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THE CONSTRUCTION OF SOFT SETS FROM FUZZY SUBSETS

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ABSTRACT

Molodtsov introduced the concept of soft sets formed from fuzzy subsets in 1999. The soft set formed from a fuzzy subset is a particular form of a soft set on its parameter set. On a soft set formed from a fuzzy subset, the parameter used is the image of a fuzzy subset which is then mapped to the collection of all subsets of a universal set. This research explains the construction of soft sets formed from fuzzy subsets. We provide the sufficient condition that a soft set formed from a fuzzy subset is a subset of another soft set. Also, give some properties of the soft sets formed from a fuzzy subset related to complement and operations concepts in soft sets

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1. INTRODUCTION

A set is a well-defined collection of objects that can be distinguished as which are members and which are not members. Members of a set are objects that have certain similarities [1]. The level of similarity of objects is relative, so there is uncertainty in grouping these objects. This case is the basis for grouping objects using varying degrees of similarity. The grouping process, which tends to show varying levels of similarity, makes it difficult to group using concepts from classical set theory. Therefore, a more relevant theory is needed to help solve the problem [2].

One of the theories that can assist in solving problems regarding uncertainty is the fuzzy subset theory. The fuzzy subset theory written by Zadeh [3], [4] solves uncertainty caused by a set's unclear properties and character. In the fuzzy subset theory, there is a value of membership or degree of membership indicating an object's membership level to a particular group.

Several years after using the concept of fuzzy subsets, Molodtsov [2] described the weaknesses in the fuzzy subset theory. According to Molodtsov, fuzzy subset theory still has difficulties determining the membership function in each case caused by the inadequacy of the parameterization tools in theory. Therefore, to overcome this, Molodtsov (1999) [5] introduced a new theory known as the soft set theory. Molodtsov explained that a soft set is a collection of parameterization subsets in a universe set.

In 1999 Molodtsov also introduced the concept of a soft set formed from a fuzzy subset which was later clarified by Aktaş & Çağman [6]. Let $\mu: U \to [0,1]$ be a fuzzy set over a set U and μ_α be an α-level subset. We can defined a soft set (F_μ, A) with $F_\mu: A \to P(U)$, $\alpha \mapsto \mu_\alpha$ and $A = Im(\mu)$.

As science progressed, fuzzy subsets and soft sets developed into new concepts and applications in decision problems. Maji *et al.* [7] defined a hybrid model called fuzzy soft sets. This new model combines fuzzy with soft sets and generalized soft sets. Irfan Ali and Shabir [8] developed the theory. They give De Morgan-type laws, as given in the Maji et al. paper, which are generally untrue. They also provide some new definitions and operations for fuzzy soft sets, making it very easy to prove the existence of De Morgan type laws in fuzzy soft sets. To address decision-making problems based on fuzzy soft sets, Feng *et al.* introduced the concept of level soft sets of fuzzy soft sets. They initiated an adjustable decision-making scheme using fuzzy soft sets [9], followed by a generalized soft fuzzy set [10] and its application to the student ranking system [11].

Research on soft sets has also been developed by integrating other fields, including algebra, and its applications in the real world. In the field of algebra, they include soft matrices introduced by Çağman and Enginoğlu [12], soft groups introduced by Aktaş and Cağman [6], soft semiring by Feng *et al.* [13], soft rings by Acar *et al.* [14], and soft modules by Sun *et al.*[15], which until now continue to develop, can see in [16], [17], [18], [19], [20], [21], [22], and [23]. Some applications of soft sets in the real world can see in [24], [25], [26], [27], [28], [29], [30], [31], [32] and much more.

Based on the explanation of the concept of fuzzy subsets, soft sets, and soft sets formed from fuzzy subsets, several questions were raised, how is the construction of a soft set formed from a fuzzy subset? Then, the subset and complement properties that apply to soft sets also apply to soft sets formed from fuzzy subsets. Furthermore, to prove the properties of the intersection, union, OR, and AND operations of two soft sets formed from fuzzy subsets.

2. RESEARCH METHODS

In this research, the steps used are as follows.

1. Explaining the definition of fuzzy subsets and α −cut on fuzzy subsets.
2. Explaining the definition of soft sets.
3. Proving and giving examples to the propositions regarding soft sets formed from fuzzy subsets.
4. Proving and giving examples to the propositions related to the properties of subsets, complements, and the operations of intersection, union, OR, and AND on soft sets formed from fuzzy subsets.
5. Writing a conclusion.

2.1 Fuzzy Subsets

A function from an empty set U to interval $[0,1]$ is called a subset fuzzy of U that defined as follows.

Definition 1. [3] Let U be a non-empty set. A fuzzy subset μ of U is defined as a mapping

$$\mu: U \rightarrow [0,1].$$

The function μ is called a fuzzy subset of U and can be expressed by $\mu = \{(u, \mu(u)) | u \in U\}$, where $\mu(u)$ is the membership degree of $u \in U$ for a fuzzy subset μ. The collection of all fuzzy subsets of U denoted by $\mathcal{F}(U)$, i.e., $\mathcal{F}(U) = \{\mu | \mu: U \rightarrow [0,1]\}$.

Analog with the concept of set, in the concept of fuzzy subset there are the concepts of fuzzy subsubset and fuzzy complement, and the concept of intersection and union operations that given as follows.

Definition 2. [3] Let $\mu, \nu \in \mathcal{F}(U)$. If $\mu(u) \leq \nu(u)$ for all $u \in U$, then μ is contained in ν and can be written $\mu \subseteq \nu$ $(\nu \supseteq \mu)$. *If* $\mu \subseteq \nu$ *and* $\mu \supseteq \nu$ then μ is equal to ν and can be written $\mu = \nu$.

Definition 3. [3] Let μ be a fuzzy subset of U. The complement of μ is the fuzzy subset μ^c, where

$$\mu^c(u) = 1 - \mu(u)$$

Definition 4. [3] Let $\mu, \nu \in \mathcal{F}(U)$. The intersection and union of μ and ν is the fuzzy subsets $\mu \cap \nu$ and $\mu \cup \nu$, where

$$(\mu \cap \nu)(u) = \min\{\mu(u), \nu(u)\} = \mu(u) \wedge \nu(u)$$
$$(\mu \cup \nu)(u) = \max\{\mu(u), \nu(u)\} = \mu(u) \vee \nu(u)$$

2.2 α −Cut on Fuzzy Subsets

Let $\boldsymbol{\mu}$ be any fuzzy subset of $\boldsymbol{U}$. The subset of $\boldsymbol{U}$, that the membership degree is more or equal to any $\boldsymbol{\alpha} \in [\mathbf{0}, \mathbf{1}]$ is called $\boldsymbol{\alpha}$-level subset, defined as following.

Definition 5. [3] Let $\mu \in \mathcal{F}(U)$. For all $\alpha \in [0,1]$ can be defined α-level subset $(\alpha - cut)$ of μ, which is denoted by μ_α, i.e.

$$\mu_\alpha = \{u | u \in U, \mu(u) \geq \alpha\}.$$

There is some properties about α-level subset connecting with properties of subset fuzzy, given as following theorem.

Theorem 1. [33] *Let* $\mu, \nu \in \mathcal{F}(U)$, *for all* $\alpha, \beta \in [0,1]$ *the following properties hold true*

1) $\mu \subseteq \nu \Rightarrow \mu_\alpha \subseteq \nu_\alpha$
2) $\alpha \leq \beta \Rightarrow \mu_\beta \subseteq \mu_\alpha$
3) $\alpha = \beta \Rightarrow \mu_\beta = \mu_\alpha$
4) $\mu = \nu \Leftrightarrow \mu_\alpha = \nu_\alpha$.

Theorem 2. [34] *Let* $\mu, \nu \in \mathcal{F}(U)$, *for all* $\alpha \in [0,1]$, *the following properties hold true*

1) $(\mu \cup \nu)_\alpha = \mu_\alpha \cup \nu_\alpha$
2) $(\mu \cap \nu)_\alpha = \mu_\alpha \cap \nu_\alpha$.

2.3 Soft Sets

In 1999, Molodsov introduced the concept of a soft set which is a pair consisting of a function from a set of parameters $\boldsymbol{A}$ to the power set of a universal set $\boldsymbol{U}$ and $\boldsymbol{A}$. The formal definition of a soft set is provided below

Definition 6. [2] Let U be a universal set and A be a set of parameters. A pair (F, A) is called a soft set over U where F is a mapping given by

$$F: A \rightarrow P(U)$$

For $a \in A$, $F(a)$ may be considered as the set a-approximate elements of the soft set (F, A). A soft set over U can be expressed by $(F, A) = \{(a, F(a)) | a \in A\}$.

The connection between two soft sets is given by this following definition.

Definition 7. [35] Let (F, A) and (G, B) be two soft sets over U. Then (F, A) is called a soft subset of (G, B) denoted by $(F, A) \mathrel{\widehat{\subseteq}} (G, B)$, if

1) $A \subseteq B$ and

2) for all $a \in A$, $F(a) \subseteq G(a)$.

Definition 8. [35] Let (F, A) and (G, B) be two soft sets over U. Then (F, A) and (G, B) are said to be equal, denoted by $(F, A) = (G, B)$, if $(F, A) \widehat{\subseteq} (G, B)$ and $(G, B) \widehat{\subseteq} (F, A)$.

The concept of soft sets includes definitions for complement and relative complement and operations such as intersection, union, *OR* and *AND* between two soft sets. These definitions are presented in a specific order.

Definition 9. [36] Let $A = \{a_1, a_2, a_3, \dots, a_n\}$ be a set of parameters. The complement of A denoted by $\neg A = \{\neg a_1, \neg a_2, \neg a_3, \dots, \neg a_n\}$ where $\neg a_i$ is "not a_i" and $\neg(\neg a_i) = a_i$, for all $i = 1,2, \dots, n$.

Definition 10. [37] The relative complement of a soft set (F, A) is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, A)$, where $F^C: A \to P(U)$ is a mapping given by $F^C(a) = U - F(a)$, for all $a \in A$.

Definition 11. [13] Bi-intersection of two soft sets (F, A) and (G, B) over U is defined to be the soft set (H, C) where $C = A \cap B$ and for all $x \in C, H(x) = F(x) \cap G(x)$. The bi-intersection of (F, A) and (G, B) is denoted by $(F, A) \widehat{\cap} (G, B) = (H, C)$.

Definition 12. [36] Let (F, A) and (G, B) be two soft sets over U. The union of (F, A) and (G, B) is defined to be a soft set (I, D), where $D = A \cup B$ and for all $x \in D$ satisfying the following conditions

$$I(x) = \begin{cases} F(x), & x \in A - B \\ G(x), & x \in B - A \\ F(x) \cup G(x), & x \in A \cap B. \end{cases}$$

The union of (F, A) and (G, B) is denoted by $(F, A) \sqcup (G, B) = (I, D)$.

Definition 13. [36] Let (F, A) and (G, B) be two soft sets over U. Operation *OR* from (F, A) and (G, B), denoted by $(F, A) \widehat{\vee} (G, B)$, is defined to be a soft set $(H, A \times B)$, where $H(a, b) = F(a) \cup G(b)$, for all $(a, b) \in A \times B$.

Definition 14. [36] Let (F, A) and (G, B) be two soft sets over U. Operation *AND* from (F, A) and (G, B), denoted by $(F, A) \widehat{\wedge} (G, B)$, is defined to be a soft set $(I, A \times B)$, where $I(a, b) = F(a) \cap G(b)$, for all $(a, b) \in A \times B$.

3. RESULT AND DISCUSSION

This section explains that a soft set can be formed from the fuzzy subsets, with the parameter set being the level subset obtained from the fuzzy subset. This section also provides the properties of soft sets formed by fuzzy sets related to subsets and operations concepts in the soft sets.

3.1 The Construction of Soft Sets from Fuzzy Subsets

According to the following proposition, a soft set can be created from a fuzzy subset, where a parameter of the soft set represents the image of the fuzzy subset.

Proposition 1. *Let U be a universal set, $\mu: U \to [0,1]$ be a fuzzy set where $A = Im(\mu) \subseteq [0,1]$. A pair (F_μ, A) is a soft set where $F_\mu: A \to P(U)$ which is defined as $F_\mu(\alpha) = \mu_\alpha$, for all $\alpha \in A$. Furthermore, soft set (F_μ, A) is called a soft set over U, formed from a fuzzy subset μ.*

Proof. Let α_1, α_2 be any element of A where $\alpha_1 = \alpha_2$, so based on **Theorem 1** and the definition of F_μ, it is obtained that $\mu_{\alpha_1} = \mu_{\alpha_2} \Leftrightarrow F_\mu(\alpha_1) = F_\mu(\alpha_2)$, thus $F_\mu: A \to P(U)$ is well defined. Therefore, $F_\mu: A \to P(U)$ is a function. In other words, based on **Definition 6**, it was proved that (F_μ, A) is a soft set over the universal set U. ■

Example 1. Let $\mu = \{(u_1, 0.4), (u_2, 0.8), (u_3, 0.1), (u_4, 0.4), (u_5, 0.7)\}$. Thus, the soft set (F_μ, A) was obtained as follows.

$$(F_\mu, A) = \left\{ \begin{matrix} (0.1, \{u_1, u_2, u_3, u_4, u_5\}), (0.4, \{u_1, u_2, u_4, u_5\}), \\ (0.7, \{u_2, u_5\}), (0.8, \{u_2\}) \end{matrix} \right\}$$

3.2 Properties of Subsets and Complements of Soft Sets Formed from Fuzzy Subsets

In this section, we will talk about the properties of subsets and complements that are applicable to soft sets made from fuzzy subsets. These properties are presented in the following proposition.

Proposition 2. *Let (F_μ, A) and (G_ν, B) be two soft sets formed from fuzzy subsets μ and ν, respectively, over a universal set U. The soft set (F_μ, A) is a subset of (G_ν, B) if $A \subseteq B$ and $\mu \subseteq \nu$.*

Proof. It is known that $A \subseteq B$ and based on **Theorem 1**, if $\mu \subseteq \nu$, then $\mu_\alpha \subseteq \nu_\alpha$. In other words, based on **Proposition 1**, it is obtained that $F_\mu(\alpha) \subseteq G_\nu(\alpha)$ for all $\alpha \in A$. Therefore, based on **Definition 7**, it is obtained that $(F_\mu, A) \tilde{\subseteq} (G_\nu, B)$. ■

Example 2. Based on Example 1, a fuzzy subset μ is obtained. Next, let the fuzzy subset ν be given by $\nu = \{(u_1, 0.7), (u_2, 0.9), (u_3, 0.1), (u_4, 0.4), (u_5, 0.8)\}$. From Example 1 we have $A = Im(\mu) = \{0.1, 0.4, 0.7, 0.8\}$. Since $B = Im(\nu) = \{0.1, 0.4, 0.7, 0.8, 0.9\}$, it is obtained that $A \subseteq B$ and $\mu(u) \leq \nu(u)$ for all $u \in U$. Furthermore, based on **Definition 2**, it is obtained that $\mu \subseteq \nu$.

Next, it is obtained that $(G_\nu, A) = \left\{\begin{matrix}(0.1, \{u_1, u_2, u_3, u_4, u_5\}), (0.4, \{u_1, u_2, u_4, u_5\}), \\ (0.7, \{u_1, u_2, u_5\}), (0.8, \{u_2, u_5\})\end{matrix}\right\}$

Consequently, $F_\mu(\alpha) \subseteq G_\nu(\alpha)$ for all $\alpha \in A$. Thus, it was obtained that $(F_\mu, A) \tilde{\subseteq} (G_\nu, B)$.

Based on **Definition 10**, it is known that the complement of a soft set (F, A) is defined as (F^C, A). Next, in this research, it is defined that $(F_\mu, A)^C = (F_\mu^C, A)$ where $F_\mu^C(\alpha) = U - F_\mu(\alpha)$ for all $\alpha \in A$. The following proposition states $(F_\mu, A)^C$ as a soft set over a universal set U that is formed from a fuzzy subset μ.

Proposition 3. *If (F_μ, A) is a soft set formed from a fuzzy subset μ over a universal set U, then $(F_\mu, A)^C$ is a soft set formed from a fuzzy subset μ over a universal set U, where $(F_\mu, A)^C = (F_\mu^C, A)$.*

Proof. Let α_1, α_2 be any element of A where $\alpha_1 = \alpha_2$, so based on **Theorem 1** and the definition of F_μ, It is obtained that $\mu_{\alpha_1} = \mu_{\alpha_2} \Leftrightarrow F_\mu(\alpha_1) = F_\mu(\alpha_2)$. Consequently, based on the definition of F_μ^C, it is obtained that $U - F_\mu(\alpha_1) = U - F_\mu(\alpha_2) \Leftrightarrow F_\mu^C(\alpha_1) = F_\mu^C(\alpha_2)$. Therefore, $F_\mu^C : A \to P(U)$ is well-defined. Thus, $F_\mu^C : A \to P(U)$ is a function. In other words, based on **Proposition 1**, it is proved that $(F_\mu, A)^C$ is a soft set formed from a fuzzy subset μ over universe U. ■

Next, the soft set $(F_\mu, A)^C$ is called the complement of a soft set (F_μ, A). Based on **Definition 3,** It is knownthat for any fuzzy subset μ, there is always a complement of μ that is denoted by μ^C. The complement of a soft set formed from a fuzzy subset is not equal to a soft set formed from the complement of a fuzzy subset. The example and proposition below illustrate this concept..

Example 3. Based on Example 1, a fuzzy subset μ is obtained. Therefore, $(F_\mu^C, A) = \left\{\begin{matrix}(0.1, \emptyset), (0.4, \{u_3\}), \\ (0.7, \{u_1, u_3, u_4\}), (0.8, \{u_1, u_3, u_4, u_5\})\end{matrix}\right\}$ and $(F_{\mu^C}, A) = \left\{\begin{matrix}(0.1, \{u_1, u_2, u_3, u_4, u_5\}), (0.4, \{u_1, u_3, u_4\}), \\ (0.7, \{u_3\}), (0.8, \{u_3\})\end{matrix}\right\}$. It is obtained that $(F_\mu^C, A) \neq (F_{\mu^C}, A)$. In other words, $(F_\mu, A)^C \neq (F_{\mu^C}, A)$.

Proposition 4. *If (F_μ, A) is a soft set formed from a fuzzy subset μ over a universal set U, then $(F_\mu, A)^C \neq (F_{\mu^C}, A)$.*

It is known that based on **Definition 9**, for any parameter set A, there is always a complement of A denoted by $\neg A$. Next, if a soft (F_μ, A) exists, then $\neg F_\mu : \neg A \to P(U)$, defined by $\neg F_\mu(\neg\alpha) = F_\mu(1 - \alpha)$ for all $\alpha \in A$, can be formed. It can be shown that $(\neg F_\mu, \neg A)$ is a soft set formed from a fuzzy subset μ over universe U, stated in the following proposition.

Proposition 5. *If (F_μ, A) is a soft set formed from a fuzzy subset μ over a universe U, then $(\neg F_\mu, \neg A)$ is a soft set formed from a fuzzy subset μ over a universe U.*

Proof. Let α_1, α_2 be any element of A where $\alpha_1 = \alpha_2$. Consequently, $1 - \alpha_1 = 1 - \alpha_2$ so based on **Theorem 1** and the definition of $\neg F_\mu$, It is obtained that $\mu_{(1-\alpha_1)} = \mu_{(1-\alpha_2)} \Leftrightarrow \neg F_\mu(\neg\alpha_1) = \neg F_\mu(\neg\alpha_2)$. Therefore,

$\neg F_\mu : \neg A \to P(U)$ is well-defined. Thus, $\neg F_\mu : \neg A \to P(U)$ is a function. In other words, based on **Proposition 1**, it is proved that $(\neg F_\mu, \neg A)$ is a soft set formed from a fuzzy subset μ over universe U. ■

Hereafter, $(\neg F_\mu, \neg A)$ is called the negation of a soft set (F_μ, A).

Proposition 6. *Let (F_μ, A) be a soft set formed from a fuzzy subset μ over a universe U. If $\mu(u) \neq 1-\alpha$ for all $u \in U$ then $\neg F_\mu^C(\neg\alpha) = F_{\mu^C}(\alpha)$ for all $\alpha \in A$.*

Proof. Let α be any element of A. Based on **Proposition 1**, **Proposition 3**, and **Proposition 5**, it is obtained that

$$\begin{aligned}\neg F_\mu^C(\neg\alpha) &= U - \left(\neg F_\mu(\neg\alpha)\right)\\ &= U - F_\mu(1-\alpha)\\ &= U - \mu_{(1-\alpha)}\\ &= U - \{u \in U | \mu(u) \geq 1-\alpha\}\\ &= \{u \in U | \mu(u) < 1-\alpha\}.\end{aligned}$$

On the other hand, based on **Definition 3**, **Proposition 1**, and **Proposition 4**, it is obtained that

$$\begin{aligned}F_{\mu^C}(\alpha) &= (\mu^C)_\alpha\\ &= \{u \in U | \mu^C(u) \geq \alpha\}\\ &= \{u \in U | 1-\mu(u) \geq \alpha\}\\ &= \{u \in U | \mu(u) \leq 1-\alpha\}.\end{aligned}$$

Because $\mu(u) \neq 1-\alpha$ for all $u \in U$, it is obtained that

$$F_{\mu^C}(\alpha) = \{u \in U | \mu(u) < 1-\alpha\}.$$

Thus, it is proven that if $\mu(u) \neq 1-\alpha$ for all $u \in U$ then $\neg F_\mu^C(\neg\alpha) = F_{\mu^C}(\alpha)$ for all $\alpha \in A$. ■

Example 4. Let $\mu = \{(u_1, 0.5), (u_2, 0.7), (u_3, 0.2), (u_4, 1), (u_5, 0.9), (u_6, 0)\}$.

Therefore, $(\neg F_\mu^C, \neg A) = \left\{\begin{matrix}(\neg 0, \{u_1, u_2, u_3, u_5, u_6\}), (\neg 0.2, \{u_1, u_2, u_3, u_6\}), (\neg 0.5, \{u_3, u_6\}),\\ (\neg 0.7, \{u_3, u_6\}), (\neg 0.9, \{u_6\}), (\neg 1, \emptyset)\end{matrix}\right\}$.

On the other hand, because $\mu(u) \neq 1-\alpha$ for all $u \in U$, then it is obtained that

$$(F_{\mu^C}, A) = \left\{\begin{matrix}(0, \{u_1, u_2, u_3, u_5, u_6\}), (0.2, \{u_1, u_2, u_3, u_6\}), (0.5, \{u_3, u_6\}),\\ (0.7, \{u_3, u_6\}), (0.9, \{u_6\}), (1, \emptyset)\end{matrix}\right\}.$$

Consequently, $\neg F_\mu^C(\neg\alpha) = F_{\mu^C}(\alpha)$ for all $\alpha \in A$.

3.3 Operations on Soft Sets Formed from Fuzzy Subsets

The operations presented in this section consisted of intersection, union, OR, and AND operations, which apply to soft sets formed from fuzzy subsets. These operations are presented in the following proposition.

Proposition 7. *Let (F_μ, A) and (G_ν, B) be two soft sets formed from fuzzy subsets μ and ν, respectively, over a universal set U. The soft set $(H_{\mu\cap\nu}, C) \,\tilde{\subseteq}\, (F_\mu, A) \,\tilde{\cap}\, (G_\nu, B)$ if $Im(\mu) \subseteq Im(\nu)$ or $Im(\nu) \subseteq Im(\mu)$ where $C = Im(\mu \cap \nu)$.*

Proof. Based on **Definition 7**, To prove that $(H_{\mu\cap\nu}, C) \,\tilde{\subseteq}\, (F_\mu, A) \,\tilde{\cap}\, (G_\nu, B)$, it must be proven that $C \subseteq A \cap B$ and $H_{\mu\cap\nu}(\gamma) \subseteq (F_\mu \cap G_\nu)(\gamma)$ for all $\gamma \in C$.

(i) Let x be any element of $\mathrm{Im}(\mu \cap \nu)$ so there exists $u \in U$ such that $x = (\mu \cap \nu)(u)$, then based on **Definition 4**, it is obtained that $x = \min\{\mu(u), \nu(u)\}$.

1) Assuming that $\mu(u) \leq \nu(u)$, it is obtained that $x = \mu(u)$ which means $x \in \mathrm{Im}(\mu)$. Next, as $\mathrm{Im}(\mu) \subseteq \mathrm{Im}(\nu)$ then $x \in \mathrm{Im}(\nu)$. Hence, it is obtained that $x \in \mathrm{Im}(\mu)$ and $x \in \mathrm{Im}(\nu)$ in other words $x \in \mathrm{Im}(\mu) \cap \mathrm{Im}(\nu)$. Consequently, $\mathrm{Im}(\mu \cap \nu) \subseteq \mathrm{Im}(\mu) \cap \mathrm{Im}(\nu)$.

2) Assuming that $\nu(u) \leq \mu(u)$, it is obtained that $x = \nu(u)$ which means $x \in \mathrm{Im}(\nu)$. Next, as $\mathrm{Im}(\nu) \subseteq \mathrm{Im}(\mu)$ then $x \in \mathrm{Im}(\mu)$. Hence, it is obtained that $x \in \mathrm{Im}(\mu)$ and $x \in \mathrm{Im}(\nu)$ in other words $x \in \mathrm{Im}(\mu) \cap \mathrm{Im}(\nu)$. Consequently, $\mathrm{Im}(\mu \cap \nu) \subseteq \mathrm{Im}(\mu) \cap \mathrm{Im}(\nu)$.

From 1) and 2), it is obtained that $\mathrm{Im}(\mu \cap \nu) \subseteq \mathrm{Im}(\mu) \cap \mathrm{Im}(\nu)$, so based on **Proposition 1**, it is proven that $C \subseteq A \cap B$.

(ii) Let γ be any element of C. Based on **Theorem 2**, **Definition 11**, and **Proposition 1**, it is obtained that

$$\begin{aligned} H_{\mu\cap\nu}(\gamma) &= (\mu \cap \nu)_\gamma \\ &= \mu_\gamma \cap \nu_\gamma \\ &= F_\mu(\gamma) \cap G_\nu(\gamma) \\ &= (F_\mu \cap G_\nu)(\gamma) \end{aligned}$$

It is obtained that $H_{\mu\cap\nu}(\gamma) = (F_\mu \cap G_\nu)(\gamma)$. Thus, it is proven that $H_{\mu\cap\nu}(\gamma) \subseteq (F_\mu \cap G_\nu)(\gamma)$ for all $\gamma \in C$.

From (i) and (ii), it is obtained that $(H_{\mu\cap\nu}, C) \tilde{\subseteq} (F_\mu, A) \tilde{\cap} (G_\nu, B)$.■

Example 5. Let $\mu = \{(u_1, 0.8), (u_2, 0.1), (u_3, 0.7), (u_4, 0.4), (u_5, 0.7)\}$ and $\nu = \{(u_1, 0.7), (u_2, 0.9), (u_3, 0.1), (u_4, 0.4), (u_5, 0.8)\}$. Therefore, $\text{Im}(\mu) \subseteq \text{Im}(\nu)$ and $A \cap B = \{0.1, 0.4, 0.7, 0.8\}$. On the other hand, based on **Definition 4**, it is obtained that $\mu \cap \nu = \{(u_1, 0.7), (u_2, 0.1), (u_3, 0.1), (u_4, 0.4), (u_5, 0.7)\}$ such that $C = Im(\mu \cap \nu) = \{0.1, 0.4, 0.7\}$. Thus, $C \subseteq A \cap B$. Furthermore, it is obtained that $(H_{\mu\cap\nu}, C) = \left\{ \begin{matrix} (0.1, \{u_1, u_2, u_3, u_4, u_5\}), (0.4, \{u_1, u_4, u_5\}), \\ (0.7, \{u_1, u_5\}) \end{matrix} \right\}$ and $((F_\mu \cap G_\nu), C) = \left\{ \begin{matrix} (0.1, \{u_1, u_2, u_3, u_4, u_5\}), (0.4, \{u_1, u_4, u_5\}), \\ (0.7, \{u_1, u_5\}) \end{matrix} \right\}$.
It is obtained that $H_{\mu\cap\nu}(\gamma) = (F_\mu \cap G_\nu)(\gamma)$. Consequently, $H_{\mu\cap\nu}(\gamma) \subseteq (F_\mu \cap G_\nu)(\gamma)$ for all $\gamma \in C$. Thus, $(H_{\mu\cap\nu}, C) \tilde{\subseteq} (F_\mu, A) \tilde{\cap} (G_\nu, B)$.

Proposition 8. *Let (F_μ, A) and (G_ν, B) be two soft sets formed from fuzzy subsets μ and ν, respectively, over a universal set U. The soft set $(I_{\mu\cup\nu}, D) \tilde{\subseteq} (F_\mu, A) \tilde{\cup} (G_\nu, B)$ if $\mu_\gamma \subseteq \nu_\gamma$ when $\gamma \in B - A$ and $\nu_\gamma \subseteq \mu_\gamma$ when $\gamma \in A - B$ where $D = Im(\mu \cup \nu)$.*

Proof. Based on **Definition 7**, to prove that $(I_{\mu\cup\nu}, D) \tilde{\subseteq} (F_\mu, A) \tilde{\cup} (G_\nu, B)$, it must be proven that $D \subseteq A \cup B$ and $I_{\mu\cup\nu}(\gamma) \subseteq (F_\mu \cup G_\nu)(\gamma)$ for all $\gamma \in D$.

(i) Let x be any element of $\text{Im}(\mu \cup \nu)$ so there exists $u \in U$ such that $x = (\mu \cup \nu)(u)$, then based on **Definition 4**, it is obtained that $x = \max\{\mu(u), \nu(u)\}$.

1) Assuming that $\mu(u) \geq \nu(u)$, it is obtained that $x = \mu(u)$ which means $x \in \text{Im}(\mu)$.

2) Assuming that $\nu(u) \geq \mu(u)$, it is obtained that $x = \nu(u)$ which means $x \in \text{Im}(\nu)$.

From 1) and 2), it is obtained that $x \in \text{Im}(\mu)$ or $x \in \text{Im}(\nu)$ in other words $x \in \text{Im}(\mu) \cup \text{Im}(\nu)$. Consequently, $\text{Im}(\mu \cup \nu) \subseteq \text{Im}(\mu) \cup \text{Im}(\nu)$, so based on **Proposition 1**, it is proven that $D \subseteq A \cup B$.

(ii) Based on **Definition 12**, for all $\gamma \in D$ holds

$$(F_\mu \cup G_\nu)(\gamma) = \begin{cases} F_\mu(\gamma), & \gamma \in A - B \\ G_\nu(\gamma), & \gamma \in B - A \\ F_\mu(\gamma) \cup G_\nu(\gamma), & \gamma \in A \cap B. \end{cases}$$

1) If $\gamma \in A - B$
Let γ be any element of $A - B$ and if $\gamma \in A - B$ then $\nu_\gamma \subseteq \mu_\gamma$. Then, based on **Theorem 2** and **Proposition 1**, it is obtained that

$$\begin{aligned} I_{\mu\cup\nu}(\gamma) &= (\mu \cup \nu)_\gamma \\ &= \mu_\gamma \cup \nu_\gamma \\ &= \mu_\gamma \\ &= F_\mu(\gamma) \end{aligned}$$

It is obtained that $I_{\mu\cup\nu}(\gamma) = F_\mu(\gamma)$ when $\gamma \in A - B$.

2) If $\gamma \in B - A$
Let γ be any element of $B - A$ and if $\gamma \in B - A$ then $\mu_\gamma \subseteq \nu_\gamma$. Then, based on **Theorem 2** and **Proposition 1**, it is obtained that

$$\begin{aligned} I_{\mu\cup\nu}(\gamma) &= (\mu \cup \nu)_\gamma \\ &= \mu_\gamma \cup \nu_\gamma \\ &= \nu_\gamma \\ &= G_\nu(\gamma) \end{aligned}$$

It is obtained that $I_{\mu\cup\nu}(\gamma) = G_\nu(\gamma)$ when $\gamma \in B - A$.

3) If $\gamma \in A \cap B$

Let γ be any element of $A \cap B$. Based on **Theorem 2** and **Proposition 1**, it is obtained that

$$\begin{aligned} I_{\mu\cup\nu}(\gamma) &= (\mu \cup \nu)_\gamma \\ &= \mu_\gamma \cup \nu_\gamma \\ &= F_\mu(\gamma) \cup G_\nu(\gamma) \end{aligned}$$

It is obtained that $I_{\mu\cup\nu}(\gamma) = F_\mu(\gamma) \cup G_\nu(\gamma)$ when $\gamma \in A \cap B$.

From 1), 2), and 3), it is obtained that $I_{\mu\cup\nu}(\gamma) = \left(F_\mu \cap G_\nu\right)(\gamma)$. Thus, it is proven that $I_{\mu\cup\nu}(\gamma) \subseteq \left(F_\mu \cap G_\nu\right)(\gamma)$ for all $\gamma \in D$.

From (i) and (ii), it is obtained that $\left(I_{\mu\cup\nu}, D\right) \widetilde{\subseteq} \left(F_\mu, A\right) \widetilde{\cup} (G_\nu, B)$. ■

Example 6. Let $\mu = \{(u_1, 0), (u_2, 0.2), (u_3, 0.4), (u_4, 0.9)\}$ and $\nu = \{(u_1, 0), (u_2, 0.2), (u_3, 0.6), (u_4, 0.8)\}$. Therefore, $A \cup B = \{0, 0.2, 0.4, 0.6, 0.8, 0.9\}$. On the other hand, based on **Definition 4**, it is obtained that $\mu \cup \nu = \{(u_1, 0), (u_2, 0.2), (u_3, 0.6), (u_4, 0.9)\}$ such that $D = Im(\mu \cup \nu) = \{0, 0.2, 0.6, 0.9\}$. Thus, $D \subseteq A \cup B$. Next, it is obtained that $\left(I_{\mu\cup\nu}, D\right) = \begin{Bmatrix} (0, \{u_1, u_2, u_3, u_4\}), (0.2, \{u_2, u_3, u_4\}), \\ (0.6, \{u_3, u_4\}), (0.9, \{u_4\}) \end{Bmatrix}$ and $\left(\left(F_\mu \cup G_\nu\right), D\right) = \begin{Bmatrix} (0, \{u_1, u_2, u_3, u_4\}), (0.2, \{u_2, u_3, u_4\}), \\ (0.6, \{u_3, u_4\}), (0.9, \{u_4\}) \end{Bmatrix}$.

It is obtained that $\nu_\gamma \subseteq \mu_\gamma$ when $\gamma \in A - B$ and $\mu_\gamma \subseteq \nu_\gamma$ when $\gamma \in B - A$. Furthermore, it is obtained that $I_{\mu\cup\nu}(\gamma) = \left(F_\mu \cup G_\nu\right)(\gamma)$. Consequently, $I_{\mu\cup\nu}(\gamma) \subseteq \left(F_\mu \cup G_\nu\right)(\gamma)$ for all $\gamma \in D$. Thus, it is obtained that $\left(I_{\mu\cup\nu}, D\right) \widetilde{\subseteq} \left(F_\mu, A\right) \widetilde{\cup} (G_\nu, B)$.

Proposition 9. *If $\left(F_\mu, A\right)$ and (G_ν, B) are two soft set respectively formed from fuzzy subsets μ and ν over the universal set U then (H, C) is a soft set formed from the OR operation of $\left(F_\mu, A\right)$ and (G_ν, B) defined as*

$$H: C \to P(U)$$

where $C = A \times B$ and $H(\alpha, \beta) = F_\mu(\alpha) \cup G_\nu(\beta)$ for all $(\alpha, \beta) \in A \times B$. Furthermore, if $(I_{(\mu\cup\nu)}, D)$ is a soft set formed from the union operation on fuzzy subsets μ and ν then $I_{(\mu\cup\nu)}(\gamma) \subseteq \left(F_\mu(\alpha) \cup G_\nu(\beta)\right)$ with the sufficient condition $\gamma \geq max\{\alpha, \beta\}$ for all $\alpha \in A, \beta \in B$, and $\gamma \in D$.

Proof. Let u be any element of $(\mu \cup \nu)_\gamma$, it means $u \in \{u \in U | (\mu \cup \nu) \geq \gamma\}$, then based on the definition of $(\mu \cup \nu)(u)$ it is obtained that $u \in \{u \in U | \max\{\mu(u), \nu(u)\} \geq \gamma\}$.

a) Assuming that $\mu(u) \geq \nu(u)$, it is obtained that $u \in \{u \in U | \mu(u) \geq \gamma\}$, so that $\mu(u) \geq \gamma$. In other words, $u \in \mu_\gamma$.

b) Assuming that $\nu(u) \geq \mu(u)$, it is obtained that $u \in \{u \in U | \nu(u) \geq \gamma\}$, so that $\nu(u) \geq \gamma$. In other words, $u \in \nu_\gamma$.

From a) and b), it is obtained that $u \in \mu_\gamma$ or $u \in \nu_\gamma$, so that $u \in \mu_\gamma \cup \nu_\gamma$. Furthermore, because $\gamma \geq \max\{\alpha, \beta\}$ then $\gamma \geq \alpha$ and $\gamma \geq \beta$, so that $u \in \mu_\alpha \cup \nu_\beta$. In other words, $(\mu \cup \nu)_\gamma \subseteq \mu_\alpha \cup \nu_\beta$. Thus, it is proven that $I_{(\mu\cup\nu)}(\gamma) \subseteq \left(F_\mu(\alpha) \cup G_\nu(\beta)\right)$.■

Hence, it is obtained that the definition of OR operation of $\left(F_\mu, A\right)$ and (G_ν, B) is

$$\left(F_\mu, A\right) \widetilde{\vee} (G_\nu, B) = (H, A \times B),$$

where $H(\alpha, \beta) = F_\mu(\alpha) \cup G_\nu(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Example 7. Let $\mu = \{(u_1, 0.2), (u_2, 0.8), (u_3, 0.5)\}$ and $\nu = \{(u_1, 0.8), (u_2, 0.3), (u_3, 0.2)\}$.

It is obtained that

$$\left(F_\mu, A\right) = \{(0.2, \{u_1, u_2, u_3\}), (0.5, \{u_2, u_3\}), (0.8, \{u_2\})\},$$

$$(G_\nu, B) = \{(0.2, \{u_1, u_2, u_3\}), (0.3, \{u_1, u_2\}), (0.8, \{u_1\})\},$$

$D = \text{Im}\,(\mu \cup \nu) = \{0.5, 0.8\}$, and $\left(I_{(\mu\cup\nu)}, D\right) = \{(0.5, \{u_1, u_2, u_3\}), (0.8, \{u_1, u_2\})\}$

Thus, when $\gamma \geq \max\{\alpha, \beta\}$, obtained that $I_{(\mu\cup\nu)}(\gamma) \subseteq \left(F_\mu(\alpha) \cup G_\nu(\beta)\right)$ for all $\alpha \in A,\ \beta \in B$, and $\gamma \in D$.

Proposition 10. *If $\left(F_\mu, A\right)$ and (G_ν, B) are two soft sets respectively formed from fuzzy subsets μ and ν over the universal set U then (J, C) is a soft set formed from the AND operation of $\left(F_\mu, A\right)$ and (G_ν, B) defined as*

$$J: C \to P(U)$$

where $C = A \times B$ and $J(\alpha, \beta) = F_\mu(\alpha) \cap G_\nu(\beta)$ for all $(\alpha, \beta) \in A \times B$. Furthermore, if $(K_{(\mu\cap\nu)}, D)$ is a soft set formed from the intersection operation on fuzzy subsets μ and ν then $\left(F_\mu(\alpha) \cap G_\nu(\beta)\right) \subseteq K_{(\mu\cap\nu)}(\gamma)$ with the sufficient condition $\gamma \leq min\{\alpha, \beta\}$ for all $\alpha \in A, \beta \in B$, and $\gamma \in D$.

Proof. Let u be any element of $\mu_\alpha \cap \nu_\beta$, it means $u \in \mu_\alpha$ and $u \in \nu_\beta$. Furthermore, because $\gamma \leq \min\{\alpha, \beta\}$, it is obtained that $\gamma \leq \alpha$ and $\gamma \leq \beta$, so that $u \in \mu_\gamma$ and $u \in \nu_\gamma$. Then based on **Definition 5**, $\mu(u) \geq \gamma$ and $\nu(u) \geq \gamma$ are obtained, thus $u \in \{u \in U | \max\{\mu(u), \nu(u)\} \geq \gamma\}$, based on the definition of $(\mu \cap \nu)(u)$, it is obtained that $u \in \{u \in U | (\mu \cap \nu)(u) \geq \gamma\}$, so that $u \in (\mu \cap \nu)_\gamma$. In other words, $\mu_\alpha \cap \nu_\beta \subseteq (\mu \cap \nu)_\gamma$. Thus, it is proven that $\left(F_\mu(\alpha) \cap G_\nu(\beta)\right) \subseteq K_{(\mu\cap\nu)}(\gamma)$.■

Hence, it is obtained that the definition of *AND* operation of (F_μ, A) and (G_ν, B) is

$$(F_\mu, A) \tilde{\wedge} (G_\nu, B) = (J, A \times B),$$

where $J(\alpha, \beta) = F_\mu(\alpha) \cap G_\nu(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Example 8. Based on Example 7, It is obtained $(F_\mu, A), (G_\nu, B)$, $D = \text{Im}\,(\mu \cap \nu) = \{0.2, 0.3\}$, and $(K_{(\mu\cap\nu)}, D) = \{(0.2, \{u_1, u_2, u_3\}), (0.3, \{u_2\})\}$

Thus, when $\gamma \leq \min\{\alpha, \beta\}$, obtained that $\left(F_\mu(\alpha) \cap G_\nu(\beta)\right) \subseteq K_{(\mu\cap\nu)}(\gamma)$ for all $\alpha \in A$, $\beta \in B$, and $\gamma \in D$.

4. CONCLUSIONS

Based on the result and discussion, it is obtained that every fuzzy subset can be formed as a soft set, with the parameter set being the image of that fuzzy subset. The sufficient condition for a soft set formed from the fuzzy subset μ is a subset of the soft set formed from the fuzzy subset ν over the same universal set if μ is a fuzzy subsubset of ν. Furthermore, the complement of a soft set formed from a fuzzy subset is also a soft set formed from a fuzzy subset. If $(K_{(\mu\cap\nu)}, Im(\mu \cap \nu))$ and $(I_{(\mu\cup\nu)}, Im(\mu \cup \nu))$ are soft sets formed from the intersection and union operations on fuzzy subsets μ and ν, respectively, then $\left(F_\mu(\alpha) \cap G_\nu(\beta)\right) \subseteq K_{(\mu\cap\nu)}(\gamma)$ with the sufficient condition $\gamma \leq \min\{\alpha, \beta\}$ *for all* $\alpha \in Im(\mu), \beta \in Im(\nu)$, and $\gamma \in Im(\mu \cap \nu)$. If $\gamma \geq \max\{\alpha, \beta\}$ for all $\alpha \in Im(\mu), \beta \in Im(\nu)$, and $\gamma \in Im(\mu \cup \nu)$ then $I_{(\mu\cup\nu)}(\gamma) \subseteq \left(F_\mu(\alpha) \cup G_\nu(\beta)\right)$.

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