a passive inerter which has brought a 2% improvement to vehicle's vertical acceleration. It was also noticed that the RMS dynamic tire load increased for both passive suspension system and suspension with passive inerter. This showed that minimization of vertical body acceleration is accompanied by the reduction in tire-road holding ability, which is the increment of dynamic tire load. However, the increment of RMS dynamic tire load of a suspension equipped with variable inerter is lower than suspension equipped with a passive inerter. Therefore, a variable inerter was proved that it can enhanced the vehicle ride performance as it did with a suspension system equipped a passive inerter.

5. Conclusion

A variable ball-screw inerter which exhibits a non-linear relationship with respect to its operating rotational speed is designed and modeled. It mainly consists of three major parts, which are cylindrical flywheel, cuboid hollow slots attached with spherical sliders and springs. The working principle of this variable inerter is that the moment of inertia of flywheel is changed due to centrifugal force when rotating under certain range of rotational speeds forcing position change of sliders, which leads to changing of inertance. The characteristic of a variable inerter is not only affected by the position of sliders, but is also dependent on its material and design parameters such as length of hollow slots introduced, thickness of flywheel, ball-screw pitch and spring constant embedded. The result showed that the vertical body acceleration, as well as dynamic tire load of a vehicle suspension system implemented with a variable inerter are reduced. It also has shorter rise time and settling time in both aspects when compared to vehicle suspension system without inerter and equipped with passive inerter. Indirectly, a variable inerter had brought improvement to ride comfort and road holding ability. However, the improvement was only slight as compared to conventional vehicle suspension system as the selection on the parameters of variable inertia flywheel is based on parametric analysis only. Multiobjective optimization can be applied to determine the optimum design parameters of variable inertia flywheel for best suspension performance.

Acknowledgement

This research is supported by Fundamental Research Grand Scheme (FRGS/1/2020/TK0/UM/02/23), Ministry of Higher Education of Malaysia. The authors thank the funder and also Universiti Malaya for supporting this research.

References

[1] M. F. Soong, R. Ramli, and M. Liza, "Using gear mechanism in vehicle suspension as a method of altering suspension characteristic," *Journal of Vibration and Control*, vol. 21, no. 11, pp. 2187–2199, 2015, doi: 10.1177/1077546313508298.

[2] M. C. Smith, “Synthesis of Mechanical Networks: The Inerter,” *IEEE Trans Automat Contr*, vol. 47, no. 10, pp. 1648–1662, 2002, doi: https://doi.org/10.1109/TAC.2002.803532.

[3] F. A. Firestone, “A New Analogy Between Mechanical and Electrical Systems,” *J Acoust Soc Am*, vol. 4, no. 3, pp. 249–267, 1933, doi: 10.1121/1.1915605.

[4] M. C. Smith and F. C. Wang, “Performance Benefits in Passive Vehicle Suspensions Employing Inerters,” in *Proceeding of the IEEE Conference on Decision and Control*, 2004. doi: 10.1109/CDC.2003.1272954.

[5] M. F. Soong, R. Ramli, and M. Liza, “Vehicle suspensions with parallel inerter: effectiveness in improving vibration isolation,” *Journal of Vibroengineering*, vol. 16, no. 1, pp. 256–265, 2014.

[6] X. Yang, L. Yan, Y. Shen, H. Li, and Y. Liu, “Dynamic performance analysis and parameters perturbation study of inerter–spring–damper suspension for heavy vehicle,” *Journal of Low Frequency Noise, Vibration and Active Control*, vol. 40, no. 3, pp. 1335–1350, 2021, doi: 10.1177/1461348420962898.

[7] M. F. Soong, R. Ramli, A. A. Saifizul, K. Y. Goh, and S. X. Long, “Investigation of inerter-based suspension systems for heavy vehicles,” *PLoS One*, vol. 18, no. 1, pp. e0280290-, Jan. 2023, [Online]. Available: https://doi.org/10.1371/journal.pone.0280290

[8] X. P. Li, F. J. Li, Z. Cao, and L. X. Yang, “Vibration Performance of Two-Stage Vehicle Suspension with Inerters,” *Dongbei Daxue Xuebao/Journal of Northeastern University*, vol. 40, no. 10, 2019, doi: 10.12068/j.issn.1005-3026.2019.10.015.

[9] Z. Ge, W. Wang, G. Li, and D. Rao, “Design, Parameter Optimisation, and Performance Analysis of Active Tuned Inerter Damper (TID) Suspension for Vehicle,” *J Sound Vib*, vol. 525, 2022, doi: 10.1016/j.jsv.2022.116750.

[10] M. F. Soong, R. Ramli, L. Mahadi, and A. Saifizul, “Ride improvement of vehicle suspensions with switchable inerter based on force cancellation strategy,” *Journal of Vibroengineering*, vol. 19, no. 2, pp. 1260–1272, 2017, doi: 10.21595/jve.2016.17486.