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Multi-Commodity Flow Models for Logistic Operations within a Contested Environment

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Abstract

Today's military logistics officers face a difficult challenge, generating route plans for mass deployments within contested environments. The current method of generating route plans is inefficient and does not assess the vulnerability within supply networks and chains. There are few models within the current literature that provide risk-averse solutions for multi-commodity flow models. In this thesis, we discuss two models that have the potential to aid military planners in creating route plans that account for risk and uncertainty. The first model we introduce is a continuous time model with chance constraints. The second model is a two-stage discrete time model with random attack scenarios. Both models demonstrate an ability to yield optimal route plans that are resilient in a contested environment.

Dedication

I dedicate this work to my family and friends who have supported me throughout my time at Clemson. Specifically I want to dedicate this work to my mother and sister, who have always had an ear to listen and a shoulder to lean on when things got hard. I take such pride in being a part of our small family.

I write this work in honor of my late grandparents, Audrey and Harold. I look forward to the day we meet again.

Acknowledgments

I would first like to thank my research advisor, Dr. Yongjia Song. He took me on as a research student while I was still an undergraduate and I have now studied under him for the last year and a half. I owe so much of my academic growth to him. From the start of this process to the end he has not given up on me, even when the learning curve was steep and the progress was slow. I will always be grateful for his mentorship and guidance and will keep the lessons I have learned from this project with me for the rest of my life. It has been a privilege to work on a project that aligns so well with the world I'll soon be working in.

I also would like to thank my academic advisor Dr. Wayne Goddard, who has continuously advocated for me since the day we first met 4 years ago. I would not have completed my B.S. or M.S. without his leadership and guidance. I will be forever grateful to have been paired with him four years ago.

I also owe thanks to Dr. Matthew Saltzman, who agreed to be a member of this committee and graciously offered his time and expertise to see this project to the end.

I would like to thank my ROTC instructors, who are the reason I have a job to go on to after earning this degree. Specifically I owe thanks to Colonel King, Lieutenant Colonel Busch, Major Blanton, Captain Pierce, and Mrs. Abbie for constantly pushing me to achieve more (and bearing with me when I changed my career preferences 24 hours before the deadline). You all saw something in me before I saw it in myself.

I would like to thank Dr. Michael Burr for teaching me the importance of slowing down. It took me a long time to understand, but I will always appreciate it.

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Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Today, logistics officers in the United States Navy face a significant challenge: developing timely, optimal, and risk-averse logistics deployment plans within a reasonable time frame. For any major conflict, Military Supply Chains (MSCs) are critical to mission success [Sani et al., 2022]. To meet theater demand, military planners use a combination of rule-based planning software and provided data to generate routing plans. However, these plans are not guaranteed to be optimal. In addition, there is no systematic way to generate plans resilient against uncertain attacks within a contested environment [Salmeron et al., 2009]. For these reasons, finding both optimal and resilient routing plans is of great importance, with the ultimate goal of developing a model that can provide resilient routing plans for large-scale deployments under uncertainty. The work of this thesis extends two deterministic models in an effort to provide a more robust solution when generating route plans for operations within contested environments.

1.1 Contested Environments

The United States defines a contested logistics environment as "an environment in which the armed forces engage in conflict with an adversary that presents challenges in all domains and directly targets logistics operations, facilities, and activities in the United States, abroad, or in transit from one location to the other" [Serrano et al., 2023]. This definition highlights that within a contested environment, operations can be targeted at any point along the supply chain. With modern advances in technology, the deployment of military assets is increasingly more "visible and vulnerable"

[Koprowski, 2005]. We define connectors to be the ships and aircraft that can transport personnel and cargo. While there are connectors that have increased stealth capabilities, the survivability of a connector does not directly imply mission success. Often the “most survivable assets” are too slow and have the lowest capacity for PMC [Bengigi et al., 2020]. Therefore, successful wide-scale deployments require more than just relying on the stealth capabilities of connectors; they demand robust and risk-averse logistics planning to bridge this gap.

1.2 Modern Military Logistics in Contested Environments

As described in [Yakıcı et al., 2018], the current logistics planning process is inefficient. When planning large-scale deployments, planners sometimes require up to of 12 hours to generate manual solutions. To make routing decisions, planners are provided with Time Phase Force Deployment Data (TPFDD) and the Joint Flow Analysis System for Transportation (JFAST) as tools to aid in creating deployment plans. The TPFDD typically provides a comprehensive list of all military personnel units and cargo (PMC) required for deployment. The JFAST then acts as a rule-based planning software that generates plans using heuristics [Yakıcı et al., 2018]. However, these heuristics are unreliable, assessing PMC and connector vehicle pairings by priorities and schedules. This process yields results of an “unknown quality” that do not consider vulnerability within the network [Koprowski, 2005]. Despite the lack of optimality, these solutions may be acceptable when providing routing plans within the Continental United States, (CONUS) since at this current time we can be sure that these routes are not transiting within contested environments. Accepting these solutions when creating routing plans outside the continental United States, however, has the potential to result in costly consequences.

In any conflict, the on-time delivery of PMC is critical to executing the mission. Failure to move assets in theater before the onset of conflict can leave troops vulnerable and, in some cases, ill-equipped to engage effectively. In [Sani et al., 2022], the authors emphasize that “a conflict can be lost due to a disruption in the supply chain”. They also define the disruption of a MSC as a “a combination of an unanticipated triggering event and the subsequent effects that risk material flow and normal business operations significantly”. Since MSCs are critical to mission success, they become a prime target for adversaries. While a disruption in a MSC may be initially problematic, this disruption can propagate and become even more critical as time passes [Sani et al., 2022].

This makes risk-averse logistics planning the backbone for successful operations when routing in a contested environment.

Currently, there is no mathematical way to handle risk when planning wide-scale deployments. Commanders rely on their logistics officers' intuition and expertise to generate risk-averse plans. There have been publications that offer differing approaches to generate resilient routing plans when encountered with a contested environment. For instance, this following cycle is shown as one example to mitigating risk within MSCs while generating routing plans in [Sani et al., 2022]:

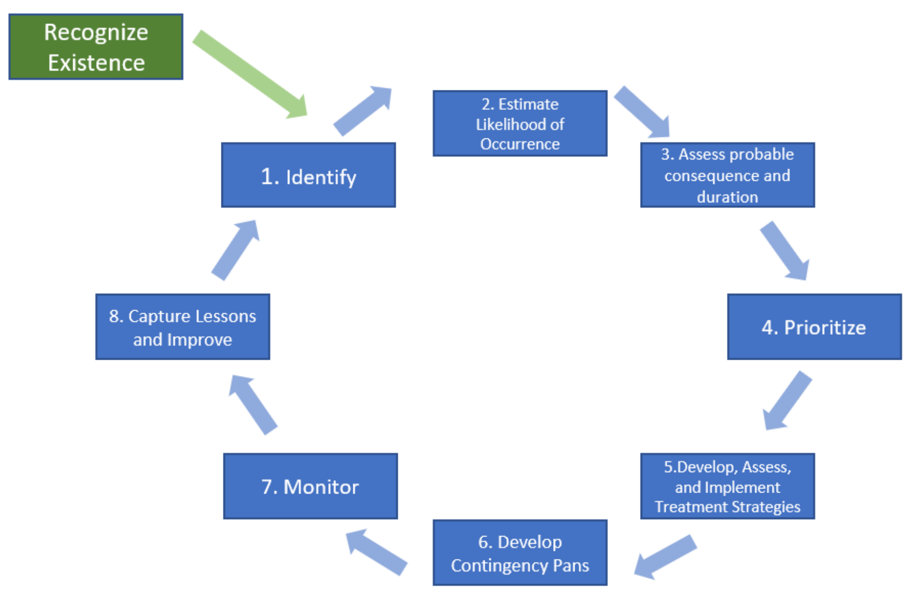

Figure 1.1: MSC Risk Mitigation Loop [Sani et al., 2022]

While this is not the only process proposed to achieve a greater resilience in MSCs, most processes presented propose a general plan to achieve greater resilience. For example, the "distributed fleet architecture" proposed in [Bengigi et al., 2020], which advocates distributing risk by employing a larger number of less expensive connectors. While these processes have potential, they also present another problem, further burdening military logistics officers and extending the already extensive time it takes to produce a route plan.

In examining the tactics proposed across the literature, it is clear that generating optimal, robust, and risk-averse routing plans for operations within contested environments is the "new frontier" of military logistics planning. This is the problem that motivates the remainder of this thesis. In Chapter 2 we explore what has been published and explored in respect to this problem. We then move to Chapter 3, where we introduce a continuous time model with chance constraints and Chapter 4 where we introduce a two-stage discrete time model under random attacks. In Chapter 5 we discuss how we tested these models and the results of those tests. Finally, in Chapter 6 we conclude and discuss the potential for future work.

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Chapter 2

Literature Review

2.1 Modelling Methodology

Using optimization models to model military logistics is not a new concept and has been done for quite some time. Modeling military logistics within contested environments or under uncertainty, however, is far less common. Models available in the literature solve the problem of routing within contested environments in multiple different ways. A few of these models are presented in this chapter. The authors of [Sani et al., 2022] present a comprehensive study of logistics models that aim to increase resilience in MSCs. The following three models are discussed in this paper, along with others that aim to reduce risk in MSCs.

In [Koprowski, 2005] the authors present a two-sided optimization model that searches for the best deployment plan while considering "worst-case interdiction". In this formulation, the first integer linear program (ILP), titled LIFTER, takes the TPFDD as input and generates an optimal routing plan. The second ILP, ATTACKER, is then run with an objective of maximally disrupting the plan LIFTER has created. LIFTER is then re-optimized to generate the final deployment plan. When compared to the JFAST solution under attack, the LIFTER solution delivers all cargo while the JFAST solution only delivers 7%.

In [Xu et al., 2016], the authors present a defender-attacker model that targets military supply chains. The defender first decides what pre-attack strategies they will take. Then the attacker must decide what resources to "contest the defender" with. This process is repeated until equilibrium is reached. This model considers the risk of enemy attack in order to mitigate risk in a

MSC [Sani et al., 2022].

We can look back as far as 1956 to find one of the first examples of a military logistics model optimizing under stochastic uncertainty. In [Ferguson and Dantzig, 1956], the authors develop a stochastic model to account for uncertainty in passenger demand in aircraft scheduling. Since this publication computational power has increased, and with it, the ability to handle more complex stochastic programming models. Despite these technological advancements, the utilization of stochastic programming to generate routing solutions in military logistics remains relatively scarce today. A few examples include the the work done in [Goggins, 1995], which uses a stochastic optimization model to incorporate uncertainty in aircraft reliability to optimize aircraft throughput. This formulation minimizes bottlenecks and allows users to assess the appropriate deployment schedule from the deterministic model and the stochastic model. This model does not account for routing within a contested environment but rather focuses on the idea of random and non-targeted disruptions.

Additional examples are cited in [Salmeron et al., 2009], however with limitations. The authors state that the stochastic models presented by [Alexander, 1999] and [Loh, 2000] can only handle small data-sets and are overly restrictive on attack recourse. The model presented in [Salmeron et al., 2009], however, provides a good base for the work in this thesis. The model is titled the Stochastic Sealift Model (SSDM) and focuses on the optimal routing of sealift connectors subject to biological attack. In the model they present dynamic rerouting at the time of attack, solving the issue of past models being overly restrictive. The model we present in chapter 4 is based on the work done in this paper, with some significant changes. For any location attacked in the SSDM model, that location shuts down for a period of time and then follows a recovery period before reopening. Additionally, the severity for each attack is fixed. In our model the capacity at each location is permanently reduced post-attack and these reductions are random, not fixed. Additionally, connectors in the SSDM operate between Seaports of Embarkation (SPOEs) and Seaports of Debarkation (SPODs) with complex arrival and departure times. In our model, the connectors operate on a three-layer network and have simple arrival and departure times. These differences are shown explicitly in figure 2.1.

Differences	Our Model	Stochastic Sealift Deployment Model (SSDM)
Recovery	Locations attacked remain open, but with permanent reduced capacity	Locations attacked are temporarily shutdown and restored using a recovery schedule
Network	Connectors move on three-layer network	Connectors only move from starting locations to transshipment locations (SPOEs to SPODs)
Delay Time	Simple arrival and departure timing – no delay	In-depth arrival, unloading, and departure times
Attack Severity	Severity of attack is random	Severity of attack is fixed

Figure 2.1: Differences between the SSDM and the Two-Stage Discrete Time Model Under Random Attack Scenarios

2.2 Network Interdiction and Chance Constraints

We can define network interdiction as "actions that serve to block or otherwise inhibit an adversary's operations, and often regards attacks against supply chain operations or communications" [Smith and Song, 2020]. In the context of MSCs and contested environments, network interdiction can range from adversaries targeting a source node so that PMC shipments can't be made, to targeting a connector en-route so that mission critical cargo does not arrive at its destination location. Military planners classify areas for potential interdiction using threat rings and sensors. If a connector enters a threat ring/sensor zone then there is an increased risk for possible interdiction. We define risk as "the product of the probability of an event occurring and its consequences…risk is about uncertainty and impact" [Kaddoussi et al., 2011].

To constrain the risk of an event occurring, say the possibility of interdiction within a threat ring, we can use a chance constraint to restrict the probability that event occurring. Take the following linear program:

$$\min f(x) \tag{2.1}$$

$$\text{s.t. } g(x, \zeta) \geq 0 \tag{2.2}$$

where equation (2.1) is the objective function and (2.2) represents the constraint mapping of a random vector ζ and decision variable x. We can reformulate this model to include a chance constraint as follows:

$$\min f(x) \tag{2.3}$$

$$\text{s.t. } P(g(x,\zeta) \geq 0) \geq p, \; p \in [0,1] \tag{2.4}$$

where p is the tolerance probability and $P(g(x,\zeta) \geq 0)$ is the probability of $g(x,\zeta) \geq 0$ occurring [Cha, 2021]

2.3 Two-Stage Stochastic Programming

A two-stage stochastic programming model optimizes under some kind of uncertainty. It requires decision makers to make decisions in two stages. The first stage decision represents the "here and now" solution and the second stage solution represents the "wait and see" solution [Upadhya, 2022]. In the context of operating in a contested environment, the "here and now" decision would be a route plan without any attack considerations. The "wait and see" solution would then be the recourse decisions a planner would make should a specific attack occur. The second-stage solution is always based on what has happened in the first stage solution, which is what makes the decisions made in the first stage solution vital to the overall output of the model.

There is a key factor that makes a two-stage stochastic model unique in comparison to a multi-stage stochastic model. The two-stage model has at most one uncertain event for every scenario, causing it to grow quadratically as opposed to exponentially with the number of time periods [Salmeron et al., 2009]. A standard two-stage model can be written as:

$$\min z = c^T x + E_\zeta Q(x,\zeta) \tag{2.5}$$

$$\text{s.t. } Ax = b \tag{2.6}$$

$$x \geq 0 \tag{2.7}$$

where,

$$Q(x,\zeta) = \min\{q^t y | Wy = h - Tx, y \geq 0\} \tag{2.8}$$

Here, the first term of equation (2.5) represents the first stage decisions and the second term represents the expectation of the second stage decisions. We define ζ to represent the uncertain data for the second stage decisions and W to be the fixed recourse [Ahmed, 2019]. We define T, h and q to be the realization of the random data [Ahmed,]. If we let $Z(x) = E_{\zeta}Q(x, \zeta)$, then we arrive at our final two-stage stochastic model:

$$\min\ z = c^T x + Z(x) \tag{2.9}$$

$$\text{s.t. } Ax = b \tag{2.10}$$

$$x \geq 0 \tag{2.11}$$

Chapter 3

Continuous Time Model with Chance Constraints

3.1 Motivations and Assumptions

This work began by extending and modifying the deterministic model presented in [Yakıcı et al., 2018] to incorporate a chance risk constraint that yields risk-averse solutions. In this model we have made some key assumptions. The first assumption is that there is no transshipment. Once a connector picks up a PMC load, this load will not be dropped off until arrival at its demand destination. Additionally, we assume that each PMC request is unique. For instance, 20 pallets may be requested for delivery at Naval Air Station (NAS), Pensacola and 20 pallets may be requested for delivery at Camp Lejune. Request one will be defined as k_1 and request two will be defined as k_2, despite the fact that they are both the same type of PMC. We also assume location distances are constant. While in reality ships do have the ability to move during a deployment, we do not consider this for simplicity. We also define "deck delay" to be the time required for each connector to remain on-site before departing. It represents the time required for connector refueling, unloading, and loading. Additionally, we restrict operations at each location to be conducted during two time blocks, defined in the formulation as block one and block two.

With respect to our risk and chance constraints, we assume that sensor locations and their relative risk factors for each connector are known. We also assume that we are provided an Operational Risk Management (ORM) matrix (shown in section 3.2.4). It is not unreasonable to assume the ORM matrix is provided as this is regular practice in the military.

3.2 Formulation

As previously mentioned, this formulation is extended from [Yakıcı et al., 2018]. In our extension, we rewrite the flow-balance constraints similar to those presented in [Wolfinger and Salazar-González, 2021] when transshipment is explicitly concerned.

3.2.1 Sets, Parameters, and Variables

Sets:

$h \in H$	Connector type
$k \in K$	PMC type
$i, j \in I$	Location (ships and airfields)
$\ell \in L := \{1, 2, \ldots, \lvert L \rvert\}$	connector stops (index from one to $\lvert L \rvert$)
$(i, j) \in ARCS$	All location pairings
$(i, j) \in HARC_h$	Allowable location pairings for connector h

Parameters:

$cap_{h,k}$	Capacity for all connectors of type h with PMC type k
$cost_h$	Operating cost for connectors of type h
$costm_\ell$	Penalty incurred for flying leg ℓ
$delay_{\ell,h}$	Minimum deck delay for connector h at stop ℓ
dem_k	Demand for PMC type k, which is requested to be routed from location i_k^p to j_k^p
i_h^c/j_h^c	Starting point/destination of connector h
i_k^p/j_k^p	Pickup/drop-off location of PMC type k
$maxflt$	Max. time for flight operations [minutes]
$net1_i/net2_i$	Earliest time location i can conduct flight operations in time block 1/block2
$neth_h/nlth_h$	No earlier/later than operating time for connector h
$net_{i,h}/nlt_{i,h}$	No-earlier/later-than arrival time for connector h in location i in time block 1: $net_{i,h} = \max\{neth_h, net1_i\}$, $nlt_{i,h} = \min\{nlth_h, nlt1_i\}$
$nlt2_{i,h}$	No-later-than arrival time at location i for connector h in block 2: $nlt2_{i,h} = \min\{nlth_h, nlt2_i\}$
pen_k	Penalty for each unit of undelivered PMC type k from i_k^p to j_k^p
$trans_{i,j,h}$	Connector h's transit time from location i to j
$P_{i,j,h}$	Probability of failure by attrition along arc (i, j) by connector h
$P_{i,h}$	Probability of failure at location i (within the range of hostile weapons) by connector h
ϵ_h	Maximum tolerable risk of failure for routing connector h
$t_{0,h}$	Start time for connector h, default value is zero
$\delta_{h,k}$	Risk threshold for connector h carrying PMC type k
ρ_k	Risk value associated with the severity of loss of PMC type k

Variables:

$x_{i,j,\ell,h}$	If connector h goes from i to j as the ℓ^{th} leg (location j would be stop ℓ)
$w_{\ell,h}$	If connector h ever goes on the ℓ^{th} leg
$t_{\ell,h}$	Arrival time of connector h at the end of its ℓ^{th} leg (stop ℓ)
e_k	Amount of unserved demand, i.e., units of PMC type k not moved from location i_k^p to j_k^p
$f_{i,j,\ell,h,k}$	Units of PMC type k moved on connector h from i as stop $\ell-1$ to j as stop ℓ (so that arc (i,j) is the ℓ^{th} leg for connector h)
$y_{i,j,\ell,h,k}$	One if PMC type k moved on connector h from i as stop $\ell-1$ to j as stop ℓ, zero otherwise
$BL_{i,\ell,h}$	One if connector h flies to i as the ℓ^{th} leg landing in time Block 2, zero otherwise
$BT_{i,\ell,h}$	One if connector h flies from i as the ℓ^{th} leg taking off in time Block 2, zero otherwise

3.2.2 Model

$$\text{Minimize } z = \sum_{h\in H}\sum_{(i,j)\in HARC_h,\ell\in L} cost_h \cdot trans_{i,j,h}\cdot x_{i,j,\ell,h} + \sum_{k\in K} pen_k\cdot e_k + \sum_{h\in H}\sum_{\ell\in L} costm_\ell \cdot w_{\ell,h} \tag{3.1}$$

$$\text{s.t. } t_{\ell,h} \geq t_{\ell-1,h} + \sum_{(i,j)\in HARC_h} (trans_{i,j,h} + delay_{\ell,h})x_{i,j,\ell,h} - nlth_h(1-w_{\ell,h}),\quad \forall h\in H, \forall \ell = 1,2,\dots,L \tag{3.2}$$

$$BL_{j,\ell,h} \leq \sum_{i|(i,j)\in HARC_h} x_{i,j,\ell,h},\ \forall j\in I, \ell\in L, h\in H \tag{3.3}$$

$$BT_{i,\ell,h} \leq \sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h},\ \forall i\in I, \ell\in L, h\in H \tag{3.4}$$

$$t_{\ell,h} \geq \sum_{(i,j)\in HARC_h} net_{j,h}x_{i,j,\ell,h} + \sum_j max(0, net2_j - net_{j,h})BL_{j,\ell,h},\quad \forall \ell\in L, h\in H \tag{3.5}$$

$$t_{\ell,h} \leq \sum_{(i,j)\in HARC_h} nlt_{j,h}x_{i,j,\ell,h} + \sum_j max(0, nlt2_{j,h} - nlt_{j,h})BL_{j,\ell,h},\quad \forall \ell\in L, h\in H \tag{3.6}$$

$$t_{\ell,h} \geq \sum_{(i,j)\in HARC_h} (net_{i,h} + trans_{i,j,h})x_{i,j,\ell,h} + \sum_i max(0, net2_i - net_{i,h})BT_{i,\ell,h},\quad \forall \ell\in L, h\in H \tag{3.7}$$

$$t_{\ell,h} \leq \sum_{(i,j)\in HARC_h} (nlt_{i,h} + trans_{i,j,h})x_{i,j,\ell,h} + \sum_i max(0, nlt2_{i,h} - nlt_{i,h})BT_{i,\ell,h},\quad \forall \ell\in L, h\in H \tag{3.8}$$

$$\sum_{j|(i_h^c,j)\in HARC_h} x_{i_h^c,j,1,h} = w_{1,h} = \sum_{\ell\in L}\sum_{i|(i,j_h^c)\in HARC_h} x_{i,j_h^c,\ell,h},\ \forall h\in H \tag{3.9}$$

$$w_{\ell,h} = \sum_{(i,j)\in HARC_h} x_{i,j,\ell,h},\ \forall h\in H, \ell\in L \tag{3.10}$$

$$w_{\ell,h} \geq w_{\ell+1,h}, \ \forall \ell = 1,2,\dots,|L|-1, \forall h \in H \tag{3.11}$$

$$w_{\ell+1,h} \geq w_{\ell,h} - \sum_{(i,j_h^c)\in HARC_h} x_{i,j_h^c,\ell,h}, \ \forall \ell = 1,2,\dots,|L|-1, \forall h \in H \tag{3.12}$$

$$\sum_{i|(i,j)\in HARC_h} x_{i,j,\ell,h} + 1 - w_{\ell+1,h} \geq \sum_{i|(j,i)\in HARC_h} x_{j,i,\ell+1,h}, \quad \forall j \in I, h \in H, \ell = 1,2,\dots,|L|-1 \tag{3.13}$$

$$\sum_{i|(i,j)\in HARC_h} x_{i,j,\ell,h} \leq \sum_{i|(j,i)\in HARC_h} x_{j,i,\ell+1,h} + 1 - w_{\ell+1,h}, \quad \forall j \in I, h \in H, \ell = 1,2,\dots,|L|-1 \tag{3.14}$$

$$\sum_{h\in H} \sum_{i|(i,j_k^p)\in HARC_h} \sum_{\ell\in L} f_{i,j_k^p,\ell,h,k} \geq dem_k - e_k, \ \forall k \tag{3.15}$$

$$f_{i,j,\ell,h,k} \leq cap_{h,k} \cdot x_{i,j,\ell,h}, \ \forall h \in H, k \in K, \ell \in L, (i,j) \in HARC_h \tag{3.16}$$

$$\sum_{h\in H} \sum_{j|(j,i_k^p)\in HARC_h} \sum_{\ell\in L} f_{j,i_k^p,\ell,h,k} = \sum_{h\in H} \sum_{i|(j_k^p,i,h)\in HARC_h} \sum_{\ell\in L} f_{j_k^p,i,\ell,h,k} = 0, \ \forall k \in K \tag{3.17}$$

$$\sum_{j|(j,i)\in HARC_h} f_{j,i,\ell,h,k} = \sum_{j|(i,j)\in HARC_h} f_{i,j,\ell+1,h,k}, \quad \forall h \in H, \forall k \in K, \forall i \in I \setminus \{i_k^p, j_k^p\}, \forall \ell = 1,2,\dots,|L|-1 \tag{3.18}$$

$$\sum_{h\in H} \sum_{\ell\in L} \sum_{j|(i_k^p,j)\in HARC_h} f_{i_k^p,j,\ell,h,k} \leq dem_k, \ \forall k \in K \tag{3.19}$$

$$\sum_{j|(i,j)\in HARC_h} f_{i,j,0,h,k} = 0, \ \forall h \in H, \forall i \neq i_k^p, \forall k \in K \tag{3.20}$$

$$\sum_{(i,j)\in HARC_h} \sum_{\ell\in L} \ln(1-P_{i,j,h}) x_{i,j,\ell,h} + \sum_{i\in I} \sum_{\ell\in L} \ln(1-P_{i,h}) \sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h} \geq \ln(1-\epsilon_h), \ \forall h \in H \tag{3.21}$$

$$f_{i,j,\ell,h,k} \leq cap_{h,k} \cdot y_{i,j,\ell,h,k}, \ \forall h \in H, k \in K, \ell \in L, (i,j) \in HARC_h \tag{3.22}$$

$$P_{i,j,h} \cdot \sum_{k\in K} \rho_k f_{i,j,\ell,k,h} \leq \delta_{k,h}, \ \forall \ell \in L, (i,j) \in HARC_h, h \in H \tag{3.23}$$

$$x_{i,j,\ell,h} \in \{0,1\}, \ \forall h \in H, \ell \in L, (i,j) \in HARC_h \tag{3.24}$$

$$y_{i,j,\ell,h,k} \in \{0,1\}, \ \forall h \in H, \ell \in L, k \in K, (i,j) \in HARC_h \tag{3.25}$$

$$w_{\ell,h} \in \{0,1\}, \ \forall h \in H, \ell \in L \tag{3.26}$$

$$0 \leq t_{l,h} \leq maxflt, \ \forall \ell \in L, h \in H \tag{3.27}$$

$$e_k \geq 0, \ \forall k \in K \tag{3.28}$$

$$f_{i,j,\ell,h,k} \geq 0, \ \forall h \in H, \ell \in L, k \in K, (i,j) \in HARC_h \tag{3.29}$$

$$BL_{i,\ell,h} \in \{0,1\}, \ \forall i \in I, \ell \in L, h \in H \tag{3.30}$$

$$BT_{i,\ell,h} \in \{0,1\}, \ \forall i \in I, \ell \in L, h \in H. \tag{3.31}$$

3.2.3 Constraint Descriptions

Equation (3.1) minimizes the total connector operating cost, the penalty for unmet PMC demand, and the additional cost added to "encourage early use of legs". Constraint (3.2) restricts the earliest landing time for connector h on leg ℓ (note that for $\ell = 1$, $t_{\ell-1,h} = 0$, and $delay_{\ell-1,h} = 0$). Constraints (3.3) and (3.4) ensure that in Block 2, $BL_{j,\ell,h} = 1$ / $BT_{j,\ell,h} = 1$ (respectively) only when connector h is routed appropriately. Constraints (3.5) and (3.6) restrict the earliest/latest possible landing times for connector h in block 2 at location j. Constraints (3.7) and (3.8) restrict the earliest/latest possible takeoff times for connector h in block 2 at location j. Constraint (3.9) ensures that connector h's first take off occurs at a "PMC coordinator-defined" location (HARC). Constraint (3.10) builds the relationship between x variables and w variables. Constraint (3.11) ensures connector h takes consecutive active legs. Constraint (3.12) models the logical constraint: if connector h has an active leg ℓ and it does not travel to its destination j_h^c, then there must be another active leg $\ell+1$. Constraints (3.13) and (3.14) ensure conservation of flow for connector h from leg ℓ to $\ell+1$. Constraint (3.15) measures unmet PMC demand. Constraint (3.16) restricts the total amount of PMC departing each destination. Constraint (3.17) forbids a load of PMC demand to enter (leave) its own origin (destination). Constraint (3.18) ensures flow balance of PMC. Constraint (3.19) ensures that the total amount of PMC picked up does not exceed the requested amount. Constraint (3.20) ensures PMC is loaded at its source location. Constraint (3.21) constrains the maximum tolerable risk of failure for routing connector h. Constraint (3.22) ensures that binary variable $y_{i,j,\ell,h,k}$ has value 1 when a positive amount of PMC type k is transported from i to j on connector h's ℓ^{th} leg. Constraint (3.23) constrains the maximum tolerable risk of failure for routing PMC type k. Constraints (3.24), (3.25), (3.26), (3.30), and (3.31) ensure the appropriate variables are binary. Constraint (3.27) ensures the departure time of a connector is within the maximum flight time of that connector. Constraints (3.28) and (3.29) ensure the appropriate variables are non-negative.

3.2.4 Derivation of Risk Constraints

Incorporating Connector Risk In [Bengigi et al., 2020], the total probability of detection by a red sensor is defined to be $P_s = \prod_{i=0}^{n}(1 - P_{f,q})$, where $P_{f,q}$ is the probability of detection along route q, visually shown in figure 3.1.

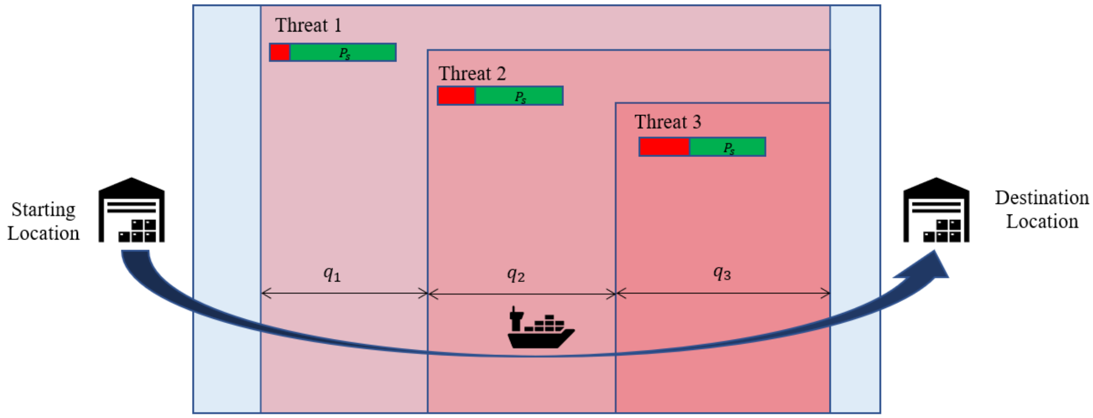

Figure 3.1: Illustration of Arc-Based Risk [Bengigi et al., 2020]

We extend this concept and define the success rate for each potential route by connector h, dictated by the $x_{i,j,\ell,h}$ variables, to be computed as follows:

$$\prod_{\substack{(i,j)|(i,j)\in HARC_h \\ \ell\in L}} (1 - P_{i,j,h}x_{i,j,\ell,h}) \times \prod_{i\in I,\ell\in L} \left(1 - P_{i,h} \sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h}\right), \tag{3.32}$$

where the first product term computes the probability of success in traveling on all the arcs on the route, and the second product term computes the probability of success in traveling through all the locations on the route. Now, given ϵ_h, the maximum tolerable risk of failure for routing connector h, we impose the following constraint:

$$\prod_{\substack{(i,j)|(i,j)\in HARC_h \\ \ell\in L}} (1 - P_{i,j,h}x_{i,j,\ell,h}) \times \prod_{i\in I,\ell\in L} \left(1 - P_{i,h} \sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h}\right) \geq 1 - \epsilon_h \tag{3.33}$$

We notice that this constraint is not linear. To solve this issue, we perform the following modification:

$$\ln\left(\prod_{\substack{(i,j)|(i,j)\in HARC_h \\ \ell\in L}} (1 - P_{i,j,h}x_{i,j,\ell,h}) \times \prod_{i\in I,\ell\in L} \left(1 - P_{i,h} \sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h}\right)\right) \geq \ln(1 - \epsilon_h), \tag{3.34}$$

which simplifies to:

$$\sum_{\substack{(i,j)|(i,j)\in HARC_h \\ \ell\in L}} \ln(1-P_{i,j,h}x_{i,j,\ell,h}) + \sum_{j\in I,\ell\in L} \ln\left(1-P_{i,h}\sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h}\right) \geq \ln(1-\epsilon_h) \quad (3.35)$$

Since $x_{i,j,\ell,h}$ is binary we know $x_{i,j,\ell,h}=0$ or $x_{i,j,\ell,h}=1$.

$$\text{When } x_{i,j,\ell,h}=0: \quad \ln(1-P_{i,j,h}\times 0)=\ln(1)=0=\ln(1-P_{i,j,h})\times 0=\ln(1-P_{i,j,h})x_{i,j,\ell,h}$$

$$\text{When } x_{i,j,\ell,h}=1: \quad \ln(1-P_{i,j,h}x_{i,j,\ell,h})=\ln(1-P_{i,j,h})\times 1=\ln(1-P_{i,j,h})x_{i,j,\ell,h}$$

Thus,

$$\sum_{\substack{(i,j)|(i,j)\in HARC_h \\ \ell\in L}} \ln(1-P_{i,j,h}x_{i,j,\ell,h}) = \sum_{\substack{(i,j)|(i,j)\in HARC_h \\ \ell\in L}} \ln(1-P_{i,j,h})x_{i,j,\ell,h} \quad (3.37)$$

Additionally, let's consider the second term in this constraint. The sum $\sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h}$ tells us whether connector h has traveled from i to j on a specific leg ℓ. As such, this sum is either 1 or 0 given a fixed leg ℓ. Because of this fact, we are able to rewrite the second term in the same way we rewrite (3.37). Thus, this second term simplifies to the following:

$$\sum_{i\in I,\ell\in L} \ln\left(1-P_{i,h}\sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h}\right) = \sum_{i\in I,\ell\in L} \ln(1-P_{i,h}) \sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h}. \quad (3.38)$$

Combining (3.37) and (3.38), we arrive at a linear representation of the original risk constraint (3.33), constraining the risk of each potential connector route as follows,

$$\sum_{\substack{(i,j)|(i,j)\in HARC_h \\ \ell\in L}} \ln(1-P_{i,j,h})x_{i,j,\ell,h} + \sum_{i\in I,\ell\in L} \ln(1-P_{i,h}) \sum_{j|(i,j)\in HARC_h} x_{i,j,\ell,h} \geq \ln(1-\epsilon_h) \quad (3.39)$$

written as (3.21) in the MIP formulation.

Calculating $P_{i,j,h}$ and $P_{i,h}$ To calculate the probability of failure at a location i or along a route from i to j, we consider the situation where the probability of failure is given by the sensors deployed by the red team, and the failure probability is understood as the detection probability, which depends on the sensor proximity and strength in relation to location and connector respectively. Given a set of sensors $s \in S$ deployed by the red team, let (i^s, j^s) be the two endpoints of the line segment corresponding to the intersection of arc (i, j) in the coverage area with respect to sensor s [Zabarankin et al., 2006] (see Figure 4.2 for an illustration), the location-based risk value $P_{i,h}$ for connector h at node i can be computed as:

$$P_{i,h} = \sum_{s \in S} \rho_{s,h} \times \frac{1}{\mathbf{d}(s,i)^2} \tag{3.40}$$

where $\rho_{s,h}$ represents the risk factor associated with a particular sensor $s \in S$ to connector $h \in H$, and $\mathbf{d}(\cdot,\cdot)$ gives the Euclidean distance between two points (for notational convenience, we use s to denote both the index of a target and its location).

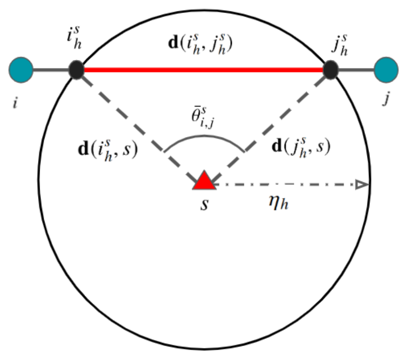

Figure 3.2: Coverage area associated with a connector k surrounding a sensor s [Margolis et al., 2022]

For the arc-based risk value $P_{i,j,h}$ for connector h to travel from node i to node j, similar to [Zabarankin et al., 2006], we calculate it as

$$P_{i,j,h} = \sum_{s \in S} \rho_{s,h} c_{i,j,s}, \tag{3.41}$$

and:

$$c_{i,j,s} := \begin{cases} \frac{\bar{\theta}_{ij}^s}{\sin \bar{\theta}_{ij}^s \mathbf{d}(i_h^s, s) \mathbf{d}(j_h^s, s)}, & \text{if } \mathbf{d}(i_h^s, j_h^s) > 0, \\ 0 & \text{otherwise,} \end{cases} \tag{3.42}$$

where $\bar{\theta}_{ij}^s$ represents the angle between two vectors, from s to i_h^s and from s to j_h^s, as illustrated in Figure 4.2.

Incorporating PMC Risk To incorporate the risk for potential PMC loss while traveling from i to j, we utilize the Operational Risk Management (ORM) matrix concept, shown in figure 3.3 [ORM, 2004]. This diagram is frequently used in the military to illustrate risk levels for specific actions or events. This matrix incorporates both the probability of an event occurring, such as the

Risk Assessment Matrix

E = Extremely High Risk H = High Risk M = Moderate Risk L = Low Risk			PROBABILITY				
			Frequent	Likely	Occasional	Seldom	Unlikely
			A	B	C	D	E
SEVERITY	CATASTROPHIC	I	E	E	H	H	M
	CRITICAL	II	E	H	H	M	L
	MARGINAL	III	H	M	M	L	L
	NEGLIGIBLE	IV	M	L	L	L	L

Table 3-1. Risk Assessment Matrix

Figure 3.3: Operational Risk Management(ORM) Matrix [ORM, 2004]

disruption of flow from i to j, and the severity of this event occurring. Here we use this to distinguish the risk associated with carrying varying types of PMC, in addition to the risk associated with flying on a particular connector through a given route.

We've already defined the probability $P_{i,j,h}$ to denote the probability of failure along the route from i to j. Here, we incorporate the ORM concept to account for the severity of losing a certain amount of a specific PMC type k (weighted by the severity of loss value ρ_k). Instead of a risk accruement perspective, we define $\delta_{h,k}$ to be the maximum allowed risk for any PMC loss that could occur at any arc:

$$P_{i,j,h} \times \sum_{k \in K} \rho_k f_{i,j,\ell,h,k} \leq \delta_{k,h}, \ \forall \ell \in L, (i,j) \in HARC_h, h \in H$$

written as (3.23) in the formulation. This process is unique as it brings the decision maker into the routing process.

Chapter 4

Two-Stage Discrete Time Model with Random Attack Scenarios

4.1 Motivations and Assumptions

In this chapter we present a two-stage stochastic model. The first stage model is based on a deterministic model presented in [Tan, 2023]. The second stage model extends the first stage model to incorporate recourse decisions made post-attack for a specific attack scenario s.

In this model we make some assumptions. The first being that there is a set of attack scenarios S available, where for each attack $s \in S$ the attack time and attack outcome is random. We assume that all attacks are "worst case", meaning that whatever percentage of capacity is destroyed is the same as the percentage connectors/PMC destroyed in inventory. Once an attack occurs, we assume that the reaction time is instantaneous for all connectors, meaning there is no delay for recourse decision making. We assume that connectors and PMC operate on a three-layer network. Connectors can travel between starting and destination layers and to and from the transshipment layer. PMC can travel to and from the transshipment layer, but not between layers. This is shown visually in figure 4.1:

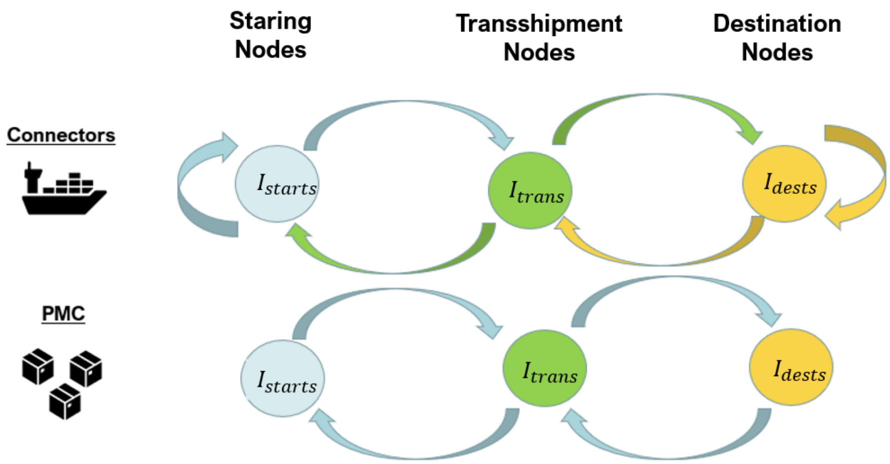

Figure 4.1: Three Layer Network for PMC and Connectors

4.2 Model Formulation

4.2.1 Deterministic Formulation

4.2.1.1 Sets, Parameters, and Variables

Sets:

$h \in H$	Connector vehicle type
$k \in K$	PMC type
$i, j \in I$	Set of all locations in the network
$I_{starts} \subset I$	Set of all starting locations
$I_{trans} \subset I$	Set of all transshipment locations
$I_{dests} \subset I$	Set of all destination locations
$(i, j) \in ARCS_h$	Allowable arcs for connector h based upon three-layer network definition
$(i, j) \in ARCS_k$	Allowable arcs for PMC k based upon three-layer network definition
$t \in T := \{1, 2, \ldots, \|T\|\}$	Time periods of operation

Parameters:

$cap_{h,k}$	Maximum amount of PMC k allowed on connector h
cap^v_h	Maximum volume for connector h
cap^w_h	Maximum weight for connector h
vol_k	Volume of PMC type k
$weight_k$	Weight of PMC type k
$cost_h$	Operating cost of connector h
$dem_{i,t,k}$	Demand for PMC type k at time t at location i
$sup_{i,k}$	Initial supply for PMC type k at location i
$start_{i,h}$	Initial number of connectors h at location i
$trans_{(i,j),h}$	Transit time from location i to j for connector h
pen_k	Penalty parameter for each unit of undelivered PMC type k

Variables:

$x_{(i,j),t,h}$	Number of connectors h beginning a route from locations i to j at time t
$IX_{i,t,h}$	Inventory of connectors h at location i at the start of time t
$y_{(i,j),t,h,k}$	Units of PMC type k transported on connector h from locations i to j beginning at time period t
$IY_{i,t,k}$	Inventory of PMC type k at location i at the start of time t
$e_{i,t,k}$	Demand for PMC type k at location i during time period t
$z_{i,t,k}$	Units of demanded PMC k delivered by time t at location i

4.2.1.2 Deterministic Model

$$\text{Minimize } z = \sum_{h\in H}\sum_{i\in I}\sum_{j\in I}\sum_{t\in T} cost_{(i,j),t,h}\cdot x_{(i,j),t,h} + \sum_{t\in T}\sum_{k\in K} pen_k \cdot \sum_{i\in I}(e_{i,t,k} - z_{i,t,k}) \tag{4.1}$$

$$\text{s.t. } IX_{i,t+1,h} = IX_{i,t,h} - \sum_{j\in I|(i,j)\in ARCS_h} x_{(i,j),t,h} + \sum_{j\in I|(j,i)\in ARCS_h} x_{(j,i),t-trans_{(j,i),h}+1,h}$$
$$\forall h \in H, t\in T, i \in I \tag{4.2}$$

$$\sum_{j\in I|(i,j)\in ARCS_h} x_{(i,j),t,h} \le IX_{i,t,h},\ \forall i\in I, t\in T, h\in H \tag{4.3}$$

$$IY_{i,t+1,k} = IY_{i,t,k} - \sum_{j\in I|(i,j)\in ARCS_k}\sum_{h\in H} y_{(i,j),t,h,k}$$
$$+ \sum_{j\in I|(j,i)\in ARCS_k}\sum_{h\in H} y_{(j,i),t-trans_{(j,i),h}+1,h,k} - z_{i,t,k},$$
$$\forall i\in I, t\in T, k\in K \tag{4.4}$$

$$\sum_{j \in I | (i,j) \in ARCS_k} \sum_{h \in H} y_{(i,j),t,h,k} \leq IY_{i,t,k} - z_{i,t,k}, \ \forall t \in T, k \in K, i \in I \tag{4.5}$$

$$y_{(i,j),t,h,k} \leq cap_{h,k} \cdot x_{(i,j),t,h}$$
$$\forall (i,j) \in I, t \in T, h \in H, k \in K \tag{4.6}$$

$$y_{(i,j),t,h,k} \cdot vol_k \leq cap_h^v \cdot x_{(i,j),t,h}$$
$$\forall (i,j) \in I, t \in T, h \in H, k \in K \tag{4.7}$$

$$y_{(i,j),t,h,k} \cdot weight_k \leq cap_h^w \cdot x_{(i,j),t,h}$$
$$\forall (i,j) \in I, t \in T, h \in H, k \in K \tag{4.8}$$

$$IY_{i,0,k} = sup_{i,k}, \ \forall i \in I, k \in K \tag{4.9}$$

$$IX_{i,0,h} = start_{i,h}, \ \forall i \in I, h \in H \tag{4.10}$$

$$e_{i,0,k} = dem_{i,0,k}, \ \forall i \in I, k \in K \tag{4.11}$$

$$e_{i,t,k} = e_{i,t-1,k} - z_{i,t-1,k} + dem_{i,t,k}, \ \forall i \in I, t \in T \setminus \{0\}, k \in K \tag{4.12}$$

$$z_{i,t,k} \leq IY_{i,t,k}, \ \forall i \in I, t \in T, k \in K \tag{4.13}$$

$$z_{i,t,k} \leq e_{i,t,k}, \ \forall i \in I, t \in T, k \in K \tag{4.14}$$

$$e_{i,t,k}, z_{i,t,k}, y_{(i,j),t,h,k}, IY_{i,t,k}, x_{(i,j),t,h}, IX_{i,t,k} \geq 0,$$
$$\forall (i,j) \in I, t \in T, h \in H, k \in K \tag{4.15}$$

$$x_{(i,j),t,h}, IX_{i,t,h} \in \mathbb{Z}_+, \ \forall (i,j) \in I, t \in T, h \in H \tag{4.16}$$

4.2.1.3 Constraint Explanations

Equation (4.1) is the deterministic model objective. It minimizes the overall connector operating cost and penalizes unmet demand to prioritize the delivery of requested items. Constraint (4.2) ensures the balance of connectors in inventory at each location from t to $t+1$. It is important to note that inventory for the $t+1$ time period is counted after connectors arrive at time $t+1$ but before connectors depart at time $t+1$. Constraint (4.3) requires the number of connectors h leaving a location to be no larger than the number of connectors h in inventory at the time of departure. Constraint (4.5), like constraint (4.2), ensures the balance of PMC units in inventory at each location from t to $t+1$. Constraint (4.4), like constraint (4.3), requires the amount of PMC k leaving a location to be no larger than the amount of PMC k in inventory at the time of departure. Constraints (4.6), (4.7), and (4.8) ensure connectors are within their unit, volume, and weight capacity limits for PMC items travelling onboard. Constraints (4.10) and (4.9) store the initial inventory for connectors and PMC at $t = 0$. Constraint (4.11) stores the initial PMC demand signals at $t = 0$ for each location. Constraint (4.12) tracks demand remaining to be satisfied for each time period. Constraints (4.13) and (4.14) restrict the satisfied demand to be no more than the inventory at each location and the demand requested at each location. Constraint (4.15) requires all variables to be non-negative and constraint (4.16) requires the connector routing and inventory variables to be integer.

4.2.2 Second Stage Formulation

Understanding the sequence of events at the time of attack, t_s is critical to understating the link between the first stage variables and second stage model.

4.2.2.1 Sequence of Events

1. Begins with the starting inventory at $t_s - 1$
2. Connectors and PMC arrive at the location (from the original plan) at t_s
3. Attack occurs at t_s
4. Inventory for PMC & Connectors is recounted to account for lost connectors and PMC
5. Recourse routing begins

4.2.2.2 Sets, Parameters, and Variables

Scenario-Based Sets:

$s \in S$	Attack scenario
$i, j \in I_s \subset M$	Locations targeted by attack s
$t \in T_s^* \subset T$	Time periods, starting from t_s to $\|T\|$ where t_s is the time period of the attack ($T_s^* := \{t_s, \ldots, \|T\|\}$

Scenario-Based Parameters:

p_s	The probability that attack s occurs
$\delta_{i,h,s}/\delta_{i,k,s}$	Percentage of connectors h/PMC k at location $i \in I_s$ destroyed during attack s

Scenario-Based Recourse Decision Variables:

$x_{(i,j),t,h,s}$	Number of connectors h beginning a route from locations i to j at time t under attack scenario s
$IX_{i,t,h,s}$	Inventory of connectors h at location i at the start of time t under attack scenario s
$y_{(i,j),t,h,k,s}$	Units of PMC type k transported on connector h from locations i to j beginning at time period t under attack scenario s
$IY_{i,t,k,s}$	Inventory of PMC type k at location i at the start of time t under attack scenario s
$e_{i,t,k,s}$	Demand for PMC type k at location i during time period t under attack scenario s
$z_{i,t,k,s}$	Units of demanded PMC k delivered by time t at location i under attack scenario s

4.2.2.3 Second Stage Recourse Model Under Attack s

$$
\begin{aligned}
Z_s(\mathbf{x}) : \text{Minimize } z = &\sum_{t \geq t_s} \left(\sum_{h \in H} \sum_{i \in I} \sum_{j \in I | (i,j) \in ARCS_h} cost_{i,j,t,h} \cdot x_{i,j,t,h,s} + \sum_{i \in I} \sum_{k \in K} pen_k \cdot (e_{i,t,k,s} - z_{i,t,k,s}) \right) \\
&- \sum_{t \geq t_s} \left(\sum_{h \in H} \sum_{i \in I} \sum_{j \in I | (i,j) \in ARCS_h} cost_{i,j,t,h} \cdot x_{i,j,t,h} + \sum_{i \in I} \sum_{k \in K} pen_k \cdot (e_{i,t,k} - z_{i,t,k}) \right)
\end{aligned} \tag{4.17}
$$

$$
\begin{aligned}
\text{s.t. } IX_{i,t+1,h,s} = IX_{i,t,h,s} &- \sum_{j \in I | (i,j) \in ARCS_h} x_{i,j,t,h,s} \\
&+ \sum_{j \in I | (j,i) \in ARCS_h} x_{j,i,t-trans_{j,i,h}+1,h,s} \cdot \mathbf{1}_{t-trans_{j,i,h}+1 \geq t_s}, \\
&\forall h \in H, t \geq t_s, i \in I_{trans}
\end{aligned} \tag{4.18}
$$

$$
\begin{aligned}
IX_{i,t+1,h,s} = IX_{i,t,h,s} &- \sum_{j \in I | (i,j) \in ARCS_h} x_{i,j,t,h,s} \\
&+ \sum_{j \in I | (j,i) \in ARCS_h} x_{j,i,t-trans_{j,i,h}+1,h,s} \cdot \mathbf{1}_{t-trans_{j,i,h}+1 \geq t_s} \\
&+ \sum_{j \in I_{trans} | (i,j) \in ARCS_h} x_{i,j,2t_s-t-1,h} \cdot \mathbf{1}_{\substack{2t_s-t-1<t_s, \\ 2t_s-t-1+\text{trans}_{i,j,h}>t_s, \\ 2t_s-t-1\geq 0}} \\
&+ \sum_{j \in I} x_{j,i,t-trans_{j,i,h}+1,h} \cdot \mathbf{1}_{t-trans_{j,i,h}+1<t_s},
\end{aligned} \tag{4.19}
$$

$$
\forall h \in H, t \geq t_s, i \in I_{starts} \cup I_{dests} \tag{4.20}
$$

$$
y_{i,t_s,h,s} \leq (1 - \delta_{i,h,s}) y_{i,t_s,h}, \ \forall i \in I_s, h \in H \tag{4.21}
$$

$$
y_{i,t_s,h,s} \geq (1 - \delta_{i,h,s}) y_{i,t_s,h} - 1 + 10^{-3}, \ \forall i \in I_s, h \in H \tag{4.22}
$$

$$
y_{i,t_s,h,s} = y_{i,t_s,h}, \ \forall i \in I \setminus I_s, h \in H \tag{4.23}
$$

$$
\sum_{j \in I | (i,j) \in ARCS_h} x_{i,j,t,h,s} \leq IX_{i,t,h,s}, \ \forall i \in I, t \geq t_s, h \in H \tag{4.24}
$$

$$\sum_{j \in I | (i,j) \in ARCS_k} \sum_{h \in H} y_{i,j,t,h,k,s} \leq IY_{i,t,k,s} - z_{i,t,k,s}, \ \forall t \geq t_s, k \in K, i \in I \quad (4.25)$$

$$IY_{i,t_s,k,s} = (1 - \delta_{i,k,s}) * IY_{i,t_s,k}, \ \forall i \in I_s, k \in K \quad (4.26)$$

$$IY_{i,t_s,k,s} = IY_{i,t_s,k}, \ \forall i \in I \setminus I_s, k \in K \quad (4.27)$$

$$IY_{i,t+1,k,s} = IY_{i,t,k,s} - \sum_{j \in I | (i,j) \in ARCS_k} \sum_{h \in H} y_{i,j,t,h,k,s} + \sum_{j \in I | (j,i) \in ARCS_k} \sum_{h \in H} y_{j,i,t-trans_{i,j,h}+1,h,k,s} \cdot \mathbf{1}_{t-trans_{j,i,h}+1 \geq t_s}, \quad \forall i \in I_{trans}, t \geq t_s, k \in K \quad (4.28)$$

$$IY_{i,t+1,k,s} = IY_{i,t,k,s} - \sum_{j \in I | (i,j) \in ARCS_k} \sum_{h \in H} y_{i,j,t,h,k,s} + \sum_{j \in I | (j,i) \in ARCS_k} \sum_{h \in H} y_{j,i,t-trans_{i,j,h}+1,h,k,s} \cdot \mathbf{1}_{t-trans_{j,i,h}+1 \geq t_s} + \sum_{j \in I} y_{i,j,2t_s-t-1,h,k} \cdot \mathbf{1}_{\substack{2t_s-t-1<t_s, \\ 2t_s-t-1+\text{trans}_{i,j,h}>t_s, \\ 2t_s-t-1 \geq 0}} + \sum_{j \in I} y_{j,i,t-trans_{j,i,h}+1,h,k} \cdot \mathbf{1}_{t-trans_{j,i,h}+1<t_s}, \quad \forall i \in I_{starts} \cup I_{dests}, t \geq t_s, k \in K \quad (4.29)$$

$$y_{i,j,t,h,k,s} \leq cap_{h,k} \cdot x_{i,j,t,h,s} \quad \forall i,j \in I, t \geq t_s, h \in H, k \in K \quad (4.30)$$

$$y_{i,j,t,h,k,s} \cdot vol_k \leq cap^v_h \cdot x_{i,j,t,h,s} \quad \forall i,j \in I, t \geq t_s, h \in H, k \in K, s \in S \quad (4.31)$$

$$y_{i,j,t,h,k,s} \cdot weight_k \leq cap^w_h \cdot x_{i,j,t,h,s} \quad \forall i,j \in I, t \geq t_s, h \in H, k \in K, s \in S \quad (4.32)$$

$$IX_{i,t,h,s} \leq (1 - \delta_{i,t,h,s}) \cdot cap_{i,h}, \forall i \in I_s, t \geq t_s, h \in H \quad (4.33)$$

$$IX_{i,t,h,s} \leq cap_{i,h}, \ \forall i \notin I_s, t \geq t_s, h \in H \quad (4.34)$$

$$IY_{i,t,k,s} \leq cap_{i,k} \cdot (1 - \delta_{i,t,k,s}), \ \forall i \in I_s, t \geq t_s, h \in H \quad (4.35)$$

$$IY_{i,t,k,s} \leq cap_{i,k}, \ \forall i \notin I_s, t \geq t_s, h \in H \quad (4.36)$$

$$e_{i,t,k,s} = e_{i,t-1,k,s} - z_{i,t-1,k,s} + dem_{i,t,k,s}, \ \forall i \in I, t > t_s, k \in K \quad (4.37)$$

$$z_{i,t,k,s} \leq IY_{i,t,k,s}, \ \forall i \in I, t \geq t_s, k \in K \quad (4.38)$$

$$z_{i,t,k,s} \leq e_{i,t,k,s}, \ \forall i \in I, t \geq t_s, k \in K \quad (4.39)$$

$$e_{i,t_s,k,s} = e_{i,t_s,k}, \ \forall i \in I, k \in K \quad (4.40)$$

$$e_{i,t,k,s}, y_{i,j,t,h,k,s}, z_{i,t,k,s}, IY_{i,t,k,s} \geq 0, \ \forall i,j \in I, t \geq t_s, h \in H, k \in K \quad (4.41)$$

$$x_{i,j,t,h,s}, IX_{i,t,h,s} \in \mathbb{Z}_+, \ \forall i,j \in I, t \geq t_s, h \in H \quad (4.42)$$

4.2.2.4 Constraint Explanations

Equation (4.17) minimizes the cost of connector operation and unmet demand under scenario s within the second stage formulation and "pays back" the cost of the first stage variables unused for $t \geq t_s$. Constraint (4.18) represents the connector inventory balance constraint for connectors at transshipment locations. Constraint (4.20) represents the connector inventory balance constraint for starting and destination locations. This constraint includes the arrival of connectors that depart before t_s and arrive after t_s. Constraints (4.21) and (4.22) enforce a reduction in connector inventory for all attacked locations based on the percentage of connector capacity destroyed at location i and the number of connectors in inventory at location i at t_s. Constraint (4.23) Initializes the amount of connectors in inventory at t_s for non-attacked locations. Constraint (4.24) ensures the number of connectors leaving location i is not greater than the number of connectors available in inventory at location i. Constraint (4.25) ensures the amount of PMC leaving location i is not greater than the amount of PMC available in inventory at location i. Constraint (4.26) enforces a reduction in PMC inventory for all attacked locations based on the percentage of PMC capacity destroyed at location i and the amount of PMC in inventory at location i at t_s. Constraint (4.27) Initializes the amount of PMC in inventory at t_s for non-attacked locations. Constraint (4.28) represents the PMC inventory balance constraint for connectors at transshipment locations. Constraint (4.29) represents the PMC inventory balance constraint for starting and destination locations. This constraint includes the arrival of PMC departing before t_s and arriving after t_s. Constraints (4.30), (4.31), and (4.32) ensure connectors are within their unit, weight, and volume capacity limits for PMC items travelling onboard. Constraint (4.33) enforces the new capacity for connectors at attacked locations. Constraint (4.34) enforces the capacity for connectors at non-attacked locations. Constraint (4.35) enforces the new capacity for PMC at attacked locations. Constraint (4.37) tracks demand remaining to be satisfied for each time period. Constraints (4.38) and (4.39) restrict the satisfied demand to be no more than the inventory at each location and the demand requested at each location. Constraint (4.40) Sets the demand quota at t_s. Constraint (4.41) requires all variables to be non-negative and constraint (4.42) requires the connector routing and inventory variables to be integer.

4.2.2.5 Special Recourse Cases at t_s

There are a few special cases that prevent us from directly linking the first stage model to the second stage model. These special cases occur when a connector departs prior to t_s and is not due to arrive until after t_s. Because the attack occurs at t_s, there is a reduction of capacity for PMC and connectors at the attacked locations, thus if we allow connectors to continue routes to these locations, there is a potential for there to be no space for them upon arrival. This causes issues with the feasibility of the model. To avoid this, we send all connectors and PMC en-route to transshipment locations back to their starting locations (where there is no possibility of attack). This solves the routing consistency issue for connectors en-route to transshipment locations. Now, if a connector is departing a transshipment location, then sending it back to its starting location at t_s may also introduce a feasibility issue (due to the possibility of a reduced location capacity for PMC and connectors). Since it's already on a route away from danger, we allow it to continue its route from the first to the second stage model. Additionally, if a connector is travelling between the starting layer or between the destination layer, we also allow it to keep its route from the first to the second stage model. Visually these cases are shown below, where t_d is the time of departure, t_s is the time of attack, and t is the original time of arrival in the second stage model prior to the recourse:

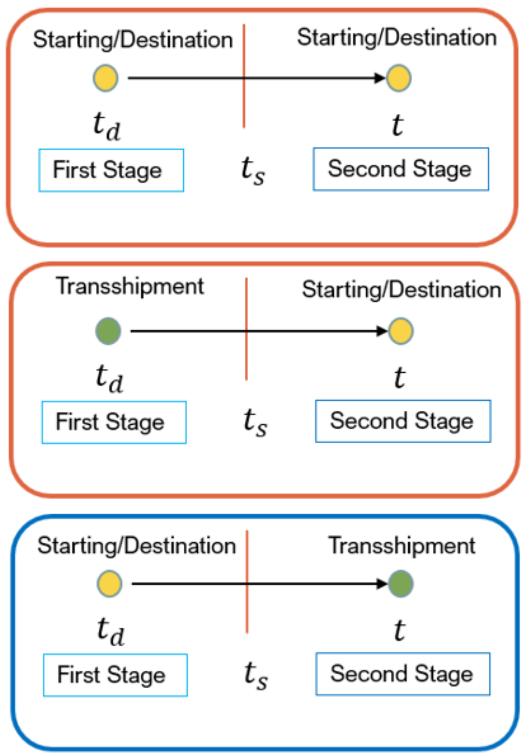

Figure 4.2: Special Recourse Cases

It's not difficult to link the first two special cases (connectors travelling between layers and connectors departing transshipment nodes). Linking the third case however, requires some additional thought. In order to determine the time of return, we need to know the total routing time. At t_s we know that $x_{i,j,t_d,h}$ has been transiting for $t_s - t_d$ time periods. We assume that it takes no additional time for the connector to turn around, hence the return time is also $t_s - t_d$. Therefore the total transit time is $2 \cdot (t_s - t_d)$. To get the time of return we add this value to the original time of departure, t_d. Thus we get the following:

$$t_{return} = 2 \cdot (t_s - t_d) + t_d$$

$$t_{return} = 2t_s - t_d$$

These recourse decisions are modeled in constraints (4.20) and (4.29).

4.2.3 Full Two-stage Stochastic Programming Model

$$\text{Minimize } z = \sum_{h \in H} \sum_{i \in I} \sum_{j \in I} \sum_{t \in T} (cost_{i,j,t,h} \cdot x_{i,j,t,h}) + \sum_{t \in T} \sum_{k \in K} \sum_{i \in I} pen_k \cdot (e_{i,t,k} - z_{i,t,k}) + \sum_{s \in S} p_s Z_s(\mathbf{x}) \tag{4.43}$$

$$\text{s.t. } (4.2) - -(4.16)$$

4.2.3.1 Constraint Explanations

Equation (4.43) minimizes the objective of the deterministic model and the expectation of second-the stage model under all scenarios s. Constraints (4.2) – (4.16) follow their descriptions in section 4.2.1.3.

Chapter 5

Experiment Setup

5.1 Continuous Time Model

With this model, our goal is to evaluate the effectiveness of the chance constraints for both PMC and connectors. We need to know how they affect the routes of both connectors and PMC, as well as how they affect the overall objective cost as we raise and lower the tolerable risk for PMC and connector routing.

5.1.1 Data Preparation

We used scenario instances based on real military logistics scenarios to test the models. While the real information is sensitive, the network instance, PMC demand requests, and available connectors were created with actual military scenarios in mind. The specific information used to test this formulation is shown in the table below:

Scenario Information			
Amount of Connectors	Number of Locations	Total Packages of PMC	Total Volume of PMC (ft^3)
2	11	113	10767

Table 5.1: Continuous Time Scenario Information

Additionally, sensor locations and ranges are randomly selected. In this case, there are two sensors selected. Since the actual scenario information is classified, the risk factor for each connector and sensor is also randomly selected from 1 to 9 (as there is no way to know the actual value). For PMC, we assign each item a value from 1 to 4 to represent the severity of loss as demonstrated in the ORM. Again, this is selected randomly as there is no way to know the actual value. 1 represents a negligible item, 2 represents a marginal item, 3 represents a critical item, and 4 represents a catastrophic item. For all simulations, the model was run using optimization software Gurobi 10.0.1.

5.1.2 Results

We begin by testing how the model behaves when exclusively raising and lowering the maximum tolerable risk of failure for connectors, titled "eRiskTol" in the following tables. We test this value at 0.25, 0.5, 0.75, and at 1 where 0.25 represents low tolerable risk and 1 represents higher tolerable risk. The output of these runs is shown in tables 5.2 and 5.3.

eRiskTol	fRiskTol	OBJ Cost	Connectors Used	# of Packages Undelivered	Vol. PMC Undelivered
1	100	**12268.98**	0, 1 (Air)	2 out of 113	595 out of 10767
0.75	100	**46027.88**	0, 1 (Air)	10 out of 113	2832 out of 10767
0.5	100	**46076.88**	0, 1 (Air)	10 out of 113	2832 out of 10767
0.25	100	**66096.61**	0, 1 (Air)	15 out of 113	2764 out of 10767

Table 5.2: Sensitivity Analysis on Constraint (3.21) Pt. 1

eRiskTol	fRiskTol	#of Pacakges Undelivered				Legs Used	Reason for OBJ Change
		Negligible	Marginal	Critical	Catastrophic		
1	100	–	1 out of 3	–	1 out of 89	7,7	Increased Delivery
0.75	100	3 out of 19	3 out of 3	1 out of 2	3 out of 89	7,7	Route Change
0.5	100	3 out of 19	3 out of 3	1 out of 2	3 out of 89	7,7	Increased Delievery
0.25	100	9 out of 19	3 out of 3	1 out of 2	2 out of 89	7,7	

Table 5.3: Sensitivity Analysis on Constraint (3.21) Pt. 2

We notice that as the tolerable risk increases, the objective value decreases. In this run we hold the PMC tolerable risk constant at 100, unconstraining this constraint as we test the effects of changing the connector tolerable risk. As the tolerable risk increases from low to medium low risk, the objective cost decreases as we increase the number of PMC items delivered. We also notice a drop in the objective due to item delivery from medium-high risk to high risk. As we increase the tolerable risk, we also notice a route change for one of the connectors, which also contributes to the drop in the objective value. The low-risk solution never enters the sensor zone. The medium-low risk and medium-high risk solutions both enter the sensor zone once; however the medium-high risk solution spends a greater amount of time in the sensor zone. The high risk solution enters the sensor zone 3 times for sensor 0 and once for sensor 1. This demonstrates that as we raise the tolerable risk of failure for a given connector, that connector will choose riskier routes in order to achieve a more cost-effective route. This visualization is shown in figure 5.1.

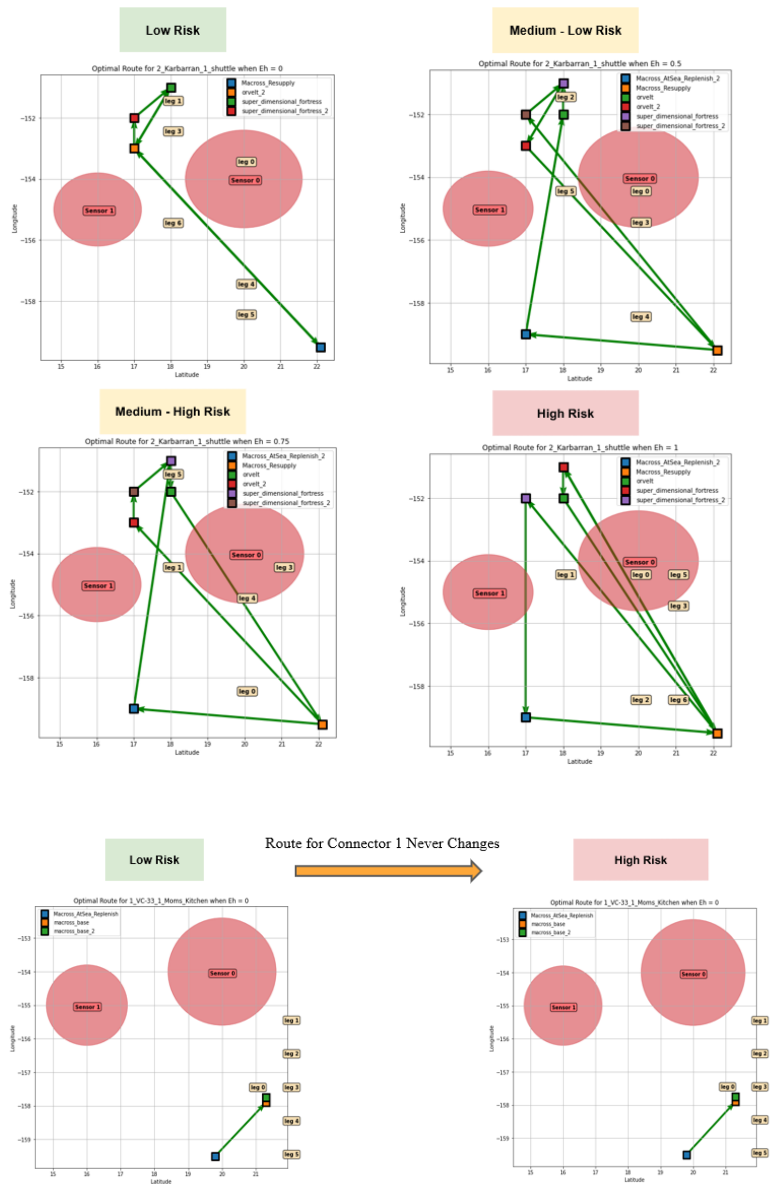

Figure 5.1: Visualization of the Sensitivity Analysis of Constraint (3.21)

Next we test how the model behaves when exclusively raising and lowering the maximum tolerable risk of failure for PMC, titled “fRiskTol” in the following tables. We test this value at 1, 10, 40, and at 1000 where 1 represents low tolerable risk and 100 represents high tolerable risk. The output of these runs is shown in tables 5.4 and 5.5 .

eRiskTol	fRiskTol	OBJ Cost	Connectors Used	# of Packages Undelivered	Vol. PMC Undelivered
1	100	**12268.98**	0, 1 (Air)	2 out of 113	595 out of 10767
1	40	**40210.98**	0, 1 (Air)	9 out of 113	1032 out of 10767
1	10	**42588.06**	0, 1 (Air)	9 out of 113	1032 out of 10767
1	1	**46020.88**	0, 1 (Air)	10 out of 113	2832 out of 10767

Table 5.4: Sensitivity Analysis on Constraint (3.23) Pt. 1

eRiskTol	fRiskTol	#of Pacakges Undelivered				Legs Used	Reason for OBJ Change
		Negligible	Marginal	Critical	Catastrophic		
1	100	–	1 out of 3	–	1 out of 89	7,7	Increased Delivery
1	40	7 out of 19	1 out of 3	–	1 out of 89	7,7	Delivery Change
1	10	7 out of 19	–	–	2 out of 89	7,7	Increased Delivery
1	1	3 out of 19	3 out of 3	1 out of 2	3 out of 89	7,7	

Table 5.5: Sensitivity Analysis on Constraint (3.23) Pt. 2

Again, we notice that as the tolerable risk increases, the objective value decreases. In this run we hold the connector tolerable risk constant at 1, unconstraining this constraint as we test the effects of changing the PMC tolerable risk. As we increase the tolerable risk from low to medium-low, we notice increased PMC delivery and a route change for connector 2 (connector 1's route never changes). Going from medium-low risk to medium-high risk there is no route change, however there is a change in the items delivered. From medium-low risk to medium-high risk one less "marginal" item is delivered and one more "catastrophic" item is delivered. Going from medium-high risk to high risk, we again see an objective drop. This is due to an increase in the number of items delivered and another route change. Because the connector tolerable risk is unconstrained, the connectors enter the sensor zones in every route. That being said, as the PMC tolerable risk increases, the number of times the connector enters the sensor zone increases as well. We also notice that as the tolerable risk increases the distribution of negligible, marginal, critical, and catastrophic PMC deliveries change as well. The route visualization for this run is shown in figure 5.2.

Now we test how the model behaves when raising and lowering both the maximum tolerable risk of failure for connectors and for PMC. The results of this run are shown in tables 5.6 and 5.7. In this run we notice that the connector routes, shown in figure 5.3, are exactly the same as those in figure 5.1, the run where maximum tolerable risk of failure for connectors was exclusively modified. The difference between that run and this run is the number of catastrophic items undelivered for the low risk solution. This tells us that the PMC risk constraint is heavily influenced by the maximum tolerable risk for the connector constraint.

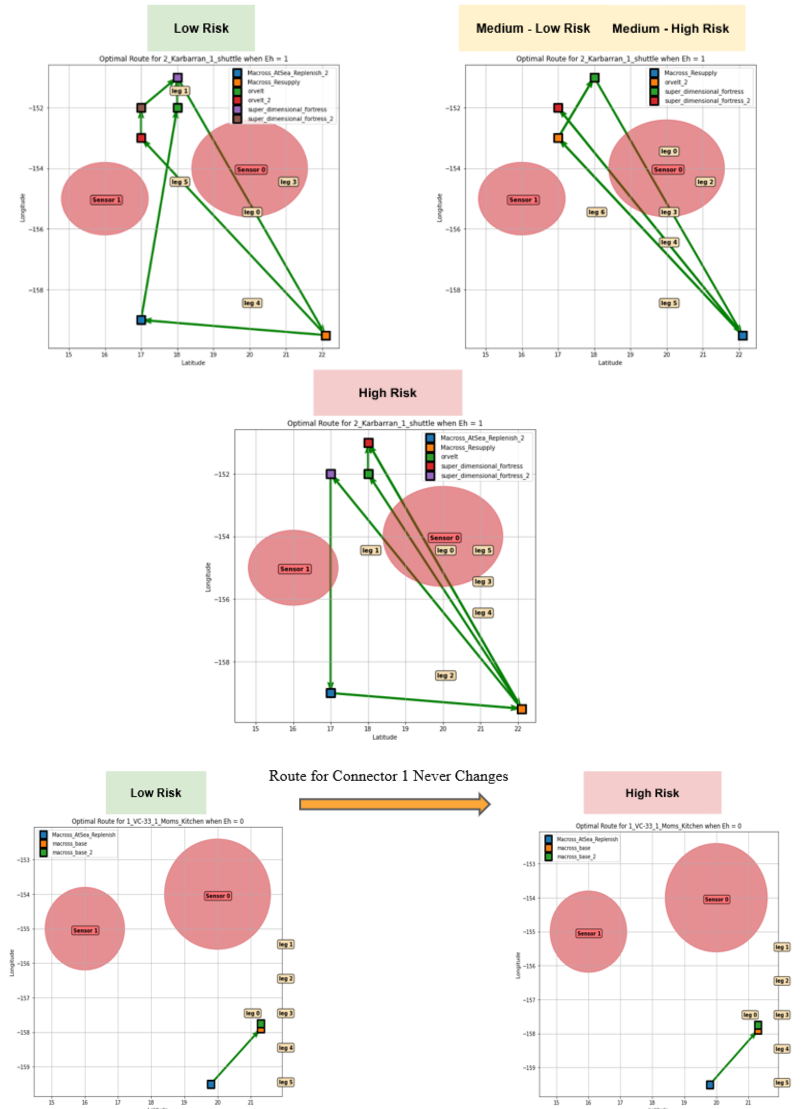

Figure 5.2: Visualization of the Sensitivity Analysis of Constraint (3.21)

eRiskTol	fRiskTol	OBJ Cost	Connectors Used	# of Packages Undelivered	Vol. PMC Undelivered
1	100	**12268.98**	0, 1 (Air)	2 out of 113	595 out of 10767
0.75	80	**46027.88**	0, 1 (Air)	10 out of 113	2832 out of 10767
0.5	40	**46076.88**	0, 1 (Air)	10 out of 113	2832 out of 10767
0.25	10	**66096.61**	0, 1 (Air)	16 out of 113	2764 out of 10767

Table 5.6: Sensitivity Analysis of Constraint (3.21) and (3.23) Pt. 1

eRiskTol	fRiskTol	#of Pacakges Undelivered				Legs Used	Reason for OBJ Change
		Negligible	Marginal	Critical	Catastrophic		
1	100	–	1 out of 3	–	1 out of 5	7,7	Increased Delivery
0.75	80	3 out of 113	3 out of 3	1 out of 2	3 out of 5	7,7	Route Change
0.5	40	3 out of 113	3 out of 3	1 out of 2	3 out of 5	7,7	Increased Delivery
0.25	10	9 out of 113	3 out of 3	1 out of 2	3 out of 5	7,7	

Table 5.7: Sensitivity Analysis of Constraint (3.21) and (3.23) Pt. 2

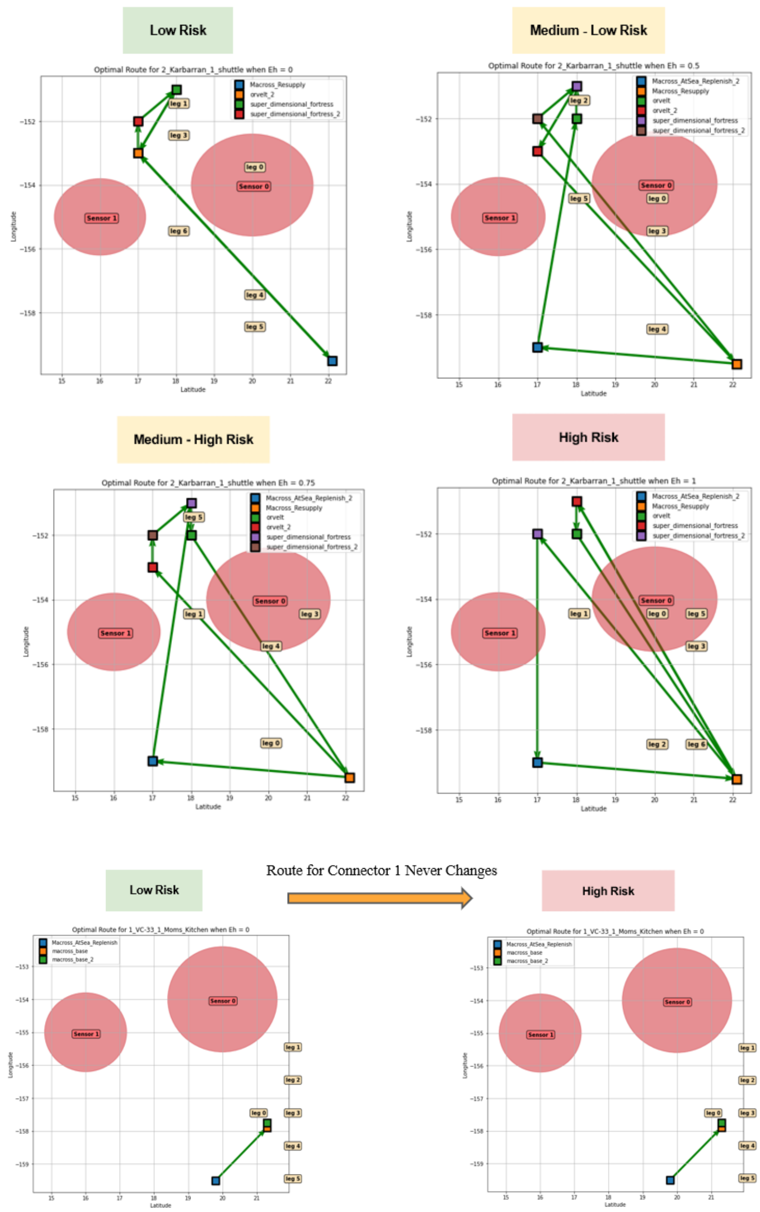

Figure 5.3: Visualization of the Constraints Sensitivity Analysis (3.21) & (3.23)

5.2 Two-Stage Discrete Time Model

With this model, out goal is to evaluate the routing solutions of the two-stage model as compared to the deterministic formulation. We need to know if the two-stage model generates solutions that are more risk-averse when compared to a plain deterministic solution.

5.2.1 Data Preparation

Like with the continuous time model, we used scenarios based on real-world military logistics scenarios. This data was modified to include capacities at locations for PMC and connectors. The location coordinates were also modified to be based on real-world locations, as see in figure 5.4. The specific information used to test this formulation is shown in table 5.8:

Scenario Information			
Amount of Connectors	Number of Locations	Total Packages of PMC	Total Volume of PMC (ft^3)
2	9	450	244,730

Table 5.8: Discrete Time Scenario Information

In this scenario, the locations are broken up into the three-layer network where there are three starting, transshipment, and destination locations. This network is shown in figure 5.4, where the blue locations represent starting locations, the green locations represent transshipment locations, and the dark blue locations represent destination locations. For each attack scenario, the attack time and destruction values are randomly selected. We assume that each attack is equally likely to occur.

5.2.2 Results

To test the effectiveness of this model, the following solutions were compared: the deterministic solution, the two-stage solution, the two-stage solution using the deterministic solution as the first stage solution, and the average of the deterministic solution under known attacks. The deterministic solution was simply calculated by running the base deterministic model. The two-stage model was tested by inputting three attack sets, one with 3 attacks, 5 attacks, and 7 attacks, and then run until a 15%, 25 % and 30 % gap respectively. The two-stage model with the deterministic solution as the first stage solution was run by taking the deterministic solution and setting it equal to the first stage variables in the two stage model. Then the model was run with the 3 attack sets, one with 3 scenarios, 5 scenarios, and 7 scenarios. The average of the deterministic solution under

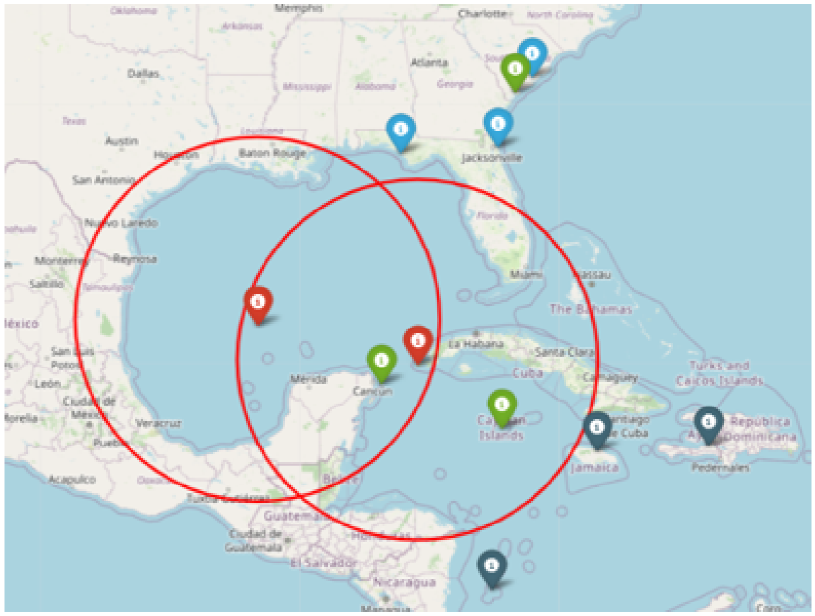

Figure 5.4: Discrete Time Scenario Three-Layer Network

known attacks was tested by running the deterministic solution with each attack scenario in each attack set used as input into the model. For each attack set, the average of the returned objective value was calculated to be weighted average of the deterministic solution under the known attacks. The results of these tests are shown in figure 5.5.

	Deterministic Model		Two-Stage Model		Two Stage with Optimal Deterministic as 1st Stage		Weighted Average of Deterministic Model Under Known Attacks	
Attack Nums	OBJ	Gap	OBJ	%Gap	OBJ	%Gap	OBJ	%Gap
3	175.27	14.73%	190.85	13.63	1,055,100.56	1.82	182.65	13.44
5	175.27	14.73%	217.95	22.82	1,018,023.46	9.62	195.866	14.09
7	175.27	14.73%	78,092.63	30	2,610,402.72	19.64	224.830	14.69

Figure 5.5: Discrete Time Scenario Results

We notice that for every attack set, the lowest objective value is from the deterministic model. This makes sense as this solution does not account for any attack scenarios and simply finds the most optimal route plan. The next best solution for every attack set is the weighted average of the deterministic model results under the known attacks. The third best objective value is the two-stage model and the fourth best objective value is the two-stage model with the deterministic model result inputted as the first stage solution. This tells us that when operating in a contested environment, the two-stage model outperforms the deterministic model. Figure 5.6 better quantifies this: We test the two-stage model with the deterministic model input to assess how well that routing

Attack Num	Two Stage Model OBJ	Two Stage with Optimal Deterministic as 1st Stage	Notes
3	190.85	1,055,100.56	Two Stage 5,528 times better
5	217.95	1,018,023.46	Two Stage 4,671 times better
7	78,092.63	2,610,402.72	Two-Stage 33 times better

Figure 5.6: Two-Stage Results Comparison

plan would hold up under various possible attacks. As shown, the objective for the two-stage with the deterministic model as input at best, performs 33 times worse than the two-stage model. It's worth nothing that for 7 attacks, the model gap is at 30% for the two-stage model and at 19.64% for the two-stage model with the deterministic model, giving an advantage to the deterministic input. However, even with this advantage it still significantly performs worse than the two-stage solution.

Chapter 6

Conclusions and Discussion

6.1 Conclusions

The goal of this work was to investigate two separate models that handle multi-commodity logistics planning within in contested environments. The first model we investigated was a continuous time model with chance constraints. We found that this model yielded risk-adverse solutions when using the chance constraints. As the maximum tolerable risk of failure for PMC and connectors decreased, the route changed to avoid sensor zones. The second model we investigated was the two-stage discrete time model with random attack scenarios. On small-scale scenarios, this model has demonstrated great promise. Solutions from the two-stage model when compared to those of the deterministic base model were significantly more risk-adverse when routing in a contested environment. The two-stage solutions avoided unnecessary PMC loss from potential attacks and at worst, performed 33 times better than the deterministic model when the deterministic solution was used as the first stage solution input. At this time it is unclear how the two-stage model will perform using to-scale military logisitics scenarios, leaving the door open for future work.

6.2 Future Work

There is great potential for future work for both models. While the continuous time model is not practical because it lacks transshipment, there is value in incorporating the chance constraints for both PMC and connectors into the base deterministic model used in the Two-Stage model.

While both the continuous time model and the two-stage model handle routing within a contested environment, they go about it in different ways. Implementation of the deterministic model with the chance constraints from the continuous time model would be beneficial in an environment where sensor locations are known, but potential attacks are not. In a setting where routing without detection is the greatest priority, using this model may be more beneficial than using that of the two-stage model.

Additionally, there is still a great amount of exploration left to do with the two-stage model. Due to limiting time factors, it remains to be tested on medium to large scale deployment instances due to how long it takes to run. It also has been tested with up to 7 attack scenarios. There is an opportunity to test model performance with an increasing number of attack scenarios to hone the solutions. Right now the attack scenarios are considered equally likely. With more knowledge of the contested environment and adversary, the set of attacks and likelihood that they occur could be modified to more accurately reflect a real-world scenario.

6.3 Closing Thoughts

Routing in contested environments is a complicated process that does not have a "one-size fits all" solution. This being said, models like the Continuous Time Model with Chance Constraints and the Two-Stage Discrete Time Model with Random Attack Scenarios show potential for a new way of generating route plans that are both optimal and risk-averse.

Appendices

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