Diagnosing and Remediating Mathematical Common Student Errors in e-Assessment Questions (A Case Study)

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Abstract

Mistakes and misconceptions in mathematics are as common as those in any real life situation. The errors frequently made by a myriad of students, known as Common Students Errors (CSEs), heavily influence their ability to learn and retain mathematics. Identifying and correcting CSEs on handwritten work is straightforward since teachers have access to the students' intermediate workings. One major drawback of this approach is the time and effort it takes to mark and to get feedback back to the individual student.

In recent years, e-Assessments have become a pivotal method of providing mathematical assessments in education. However, there is increasing concern that e-Assessment cannot act like a human-marker to identify and correct CSEs in mathematical e-Assessment questions. Therefore, how to diagnose and remediate CSEs in e-Assessment questions has long been a question of great interest in research in e-Assessments.

This thesis explores a method to diagnose and remediate CSEs in Engineering Mathematics (EM) e-Assessment questions. Firstly, this study contributes to existing knowledge of mathematical CSEs by systematically gathering and taxonomically compiling CSEs in EM e-Assessment questions and producing an interactive book. Secondly, different features of e-Assessment questions, to capture mathematical CSEs and provide enhanced feedback to correct those CSEs, are developed. Thirdly, new light is shed on our understanding of parameter selection restrictions of some mathematical e-Assessment questions which have at least one CSE. Fourthly, it offers some important insights into students' perceptions on the CSE enhanced feedback by analysing a questionnaire. Finally, it provides strong empirical confirmation that the CSE enhanced feedback has successfully corrected some mathematical CSEs in the majority of students who participated in this study.

The outcomes of this thesis contribute in several ways to our understanding of mathematical CSEs and addressing them in e-Assessment questions, and provides a basis for further research.

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1 Introduction

1.1 Motivation

Making mistakes and having misconceptions in mathematics are as common as those in any real life situations. Students, when answering a mathematical question, may make a mistake in their answer for a variety of reasons. Some of these reasons can be listed as carelessness, random errors, calculation errors, misreading the questions, misinterpretation of symbols or texts, lack of awareness or inability to check the answer given, lack of relevant experience or knowledge related to that mathematical topics or concepts, and as a result of a misconception. Some of these errors are frequently made by a considerable number of students (Rushton, 2014). Those frequently made errors are sometimes referred to as Common Student Errors (CSEs).

When such CSEs are made in handwritten work, in general, the teacher is able to identify the mistake(s) during the marking process and give detailed written feedback on the student's script. However, this approach is time-consuming and needs a lot of teachers' effort and subsequently it creates delays in providing feedback to the individual student.

E-Assessment has become a standard method to provide formative and summative assessments in many universities all around the world due to the unprecedented advancement in information technology (Sangwin, 2013). A few of the advantages of e-Assessment are that it can provide instant tailored feedback to help students improve their knowledge and performance, students can access online tests in different geographical locations at different times, and can undertake such tests many times to assess and refine their knowledge. Moreover, it allows educators to identify areas in which more help is needed and then to take necessary actions to address the difficult areas in the subject. It is found that students learn from e-Assessment feedback and enhance their technical knowledge by using it (Gill and Greenhow, 2008). Therefore, e-Assessments which provide effective feedback and select questions based on pedagogic principles should be promoted as a learning resource (Greenhow, 2015).

The use of e-Assessment would be more beneficial if it could detect and report

CSEs, and provide effective and tailored feedback to correct students' misconceptions instantly. This would allow e-Assessments to perform more like a human marker (Walker, Gwynllyw, and Henderson, 2015). However, how to detect CSEs when students answer e-Assessment questions incorrectly, and how to improve the feedback provided to correct CSEs have not been fully studied.

Therefore, the motivation for the research presented in this thesis, the Common Student Error Project (CSE Project) at the University of the West of England, Bristol (UWE Bristol), is to explore a method to detect CSEs and to provide enhanced feedback (EFB) in Engineering Mathematics (EM) e-Assessment questions. The method is developed for EM questions using the Dewis e-Assessment system, developed at UWE Bristol, as the demonstration platform. However, this approach can be adopted easily for other e-Assessment systems and in other contexts and disciplines.

1.2 Aims and objectives

This project is concerned with the further development of the in-house algorithmic Dewis e-Assessment System. The main aim of this research is to develop a computer-assisted method to detect CSEs in EM e-Assessment questions and to provide comprehensive individualised feedback through the Dewis e-Assessment System.

Dewis is a fully algorithmic open source e-Assessment system which was primarily designed for numerate e-Assessments. It facilitates formative and summative e-Assessments in a variety of fields such as Business, Computer Science, Nursing, Software Engineering, Engineering, Mathematics and Statistics (Gwynllyw and Henderson, 2009; Gwynllyw and Henderson, 2012). Due to the algorithmic nature of Dewis, it has the potential to mark the students' answer against alternative answers. This means that it has the potential to detect CSEs in student answers but this was not a feature that had been implemented widely when this research started. This research gap led to the instigation of the CSE Project at UWE Bristol which started in 2017. Primarily, this research is focused on EM e-Assessments delivered by Dewis to first year Engineering students at UWE Bristol.

The main aim of this project is to introduce a method to detect CSEs and to provide tailored feedback in EM e-Assessment questions. The core research questions to be addressed in the CSE Project at UWE Bristol are:

1. How to detect mathematical CSEs in traditional assessments and e-Assess-

ments questions?

2. What mathematical CSEs do first year EM students make in e-Assessment questions?

3. How to improve the e-Assessment feedback in order to address these CSEs?

4. How to measure the effectiveness of detecting and addressing CSEs in mathematical e-Assessment questions?

1.3 Original contributions

The noteworthy original contributions of this thesis, diagnosing and remediating mathematical CSEs in e-Assessment questions, are as follows:

1. Development of the "Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessment (CSE Book)" [see Section 3.2, Section 4.1.1 , Section 5.1, and Appendix C];

2. Development of the features in the Dewis e-Assessment system to detect CSEs and also to improve feedback to address them [see Section 3.3 and Section 5.2];

3. Identification of the necessity of careful selection of parameter values when a question has identified CSEs [see Section 3.3.4 and Section 5.3];

4. Evaluation of the students' perceptions of enhanced e-Assessment feedback addressing mathematical CSEs [see Section 3.4, Section 3.5, Section 4.2 and Section 5.4];

5. Impact and effectiveness of remediating mathematical CSEs in EM e-Assessments questions [see Section 3.6, Section 4.3 and Section 5.5].

1.4 List of Publications

1. Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2020). Using E-Assessment to Address Mathematical Misconceptions in Engineering Students. *International Journal of Information and Education Technology*. 10.5, pp. 356–361 [see Section D.1 in Appendix D];

2. Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2021). Students' Perceptions of Enhanced e-Assessment Feedback Addressing Common Student Errors in Mathematics. *MSOR Connections*. 19.2, pp. 10–27 [see Section D.2 in Appendix D];

3. Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2022). Gathering and Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification. *MSOR Connections*. 20.3, pp. 55–71 [see Section D.3 in Appendix D];

4. Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2022a). *Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)*. [Available from: `https://uwe-repository.worktribe.com/output/9303961`] [see Section C.1 in Appendix C];

5. Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2023). Correct for the wrong reason: why we should know more about Mathematical Common Student Errors in e-Assessment questions. MSOR Connections. [Accepted] [see Section D.4 in Appendix D]

6. Kinnear, G., Jones, I., Sangwin, C., Alarfaj, M., Davies, B., Fearn, S., Foster, C., Heck, A., Henderson, K., Hunt, T., Iannone, P., Kontorovich, I., Larson, N., Lowe, T., Meyer, J.C., O'Shea, A., Rowlett, P., Sikurajapathi, I., Wong, T., 2021. A Collaboratively-Derived Research Agenda for e-Assessment in Undergraduate Mathematics *Int. J. Res. Undergrad. Math. Ed.*. [Available from `https://doi.org/10.1007/s40753-022-00189-6`].

1.5 Declaration

This thesis, and the publications cited in the coming chapters where I am the lead author, are solely my own work.

1.6 Thesis outline

This thesis is organised into six chapters as follows:

Chapter 1 starts by providing a brief introduction to the CSE Project at UWE Bristol by highlighting the motivation, aims and objectives, original contributions, and the publications of the project. Chapter 2 gives a literature review as a way of introducing the reader to the background knowledge regarding the CSE Project at UWE Bristol. In Chapter 3, the methodologies used to diagnose, detect and remediate mathematical CSEs in EM e-Assessment questions is thoroughly described. Chapter 4 analyses the mathematical CSEs gathered in Stage One of the CSE Project to develop a taxonomy of mathematical CSEs in EM e-Assessments questions. Then it analyses a questionnaire data to evaluate the students' perceptions of the enhanced e-Assessment feedback addressing mathematical CSEs in EM e-Assessment questions. This chapter also analyses the Dewis e-Assessment data to find the effectiveness of diagnosing and remediating mathematical CSEs in e-Assessment questions. Chapter 5 discusses the results of this research while highlighting the various main contributions to the knowledge. Finally, Chapter 6 makes conclusive notes regarding the findings of the research, limitations, further work and summary of the whole research. This chapter ends with a brief reflection.

2 Literature Review

A critical understanding of the current state of knowledge in the fields of CSEs, traditional assessment and feedback in education, e-Assessments in education, the Dewis e-Assessment System and the Dewis Question editor is important for this research. Some of the literature review of these topics are discussed in this chapter.

2.1 Introduction to Mathematical Common Student Errors (CSEs)

This section focuses on the literature review of research into mathematical CSEs, some example CSEs in different areas of mathematics, the importance of understanding mathematical misconceptions, different ways of identifying and dealing with misconceptions, as well as taxonomies of mathematical CSEs.

2.1.1 Mathematical CSEs

Mathematical misconceptions are as common as any other phenomena in real life situations. Mathematical concepts could be misunderstood by students even though they are well taught, and these misunderstood concepts could have disastrous effects on learners (McDonald, 2010).

Students may arrive at an incorrect answer when answering a mathematical question due to a variety of reasons. This could be the result of carelessness, random errors, calculation errors, misreading the question, misinterpretation of symbols or texts, lack of awareness or inability to check the answer given, lack of relevant experience or knowledge related to that mathematical topics or concepts and the result of a misconception (Hansan, 2014; Rushton, 2014). Such errors lead to incorrect answers or loss of accuracy marks. Many of these errors are made by just a few students. However, some of these errors are commonly made by a considerable number of students (Rushton, 2014). These frequently made errors are sometimes referred to as CSEs.

Researchers express different opinions about the difference between errors and misconceptions in literature. According to Confrey (1990) the reasons for both errors and misconceptions are the rules and beliefs that students hold. They argue that the difference between errors and misconceptions is that misconceptions are attached to particular theoretical positions. However, Nesher (1987) uses the term 'misconceptions' to describe systematic errors without reference to a theoretical position.

Within the literature, different terms are used to refer to misconceptions or errors. Rees and Barr (1984) use the term 'mal-rule' to refer to an understandable but incorrect implementation of a process resulting from a student's misconception. For example, a classic mal-rule students make is to answer a^2+b^2 when asked to expand $(a+b)^2$. Mal-rules can be classified as manipulative, parsing, execution/ clerical and random (Sleeman, 1984).

The term 'bug' is used by VanLehn (1982) to refer to a systematic error resulting from wrong steps in the calculation procedure. A Borrow Across-Zero bug is a systematic error caused by a student having trouble with borrowing, especially in the presence of zeros (VanLehn, 1982). For example, a student answering 98 when asked to calculate $305-117$ would be considered as a Borrow Across-Zero bug. In the aforementioned calculation, the student skips the step where the zero changed to nine during borrowing across zero (VanLehn, 1982).

Some Mathematics Education research has explored possible causes and effects of certain mathematical misconceptions and the impact that they have on students' future learning (Booth et al., 2014; Confrey, 1990; Fischbein, 1989; Nesher, 1987; Brown and Burton, 1978). After having investigated bugs (misconceptions) in high school algebra problems, Brown and Burton (1978) discussed possible arithmetic bugs which might lead to some specific algebraic bugs.

Booth et al. (2014) conducted a study to assess algebraic misconceptions that algebra students make at school. They concluded that students who make specific persistent errors due to underlying misconceptions in arithmetic may need additional intervention since misconceptions are not corrected through typical instruction. They conclude that these additional interventions can be carried out by targeting individual misconceptions or by improving conceptual understanding throughout the algebra course. The findings of Brown and Burton (1978) and the findings of Booth et al. (2014) hold the same conclusions, that students' arithmetic misconceptions affect their algebraic thinking and obstruct their performance and learning of algebra.

2.1.2 CSEs in different areas of mathematics: some examples

Some research has been conducted to identify misconceptions in different areas of mathematics. Some of the errors in fractions, algebra, indices, logarithms, and trigonometry documented in published literature are presented below:

- The addition or subtraction of two fractions is equal to addition of numerators divided by addition of denominators of the given fractions (McDonald, 2010; Rees and Barr, 1984; Rushton, 2014);
  e.g. $\frac{1}{2}+\frac{1}{4}$ is equal to $\frac{2}{6}$
- The sum of the squares is equal to the square of the sum (Rees and Barr, 1984; McDonald, 2010; Walker, Gwynllyw, and Henderson, 2015);

  e.g. a^2+b^2 is equal to $(a+b)^2$
- The difference of squares is equal to the square of the difference (McDonald, 2010);

  e.g a^2-b^2 is equal to $(a-b)^2$
- Multiply out only a part of the brackets (Rushton, 2014);

  e.g. $3(2x-5)$ is equal to $6x-5$
- The double of a term is equal to the square of the term (Rees and Barr, 1984);

  e.g. $2x$ is equal to x^2
- The square root of the difference of squares is equal to the difference of the terms (McDonald, 2010);

  e.g. $\sqrt{a^2-b^2}$ is equal to $a-b$
- The quotient of two indices is found by dividing the powers (McDonald, 2010);

  e.g. $\frac{a^5}{a^3}$ is equal to $a^{\frac{5}{3}}$
- The logarithm of a quotient is equal to the quotient of the logarithms (McDonald, 2010);

  e.g. $\log\left(\frac{a}{b}\right)$ is equal to $\frac{\log(a)}{\log(b)}$

- The tangent of twice an angle is equal to two times the tangent of the angle (Similar errors arise with other trigonometric functions such as sin, cos, etc.) (McDonald, 2010);

  e.g. $\tan(2A)$ is equal to $2\tan(A)$

2.1.3 Importance of understanding mathematical misconceptions

According to Hiebert and Lefevre (1986) there are two main types of knowledge that learners gain: conceptual and procedural. Conceptual knowledge is an understanding of the principles and relationships that underlie a domain (Rittle-Johnson and Siegler, 1998), or to know 'why' something happens in a particular way (Hiebert and Lefevre, 1986). Procedural knowledge is an ability to execute action sequences to solve problems (Rittle-Johnson and Siegler, 1998), or to know 'how' something happens in a particular way (Hiebert and Lefevre, 1986).

Star (2015) believes these two types of knowledge lie on a continuum and cannot always be separated, and mathematical competence depends on learners developing and linking their knowledge of concepts and procedure. Rittle-Johnson and Siegler (1998) have found that conceptual and procedural knowledge influence one another. Depending on the mathematical content, the two knowledges develop iteratively and one or the other may come first (Rittle-Johnson and Siegler, 1998). However, for many mathematical domains it is essential to have correct conceptual knowledge in order to develop correct procedural skills (Booth et al., 2017).

It is problematic when mathematics learners hold many faulty conceptual ideas or misconceptions at various points in their learning process. These misconceptions or flawed conceptual knowledge might impact students' performance and learning (Booth et al., 2017). A critical part of success in mathematics is to have a good procedural knowledge in order to carry out procedures to solve problems (Rittle-Johnson, Siegler, and Alibali, 2001; Kilpatrick, Swafford, and Findell, 2001). In their study, Rittle-Johnson and Siegler (1998) found that conceptual knowledge and procedural skill are related.

Durkin and Rittle-Johnson (2015) suggested that lack of conceptual understanding and strongly held misconceptions lead students to make errors when solving mathematical problems. They found that, sometimes, these strongly held misconceptions are very difficult to correct with instruction. Persistent errors are often an indication that a student holds a particular underlining misconception (Cangelosi et al., 2013).

Several education researchers have explored how conceptual understanding of algebra (Booth, Koedinger, and Siegler, 2007; Anderson, 1989; Vanlehn and Jones, 1997), and calculus (Muzangwa and Chifamba, 2012; Bezuidenhout, 2001) impact students' performance and learning. Within the topic of algebra, it has been found that students with stronger conceptual knowledge are better at solving equations, and are better at learning new procedures than peers with flawed conceptual knowledge (Booth, Koedinger, and Siegler, 2007; Anderson, 1989; Vanlehn and Jones, 1997). Booth and Koedinger (2008) found that students with flawed conceptual knowledge about the equals sign or negative signs frequently solve equations incorrectly and hence have greater difficulty learning how to solve equations correctly. They suggest that equation solving skills can be improved by correcting these misconceptions.

In their research into errors and misconceptions in an undergraduate course in calculus, Muzangwa and Chifamba (2012) found that a majority of errors are due to knowledge gaps in basic algebra. Also, it is found that poor understanding of the basic themes of calculus, like limits of functions and their representation, are the main causes of some misconceptions in calculus. Not only that, Bezuidenhout (2001) has suggested that, sometimes misconceptions in calculus are a result of teaching approaches which pay more attention to procedural knowledge as opposed to conceptual understanding of calculus. Presenting information in a variety of modes rather than a single mode could be used to develop concepts from an intuitive stage to an analytic stage (Felder and Henriques, 1995). As an example, the graph of a function, algebraic methods, a table of values, or verbal descriptions could be used to explain the limit of a function (Muzangwa and Chifamba, 2012).

2.1.4 Different ways of identifying and dealing with misconceptions

Education research has proposed several methods of identifying when students have misconceptions, and different kinds of intervention methods to correct mathematical misconceptions and improve students' conceptual understanding.

Hart (1981) has written a series of problems in secondary school mathematics, spanning a wide range of difficulties, which are free from technical terms in order to test the understanding rather than the repetition of skills. They have tested these questions on students in conjunction with interviews to understand students' errors in topics such as measurement, number operations, place value and decimals, fractions, positive and negative numbers, ratio and proportion, algebra, graphs, reflections and rotations, vectors and matrices.

Rees and Barr (1984) have identified several tasks in mathematics which cause difficulty to students over a wide age range. They name these tasks ‘core of common difficulty’. After identifying these ‘core of common difficulty’ tasks in mathematical problems, Rees and Barr (1984) asked students to ‘talk through’ their solutions in order to diagnose students’ errors. In their book, Rees and Barr (1984) have proposed a method of helping students with difficulties: “Give the diagnostic assessment, Look, Listen, Learn, Prescribe”. They have given some examples on the topics of natural numbers, decimal fractions, common fractions, percentage-ratio-proportion, similarity, measurements of the circle, algebra, and statistics to show how to employ their proposed method. Each prescription for each difficulty shows how teachers can enhance students’ development of ‘Concepts, Competence and Confidence’ by being ‘Aware, Explicit and Flexible’.

Williams and Ryan (2000) analysed mathematical misunderstandings of cohorts of children by inspecting the scripts of the statutory national mathematics tests taken by all 7 year-old and 14 year-old students in England and Wales in 1997. They identified and described the common errors students made on these tests and these were reported to all primary and secondary schools in England and Wales. They believe that it might help teachers to understand the common errors and perhaps teach more effectively.

Ryan and Williams (2007) have used specifically designed questions to identify errors and misconceptions of students aged 4-15 across the mathematics curriculum. In their book, they systematically develop concepts for teachers to use in organising their understanding and knowledge of children’s mathematics and offer practical guidance for classroom teaching.

Teachers can utilise more in-class exercises as opposed to lectures to identify students’ misconceptions and assess how pervasive they are (Savion, 2009). Berrett (2012) highlights that one benefit of the ‘flipped’ classroom approach where students watch recorded video lectures at home and engage in problem-based learning activities in class (Henderson, 2017; Maciejewski, 2016; Mazur, 1997), is that students’ misconceptions are more likely to arise while they are with the instructor. In which case the instructor can respond to them easily and promptly (Berrett, 2012).

Several ways that the instructor can quickly assess students’ understanding of a topic are discussed by Sevian (2011) and Cotner, Baepler, and Kellerman (2008). ‘Clickers’ can be used in classes to not only make all students engage in the activity, to assess their preparedness, and elicit discussions but also can be used to identify misconceptions, to gauge understanding of concepts and to identify students who need additional help (D'Inverno, Davis, and White, 2003; Sevian, 2011).

Further, Cotner, Baepler, and Kellerman (2008) have advocated a single and efficient way for learners to self-assess their progress in a course which is to structure significant small-group discussions called 'The Immediate Feedback Assessment Technique' (IF-AT). It is a relatively simple, special kind of scoring sheet. IF-AT is a low-tech tool for providing immediate feedback, targeting student misconceptions, and generating group discussion. IF-AT forms use a multiple-choice question sheet with a thin opaque film covering the answer options. Students scratch off the coating of the rectangle corresponding to their first-choice answer. If the answer is correct, a star symbol appears indicating their answer is correct. The student's learning is immediately reinforced, the student receives full credit for the answer, and moves on to the next question. If it is incorrect, they must re-read the question and the remaining answer options and scratch off until they arrive at the correct answer. The student earns partial credit for multiple attempts and learns the correct response for each question while taking the test (Cotner, Baepler, and Kellerman, 2008).

Another method of diagnosing misconceptions is discussed by Cakir (2008) who advocates assessing students' misconceptions through a quiz-style assessment both before and after instruction. Yip (1998) has suggested that this diagnosing method will provide continuous feedback on the effectiveness of the teaching strategies used for teachers.

Hiebert et al. (1996) argue that students' conceptual understanding can be enhanced by reforming the mathematics curriculum and instruction by allowing students to engage in resolving real-world problems. This method encourages students to problematise the subject rather than mastering skills and applying them.

Ma (1999) suggests students' conceptual misunderstandings can be addressed by re-teaching fundamental concepts and principles. Re-teaching is a post-instructional action or strategy initiated by teachers to support students who did not learn content, concepts or procedures from 'first' teaching and learning activities. It provides second-chances for students to try again to learn the content, concept, skill or procedure, as well as for teachers to refine and target their instruction (Bellert, 2015).

Rittle-Johnson and Star (2007) found that having students compare multiple solution methods can be used to improve students' conceptual understanding in algebra. Hilbert et al. (2007) have suggested that encouraging students to explain correct examples is very effective in increasing students' conceptual knowledge. This method also increases students' ability to solve both similar and more difficult problems (Renkl et al., 1998). Further research (Adams et al., 2014; Booth et al., 2013) suggested that explaining correct and incorrect examples further increased students' conceptual understanding. It has been found that practice in identifying, explaining, and correcting errors may help students process problems at a deeper level, and thereby help them overcome misconceptions (Adams et al., 2014; Booth et al., 2013; Durkin and Rittle-Johnson, 2012).

Booth et al. (2017) have proposed a method called 'Combining self-explanation, worked examples, and learning from errors method' which is a single effective intervention to correct mathematical misconceptions and improve students conceptual understanding. Simply, it involves explaining correct and incorrect worked examples during problem solving practice. These worked examples are examples of a fictitious learner's problem solution. Students are asked to explain the example in response, about particular errors made in solutions, or about how the fictitious learner might be thinking about the problem.

2.1.5 Taxonomies of mathematical CSEs

The theoretical study of classification, including its bases, principles, procedures and rules is called a taxonomy (Ford, Gillard, and Pugh, 2018; Simpson, 1961). The entities in a successful taxonomy can be verifiable by observation and will offer both an appropriate and suitable class for each entity (Ford, Gillard, and Pugh, 2018; Bailey, 1994).

There has been recent research into theorising student errors supported by empirical studies in the topics of natural number bias (Obersteiner et al., 2013), overgeneralisation (Knuth et al., 2006) and visual saliency (Kirshner and Awtry, 2004). Rushton (2014) conducted a study of common errors in Mathematics made in certain General Certificate of Secondary Education mathematics papers taken by candidates in England, including an internationally available version, and errors were catalogued into themes and sub-themes.

Some researchers have analysed students' errors to classify them into certain categorisations based on an analysis of students' behaviours. Donaldson (1963), cited by Orton (1983) has described three types of error as follows:

1. Structural errors: these errors arise from some failure to appreciate the re-

lationships involved in the problem or to grasp some principle essential to solution;

2. Arbitrary errors: Arbitrary errors are those in which the subject behaved arbitrarily and failed to take account of the constraints laid down in what was given;

3. Executive errors: Executive errors are those which involve failure to carry out manipulations, though the principles involved may have been understood.

Radatz (1979) has classified various causes of errors that cut across mathematical content topics. Errors in five main categories were identified by examining the mechanisms used in obtaining, processing, retaining, and reproducing the information contained in mathematical tasks. Those five main categories of errors can be listed as follows:

1. Errors due to language difficulties: Mathematics is like a 'foreign language' for students who need to know and understand mathematical concepts, symbols, and vocabulary. Misunderstanding the semantics of mathematics language may cause students' errors at the beginning of problem solving;

2. Errors due to difficulties in obtaining spatial information: Representation of a variety of iconic instructions, diagrams, and visualisations in mathematical tasks makes heavy demand on students' spatial abilities and capacity for visual discrimination;

3. Errors due to deficient mastery of prerequisite skills, facts, and concepts: Students may forget or be unable to recall related information in solving problems;

4. Errors due to incorrect associations or rigidity of thinking: Students who have inadequate flexibility in decoding and encoding new information may use the same technique even though the fundamental conditions of the mathematical task have changed;

5. Errors due to the application of irrelevant rules or strategies: These errors might happen when students have experiences in successfully applying comparable rules and strategies in other content areas.

Gill and Greenhow (2008) have categorised the mistakes that university students made in mechanics. They have produced the following taxonomy of errors and their classification:

1. Assumption: Students assume certain things that are not true, for example, in projectile questions, that vertical velocity is equal to initial velocity;
2. Calculation: Method correct but calculation errors are made;
3. Copying errors: Copying values incorrectly;
4. Definition: Not knowing the definition of terms given in question text, e.g. magnitude;
5. Formulas: Incorrectly stating/recalling formulas;
6. Incorrect values used: Using incorrect values in a method, for example, when substituting values into formulas;
7. Knowledge: Knowledge students are lacking that would enable them to answer questions;
8. Methodology: Students attempt to use an incorrect method to answer a question;
9. Modelling: Unable to model a particular situation/arrangement, i.e. unable to identify all forces or the correct forces acting on the particle;
10. Procedural: The method student attempts to use is correct but he/she can only do initial/certain stages of the method. They stop halfway through when they do not know the stages that follow or when they are unable to interpret initial results;
11. Reading: Reading the question text incorrectly and confusing the value of variables;
12. Trigonometry errors: Basic definitions of cosine, sine and tangent are incorrect. This is most apparent in questions where students are required to resolve forces.

Ford, Gillard, and Pugh (2018) developed a taxonomy of errors made by undergraduate mathematics students. In their study, they gathered errors by firstly recalling obvious mathematical errors that occur among mathematics undergraduates and secondly by analysing a selection of students' paper-based exam scripts from first year undergraduate mathematics courses. They have identified the following six main error categories:

1. Errors of slips of action (S);
2. Errors of understanding (U);
3. Errors in choice of method (CM);
4. Errors in the use of a method (UM);
5. Errors related to proof (P);
6. Errors in students' communication of their mathematical solutions (C).

This taxonomy proposed by Ford, Gillard, and Pugh (2018) has sub-themes which sum up most of the CSEs categories presented in this literature review. Therefore, this taxonomy will be used in the first stage of the CSE Project to select and categorise only those CSEs which are relevant to e-Assessments (See Section 3.2).

2.2 Introduction to traditional assessments and feedback in education

This section looks into the literature on traditional assessments and feedback in education and conditions for effective assessments and feedback which will be useful when producing CSE EFB for the e-Assessment questions, and drawbacks of using traditional assessments and feedback.

2.2.1 Traditional assessments and feedback

Assessment plays a vital role in learning and teaching. "What is assessed defines what is taught and how it is learnt" (JISC, 2007). Boud (1995) has highlighted that whilst students may be able to escape the effects of poor teaching they cannot escape the effects of poor assessment. Assessment is the most effective feature on the learning of the student (Gibbs and Simpson, 2005; Yüksel and Gündüz, 2017). Assessments can be designed to help students understand their own learning behaviour, as well to provide teachers with an understanding of the impact of their teaching (Crisp, 2007). It shapes institutional practice and it also affects a learner's view of the value of engaging in learning. Getting assessment 'right' promotes the well-being of learners and institutions, and widening participation in education (JISC, 2007).

Teachers use assessments to determine whether students have met their learning objectives. Formative assessments and summative assessments are the two ways that teachers measure what their students are learning (Connors, 2021; Crisp, 2007; JISC, 2007). Formative assessments are used throughout a class or course to monitor student learning and provide ongoing feedback to teachers and students. It helps teachers to identify misconceptions, struggles, and learning gaps in students learning, while assessing ways to close such gaps. Some researchers consider that the main purpose of formative assessments is to define the gap between the students' real performance and desired performance (Sadler, 1987; Sadler, 1998; Lipnevich et al., 2014). It helps students to identify their strengths and weaknesses, and take ownership of their learning. It also provides information to the faculty about the areas students are struggling with so that sufficient support can be put in place. Formative assessment is sometimes called 'assessment for learning' (McMillan, 2013; Yüksel and Gündüz, 2017; McDowell et al., 2011; Sambell, McDowell, and Montgomery, 2013).

On the other hand, summative assessments evaluate student learning, knowledge, proficiency, or success at the conclusion of a unit, course, or programme (Connors, 2021; Crisp, 2007; JISC, 2007; Yüksel and Gündüz, 2017). Summative assessments are almost always formally graded and often heavily weighted. Summative assessment is sometimes referred to as 'assessment of learning'. According to Earl (2003) (cited by Yüksel and Gündüz (2017)), it is an assessment method used to certify students' learning, submitting reports to the students about their development, and giving signs to the students about their own positions comparing themselves with other students.

In the the early 1970s, Snyder (1971) and Miller (1974) found that what influenced students most was not the teaching but the assessment. Assessment is the best way of identifying the learners' needs and to help students learn and improve their academic performance. It can be used to instil desire to progress further if linked to appropriate resources, good quality, and timely feedback (JISC, 2007). According to Stödberg (2012), assessments determine the extent of students' skill and knowledge in order to ensure that they have achieved the desired learning outcomes.

Assessment is considered as an integral part of students' learning. Not only does it promote student learning but it also allows them to receive support in order to improve their learning (JISC, 2010). In literature, these characteristics are normally attributed to formative assessments (Deeley et al., 2019), however, if students receive constructive feedback for their summative assessments it may also be a great opportunity for them to learn. Characteristics of assessment for learning, such as providing ongoing feedback, can be utilised in all assessments (Taras, 2002).

Another powerful influence that makes a difference to student achievement is feedback (Hattie, 1987). Reviewing over 250 studies across a range of subjects, levels, situations and students abilities, Black and Wiliam (1998) have found that a formative feedback has positive influence on students' learning and performance. Feedback actually has extraordinarily larger positive effects on student's learning than other aspects of teaching (Black and Wiliam, 1998).

A number of researchers highlighted the importance of feedback for learning. Buchanan (2000) suggests that feedback has a role in fostering meaningful interaction between student and instructional materials whilst Yorke (2001) emphasises that feedback contributes to student development and retention.

2.2.2 Conditions for effective assessments and feedback

Several educational researchers have identified conditions under which assessment and effective feedback supports students' learning (Sadler, 1989; Gibbs and Simpson, 2005; Nicol and Macfarlane, 2006). According to Sadler's theoretical analysis of the role of feedback, the ultimate goal of feedback is to teach students to monitor their own performance. Students can utilise feedback to self-assess to learn and direct their future learning as follows (Sadler, 1989):

1. Students must gain knowledge of what is required;
2. Students must compare their own performance with the required standard;
3. Students must do something which will close the gap between their own performance and the required standard.

Gibbs and Simpson (2005) have outlined the following ten conditions which refer to two relatively distinct categories of influence (influence of the design of assessment systems and assignments on how much students study, what they study and on the quality of their engagement; and the influence of feedback on learning).

1. Sufficient assessed tasks are provided for students to capture sufficient study time;

2. These tasks are engaged with by students, orienting them to allocate appropriate amounts of time and effort to the most important aspects of the course;
3. Tackling the assessed task engages students in productive learning activity of an appropriate kind;
4. Sufficient feedback is provided, i.e. both often enough and in enough detail;
5. The feedback focuses on students' performance, on their learning and on actions under the students' control, rather than on the students themselves and on their characteristics;
6. The feedback is timely in that it is received by students while it still matters to them and in time for them to pay attention to further learning or receive further assistance;
7. Feedback is appropriate to the purpose of the assignment and to its criteria for success;
8. Feedback is appropriate in relation to students' understanding of what they are supposed to be doing;
9. Feedback is received and attended to;
10. Feedback is acted upon by the student.

Nicol and Macfarlane (2006) have identified seven principles of good feedback practice which might strengthen the students' capacity to self-regulate their own performance. The seven principles of good feedback practice defined in Nicol and Macfarlane (2006) are as follows:

1. Helps clarify what good performance is (goals, criteria, expected standards);
2. Facilitates the development of self-assessment (reflection) in learning;
3. Delivers high quality information to students about their learning;
4. Encourages teacher and peer dialogue around learning;
5. Encourages positive motivational beliefs and self-esteem;
6. Provides opportunities to close the gap between current and desired performance;

7. Provides information to teachers that can be used to help shape teaching.

Considering these overlapping properties of effective feedback proposed by Sadler (1989), Gibbs and Simpson (2005), and Nicol and Macfarlane (2006), Robinson (2015) has identified ten possible aims of feedback as follows:

1. Clarify what good performance is;
2. Correct errors;
3. Developing students' understanding through explanations;
4. Prompt further studies by the student, to provide opportunities to close the gap between the student's current work and the required standard, or to provide further learning beyond the work on which feedback is given;
5. Promote the development of generic skills by focusing on the use of skills more than on the content;
6. Deliver high quality information to students about their learning;
7. Promote awareness of the learning processes involved;
8. Facilitate the development of self-assessment and reflection;
9. Promote dialogue between tutors and peers about the learning;
10. Encourage students to continue studying, particularly by encouraging positive motivational belief and self-esteem.

In a broader perspective, assessment for learning, assessment of learning, learning from assessments and feedback help students to deepen their understanding and learning, and develop their attributes and skills, and widen employment and lifelong learning (Deeley, 2014).

2.2.3 Drawbacks of using traditional assessment and feedback

Despite the above literature highlighting the importance of assessment and feedback, students and teachers still possess some negative thoughts on them. According to the National Student Survey (NSS) data, student dissatisfaction with assessment and feedback is a significant challenge for most Higher Education Institutions in the United Kingdom. Two key statements relating to the effectiveness of feedback

“Feedback on my work has been timely” and “ I have received helpful comments on my work” received lower scores compared to most of the other questions on the survey (OFS, 2022). Related studies on the NSS highlights that regardless of overall satisfaction trends, students typically demonstrate much less satisfaction with assessment and feedback than with other measures since the NSS first started in 2005 (Deeley et al., 2019).

Teachers spend a considerable amount of time providing written comments on students’ assignments. Sometimes students may not collect their feedback, read or heed the feedback made by their teachers. These situations may convey to the teacher that their efforts are wasted (Deeley et al., 2019). Even though teachers put considerable time and effort in to provide feedback, it seems that for many students feedback has no impact on their learning and is of little use or value to them (Sadler, 2010; Lunt and Curran, 2010). Lunt and Curran (2010) suggest that these findings show that there is a mismatch between staff and students’ understanding of feedback.

Various studies (Hattie and Timperley, 2007; Jönsson, 2013; O’Donovan, Rust, and Price, 2016) have suggested that the relevance of feedback can be lost if it is given too late. Over the years, the class sizes in higher education have increased significantly and hence the feedback to individual students have declined and teachers spend much of their time marking assessments (Gibbs and Simpson, 2005). Providing comprehensive and useful feedback under such time pressures might lead to a problem with the quantity and quality of feedback provided (Gibbs and Simpson, 2005).

Further, assessment activities are expensive and produce stress for both teachers and students who are involved in the process (Crisp, 2007). The overall assessment process requires a substantial amount of time and effort by students in terms of responding to assessment tasks, and teachers in terms of setting and marking or grading assessments (Stödberg, 2012). Preparation and marking of traditional paper-based assessments is an expensive and long process and it also requires a significant amount of time and effort by teachers. To mitigate this situation, the use of information technology to conduct assessment has significantly risen in higher education (Sangwin, 2004; Stödberg, 2012; Sangwin, 2015; Rolim and Isaias, 2019).

2.3 Introduction to e-Assessments in education

This section gives a literature review on e-Assessments, and the advantages and challenges of using e-Assessments in education, and research in addressing CSEs in e-Assessment questions.

2.3.1 E-Assessments

The Joint Information Systems Committee (JISC) Qualifications and Curriculum Authority (QCA) define e-Assessments as "the end-to-end electronic assessment processes where ICT is used for the presentation of assessment activity, and the recording of responses. This includes the end-to-end assessment process from the perspective of learners, tutors, learning establishments, awarding bodies and regulators, and the general public" (JISC, 2007).

Different literature suggest different definitions for e-Assessments. For Ridgway, McCusker, and Pead (2004), e-Assessments can have different forms such as, automating administrative procedures, digitising paper-based systems, and online testing which extends from multiple-choice tests to interactive assessments of problem-solving skills. Broadly speaking, it can be defined as the use of a computer as part of any activity related to assessment (Jordan, 2013). However, Timmis et al. (2016) thoroughly reviewed technology-enhanced assessment (e-Assessments) to clarify the terminology of e-Assessments. They argued that the terminology had changed significantly as the result of increased understanding of the potential use of technology in assessment. They suggested that the terminology evolved from computer-based testing to computer assisted (or aided) assessment to online assessment and, finally, to e-Assessment. For Harley et al. (2021) a computer-based examination is an examination performed with the help of computers, not necessarily online. For the purpose of this thesis, e-Assessments which involves the use of technology to present and deliver assessment tasks, receive students' responses, and record these responses as well as to provide feedback to students are considered.

In the early 1920's it was recognised that technology could be used in assessments when Pressey designed several machines for the automatic testing of intelligence and information (Skinner, 1958; Pressey, 1928). One of those machines was used for multiple-choice tests. The student was required to press the button corresponding to their first choice of answer. If their choice was correct, the device moved on to the next item. If their choice was incorrect the error was tallied and the student was required to make choices until they got the question right (Skinner, 1958). Pressey (1928) emphasised that not only could these machines be used to test and score but could also be used to teach. Additionally, Pressey (1928) pointed out that the use of machines for testing could enable the teacher to utilise their time in developing students' enthusiasms, and clear thinking. At the same time, in the 1920's the use of standardised assessment, and automatic scoring technology was started in schools, and due to that, large-scale testing became convenient and cost-effective (Audette, 2005).

The use of computers to support assessments was first attempted in the 1960's. Two of these first attempts are: PLATO (Programmed Logic for Automatic Teaching Operations), and TICCIT (Time-shared, Interactive, Computer-Controlled, Information Television) (Alruwais, Wills, and Wald, 2018).

In the 1990's, a massive change occurred in the education sector when the World Wide Web was introduced (Llamas-Nistal et al., 2013). This led to many companies introducing their own e-Assessment Systems. JISC was established in 1993 to support post-16 and higher education and research by providing leadership in the use of information and communications technology in support of learning, teaching, research and administration in the United Kingdom (JISC, 2007).

Two types of e-Assessments are described by JISC (2007): computer-based assessment (CBA) and computer-assisted assessment (CAA). CBAs are assessments delivered and marked by computers. CAAs are practices that depend on computers, such as online discussion forums for peer assessment, audience response systems in group work, completion and submission of work electronically, or storage of work in an e-portfolio. However, in the literature, these terms are often viewed as interchangeable (JISC, 2007).

Easy accessibility and the advantages of e-Assessments have led many university mathematics departments to conduct formative and summative assessments in the form of e-Assessments (Sangwin, 2013). Over the past years, several e-Assessment Systems, such as STACK (Sangwin, 2004), Dewis (Gwynllyw and Henderson, 2009), Math e.g. (Greenhow and Kamavi, 2012), and Numbas (Foster, Perfect, and Youd, 2012) have been developed at several universities in the UK.

The Covid-19 pandemic (WHO, 2020) has engendered significant disruption in education activities all over the world (Daniel, 2020). This situation forced many universities to suspend conventional education temporarily and to make an urgent transition to emergency distance learning, technology enhanced learning and e-Assessments (OECD, 2021; Bothwell, 2020). However, many universities faced different challenges such as technical inadequateness, lack of qualified online tools, inexperience of instructors and students in distance education (Senel and Senel, 2021). One of the biggest challenges of education during the pandemic was assessment and grading (Council of the European Union, 2020). These hardships brought up great opportunities for researchers to further investigate e-Assessments usage in universities in the topics of assessment approaches used in the pandemic, students perceptions on these assessments, and the pros and cons of using these practices (Senel and Senel, 2021; Divjak, Žugec, and Anicic, 2022).

2.3.2 The advantages of using e-Assessments

Some universities have adopted e-Assessments (paper-less) instead of traditional examinations to develop, accurate and faster methods to assess students (Alruwais, Wills, and Wald, 2018). Properly performing e-Assessments are hugely beneficial for both teachers and students (Gwynllyw and Henderson, 2009). This section looks into the literature on some of the advantages of using e-Assessments.

One of the benefits of using e-Assessment is its capability to provide instant and tailored feedback which helps students improve their learning (Gwynllyw and Henderson, 2009; Crews and Curtis, 2011). E-Assessments have enhanced students' learning outcomes by producing immediate and direct feedback for them (Gilbert, Whitelock, and Gale, 2011; JISC, 2010; Gwynllyw and Henderson, 2009).

Gibbs and Simpson (2005) state that feedback can be used to correct errors and develop understanding through explanations. Even though errors are corrected accurately, feedback will not be useful if it is provided to student too late. Students should receive the feedback in a timely manner while it still matters to them so that they can pay attention to further learning or receive further assistance (Gibbs and Simpson, 2005). Provision of timely feedback may close the gap between actual and desired performance levels of students (Nicol, 2007). Dermo (2009) and Gikandi, Morrow, and Davis (2011) posit that high quality and accurate feedback delivered in a timely manner plays an important role in students' learning.

Students could use instant feedback to organise self-learning by noticing deficiencies and mistakes (Senel and Senel, 2021). E-Assessments and their instant feedback provide opportunities and motivation for students to practise and excel in their subject (Broughton, Robinson, and Hernandez-Martinez, 2013). In addition, by reviewing and studying this feedback, students can identify their weakness as well as their strengths in order to achieve continuous improvement in their learning

(Gill and Greenhow, 2008; Daly et al., 2010; Eynon, 2008; Russell et al., 2006). Creating a system to assess students which achieves educational goals and helps students to develop their skills will be useful for society in the long-run (Ridgway, McCusker, and Pead, 2004).

Gill and Greenhow (2008) conducted a study to find out the effectiveness of e-Assessment feedback and found that students improve their performance by engaging with the feedback provided in e-Assessments. Therefore, Greenhow (2015) suggests that e-Assessments which select questions based on pedagogic principles should be promoted as a learning tool due to its capability of providing effective feedback.

Teachers also have the benefit of being able to provide instant feedback when they integrate e-Assessments into their teaching. Teachers can easily provide formative feedback instantly to large student cohorts using e-Assessments (Bull, 2013). Further, in their book, Bull (2013) highlights that academic staff can quickly assess the students' performance data and whether students have understood the lesson being taught. This will also help teachers identify if there are any misconceptions or gaps in students' knowledge.

E-Assessments can be accessed in different geographical locations at any time, and students can undertake online tests several times to improve their learning (Gwynllyw and Henderson, 2009). Ridgway, McCusker, and Pead (2004) state that students favour e-Assessments because they provide more controlled, friendly interfaces and present tests as games and simulations, which resemble learning environments and recreational activities. E-Assessments can be accessed using just a personal computer, and an internet connection (Carlbring et al., 2007).

JISC (2007) highlights that e-Assessments can support personalisation. Since e-Assessments can be assessed at any time and anywhere, it provides special benefits for learners who have difficulties in learning due to distance, disability, illness, or work commitments. Most e-Assessments have the potential to offer extra supporting materials to those students who require additional learning support and study skills (Bull, 2013).

Most e-Assessment Systems have the capability of producing practice tests for learners. Students find the availability of these practice tests very useful study resources to support their learning (Mccabe, 2009). These practice e-tests foster students to take responsibility for their own learning, and support students to reach maturity for learning at university level (Broughton, Robinson, and Hernandez-Martinez, 2013). By checking students' access data, some lecturers found that a surprising number of students engage in these practice e-tests (Broughton, Robinson, and Hernandez-Martinez, 2013).

Tomlinson (2004) highlights that one benefit of using e-Assessments is the potential to test learners in both structured and unstructured environments. It can provide different kinds of tests with different types of questions such as quick multiple-choice questions (MCQ), short answer questions, and long answer questions.

Another advantage of using e-Assessments is that it reduces the marking load for lecturers (Bull, 2013). Lecturers who teach one of the largest mathematics and engineering undergraduate cohorts found the use of e-Assessments very efficient, convenient and freed up their time (Broughton, Robinson, and Hernandez-Martinez, 2013). Also some e-Assessment Systems support the use of random parameters within a coded question (Sangwin, 2004; Gwynllyw and Henderson, 2009; Greenhow and Kamavi, 2012; Foster, Perfect, and Youd, 2012). This feature in e-Assessments mean that within the code of one question it can generate a class of questions (Greenhow, 2015), and the randomisation of question reduces the potential of cheating (Bull, 2013).

The evidence reviewed here suggests that there are huge benefits of using e-Assessments in education. However, other literature acknowledges that there are also some disadvantages of using e-Assessments in education.

2.3.3 The challenges of using e-Assessments

The challenges of using e-Assessment in education have been explored in several studies, and some of these challenges are summarised in this section.

First of all, the main disadvantage of using e-Assessment Systems is that the initial implementation of e-Assessment is very time-consuming, and expensive. Also the management of e-Assessment Systems requires continuous monitoring of its hardware and software to avoid failure in delivering required assessments (Bull, 2013). In addition, it requires a high level of coordination of academics, support staff, computer services and administrators to deliver an assessment (Bull, 2013).

Students' low level of computer literacy, or their lack of experience with computer or online assessment process are some of the critical challenges in using e-Assessment in education (Osuji, 2012; Donovan, Mader, and Shinsky, 2007). Also, students having less access to computers and the internet is another challenge when implementing e-Assessments in higher education (Crews and Curtis, 2011). Ridgway, McCusker, and Pead (2004) and Jordan and Mitchell (2009) state that most teachers have limited knowledge on e-Assessments. Therefore, training teachers to be confident to use e-Assessment Systems is also one of the challenges in using e-Assessments (Alruwais, Wills, and Wald, 2018). Ridgway, McCusker, and Pead (2004) have stressed that "it is important that e-Assessment does not create a 'digital divide' which privileges some students over others on the basis of opportunities of access". However, these challenges can be addressed by providing some computer appreciation training to make students and teachers computer literate (Osuji, 2012), and provide students with well-equipped computer labs and internet access at universities (Alruwais, Wills, and Wald, 2018).

Another challenge of using e-Assessments is that there are not enough large computer laboratories to accommodate large cohorts in universities. Therefore, some lecturers find a lack of resources to invigilate online summative assessments (Broughton, Robinson, and Hernandez-Martinez, 2013). One of the other challenges of using e-Assessments is the time it takes to design and code good quality questions. Educators who design questions find that to code and develop a good quality questions and their feedback in e-Assessment requires lots of their time and energy. Hence, it is very expensive considering the amount of time and effort they need to invest (Gill and Greenhow, 2008).

Further, some lecturers have found that most of the existing e-Assessment questions do not have the ability to test all aspects of the syllabus. Therefore, to design new questions sometimes requires a considerable amount of time to learn different techniques of coding e-Assessment questions (Broughton, Robinson, and Hernandez-Martinez, 2013).

However, these challenges can be minimised since e-Assessment questions code can be reused in subsequent tests. Furthermore, most of e-Assessment Systems can randomly generate question parameters, so coding a question actually means it produces a selection of randomly generated questions on the fly (Walker, Gwynllyw, and Henderson, 2015).

One of the technology related issues in using e-Assessments is that there is a lack of interoperability between different e-Assessment Systems (Bull, 2013). Bull (2013) states that "interoperability describes the capacity for different systems to share information and services such that two or more networks can communicate with each other to exchange data in a common file". The Instructional Management Systems (IMS) Question and Test Interoperability (QTI) have started a project to address this challenge (Bull, 2013). The evaluation of this project in 2002 indicated that vendors of different e-Assessment Systems had developed systems which still allowed them to exchange only simplest form of questions (Sclater, Low, and Barr, 2002). This ongoing project has been exploring the methods of increasing interoperability and efficiencies within the e-Assessments industry (1EdTech, 2023).

E-Assessments cannot act very flexibly like a human marker when faced with ill-posed or unanticipated student responses (Greenhow, 2015). Detecting CSEs on traditional paper-based assignments compared to e-Assessments is more straightforward since the human marker has access to the students' intermediate workings and thus can easily spot when a CSE has been made. E-Assessment Systems cannot easily point out CSEs on student answers since typically few intermediate working steps are submitted, and also, each student attempts a different but equivalent version of the question due to the use of random parameters (Walker, Gwynllyw, and Henderson, 2015). That paper suggest that an e-Assessment would act more like a human marker, if it could detect and report CSEs, and provide effective and tailored feedback instantly by correcting students' misconceptions. This suggestion is one of the insights that prompted the research questions of this thesis.

2.3.4 Research in addressing CSEs in e-Assessment questions

Some research has been carried out to identify CSEs in e-Assessment questions in and this is discussed in this section.

Gill and Greenhow (2008) have carried out research to identify the types of mistakes that students actually make and common misconceptions that they have in the area of mechanics by examining several years' worth of paper-based exam scripts. Then they have produced a taxonomy of error types to categorise those mistakes and misconceptions. Subsequently, they have used these CSEs to code distractors in multiple-choice e-Assessment questions and to provide tailored feedback if a particular distracter is chosen by the student.

Jordan (2014) has analysed thousands of student responses in computer-marked assessment questions to find out the errors made by adult distance learners of science. The common errors were observed by looking at the responses of the students for the respective questions. Then the questions for subsequent years were improved using this information and provided targeted feedback to commonly incorrect responses.

Walker, Gwynllyw, and Henderson (2015) carried out two case studies to identify CSEs in post-submission e-Assessment questions in logarithms and indices for first-year computing students at UWE Bristol, and SturmLiouville problems for second-year mathematics students at Leeds University. They have used answer data stored in the Dewis e-Assessment System to detect previously unsuspected CSEs. Having identified new CSEs, the feedback to the question was amended to provide detailed, tailored feedback to the questions.

The above research on detecting CSEs in e-Assessment provided motivation to start the CSE Project at UWE Bristol in order to extend the functionality of the Dewis e-Assessment to diagnose and remediate mathematical CSEs in e-Assessment questions.

2.4 Dewis e-Assessment System

A literature review on the development and features of Dewis e-Assessment System at UWE Bristol is summarised in this section.

Dewis is a fully algorithmic open-source e-Assessment System, which was primarily designed and developed for numerate e-Assessments by a team of Mathematicians, Statisticians and Software Engineers at UWE Bristol (Gwynllyw and Henderson, 2009; Gwynllyw and Henderson, 2012; Gwynllyw, Weir, and Henderson, 2016). It was mainly designed for the assessment of mathematics and statistics and supports different question input types such as numerical inputs, matrices, vectors, algebraic expressions, multiple-choice, multiple-selection, graphical input, and computer programs. It has a number of student-friendly features, such as shut-down recovery and pre-processing checks on student input.

Dewis employs an algorithmic approach which enables the separate solution, marking and feedback algorithms to respond dynamically to a student's input. The question parameters are randomised and generated at the point of delivery; therefore, no two students receive exactly the same question. Depending on the nature of the e-Assessment, students can practice the same question several times with different parameters in order to gain mastery.

Each Dewis question has full feedback bespoke to that question and its specific randomly generated parameters. The feedback not only supplies the correct answer but a fully worked solution showing how the correct answer was obtained.

Furthermore, Dewis has a data lossless feature. All data relating to every assessment attempt is recorded on the Dewis server. This enables the academic to track efficiently how a student or cohort of students has performed on a particular e-Assessment (Walker, Gwynllyw, and Henderson, 2015). The highly developed reporting system enables tracking of module cohorts, tutorial groups and individual students. Dewis has another useful feature called re-mark. Academics can use this feature to perform a 're-mark' of an whole assessment, if they decided to alter a question code at the end of an assessment.

Academics can manage their e-Assessments through a web-based 'Academics' Management Tool'. The sub-categories of this are Users' Manager, Assessment Manager, Public Question Bank, and Private Question Bank. Within the Users' Manager academics can manage the student users registered on the system such as to register those students who need more time on assessments due to accessibility issues. The Assessment Manager allows the academic to create and manage assessments. All the Dewis questions, which have passed moderation processes, reside in the Public Question Bank. Academics can view and try-out Dewis questions in the Public Question Bank. The question bank lists questions according to their general classification or according to the keywords associated with them. Academics may copy any public question to their Private Question Bank in order to view or edit the question's code. In addition, the Private Question Bank allows academics to create, manage, or alter questions that are bespoke to their own module.

Over the past decade, Dewis has been used very successfully to facilitate both formative and summative e-Assessments across a number of modules, delivered to students in a wide range of fields, e.g. Business, Computer Science, Nursing, Software Engineering, Engineering and Mathematics and Statistics. One aim of the CSE project is to enhance the potential of Dewis, by developing and using additional features allowing Dewis to detect CSEs and to provide instant tailored feedback.

The intended methodology of this research is to alter some existing original question codes to capture identified CSEs. The amendments to question codes are carried out in the Question Editor in Dewis, therefore, in the following section, the features of the Question Editor in Dewis are discussed.

2.5 The Dewis Question Editor

In this section, the process of editing questions in the Dewis Question Editor within the Private Question Bank is reviewed using an existing question. It should be noted that, the Dewis Question Editor space is used solely by academics, and it is not the student view of the question.

For illustration purposes, the question considered involves finding the value of $f(c)$ for the given function $f(t) = Au(t-a) + Bu(t-b)$ where $u(t)$ represents the Unit Step Function. The randomly generated values of A, B, a, b and c are supplied to the student and the student is required to calculate the value of the integer $f(c)$. For each question in the Private Question Bank there are two menu options, namely, 'Edit Source Code' and 'Tryout' (see Figure 2.1). The selected question can be viewed by clicking the 'Tryout' button and an instance of the question is shown in Figure 2.2.

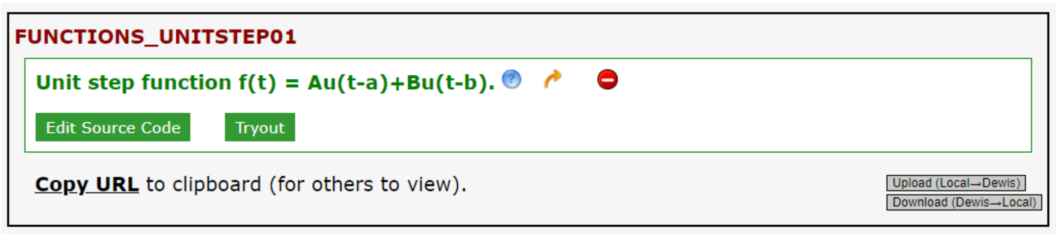

Figure 2.1: Menu Options for a typical Dewis question

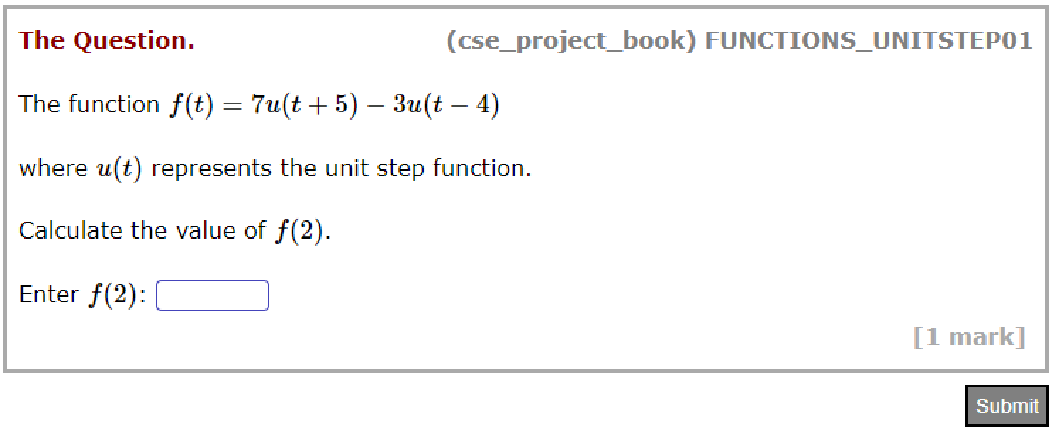

Figure 2.2: Dewis question involving Unit Step Functions

The source code can be viewed and edited by clicking the 'Edit Source Code' button. This opens an editing window as shown in Figure 2.3. Alternatively, the source code can be downloaded to the users' computer and opened in any preferred text editor. In this case, the question code can be uploaded back to Dewis once the alterations have been made (Dewis, 2022).

Figure 2.3 shows that the code is partitioned into nine main areas, identified by their tag names. These areas are as follows: *Settings*, *Descriptor*, *Parameters*, *Question*, *Solution*, *Marking*, *Scores*, *Report*, and *Feedback* [*Feedback(0)*, *Feedback(1)*, *Feedback(2)*, *Feedback(3)*]. The purpose of each tag is briefly reviewed in the following sections. This review is conducted only for the example question selected. More information about the Dewis Question Editor can be found in Dewis (2022).

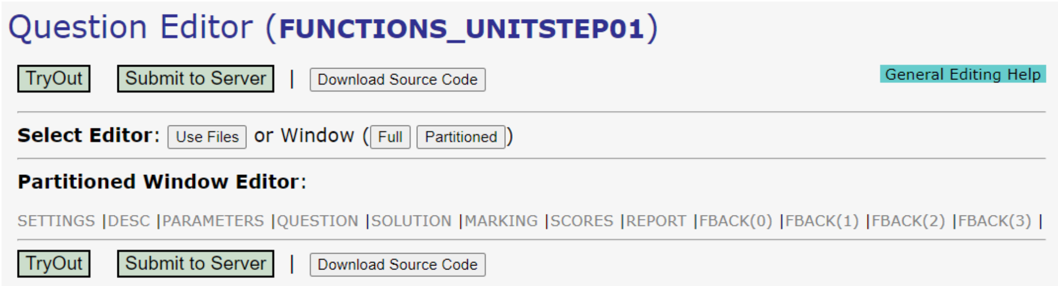

Figure 2.3: Dewis Question Editor menu options

Settings Tag

The *Settings* tag defines some basic question settings such as how many answer inputs there will be in this question and whether the question uses mathematical symbols and/or graphics. Figure 2.4 shows the four settings for the above example question. They can be described as follows:

```
<SETTINGS>

     <NUM_INPUTS>1
     <NUM_FLAGS>1
     <USES_MATHML>1
     <USES_SVG>0

</SETTINGS>
```

Figure 2.4: Example of the *Settings* Tag in the Dewis Question Editor

- $<$ NUM_INPUTS $>$ - The number of answer inputs required in the question is given here. This question requires one answer, namely the integer $f(c)$. Therefore, the number of inputs required to be entered by the student is set to one.

- $<$ NUM_FLAGS $>$ - In Dewis question code, the `$fn` parameters are the Performance Indicators (PIs), also known as Performance Flgs (PFs), that hold the information about the performance of a student's answer. For this question, *Settings* indicates that there is one PI which is associated with the student's one input. For more complex marking schemes such as partial marking, or follow-on marking, one question input could be associated with more than one performance indicator. More information about complex marking schemes can be found in Dewis (2022).

- $<$ USES_MATHML $>$ - Setting to state whether the question uses mathematical symbols, which are rendered using MathJax (MathJax, 2022). It is allocated a value of 1 for ‘yes’ and 0 for ‘no’. This example question uses mathematical symbols and therefore, $<$ USES_MATHML $>$ is set to 1.

- $<$ USES_SVG $>$ - This is used to indicate whether the question uses dynamic graphics, which are rendered using JSXGraph (JSXGraph, 2022). It is allocated a value of 1 for ‘yes’ and 0 for ‘no’. In this example question, it is set to 0 since no dynamic graphics are used.

Descriptor Tag

The *Descriptor* tag is used to enter some descriptor text for the question. Figure 2.5 shows that detailed and brief descriptions are used to give information about the question. Keywords are declared so that the system can categorise the question in the Public Question Bank.

Details about the author of the question and the date that the question was last updated can be included in this section. Note that the question author has been removed for confidentiality from Figure 2.5.

```
<DESCRIPTOR>

<CATEGORY>functions</CATEGORY>

<KEYWORDS> ENGINEERING-MATHEMATICS,uwe-
ufmfj9-30-1</KEYWORDS>

<BRIEF>
Unit step function f(t) = Au(t-a)+Bu(t-b)
</BRIEF>

<DETAILED>
Given that f(t) = Au(t-a)+Bu(t-b),
where u(t) is the unit step function,
find the value of f(c)

This question was designed by ....
Last Change: Sept 15, 2013.
</DETAILED>

</DESCRIPTOR>
```

Figure 2.5: Example of the *Descriptor* Tag in the Dewis Question Editor

Parameters Tag

This tag defines and populates the global question parameters that will be used by the subsequent tags. Typically, these parameters are generated randomly. Figure 2.6 shows that this example question uses five global question parameters namely `$p1`, `$p2`, `$p3`, `$p4` and `$p5` to generate values for b, a, c, A, and B, respectively. These global parameters can be readable by subsequent tags such as the *Solution* tag and the *Feedback* tag.

Local parameters can also be created in the *Parameters* tag. In Dewis, any local Perl variable needs to be explicitly declared using the '`my`' command (Perl, 2022). To populate any global or local variable there are a number of in-built functions which are supplied in Dewis. Two key popular functions for generating values for the parameters are `integerRandom()` and `integerRandomBar()`. As an example, in this question the value for the global integer parameter `$p2` is selected randomly between -5 and +5. Further, the global integer parameter `$p4` is selected using the `integerRandomBar()` function, which randomly generates a non-zero integer value for `$p4` between -5 and +5. More information about parameter functions can be found in Dewis (2022).

```
<PARAMETERS>

<GLOBAL> $p1  $p2  $p3  $p4  $p5 </GLOBAL>

$p2 = integerRandom(-5,5);          # a
$p1 = integerRandom(-9,$p2-1);      # b
$p3 = integerRandom(-12,12);        # c
$p4 = integerRandomBar(-5,5,0);     # A
$p5 = integerRandomBar(-5,5,0);     # B

</PARAMETERS>
```

Figure 2.6: Example of the *Parameters* Tag in the Dewis Question Editor

Question Tag

The question that will be presented to the students is created in this tag. Figure 2.7 shows the *Question* Tag for the example question. The majority of the code is written in basic HTML with some LaTeX. Typically, very little, if any, Perl is used in this tag.

In the *Question* Tag, the global variables (for this question `$p1`, `$p2`, `$p3`, `$p4` and `$p5`) set up in the *Parameters* tag are used to define the question. Here '`prettyPrint`' is used to tidy up algebraic entries which are display to the students in an assessment. The question that is sent to the browser is written within the <PRINT> ... </PRINT> tags. Further, mathematical symbols and mathematical equations which are sent to the browser are written within the $...$ tags. A <PROMPT> ... </PROMPT> is used for each question input. The *ID* identifies the prompt identity, in this question, `ID = '1'` corresponds to the input for $f(c)$. The function '`displayMaxMark(1)`' is used to display the total marks available for this question (which is 1 in this case).

```
<QUESTION>

my $arg1=prettyPrint("t-$p1");
my $arg2=prettyPrint("t-$p2");

my $fun=prettyPrint("$p4 u($arg1) + $p5 u($arg2)");

<PRINT>
The function <MATH> f(t) = $fun </MATH>
<p></p>
where <MATH> u(t) </MATH> represents the unit step
function.
<p></p>
Calculate the value of <MATH> f($p3) </MATH>.
<p></p>
Enter <MATH> f($p3) </MATH>:<PROMPT ID='1'></PROMPT>
</PRINT>

displayMaxMark(1);

</QUESTION>
```

Figure 2.7: Example of the *Question* Tag in the Dewis Question Editor

Solution Tag

The correct solution(s) is populated in the *Solution* tag. Figure 2.8 shows the *Solution* tag for the example question. The global parameters `$cn` are used to contain the correct answer(s). In this example, `$c1` is used to populate the correct answer for the input with `ID = '1'`, namely $f(c)$. Note that, the subroutine `unitStep` is defined within this tag in order to simplify the calculation of `$c1`.

```
<SOLUTION>

$c1 = $p4*unitStep($p3-$p1) + $p5*unitStep($p3-$p2);

sub unitStep
{
    # passing argument
    my $arg = $_[0];
    # calculate the unit step function of $arg
    my $value = ($arg>=0)?1:0;
    return ($value);
}

</SOLUTION>
```

Figure 2.8: Example of the *Solution* Tag in the Dewis Question Editor

Marking Tag

The question is marked in the *Marking* tag. Figure 2.9 shows the *Marking* tag for the example question. The submitted answer for this question is held in the variable `$a1`, which corresponds to the students' answer to the prompt `ID='1'`.

For this particular question, it simply involves comparing the student's answer `$a1` with the corresponding correct answer `$c1`. The PI, `$f1` is allocated a value representing the marking outcome. For integer inputs, each flag can take one of three values -1, 0, 1 representing the student 'not answering', 'answering but incorrectly' or 'answering correctly', respectively. There is no fixed rule that determines which `$fn` parameter is linked to which student input (Dewis, 2022).

```
<MARKING>

<MARK_INTEGER FLAG='$f1' ANS='$a1' COR='$c1' >

</MARKING>
```

Figure 2.9: Example of the *Marking* Tag in the Dewis Question Editor

Scores Tag

The *Scores* tag allocates marks based on the values of the PIs. Figure 2.10 shows the *Scores* tag for the example question. The scores based on the value of `$f1` are constructed here. For this question, answering the question correctly is worth one mark. Each PI has the potential of contributing to the marks allocation and more details can be found in Dewis (2022).

```
<SCORES>

<F1 VALUE='1' SCORE='1'>

</SCORES>
```

Figure 2.10: Example of the *Scores* Tag in the Dewis Question Editor

Report Tag

The *Report* tag gives a brief report based on the students' submitted answers. Figure 2.11 shows the *Report* tag for the example question. The value of `$f1` is used to determine the response given here. < PRINT_REPORT ...> has three different report statements for each value of `$f1`. Table 2.1 shows the three possible report statements for this example question. More details about the *Report* tag can be found in Dewis (2022).

```
<REPORT>

<PRINT_REPORT FLAG='$f1' TEXT="<MATH> f($p3) </MATH>"
ANS=' $a1' >

</REPORT>
```

Figure 2.11: Example of the *Report* Tag in the Dewis Question Editor

Table 2.1: Report Outcomes

`$f1` **Value**	**Sample report**
1	Your answer for f(2) is 7. Your answer is correct.
0	Your answer for f(2) is 55. Your answer is incorrect.
-1	You did not supply an answer for f(2).

Feedback Tag

The *Feedback* tags reproduce the question, provide a model solution for the question and reproduce the content of the *Report* section. There are four *Feedback* tags and these can be used to provide additional marking analysis. In this research, one of these *Feedback* tags is used to provide CSE EFB for the detected CSEs. Figure 2.12 shows an example Feedback Report produced for an incorrect answer submission for the example question.

Having studied the current literature which is important to this thesis, the CSE Project at UWE Bristol started to develop a method to diagnose and remediate mathematical CSEs in e-Assessment questions using Dewis as a demonstration platform. A detailed methodology for providing CSE EFB is discussed in Chapter 3.

The Feedback RETRY

For this catalogue version, you scored 0 marks out of a maximum possible of 1 (0%).

The Question

The function $f(t) = 7u(t+5) - 3u(t-4)$

where $u(t)$ represents the unit step function.

Calculate the value of $f(2)$.

The Solution

We present two ways to look at this feedback:

Approach 1

We have $f(2) = 7u(2+5) - 3u(2-4)$ having replaced the t in the original expression by 2.

Hence, we have $f(2) = 7u(7) - 3u(-2)$.

We know that $u(7) = 1$ and $u(-2) = 0$ and hence we have:

$f(2) = 7(1) - 3(0) = 7$.

Approach 2

The unit step function satisfies the following:

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Hence, we have:

$$u(t+5) = \begin{cases} 0 & t < -5 \\ 1 & t \geq -5 \end{cases}$$

Similarly, we have:

$$u(t-4) = \begin{cases} 0 & t < 4 \\ 1 & t \geq 4 \end{cases}$$

Therefore $f(t) = 7u(t+5) - 3u(t-4)$ is given by:

$$f(t) = \begin{cases} 0 & t < -5 \\ 7 & -5 \leq t < 4 \\ 7-3 & t \geq 4 \end{cases}$$

Therefore, the solution is :
$f(2) = 7$

The Report

Your answer for $f(2)$ is 55.
Your answer is **not** correct.

You scored 0 marks for this question.

RETRY

Figure 2.12: Example of the Feedback Report

3 Methodology

This research, the CSE Project at UWE Bristol, takes the form of a case study. Both quantitative and qualitative research methods were used in this investigation. This chapter discusses the specific methodologies by which the different stages of this research and analyses were conducted.

3.1 Introduction to the CSE Project at UWE Bristol

The CSE Project at UWE Bristol began in 2017 with an aim of developing a technique to detect CSEs and to provide tailored feedback in Dewis e-Assessment questions, used in a first year EM module (Sikurajapathi, Henderson, and Gwynllyw, 2023; Sikurajapathi, Henderson, and Gwynllyw, 2022a; Sikurajapathi, Henderson, and Gwynllyw, 2022b; Sikurajapathi, Henderson, and Gwynllyw, 2021; Sikurajapathi, Henderson, and Gwynllyw, 2020).

This research sought to answer the following specific research questions:

1. How to detect mathematical CSEs in traditional assessments and e-Assessments?
2. What mathematical CSEs do first year EM students make in e-Assessment questions?
3. How to improve the e-Assessment feedback in order to address these CSEs?
4. How to measure the effectiveness of detecting and addressing CSEs in mathematical e-Assessment questions?

There are several benefits to answering these research questions. Even though this research has been done in a particular context using the Dewis e-Assessment System, the research outcomes contribute to the knowledge to inform more general practice in assessment and learning. For example, the collection of mathematical CSEs compiled during this research, which is published in UWE Bristol repository (Sikurajapathi, Henderson, and Gwynllyw, 2022a), is not only beneficial for first year EM students and lecturers, but also it is equally beneficial for secondary, and first year university level mathematics students and teachers. The CSE collection, titled 'Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)' can be used to correct students' mathematical misconceptions either in hand-written assessments or e-Assessment questions (Sikurajapathi, Henderson, and Gwynllyw, 2022a). The CSE Book can also be found in Appendix C of this thesis.

Further, this CSE detecting technique will be beneficial to several disciplines and organisations that either use Dewis or any other e-Assessment System which is capable of giving dynamic feedback based on a student's answer. The new knowledge raised from this research can be used in those e-Assessment Systems so that they may emulate a human marker, providing instant CSE EFB highlighting possible CSEs. This will help students correct their mathematical misconceptions. Also, teachers can use the findings to identify areas in which more help is needed in student learning. Integrating the research outcomes from the CSE Project into other e-Assessment Systems will be beneficial to generations to come (Sikurajapathi, Henderson, and Gwynllyw, 2023; Sikurajapathi, Henderson, and Gwynllyw, 2022a; Sikurajapathi, Henderson, and Gwynllyw, 2022b; Sikurajapathi, Henderson, and Gwynllyw, 2021; Sikurajapathi, Henderson, and Gwynllyw, 2020).

3.1.1 The Structure of the CSE Project at UWE Bristol

The overall structure of the CSE Project takes the form of five stages:

1. Stage One: Data (CSEs) Collection;
2. Stage Two: CSE Code Development;
3. Stage Three: CSE Code Trial Phase;
4. Stage Four: Students' Perceptions on CSE EFB;
5. Stage Five: Impact of the CSE Project.

The following sections describe and discuss the methodologies used in the five stages of this investigation.

3.2 CSE Project Stage One: Data (CSEs) Collection

This section discusses the methodology used in collecting first year EM CSEs from e-Assessment questions.

3.2.1 Primary data set overview

The first year EM module was selected to collect CSEs for this research. There are several reasons for selecting this particular module for this study. EM was one of the longest running modules at UWE Bristol which used Dewis to deliver all of its e-Assessments since 2009 (Gwynllyw and Henderson, 2012; Gwynllyw and Henderson, 2009). Also, I had been conducting the tutorial classes for this module for a year when I first started this research. Therefore, I had a good understanding of the subject content, assessments and e-Assessment questions of this module. Further, while working with students at tutorial classes, I had already gained a clear idea as to which kinds of CSEs the students tended to make when solving a problem. Therefore, selecting the EM module for data collection was most appropriate at the time this research was started.

A grade C in A-level mathematics or other equivalent qualification in mathematics is one of the entry requirements for all undergraduate engineering degrees at UWE Bristol. In this EM module, although the majority of students had a grade C or higher in A-level mathematics there was a substantial proportion of students who had equivalent qualifications such as BTEC, Foundation Engineering degrees, Access courses and international qualifications.

EM was a 30-credit module making up a quarter of the credit for the first year (Level 4) and was delivered to a large and diverse student cohort at UWE Bristol. Students learnt mathematical techniques that would support their engineering studies, including learning to program in Matlab. As well as the Matlab weekly PC sessions in Semester 1, all students received two hours of lectures, supported by a one-hour tutorial each week. In addition, all students had a scheduled weekly two-hour Peer Assisted Learning (PAL) session (Falchikov, 2001). These were run by second year (Level 5) PAL tutors in which whole course support was offered.

The module was assessed through coursework (25%) and examination (75%). The coursework was designed to encourage engagement in the module. The Matlab assignment comprises 50% of the coursework mark, whilst e-Assessments delivered throughout the year comprise the remaining 50% coursework mark. Revision e-Assessments were made available to students a few weeks before the e-examination, and students were allowed unlimited attempts on these revision tests.

Since EM started to deliver e-Assessments using Dewis, a substantial library of Dewis questions to support the teaching of EM has been built. This question library resource has enabled the module team to try out different delivery patterns of e-

Assessment in order to improve year-long student engagement with the module and hence improve attainment levels.

Since the 2015-2016 academic year, 22 weekly e-Assessments had been used on the module and students were given access to these e-Assessments throughout the year and were allowed unlimited attempts. The e-Assessment coursework mark was calculated from the top 20 marks from the 22 weekly tests (twelve tests in Semester 1 and ten in Semester 2). All weekly tests were open from the start of the module.

In January, at the end of the first semester, students were required to take a two-hour e-examination, sat under controlled conditions and questions on this e-examination were based on the questions students had already encountered in their weekly tests (Henderson, Gwynllyw, and Hooper, 2016). Due to the lack of available computers, this January e-examination was delivered in two sessions. Approximately half the students were timetabled for the morning session and the other half for the afternoon.

For each separate run of the e-examination, the parameters of the questions were fixed in order to ensure fairness. This approach also meant that, at the start of the e-examination, students were given a hard-copy of the specific questions that they were attempting. Students valued this, as some found it easier to work from a paper copy than from the screen.

Both e-examination versions contained a mixture of input types: numerical, algebraic and drop-down. The question structure and subject content were the same for both papers but different numeric parameters were used in each case to make the two e-examinations different but of comparable difficulty.

For this research, CSEs were collected from the 2017-18 and 2018-19 e-examinations.

3.2.2 Data collection and protection procedure

For the 2017-18 e-examination, a total of 298 students sat the e-examination, 148 in the morning and 150 in the afternoon. A total of 321 students sat the 2018-19 e-examination, 168 in the morning and 153 in the afternoon.

Each e-examination (2017-18 and 2018-19) contained 19 questions. The official submission was electronic but students were given examination booklets in order to write their rough workings to questions and these booklets were collected at the end of the e-examination.

The first step in identifying mathematical CSEs was to analyse the submissions from the January e-examination. All data relating to every assessment attempt is recorded on the Dewis server. This feature on Dewis enables academics to efficiently track how a student or cohort of students has performed on a particular e-Assessment.

Prior to commencing the study, to protect students' identities, their student identification number (ID) were pseudonymised according to General Data Protection Regulation (GDPR) compliance rules. My supervisor (Dr. Karen Henderson) and I had a meeting with a GDPR officer at UWE Bristol to seek expert advice to confirm that this pseudonymisation procedure satisfies GDPR requirements (Sikurajapathi et al., 2019). Having obtained the GDPR guidance, the students' IDs in the rough working booklets and the Dewis-stored data were pseudonymised.

The next step was to use the Dewis Reporter output to select the most common incorrect answers (MCIAs) to each question on the 2017-18 and 2018-19 e-examinations. Having completed the identification of the MCIAs, the written answer scripts of the students who submitted each of the MCIAs were carefully examined. The aim of this process was to understand what kind of mistake led the students to arrive at those MCIAs. Having access to the students' workings was invaluable for this process.

For each MCIA, the CSE percentage is calculated as follows:

$$\text{CSE percentage} = \frac{\text{Number of CSE answers}}{\text{Number of incorrect answers}}\%$$

If the CSE percentage is 4% or more, then that MCIA is considered as a CSE in this study.

Altogether 65 CSEs were found in 27 questions. The names of Dewis questions, their identification codes, number of identified CSEs of each question and their identification codes are shown in Table 3.1. These names of Dewis questions and identification codes will be used in this thesis.

Table 3.1: Names of Dewis questions, their identification codes, number of identified CSEs for each question and their identification codes

Dewis question name	**ID**	**No. of CSE**	**CSE ID**
ALGEBRA_COMPLETESQUARE01	Q1	3	Q1 CSE1

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Table 3.1 *Continued from previous page*

Dewis question name	**ID**	**No. of CSE**	**CSE ID**
			Q1 CSE2
			Q1 CSE3
FUNCTIONS_UNITSTEP01	Q2	1	Q2 CSE1
FUNCTIONS_TRIGONOMETRY01	Q3	2	Q3 CSE1
			Q3 CSE2
FUNCTIONS_TRIGONOMETRY02	Q4	1	Q4 CSE1
FUNCTIONS_TRIGONOMETRY03	Q5	2	Q5 CSE1
			Q5 CSE2
CALCULUS_DIFFERENTIATION_STANDARD02	Q6	2	Q6 CSE1
			Q6 CSE2
CALCULUS_INTEGRATION_MEANVALUE01	Q7	3	Q7 CSE1
			Q7 CSE2
			Q7 CSE3
CALCULUS_INTEGRATION_MEANVALUE02	Q8	2	Q8 CSE1
			Q8 CSE2
COMPLEXNUMBERS_CARTESIANMODULUS01	Q9	1	Q9 CSE1
COMPLEXNUMBERS_CARTESIANDIVISION01	Q10	1	Q10 CSE1
COMPLEXNUMBERS_POLARDIVISION01	Q11	4	Q11 CSE1
			Q11 CSE2
			Q11 CSE3
			Q11 CSE4
COMPLEXNUMBERS_CARTESIANTOPOLAR01	Q12	4	Q12 CSE1
			Q12 CSE2
			Q12 CSE3
			Q12 CSE4
SERIES_GEOMETRIC01	Q13	1	Q13 CSE1
SERIES_MACLAURIN01	Q14	3	Q14 CSE1
			Q14 CSE2
			Q14 CSE3
ENGINEERING_CENTREMASS01	Q15	2	Q15 CSE1
			Q15 CSE2

Continued on next page

Table 3.1 *Continued from previous page*

Dewis question name	ID	No. of CSE	CSE ID
CALCULUS_INTEGRATION _VOLUMEREVOLUTION01	Q16	6	Q16 CSE1
			Q16 CSE2
			Q16 CSE3
			Q16 CSE4
			Q16 CSE5
			Q16 CSE6
ENGINEERING_DIMENSIONS01	Q17	3	Q17 CSE1
			Q17 CSE2
			Q17 CSE3
CALCULUS_DIFFERENTIATION_STANDARD01	Q18	1	Q18 CSE1
CALCULUS_DIFFERENTIATION_CHAINRULE01	Q19	2	Q19 CSE1
			Q19 CSE2
CALCULUS_DIFFERENTIATION_CHAINRULE02	Q20	2	Q20 CSE1
			Q20 CSE2
CALCULUS_DIFFERENTIATION_PRODUCTRULE01	Q21	1	Q21 CSE1
CALCULUS_DIFFERENTIATION_IMPLICT01	Q22	3	Q22 CSE1
			Q22 CSE2
			Q22 CSE3
CALCULUS_PARTIAL_DIFFERENTIATION01	Q23	4	Q23 CSE1
			Q23 CSE2
			Q23 CSE3
			Q23 CSE4
CALCULUS_PARTIAL_DIFFERENTIATION02	Q24	2	Q24 CSE1
			Q24 CSE2
SERIES_BINOMIAL01	Q25	2	Q25 CSE1
			Q25 CSE2
CALCULUS_INTEGRATION_PARTS01	Q26	4	Q26 CSE1
			Q26 CSE2
			Q26 CSE3
			Q26 CSE4
CALCULUS_INTEGRATION_SUBSTITUTION01	Q27	3	Q27 CSE1

Continued on next page

Table 3.1 *Continued from previous page*

Dewis question name	**ID**	**No. of CSE**	**CSE ID**
			Q27 CSE2
			Q27 CSE3

To compare the statistical measures of consistency of the number of students who did the respective CSE across the years, the rate of each CSE answer out of total number of students who sat the e-examinations (CSE Rate) in each year was calculated as follows:

$$\text{CSE Rate} = \frac{\text{Number of CSE answers}}{\text{Total number of students who sat the e-examination}}$$

The summary of the number of incorrect answers, number of CSEs found, their CSE percentage and CSE Rate for each CSE are documented in Table 3.2.

Statistical measures of consistency on the CSE Rate across the years 2017-18 and 2018-19 (presented in Table 3.2) were carried out using 'the partially overlapping samples t-test' in the R package (Derrick, Toher, and White, 2017; R Statistical Software, 2023). This test performs a comparison of means using the partially overlapping t-test, for two samples each with paired and unpaired observations. This test was chosen to perform the statistical measures of consistency because the CSE Rates across the years were either paired or unpaired. This R function calculates the test statistic, the degrees of freedom, and the p-value. Statistically, if the p-value is greater than 0.05, then there is no evidence of a difference between the two samples. On the other hand, if the p-value is less than or equal to 0.05, then there is evidence of a significant difference between the two samples (Derrick, Toher, and White, 2017; Watson and Petrie, 2010).

This test was carried out on the CSE Rate of the e-examinations in the 2017-18 and 2018-19. It should be noted that, 298 students sat the 2017-18 e-examination, and 321 students sat the 2018-19 e-examination. The p-value for the two groups is 0.2046. This suggests that there is no evidence that there is a difference between the number of each CSEs made by the students in the e-examinations in 2017-18 and 2018-19.

Table 3.2: Summary of CSEs found during the CSE Project at UWE Bristol

CSE ID	2017-2018 (298)				2018-2019 (321)			
	No. of incorrect answers	**No. of CSE answers**	**CSE %**	**CSE Rate**	**No. of incorrect answers**	**No. of CSE answers**	**CSE %**	**CSE Rate**
Q1 CSE1	56	28	50%	0.09	57	33	58%	0.10
Q1 CSE2	45	6	13%	0.02	51	14	27%	0.04
Q1 CSE3	26	4	15%	0.01	24	2	8%	0.01
Q2 CSE1	86	35	41%	0.12	100	32	32%	0.10
Q3 CSE1	13	5	38%	0.02	19	9	47%	0.03
Q3 CSE2					29	7	24%	0.02
Q4 CSE1	61	37	61%	0.12				
Q5 CSE1					81	47	58%	0.15
Q5 CSE2					84	9	11%	0.03
Q6 CSE1					33	7	21%	0.02
Q6 CSE2					33	5	15%	0.02
Q7 CSE1	183	52	28%	0.17				
Q7 CSE2	183	44	24%	0.15				
Q7 CSE3	183	8	4%	0.03				

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Table 3.2 *Continued from previous page*

CSE ID	2017-2018 (298)				2018-2019 (321)			
	No. of incorrect answers	No. of CSE answers	CSE %	CSE Rate	No. of incorrect answers	No. of CSE answers	CSE %	CSE Rate
Q8 CSE1					121	10	8%	0.03
Q8 CSE2					121	10	8%	0.03
Q9 CSE 1	57	40	70%	0.13				
Q10 CSE1	109	9	8%	0.03	95	9	9%	0.03
Q11 CSE1	197	66	34%	0.22				
Q11 CSE2	197	9	5%	0.03				
Q11 CSE3	197	17	9%	0.06				
Q11 CSE4	73	11	15%	0.04	92	14	15%	0.04
Q12 CSE1					88	19	22%	0.06
Q12 CSE2					88	13	15%	0.04
Q12 CSE3					88	14	16%	0.04
Q12 CSE4					88	11	13%	0.03
Q13 CSE1	67	34	51%	0.11				
Q14 CSE1	116	28	24%	0.09	122	29	24%	0.09

Continued on next page

Table 3.2 *Continued from previous page*

CSE ID	2017-2018 (298)				2018-2019 (321)			
	No. of incorrect answers	No. of CSE answers	CSE %	CSE Rate	No. of incorrect answers	No. of CSE answers	CSE %	CSE Rate
Q14 CSE2	59	11	19%	0.04	80	16	20%	0.05
Q14 CSE3					53	5	9%	0.02
Q15 CSE1	28	7	25%	0.02	64	12	19%	0.04
Q15 CSE2					64	11	17%	0.03
Q16 CSE1	107	9	8%	0.03	135	3	2%	0.01
Q16 CSE2	107	9	8%	0.03	135	13	10%	0.04
Q16 CSE3	107	9	8%	0.03	135	8	6%	0.02
Q16 CSE4	107	7	7%	0.02				
Q16 CSE5	107	5	5%	0.02	135	7	5%	0.02
Q16 CSE6					135	6	4%	0.02
Q17 CSE1					13	6	46%	0.02
Q17 CSE2					46	9	20%	0.03
Q17 CSE3					20	8	40%	0.02
Q18 CSE1	59	30	51%	0.10				

Continued on next page

Table 3.2 *Continued from previous page*

CSE ID	2017-2018 (298)				2018-2019 (321)			
	No. of incorrect answers	No. of CSE answers	CSE %	CSE Rate	No. of incorrect answers	No. of CSE answers	CSE %	CSE Rate
Q19 CSE1	56	7	13%	0.02				
Q19 CSE2	56	10	18%	0.03				
Q20 CSE1	73	6	8%		0.02			
Q20 CSE2	73	22	30%	0.07				
Q21 CSE1					73	13	18%	0.04
Q22 CSE1	158	18	11%	0.06	180	15	8%	0.05
Q22 CSE2	158	12	8%	0.04	180	5	3%	0.02
Q22 CSE3	158	10	6%	0.03	180	2	1%	0.01
Q23 CSE1	103	17	17%	0.06				
Q23 CSE2	103	7	7%	0.02				
Q23 CSE3	103	6	6%	0.02				
Q23 CSE4	103	5	5%	0.02				
Q24 CSE1					145	32	22%	0.10
Q24 CSE2					145	11	8%	0.03

Continued on next page

Table 3.2 *Continued from previous page*

CSE ID	2017-2018 (298)				2018-2019 (321)			
	No. of incorrect answers	No. of CSE answers	CSE %	CSE Rate	No. of incorrect answers	No. of CSE answers	CSE %	CSE Rate
Q25 CSE1					165	9	5%	0.03
Q25 CSE2					165	8	5%	0.02
Q26 CSE1	143	13	9%	0.04				
Q26 CSE2	143	13	9%	0.04				
Q26 CSE3	143	11	8%	0.04				
Q26 CSE4	143	8	6%	0.03				
Q27 CSE1					192	19	10%	0.06
Q27 CSE2					192	14	7%	0.04
Q27 CSE3					192	10	5%	0.03

3.2.3 Categorisation of mathematical CSEs in e-Assessments

All of the CSEs found over the course of the CSE project were documented in a systematic order in the CSE book together with their mathematical taxonomy coding. The general taxonomy proposed by Ford, Gillard, and Pugh (2018) was used to select and categorise only those CSEs which are relevant to e-Assessment. The CSEs found during the CSE Project fall into just four of the error categories (S, U, CM and UM) based on the Ford, Gillard, and Pugh (2018) taxonomy (see Section 2.1.5). A detailed discussion on analysing these CSEs is discussed in Chapter 4.

3.2.4 CSE recording template

Each CSE found to date has been recorded using the template as shown in Figure 3.1. The template contains seven areas and each area and its contents are described in detail below.

Area 1: The link to the online Dewis e-Assessment question is available here. The reader may try out the question by clicking the Question hyperlink. By attempting the question and answering with a relevant CSE response, it is possible to see how Dewis detects the CSE and provides instant tailored feedback to address the CSE made.

Area 2: In this area, a screen-shot of the Dewis question is given.

Area 3: The correct solution to the question is presented in brief here.

Area 4: The taxonomy code of the CSE, which is presented in Area 5, is given here (see Section 4.1 for the taxonomy coding).

Area 5: A sample of the CSE and the incorrect answer(s) that led from it is presented here. At the top of this area, the CSE error is summarised by a statement which is presented in red text. Then, the detailed steps of the exact way the CSE is made and the solution as written by students in their rough work booklets is presented. A tilde ($\sim$) on the CSE answer was used to differentiate it from the correct answer. For example, in Figure 3.1, the CSE answer for this question is denoted as, $\tilde{f}(2) = 55$, in red text.

Area 6: In this section, the number of CSE answers made, the total incorrect answers made in the question and the CSE percentage for each year are presented as No. of CSEs / No. of incorrect answers (CSE %). For example, in

Figure 3.1, in the 2017-18 exam, this particular CSE was made by 35 out of the 86 students who gave an incorrect answer to this question; therefore, the CSE percentage is 41%. This data is presented in this area as 35/86 (41%). Similarly, the data for 2018-19 is presented as 32/100 (32%).

Area 7: The academic year that the data was collected from is presented here. Figure 3.1 shows that 35/86 (41%) and 32/100 (32%) presented in Area 6 relate to the years 2017-18 and 2018-19 presented in Area 7 respectively.

3.3 CSE Project Stage Two: CSE Code Development

In this section, the methodology used to amend the original question code in order to capture CSEs is discussed. The answers (inputs) to the questions were either integer, floating-point or algebraic, and the amendments were done differently depending on the input type. Three questions, which have 6 identified CSEs, had integer inputs. Answers to 14 questions, which have 35 identified CSEs, were floating-point inputs. The remaining 10 questions, which have 24 CSEs, had algebraic inputs (see Table B.1 in Appendix B for more details).

During the course of the CSE Project, 27 original Dewis questions were amended in order to capture 65 CSEs in total. All these CSEs were taxonomically and systematically documented in Sikurajapathi, Henderson, and Gwynllyw (2022a). These amended questions are stored in Dewis under the folder *cse_project_book* and individual questions can be attempted via the links given in Sikurajapathi, Henderson, and Gwynllyw (2022a). A summary of the questions, CSE percentages and where to find them in the CSE Book (Sikurajapathi, Henderson, and Gwynllyw, 2022a) can be found in Table B.1 in Appendix B.

One example of each input category is selected in the following three subsections to discuss how the question codes were amended to capture integer, floating-point and algebraic CSEs answers respectively.

3.3.1 Integer answer capture

In this section, the code used to capture CSEs of integer answers is discussed. For this discussion, the question presented in Figure 3.2, which was given in the morning session of the January e-examination in the 2017-18 academic year, is chosen.

Question ①			
The function $f(t) = 7u(t+5) - 3u(t-4)$ where $u(t)$ represents the unit step function. Calculate the value of $f(2)$. ② Enter $f(2)$:			
Correct Solution			
$f(t) = 7u(t+5) - 3u(t-4)$ $f(2) = 7u(2+5) - 3u(2-4)$ $= 7u(7) - 3u(-2)$ $= 7 \times 1 - 3 \times 0$ $f(2) = 7$ ③			
CSE 1 related to this question	**CSE Taxonomy Code:**	**U1** ④	
Answer was derived by assuming $u = 1$ and not a function. $f(t) = 7u(t+5) - 3u(t-4)$ $f(2) = 7u(7) - 3u(-2)$ $\tilde{f}(2) = 7(7)u - 3(-2)u$ $\tilde{f}(2) = 49u + 6u$ $\tilde{f}(2) = 55u$ since $u = 1$ $\tilde{f}(2) = 55$ ⑤			
No. of CSEs /No. incorrect answers (CSE %)	35/86 (41%) 32/100 (32%) ⑥	**Date collected**	2017-18 2018-19 ⑦

Figure 3.1: CSE recording template of Q2 CSE1 related to FUNCTIONS_UNIT-STEP01 question on Dewis which involves an integer answer

The same question, but with different parameters, was given in the afternoon session of the January e-examination in the 2017-18 academic year and is shown in Figure 3.3. Only one CSE was identified for this question.

A brief discussion about the question requiring an integer answer and its CSE answer

This question (See Figures 3.2 and 3.3) involves finding the value of $f(c)$ for the given function $f(t) = Au(t-a) + Bu(t-b)$ where $u(t)$ represents the Unit Step Function. The values of A, B, a, b and c are supplied to the student and the student is required to calculate the value of $f(c)$.

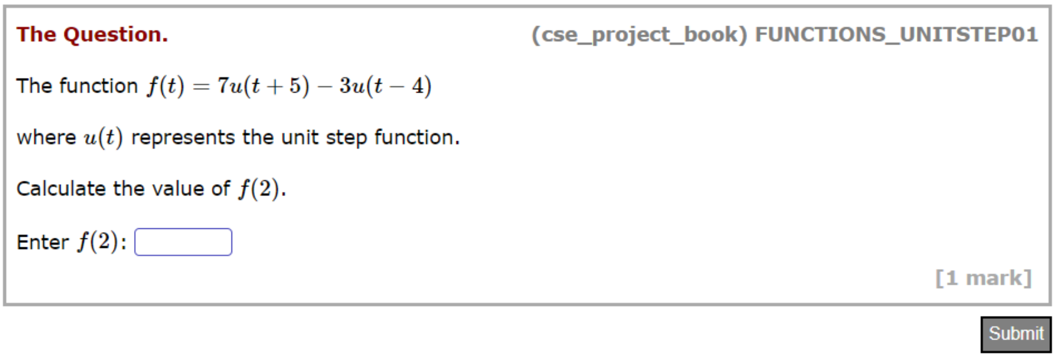


Figure 3.2: Dewis question requiring an integer answer given in the morning session of the 2017-18 e-examination

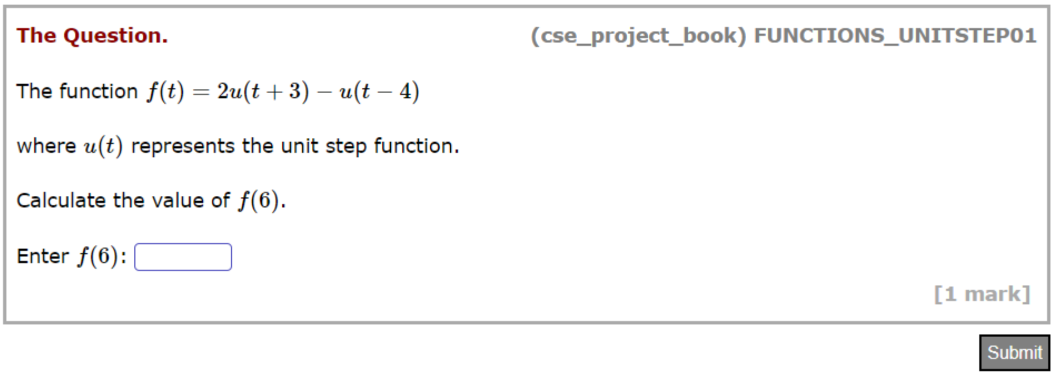


Figure 3.3: Dewis question requiring an integer answer given in the afternoon session of the 2017-2018 e-examination

The correct answer for the question presented in Figure 3.2 is $f(2) = 7$ and for the question question presented in Figure 3.3 is $f(6) = 1$. Finding the students who attempted the question but answered incorrectly, as opposed to those students who did not answer the question, using the Dewis reporter was straightforward. In both cases students scored zero marks but as mentioned in Section 2.5, since the PI was allocated a value representing the marking outcome, all that was required was to list those students who received a PI value of 0 (answered incorrectly).

Dewis-stored data indicated that, in the morning session, 12 out of the 44 students who gave an incorrect answer to the question (see Figure 3.2) gave the answer as $f(2) = 55$. Similarly, Dewis-stored data indicated that, in the afternoon session, 23 out of the 42 students who gave an incorrect answers to the question (see Figure 3.3) gave the answer as $f(2) = 16$.

Careful examination of the students' hand-written scripts indicated that these students incorrectly assumed that u is not a function but is equal to 1. Accordingly, the CSE caused them to arrive at the incorrect answers of $\tilde{f}(2) = 7\times1\times(2+5)-3\times1\times(2-4) = 55$ and $\tilde{f}(6) = 2\times1\times(6+3)-1\times(6-4) = 16$ for the morning session (see Figure 3.2) and the afternoon session (see Figure 3.3) respectively.

Therefore, altogether 35 out of 86 students who gave an incorrect answer to either the morning or afternoon questions incorrectly assumed that u is not a function but is equal to 1. This means the CSE percentage for the 2017-18 academic year is 41%.

The same question, but with different parameters, was given in the 2018-19 e-examination. Dewis data and students' hand-written scripts indicated that the same CSE was made by 16 out of the 45 students who gave an incorrect answer to the question in the morning session. The same CSE was made by 16 out of the 55 students students who gave an incorrect answer to the question in the afternoon session. Therefore, altogether 32 out of the 100 students who gave an incorrect answer to either the morning or afternoon question, incorrectly assumed that u is not a function but is equal to 1. This means the CSE percentage for the 2018-19 academic year is 32%.

All of the information related to this CSE, which is presented in section 3.1.2 in the CSE Book Sikurajapathi, Henderson, and Gwynllyw (2022a), is shown in Figure 3.1.

Editing the original question code requiring an integer answer to capture the CSE

Having identified the CSE involved in the question discussed above (see Figure 3.2 and Figure 3.3), the original question code was amended to capture and report on this CSE. The original code of this question, which is presented in Section 2.5, is referred to here in order to discuss the amendments made to capture and report on the CSE. In this methodology, the main changes occur in *Settings*, *Solution*, *Marking*, *Report*, and *Feedback (3)* Tags and these amendments are discussed in the following sections.

Editing the *Settings* Tag of the question to capture the integer CSE

The *Settings* Tag in the original question is shown in Figure 2.4. To capture the CSE answer in this question, one more PI was needed. Therefore, the < NUM_FLAGS > in the original question (which is one) was changed to two in the *Settings* Tag. The *Settings* Tag of the amended code is shown in Fiqure 3.4.

```
<SETTINGS>

     <NUM_INPUTS>1
     <NUM_FLAGS>2
     <USES_MATHML>1
     <USES_SVG>0

</SETTINGS>
```

Figure 3.4: Amended *Settings* Tag for FUNCTIONS_UNITSTEP01 question on Dewis

Editing the *Solution* Tag of the question to capture the integer CSE

The *Solution* Tag in the original question is shown in Figure 2.8. To calculate the CSE answer in this question, one GLOBAL parameter `$x1` was introduced as shown in the Figure 3.5. The CSE answer was stored as `$x1` and was calculated as `$x1 = $p4*($p3-$p1) + $p5*($p3-$p2)`. That is the CSE answer result from setting the Unit Step Function equal to the value of 1 and t equal to `$p3` in the given function $f(t)$.

Editing the *Marking* Tag of the question to capture the integer CSE

The *Marking* Tag in the original question is shown in Figure 2.9. To mark the CSE answer of this question, the second flag set in the *Settings* section was utilised. The marking of the CSE answer was carried out as shown in Figure 3.6 using the < MARK_INTEGER FLAG > command. Here the students' answer `$a1` was marked against the CSE answer `$x1` and the outcome is stored in `$f2`.

```
<SOLUTION>

<GLOBAL> $x1 </GLOBAL>

$c1 = $p4*unitStep($p3-$p1) + $p5*unitStep($p3-$p2);

sub unitStep
{
    # passing argument
    my $arg = $_[0];
    # calculate the unit step function of $arg
    my $value = ($arg>=0)?1:0;
    return ($value);
}

#=====================================================
#CSE1 Answer
#CSE answer resulted substituting u=1 and
#t=$p3 in f(t)=p4*u(t-p1)+p5*u(t-p2)
#=====================================================

$x1= ($p4*($p3-$p1)) + ($p5* ($p3-$p2));

</SOLUTION>
```

Original code

CSE code

Figure 3.5: Amended *Solution* Tag for FUNCTIONS_UNITSTEP01 question on Dewis

```
<MARKING>

<MARK_INTEGER FLAG='$f1' ANS='$a1' COR='$c1'>

#================================================
#CSE1 MARKING
#================================================

<MARK_INTEGER FLAG='$f2' ANS='$a1' COR='$x1'>

</MARKING>
```

Original code

CSE code

Figure 3.6: Amended *Marking* Tag for FUNCTIONS_UNITSTEP01 question on Dewis

Editing the *Report* Tag of the question to capture the integer CSE

The *Report* Tag in the original question is shown in Figure 2.11. The *Report* Tag of the CSE code was written as shown in Figure 3.7. A conditional statement was used to notify to the student the possible CSE which may have caused their incorrect answer. Figure 3.8 shows the Report output when the CSE answer for the question

```
<REPORT>

<PRINT_REPORT FLAG='$f1' TEXT="<MATH> f($p3) </MATH>"
ANS=' $a1' >
#===================================================
# REPORT when CSE1 is triggered
#===================================================

if($f1 == 0 && $f2 > 0)
{
     <PRINT>
<div style='font-size:100%;font-weight:normal;border:2px
orangered
solid;padding:10px;margin:10px;width:95%;background-
color:white;color:white;line-height:20px'>

<span style='color:blue'> We note that your incorrect
answer seems to have been caused by a misunderstanding of
the unit step function.  <p></p>
Please refer to the report section of the feedback to see
what could have gone wrong when you answered the question
</span>.<p></p>
</div>
     </PRINT>
}

</REPORT>
```

Original code

CSE code

Figure 3.7: Amended *Report* Tag for FUNCTIONS_UNITSTEP01 question on Dewis

shown in Figure 3.2, ($\tilde{f}(2) = 55$), is submitted.

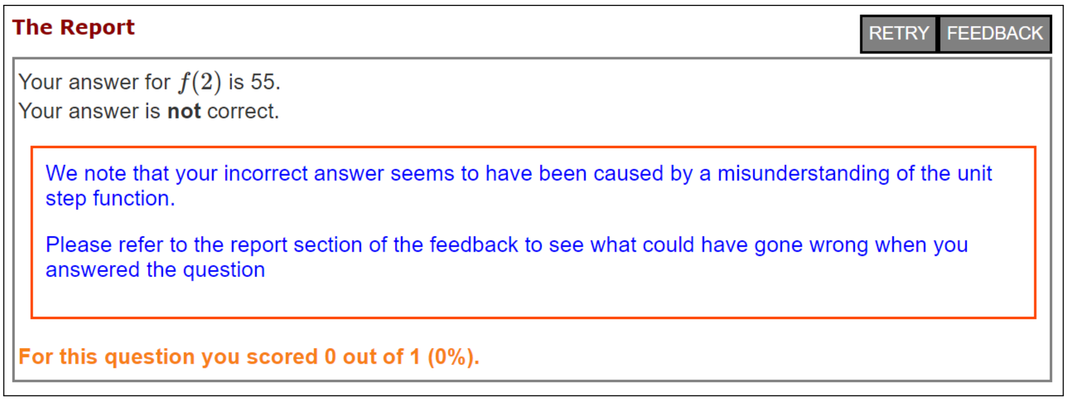


Figure 3.8: Amended Report Output for the FUNCTIONS_UNITSTEP01 question shown in Figure 3.2

Editing the *Feedback (3)* Tag of the question to capture the integer CSE

The *Feedback (3)* Tag was used to provide CSE EFB for the detected CSEs. Figure 3.9 shows the CSE EFB given to the students when the CSE answer for the question show in Figure 3.2, ($\tilde{f}(2) = 55$), is submitted. The CSE EFB for this ques-

The Report

Your answer for $f(2)$ is 55.
Your answer is **not** correct.

Please note that your answer seems to have been derived by assuming that u equals to 1. This is incorrect.

Please note $u(t)$ is the **unit step function,** which satisfies the following:

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

That means

- when t is negative, $u(t) = 0$, and
- when t is greater than or equal to zero, $u(t) = 1$.

We have been given the function,

$f(t) = 7u(t+5) - 3u(t-4) \longrightarrow$ Ⓐ

Substitute $t = 2$ into Ⓐ to find $f(2)$.

This gives,

$f(2) = 7u(2+5) - 3u(2-4)$

Hence, we have $f(2) = 7u(7) - 3u(-2)$

We know that,

- when t is negative $u(t) = 0$, and
- when t is greater than or equal to zero $u(t) = 1$.

So, $u(7) = 1$ and $u(-2) = 0$ and hence we have,

$f(2) = 7u(7) - 3u(-2) = 7 \times 1 - 3 \times 0 = 7.$

You scored 0 marks for this question.

Figure 3.9: CSE EFB Output for FUNCTIONS_UNITSTEP01 question shown in Figure 3.2.

tion was prepared in detail as shown in Figure 3.9. The colour blue was selected as the text colour for the CSE EFB to get the attention of the student. At the beginning of the CSE EFB, what might have gone wrong when solving the question was emphasised in a red box. Additionally, the definition of the Unit Step Function was recalled and the value of the function $u(t)$ for different values of t is explained in a simplified way.

Underneath the red box, a detailed solution of the question was presented. Here, some standard procedures for solving mathematical questions were employed such as labelling equations and referring to them in the solution.

3.3.2 Floating-point answer capture

In this section the codes used to capture CSEs of floating-point answers are discussed. For this discussion the question presented in Figure 3.10, which has floating-point answers, is used. This question was given in the morning session of the January e-examination in the 2017-18 academic year.

The same question, but with different parameters, was given in the afternoon session of the January e-examination in the 2017-18 academic year and is shown in Figure 3.11.

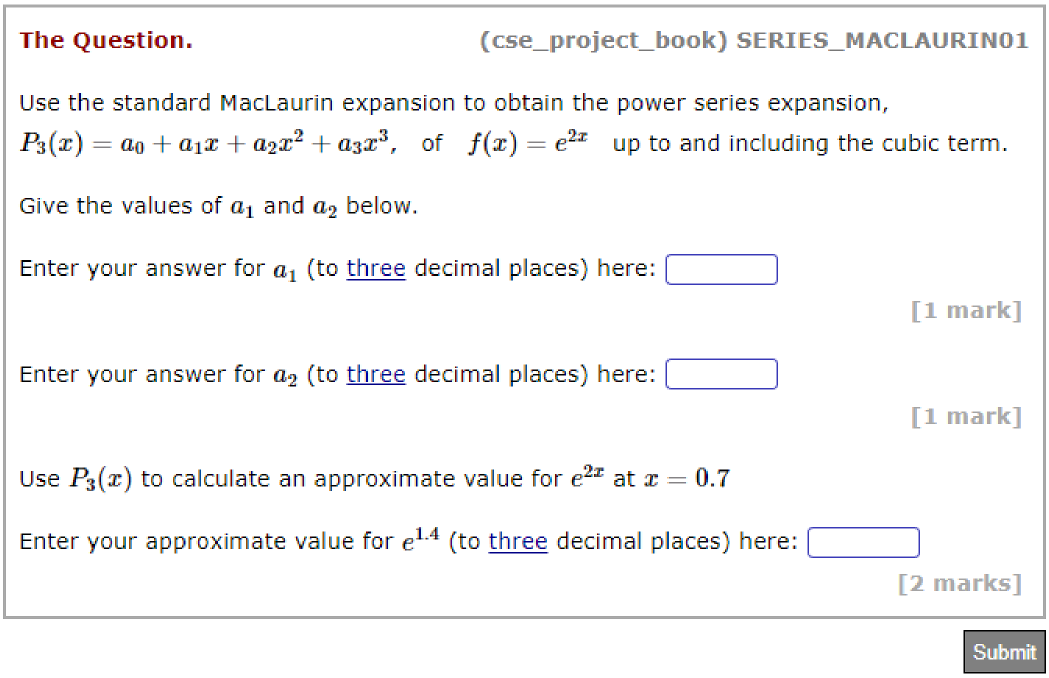
A Brief Discussion about the question requiring a floating-point answer and its CSE answer

This question (See Figures 3.10 and 3.11) involves using the standard Maclaurin expansion to obtain the power series expansion $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ of a given exponential function, e^{ax}. Students are required to calculate the values of a_0, a_1, a_2 and a_3, and then calculate the approximate value of e^{ax} at a given value of x correct to three decimal places. Three CSEs were identified for this question. For this discussion, only the CSE which involves giving the exact value instead of the approximate value of e^{ax} at a given x value is considered.

The correct answer for the question presented in Figure 3.10 is $P_3(0.7) = 3.837$ and for the question presented in Figure 3.11 is $P_3(0.5) = 4.188$. Those students who answered the question incorrectly were identified by checking the students who received a PI value of 0 for `$f3` as opposed to those who received 0 marks for the third answer of the question.

Using the Dewis-stored data it was found that in the morning session, 12 out of the 59 students who gave an incorrect answer to the question (see Figure 3.10) gave the answer as $P_3(0.7) = 4.055$. Similarly, in the afternoon session, 16 out of the 57 students who gave an incorrect answer to the question (see Figure 3.11) entered the answer as $P_3(0.5) = 4.482$.

The reason for these MCIAs were identified by examining the students' handwritten scripts. It was found that, those students did not use $P_3(x)$ to calculate their answer. Some of them just wrote the CSE answer on their script and others had



The Question. (cse_project_book) SERIES_MACLAURIN01

Use the standard MacLaurin expansion to obtain the power series expansion,
$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, of $f(x) = e^{2x}$ up to and including the cubic term.

Give the values of a_1 and a_2 below.

Enter your answer for a_1 (to three decimal places) here:

[1 mark]

Enter your answer for a_2 (to three decimal places) here:

[1 mark]

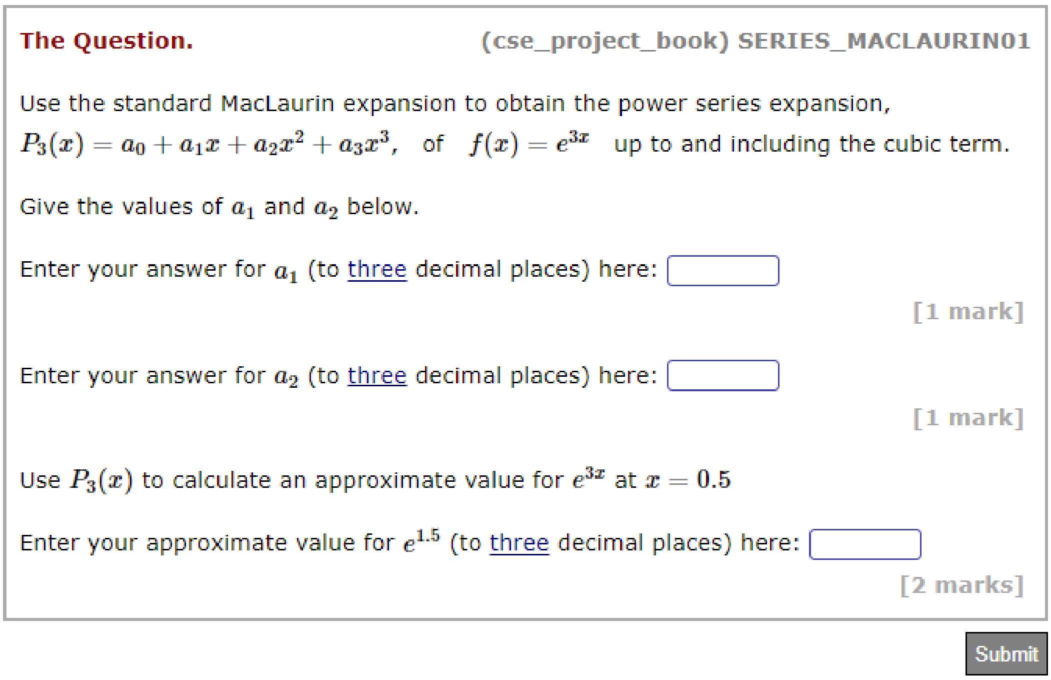
Use $P_3(x)$ to calculate an approximate value for e^{2x} at $x = 0.7$

Enter your approximate value for $e^{1.4}$ (to three decimal places) here:

[2 marks]

Submit

Figure 3.10: Dewis question requiring floating-point answers given in the morning session of the 2017-18 e-examination



The Question. (cse_project_book) SERIES_MACLAURIN01

Use the standard MacLaurin expansion to obtain the power series expansion,
$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, of $f(x) = e^{3x}$ up to and including the cubic term.

Give the values of a_1 and a_2 below.

Enter your answer for a_1 (to three decimal places) here:

[1 mark]

Enter your answer for a_2 (to three decimal places) here:

[1 mark]

Use $P_3(x)$ to calculate an approximate value for e^{3x} at $x = 0.5$

Enter your approximate value for $e^{1.5}$ (to three decimal places) here:

[2 marks]

Submit

Figure 3.11: Dewis question requiring floating-point answers given in afternoon session of the 2017-18 e-examination

not written any answers on their script. A highly anticipated reason for this MCIA was that the exact value of e^{ax} was given instead of the approximate value of e^{ax} at

$x = c$. As it was expected, the exact values of $e^{1.4}$ and $e^{1.5}$ to three decimal places correspond to, $e^{1.4} = 4.055$ and $e^{1.5} = 4.482$ respectively.

Altogether 28 out of 116 students who gave an incorrect answer to either the morning or afternoon questions incorrectly, entered the exact value instead of the approximate value of e^{ax} at the given value of x. This means that the CSE percentage of this question for the 2017-18 academic year is 24%.

The same question, but with different parameters, was given in the 2018-19 e-examination. Using Dewis data and students' hand-written scripts it was found that the same CSE was made by 16 out of the 57 students who gave an incorrect answer to the question in the morning session. The same CSE was made by 13 out of the 65 students who gave an incorrect answer to the question in the afternoon session. Therefore, altogether 29 out of the 122 students made this CSE in the 2018-19 e-examination, which means the CSE percentage for the 2018-19 academic year is also 24%.

All of the information related to this CSE, which is presented in section 4.1.3 in the CSE Book (Sikurajapathi, Henderson, and Gwynllyw, 2022a), is shown in Figure 3.12.

Editing the original question code requiring a floating-point answer to capture the CSE

The original question code was amended to capture and report on this CSE. In this methodology, the main amendments were done in the *Settings*, *Solution*, *Marking*, *Report*, and *Feedback (3)* Tags of the question and these amendments are discussed in the following sections.

Editing the *Settings* Tag of the question to capture the floating-point CSE answer

To capture the CSE answers in this question, one more PIs was needed, so the value of the < NUM_FLAGS> was increased by one.

Editing the *Solution* Tag of the question to capture the floating-point CSE answer

After making the amendments, the *Solution* Tag is as shown in Figure 3.13. The respective correct answers for the three sub-questions of this question were stored

Question

Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, of $f(x) = e^{4x}$ up to and including the cubic term.

Give the values of a_1 and a_2 below.

Enter your answer for a_1 (to three decimal places) here: [ ]

Enter your answer for a_2 (to three decimal places) here: [ ]

Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$

Enter your approximate value for $e^{0.8}$ (to three decimal places) here: [ ]

Correct Solution

Using MacLaurin expansion,

$$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6} \ldots$$

$$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$

$$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$

$$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**CM1**

Giving the exact value of e^{n} instead of the approximate value.

$$e^{0.8} = 2.226$$

No. of CSEs /No. incorrect answers **(CSE %)**	28/116 **(24%)** 29/122**(24%)**	**Date collected**	**2017-18** **2018-19**

Figure 3.12: Q14 CSE1 related to SERIES_MACLAURIN01 question on Dewis which involves a floating-point answer

as `$x1`, `$x2` and `$x3` in the original question code. Therefore, to calculate this particular CSE answer for this question, the GLOBAL parameters `$x4` and `$x5` were introduced as shown in the Figure 3.13. The parameter `$x4` was used to store the machine precision value of the CSE answer and was calculated as `$x4 = exp ($p1*$p2)`, where `$p1` represents a and `$p2` represents the value of x at which the approximate value of e^{ax} is sought. The command `stringDp($x4, 3)` was used to express `$x4` to three decimal places and this value was stored against the global parameter `$x5`.

```
<SOLUTION>
<GLOBAL> $x1  $x2  $x3  $x4  $x5  $x6  $x7  $x8  $x9
$x10  $x11  $x12  $x13  $x14 </GLOBAL>

sub fac { $_[0]>1?$_[0]*fac($_[0]-1):1; }

my $p3Plus1 = $p3 + 1;
my $kb = $p1 * $p2;
my $sum = 0;
my $valu;

for (my $ii=0; $ii<$p3Plus1; $ii++)
{
     $valu = ($kb)**$ii/fac($ii);
     $sum = $sum + $valu;
}
$x1= $p1;
$c1 = stringDp($x1,3);

my $coeff2= $p1*$p1/2;
$x2= $coeff2;
$c2 = stringDp($x2,3);

$x3 = $sum;
$c3 = stringDp($x3,3);

#=====================================================
#----------- CSE1 Answer
#Found exact value of exp instead of approximated value
#=====================================================
$x4 = exp($p1*$p2); #CSE1
$x5 = stringDp($x4,3); #CSE1
</SOLUTION>
```

Original code

CSE code

Figure 3.13: Amended *Solution* Tag for SERIES_MACLAURIN01 question question on Dewis

Editing the *Marking* Tag of the question to capture the floating-point CSE answer

The *Marking* Tag was amended to capture this CSE as shown in Figure 3.14. The respective correct answers for the three sub-questions of this question were marked against the PIs `$f1`, `$f2` and `$f3` in the original question code. It should be noted that, the PI value which was used to mark the third part of the question was `$f3`. Since the required answer for the third part of this question was a floating-point answer, the function < MARK_FLOAT_DP FLAG > was used to mark the students' answer. Here the students' answer of `$a3` for the third part of the question was marked against the correct answer `$c3` and the outcome was stored in `$f3` in the original question.

The marking of this particular CSE was carried out by utilising the newly introduced PI `$f4`. Here the students' answer was marked against the CSE answer.

```
<MARKING>

<MARK_FLOAT_DP FLAG='$f1' ANS='$a1' COR='$c1'
PREC='3' LONG='$x1' >
<MARK_FLOAT_DP FLAG='$f2' ANS='$a2' COR='$c2'
PREC='3' LONG='$x2' >
<MARK_FLOAT_DP FLAG='$f3' ANS='$a3' COR='$c3'
PREC='3' LONG='$x3' >

#============================================
#----------- CSE1 MARKING----------------
#============================================

<MARK_FLOAT_DP FLAG='$f4' ANS='$a3' COR='$x5'
PREC='3' LONG='$x4' >

</MARKING>
```

Original code

CSE code

Figure 3.14: Amended *Marking* Tag for SERIES_MACLAURIN01 question on Dewis

Editing the *Report* Tag of the question to capture the floating-point CSE answer

After making amendments to the report tag (see Figure 3.15), an illustration of the Report output is shown in Figure 3.16 when the CSE answer for the question shown in Figure 3.10, ($\tilde{P}_3(0.7) = 4.055$), is submitted. The first four lines of text were already in the original Report output. The newly added CSE Report output was displayed using blue text in a red box as shown in Figure 3.16.

Editing the *Feedback (3)* Tags of the question to capture the floating-point CSE answer

The CSE EFB for the detected CSE was provided using the *Feedback (3)* tag. Figure 3.17 shows the CSE EFB given to students when the CSE answer for the question shown in Figure 3.10, ($\tilde{P}_3(0.7) = 4.055$), is submitted. The CSE EFB for this

```
<REPORT>

<PRINT_REPORT FLAG='$f1' TEXT="<MATH> a_1 </MATH>"
ANS='$a1' >
<PRINT_REPORT FLAG='$f2' TEXT="<MATH> a_2 </MATH>"
ANS='$a2' >
<PRINT_REPORT FLAG='$f3' TEXT="<MATH> P_{$p3}($p2)
</MATH>" ANS='$a3' >

#========================================
#----------- REPORT when CSE1 is triggered
#========================================

my $kpoint1 = $p1*$p2;

if(($f3 == 0)&&($f4 > 0))
{
     <PRINT>
<div style='font-size:100%;font-weight:normal;border:2px
orangered solid; padding:10px; margin:10px; width:95%;
background-color:white;color:white;line-height:20px'>

<span style='color:blue'>
Your incorrect answer, <MATH> e^{$kpoint1} = $a3 </MATH>
seems to have derived by calculating the <span style=
'color:red' > <b> exact value </b> of <MATH> e^{$kpoint1}
</MATH>. </span><p></p> <p></p>
Here, you are asked to use <MATH> P_{$p3}(x) </MATH> to
calculate <b> an approximate value </b> for <MATH>
e^{$kpoint1}. </MATH>  <p></p>
Please refer to the report section of the FEEDBACK  for
the detailed workings of this problem.</span><p></p>
</div>
     </PRINT>
}
</REPORT>
```

Original code

CSE code

Figure 3.15: Amended *Report* Tag for SERIES_MACLAURIN01 question shown in Figure 3.10

question, as shown in Figure 3.17, was prepared using the same principles as explained for the integer CSE example.

3.3.3 Algebraic answer capture

The code used to capture CSEs of questions with algebraic answers is discussed in this section. The question which requires an algebraic answer and depicted in Figure 3.18 is used as an example for the discussion here. This question was given in the morning session of the January e-examination in the 2017-18 academic year.

The same question, but with different parameters, was given in the afternoon session of the January e-examination in the 2017-18 academic year and is shown in Figure 3.19.

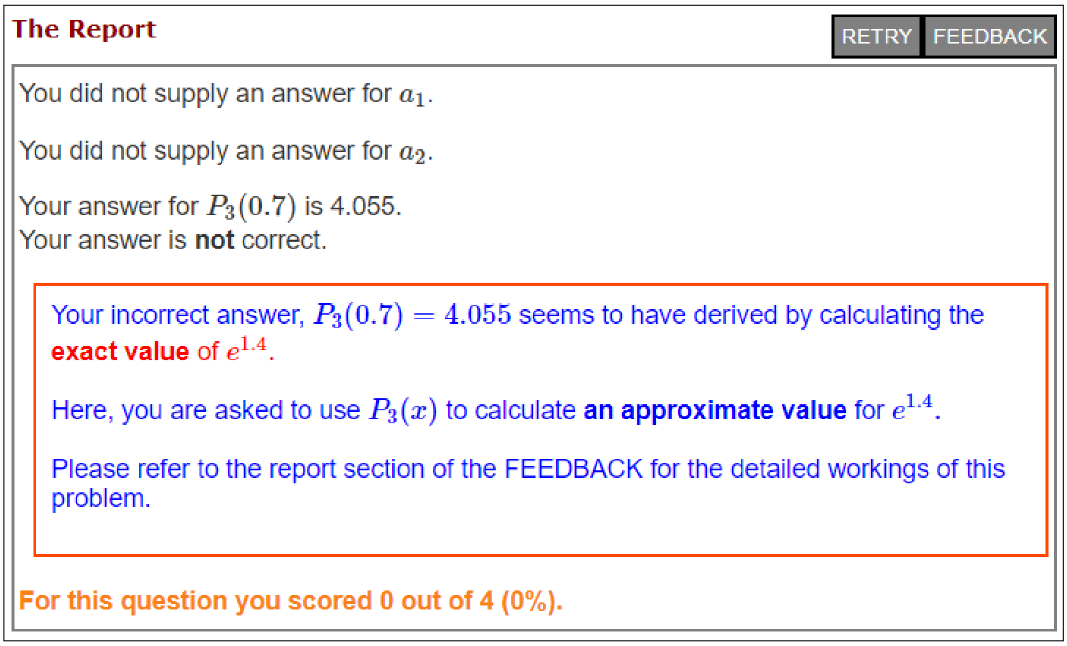


Figure 3.16: Amended Report Output for SERIES_MACLAURIN01 question shown in Figure 3.10

A brief discussion about the question requiring an algebraic answer and its CSE answer

The question in Figures 3.18 and 3.19 involves calculating $\frac{dy}{dx}$, of an equation in the form $ax^b + cx^d y^e = ky^f$, where a, b, c, d, e, f and k are constants, using implicit differentiation.

Students are required to submit an algebraic answer as a function of x and y. Three CSEs were identified for this question. In this discussion, only the CSE involved in $\frac{d}{dx}(f(x)g(y))$ as $\frac{d}{dx}(f(x))\frac{d}{dx}(g(y))$ is considered.

The correct answers for the questions presented in Figure 3.18 and Figure 3.19 are $\frac{dy}{dx} = \frac{6x^2 + y^4}{6y - 4xy^3}$ and $\frac{dy}{dx} = \frac{4x^3 + 4xy^3}{5 - 6x^2y^2}$ respectively.

Using the Dewis reporter, it was found that in the morning session, 12 out of the 78 students who gave an incorrect answer to the question (see Figure 3.18) entered their answer as $\frac{dy}{dx} = \frac{6x^2}{6y - 4y^3}$. In the afternoon session, 6 out of the 80 students who gave an incorrect answer to the question (see Figure 3.19) entered the answer as $\frac{dy}{dx} = \frac{4x^3}{5 - 12xy^2}$.

The students' hand-written scripts were examined to find out the reason behind these MCIAs and it was found that this error occurred since the students took

The Report

You did not supply an answer for a_1.

You did not supply an answer for a_2.

Your answer for $P_3(0.7)$ is 4.055.
Your answer is **not** correct.

> Your incorrect answer, $P_3(0.7) = 4.055$ seems to have derived by calculating the **exact value** of $e^{1.4}$.
>
> Here. you are asked to use $P_3(x)$ to calculate **an approximate value** for $e^{1.4}$.

The expansion of $f(x) = e^{2x}$ up to and including the cubic term, $P_3(x)$ is,

$$P_3(x) = e^{2x} = 1 + (2x) + \frac{(2x)^2}{2} + \frac{(2x)^3}{6} \longrightarrow \text{Ⓐ}$$

Using Ⓐ to calculate an approximation value for $f(x) = e^{2x}$ at $x = 0.7$,

$$P_3(0.7) = e^{1.4} = 1 + (1.4) + \frac{(1.4)^2}{2} + \frac{(1.4)^3}{6}$$

$P_3(0.7) = 3.837$ correct to three decimal places.

You scored 0 marks for this question.

Figure 3.17: CSE EFB Output for SERIES_MACLAURIN01 question shown in Figure 3.10.

The Question. (cse_project_book) CALCULUS_DIFFERENTIATION_IMPLICT01

Given

$2x^3 + xy^4 = 3y^2$,

find the derivative $\frac{dy}{dx}$ as a function of x and y.

$\frac{dy}{dx} =$ [ ]

Submit

Figure 3.18: Dewis question requiring an algebraic answer given in the morning session of the 2017-18 e-examination

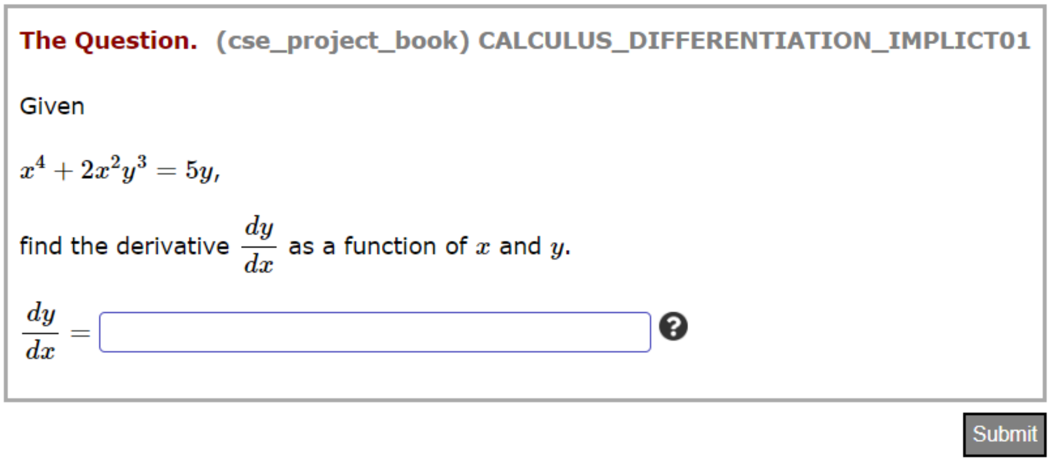


Figure 3.19: Dewis question requiring an algebraic answer given in afternoon session of the 2017-18 e-examination

$\frac{d}{dx}(cx^d y^e)$ as $\frac{d}{dx}(cx^d)\frac{d}{dx}(y^e)$ or more generally $\frac{d}{dx}(f(x)g(y))$ as $\frac{d}{dx}(f(x))\frac{d}{dx}(g(y))$.

Altogether 18 out of 158 students who gave an incorrect answer to either the morning or afternoon questions made this CSE. The CSE percentage for this particular CSE for the 2017-18 academic year is 11%.

The same question, but with different parameters, was given in the 2018-19 e-examination. Using Dewis data and students' hand-written scripts it was found that the same CSE was made by 5 out of 93 and 10 out of 87 students who gave an incorrect answer to the question in the morning and afternoon session respectively. Therefore, altogether 15 out of the 180 students who answered this question incorrectly made this CSE in the 2018-19 e-examination, resulting the CSE percentage for that year being 8%.

All of the information of this CSE is presented on in section 3.2.2 in the CSE Book (Sikurajapathi, Henderson, and Gwynllyw, 2022a) and also is shown in Figure 3.20.

Editing the original question code requiring an algebraic answer to capture the CSE

The original question code was amended to capture and report on this CSE. In this methodology, the main amendments were done in the *Settings*, *Marking*, *Report*, and *Feedback (3)* Tags of the question. It should be noted that since the CSE answer is an algebraic input both the CSE answer and its marking were done in the *Marking* Tag.

Question

Given

$x^4 + 2x^2y^3 = 5y,$

find the derivative $\frac{dy}{dx}$ as a function of x and y.

$\frac{dy}{dx} =$ [ ]

Correct Solution

$$x^4 + 2x^2y^3 = 5y$$

$$4x^3 + 2x^2 \times 3y^2\frac{dy}{dx} + 4x \times y^3 = 5\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4x^3 + 4xy^3}{5 - 6x^2y^2}$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**U2**

Taking $\frac{d}{dx}(f(x)g(y)) = \frac{d}{dx}(f(x)) \times \frac{d}{dx}(g(y))$

$$x^4 + 2x^2y^3 = 5y$$

$$4x^3 + 4x \times 3y^2\frac{dy}{dx} + y^4 = 5\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4x^3}{5 - 12xy^2}$$

No. of CSEs /No. incorrect answers **(CSE %)**	18/158 **(11%)** 15/180 **(8%)**	**Date collected**	**2017-18** **2018-19**

Figure 3.20: Q22 CSE1 related to CALCULUS_DIFFERENTIATION_IMPLICT01 question on Dewis which involves an algebraic answer

Editing the *Settings* Tag of the question to capture the algebraic CSE answer

To capture the CSE answers in this question, one more PIs was needed so the value of $<$ NUM_FLAGS $>$ was increased by one.

Editing the *Marking* Tag of the question to capture the algebraic CSE answer

The *Marking* Tag was amended to capture this CSE is shown in Figure 3.21. Since the required answer for this question was an algebraic answer, the function < MARK_-FUNCTION > was used to mark the students answer against the correct answer which was set up in the *Solution* Tag in the original code. The outcome was stored in `$f1` in the original code.

The CSE answer was set to be `$cse1` and a new local parameter `$a1Alt1` was set up as `$a1 -($cse1)+$correct`. Then, `$a1Alt1` was marked against the correct answer. The marking of this particular CSE was carried out by utilising the newly introduced PI, `$f2`. (Please note that, a new way of marking CSEs for algebraic questions has been introduced in Dewis after this thesis work was completed.)

Editing the *Report* Tag of the question to capture the algebraic CSE answer

The amended *Report* Tag is shown in Figure 3.22. The Report output is shown in Figure 3.23 when the CSE answer for the question shown in Figure 3.18, namely, $\frac{\tilde{d}y}{dx} = \frac{6x^2}{6y-4y^3}$, is submitted. The first three lines of text were already in the original Report output. The newly added CSE Report output was displayed in blue text in a red box as shown in Figure 3.23.

Editing the *Feedback (3)* Tag of the question to capture the algebraic CSE answer

The CSE EFB for the detected CSE was provided using the *Feedback (3)* tag. Figure 3.24 shows the CSE EFB given to students when the CSE answer of $\frac{\tilde{d}y}{dx} = \frac{6x^2}{6y-4y^3}$, for the question shown in Figure 3.18, is submitted.

The CSE EFB for this question was prepared in a similar way as described in Section 3.3.1 and Section 3.3.2. The CSE EFB for this question is shown in Figure 3.24.

3.3.4 Credibility of the Amended Codes

The amended CSE capture code for each question was validated by re-marking the e-examination for the 2017-18 cohort. This was done by checking that the additional PIs were populated for those students who had already been identified as

```
<MARKING>

<MARK_FUNCTION FLAG='$f1' ID='1' ANS='a1' >        } Original code

#========================================
#--------------CSE1 MARKING ----------------
#========================================
$f2 = -1;
 if ($f1==0)
{
my $numerator = '';
my $denominator = '';
my ($aa,$bb,$cc,$dd,$ee,$kk,$ff) = ($p1,$p2,$p3,$p4,$p5,
$p6,$p7);

$numerator = "$aa * $bb * XX^($bb-1) + $cc * $dd *
XX^($dd-1) * YY^$ee";
$denominator = "$kk * $ff * YY^($ff-1)- $cc * $ee *
XX^($dd) * YY^($ee-1)";
my $correct = "($numerator)/($denominator)";

my $cse1numerator = '';
my $cse1denominator = '';
$cse1numerator = "$aa * $bb * XX^($bb-1)" ;
$cse1denominator = "$kk * $ff * YY^($ff-1)- $cc *$dd* $ee
* XX^($dd-1) * YY^($ee-1)";

my $cse1 = "($cse1numerator)/($cse1denominator)"; #cse1
my $a1Alt1 = "$a1 - ($cse1) + $correct";
$a1Alt1 =~ s/XX/x/g;
$a1Alt1=~ s/YY/y/g;

<MARK_FUNCTION ID='1' ANS='$a1Alt1' FLAG='f2'>
}

</MARKING>
```

CSE code

Figure 3.21: Amended *Marking* Tag for CALCULUS_DIFFERENTIATION_IMPLICT01 question on Dewis

making CSEs on the e-examination. Once this process had been completed satisfactorily, the weekly tests and revision tests were also re-marked, using the amended codes.

Some restrictions related to the parameter selections of the questions which have CSEs were found during this process. These findings and the methodology used in addressing these restrictions are presented in this section.

Two categories of parameter selection restriction were identified. Firstly for some questions, there were particular random parameters for which the correct answer and the CSE answer were the same. Secondly, it was found that for one question with more than one CSE, it was possible for two CSEs to be triggered simultaneously for some particular parameters.

```
<REPORT>

<PRINT_REPORT  FLAG='$f1'  TEXT="<MATH>dy/dx</MATH>"
ANS='$a1'  >

#=========================================================
#---------- REPORT when CSE1 is triggered---------
#=========================================================


if (($f1 == 0) && ($f2 == 1) &&($REPORT_OUTCOME))
{
     <PRINT>

<div style='font-size:100%;font-weight:normal;border:2px
orangered
solid;padding:10px;margin:10px;width:95%;background-
color:white;color:white;line-height:20px'>
<span style='color:blue'>

We note that your incorrect answer seems to have been
caused by  incorrectly differentiating <b> the second
term </b>
of the given equation.
<p></p>
Please refer to the report section of the feedback to see
what could have gone wrong when you solved the question.
</span><p></p>

</div>

     </PRINT>
}
</REPORT>
```

Original code

CSE code

Figure 3.22: Amended *Report* Tag for CALCULUS_DIFFERENTIATION_IMPLICT01 question shown in Figure 3.18

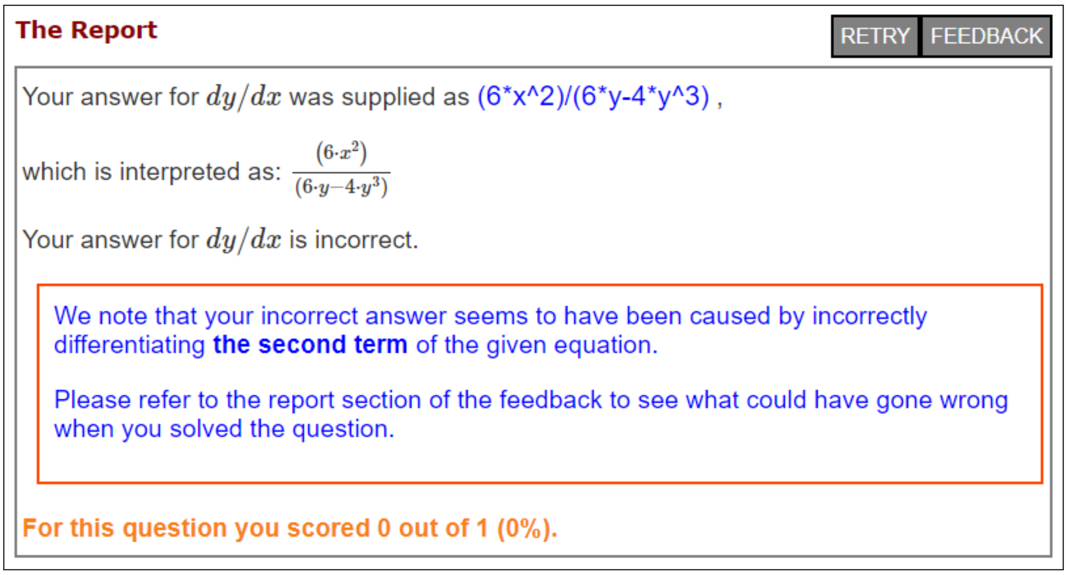
Same correct answer and CSE answer

A few instances where the correct answer and the CSE answer were the same are presented in this section.

Case 1

As shown in Table 3.1, there is one CSE associated with the question FUNCTION_UNITSTEP01, namely Q2 CSE1, and this is presented in Figure 3.1. This CSE occurs by assuming that the unit step function, u, is equal to 1 and is not a function.

Whilst re-marking the weekly tests, it was noted that for some parameters the correct answer and Q2 CSE1 answer of this question were the same. This occurs for example, for the function, $f(t) = 2u(t+7) - 5u(t+1)$ when the value of $f(4)$


The Report RETRY FEEDBACK

Your answer for dy/dx was supplied as (6*x^2)/(6*y-4*y^3) ,

which is interpreted as: $\frac{(6\cdot x^2)}{(6\cdot y-4\cdot y^3)}$

Your answer for dy/dx is incorrect.

We note that your incorrect answer seems to have been caused by incorrectly differentiating **the second term** of the given equation.

Please refer to the report section of the feedback to see what could have gone wrong when you solved the question.

For this question you scored 0 out of 1 (0%).

Figure 3.23: Amended Report Output for CALCULUS_DIFFERENTIATION_IMPLICT01 question shown in Figure 3.18

is asked for. In this case, the correct answer can be calculated as,

$$\begin{aligned} f(4) &= 2u(4+7) - 5u(4+1) \\ &= 2u(11) - 5u(5) \\ &= 2\times 1 - 5\times 1 \\ &= -3 \end{aligned}$$

and the CSE answer can be calculated as,

$$\begin{aligned} \tilde{f}(4) &= 2u(4+7) - 5u(4+1) \\ &= 2u(11) - 5u(5) \\ &= 2\times u\times 11 - 5\times u\times 5 \\ &= 22u - 25u \\ &= -3u \\ &= -3 \end{aligned}$$

It can be seen that, both the correct answer and the CSE answer are equal to -3 for this particular parameter selection.

The Report

Your answer for dy/dx was supplied as (6*x^2)/(6*y-4*y^3) ,

which is interpreted as: $\frac{(6 \cdot x^2)}{(6 \cdot y - 4 \cdot y^3)}$

Your answer for dy/dx is incorrect.

We note that your incorrect answer seems to have been caused by differentiating **the second term** of the given equation incorrectly.

Note that the given function, $2x^3 + xy^4 = 3y^2 \longrightarrow$ Ⓐ, is an implicit function,

i.e. it is quite difficult to re-arrange this function so that y can be expressed as a function of x.

To find the derivative $\frac{dy}{dx}$ as a function of x and y, we have to differentiate Ⓐ with respect to x.

Differentiating each term of Ⓐ with respect to x gives,

$$\frac{d}{dx}(2x^3) + \frac{d}{dx}(xy^4) = \frac{d}{dx}(3y^2) \longrightarrow \text{Ⓑ}$$

Please note that,

The first term of Ⓑ is a function of x and hence differentiating it with respect to x is straightforward.

The second term of Ⓑ is a product, and we need to use the product rule to and the chain rule to differentiate it.

The third term of Ⓑ is a function of y and we need to use the chain rule to differentiate it.

$$\frac{d}{dx}(2x^3) + \frac{d}{dx}(xy^4) = \frac{d}{dx}(3y^2)$$

$$\frac{d}{dx}(2x^3) + [x\frac{d}{dx}(y^4) + y^4\frac{d}{dx}(x)] = \frac{d}{dx}(3y^2),$$ [Note that, $\frac{d}{dx}(xy^4) = x\frac{d}{dx}(y^4) + y^4\frac{d}{dx}(x)$]

$$6x^2 + [x\frac{d}{dy}(y^4)\frac{dy}{dx} + y^4] = \frac{d}{dy}(3y^2)\frac{dy}{dx}$$

$$6x^2 + 4xy^3\frac{dy}{dx} + y^4 = 6y\frac{dy}{dx},$$ [Note that, $\frac{d}{dy}(y^4) = 4y^3$ and $\frac{d}{dx}(3y^2) = 6y$]

Rearranging and making $\frac{dy}{dx}$ the subject gives,

$$(6y - 4xy^3)\frac{dy}{dx} = 6x^2 + y^4$$

$$\frac{dy}{dx} = \frac{6x^2 + y^4}{6y - 4xy^3}$$

You scored 0 marks for this question.

Figure 3.24: CSE EFB Output for the CALCULUS_DIFFERENTIATION_IMPLICT01 shown in Figure 3.18.

Case 2

There was only one CSE found related to the question SERIES_GEOMETRIC01 (see Table 3.1), namely Q13 CSE1, and this is shown in Figure 3.25. This CSE occurs by finding the sum of the first four terms instead of the sum of the infinite series.

Question			
Consider the following geometric series, S, where: $S = 2 + 2(0.7) + 2(0.7)^2 + 2(0.7)^3 \cdots$. Write down the first term, a and the common ratio, r in the boxes below. Enter a: [ ] Enter r: [ ] Hence calculate the sum, S and enter your result in the box below. Enter S (to three decimal places) here: [ ]			
Correct Solution			
The first term $a = 2$ The common ratio $r = 0.7$ The sum of an infinite series (S) exists, provided $\lvert r \rvert < 1$ $S = \frac{a}{1-r}$ $= 6.667$			
CSE 1 related to this question		**CSE Taxonomy Code:**	**CM1**
Finding the sum of first four terms instead of the sum of the infinite series. $\tilde{S} = \frac{a(1-r^n)}{1-r}$ $\tilde{S} = \frac{2(1-0.7^4)}{1-0.7}$ $\tilde{S} = 5.066$			
No. of CSEs /No. incorrect answers **(CSE %)**	34/67**(51%)**	**Date collected**	**2017-18**

Figure 3.25: Q13 CSE1 of Question SERIES_GEOMETRIC01

During the re-marking process it was found that, for some question parameters, the correct answer and Q13 CSE1 answer of SERIES_GEOMETRIC01 were the same. As an example, this happens when the sum of the infinite series, $S = 2 +$

$2(0.1)+2(0.1)^2+2(0.1)^3+...$ is asked for. In this case, the correct answer can be calculated as:

$$\begin{aligned} S &= \frac{a}{1-r} \\ &= \frac{2}{1-0.1} \\ &= 2.22222... \\ &= 2.222 \text{ correct to 3 dp} \end{aligned}$$

and the CSE answer can be calculated as,

$$\begin{aligned} \tilde{S} &= \frac{a(1-r^n)}{(1-r)} \\ &= \frac{2(1-0.1^4)}{(1-0.1)} \\ &= 2.222 \end{aligned}$$

It can be seen that, to 3 decimal places both the correct answer and the CSE answer are the same in this case.

Case 3

As shown in Table 3.1, there are three CSEs found for the question SERIES_-MACLAURIN01, namely Q14 CSE1, Q14 CSE2, and Q14 CSE3. Q14 CSE1 is presented in Figure 3.12. In this question, students are asked to find the power series expansion, $P_3(x)$, of $f(x) = e^{ax}$, up to and including the cubic term, and to use $P_3(x)$, to calculate an approximate value for $f(x)$ at $x = c$, correct to three decimal places. The parameters a and c are generated randomly for each instance of the question. Q14 CSE1 is to give the exact value of e^{ax} instead of the approximate value of e^{ax} at $x = c$.

It was found that for some question parameters the correct answer and the Q14 CSE1 answer of this question were the same, to three decimal places. For example, both the approximate value and the exact value of e^{2x} at $x = -0.1$ are equal to 0.819, correct to 3 dp. In this case, the correct answer can be calculated as follows:

$$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

$$
\begin{aligned}
&= 1+2x+\frac{(2x)^2}{2}+\frac{(2x)^3}{6} \\
P_3(-0.1) &= 1+2(-0.1)+\frac{(2\times -0.1)^2}{2}+\frac{(2\times -0.1)^3}{6} \\
&= 0.818667... \\
&= 0.819 \text{ correct to 3 dp.}
\end{aligned}
$$

The Q13 CSE1 answer (exact value for e^{2x} at $x = -0.1$) is,

$$
\begin{aligned}
\tilde{P}_3(-0.1) &= 0.819 \\
&= 0.818731... \\
&= 0.819 \text{ correct to 3 dp.}
\end{aligned}
$$

It can be seen that, to 3 decimal places both the correct answer and the CSE answer are the same in this case.

Case 4

As shown in Table 3.1, there are three CSEs found for the question CALCULUS_-INTEGRATION_MEANVALUE01, namely Q7 CSE1, Q7 CSE2, and Q7 CSE3. Q7 CSE1 is presented in Figure 3.26. Q7 CSE1 occurs by evaluating the mean value of $f(t)$ using degrees instead of radians.

During the re-marking process, it was found that for some parameters the correct answer and the Q7 CSE1 answer of this question were the same to two decimal places. For example, both the correct answer and the Q7 CSE1 for finding the mean value of $f(t) = -3\sin(5t)$ in the interval $3 < t < 7$ are equal to -0.02 as shown below. In this case, the correct mean value, m, can be calculated as:

$$
\begin{aligned}
m &= \frac{1}{(7-3)}\int_3^7 -3\sin(5t)dt \\
&= \frac{1}{4}\left[\frac{3}{5}\cos(5t)\right]_3^7 \\
&= \frac{3}{20}[\cos(35)-\cos(15)] \\
&= \frac{3}{20}[-0.9037+0.7596] \\
&= -0.021615...
\end{aligned}
$$

Question

Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places.

Enter correct to two decimal places: [ ]

Correct Solution

$$f(t) = 4\sin(10t)$$

Let the mean value of $f(t) = m$

$$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$

$$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$

$$= -\frac{1}{10}((\cos(60) - \cos(20))$$

$$= 0.14$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**UM2**

Substituting for t in degrees

$$f(t) = 4\sin(10t)$$

Let the mean value of $f(t) = m,$

$$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$

$$\tilde{m} = 0.04$$

No. of CSEs /No. incorrect answers **(CSE %)**	52/183 **(28%)**	**Date collected**	**2017-18**

Figure 3.26: Q7 CSE1 of Question CALCULUS_INTEGRATION_MEAN-VALUE01

$$= -0.02 \text{ correct to 2 dp.}$$

The Q7 CSE1 answer, $\tilde{m}$ is

$$\tilde{m} = \frac{1}{(7-3)}\int_3^7 -3\sin(5t)dt$$

$$
\begin{aligned}
&= \frac{1}{4}\left[\frac{3}{5}\cos(5t)\right]_3^7 \\
&= \frac{3}{20}\left[\cos(35^o) - \cos(15^o)\right] \\
&= \frac{3}{20}\left[0.8192 - 0.9659\right] \\
&= -0.022005... \\
&= -0.02 \text{ correct to 2 dp.}
\end{aligned}
$$

It can be seen that, to 2 decimal places both the correct answer and the CSE answer are the same in this case.

Case 5

As shown in Table 3.1, there are six CSEs found for the question CALCULUS_INTEGRATION_VOLUMEREVOLUTION01 and two of these CSEs are discussed in Cases 5 and Case 6. Q16 CSE5 is considered in this case and is presented in Figure 3.27 and Q16 CSE4 is considered in Case 6. Q16 CSE5 results from finding the volume of revolution by substituting the upper and lower limits without integrating.

During the re-marking process, it was found that for some question parameters the correct answer and the Q16 CSE5 answer of this question were the same. This occurs for the curve $y = 6x$ rotated about the x-axis between $x = 0$ and $x = 3$. In this case, the correct answer, V, can be calculated as:

$$
\begin{aligned}
V &= \pi \int_0^3 (6x)^2 dx \\
&= \pi \int_0^3 36x^2 dx \\
&= 36\pi \left[\frac{x^3}{3}\right]_0^3 \\
&= 12\pi \left[3^3 - 0\right] \\
&= 1017.87602... \\
&= 1017.88 \text{ correct to 2 dp.}
\end{aligned}
$$

The Q16 CSE5 answer, $\tilde{V}$ is

$$
\tilde{V} = \pi \left[(6x)^2\right]_0^3
$$

Question

Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]

Correct Solution

The volume of revolution of y over the range $a < x < b$ given by

$$V = \pi \int_a^b y^2 dx$$

For given data,

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$= \pi \int_1^4 0.64\, x^3 dx$$

$$= 128.18$$

CSE 5 related to this question	**CSE Taxonomy Code:**	**UM2**

Substitute upper and lower limits without integrating

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$\bar{V} = 0.64\, \pi \left[\frac{x^3}{3}\right]_1^4$$

$$= 126.67$$

No. of CSEs /No. incorrect answers (CSE %)	5/107 (5%) 7/135 (5%)	**Date collected**	**2017-18** **2018-19**

Figure 3.27: Q16 CSE5 of Question CALCULUS_INTEGRATION_VOLUMEREVOLUTION01

$$= 36\pi\left[x^2\right]_0^3$$

$$= 36\pi\left[3^2 - 0\right]$$

$$= 1017.87602...$$

$$= 1017.88 \text{ correct to 2 dp.}$$

It can be seen that both the correct answer and the CSE answer are the same up to two decimal places in this case.

Case 6

Q16 CSE4 is presented in Figure 3.28 and is caused by taking $(x^p)^q$ to be x^{p^q}.

Question

Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: ______

Correct Solution

The volume of revolution of y over the range $a < x < b$ given by

$$V = \pi \int_a^b y^2 dx$$

For given data,

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$= \pi \int_1^4 0.64\, x^3 dx$$

$$= 128.18$$

CSE 4 related to this question	**CSE Taxonomy Code**:	**S3**

Taking $(x^p)^q = x^{p^q}$

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$\tilde{V} = \pi \int_1^4 0.8^2\, x^{1.5^2} dx$$

$$\tilde{V} = \pi \int_1^4 0.64\, x^{2.25} dx$$

$$\tilde{V} = 0.64\, \pi \left[\frac{x^{3.25}}{3.25}\right]_1^4 = 55.38$$

No. of CSEs /No. incorrect answers (CSE %)	7/107 (7%)	Date collected	2017-18

Figure 3.28: Q16 CSE4 of Question CALCULUS_INTEGRATION_VOLUMEREVOLUTION01

During the re-marking process, it was found that for some parameters the correct answer and the Q16 CSE4 answer of this question were the same. For example, when finding the volume of revolution for the curve $y = 0.6x^2$ rotated about x-axis between $x = 1$ and $x = 4$. In this case, the correct answer V can be calculated as:

$$\begin{aligned} V &= \pi \int_1^4 (0.6x^2)^2 dx \\ &= \pi \int_1^4 0.36x^4 dx \\ &= 0.36\pi \left[\frac{x^5}{5}\right]_1^4 \\ &= \frac{0.36}{5}\pi \left[4^5 - 1\right] \\ &= 231.3971... \\ &= 231.40 \text{ correct to 2 dp} \end{aligned}$$

The Q16 CSE5 answer will be the same for this particular parameter selection because $x^{2^2} = (x^2)^2 = x^4$.

Case 7

As shown in Table 3.1, there are two CSEs found for the question CALCULUS_-INTEGRATION_MEANVALUE02, namely Q8 CSE1 and Q8 CSE2. Q8 CSE1 is presented in Figure 3.29. This is caused by evaluating the mean value of the given function as $\int_a^b f(t)dt$ instead of $\frac{1}{b-a}\int_a^b f(t)dt$. That is the divisor $(b-a)$ is missing in the calculation.

During the re-marking process, it was found that for some parameters, the correct answer and the Q8 CSE1 answer of this question were the same. For example, this is the case when finding the mean value of the function $f(t) = 3t^5 + 4$ in the interval $1 < t < 2$. In this case, the correct answer m can be calculated as:

$$\begin{aligned} m &= \frac{1}{2-1}\int_1^2 (3t^5 + 4)dt \\ &= \left[\frac{t^6}{2} + 4t\right]_1^2 \\ &= 35.5000 \\ &= 35.50 \text{ to 2 dp} \end{aligned}$$

Question

Given that $f(t) = 3t^5 + 4$, find the mean value of $f(t)$ in the interval $1 < t < 3$, correct to two decimal places.

Enter correct to two decimal places: [ ]

Correct Solution

The mean value, m, of $f(t)$ in the interval $a < t < b$ is given by

$$\frac{1}{b-a}\int_a^b f(t)dt$$

$$m = \frac{1}{(3-1)}\int_1^3 (3t^5 + 4)dt$$

$$= \frac{1}{2}\left[\frac{t^6}{2} + 4t\right]_1^3$$

$$= 186.00$$

CSE 1 related to this question	CSE Taxonomy Code:	U5

Taking the mean value as $\int_a^b f(t)dt$ instead of $\frac{1}{b-a}\int_a^b f(t)dt$

$$\tilde{m} = \int_1^3 (3t^5 + 4)dt$$

$$= \left[\frac{t^6}{2} + 4t\right]_1^3$$

$$= 372$$

No. of CSEs /No. incorrect answers (CSE %)	10/121 (8%)	Date collected	2018-19

Figure 3.29: Q8 CSE1 of Question CALCULUS_INTEGRATION_MEAN-VALUE02

and the Q8 CSE1 $\tilde{m}$ will be the same for this parameter selection since $b - a = 2 - 1 = 1$.

Same two CSE Answers

During the re-marking process, it was found that for one question with more than one CSE, it was possible for two CSEs to be triggered simultaneously for some particular parameters. That case is discussed in the following section.

Case 8

As shown in Table 3.1, there are three CSEs found for the question CALCULUS_-INTEGRATION_MEANVALUE01, namely Q7 CSE1, Q7 CSE2, and Q7 CSE3.

Q7 CSE2 and Q7 CSE3 are presented in Figure 3.30 and Figure 3.31 respectively. Q7 CSE2 occurs by directly substituting the midpoint of the range of t in degrees into the given function $f(t)$ to evaluate the mean value of the function. Q7 CSE3 is to substitute the end values of the range of t into the given function $f(t)$ in degrees and then take the average. It should be noted that, the function $f(t)$ can involve either the sine or cosine form each time the question is generated.

During the re-marking process, it was found that for some parameters the Q7 CSE2 answer and the Q7 CSE3 answer of this question were the same. For example, both the Q7 CSE2 answer and the Q7 CSE3 for finding the mean value of $f(t) = 4\cos(3t)$ in the interval $5 < t < 7$ are equal to 3.80 correct to two decimal places in degrees as shown below.

For these parameter values the mid-point of the range of t is 6. Therefore, in this case the Q7 CSE2 answer, $\tilde{m_2}$, can be calculated as follows:

Let the mean value of $f(t)$ is $\tilde{m_2}$. The middle point of the range of t is 6.

$$\begin{aligned}\tilde{m_2} &= 4\cos(3\times 6^o)\\ &= 3.804226...\\ &= 3.80 \text{ correct to 2 dp.}\end{aligned}$$

The Q7 CSE3 answer, $\tilde{m_3}$ is,

$$\begin{aligned}\tilde{m_3} &= \frac{4\cos(3\times 5^o)+4\cos(3\times 7^o)}{2}\\ &= 3.799012...\\ &= 3.80 \text{ correct to 2 dp.}\end{aligned}$$

Question

Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places.

Enter correct to two decimal places: [ ]

Correct Solution

$$f(t) = 4\sin(10t)$$

Let the mean value of $f(t) = m$

$$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$

$$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$

$$= -\frac{1}{10}((\cos(60) - \cos(20))$$

$$= 0.14$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**U7, UM2**

Directly substituting the midpoint of the range of t in degrees into the given function.

$$f(t) = 4\sin(10t)$$

Let the mean value of $f(t) = m$. The middle point of the range of $t = 4$

$$\tilde{m} = 4\sin(10 \times 4^o) = 4\sin(40^o)$$

$$\tilde{m} = 2.57$$

No. of CSEs /No. incorrect answers (CSE %)	44/183 **(24%)**	**Date collected**	**2017-18**

Figure 3.30: Q7 CSE2 of Question CALCULUS_INTEGRATION_MEAN-VALUE01

It can be seen that, to 2 decimal places, both the Q7 CSE2 answer and the Q7 CSE3 answer of this question are the same in this case.

Question			
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places:			
Correct Solution			
$$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$			
CSE 3 related to this question	**CSE Taxonomy Code:**	**U7, UM2**	
Substituting the end values of the range t into the given function in degrees and then taking the average. $f(t) = 4\sin(10t)$, let the mean value of $f(t) = m$ $$\bar{m} = \frac{4\sin(10\times 2^o) + 4\sin(10\times 6^o)}{2}$$ $$= 2.42$$			
No. of CSEs /No. incorrect answers (CSE %)	8/183 **(4%)**	**Date collected**	**2017-18**

Figure 3.31: Q7 CSE3 of Question CALCULUS_INTEGRATION_MEAN-VALUE01

Resolution

For cases 1-5, without students' rough working, there is no way of learning whether the student arrived at the final answer by following the correct approach or by making the CSE. Therefore, to avoid these scenarios occurring for future instances of the questions, the CSE codes were amended further to select parameters in a way that the correct answer never equals the CSE answer using a while loop.

For case 6, the value of the power (say p) of x in the function y in the original code was selected randomly from the following list of values, 0.25, 0.5, 1, 1.25, 1.5, 2. It can easily be seen that the correct answer of this question is equal to the Q16 CSE4 if and only if $p = 2$. Therefore, the value of 2 was removed from the list of possible values given for p in the amended code.

For case 7, by looking at the correct answer and the Q8 CSE1, answer it can be concluded that these will be equal if and only if $b - a = 1$. That is, the difference of the upper and lower limit, is 1. To avoid the correct answer being equal to the Q8 CSE1 answer, the value of b which resulted in the amended code is randomly selected so that b does not equal $1 + a$.

After finding these cases, all of the other CSE codes were amended to avoid correct answers being equal to the CSE answers. As a further precaution, the codes were amended by providing CSE EFB only when the PI value of the correct answer is zero and the the PI of the CSE answer is one. That means, a respective CSE EFB is given only when a student gives an incorrect answer which is the respective CSE answer.

When a question has more than one CSE, such as in case 8, the codes were amended to provide CSE EFB only when the PI value of the correct answer is zero and the PI of exactly one CSE answer is triggered.

3.4 CSE Project Stage Three: CSE Code Trial Phase

This section discusses the methodology used in Stage Three of the CSE Project. The aim of this stage was to use the improved CSE EFB questions in the weekly tests and revision tests for the 2019-20 cohort of EM students. The methodology of testing the improved CSE EFB questions and the data collection is discussed in the following sections.

3.4.1 Weekly trial phase

After agreement with the module leader of the EM Module, 16 questions which capture 29 CSEs altogether were used in the weekly tests for the 2019-20 cohort of EM students. The CSEs of the selected questions are shown in Table 3.3. It should be noted that not all of the CSEs related to the chosen questions were coded by the time of the weekly trial phase. The CSEs which were not ready are Q3 CSE2, Q14 CSE3, Q15 CSE2, Q16 CSE3, Q16 CSE4, Q16 CSE5, Q16 CSE6 and Q25 CSE2.

In addition, Table 3.3 shows the number of distinct students who gave a CSE answer and received the respective CSE EFB in the weekly tests. These data were collected by looking at how many students triggered the PI designated to the CSE answer on submission to Dewis. Since each weekly test can be attempted by a student several times, Dewis-stored data were carefully examined to gather how many distinct students made the respective CSE.

Table 3.3 shows that almost all of the CSEs were triggered in the weekly tests except for CSE3 of question Q7 (CALCULUS_INTEGRATION_MEANVALUE01) and CSE1 of question Q15 (ENGINEERING_CENTREMASS01).

Table 3.3: CSEs triggered in the weekly tests for the 2019-2020 cohort of EM

CSE ID	**No. of distinct CSE answers**
Q1 CSE1	64
Q1 CSE2	5
Q1 CSE3	2
Q2 CSE1	44
Q3 CSE1	2
Q4 CSE1	49
Q7 CSE1	22
Q7 CSE2	3
Q7 CSE3	0
Q9 CSE1	8
Q10 CSE1	5
Q14 CSE1	26
Q14 CSE2	14
Q15 CSE1	0
Q16 CSE1	7
Q16 CSE2	7
Q18 CSE1	5

Continued on next page

Table 3.3 *continued from previous page*

CSE ID	No. of distinct CSE answers
Q19 CSE1	2
Q19 CSE2	2
Q20 CSE1	2
Q20 CSE2	14
Q22 CSE1	8
Q22 CSE2	2
Q22 CSE3	8
Q25 CSE1	34
Q26 CSE1	1
Q26 CSE2	9
Q26 CSE3	1
Q26 CSE4	1

3.4.2 Revision trial phase

Ten questions which capture 21 CSEs were used in the revision tests. All of these 10 questions were used in a previous weekly test. The questions selected for the revision tests are shown in the Table 3.4. Table 3.4 shows the number of distinct students who gave a CSE answer and received the respective CSE EFB in the revision tests.

Table 3.4 shows that almost all of the CSEs were triggered in the revision tests except CSE1 of question Q15 (ENGINEERING_CENTREMASS01) and CSE1 of question Q19 (CALCULUS_DIFFERENTIATION_CHAINRULE01).

Table 3.4: CSEs triggered in the revision tests for the 2019-2020 cohort of EM

CSE ID	No. of distinct answers
Q1 CSE1	58
Q1 CSE2	6

Continued on next page

Table 3.4 *continued from previous page*

CSE ID	No. of distinct answers
Q1 CSE3	2
Q2 CSE1	29
Q10 CSE1	3
Q14 CSE1	4
Q14 CSE2	4
Q15 CSE1	0
Q16 CSE1	10
Q16 CSE2	6
Q19 CSE1	0
Q19 CSE2	6
Q20 CSE1	3
Q20 CSE2	14
Q22 CSE1	14
Q22 CSE2	4
Q22 CSE3	6
Q26 CSE1	2
Q26 CSE2	20
Q26 CSE3	1
Q26 CSE4	3

The data collected in the trial phase is analysed in Chapter 4 and the findings of this data are discussed in Chapter 5.

After the trial phase the next step was to gather students' perceptions on the CSE EFB they received and this is discussed in the following section.

3.5 CSE Project Stage Four: Students' Perception on CSE EFB

In this section, the methodology used in Stage Four of the CSE Project is discussed. Finding students' perceptions on the CSE EFB delivered through Dewis is the prime purpose of this stage.

3.5.1 Questionnaire design techniques

Traditionally, the first choice of data collection in research was paper questionnaires (Ebert et al., 2018). However, tremendous incremental usage in internet and computer-mediated communication has led to an increase in the use of online surveys. Recent developments of survey authoring software packages and online survey services make online survey research much easier (Wright, 2005). Using online questionnaires has numerous benefits in terms of cost, time, ease of administration, data collation and analysis (Dillman, 2007).

Questionnaire methodology was employed to gather students' perceptions on the CSE EFB delivered through Dewis. The questionnaire was prepared to collect both quantitative and qualitative data through closed questions and open-ended questions. In particular, this questionnaire sought to answer the following research questions:

1. How and to what extent does the current enhanced feedback help students to change their conceptual understanding and facilitate their understanding of the subject?

2. What are their views on the user-friendly features of the enhanced feedback?

3.5.2 Ethical consideration

The questionnaire was designed in accordance with policy, procedures and guidance of the Faculty Research Ethics Committee (FREC) at UWE Bristol. The questionnaire distribution and collection of data for the research was commenced after receiving written approval from FREC to undertake research involving human participants.

3.5.3 Questionnaire design and distribution

The questions in the questionnaire, shown in Figure 3.32, fell into two groups: Likert-scale and open-ended. The first four questions on the questionnaire were closed questions. These questions were prepared using a 5-point Likert-scale ranging from "Strongly agree" to "Strongly disagree". There were three open-ended questions on the questionnaire and a comment box was provided for students to input their responses to each of these questions. The questionnaire was administrated via Qualtrics software (Qualtrics, 2005). Qualtrics is a web-based survey software

UWE Bristol University of the West of England

QUESTIONNAIRE

Evaluating the effectiveness of the enhanced feedback on the Dewis e-Assessment System

This questionnaire has a number of questions asking you for your feedback on the enhanced feedback you received on Engineering Mathematics weekly test (*include assessment number here*) on the Dewis e-assessment system.

Please tick (√) in the appropriate column alongside the question number on the questionnaire.

Do not worry about projecting a good image. Your answers are **CONFIDENTIAL.**
Thank you for your cooperation.

		Strongly disagree	disagree	Neutral	Agree	Strongly agree
1	The enhanced feedback I received on weekly test (*number*) improved my mathematical understanding.					
2	The enhanced feedback makes me feel confident/comfortable with Engineering Mathematics.					
3	The information in the enhanced feedback is relevant to the question asked.					
4	I am satisfied with the overall structure of the enhanced feedback.					

5. What do you **like** about the enhanced feedback you received?

6. What do you **dislike** about the enhanced feedback you received?

7. Do you have any suggestions for improvement?

Figure 3.32: Example of the questionnaire sent out to students

tool which can be used to conduct publicly available surveys, or to give specific users access to a survey. Qualtrics is an approved UWE Bristol platform for conducting quantitative research online. Qualtrics is freely available to all UWE Bristol staff and students.

Using online questionnaires has numerous benefits in terms of cost, time, ease of administration, data collation and analysis (Dillman, 2007). In addition to the aforementioned advantages, Qualtrics offers the option of reaching participants via email link to the questionnaire, which allows participants to respond anonymously.

The questions were designed to avoid long, double-barrelled, technical, ambiguous, leading or double negative questions or statements. In order to make the questionnaire short and clear, lengthy questions were avoided to make sure that the questionnaire fitted on one page (Dillman, 2007). Great care was taken to make the questionnaire visually appealing (Frankfort-Nachmias, 1996). The UWE logo was inserted at the top of the questionnaire to make it more professional and institution-related. In the invitation email, it was specifically stated how the participants' responses would be used in the future development of Dewis and hence be valued as a whole by the UWE Bristol community (Oppenheim, 1992).

As suggested by Dillman (2007), in order to maximise response and completion rates, a clear indication of how long the questionnaire would take to complete was given in the invitation email. Further, clear instructions were included, together with the purpose of the questionnaire and important information related to the research which were available in a separate 'Participant Information Sheet' (PIS). A link to the PIS, which was placed on the CSE Project web page (CSE Project at UWE, 2019b), was included in the 'Informed Consent' section at the beginning of the questionnaire.

For each weekly test, the students who had received CSE EFB on each question were identified by analysing the additional PIs in the Dewis Reporter. At the end of each weekly test, the questionnaire was sent to those identified students. There were some students who received CSE EFB, and hence the questionnaire, in more than one week. The total number of questionnaires sent by the end of the semester was 336 and these were sent to 196 distinct students who had received CSE EFB in at least one of their weekly tests.

At the end of the revision test, 129 distinct students who had received CSE EFB for this test were identified. In order to gather more responses from the students at the end of the semester, the questionnaire was sent to all of the students who had received CSE EFB in either the weekly or revision tests. There were 78 students who received CSE EFB for both the weekly and revision tests. Therefore, in order to avoid sending the questionnaire to those students twice, the questionnaire was sent to the 247 distinct students who had received CSE EFB for either the end of semester revision test or the weekly tests. The 2019-20 cohort 74.8% of students made at least one CSE in either their weekly tests or the revision test.

The weekly questionnaires and end of semester questionnaire were completed by 33 and 26 participants respectively. In total, the questionnaire was completed by 59 participants.

3.5.4 Quantitative and qualitative data analysis techniques

The first four questions of the questionnaire were in Likert-scale format. Therefore, the participants responses to the Likert-scale questions were analysed qualitatively by particularly looking for the agreement percentage (AP). The AP is the number of participant that selected “Agree” or “Strongly Agree” divided by the sum of those participants selecting a response on that question.

Thematic analysis techniques were used to analyse the three open-ended questions in the questionnaire. Thematic analysis is a widely used qualitative method. It is used to analyse qualitative data gathered in the form of open-ended responses to questionnaires (Castleberry and Nolen, 2018).

Thematic analysis is used for identifying, analysing and reporting patterns or themes within data. In their paper, Braun and Clarke (2006) describe how to conduct thematic analysis in six phases (Familiarising yourself with your data, Generating initial codes, Searching for themes, Reviewing themes, Defining and naming themes, Producing the report). Following the six phases as described by Braun and Clarke (2006), thematic analysis was conducted on the responses to the open-ended questions on the questionnaire. The results of both the quantitative analysis and the qualitative analysis are presented in Chapter 4.

3.6 CSE Project Stage Five: Impact of the CSE Project

In this section, the methodology used in Stage Five of the CSE Project is discussed. Determining the efficacy of diagnosing and remediating mathematical CSEs in e-Assessment questions is the main purpose of this stage.

Eight questions which have 17 identified and coded CSEs were used in the end of semester e-examination for the 2019-20 cohort. Since the original question codes were used in the e-examination, the re-marking facility in Dewis was used to re-mark the students answers against these amended eight question codes after the 2019-20 e-examination. This was the most efficient method of finding the number of students who made CSEs in their e-examination answers as opposed to going through all of the 328 students' handwritten scripts.

Each of these eight questions were used in previously weekly and revision tests. The summary of the questions selected for the e-examination, CSE code reference, number of incorrect answers, number of CSE answers found in each questions and their CSE percentage are documented in Table 3.5.

It should be noted that Table 3.5 contains those students who made the relevant CSE for the first time in the e-examination as well as those students who repeated the same CSE at the e-examination even though they had received CSE EFB during weekly and revision tests. The students who made the CSE for the first time at the e-examination had not seen the CSE EFB before the e-examination. Since the main purpose of this stage is to investigate the success of the CSE Project, a thorough analysis of the students who received CSE EFB either in weekly or revision tests before the e-examination is carried out in Chapter 4.

Table 3.5: Details of identified CSEs found during the re-marking of the 2019-20 e-examination

CSE ID	**No. of incorrect answers**	**No. of CSE answers**	**CSE %**
Q1 CSE1	55	25	45%
Q1 CSE2	59	10	17%
Q1 CSE3	29	2	7%
Q2 CSE1	55	9	16%
Q10 CSE1	86	3	3%
Q14 CSE1	117	29	25%
Q14 CSE2	70	11	16%
Q15 CSE1	48	16	33%
Q16 CSE1	125	7	6%
Q16 CSE2	125	12	10%
Q22 CSE1	144	7	5%
Q22 CSE2	144	4	3%
Q22 CSE3	144	5	3%
Q26 CSE1	143	10	7%

Continued on next page

Table 3.5 *continued from previous page*

CSE ID	**No. of incorrect answers**	**No. of CSE answers**	**CSE %**
Q26 CSE2	143	8	6%
Q26 CSE3	143	10	7%
Q26 CSE4	143	8	6%

4 Data Analysis

This chapter is divided into three sections. The first section analyses the mathematical CSEs gathered in Stage One of the CSE Project to develop a taxonomy of mathematical CSEs in EM e-Assessments questions. In the second section, using the data obtained from the questionnaire in Stage Four of the CSE Project, the students' perceptions on the CSE EFB is analysed to validate the effectiveness of the CSE EFB produced in this project. The third section analyses the Dewis-stored data gathered in Stage Five of the CSE Project to examine the impact of diagnosing and remediating mathematical CSEs in e-Assessment questions.

4.1 Analysis of mathematical CSEs in e-Assessments questions

4.1.1 Development of a taxonomy of mathematical CSEs in e-Assessments

This section analyses the mathematical CSEs gathered in Stage One of the CSE Project to develop a taxonomy of mathematical CSEs in EM e-Assessments questions.

All 65 CSEs found in the course of the CSE project were documented in a systematic order in the CSE book together with their mathematical taxonomy coding. The general taxonomy proposed by Ford, Gillard, and Pugh (2018) were adapted to select and categorise only those CSEs which are relevant to e-Assessments. The 65 CSEs found during the CSE project only fell into four of the error categories (S, U, CM and UM) from the Ford, Gillard, and Pugh (2018) taxonomy (see Section 2.1.5).

Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C) were not found among the CSEs made by the EM students, due to the nature of the questions asked and the nature of the system used to deliver the questions.

None of the e-Assessment questions delivered by Dewis involve mathematical theorems and proofs and hence Errors related to proof (P) were not viable in this CSE collection. Further, the e-examination did not contain questions that required student's communication of their mathematical solutions, correct use of notation or labelling and qualitative judgements on clarity of expression. Therefore, errors in student's communication of their mathematical solutions (C) were not found in this CSE collection. Further, in some cases, a few of the CSEs found fell into two categories due to the mix of misconceptions made by the students as they arrived at their incorrect answer.

Under the category Errors of slip of action (S), three main errors, namely copying error, careless errors on simple calculations, and incorrect algebraic manipulation were identified. A total of 13 out of 65 CSEs were found to fall into the Errors of slip of action category (S).

Seven main errors were identified under the Errors of understanding (U) category, such as confusing different mathematical structures, incorrect argument, lack of consideration of potential indeterminate forms, proposed solution is not viable, definition/method/theorem not recalled correctly, partial solution given, and incorrect assumptions. In total 45 CSEs are in the Errors of understanding category.

Only one main error was found in each of the Errors in choice of method (CM) and Errors in use of method (UM) categories. Three CSEs were grouped into the main error of applying an inappropriate formula/method/theorem in CM. There were 9 CSEs which fell into Error in use of an appropriate definition/method/theorem in the UM category.

All the codes, errors and examples found in this CSE collection process are shown in Table 4.1.

Table 4.1: Taxonomy of Mathematical Common Student Errors in e-Assessments

Main category	**Code**	**Error**	**Examples**
Slip of action	S1	Copying error	Incorrect copying of the question
			Mistake copying/submitting answer into e-Assessment
			Incorrect interpretation of the question
	S2	Careless error on simple calculations	Overlooking negative signs
			Omission of denominator

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Table 4.1 *continued from previous page*

Main category	Code	Error	Examples
	S3	Incorrect algebraic manipulations	Incorrect division of two complex numbers
			Sum of product is split as a product of two sums
			Incorrect handling of powers
Errors of understanding	U1	Confusing different mathematical structures	Confusing the structure of completing the square and the quadratic equation
			Stating that a unit step function is a number
	U2	Incorrect argument	Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives
			Taking the integration of the product of two functions as the product of individual integrals
	U3	Lack of consideration of potential indeterminate forms	Taking the square of a negative number to be negative
	U4	Proposed solution is not viable	Angle is not within the given range
	U5	Definition/method/ theorem not recalled correctly	Method of completing the square is not recalled correctly
			Definition of waveform properties not recalled correctly
			Method of differentiating a standard function is not recalled correctly
			Method of solving trigonometry equation is not recalled correctly

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Table 4.1 *continued from previous page*

Main category	Code	Error	Examples
			Chain rule is not recalled correctly
			Method of Partial differentiation not recalled correctly
			Method of differentiating implicit functions is not recalled correctly
			Mean value theorem is not recalled correctly
			Method of calculating the argument of a complex number is not recalled correctly
			Binomial theorem is incorrectly followed
			Definition of Centre of Mass is not recalled correctly
			Method of finding the principle value of the argument of a complex number is not recalled correctly
			Method of integrating not recalled correctly
			Definition of volume of revolution is not recalled correctly
	U6	Partial solution given	Correct workings but unfinished solution
	U7	Incorrect assumptions	Incorrect assumptions on the mean value theorem
			Taking dimension of velocity as $[v] = [MT^{-1}]$
Errors in choice of method	CM1	Applying an inappropriate formula/method/theorem	Uses a method which is not relevant in the situation

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Table 4.1 *continued from previous page*

Main category	Code	Error	Examples
			Uses a formula which is not relevant in the situation
Errors in use of method	UM2	Error in use of an appropriate definition/method/ theorem	Error in the use of the chain rule
			Error in use of partial differentiation method
			Incorrect units applied
			Method finding the volume of revolution is incorrectly followed

4.1.2 CSE examples in each taxonomical category

In this section examples of CSEs in each taxonomical category (Slip of action, Errors of understanding, Errors in choice of method, and Errors in use of method) are presented. These and the rest of the CSEs found in the CSE Project can be found in UWE Bristol's Research Repository (Sikurajapathi, Henderson, and Gwynllyw, 2022a) as well as in Appendix C of this thesis.

A CSE due to Slip of Action

Figure 4.1 shows a CSE related to a question in Algebra (Completing the Square) (see Section 2.1.1 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). Students' answer scripts indicated that even though students had solved the question correctly, they submitted incorrect answers for b which corresponded to the negative of the correct value of b. Therefore, this CSE can be considered as a copying error in the Slip of action category when submitting answers into the e-Assessment. In 2017-2018, 28 students, out of the 56 who answered this question incorrectly (50%) made this CSE. In 2018-19, 33 students from 57 who answered this question incorrectly (58%) made the same mistake.

A CSE due to Errors of Understanding

Figure 4.2 shows a CSE related to a question on complex numbers (rectangular form) (see Section 3.3.1 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in

Question

The expression

$t^2 - 12t + 40$

can be expressed in the form:

$a(t-b)^2 + c$

where a, b and c are constants.

Calculate the values of these constants - note that all these solutions are integers:

Enter the value of a [ ]
Enter the value of b [ ]
Enter the value of c [ ]

Correct Solution

$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$
$$= (t-6)^2 + 4$$
$$a = 1, b = 6 \text{ and } c = 4$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**S1**

Give answer $\tilde{b}$ which corresponds to the negative of the correct value of b.

$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$
$$= (t-6)^2 + 4$$
$$\tilde{b} = -6 \text{ and } c = 4$$

No. of CSEs /No. incorrect answers (CSE %)	28/56 (**50%**) 33/57 (**58%**)	**Date collected**	**2017-18** **2018-19**

Figure 4.1: Q1 CSE1 of Question ALGEBRA_COMPLETESQUARE01 due to Slip of Action

Appendix C). Students' answer scripts indicated that the square of a negative number was taken to be negative. Therefore, this CSE can be considered as a lack of consideration of potential indeterminate forms. In 2017-18, 40 students, out of the 57 who answered this question incorrectly (70%) triggered this CSE. It should be noted this question was not used in the 2018-19 e-examination.

Question			
Find the modulus $\lvert z \rvert$ of the complex number $z = -2 + 5j$, correct to two decimal places. Enter $\lvert z \rvert$ correct to 2 decimal places:			
Correct Solution			
$z = -2 + 5j$ $\lvert z \rvert = \sqrt{(-2)^2 + 5^2}$ $= \sqrt{4 + 25}$ $= \sqrt{29}$ $\lvert z \rvert = 5.39$			
CSE 1 related to this question	**CSE Taxonomy Code:**	**U3**	
Taking $(-n)^2 = -n^2$ $z = -2 + 5j$ $\lvert z \rvert = \sqrt{(-2)^2 + 5^2}$ $\widetilde{\lvert z \rvert} = \sqrt{-4 + 25}$ $= \sqrt{21}$ $\widetilde{\lvert z \rvert} = 4.58$			
No. of CSEs /No. incorrect answers (CSE %)	40/57**(70%)**	**Date collected**	**2017-18**

Figure 4.2: Q9 CSE1 of Question COMPLEXNUMBERS_CARTESIANMODULUS01 due to Error of Understanding

A CSE due to Errors in Choice of Method

Figure 4.3 shows a CSE related to a question on infinite geometric series (see Section 4.1.2 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). Students' answer scripts indicated that 34 students out of 67 who answered this question incorrectly (51%) used the formula to find the sum of the first four terms instead of the formula to find the sum of the infinite series. Therefore, this CSE can be considered as applying an inappropriate formula in Error in Choice of Method.

Question

Consider the following geometric series, S, where:

$S = 2 + 2(0.7) + 2(0.7)^2 + 2(0.7)^3 \cdots$.

Write down the first term, a and the common ratio, r in the boxes below.

Enter a: [ ]

Enter r: [ ]

Hence calculate the sum, S and enter your result in the box below.

Enter S (to three decimal places) here: [ ]

Correct Solution

The first term $a = 2$. The common ratio $r = 0.7$

The sum of an infinite series (S) exists, provided $|r| < 1$

$$S = \frac{a}{1-r}$$
$$= 6.667$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**CM1**

Finding the sum of first four terms instead of the sum of the infinite series.

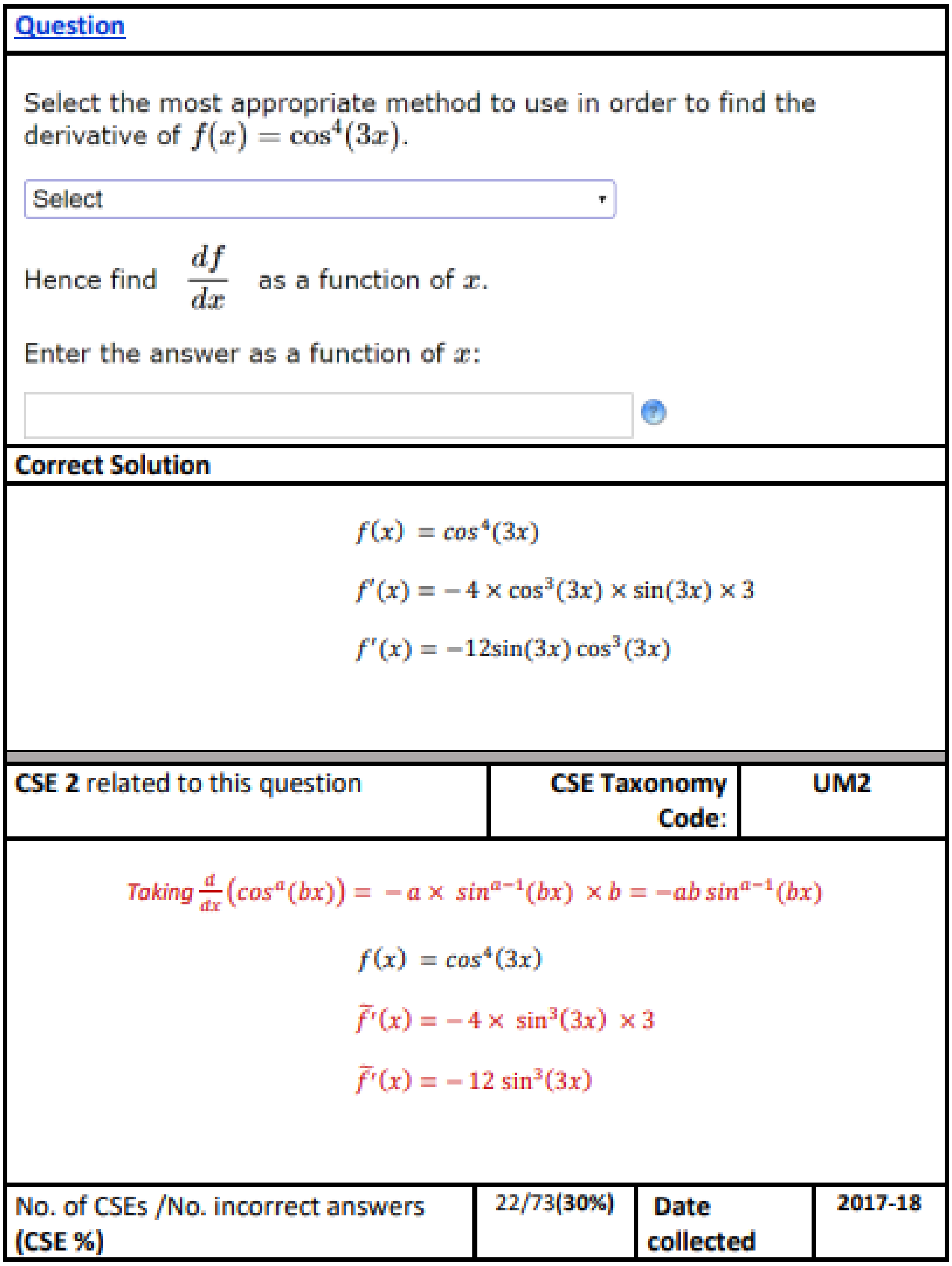
$$\tilde{S} = \frac{a(1-r^n)}{1-r}$$
$$\tilde{S} = \frac{2(1-0.7^4)}{1-0.7}$$
$$\tilde{S} = 5.066$$

No. of CSEs /No. incorrect answers (CSE %)	34/67(**51%**)	**Date collected**	**2017-18**

Figure 4.3: Q13 CSE1 of Question SERIES_GEOMETRIC01 due to Errors in Choice of Method

A CSE due to Errors in Use of Method

Figure 4.4 shows a CSE related to differentiating a function of the form $f(x) = \cos^a(bx)$ (See Section 5.1.2 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). 22 students out of 73 (30%) incorrectly answered that the derivative of $f(x)$ is $-ab\sin^{a-1}(bx)$ due to an error in the use of the Chain Rule. Therefore, this CSE can be considered as an Error in Use of Method.



Question

Select the most appropriate method to use in order to find the derivative of $f(x) = \cos^4(3x)$.

Select

Hence find $\frac{df}{dx}$ as a function of x.

Enter the answer as a function of x:

Correct Solution

$$f(x) = cos^4(3x)$$
$$f'(x) = -4 \times \cos^3(3x) \times \sin(3x) \times 3$$
$$f'(x) = -12\sin(3x)\cos^3(3x)$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**UM2**

$$Taking\ \frac{d}{dx}(cos^a(bx)) = -a \times sin^{a-1}(bx) \times b = -ab\, sin^{a-1}(bx)$$
$$f(x) = cos^4(3x)$$
$$f'(x) = -4 \times \sin^3(3x) \times 3$$
$$f'(x) = -12\sin^3(3x)$$

No. of CSEs /No. incorrect answers **(CSE %)**	22/73**(30%)**	**Date collected**	**2017-18**

Figure 4.4: Q20 CSE2 of Question CALCULUS_DIFFERENTIATION_CHAIN-RULE02 due to Errors in Use of Method

4.2 Analysis of students' perceptions of mathematical CSE EFB in e-Assessment questions

In this section, using the data obtained from the questionnaire in Stage Four of the CSE Project, the students' perceptions on the CSE EFB is analysed to validate the effectiveness of the CSE EFB produced in this project.

As mentioned in Chapter 3, there were 59 participants who completed the questionnaire. The data analysis on the Likert-scale questions and the Open-ended questions are discussed in the following sections.

4.2.1 Quantitative data analysis of Likert-scale questions

In the following sections, each of the Likert-scale questions in the questionnaire are discussed separately. The bar-chart figures are used to show the percentages of each Likert-scale response with the AP for each statement. It should be noted that the percentages do not always total to 100% due to rounding.

Q1: The enhanced feedback I received on weekly test [x] improved my mathematical understanding

Figure 4.5 presents the participants' responses to the statement "The enhanced feedback I received on weekly test [x] improved my mathematical understanding" in the questionnaire. This shows that the majority of participants either strongly agreed or agreed that the CSE EFB they received improved their mathematical understanding. The AP of the participants to the statement is 88% and Figure 4.5 indicates the participants' positive appreciation towards the conceptual change afforded by the CSE EFB.

Q2. The enhanced feedback makes me feel confident/comfortable with Engineering Mathematics

Figure 4.6 shows the participants' responses to the statement "The enhanced feedback makes me feel confident/comfortable with Engineering Mathematics" for the weekly questionnaire and/or the end of semester questionnaire. The results show that the majority of the participants agreed with this statement and the AP of the participants to the statement is 73%.

Q3. The information in the enhanced feedback is relevant to the question asked

The third statement of the questionnaire is looking at how students feel about the relevance of the CSE EFB. Figure 4.7 shows the questionnaire responses to the question "The information in the CSE EFB is relevant to the question asked" for the weekly questionnaire and/or the end of semester questionnaire.

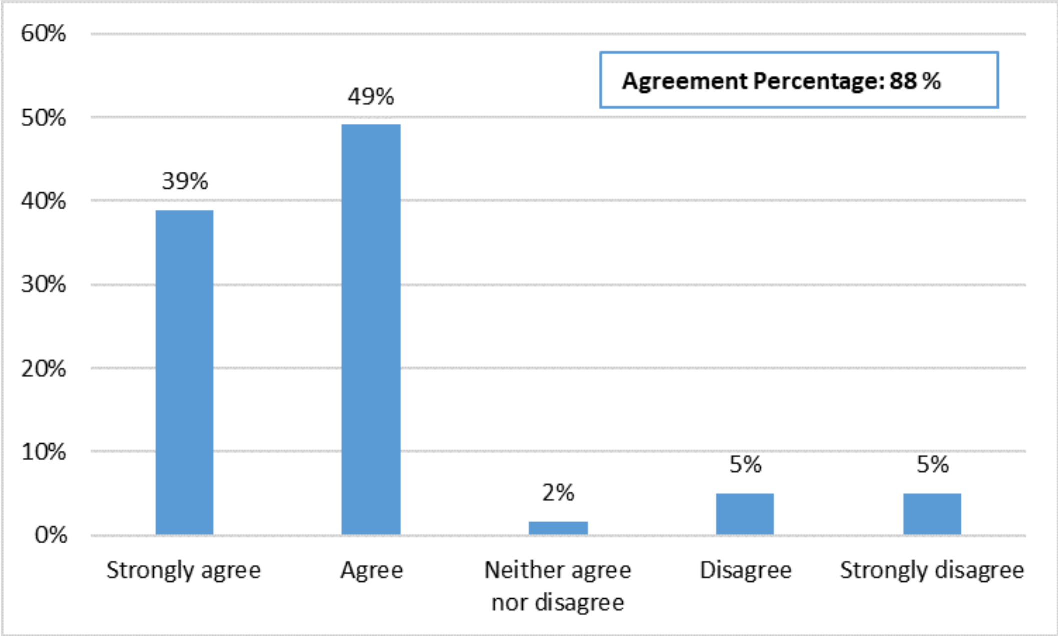


Figure 4.5: Questionnaire responses to the question "The enhanced feedback I received improved my mathematical understanding"

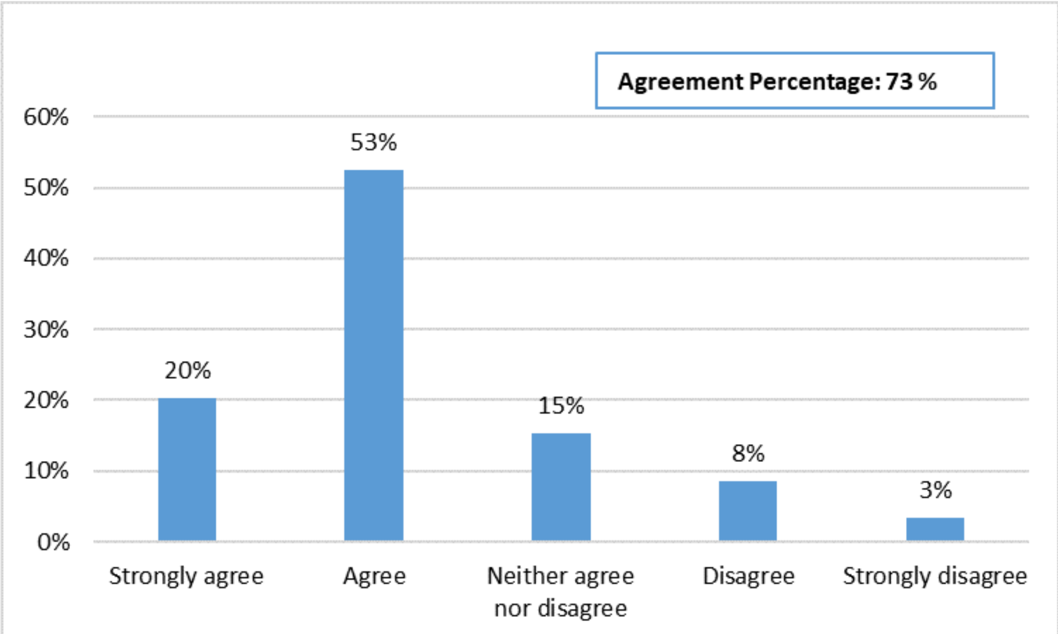


Figure 4.6: Questionnaire responses to the question "The enhanced feedback makes me feel confident/comfortable with Engineering Mathematics"

What stands out in Figure 4.7 is that almost all of the participants agreed or strongly agreed that the information in the CSE EFB is relevant to the question asked (AP 95%).

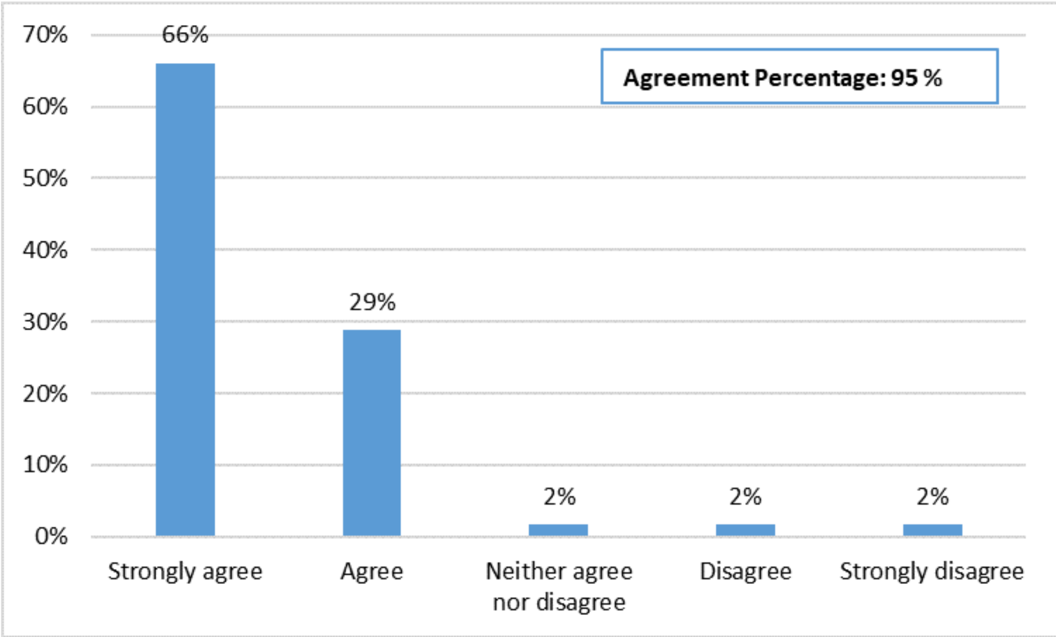


Figure 4.7: Questionnaire responses to the question “The information in the enhanced feedback is relevant to the question asked”

Q4. I am satisfied with the overall structure of the enhanced feedback

Figure 4.8 shows the questionnaire responses to the question “I am satisfied with the overall structure of the enhanced feedback”. The majority of those who responded to this statement indicated that they were satisfied with the overall structure of the CSE EFB. Figure 4.8 shows that the AP for this statement is 87%.

4.2.2 Qualitative Data Analysis of Open-ended questions

In the following sections, each of the open-ended questions in the questionnaire are discussed separately. Using thematic analysis, overarching themes and sub-themes were identified for each question.

Q5. What do you like about the enhanced feedback you received?

The prime purpose of this question was to capture what students like about the CSE EFB in order to keep those features unchanged when new CSE EFB is constructed in the future.

Two overarching themes, Conceptual change and User-friendly features, emerged from a detailed thematic analysis of the texts of students’ responses to this question. The sub-themes which emerged from the two aforementioned main themes are sum-

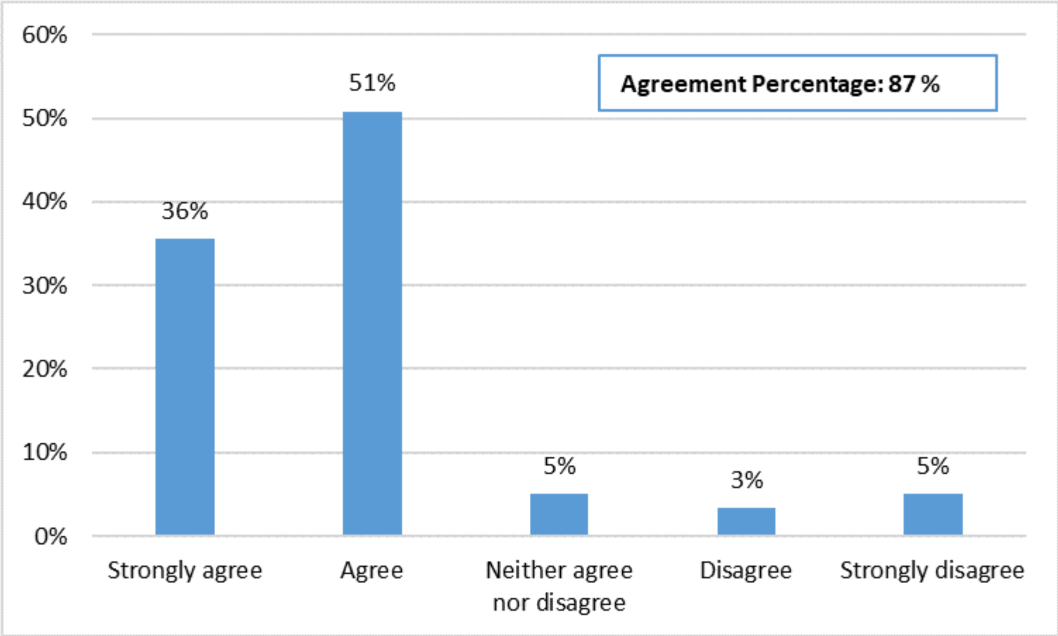


Figure 4.8: Questionnaire responses to the question "I am satisfied with the overall structure of the enhanced feedback"

marised in Table 4.2.

The Conceptual change theme highlighted three sub-themes which examined perception on Correct CSE capture (Correct capture), Facilitating learning (Beneficial), and Relevance of the content on CSE EFB (Relevant).

Under the sub-theme Correct capture, many participants felt that the CSE EFB they received cleared up their doubts. Further, they claimed that the feedback made them understand why and where they went wrong. The majority of the participants' appreciated the way in which the CSE EFB helped them to change their misunder-

Table 4.2: Themes resulting from thematic analysis on student responses to the question "What do you like about the enhanced feedback you received?"

Main themes	Sub-themes
Conceptual change	Correct CSE capture (Correct capture)
	Facilitate learning (Beneficial)
	Relevance of the content on CSEs enhanced feedback (Relevance).
User-friendly features	Coherent structure
	Accessibility

standings/misconceptions of the mathematical concepts and to improve their learning. In the Beneficial sub-theme, a significant number of participants mentioned the benefit they received from the CSE EFB in improving their understanding. Further, they noted the usefulness and helpfulness of the feedback to their learning and understanding of the subject. In the Relevance sub-theme, a couple of participants mentioned how relevant the received feedback was in their learning. Table A.1 in Appendix A contains several examples of such quotes which emerged from the Conceptual Change theme and three examples of such quotes are given below:

> “It makes you feel conscious of errors you made. The fact that it tells you what you’ve done based on your final input is clever."
>
> “The enhanced feedback got right to the reason the answer was wrong rather than lingering on things already explained above in the solution."
>
> “The Feedback which I received helped me to understand where I was most likely to make errors and showed the correct way of working out solutions."

The User-friendly features theme contained two sub-themes: Coherent Structure and Accessibility. Under the sub-theme Coherent Structure, it emerged that many participants liked the structure of the CSE EFB and particularly highlighted its step-by-step, clear and concise explanations. Several participants appreciated the accessibility features of the CSE EFB. In particular they commented on its instant availability, quick accessibility, and visibility in different colours. Table A.2 in Appendix A shows multiple examples of those quotes which arose from the User-friendly Features theme. One example from Table A.2 in Appendix A is reproduced here:

> “The total feedback was overall concise and accessible."

Q6. What do you dislike about the enhanced feedback you received?

This question was looking for what students disliked about the CSE EFB in order to amend and improve the features of future CSE EFB. Thematic analysis on the responses for this question highlighted four main themes: Everything is alright, Short explanations, Less accessibility features and Not helpful.

It was encouraging to see that the majority of the participants said that they were satisfied with the current CSE EFB and did not indicate any aversion to it.

Some comments from the Everything is alright theme can be found in Table A.3 in Appendix A.

A few participants indicated that the CSE EFB is very short for some questions and suggested that they would prefer to have more detailed feedback, which would improve it in the future. Two such examples of participants' comments are shown below:

> "Some answers can be quite brief so more in depth answers would be great."
>
> "Needs more steps for the student to fully understand what is happening throughout the equation."

The participant who made the above comment also disagreed to all of the Likert-scale statements on the questionnaire except for statement Q3. Further comments on the Short explanations theme can be found in Table A.4 in Appendix A.

A few comments related to the Less accessibility features theme. Some participants mentioned the issue of visibility of the current CSE EFB and gave some useful suggestions for increasing its visibility. One participant suggested moving the CSE EFB to the general Solution section rather than including it in a separate section (Report section) to avoid scrolling past the CSE EFB. These inputs were very valuable and can be incorporated and used to address issues raised in the future development of the CSE Project. Table A.5 in Appendix A shows multiple examples of quotes which arose from this theme. One example of which is given here:

> "The incorrect answer could be written right after the correct one rather than right at the very bottom so that it would be easier to understand."

However, only one participant found the CSE EFB not to be useful and stated that "It doesn't help me to learn anything." The same participant strongly disagreed to statements Q1 and Q2 and disagreed to statement Q4. However, the participant agreed with statement Q3.

Q7. Do you have any suggestions for improvement?

The last item of the questionnaire was "Do you have any suggestions for improvement?" This questions was looking for participants' views on what is lacking in the feedback and for ideas on how to further develop the CSE EFB in the future.

Table 4.3: Themes resulting from thematic analysis of student responses to the question “Do you have any suggestions for improvement?”

Main themes	Sub-themes
Everything is alright	Everything is alright
Suggestions to improve current features	Detailed Explanations
	More Accessibility features
Suggestions for future directions	Enhanced feedback for all the other questions
	New ideas for further improvement

Thematic analysis on the responses for this question revealed three main themes: Everything is alright, Suggestions to improve current features, and Suggestions for future directions. Table 4.3 summarises these themes and all of the sub-themes which emerged from this question.

Most of the participants were satisfied with the CSE EFB they received and did not give any suggestions for further improvements. A few responses received under the Everything is alright theme can be found in Table A.6 in Appendix A, and one of these is presented here.

> “I think it is as good as it can be. Thank you!"

A few participants echoed the same suggestions that we received for question 6 of the questionnaire. Namely, they suggested providing detailed feedback and making the feedback more accessible, readable and efficient. A few responses received in the Suggestions to improve current features theme are detailed in Table A.7 in Appendix A and two of these are presented here.

> “Include all steps, even if they seem unimportant."
>
> “Provide enhanced feedback not just on hard questions but on easy ones too."

One participant who disagreed to the Likert-scale statements Q1, Q2 and Q4 but agreed to Likert-scale statement Q3 suggested “To make it more readable and a more efficient design”.

Some participants provided useful suggestions for future directions, which emerged as a main theme. Within this theme, the comments were categorised into two sub-themes, namely “Enhanced feedback for all the other questions” and “New ideas for further improvement”.

A few of the comments given in this theme are recorded in Table A.8 in Appendix A, and one of these comments is as follows:

> “I would also like to know the subject of each question so that I could Google anything that I didn’t understand. Another option would be to have a link to the lectures that covered each question, so that if I got a question wrong I could know what lecture covered that topic."

4.3 Analysis of the effectiveness of diagnosing and remediating mathematical CSEs in e-Assessment questions

This section analyses the Dewis-stored data gathered in Stage One and Stage Five of the CSE Project to examine the effectiveness of diagnosing and remediating mathematical CSEs in e-Assessment questions. Firstly, a data comparison of the CSE occurrences before and after the CSE EFB feature on Dewis is performed. Secondly, further investigation is carried out to find out the success of the CSE Project using a thorough analysis of those students who received CSE EFB either in weekly or revision tests before the e-examination. These analyses and their outcomes are presented in this section to validate the effectiveness of diagnosing and remediating mathematical CSEs in e-Assessment questions.

4.3.1 Quantitative analysis on the data of before and after CSE EFB feature on Dewis

In this section, analysis of the CSE occurrences before and after the CSE EFB feature on Dewis is performed. The data gathered from the 2017-18, 2018-19 and 2019-20 e-examinations of EM are utilised for the analysis.

As mentioned in Section 3.6, eight questions which have 17 identified and coded CSEs were used in the end of semester e-examination for the 2019-20 cohort. Each of these eight questions were used previously in weekly and revision tests for the 2019-20 cohort.

These eight questions were already used either in the 2017-18 or the 2018-19 end of semester controlled conditions e-examinations. The summary of the questions selected for the e-examination in 2019-20, CSE ID, number of incorrect answers, number of CSE answers for each question, their CSE percentage and CSE

Rate for the controlled condition e-examinations in 2017-18, 2018-19 and 2019-20 are presented in Table 4.4.

The data for 2017-18 and 2018-19 were taken from the first stage of the CSE Project when the students' answer scripts were examined by hand. For the 2019-2020 cohort, the total number of incorrect and CSE answers triggered by students were gathered by re-marking the 2019-20 e-examination. Re-marking had to be done because for the e-examination, original question codes were used instead of the amended CSE EFB questions. This was the decision of the module leader and therefore, the 2019-20 e-examination data was gathered by utilising the re-marking feature of the Dewis e-Assessment System.

Table 4.4: The total number of incorrect and CSE answers made by students in the 2017-2018, 2018-2019 and the 2019-2020 e-examinations

CSE ID	2017-2018 Pre-CSE EFB				2018-2019 Pre- CSE EFB				2019-2020 Post-CSE EFB			
	No. of incorrect answers	**No. of CSE answers**	**CSE %**	**CSE Rate**	**No. of incorrect answers**	**No. of CSE answers**	**CSE %**	**CSE Rate**	**No. of incorrect answers**	**No. of CSE answers**	**CSE %**	**CSE Rate**
Q1 CSE1	56	28	50%	0.09	57	33	58%	0.10	55	25	45%	0.08
Q1 CSE2	45	6	13%	0.02	51	14	27%	0.04	59	10	17%	0.03
Q1 CSE3	26	4	15%	0.01	24	2	8%	0.01	29	2	7%	0.01
Q2 CSE1	86	35	41%	0.12	100	32	32%	0.10	55	9	16%	0.03
Q10 CSE1	109	9	8%	0.03	95	9	9%	0.03	86	3	3%	0.01
Q14 CSE1	116	28	24%	0.09	122	29	24%	0.09	117	29	25%	0.09
Q14 CSE2	59	11	19%	0.04	80	16	20%	0.05	70	11	16%	0.03
Q15 CSE1	28	7	25%	0.02	64	12	19%	0.04	48	16	33%	0.05
Q16 CSE1	107	9	8%	0.03	135	3	2%	0.01	125	7	6%	0.02
Q16 CSE2	107	9	8%	0.03	135	13	10%	0.04	125	12	10%	0.04
Q22 CSE1	158	18	11%	0.06	180	15	8%	0.05	144	7	5%	0.02
Q22 CSE2	158	12	8%	0.04	180	5	3%	0.02	144	4	3%	0.01
Q22 CSE3	158	10	6%	0.03	180	2	1%	0.01	144	5	3%	0.02
Q26 CSE1	143	13	9%	0.04					143	10	7%	0.03

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Table 4.4 – *Continued from previous page*

CSE ID	2017-2018 Pre-CSE EFB				2018-2019 Pre-CSE EFB				2019-2020 Post-CSE EFB			
	No. of incorrect answers	**No. of CSE answers**	**CSE %**	CSE **Rate**	**No. of incorrect answers**	**No. of CSE answers**	**CSE %**	CSE **Rate**	**No. of incorrect answers**	**No. of CSE answers**	**CSE %**	CSE **Rate**
Q26 CSE2	143	13	9%	0.04					143	8	6%	0.02
Q26 CSE3	143	11	8%	0.04					143	10	7%	0.03
Q26 CSE4	143	8	6%	0.03					143	8	6%	0.02

It should be noted that the column which shows the number of CSE answers in the 2019-20 cohort in Table 4.4 includes students who made the respective CSE for the first time in the e-examination as well as those students who repeated the same CSE at the e-examination even though they had received CSE EFB during weekly and revision tests.

It can be seen from the data in Table 4.4, that nine CSEs have the lowest CSE percentage in the 2019-20 cohort (namely Q1 CSE1, Q1 CSE3, Q2 CSE1, Q10 CSE1, Q14 CSE2, Q22 CSE1, Q22 CSE2, Q26 CSE1 and Q26 CSE2) compared to the respective CSE percentages in the 2017-18 and 2018-19 cohorts. Even though this indicates a promising result of positive effectiveness of the CSE EFB, thorough analyses will be done hereafter to confirm this indication.

The partially overlapping samples t-test was carried out on the CSE Rate presented in Table 4.4. The p-value for the years 2017-18 and 2018-19 is 0.9011. This suggests that there is no evidence that there is a difference between the number of each CSE made by students in the e-examinations in the pre-CSE EFB years (2017-18 and 2018-19).

The same test on the CSE Rate in the pre-CSE EFB years versus the post-CSE EFB year was performed. The p-value for 2017-18 versus 2019-20 is 0.0485 and the p-value for 2018-19 versus 2019-20 is 0.0403. Since in the both cases the p-value is less than or equal to 0.05, it suggests that there is evidence of a significant difference between the pre and post CSE EFB groups. The findings of these statistical measures of consistency on the CSE Rate across the pre and post years of CSE EFB suggest that there is a positive effect of the CSE EFB.

Further, the CSE percentages of Q14 CSE1, Q26 CSE3 and Q26 CSE4 in the 2019-20 cohort are comparable with the respective CSE percentages in the 2017-18 and 2018-19 cohorts. It should be noted that two distinct students who received CSE EFB for Q26 CSE3 either in the weekly or revision tests (see Table 3.3 and Table 3.4) did not sit the 2019-20 e-examination. This means that all 10 students did Q26 CSE3 for the first time in the 2019-20 e-examination.

Q26 CSE3 is presented in Figure 4.9 (see Section 3.5.27 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). Q26 CSE3 belongs to one of the main taxonomical categories (see Table 4.1) called Errors of understanding (U). Q26 CSE3 occurs due to not recalling the method of integration correctly (U5). In particular, the reason for Q26 CSE3 is missing out x in front of the trigonometric function when integrating the function $x\cos(ax)$, and hence getting the CSE answer

$\dfrac{\sin(ax)}{a}$.

Question

Evaluate the following:

$$\int x\cos(3x)dx$$

as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:

Correct Solution

Use integration by parts

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Take

$$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$

$$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$

$$I = \int x\cos(3x) = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3}dx$$

$$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 3 related to this question	**CSE Taxonomy Code**:	**U5**

Missed out x in front of the trigonometric function

$$I = \int x\cos(3x)dx = \left(\frac{\sin(3x)}{3}\right)$$

No. of CSEs /No. incorrect answers **(CSE %)**	**11/143 (8%)**	**Date collected**	**2017-18**

Figure 4.9: Q26 CSE3 of Question CALCULUS_INTEGRATION_PARTS01

Further investigation of the data in the weekly and revision tests of the 10 students who did Q26 CSE3 for the first time in the 2019-20 e-examination revealed some interesting and possible reasons for doing that CSE at the e-examination. Three out of 10 students who did Q26 CSE3 for the first time in the 2019-20 e-examination got the question correct in at least one attempt in the weekly test but did not answer the question at the revision test. Five of the remaining seven students either did not do the test or did not answer the question either in the weekly or revision tests. The other two students either did not do the test, did not answer or submitted an incorrect answer for the question in either the weekly or revision tests. The investigation disclosed that most of the students who did Q26 CSE3 for the first time in the 2019-20 e-examination missed the chance of getting their misconception corrected due to not attempting the weekly or revision tests before the e-examination. If they had attempted these tests before the e-examination, Q26 CSE3 could have been corrected through CSE EFB.

The CSE percentage of Q15 CSE1 in the 2019-20 cohort has the highest respective CSE percentages (33%) compared to the 2017-18 and 2018-19 cohorts. However, it should be noted that, Q15 CSE1 was not triggered during either the weekly or revision tests in 2019-20. This indicates that all 16 students did Q15 CSE1 for the first time in the e-examination.

Q15 CSE1 is presented in Figure 4.10 (see Section 2.3.2 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). Q15 CSE1 is an Incorrect algebraic manipulation error (S3) in Slip of action main category of the taxonomy (see Table 4.1). In particular, Q15 CSE1 occurs due to splitting the sum of a product as a product of two sums.

Further investigation was carried out to find more about these 16 students who did Q15 CSE1 for the first time in the 2019-20 e-examination. It revealed that Q15 CSE1 has a similar trend as for the Q26 CSE3 presented above. Four out of 16 students who did Q15 CSE1 for the first time in the e-examination answered the question correctly in at least one attempt in the weekly test but either did not do the revision test or did not answer the question at the revision test. The remaining 12 students either did not do the test or did not answer the question either in the weekly or revision tests. This investigation reveals that the majority of the students who did Q15 CSE1 for the first time in the 2019-20 e-examination did not have a chance to get their misconception corrected because they did not do the test or did not answer the weekly or revision tests before the 2019-20 e-examination. If they had done these tests before the e-examination, Q15 CSE1 could have been corrected through CSE EFB.

Further, Q16 CSE2 in the 2019-20 cohort has a higher respective CSE percentage (10%) compared to the 2017-18 and 2018-19 cohorts. Q16 CSE2 is presented in Figure 4.11 (see Section 3.5.29 of Sikurajapathi, Henderson, and Gwynllyw (2022a)

Question

Masses of 4 kg, 6 kg and 10 kg are located at points with co-ordinates (-2,5) , (1,-4) and (3,1) respectively.

Find the co-ordinates of their Centre of Mass,$(\bar{x}, \bar{y})$, correct to one decimal place.

Enter $\bar{x}$:

Enter $\bar{y}$:

Correct Solution

$$\bar{x} = \frac{\sum_{i=1}^{3} m_i x_i}{\sum_{i=1}^{3} m_i} = \frac{4 \times (-2) + 6 \times 1 + 10 \times 3}{(4+6+10)} = 1.4$$

$$\bar{y} = \frac{\sum_{i=1}^{3} m_i y_i}{\sum_{i=1}^{3} m_i} = \frac{4 \times 5 + 6 \times (-4) + 10 \times 1}{(4+6+10)} = 0.3$$

CSE 1 related to this question	CSE Taxonomy Code:	S3

Taking

$$\sum_{i=1}^{n} m_i x_i = \sum_{i=1}^{n} m_i \sum_{i=1}^{n} x_i$$

and $$\sum_{i=1}^{n} m_i y_i = \sum_{i=1}^{n} m_i \sum_{i=1}^{n} y_i$$

$$\bar{\bar{x}} = \frac{(4+6+10) \times (-2+1+3)}{(4+6+10)} = 2$$

$$\bar{\bar{y}} = \frac{(4+6+10) \times (5-4+1\ )}{(4+6+10)} = 2$$

No. of CSEs /No. incorrect answers (CSE %)	7/28 (25%) 12/64(19%)	Date collected	2017-18 2018-19

Figure 4.10: Q15 CSE1 of Question ENGINEERING_CENTRMASS01

in Appendix C). This error also belongs to the Errors of understanding category (U) which happens due to not recalling the definition of the volume of revolution correctly (U5). The error was finding the value of the definite integral of just the given function y between a given region of x.

The Dewis-stored data revealed that all 12 students did Q16 CSE2 for the first time in the 2019-20 e-examination. Only two of these 12 students answered the question correctly in at least one attempt in the weekly test but did not answer the

Question

Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]

Correct Solution

The volume of revolution of y over the range $a < x < b$ given by

$$V = \pi \int_a^b y^2 dx$$

For given data,

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$= \pi \int_1^4 0.64\, x^3 dx$$

$$= 128.18$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**U5**

Taking $V = \int_a^b y\, dx$

$$\tilde{V} = \int_1^4 0.8\, x^{1.5} dx$$

$$= 0.8 \left[\frac{x^{2.5}}{2.5}\right]_1^4$$

$$= 9.92$$

No. of CSEs /No. incorrect answers **(CSE %)**	9/107 **(8%)** 13/135 **(10%)**	**Date collected**	**2017-18** **2018-19**

Figure 4.11: Q16 CSE2 of Question CALCULUS_INTEGRATION_VOLUMEREVOLUTION01

question at the revision test. The rest of the students either did not do the test or did not answer the question either in the weekly or revision tests. It is apparent that, the majority of the students who did Q16 CSE2 for the first time in the e-examination missed the chance of getting their misconception corrected due to not attempting the weekly or revision tests before the 2019-20 e-examination. If they had done these tests before the 2019-20 e-examination, Q16 CSE2 could have been corrected through CSE EFB.

The evidence presented thus far supports the idea that the majority of the students who made CSEs at the 2019-20 e-examination did not at least attempt the weekly or revision tests before the e-examination. The purpose of this investigation is to find out the effectiveness of the CSE Project, so further investigation is needed on those students who received CSE EFB either in weekly or revision tests before the 2019-20 e-examination. These investigations are presented in the following section.

4.3.2 Quantitative data analysis of success of the CSE Project at UWE Bristol

Dewis-stored data on the weekly tests, revision tests, and re-marking data of the controlled conditions e-examinations are analysed here to find the success of diagnosing and remediating mathematical CSEs in e-Assessment questions.

As mentioned in Section 4.3.1, the number of CSE answers for each question in 2019-20 in Table 4.4 includes both those students who made the relevant CSE for the first time in the e-examination as well as those students who repeated the same CSE at the e-examination. Since the main purpose of this stage of the research is to investigate the success of the CSE Project, a thorough analysis was carried out on the data of students who received CSE EFB either in weekly or revision tests before the e-examination. Table 4.5 contains data gathered about those students who had received CSE EFB either in weekly or revision tests before they sat the e-examination.

It should be worth reiterating here that Table 4.5 shows the success rate of just 15 of 17 CSEs in the eight questions used in the 2019-20 e-examination. This is because during either the weekly or revision tests, Q15 CSE1 was not triggered. Further, the two distinct students who received CSE EFB for Q26 CSE3 either in weekly or revision test did not sit the 2019-20 e-examination.

The second column of Table 4.5 shows the number of students who made CSEs in either the weekly tests or the revision tests and received the CSE EFB before the e-examination (N). The third, fourth, fifth and sixth columns show how many of the N students answered the question correctly (n), made the same CSE again, made a different error, and did not answer the question in the controlled conditions 2019-20 e-examination respectively. The last column shows the success rate of correcting mathematical CSEs in e-Assessment questions. The success rate was calculated using the following formula:

$$\text{Success rate} = \frac{n}{N}\%$$

Table 4.5: Performance data at the 2019-20 e-examination of the students who had received CSE EFB either in weekly or revision tests before the e-examination.

CSE ID	**No. of students who received CSE EFB before the e-examination (N)**	**No. of students who answered correctly (n)**	**No. of students who gave CSE answers**	**No. of students who made a different error**	**No. of students who did not answer the question**	**Success rate** $(\frac{n}{N}\%)$
Q1 CSE1	99	86	9	2	2	87%
Q1 CSE2	11	11	0	0	0	100%
Q1 CSE3	4	3	0	1	0	75%
Q2 CSE1	60	44	3	13	0	73%
Q10 CSE1	7	6	0	1	0	86%
Q14 CSE1	28	14	3	8	3	50%
Q14 CSE2	16	11	0	2	3	69%
Q16 CSE1	17	15	0	2	0	88%
Q16 CSE2	12	6	0	6	0	50%
Q22 CSE1	19	12	0	7	0	63%
Q22 CSE2	6	4	0	2	0	67%
Q22 CSE3	13	6	0	7	0	46%
Q26 CSE1	3	1	0	2	0	33%
Q26 CSE2	26	19	1	5	1	73%
Q26 CSE4	4	2	0	2	0	50%

What stands out in Table 4.5 is that most of the students who had received CSE EFB before the e-examination answered the question correctly at the 2019-20 e-examination. The success rate for 13 of the CSEs was greater than or equal to 50% and the highest success rate was 100%. Another interesting fact that emerged from Table 4.5 is that the majority of the students who got the answer incorrect did not make the same 11 CSEs (out of 15) again at the e-examination. Instead, they made a different error or did not answer the question. It is apparent from Table 4.5 that CSE EFB had a significant positive effect on diagnosing and remediating some mathematical CSEs in e-Assessment questions.

The success rate of Q1 CSE2 is 100%. Q1 CSE2 is presented in Figure 4.12 (see Section 3.5.1 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). This CSE error belongs to one of the main categories in the taxonomy (see Table 4.1) called Errors of understanding (U). Q1 CSE2 occurs due to not recalling the method of completing the square correctly (U5). The reason for making Q1 CSE2 is to add b^2 instead of subtracting b^2 when completing the square. The data in Table 4.5 suggests that this understanding error was successfully remediated during this project.

The 10 students who made Q1 CSE2 at the 2019-20 e-examination (See Table 4.4) did not make this CSE either in the weekly or revision tests. Weekly and revision tests data reveals that nine of these students answered the question correctly in at least one attempt at the weekly test and the other student got the answer incorrect in their only attempt at the weekly test. However, the latter student answered the question correctly in at least one attempt in the revision test. During the revision period, six students answered the question correctly in at least one attempt and four students did not do the revision test at all.

It is apparent that all 10 students who made Q1 CSE2 for the first time at the 2019-20 e-examination had got the question correct in at least one attempt at either the weekly or revision tests. The reason for them to make Q1 CSE2 for the first time in the 2019-20 e-examination might have been to do with forgetting the correct method of completing the square under the pressure of a controlled condition e-examination. Further, the reason for them to get the answer correct in either the weekly or revision tests might have been to do with the fact that they had access to their notes or they did these tests while they had a fresh learning memory.

Looking at Table 4.5, it is apparent that Q16 CSE1 has the next highest success rate, namely 88%. Q16 CSE1 is presented in Figure 4.13 (see Section 3.5.28 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). This error also

Question

The expression

$t^2 - 12t + 40$

can be expressed in the form:

$a(t-b)^2 + c$

where a, b and c are constants.

Calculate the values of these constants - note that all these solutions are integers:

Enter the value of a [ ]

Enter the value of b [ ]

Enter the value of c [ ]

Correct Solution

$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$

$$= (t-6)^2 + 4$$

$$a = 1, b = 6 \text{ and } c = 4$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**U5**

Incorrectly add b^2 instead of subtracting b^2 when completing the square.

$$t^2 - 12t + 40 = (t-6)^2 + 36 + 40$$

$$= (t-6)^2 + 76$$

$$\bar{c} = 76$$

No. of CSEs /No. incorrect answers **(CSE %)**	6/45 **(13%)** 14/51**(27%)**	**Date collected**	**2017-18** **2018-19**

Figure 4.12: Q1 CSE2 of Question ALGEBRA_COMPLETESQUARE01

belongs to the Errors of understanding category (U). Specifically, this error happens due to not recalling the definition of the volume of revolution correctly by missing out π in the calculation (U5). The data for Q16 CSE1 in Table 4.5 suggests that this understanding error was also successfully remediated during the CSE project.

When looking at Table 4.5, it can be seen that two of the students who had

Question

Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]

Correct Solution

The volume of revolution of y over the range $a < x < b$ given by

$$V = \pi \int_a^b y^2 dx$$

For given data,

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$= \pi \int_1^4 0.64\, x^3 dx$$

$$= 128.18$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**

Missing π

$$\tilde{V} = \int_a^b y^2 dx$$

$$\tilde{V} = \int_1^4 (0.8x^{1.5})^2 dx = \int_1^4 0.64\, x^3 dx = 40.80$$

No. of CSEs /No. incorrect answers **(CSE %)**	9/107 **(8%)** 3/135**(2%)**	**Date collected**	**2017-18** **2018-19**

Figure 4.13: Q16 CSE1 of Question CALCULUS_INTEGRATION_VOLUMEREVOLUTION01

received CSE EFB for Q16 CSE1 before the 2019-20 e-examination made a different error at the 2019-20 e-examination. This means that all seven (See Table 4.4) students made Q16 CSE1 for the first time at the 2019-20 e-examination. These students' weekly and revision tests data reveals that four of them answered the question correctly in at least one attempt at the weekly or revision tests. The other three students did not attempt either of weekly test or revision tests. This data suggests that approximately 43% of students who did Q16 CSE1 for the first time at the 2019-20 e-examination missed the chance of getting their error corrected by not attempting the weekly or revision tests. Q16 CSE1 could have been corrected if they had done these tests before 2019-20 e-examination. The most likely causes of the four students who answered the questions either in weekly or revision test making Q16 CSE1 at the 2019-20 e-examination are the same reasons given for the Q1 CSE2 before (e.g. forgetting the method, missing π in the calculation and controlled condition examination pressure).

Only two CSEs had a success rate lower than 50%, namely Q22 CSE3 (46%) and Q26 CSE1 (33%). Q22 CSE3 is presented in Figure 4.14 (see Section 3.5.12 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). Q22 CSE3 belongs to one of main categories in taxonomy (see Table 4.1) called Errors of understanding (U). Q22 CSE3 occurs due to not recalling the method of differentiating implicit functions correctly (U5). The reason for getting Q22 CSE3 is by taking y to be a constant when differentiating with respect to x and then equating the answer to $\frac{dy}{dx}$.

Table 4.5 shows that seven (out of 13) students who had received CSE EFB for Q22 CSE3 before the 2019-20 e-examination made a different error to Q22 CSE3 at the 2019-20 e-examination. Therefore, data for Q22 CSE3 in Table 4.4 suggests that all five students made Q22 CSE3 for the first time at the 2019-20 e-examination. Only two of these students got the answer correct in at least one attempt at the weekly tests. The other three did not do the test or did not answer the question in the weekly test. Moreover, all of these six students either did not do the revision test or did not answer the question in the revision tests. It is evident that 60% of students who made Q22 CSE3 for the first time at the 2019-20 e-examination missed their opportunity of getting Q22 CSE3 corrected because of not attempting either of the weekly or revision tests.

Table 4.5 shows that Q26 CSE1 has the lowest success rate. As can be seen in Table 4.5 only one out of the three students who had received CSE EFB for Q26 CSE1 before the 2019-20 e-examination answered the question correctly at the e-examination. Q26 CSE1 is presented in Figure 4.15 (see Section 3.5.26 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). Q26 CSE1 belongs to one of main categories in taxonomy (see Table 4.1) called Errors of understanding (U). Q26 CSE1 occurs due to not recalling the method of integrating correctly (U5). The reason for making Q26 CSE1 is to treating the x in front of the trigonometric function as a constant when integrating the function $x\cos(ax)$, and getting the CSE

Question

Given

$x^4 + 2x^2y^3 = 5y,$

find the derivative $\frac{dy}{dx}$ as a function of x and y.

$\frac{dy}{dx} =$ ______

Correct Solution

$$x^4 + 2x^2y^3 = 5y$$

$$4x^3 + 2x^2 \times 3y^2\frac{dy}{dx} + 4x \times y^3 = 5\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4x^3 + 4xy^3}{5 - 6x^2y^2}$$

CSE 3 related to this question	**CSE Taxonomy Code:**	**U5**

Taking y to be a constant when differenting with respect to x.

Equating the answer to $\frac{dy}{dx}$

$$x^4 + 2x^2y^3 = 5y$$

$$4x^3 + 4x \times y^3$$

Therefore,

$$\frac{dy}{dx} = 4x^3 + 4xy^3$$

No. of CSEs /No. incorrect answers **(CSE %)**	10/158 (6%)	**Date collected**	**2017-18**

Figure 4.14: Q22 CSE3 of Question CALCULUS_DIFFERENTIATION_IMPLICIT01

answer as $x\frac{\sin(ax)}{a}$. The data of Table 4.5 shows that the other two students made a different error to Q26 CSE1 at the 2019-20 e-examination.

This collective evidence suggests that CSE EFB had some influence on making students understand that Q22 CSE3 and Q26 CSE1 are errors and then not do those errors again. However, they might not have managed to learn the correct concepts for solving this question using the CSE EFB provided.

Question

Evaluate the following:

$$\int x\cos(3x)dx$$

as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:

Correct Solution

Use integration by parts

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Take

$$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$

Then

$$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$

$$I = \int x\cos(3x)\,dx = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3}dx$$

$$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**

Treating the x in front of the trigonometric function as a constant (or partial solution is given)

$$I = \int x\cos(3x)dx = x\left(\frac{\sin(3x)}{3}\right)$$

No. of CSEs /No. incorrect answers (CSE %)	13/143 (9%)	**Date collected**	**2017-18**

Figure 4.15: Q26 CSE1 of Question CALCULUS_INTEGRATION_PARTS01

All 10 students who did Q26 CSE1 (see Table 4.4) made that CSE for the first time at the 2019-20 e-examination. Their weekly and revision tests data revealed that seven of them did not attempt either of the tests, and the other three got a correct answer for this question in one at least attempt at the weekly or revision tests.

It is worth noting that, Table 4.5 shows that only four CSEs (out of 15 CSEs) were made again at the e-examination by those students who had received CSE EFB before the 2019-20 e-examination. Those CSEs were namely, Q1 CSE1, Q2 CSE1, Q14 CSE1 and Q26 CSE2. The numbers of students who made Q2 CSE1, Q14 CSE1 and Q26 CSE2 are 3, 3 and 1 respectively.

Q2 CSE1 is presented in Figure **??** (see Section 3.1.2 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). Q2 CSE1 is in one of the main categories in the taxonomy (see Table 4.1) called Errors of understanding (U). Q2 CSE1 occurs due to incorrectly assuming that a unit step function (u) is not a function but is equal to 1 (U1). Table 4.5 shows that 16 out of 60 students who had received CSE EFB for Q2 CSE1 before the 2019-20 e-examination did not answer the question correctly at the 2019-20 e-examination. Table 4.5 shows that three of them repeated Q2 CSE1 at the 2019-20 e-examination and 13 of them made a different error instead. This indicates that 3 out of 9 (33%) students who made Q2 CSE1 at the the 2019-20 e-examination (See Table 4.4) had received CSE EFB during weekly or revision tests before the 2019-20 e-examination. However, two thirds of the students (67%) did the CSE for the first time at the 2019-20 e-examination. The weekly and revision tests data of these six students revealed that none of them either took the tests or answered the question in both tests. This means that if they had tried the question and done Q2 CSE1 either in the weekly or revision tests, then that error could have been corrected using CSE EFB.

Q14 CSE1 is presented in Figure 3.12 (see Section 4.1.3 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C). Q14 CSE1 is in a main category of the taxonomy (see Table 4.1) called Errors in choice of method (CM). Q14 CSE1 is to finding the exact value of e^{ax} instead of finding the approximate value of e^{ax} at $x = c$ using the standard MacLaurin expansion. This error results by applying a method which is not valid in the situation (CM1). When analysing the data in Table 4.4 and Table 4.5, it can be see that only 10% (3/29) of students who made Q14 CSE1 at the the 2019-20 e-examination (See Table 4.4) made the same Q14 CSE1 at the the 2019-20 e-examination. On the other hand, around 90% (26/29) students did the CSE for the first time at the 2019-20 e-examination.

The weekly and revision tests data of the 26 students who did Q14 CSE1 for the first time at the 2019-20 e-examination shows that two of them got the question correct in at least one attempt at the weekly and revision tests, 11 of them got the question correct in at least one attempt at the weekly tests, but they either did not do the revision test or answer the question in the revision test. The other 13 either did not do both tests or attempt the question in the tests. It is evident that 13 of

26 students who made Q14 CSE1 for the first time at the 2019-20 e-examination missed the opportunity to get Q14 CSE1 corrected because they did not attempt either of the weekly or revision tests.

However, Q1 CSE1 was made by nine students who received CSE EFB for Q1 CSE1 before the e-examination. This error is to give the answer for *b* which corresponds to the negative of the correct value of *b*. Q1 CSE1, is presented in Figure 4.1 (see Section 2.1.1 of Sikurajapathi, Henderson, and Gwynllyw (2022a) in Appendix C), and is a Copying error (S1) in Slip of action main category in the taxonomy (see Table 4.1). The reason that nine students who received CSE EFB for Q1 CSE1 before the e-examination made the same error at the e-examination may have been to do with a mistake in copying or submitting their answer into Dewis under the pressure of a controlled condition e-examination. On the other hand, it is also worth noting that CSE EFB for Q1 CSE1 helped a large number of students (86) correct their mistakes in copying or submitting answers into the e-Assessment (see Table 4.5).

This investigation reveals that the majority of students who had received CSE EFB before the the 2019-20 e-examination, answered the question correctly at the 2019-20 e-examination. Moreover, it was discovered that majority of students who answered the question incorrectly despite having received CSE EFB before the e-examination, did not made the same CSE again at the 2019-20 e-examination. Instead, they made a different error or did not answer the question. These pieces of evidence suggest that CSE EFB had some influence on making the students learn not to do those errors again. However, they might not have managed to learn the correct concepts for answering the question using the CSE EFB provided. Further, it revealed that the majority of students who made the respective CSEs for the first time at the 2019-20 e-examination missed the opportunity of getting their misconceptions or mistakes corrected because of not attempting either of the weekly or revision tests before the the 2019-20 e-examination.

Therefore, based on the data analyses in this section, it is evident that CSE EFB greatly helped students to correct some of their mathematical CSEs. These analyses provide strong evidence to support the fact that CSE EFB made a significant positive effect on diagnosing and remediating some of mathematical CSEs in e-Assessment questions.

4.3.3 Limitations of analysing the effectiveness of the CSE Project at UWE Bristol

Several limitations to the analyses done in Section 4.3.1 and Section 4.3.2 need to be acknowledged. The analyses would have been more comprehensive if it had explored the effectiveness of diagnosing and remediating all of the 65 CSEs. However, including all of the 27 questions containing the 65 CSEs in the controlled condition e-examination was not possible. This was due to in part, the limited time duration of the e-examination and also the questions for the e-examination were selected by the EM module leader.

Further, collecting data in the subsequent years to validate the effectiveness of the CSE Project was not possible due to several reasons. In 2020-21, the EM module was replaced by a different module due to a curriculum update at UWE Bristol. Therefore, the EM module was offered for the last time only to resitting students in the 2020-21 year. Also the e-examination was moved online due to Covid-19 (WHO, 2020). Consequently, these reasons limited the analysis of the effectiveness of diagnosing and remediating all of the identified 65 CSEs.

5 Outcomes and Contributions

This chapter outlines the important findings and significant contributions made from this research, the CSE Project at UWE Bristol. The chapter is divided into five sections to summarise the major outcomes of this research.

Section 5.1 summarises the production of the interactive book called 'Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)' which has been produced as a result of this research. Section 5.2 recaps the development of features in Dewis which detect CSEs and also provides improved feedback to correct them. In Section 5.3, some restrictions related to the parameter selections of questions which have at least one CSE which were found during re-marking process are summarised. Section 5.4 recaps students' perceptions of the enhanced e-Assessment feedback addressing CSEs in mathematics which were gathered from the questionnaire conducted to evaluate the effectiveness of the CSE Project. Finally, the encouraging results which arose from quantitative analysis of Dewis-stored data and re-marking data are summarised in Section 5.5 to highlight the efficacy of diagnosing and remediating mathematical CSEs in e-Assessment questions during the CSE Project at UWE Bristol.

5.1 A collection of CSEs with taxonomy coding

This section summarises one of the main contribution of this research presented in Section 3.2. The interactive book called 'Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)' has been produced as a result of the CSE Project at UWE Bristol (Sikurajapathi, Henderson, and Gwynllyw, 2022a; Sikurajapathi, Henderson, and Gwynllyw, 2022b).

The process of creating this CSE Book was long and tedious but worthwhile. It involved various stages, namely, the systematic collection and compilation of CSEs, and the classification of them taxonomically according to a taxonomy presented in the existing literature, by examining first year EM students' rough answer scripts and e-Assessment-stored data (Sikurajapathi, Henderson, and Gwynllyw, 2022b).

The CSEs presented in the CSE book could be useful for mathematics teachers when providing feedback to students to correct CSEs. Further, institutions can utilise it in the future development of teaching and support resources to ensure that these CSEs will be addressed to help students to acquire better understanding of mathematics. Moreover, mathematics learners can try these questions online by using the respective hyper-links given in the CSE Book. If any of the identified CSEs are entered in the solution, then CSE EFB is provided to correct their misconceptions instantly.

The CSE Book, Sikurajapathi, Henderson, and Gwynllyw (2022a), is deposited in UWE Bristol Repository which is publicly available (see Appendix C). Findings of this part of the research is published in the journal paper Sikurajapathi, Henderson, and Gwynllyw (2022b) (see Section D.3 in Appendix D).

5.2 Detecting mathematical CSEs and providing CSE EFB in e-Assessment questions

This section recaps one of the main contributions of this research presented in Section 3.3. During the CSE Project, 27 original Dewis questions were amended in order to capture 65 CSEs in total. The answers (inputs) to the questions were either integer, floating-point or algebraic and the amendments were done differently depending on the input type. Three questions, which have 6 identified CSEs, had integer inputs. Answers to 14 questions, which have 35 identified CSEs, required floating-point inputs. The remaining 10 questions, which have 24 CSEs, had algebraic inputs (see Table B.1 in Appendix B for more details).

Careful attention was given to preparing the CSE EFB for each CSE. The text colour of the CSE EFB was selected as blue in order to attract the students attention. What might have gone wrong when answering the question was emphasised in a red box at the beginning of the CSE EFB. Then, the detailed solution of the question was presented underneath the red box. Standard mathematical procedures, such as labelling equations and referring to them elsewhere in the solution, were used whenever possible.

These amended questions are stored in Dewis under the folder *cse_project_book* and individual questions can be attempted via the link given in Sikurajapathi, Henderson, and Gwynllyw (2022a). Some of the findings of this part of the research is published in the journal paper Sikurajapathi, Henderson, and Gwynllyw (2020) (see Section D.1 in Appendix D).

5.3 Parameter selection restrictions in CSE coding

This section summarises one of the main contributions of this research presented in Section 3.3.4. Section 3.3.4, shows some aspects of why it is important to know more about Mathematical CSEs in e-Assessment questions, using several examples. To check the credibility of the amended question codes, they were used to re-mark the weekly tests in the 2017-18 cohort. Two categories of parameter selection restrictions of the questions which have CSEs were identified during this process. Firstly for some questions, there were particular random parameters for which the correct answer and the CSE answer were the same. In such cases, there may have been instances where some students were awarded full marks and hence thought that they had answered the question correctly when in fact they had made a CSE. Secondly, it was found that for some questions with more than one CSE it was possible for two CSEs to be triggered simultaneously for some particular parameters.

The resolutions that were used to address these issues by amending the original question code for all identified CSEs are also describes in detailed in Section 3.3.4. These findings were disseminated in the CETL-MSOR Conference 2022, and the content of the topic has been accepted for publication in the journal paper Sikurajapathi, Henderson, and Gwynllyw (2023) (see Section D.4 in Appendix D).

5.4 Students' perceptions of enhanced e-Assessment feedback addressing CSEs in mathematics

This section summarises one of the main contributions of this research presented in Section 3.4, 3.5, and 4.2. During Stage Four of the CSE Project, the students' views on the effectiveness of the CSE EFB in correcting their misconceptions and improving their Engineering Mathematics learning were sought. Further, students' satisfaction of the user-friendly features in the CSE EFB was investigated.

The results and the AP of the Likert-scale questions indicates that the majority of the participants agreed that the CSE EFB improved their mathematical understanding and made them feel confident/comfortable with Engineering Mathematics. They also indicated that the information in the CSE EFB is relevant to the question asked and that they are satisfied with the overall structure of the CSE EFB.

The responses to the Likert-scale questions and the open-ended questions showed that the majority of the participants had positive feelings toward the CSE EFB. Participants appreciated that the CSE EFB helped them to address their misunderstanding and to improve their engineering mathematics learning.

The study also gave insight into how students find the user-friendly features of the CSE EFB. Most of them had positive comments about its coherent structure and ergonomic features. One specific concern that emerged related to improving the visibility of the CSE EFB. There were some very valuable suggestions of how to improve these features, such as moving it to a more noticeable place on the feedback report, and redesigning the CSE EFB to have a more efficient and readable structure.

Some other notable suggestions were to include videos within the CSE EFB and web links to extra materials. The majority of the participants highly valued the effectiveness of the CSE EFB and suggested/wished to have CSE EFB for the rest of the questions in the Engineering Mathematics e-Assessments. These suggestions and the highly positive perception of the CSE EFB suggest that students find the enhanced feedback valuable for their learning. This part of the research and its findings were published in the journal paper Sikurajapathi, Henderson, and Gwynllyw (2021) (see Section D.2 in Appendix D).

5.5 Efficacy of diagnosing and remediating mathematical CSEs in e-Assessment questions

This section summarises one of the main contributions of this research presented in Section 3.6, and Section 4.3. In order to find out the effectiveness of the CSE Project, firstly, quantitative data analysis was carried out on the CSE occurrences before (2017-18 and 2018-19) and after (2019-20) the CSE enhanced feedback feature was incorporated into Dewis. Secondly, further investigation is carried out to find out the success of the CSE Project using a thorough analysis of those students who received CSE EFB either in weekly or revision tests before the e-examination. These analyses were carried out utilising the data gathered from the 2017-18, 2018-19 and 2019-20 e-examinations of the same module.

These data analyses revealed that that CSE EFB greatly helped many students to correct their mathematical CSEs. This investigation reveals that the majority of the students who had received CSE EFB before the the 2019-20 e-examination answered the question correctly at the 2019-20 e-examination.

Another important finding was that the majority of the students who answered the question incorrectly, even though they had received CSE EFB before the e-examination, did not made the same CSE again at the e-examination. Instead, they made a different error or did not answer the question. This evidence suggests that CSE EFB had some influence on making the students understand CSEs and not to do those errors again. However, they might not have managed to learn the correct concepts of solving this question using the CSE EFB provided.

The most interesting finding was that the majority of students who made the respective CSEs for the first time at the 2019-20 e-examination did not attempt either of the weekly or revision tests, or did not answer the respective question either in the weekly or revision tests. This phenomena recurred throughout the dataset of those students who made the respective CSEs for the first time at the 2019-20 e-examination. If they had attempted the respective questions either in the weekly or revision tests, respective CSEs of the question could have been corrected thorough CSE EFB.

The outcomes of the data analyses proves that CSE EFB greatly helped students to correct some of their mathematical CSEs. Further, there is evidence to support that CSE EFB made a significant positive effect on diagnosing and remediating some of the mathematical CSEs in e-Assessment questions.

Findings of this part of the research were disseminated at the EAMS 2022 Conference and the presentation can be found in Sikurajapathi et al. (2021).

6 Conclusions and Further Work

In this final chapter, conclusions and limitations of this research, and further work of exploration are put forward. Finally, a brief summary and reflection of the research are presented in the last section of this chapter.

6.1 Conclusions

This research set out to answer the following specific research questions:

1. How to detect mathematical CSEs in traditional assessments and e-Assessments questions?
2. What mathematical CSEs do first year EM students make in e-Assessment questions?
3. How to improve the e-Assessment feedback in order to address these CSEs?
4. How to measure the effectiveness of detecting and addressing CSEs in mathematical e-Assessment questions?

This research was carried out in five stages: Data (CSEs) Collection; CSE Code Development; CSE Code Trial Phase; Students' Perceptions on CSE EFB; Impact of the CSE Project analysis. The aims of the project have been successfully achieved, making some significant contributions to the current literature.

Firstly, during Stage One of the study, in total 619 students' hand-written scripts and Dewis-stored data were carefully analysed to collect 65 mathematical CSEs in 27 e-Assessment questions. The work collating CSEs and taxonomically classifying them is the largest study carried out so far in the area of mathematical CSEs in e-Assessment questions in EM research according to the open literature. Dissemination of this knowledge can be found in the interactive report called 'Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessment (CSE Book)' Sikurajapathi, Henderson, and Gwynllyw (2022a) (see Appendix C), and the journal paper Sikurajapathi, Henderson, and Gwynllyw (2022b) (see Section D.3 in Appendix D). The first and second research questions of this study, ‘How to detect mathematical CSEs in traditional assessments and e-Assessments?’ and ‘What mathematical CSEs do first year EM students make in e-Assessment questions?’ were successfully answered by completing this task.

Secondly, this is the largest study carried out to amend Dewis questions in order to capture 65 mathematical CSEs and to provide CSE EFB to address those CSEs. The amendments were done differently depending on the input types of e-Assessment questions: integer, floating-point or algebraic. These amended questions are stored in Dewis under the folder *cse_project_book*. The individual questions can be attempted via the link given in the CSE Book published in the UWE Bristol repository Sikurajapathi, Henderson, and Gwynllyw (2022a). Some of the findings of this part of the research is published in the journal paper Sikurajapathi, Henderson, and Gwynllyw (2020) (see Section D.1 in Appendix D).

Thirdly, this study has gone some way towards enhancing our understanding of parameter selection restrictions of mathematical e-Assessment questions which have at least one CSE. The findings related to this topic were disseminated in the CETL-MSOR Conference 2022, and have been accepted for publication in the journal paper Sikurajapathi, Henderson, and Gwynllyw (2023) (see Section D.4 in Appendix D). The third research question of this study, ‘How to improve the e-Assessment feedback in order to address these CSEs?’, was successfully answered by amending the Dewis original question codes to detect these CSEs during Stage Two (CSE Code Development) and by testing them during Stage Three (CSE Code Trial Phase).

Fourthly, the validity of this study was evaluated using a questionnaire during Stage Four. The analysis of the questionnaire has extended our knowledge of students’ perceptions of enhanced e-Assessment feedback addressing CSEs in mathematics produced during this study. The empirical findings in the questionnaire analysis were published in the journal paper Sikurajapathi, Henderson, and Gwynllyw (2021) (see Section D.2 in Appendix D).

Finally, the analysis of quantitative data undertaken in Stage Five of this research helped to understand the efficacy of remediating mathematical CSEs in e-Assessment questions in this research. The findings of this analysis were disseminated at the EAMS 2022 Conference, and the presentation can be found in Sikurajapathi et al. (2021). The fourth research question of this study, ‘How to measure the effectiveness of detecting and addressing CSEs in mathematical e-Assessment questions?’, was successfully answered during Stage Four (Students’ Perceptions on CSE EFB) and Stage Five (Impact of the CSE Project).

The findings presented in this thesis contribute in several ways to our understanding of CSEs that first year EM students make in e-Assessment questions and provide a basis for further research in mathematical CSEs in general. The methods used in developing a technique to diagnose and remediate Mathematical CSEs in e-Assessment questions may be applied to several disciplines and organisations that either use Dewis or any other e-Assessment system which is capable of giving dynamic feedback based on a students' answer. The new knowledge raised from this research can be used in those e-Assessment Systems so that they may emulate a human marker, providing instant enhanced feedback highlighting possible CSEs. This will help students to correct their mathematical misconceptions. Also, teachers can use the findings to identify areas in which more help is needed in student learning. Integrating the research outcomes from the CSE Project into other e-Assessment Systems will be useful to the mathematics and e-Assessment communities (Sikurajapathi, Henderson, and Gwynllyw, 2023; Sikurajapathi, Henderson, and Gwynllyw, 2022b; Sikurajapathi, Henderson, and Gwynllyw, 2022a; Sikurajapathi, Henderson, and Gwynllyw, 2021; Sikurajapathi, Henderson, and Gwynllyw, 2020).

6.2 Limitations

With regard to the research methods, some limitations need to be acknowledged. As mentioned in Section 4.3.3, a comprehensive analysis to find the effectiveness of remediating all the 65 CSEs was not possible due to different reasons. Including 27 questions which have 65 identified CSEs in an e-examination was not possible because of the time constraints of the e-examination. In addition, selecting the questions for the e-examination was not under my control.

To date, only a limited number of CSEs have been identified in the questions considered in this research. However, the knowledge acquired from Section 3.3.4 raises intriguing questions regarding the diagnosing and remediating mathematical CSEs in e-Assessment questions. Since it is not certain that all the CSEs related to a question have been identified, there is a possibility of the correct answer being equal to an unidentified CSE answer. Further, there is also the possibility that an identified CSE answer is equal to an unidentified CSE answer. These issues are intriguing questions which could be explored in further research.

6.3 Recommendations

Further research on mathematical CSEs in e-Assessment question would be a great help to both mathematics and mathematical e-Assessment in education. A few areas of future work are proposed in the following sections.

6.3.1 Incorporating the CSE codes into other Dewis questions

A possible future work is to use the same amended question codes in order to capture identified CSEs in other mathematical questions on Dewis. Some of the CSEs identified in this research are prone to be made in other mathematical e-Assessment questions.

For example, the CSEs 'taking the square of a negative number to be negative' or 'taking $(x^p)^q$ to be x^{p^q}' can also be made in other mathematical questions on Dewis. Capturing those identified CSEs in other questions is now straightforward as the amended question codes produced in this research can be utilised to amend new question codes. Therefore, utilisation of existing amended question codes in other mathematical questions in Dewis would be a fruitful area of further research and development.

6.3.2 Expanding the collection of mathematical CSEs in e-Assessment questions

A future study identifying more mathematical CSEs in other e-Assessment questions would be interesting. I intend to identify what type of CSEs that the first year students make in e-Assessment questions in the module Mathematics for Civil and Environmental Engineering (MCEE) in which I have been the module leader since 2021.

I have incorporated an end of year e-examination on Dewis for this module where the students are required to submit their written scripts as a PDF document after submitting the answers on the Dewis. These PDF answer scripts and Dewis-stored data can be used to identify mathematical CSEs made by MCEE students. Then, using the same methodology used in this thesis, enhanced e-Assessment feedback can be provided to address the new CSEs in MCEE. Gathering more CSEs in mathematical e-Assessment questions would make a significant contribution to the knowledge in mathematical CSEs and CSEs in e-Assessment questions.

6.3.3 Making a dynamic website to include mathematical CSEs

It would be useful to have a dynamic website to disseminate the mathematical CSEs found during this and ongoing research to a wider community. This will provide easy accessibility to researchers, teachers, and students to learn about mathematical CSEs. I intend to make this dynamic website available within the CSE Project website to include the 65 CSEs which I documented in the CSE Book during my doctoral research (CSE Project at UWE, 2019a; Sikurajapathi, Henderson, and Gwynllyw, 2022a).

These CSEs will be presented under different mathematics topics such as CSEs in Arithmetic, CSEs in Differentiation etc. in different Tabs on the website. This website will be dynamic in that it can be updated with new CSEs as and when they are found during the project discussed in Section 6.3.2. Producing a dynamic website to store all the mathematical CSEs in one place will be beneficial to mathematics teachers, and students. Further, a dynamic website would provide a useful resource to those researchers who are interested in mathematical CSEs.

6.3.4 Improving feedback of the questions in Dewis

As discussed in Section 2.2, good quality and detailed timely feedback can be used to instil desire in students to progress further and to correct their misconceptions (JISC, 2007; Gibbs and Simpson, 2005; Nicol and Macfarlane, 2006; Robinson, 2015). It was noted that feedback given in some Dewis mathematical e-Assessment questions is not effective enough to support students' learning. Possible improvements to feedback in Dewis mathematical e-Assessment questions are discussed here.

Presenting detailed feedback in a variety of forms

The question shown in Figure 6.1 involves finding the magnitude and argument of the division of two given complex numbers in polar form. The typical feedback given in Dewis for this question is shown in Figure 6.2.

In the feedback shown in Figure 6.2, finding the argument does not contain much detail. As discussed in Section 2.1.3, presenting new concepts in a variety of forms may be beneficial for students to learn difficult concepts (Felder and Henriques, 1995; Muzangwa and Chifamba, 2012). Therefore, the feedback shown in Figure 6.2 could be improved by presenting the answer for the argument of z using

The Question. (cse_project_book) COMPLEXNUMBERS_POLARDIVISION01

Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the magnitude, r, and argument, θ, of

$$z = \frac{(w_1)^3}{(w_2)^2},$$

where $-\pi < \theta \leq \pi$.

Enter your answers correct to three decimal places.

Enter $r = |z|$:

Enter $\theta = arg(z)$:

[4 marks]

Submit

Figure 6.1: COMPLEXNUMBERS_POLARDIVISION01 question in Dewis

The Solution

Standard result:

If $w_1 = r_1 e^{j\theta_1}$ and $w_2 = r_2 e^{j\theta_2}$ then

$$(w_1)^n = r_1^n e^{jn\theta_1} \quad \frac{w_1}{w_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)} .$$

So in this case

$$z = \frac{(w_1)^3}{(w_2)^2} = \frac{(r_1)^3}{(r_2)^2} e^{j(3\theta_1 - 2\theta_2)} = \frac{(1.8)^3}{(1.2)^2} e^{3.2j}$$

So the magnitude of z is $r = \dfrac{(1.8)^3}{(1.2)^2}$

and the argument of z , in the required range, is $\theta = 3.2 - 2\pi$.

The value of $r = |z|$ is 4.050000.. which, to three decimal places, is 4.050.
The value of $\theta = arg(z)$ is -3.083185.. which, to three decimal places, is -3.083.

Figure 6.2: Feedback for COMPLEXNUMBERS_POLARDIVISION01 question shown in Figure 6.1

an Argand Diagram as well. Presenting the method of finding the argument of z in both written form and on an Argand diagram would give students better chances to learn about the position of the argument of z and how it can be obtained. Also, providing the feedback in different forms might be helpful for students to correct their misconceptions.

Presenting multiple solution methods in feedback

Referring to the literature review in Section 2.2, it is easier and more helpful for students to understand new concepts if multiple solution methods to the same question are made available to them (Rittle-Johnson and Star, 2007). During this doctoral research it was observed that it is possible to provide multiple methods of solutions to some mathematical questions in Dewis.

As an example, consider the question shown in Figure 6.3 which involves differentiating $b\log(ax)$.

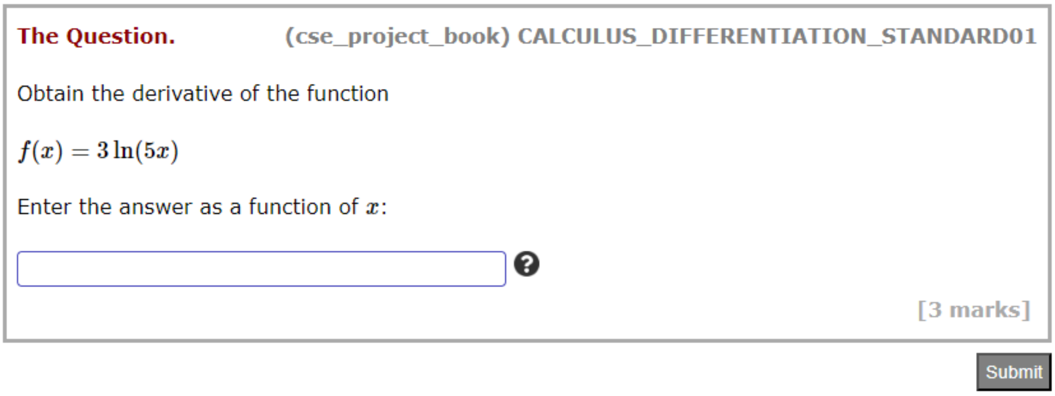

Figure 6.3: CALCULUS_DIFFERENTIATION_STANDARD01 question in Dewis

The typical feedback given in Dewis for this question is shown in Figure 6.4.

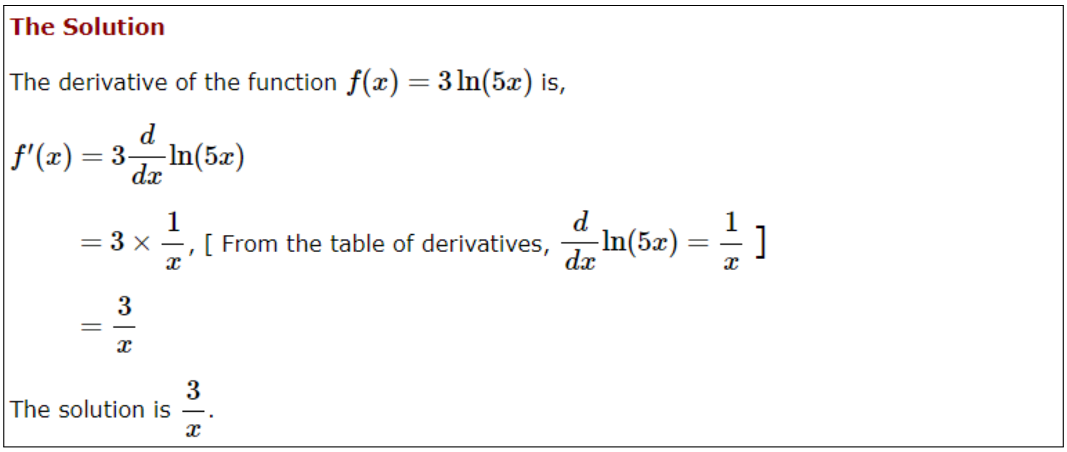

Figure 6.4: Feedback for CALCULUS_DIFFERENTIATION_STANDARD01 question shown in Figure 6.3

The current feedback shown in Figure 6.4 could be improved by providing other possible methods of differentiating $b\log(ax)$. One of the methods would be the use of the Chain rule to find the derivative of $\log(ax)$. Another method that could be presented in the feedback would be to use the properties of logarithms to write $\log(ax) = \log(a) + \log(x)$ and then differentiate.

As discussed here, the feedback of some mathematical Dewis questions could be improved by providing multiple solution methods to the same questions in future.

6.3.5 Re-formatting some questions in Dewis

Some mathematical questions in Dewis could be re-formatted to avoid misunderstanding of the questions. It can be seen that some CSEs would not have been made by the students if the question was posed in a different way. In some cases asking different questions to validate the answer will be easier for students to show their acquired knowledge in the topic. Moreover, it would be beneficial for students if there were opportunities to submit their answer in a graphical way. Possible re-formatting of three questions as examples are discussed in the following sections.

Re-structuring some questions to avoid mathematical CSEs

Figure 4.1 shows a CSE related to a question in Algebra (Completing the Square). It was noted that even though students had solved the question correctly, some students submitted incorrect answers for b which corresponded to the negative of the correct value of b. Therefore, this CSE could have been avoided by re-formatting the question by asking for a, b and c when the given algebraic expression is expressed in the form $a(t+b)^2 + c$ instead of expressing it as $a(t-b)^2 + c$.

Asking different questions to validate the answer

The question shown in Figure 3.12 involves using the standard Maclaurin expansion to obtain the power series expansion $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ of a given exponential function, e^{ax}. Students are required to calculate the values of a_0, a_1, a_2 and a_3, and then calculate the approximate value of e^{ax} at a given value of x correct to three decimal places.

It can be seen that it is enough to ask for a_1, a_2 and a_3 in order to evaluate students' knowledge of Maclaurin expansions. Further, asking for these values would not lead students to make Q14 CSE1 (submitting the exact value instead of the approximate value of e^{ax} at the given value of x).

Providing a different format to submit the answer

Some questions in Dewis can be re-formatted to make them better questions by providing different format to enter the answer(s). As an example, in the question shown in Figure 6.1, the question could be modified to facilitate the students to mark the answer for the argument of z on a given Argand Diagram. Re-formatting the question in this way would be relatively straightforward as Dewis already has the facility for submitting graphical answers such as marking points on a Cartesian Coordinate System.

Therefore, as explained in this section, some mathematical questions in Dewis could be re-formatted to make them more effective and easily understandable in the future.

6.3.6 Developing adaptability features in Dewis to track students who re-made the same CSE

The data lossless feature in Dewis means that all data relating to every assessment attempt is recorded and stored on the Dewis server (Walker, Gwynllyw, and Henderson, 2015). Therefore, it currently would be possible to track which of those students, who made a particular CSE at some point, made it again in their studies. However, the current procedure to do this is cumbersome, requiring manual downloading and searching of data. A possible future development of Dewis would be to automatically and efficiently track when a student has made a particular CSE in subsequent e-Assessment questions.

Students might be repeating the same error due to a lack of conceptual understanding or due to strongly held misconceptions (Durkin and Rittle-Johnson, 2015). Therefore, if the first CSE FEB failed to address their misconception, different forms of intervention should be done. One intervention could be to present the CSE EFB in a variety of forms in simple ways such as through the use of diagrams, and graphs (Felder and Henriques, 1995; Muzangwa and Chifamba, 2012). As suggested by Ma (1999) another intervention could be re-teaching fundamental concepts and principles to address students' conceptual misunderstandings. This could be done in the CSE EFB by including a video in which an academic addresses the relevant conceptual misunderstandings by re-teaching and thoroughly explaining the fundamental concepts and principles related to a particular CSE.

In this way, the data lossless feature could be used to correct students faulty conceptual ideas or misconceptions at various points in their learning process. This development would be beneficial for students and it could have a critical impact on their performance and learning.

6.3.7 Identifying other CSEs using the re-marking feature in Dewis

It is evident that students made other CSEs to the Dewis questions considered in this thesis which were not identified during this research (see Table 4.5). Identifying such CSEs would be beneficial for teachers as well as students. Some of these CSEs could be identified by examining wrong answers submitted by students and then anticipating the error that may have caused them to arrive at this wrong answer (Walker, Gwynllyw, and Henderson, 2015). In such cases, the CSE could be incorporated into the Dewis question code. By re-marking all of the submissions using the amended question code it would be possible to see how many students triggered the same mistake. Then CSE EFB could be produced to correct any newly identified CSEs. This methodology could be reiterated until all of the incorrect answers are exhausted.

Therefore, identifying other errors using the Dewis re-marking feature could be useful as a future iteration of this study.

6.3.8 Developing an item analysis facility to evaluate e-Assessment questions in Dewis

The difficulty and quality of mathematical questions in Dewis are at different levels and are not currently quantified. So, it is difficult to have any assurance of the reliability and validity of the questions without a proper analysis. Therefore, it would be useful if Dewis had a facility to identify reliable and valid test items (questions) and tests (e-assessments) and to produce computer-generated item analysis reports. The statistical procedures which are used to evaluate the characteristics of test items and hence to improve reliability and validity of a test is called item analysis (Anastasi, 1990).

One of the main goals of test construction is to develop a test of minimum length with the needed degree of reliability and validity for the intended users (Crocker and Algina, 2008). In the initial state of the test construction the test developers produce a large collection of test items, and subsequently use item analysis to select a subset of items of the initial collection that makes the greatest contributions to reliability and validity. Item analysis procedures determine unproductive items that should be revised, eliminated and replaced with new items. This procedure leads to a revised test with more discriminating items with higher reliability and validity (Gregory, 2007).

Several indices are used in item analysis procedure. These indices compare individual test items with the other items in the test and the context of the whole test (Cohen and Swerdlik, 2005). Some of well-known item analysis indices are Item Difficulty Index (p-value), the Item Discrimination Index (D) and the Correlational Indices of Item Discrimination (Crocker and Algina, 2008). Selecting appropriate item analysis indices depends on the test type (Crocker and Algina, 2008; Hogan, 2007).

Developing features in Dewis to identify reliable and valid test items and e-assessments, and to produce computer-generated item analysis report could be extremely useful. This feature would confirm that a test contained a good collection of items which grasp the taught material while challenging the students appropriately. Further, the information of item analysis report can be used to improve test items in the Public Question Bank on Dewis as well as to improve teaching material. In addition, the report data could be used as guidance to revise and rewrite items and hence improve future tests.

6.3.9 Revising the CSE Book to identify reasons for each CSE

The CSE Book produced in this research could be improved by revising it to include the reasons behind each CSE rather than simply describing the error. The identification of such reasons could be useful for e-Assessment question writers to predict errors when writing questions.

How to improve the CSE taxonomy in the CSE Book by finding the possible reason behind CSEs is discussed using some examples here.

Illegal commuting of operations or procedures

Careful examination of Q10 CSE1 of question COMPLEXNUMBERS_CARTESIANDIVISION01 (see Figure 6.5) and Q15 CSE1 of Question ENGINEERING_-CENTRMASS01 (see Figure 4.10) suggests that the reason behind these CSEs are due to students illegally commuting operations or procedures.

Question		
Find the real and imaginary parts of $z = \frac{2-7j}{3+5j}$ correct to three decimal places. Enter $Re(z)$: ☐ Enter $Im(z)$: ☐		
Correct Solution		
$z = \frac{2-7j}{3+5j}$ $z = \frac{(2-7j)(3-5j)}{(3+5j)(3-5j)}$ $z = \frac{-29-31j}{34}$ $Re(z) = \frac{-29}{34} = -0.853, \quad Im(z) = \frac{-31}{34} = -0.912$		
CSE 1 related to this question	**CSE Taxonomy Code:**	**S3**
Consider $z = \frac{a+bj}{c+dj}$ $Re(z) = \frac{a}{c}, Im(z) = \frac{b}{d}$ $\widetilde{Re(z)} = \frac{2}{3} = 0.667, \quad \widetilde{Im(z)} = \frac{-7}{5} = -1.4$		
No. of CSEs /No. incorrect answers **(CSE %)**	9/109 **(8%)** 9/95**(9%)**	**Date collected**: **2017-18 2018-19**

Figure 6.5: Q10 CSE1 of Question COMPLEXNUMBERS_CARTESIANDIVISION01

As shown in Figure 6.5, the generalised reason behind Q10 CSE1 is to consider the division of two sums $\frac{a+b}{c+d}$ to be equal to the sums of divisions $\frac{a}{c}+\frac{b}{d}$. Similarly, the generalised reason for Q15 CSE1 is to assume that the sum of the product is equal to the product of the sums. In both these cases, it can be concluded that the reason behind these CSEs is due to illegal commuting of operations or procedures.

Erroneous power manipulations

The reason behind a few of the CSEs in the CSE Book is due to students making erroneous power manipulations. As examples, Q16 CSE4 (see Figure 3.28) and Q16 CSE6 (see Figure 6.6) of question CALCULUS_INTEGRATION_VOLUMEREVOLUTION01 can be identified as CSEs due to erroneous power manipulations.

Question

Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]

Correct Solution

The volume of revolution of y over the range $a < x < b$ given by

$$V = \pi \int_a^b y^2 dx$$

For given data,

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$= \pi \int_1^4 0.64\, x^3 dx$$

$$= 128.18$$

CSE 6 related to this question	**CSE Taxonomy Code:**	**S3**

Taking $(x^p)^q = x^{p+q}$

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$\bar{V} = \pi \int_1^4 0.8^2\, x^{1.5+2} dx$$

$$\bar{V} = 0.64\, \pi \left[\frac{x^{4.5}}{4.5}\right]_1^4$$

$$= 228.32$$

No. of CSEs /No. incorrect answers (CSE %)	6/135 (4%)	Date collected	2018-19

Figure 6.6: Q16 CSE6 of Question CALCULUS_INTEGRATION_VOLUMEREVOLUTION01

Both of these CSEs might have been made due to lack of knowledge of the index laws. Therefore, it would be better to identify the reason for these errors instead of just describing the error in the CSE Book.

It has been noted that a similar approach (i.e. identifying the reason behind a CSE) could be carried out on the rest of the CSEs in the CSE Book to produce a better mathematical CSE taxonomy. For example, students made some CSEs in trigonometry and complex numbers due to incomplete understanding of angle quadrants. Other identified reasons behind some CSEs can be listed as errors due to erroneous log manipulations, failing to understand what a function is, applying valid rules outside their range of validity and lack of understanding of what is being asked in the question.

Identification of the reasons behind CSEs in the CSE Book might be applicable in other areas in mathematics. For example, CSEs due to illegal commutativity of operations or procedures could be made in other mathematical areas such as matrix multiplication and function composition.

Further, identifying the probable reason of a known error could be useful when writing future questions not only for first year engineering students, but also for other undergraduate levels, other STEM subjects and some social sciences subjects such as Economics and Psychology.

6.4 Summary

This research was conducted to identify what type of CSEs first year EM students make in e-Assessment questions, and to create a technique to detect such CSEs and improve the e-Assessment feedback in order to address these CSEs in Dewis e-Assessment questions.

At the beginning of this research, a critical understanding of the current state of the knowledge in the fields of CSEs, assessment and feedback in education, e-Assessments, and the Dewis e-Assessment System was acquired. The literature review related to this thesis is presented in Chapter 2.

This research was conducted through original research methods. This research was conceptualised, designed and implemented to create new knowledge in diagnosing and remediating mathematical CSEs in e-Assessment questions. Prior to undertaking the study, expert advice were sought from a GDPR officer at UWE Bristol to confirm that this data projection procedure satisfies GDPR requirements. The research was designed in five stages which were suitable for the Dewis e-Assessment System. The methodology of this research is presented in Chapter 3 in detail. On one occasion, while checking the credibility of the amended codes, the project design was adjusted in the light of emergent issues of selection of parameters (see Section 3.3.4).

The strengths and weaknesses of the project were evaluated through data analysis. A questionnaire was conducted and analysed to investigate students' perceptions on the CSE EFB delivered through Dewis. Prior to commencing the study, ethical clearance was obtained from FREC at UWE Bristol. Both quantitative and qualitative methods were used to analyse the questionnaire data. The Dewis-stored data was analysed quantitatively to examine the impact of diagnosing and remediating mathematical CSEs in e-Assessment questions. The data analysis of this project is presented in Chapter 4

This research led to creation of new knowledge in diagnosing and remediating Mathematical CSEs in e-Assessment questions. The important findings and significant contributions made from this research is presented in Chapter 5. The new knowledge created though this research was disseminated though several form of research dissemination channels: journal papers, reports, conference talks, and posters.

Four peer-reviewed research papers based on this research have been published in well-recognised journals in the field of mathematics and e-Assessments. These publications can be found in Appendix D. Further, an interactive book called 'Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)' was produced as a result of this research. The CSE book, Sikurajapathi, Henderson, and Gwynllyw (2021), is currently deposited in UWE Bristol's Repository. The CSE Book can also be found in Appendix C of this thesis.

The outcomes and contributions of this thesis have received satisfying scholarly reviews by accomplished and recognised scholars in the field. The journal papers produced during this research were cited by several scholars in the field.

References

1EdTech (2023). *Question and Test Interoperability (QTI) and Assessment*. [Accessed: 21 June 2022]. Available from: https://www.imsglobal.org/activity/qtiapip.

Adams, D. M., McLaren, B. M., Durkin, K., Mayer, R. E., Rittle-Johnson, B., Isotani, S., and Van Velsen, M. (2014). Using erroneous examples to improve mathematics learning with a web-based tutoring system. *Computers in Human Behavior*. 36, pp. 401–411.

Alruwais, N., Wills, G., and Wald, M. (2018). Advantages and Challenges of Using e-Assessment. *International Journal of Information and Education Technology*. 8.1, pp. 34–37.

Anastasi, A. (1990). *Psychological Testing*. New York: Macmillan Publishing Company.

Anderson, J. R. (1989). The analogical origins of errors in problem solving. In: *Complex information processing: The impact of Herbert*. Ed. by D. Klahr and K. Kotovsky. Lawrence Erlbaum Associates, Inc, 343–371.

Audette, B. (2005). *Beyond Curriculum Alignment: How One High School Is Using Student Assessment Data to Drive Curriculum and Instruction Decision Making*. [Accessed: 28 June 2022]. Available from: https://tinyurl.com/pk3avtm8.

Bailey, K. D. (1994). *Typologies and Taxonomies: An Introduction to Classification Techniques*. Sage Publications.

Bellert, A. (2015). Effective re-teaching. *Australian Journal of Learning Difficulties* [online]. 20.2, pp. 163–183. DOI: 10.1080/19404158.2015.1089917.

Berrett, D. (Sept. 2012). How "Flipping" the Classroom Can Improve the Traditional Lecture. *Education Digest: Essential Readings Condensed for Quick Review*. 78.1, pp. 36–41.

Bezuidenhout, J. (2001). Limits and continuity : some conceptions of first year students. *International Journal of Mathematics Education in Science and Technology*. 32.4, pp. 487–500.

Black, P. and Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education*. 5, pp. 7–74.

Booth, J. and Koedinger, K. (Jan. 2008). Key misconceptions in algebraic problem solving. *Proceedings of the 30th Annual Cognitive Science Society*, pp. 571–576.

Booth, J. L., Koedinger, K. R., and Siegler, R. S (2007). The Effect of Prior Conceptual Knowledge on Procedural Performance and Learning in Algebra. In: *Proceedings of the 29th Annual Cognitive Science Society*. Ed. by M. D. S. and T. J. G. Cognitive Science Society, pp. 137–142.

Booth, J. L., Barbieri, C., Eyer, F., and Pare-blagoev, E. (2014). Persistent and Pernicious Errors in Algebraic Problem Solving. Journal of Problem Solving. *Journal of Problem Solving*. 7, pp. 10–23.

Booth, J., Lange, K., Koedinger, K. R., and Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction*. 25, pp. 24–34.

Booth, J., Mcginn, K., Barbieri, C., and Young, L. (Oct. 2017). Misconceptions and Learning Algebra. In: *And the Rest is Just Algebra*. Ed. by S. Stewart. Chap. 4, pp. 63–78. ISBN: 978-3-319-45052-0. DOI: 10.1007/978-3-319-45053-7_4.

Bothwell, E. (2020). UK universities favour blended learning approach for 2020-21. *Times Higher Education* [online]. 2457. [Accessed: 26 December 2022]. ISSN: 0049-3929. Available from: https://www.timeshighereducation.com/news/uk-universities-favour-blended-learning-approach-2020-21.

Boud, D. (Jan. 1995). Assessment and learning: Contradictory or complimentary, pp. 35–48.

Braun, V. and Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*. 3.2, pp. 77–101.

Broughton, S., Robinson, C. L., and Hernandez-Martinez, P. (2013). Lecturers' perspectives on the use of a mathematics-based computer-aided assessment system. *Teaching mathematics and its applications*. 32.2, pp. 88–94. ISSN: 0268-3679.

Brown, J. S. and Burton, R. R. (1978). Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science* [online]. 2.2, pp. 155–192. DOI: doi.org/10.1207/s15516709cog0202_4.

Buchanan, T. (2000). The efficacy of a World-Wide Web mediated formative assessment. *Journal of Computer Assisted Learning*. 16, pp. 193–169.

Bull, J. (2013). *Blueprint for computer-assisted assessment*. London: Routledge.

Cakir, M. (2008). Constructivist approaches to learning in science and their implications for science pedagogy: A literature review. *International Journal of Environmental & Science Education*. 3.4, pp. 193–206.

Cangelosi, R., Madrid, S., Cooper, S., Olson, J., and Hartter, B. (2013). The negative sign and exponential expressions: Unveiling students' persistent errors and misconceptions. *The Journal of Mathematical Behavior* [online]. 32.1, pp. 69–82. ISSN: 0732-3123. DOI: doi.org/10.1016/j.jmathb.2012.10.002.

Carlbring, P., Gunnarsdottir, M., Hedensjo, L., Andersson, G., Ekselius, L., and Furmark, T. (2007). Treatment of social phobia: Randomised trial of internet-delivered cognitive-behavioural therapy with telephone support. *British Journal of Psychiatry* [online]. 190.2, pp. 123–128. DOI: 10.1192/bjp.bp.105.020107.

Castleberry, A. and Nolen, A. (2018). Thematic analysis of qualitative research data: Is it as easy as it sounds? *Currents in Pharmacy Teaching and Learning*. 10, pp. 807–815.

Cohen, R. J. and Swerdlik, M. E. (2005). *Psychological Testing and Assessment: An Introduction to Tests and Measurement*. New York: McGraw-Hill.

Confrey, J. (1990). A review of the research on student conceptions in mathematics, science, and programming. *Review of Research in Education*. 16, pp. 3–56.

Connors, C. B. (2021). Summative and Formative Assessments: An Educational Polarity. *Kappa Delta Pi Record* [online]. 57.2, pp. 70–74. DOI: 10.1080/00228958.2021.1890441.

Cotner, S., Baepler, P., and Kellerman, A. (2008). Scratch this! The IF-AT as a technique for stimulating group discussion and exposing misconceptions. *Journal of College Science Teaching*. 37.4, pp. 48–53.

Council of the European Union (June 16, 2020). *Council conclusions on countering the COVID-19 crisis in education and training*. [Accessed: 15 December 2022]. Council of the European Union. Available from: https://data.consilium.europa.eu/doc/document/ST-8610-2020-INIT/en/pdf.

Crews, T. and Curtis, D. (Dec. 2011). Online Course Evaluations: Faculty Perspective and Strategies for Improved Response Rates. *Assessment & Evaluation in Higher Education* [online]. 36, pp. 865–878. DOI: 10.1080/02602938.2010.493970.

Crisp, G. (2007). *The e-Assessment Handbook*. London: Continuum International Publishing Group.

Crocker, L. and Algina, J. (2008). *Introduction to Classical and Modern Test Theory*. New York: Holt, Rinehart, and Winston.

CSE Project at UWE (2019a). *CSE Project*. [Accessed: 24 June 2020]. Available from: http://www.cems.uwe.ac.uk/~bin-sikurajapa/dewis/cseproject/.

CSE Project at UWE (2019b). *CSE Project: Participant Information Sheet*. [Accessed: 24 June 2020]. Available from: https://fetstudy.uwe.ac.uk/~bin-sikurajapa/dewis/cseproject/docs/Participant_Information_Sheet.pdf.

Daly, C., Pachler, N., Mor, Y., and Mellar, H. (Aug. 2010). Exploring formative e-Assessment: Using case stories and design patterns. *Assessment & Evaluation in Higher Education* [online]. 35, pp. 619 –636. DOI: 10.1080/02602931003650052.

Daniel, S. (2020). Education and the COVID-19 pandemic. *Prospects* [online]. 49, pp. 91–96. DOI: https://doi.org/10.1007/s11125-020-09464-3.

Deeley, S. J. (2014). Summative co-assessment: A deep learning approach to enhancing employability skills and attributes. *Active Learning in Higher Education* [online]. 15.1, pp. 39–51. DOI: 10.1177/1469787413514649.

Deeley, S. J., Fischbacher-Smith, M., Karadzhov, D., and Koristashevskaya, E. (2019). Exploring the ‘wicked’ problem of student dissatisfaction with assessment and feedback in higher education. *Higher Education Pedagogies* [online]. 4.1, pp. 385–405. DOI: 10.1080/23752696.2019.1644659.

Dermo, J. (2009). E-Assessment and the student learning experience: a survey of student perceptions of e-Assessment. *British Journal of Educational Technology*. 40.2, 203–214.

Derrick, B., Toher, D., and White, P. (2017). How to compare the means of two samples that include paired observations and independent observations: A companion to Derrick, Russ, Toher and White (2017). *The Quantitative Methods for Psychology* [online]. 13.2, pp. 120–126. DOI: 10.20982/tqmp.13.2.p120. Available from: http://www.tqmp.org/RegularArticles/vol13-2/p120/p120.pdf.

Dewis (2022). *Structure of a Dewis Question*. [Accessed: 04 April 2022]. Available from: https://dewisprod.uwe.ac.uk/fixed/2022/manuals/question-editor/structure.html.

Dillman, D. (2007). *Mail and Internet surveys : the tailored design method : 2007 update with new internet, visual and mixed-mode guide*. Hoboken, New Jersey : John Wiley & Sons, Inc.

D'Inverno, R., Davis, H., and White, S. (2003). Using a personal response system for promoting student interaction. *Teaching Mathematics and Its Applications*. 22.4, pp. 163–169.

Divjak, B., Žugec, P., and Anicic, K. P. (2022). E-Assessment in mathematics in higher education: a student perspective. *International Journal of Mathematical Education in Science and Technology* [online]. 0.0, pp. 1–23. DOI: 10.1080/0020739X.2022.2117659.

Donaldson, M. (1963). *A study of children's thinking*. London: Routledge.

Donovan, J., Mader, C. E., and Shinsky, J. (2007). Online vs. Traditional Course Evaluation Formats: Student Perceptions. *Journal of Interactive Online Learning*. 6, pp. 158–180.

Durkin, K. and Rittle-Johnson, B. (2015). Diagnosing misconceptions: Revealing changing decimal fraction knowledge. *Learning and Instruction* [online]. 37, pp. 21–29. DOI: 10.1016/j.learninstruc.2014.08.003.

Durkin, K. and Rittle-Johnson, B. (2012). The effectiveness of using incorrect examples to support learning about decimal magnitude. *Learning and Instruction*. 22.3, pp. 206–214.

Earl, L. M. (2003). *Assessment as Learning: Using Classroom Assessment to Maximize Student Learning*. California: Corwin Press, Inc.

Ebert, J. F., Huibers, L., Christensen, B., and Christensen, M. B. (2018). RPaper- or Web-Based Questionnaire Invitations as a Method for Data Collection: Cross-Sectional Comparative Study of Differences in Response Rate, Completeness of Data, and Financial Cost. *Journal of Medical Internet Research*. 20.1, pp. 1–13.

Eynon, R. (Feb. 2008). The use of the world wide web in learning and teaching in higher education: Reality and rhetoric. *Innovations in Education and Teaching International* [online]. 45, pp. 15–23. DOI: 10.1080/14703290701757401.

Falchikov, N. (2001). *Learning Together: Peer Tutoring in Higher Education*. London, UK: Routledge.

Felder, R. M. and Henriques, E. R. (Mar. 1995). Learning and Teaching Styles In Foreign and Second Language Education. *Foreign Language Annals*. 28, pp. 21 –31.

Fischbein, E. (1989). Tacit models and mathematical reasoning. *For the Learning of Mathematics*. 9, pp. 9–14.

Ford, S., Gillard, J., and Pugh, M. (2018). Creating a Taxonomy of Mathematical Errors For Undergraduate Mathematics. *MSOR Connections*. 18.1, pp. 37–45.

Foster, B., Perfect, C., and Youd, A. (2012). A Completely Client-side Approach to e-Assessment and E-learning of Mathematics and Statistics. *International Journal of e-Assessment*. 2.2, pp. 1–12.

Frankfort-Nachmias, C. (1996). *Research methods in the social sciences*. 5th ed. London : Arnold.

Gibbs, G and Simpson, C (2005). Conditions Under Which Assessment Supports Students' Learning. *Learning and Teaching in Higher Education*. 1, pp. 3–31.

Gikandi, J. W., Morrow, D., and Davis, N. E. (2011). Online formative assessment in higher education: a review of the literature. *Computers & Education*. 57.4, 2333–2351.

Gilbert, L., Whitelock, D., and Gale, V (2011). *Synthesis report on assessment and feedback with technology enhancement*. Project Report. [Accessed: 28 Oct 2022]. Available from: `https://eprints.soton.ac.uk/273221/`.

Gill, M. and Greenhow, M. (2008). How effective is feedback in Computer-Aided Assessments? *Learning, Media and Technology.* 33.3, pp. 207–220.

Greenhow, M. (2015). Effective Computer-aided Assessment of Mathematics; Principles, Practice and Results. *Teaching Mathematics and Its Applications*. 34, pp. 117–137.

Greenhow, M. and Kamavi, K. (2012). Maths e.g. - a Web Assessment Application for Stem and Beyond. *Proceedings of the HEA Stem Learning and Teaching Conference* [online]. DOI: `doi.org/10.11120/stem.hea.2012.062`.

Gregory, R. J. (2007). *Psychological Testing: History, Principles, and Applications*. The United States of America: Pearson Education, Inc.

Gwynllyw, R. and Henderson, K. (Nov. 2009). "A computer aided assessment system for mathematics and statistics". In: *CETL-MSOR 2008 Conference Proceedings*, pp. 38–44.

Gwynllyw, R. and Henderson, K. (2012). "Intelligent marking in summative e-Assessment". In: *Proceedings of the HEA STEM Learning and Teaching Conference*, pp. 38–44.

Gwynllyw, R., Weir, I. S., and Henderson, K (2016). Using DEWIS and R for Multi-Staged Statistics e-Assessments. *Teaching Mathematics and Its Applications*. 35, pp. 14–26.

Hansan, A. (2014). *Children Errors in Mathematics*. 3rd ed. Prentice Hall, p. 239.

Harley, J. M., Lou, N., Liu, Y, Cutumisu, M., Daniels, L. M., Leighton, J., and Nadon, L. (2021). University students' negative emotions in a computer-based examination: the roles of trait test-emotion, prior test-taking methods and gender. *Assessment & Evaluation in Higher Education* [online]. 46.6, pp. 956–972. DOI: 10.1080/02602938.2020.1836123.

Hart, K. M. (1981). *Children's understanding of mathematics 11-16*. London: John Murray.

Hattie, J. and Timperley, H. (2007). The power of feedback. *Review of Educational Research* [online]. 77.1, 81–112. DOI: 10.3102/003465430298487.

Hattie, J. (1987). Identifying the salient facets of a model of student learning: a synthesis of meta-analyses. *International Journal of Educational Research*. 11, pp. 187–212.

Henderson, K. (2017). Using e-Assessment to support flipped-style teaching. *MSOR Connections*. 15 (2), pp. 34–41. ISSN: 1473-4869.

Henderson, K., Gwynllyw, R., and Hooper, A. (2016). Using electronic exams to provide engineering mathematics students with rapid feedback. *Proc. the 18th SEFI Mathematics Working Group Seminar on Mathematics in Engineering Education*, pp. 105–111.

Hiebert, J. and Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In: *Conceptual and procedural knowledge: The case of mathematics*. Lawrence Erlbaum Associates, Inc. Chap. 1, pp. 1–27.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., and Wearne, D. (1996). Problem Solving as a Basis for Reform in Curriculum and Instruction: The Case of Mathematics. *American Educational Research Association*. 25.4, pp. 12–21.

Hilbert, T., Renkl, A., Schworm, S., Kessler, S., and Reiss, K. (2007). Learning to teach with worked-out examples: a computer-based learning environment for teachers. *Journal of Computer Assisted Learning*. 24, pp. 316–332.

Hogan, T. P. (2007). *Psychological Testing: A Practical Introduction*. The United States of America: John Wiley and Sons, Inc.

JISC (2007). *Effective Practice with e-Assessment: An overview of technologies, policies and practice in further and higher education*. Project Report. [Accessed: 28 Oct 2022]. Available from: https://www.webarchive.org.uk/wayback/archive/20140614142816/http://www.jisc.ac.uk/whatwedo/programmes/elearningpedagogy/assessment.aspx.

JISC (2010). Effective assessment in a digital age: A guide to technology-enhanced assessment and feedback. *Technology enhanced Assessment*, pp. 26–28.

Jönsson, A. (Mar. 2013). Facilitating Productive Use of Feedback in Higher Education. *Active Learning in Higher Education* [online]. 14.1, pp. 63–76. DOI: 10.1177/1469787412467125.

Jordan, S. (Sept. 2013). E-Assessment: Past, present and future. *NDIR* [online]. 3.1, pp. 87–106. DOI: doi:10.11120/ndir.2013.00009.

Jordan, S. (2014). Adult science learners' mathematical mistakes: An analysis of responses to computer-marked questions. *European Journal of Science and Mathematics Education*. 2.2, pp. 63–86.

Jordan, S. and Mitchell, T. (2009). e-Assessment for learning? The potential of short-answer free-text questions with tailored feedback. *British journal of educational technology*. 40.2, pp. 371–385. ISSN: 0007-1013.

JSXGraph (2022). *Dynamic Mathematics with JavaScript*. [Accessed: 21 June 2022]. Available from: https:https:https://jsxgraph.uni-bayreuth.de/wp/index.html.

Kilpatrick, J., Swafford, J., and Findell, B. (2001). *Adding it up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Kirshner, D. and Awtry, T. (2004). Visual salience of algebraic transformations. *Journal for Research in Mathematics Education.* 35, pp. 224–257.

Knuth, E., Stephens, A., McNeil, N., and Alibali, M. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education.* 37, pp. 297–312.

Lipnevich, A. A., McCallen, L. N., P., M. K., and K., S. J. (2014). Mind the gap! Students' use of exemplars and detailed rubrics as formative assessment. *Instructional Science* [online]. 42.4, pp. 539–559. ISSN: 00204277, 15731952. Available from: http://www.jstor.org/stable/43575435 [Accessed Dec. 12, 2022].

Llamas-Nistal, M., Fernández-Iglesias, M., González-Tato, J., and Mikic-Fonte, F. (2013). Blended e-Assessment: Migrating classical exams to the digital world. *International Journal of Information and Education Technology*. 62, pp. 72–84.

Lunt, T. and Curran, J. (Dec. 2010). 'Are you listening please?' The advantages of electronic audio feedback compared to written feedback. *Assessment & Evaluation in Higher Education* [online]. 35.7, pp. 759–769. DOI: https://doi.org/10.1080/02602930902977772.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Routledge: New York.

Maciejewski, W. (2016). Flipping the calculus classroom: an evaluative study. *Teaching Mathematics and its Applications: An International Journal of the IMA* [online]. 35 (4), 187–201. DOI: doi.org/10.1093/teamat/hrv019.

MathJax (2022). *Beautiful and accessible math in all browsers*. [Accessed: 21 June 2022]. Available from: https:https://www.mathjax.org/.

Mazur, E. (1997). *Peer Instruction: A User's Manual*. Series in Educational Innovation. Prentice Hall, p. 253. Available from: /files/mazur/files/rep_0.pdf.

Mccabe, M. (Nov. 2009). The exponential growth of mathematics and technology at the University of Portsmouth. *Teaching Mathematics and Its Applications*. 28, pp. 222–227.

McDonald, B. (2010). *Mathematical Misconceptions*. [Accessed: 28 Nov 2022]. LAMBERT Academic Publishing. Available from: https://www.researchgate.net/publication/274080093_Mathematical_Misconceptions/link/55155e5d0cf2d70ee2701d47/download.

McDowell, L., Wakelin, D., Montgomery, C., and King, S. (2011). Does assessment for learning make a difference? The development of a questionnaire to explore the student response. *Assessment & Evaluation in Higher Education* [online]. 36.7, pp. 749–765. DOI: 10.1080/02602938.2010.488792.

McMillan, J. H. (2013). *Classroom Assessment: Principles and Practice for Effective Standards-Based Instruction*. Pearson Education.

Miller, C. M. (1974). *Up to the Mark: a study of the examination game*. Guildford: Society for Research into Higher Education.

Muzangwa, J. and Chifamba, P. (2012). Analysis of Errors and Misconceptions in the Learning of Calculus by Undergraduate Students. *Acta Didactica Napocensia*. 5.

Nesher, P (1987). Towards an instructional theory: The role of student's misconceptions. *For the Learning of Mathematics.* 7, pp. 33–40.

Nicol, D. (2007). Laying a foundation for lifelong learning: Case studies of e-Assessment in large 1st-year classes. *British journal of educational technology* [online]. 38.4, pp. 668–678. ISSN: 0007-1013. DOI: doi.org/10.1111/j.1467-8535.2006.00657.x.

Nicol, D. and Macfarlane, D. (May 2006). Formative Assessment and Self-Regulated Learning: A Model and Seven Principles of Good Feedback Practice. *Studies in Higher Education* [online]. 31, pp. 199–218. DOI: 10.1080/03075070600572090.

Obersteiner, W., Dooren, W., Hoof, J., and Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction.* 28, pp. 64–72.

O'Donovan, B., Rust, C., and Price, M. (2016). A scholarly approach to solving the feedback dilemma in practice. *Assessment & Evaluation in Higher Education* [online]. 41.6, pp. 938–949. DOI: 10.1080/02602938.2015.1052774.

OECD (2021). *The State of Higher Education*, p. 46. DOI: https://doi.org/https://doi.org/10.1787/83c41957-en.

OFS (2022). *National Student Survey-NSS*. [Accessed: 13 December 2022]. Available from: https://www.officeforstudents.org.uk/advice-and-guidance/student-information-and-data/national-student-survey-nss/.

Oppenheim, A. N. (1992). *Questionnaire design, interviewing and attitude measurement*. 2nd ed. London Pinter.

Orton, A. (1983). Students' understanding of integration. *Educational Studies in Mathematics*. 14, pp. 1–18.

Osuji, U. S. A. (Oct. 2012). The use of e-Assessments in the Nigerian higher education system. *The Turkish Online Journal of Distance Education*. 13, pp. 140–152.

Perl (2022). *Perl Programming Language*. [Accessed: 05 April 2022]. Available from: https://www.perl.org/.

Pressey, S. (1928). Educational Research and Statistics: A simple apparatus which gives tests and scores- and teach. *School and Society*. XXIII.586, pp. 373–376.

Qualtrics (2005). *Qualtrics (2019) [computer program]*. [Accessed:01 March 2021]. Available from: https://www.qualtrics.com/.

R Statistical Software (2023). *The R Project for Statistical Computing*. [Accessed: 21 June 2023]. Available from: https://www.r-project.org/.

Radatz, H. (May 1979). Error Analysis in Mathematics Education. *Journal for Research in Mathematics Education*. 10.3, pp. 163–172.

Rees, R. and Barr, G. (1984). *Diagnosis and Prescription in the Classroom: Some Common Maths Problems.* London: Harper & Row.

Renkl, A., Stark, R., Gruber, H., and Mandl, H. (1998). Learning from Worked-Out Examples: The Effects of Example Variability and Elicited Self-Explanations. *Contemporary Educational Psychology*. 23, 90–108.

Ridgway, J., McCusker, S., and Pead, D. (2004). "Literature review of e-Assessment". In: [Accessed: 30 Oct 2022]. FUTURELAB SERIES. Available from:

https://www.nfer.ac.uk/publications/FUTL64/FUTL64literaturereview.pdf.

Rittle-Johnson, B. and Siegler, R. (1998). The relation between conceptual and procedural knowledge in learning mathematics: A review. In: *The development of mathematical skills*. Ed. by D. Chris. Psychology Press A member of the Tylor & Farancis group. Chap. 4, pp. 75–110.

Rittle-Johnson, B., Siegler, R., and Alibali, M. (June 2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology* [online]. 93, pp. 346–362. DOI: 10.1037//0022-0663.93.2.346.

Rittle-Johnson, B. and Star, J. (Aug. 2007). Does Comparing Solution Methods Facilitate Conceptual and Procedural Knowledge? An Experimental Study on Learning to Solve Equations. *Journal of Educational Psychology* [online]. 99.3, pp. 561–574. DOI: 10.1037/0022-0663.99.3.561.

Robinson, M. (2015). Chapter 12: Providing effective feedback. In: *Transitions in undergraduate mathematics education*. Ed. by M. Grove, T. Croft, J. Kyle, and D. Lawson. The University of Birmingham, pp. 159–172.

Rolim, C. and Isaias, P. (2019). Examining the Use of e-Assessment in Higher Education: Teachers and Students' Viewpoints. *British Journal of Educational Technology*. 50.2, pp. 1785–1800.

Rushton, N. (2014). Common errors in mathematics. *Research Matters: A Cambridge Assessment publication.* 17, pp. 8–17.

Russell, J., Elton, L., Swinglehurst, D., and Greenhalgh, T. (Aug. 2006). Using the online environment in assessment for learning: A case-study of a web-based course in primary care. *Assessment & Evaluation in Higher Education* [online]. 31, pp. 465–478. DOI: 10.1080/02602930600679209.

Ryan, J. and Williams, J. (2007). *Children's mathematics 4–15: Learning from errors and misconceptions*. Maidenhead, UK: Open University Press.

Sadler, D. R. (1987). Specifying and Promulgating Achievement Standards. *Oxford Review of Education* [online]. 13.2, pp. 191–209. ISSN: 03054985, 14653915. Available from: http://www.jstor.org/stable/1050133 [Accessed Dec. 12, 2022].

Sadler, D. R. (1989). Formative assessment and the design of instructional systems. *Instructional Science*. 18, pp. 119–144.

Sadler, D. R. (1998). Formative Assessment: revisiting the territory. *Assessment in Education: Principles, Policy & Practice* [online]. 5.1, pp. 77–84. DOI: 10.1080/0969595980050104.

Sadler, D. R. (Aug. 2010). Beyond feedback: Developing student capability in complex appraisal. *Assessment & Evaluation in Higher Education* [online]. 35.5, 535–550. DOI: 10.1080/02602930903541015.

Sambell, K., McDowell, L., and Montgomery, C. (2013). *Assessment for learning in higher education*. London: Routledge.

Sangwin, C. (2004). Assessing Mathematics Automatically Using Computer Algebra and the Internet. *Teaching Mathematics and Its Applications*. 23.1, pp. 1–14.

Sangwin, C. (2013). *Computer Aided Assessment of Mathematics. [S.l.]* Oxford University Press.

Sangwin, C. (2015). Chapter 11: Technology-led teaching: The role of mathematical software. In: *Transitions in undergraduate mathematics education*. Ed. by M. Grove, T. Croft, J. Kyle, and D. Lawson. The University of Birmingham, pp. 145–158.

Savion, L. (2009). Clinging to discredited beliefs: The larger cognitive story. *Journal of the Scholarship of Teaching and Learning,* 9.1, pp. 81–92.

Sclater, N., Low, B., and Barr, N. (2002). Interoperability with CAA: does it work in practice? In: *Proceedings of the 6th International Computer-assisted Assessment Conference*. Ed. by M. Danson. [Accessed: 27 Dec 2022]. Loughborough: Loughborough University, pp. 317–327. Available from: https://tinyurl.com/ybvtdrvu.

Senel, S. and Senel, H. C. (2021). Remote Assessment in Higher Education during COVID-19 Pandemic. *International Journal of Assessment Tools in Education* [online]. 8.2, pp. 181 –199. DOI: 10.21449/ijate.820140.

Sevian H. ang Robinson, W. E. (2011). Clickers Promote Learning in All Kinds of Classes—Small and Large, Graduate and Undergraduate, Lecture and Lab. *Journal of College Science Teaching*. 40.3, pp. 14–18.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2020). Using e-Assessment to Address Mathematical Misconceptions in Engineering Students. *International Journal of Information and Education Technology.* 10.5, pp. 356–361.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2021). Students' Perceptions of Enhanced e-Assessment Feedback Addressing Common Student Errors in Mathematics. *MSOR Connections.* 19.2, pp. 10–27.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2022a). *Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessment (CSE Book)*. Available from: https://uwe-repository.worktribe.com/output/9303961.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2022b). Gathering and Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification. *MSOR Connections*. 20.3, pp. 55–71.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2023). Correct for the wrong reason: why we should know more about Mathematical Common Student Errors in e-Assessment questions. *MSOR Connections*. [Accepted].

Sikurajapathi, I., Henderson, K., and Button, J. (2019). *GDPR Advice Meeting, 08 July*. The University of the West of England, Bristol.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2021). *The effectiveness of remediating mathematical Common Student Errors in e-Assessment*. [Presentation]. Available from: `hhttps://eams.ncl.ac.uk/sessions/2021/the-effectiveness-of-remediating-mathematical-common-student-errors-in-e-assessments/`.

Simpson, G. G. (1961). *Principles of Animal Taxonomy*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Skinner, B. (1958). Teaching Machines: From the experimental study of learning come devices which arrange optimal conditions for self-instruction. *SCIENCE*. 128.3330, pp. 969–977.

Sleeman, D. (1984). “Mis-generalization: An Explanation of Observed Mal-rules.” In: *Proceedings of the Sixth Annual Conference of the Cognitive Science Society.* Pp. 1–8.

Snyder, B. R. (1971). *The Hidden Curriculum*. Cambridge, MA: MIT Press.

Star, J. R. (2015). Reconceptualizing procedural knowledge. *Journal for research in mathematics education*, pp. 404–411.

Stödberg, U. (2012). A research review of e-Assessment. *Assessment and Evaluation in Higher Education,* 37.5, 591–604.

Taras, M. (2002). Using Assessment for Learning and Learning from Assessment. *Assessment & Evaluation in Higher Education* [online]. 27.6, pp. 501–510. DOI: `10.1080/0260293022000020273`.

Timmis, S., Broadfoot, P., Sutherland, R., and Oldfield, A. (2016). Rethinking assessment in a digital age: opportunities, challenges and risks. *British Educational Research Journal* [online]. 42.3, pp. 454–476. DOI: `https://doi.org/10.1002/berj.3215`.

Tomlinson, M. (2004). *14–19 Curriculum and Qualifications Reform: Final Report of the Working Group on 14–19 Reform*. Project Report. [Accessed: 26 December 2022]. Available from: http://www.educationengland.org.uk/documents/pdfs/2004-tomlinson-report.pdf.

VanLehn, K. (1982). Bugs are not enough: Empirical studies of bugs, impasses and repairs in procedural skills. *Journal of Mathematical Behavior*. 3.2, pp. 3–71.

Vanlehn, K. and Jones, R. (1997). What mediates the self-explanation e ect? Knowledge gaps, schemas or analogies? In: *Proceedings of the fifteenth Annual Conference of the Cognitive Science Society*. Ed. by M. Polson. Cognitive Science Society, pp. 1034–1039.

Walker, P., Gwynllyw, R., and Henderson, K. (2015). Diagnosing student errors in e-Assessment questions. *Teaching mathematics and its Applications*. 34.3, pp. 160–170.

Watson, P. and Petrie, A. (2010). Method agreement analysis: A review of correct methodology. *Theriogenology* [online]. 73.9, pp. 1167–1179. ISSN: 0093-691X. DOI: https://doi.org/10.1016/j.theriogenology.2010.01.003. Available from: https://www.sciencedirect.com/science/article/pii/S0093691X10000233.

WHO (2020). *Coronavirus disease (COVID-19) pandemic*. [Accessed: 22 December 2022]. World Health Organization. Available from: https://www.who.int/europe/emergencies/situations/covid-19.

Williams, J. and Ryan, J. (2000). National Testing and the Improvement of Classroom Teaching: Can They Coexist? *British Educational Research Journal*. 26.1, pp. 49–73.

Wright, K. B. (2005). Researching Internet-Based Populations: Advantages and Disadvantages of Online Survey Research, Online Questionnaire Authoring Software Packages, and Web Survey Services. *Journal of Computer-Mediated Communication*. 10.3.

Yip, D. (Apr. 1998). Identification of misconceptions in novice biology teachers and remedial strategies for improving biology learning. *International Journal of Science Education* [online]. 20, pp. 461–477. DOI: 10.1080/0950069980200406.

Yorke, M. (2001). Formative Assessment and its Relevance to Retention. *Higher Education Research & Development*. 20.2, pp. 115–126.

Yüksel, H. S. and Gündüz, N. (2017). Formative and summative assessment in higher education: Opinions and practices of instructors. *European Journal of Education Studies* [online]. 3.8. DOI: 10.5281/zenodo.832999.

A Appendix: Thematics Analysis on Questionnaire Responses

Table A.1: Students' responses of Conceptual Change Theme for the question "What do you like about the enhanced feedback you received?"

Sub-themes in Conceptual Change	Students' responses
Correct CSE capture (Correct Capture)	"Told me exactly where I went wrong." "It give me a good understanding of what I did." "It also explained in detail why I was incorrect." "The fact that the feedback tells me where I actually went wrong and if I repeat the test, then I would not make the same mistake." "The enhanced feedback got right to the reason the answer was wrong." "The Feedback which I received helped me to understand where I was most likely to make errors and showed the correct way of working out solutions." "I can clearly see where I went wrong and it gives me a chance to improve." "It helps me to make me realize the mistake where I went wrong on some type of questions." "It makes you feel conscious of errors you made. The fact that it tells you what you've done based on your final input is clever." "I think it is a great model of reinforcing problems of understanding." "It made me understand more in depth."

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Table A.1 – *Continued from previous page*

Sub-themes in Conceptual Change	**Students' responses**
Facilitate learning (Beneficial)	"Very useful and well structured. Helps to answer any similar questions." "It was certainly useful to receive enhanced feedback alongside the standard feedback." "Very useful and helps to further understanding." "It helped my understanding." "I reckon that the enhanced feedback must be very helpful to those, who struggle with some questions." "I have read the feedback and it seemed very helpful and clear to me."
Relevance of the content on CSEs enhanced feedback (Relevant).	"Immediate and specific question related instead of a general explanation." "Its overall applicability to my work." "It's related to the problem."

Table A.2: Students' responses of User-friendly Features Theme for the question "What do you like about the enhanced feedback you received?"

Sub-themes in User-friendly Features	**Students' responses**
Coherent structure	"Very useful and well structured." "The total feedback was overall concise" "Step by step method." "It also explained in detail why I was incorrect." "Short and simple." "Clear and concise information." "Well detailed with every step explained thoroughly." "It shows the correct answer and detailed workings." "Clear and concise." "It's very well structured so that it is easy to understand." "Clear and concise method, made it easier to understand the question." "Write all steps of solution." "It's well explained."
Accessibility	"The total feedback was overall concise and accessible." "Its simplicity." "That it is instant." "It was in a different colour so more visible." "Immediate." "Accessible feature and introduced to the user."

Table A.3: Students' responses of 'Everything is alright' Theme for the question "What do you dislike about the enhanced feedback you received?"

Main themes	Students' responses
Everything is alright/ nothing to dislike	"Nothing." "There is not really much there to dislike, it's just maths feedback." "Nothing to dislike." "I haven't found any cons regarding the feedback." "I find it good enough." "No."

Table A.4: Students' responses of 'Short Explanations Theme' for the question "What do you dislike about the enhanced feedback you received?"

Main theme	Students' responses
Short explanations	"Some answers can be quite brief so more in depth answers would be great." "Some feedback solutions explain steps without showing the working needed for those steps." "Sometimes the workings are not easy to understand." "Needs more steps for the student to fully understand what is happening throughout the equation." "Sometimes it's unclear on how it gets from one step to another." "For some questions it is really helpful. For other questions I don't think it goes far enough to explain the workings." "I wish the enhanced feedback was more detailed."

Table A.5: Students' responses for 'Less Accessibility Features Theme' for the question "What do you dislike about the enhanced feedback you received?"

Main theme	Students' responses
Less accessibility features	"It was below the general feedback and correct answer, so it's easy to just scroll past." "It's structure" "Needs to be more organised and easier to identify where you made the mistake." "The incorrect answer could be written right after the correct one rather than right at the very bottom so that it would be easier to understand."

Table A.6: Students' responses of 'Everything is alright' Theme for the question "Do you have any suggestions for improvement?"

Main themes	Students' responses
Everything is alright/ nothing to dislike	"I think it is as good as it can be. Thank you!" "Nothing." "It's good enough." "No" "I think it is as good as it can be. Thank you!"

Table A.7: Students' responses of 'Suggestions to improve current features' Theme for the question "Do you have any suggestions for improvement?"

Sub themes	Students' responses
Detailed Explanations	"Include all steps, even if they seem unimportant." "Make it a bit clearer to understand." "More detailed feedback, especially for integration and differentiation questions." "Highlight your mistake, but show other possible common mistakes optionally. That way you can roughly know what to look out for." "Include an extra example? Time consuming so understandable if not"
More Accessibility features	"To make it more readable and a more efficient design." "I would suggest using two columns when designing the layout for the feedback. One should just show my answer. The other shows the right answer with the detailed working." "Moving the incorrect answer closer to the correction or right next to it and maybe making it easier to find the questions you got wrong rather than scrolling all the way and having to search for it."

Table A.8: Students' responses of 'Suggestions for future directions' Theme for the question "Do you have any suggestions for improvement?"

Sub themes	**Students' responses**
Enhanced feedback for all the other questions	"I would like more feedback for all question I get wrong, and with a more detailed step by step approach." "It doesn't give alternate answers with different questions as an option for more complex questions." "Not all questions has enhanced feedback." "I would prefer more feedback from Dewis, in particular more steps in how problems are solved." "Provide enhanced feedback not just on hard questions but on easy ones too."
New ideas for further improvement	"I would also like to know the subject of each question so that I could Google anything that I didn't understand." "Another option would be to have a link to the lectures that covered each question, so that if I got a question wrong I could know what lecture covered that topic." "Videos of a maths teacher doing each question and talking through each step."

B Appendix: Common Student Error Summary

Table B.1: Summary of CSEs found during the CSE Project at UWE Bristol

Question type	CSE ID	2017-2018 No. of incorrect answers	2017-2018 No. of CSE answers	2017-2018 CSE %	2018-2019 No. of incorrect answers	2018-2019 No. of CSE answers	2018-2019 CSE %	Taxonomy code	CSE book section	CSE book page No.
Integer Input	Q1 CSE1	56	28	50%	57	33	58%	S1	2.1.1	13
	Q1 CSE2	45	6	13%	51	14	27%	U5	3.5.1	34
	Q1 CSE3	26	4	15%	24	2	8%	U1	3.1.1	26
	Q2 CSE1	86	35	41%	100	32	32%	U1	3.1.2	27
	Q3 CSE1	13	5	38%	19	9	47%	U5	3.5.2	35
	Q3 CSE2				29	7	24%	U5	3.5.3	36
	Q4 CSE1	61	37	61%				U5	3.5.4	37
	Q5 CSE1				81	47	58%	U5	3.5.5	38
	Q5 CSE2				84	9	11%	U6	3.6.1	64
	Q6 CSE1				33	7	21%	U5	3.5.7	40
	Q6 CSE2				33	5	15%	U5	3.5.8	41
	Q7 CSE1	183	52	28%				UM2	5.1.4	77
	Q7 CSE2	183	44	24%				U7, UM2	3.7.1, 5.1.5	67,78
	Q7 CSE3	183	8	4%				U7, UM2	3.7.2,5.1.6	68,79

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Table B.1 *Continued from previous page*

Question type	CSE ID	2017-2018			2018-2019			Taxonomy code	CSE book section	CSE book page No.
		No. of incorrect answers	No. of CSE answers	CSE %	No. of incorrect answers	No. of CSE answers	CSE %			
Floating Point Input	Q8 CSE1				121	10	8%	U5	3.5.18	51
	Q8 CSE2				121	10	8%	U7	3.7.3	69
	Q9 CSE 1	57	40	70%				U3	3.3.1	32
	Q10 CSE1	109	9	8%	95	9	9%	S3	2.3.1	22
	Q11 CSE1	197	66	34%				U4,U6	3.4.1,3.6.2	33,65
	Q11 CSE2	197	9	5%				U5	3.5.19	52
	Q11 CSE3	197	17	9%				U5	3.5.20	53
	Q11 CSE4	73	11	15%	92	14	15%	CM1	4.1.1	71
	Q12 CSE1				88	19	22%	S2	2.2.1	18
	Q12 CSE2				88	13	15%	U5	3.5.21	54
	Q12 CSE3				88	14	16%	UM2	5.1.7	80
	Q12 CSE4				88	11	13%	S2,UM2	2.2.2,5.1.8	19,81
	Q13 CSE1	67	34	51%				CM1	4.1.2	72
	Q14 CSE1	116	28	24%	122	29	24%	CM1	4.1.3	73
	Q14 CSE2	59	11	19%	80	16	20%	S2,U6	2.2.3,3.6.3	20,66

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Table B.1 *Continued from previous page*

Question type	CSE ID	2017-2018			2018-2019			Taxonomy code	CSE book section	CSE book page No.
		No. of incorrect answers	No. of CSE answers	CSE %	No. of incorrect answers	No. of CSE answers	CSE %			
	Q14 CSE3				53	5	9%	S1	2.1.3	15
	Q15 CSE1	28	7	25%	64	12	19%	S3	2.3.2	23
	Q15 CSE2				64	11	17%	U5	3.5.23	56
	Q16 CSE1	107	9	8%	135	3	2%	U5	3.5.28	61
	Q16 CSE2	107	9	8%	135	13	10%	U5	3.5.29	62
	Q16 CSE3	107	9	8%	135	8	6%	U5	3.5.30	63
	Q16 CSE4	107	7	7%				S3	2.3.3	24
	Q16 CSE5	107	5	5%	135	7	5%	UM2	5.1.9	82
	Q16 CSE6				135	6	4%	S3	2.3.4	25
	Q17 CSE1				13	6	46%	U7	3.7.4	70
	Q17 CSE2				46	9	20%	S1	2.1.4	16
	Q17 CSE3				20	8	40%	S1	2.1.5	17
	Q18 CSE1	59	30	51%				U5	3.5.6	39
	Q19 CSE1	56	7	13%				U5	3.5.9	42
	Q19 CSE2	56	10	18%				UM2	5.1.1	74

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Table B.1 *Continued from previous page*

Question type	CSE ID	2017-2018			2018-2019			Taxonomy code	CSE book section	CSE book page No.
		No. of incorrect answers	No. of CSE answers	CSE %	No. of incorrect answers	No. of CSE answers	CSE %			
Algebraic Input	Q20 CSE1	73	6	8%				U5	3.5.10	43
	Q20 CSE2	73	22	30%				UM2	5.1.2	75
	Q21 CSE1				73	13	18%	U2	3.2.1	28
	Q22 CSE1	158	18	11%	180	15	8%	U2	3.2.2	29
	Q22 CSE2	158	12	8%	180	5	3%	U5	3.5.11	44
	Q22 CSE3	158	10	6%	180	2	1%	U5	3.5.12	45
	Q23 CSE1	103	17	17%				U5	3.5.13	46
	Q23 CSE2	103	7	7%				U5	3.5.14	47
	Q23 CSE3	103	6	6%				U5	3.5.15	48
	Q23 CSE4	103	5	5%				UM2	5.1.3	76
	Q24 CSE1				145	32	22%	U5	3.5.16	49
	Q24 CSE2				145	11	8%	U5	3.5.17	50
	Q25 CSE1				165	9	5%	S1	2.1.2	14
	Q25 CSE2				165	8	5%	U5	3.5.22	55
	Q26 CSE1	143	13	9%				U5	3.5.26	59

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Table B.1 *Continued from previous page*

Question type	CSE ID	2017-2018			2018-2019			Taxonomy code	CSE book section	CSE book page No.
		No. of incorrect answers	No. of CSE answers	CSE %	No. of incorrect answers	No. of CSE answers	CSE %			
	Q26 CSE2	143	13	9%				S2	2.2.4	21
	Q26 CSE3	143	11	8%				U5	3.5.27	60
	Q26 CSE4	143	8	6%				U2	3.2.4	31
	Q27 CSE1				192	19	10%	U5	3.5.24	57
	Q27 CSE2				192	14	7%	U5	3.5.25	58
	Q27 CSE3				192	10	5%	U2	3.2.3	30

C Appendix: Research Output Reports Deposited in UWE Repository

C.1 Sikurajapathi, I., Henderson, K. and Gwynllyw, R. (2020) *Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)*

(Published Version)

Declaration:

Indunil Sikurajapathi conducted the research, analysed the data and wrote this report. Karen Henderson and Rhys Gwynllyw organised and supervised the research.

Collection of Taxonomically Classified Mathematical Common Student Errors in E-Assessments (CSE Book)

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Content

1 Introduction

This book presents Common Student Errors (CSEs) that have been made by first year students taking Engineering Mathematics e-examinations at the University of the West of England, Bristol (UWE, Bristol). The CSEs presented in this document were collected from 2017-2019 as part of the Common Student Errors Project (CSE Project at UWE) conducted at UWE, Bristol. This research has been conducted by Indunil Sikurajapathi for her Doctoral studies under the guidance of Dr Karen Henderson and Dr Rhys Gwynllyw.

This book is organised as follows. In Section 1.1 and Section 1.2 we present a brief background of Common Student Errors (CSEs) and the Dewis e-Assessment System. Then we explain how we conducted the CSE Project at UWE, Bristol in Section 1.3 and the present the CSEs Collection found during the project in Section 1.4. The taxonomy coding utilised to classify the CSEs in e-Assessments found in this project is discussed in detail in Section 1.5. After that we provide a Guide to the CSE recording template in Section 1.6 which is used in the subsequent Sections 2-23 to present all of the 65 CSEs found to date during the CSE Project at UWE, Bristol.

One of the special features of this book is that it provides hyperlinks to each question on the Dewis e-Assessment System in order to facilitate the reader to try these questions online. If any of the identified CSEs are submitted as answers, then enhanced feedback will be provided, which aims to correct any misconceptions in a timely manner.

The information in this book may be used to inform teachers so that they can provide students with a better understanding of the mathematical skills and knowledge while teaching the subject. It may also be useful for institutions as they can utilise it in the future development of teaching materials to ensure that these CSEs will be addressed. Further, the content of this book can be used to develop support materials and resources to address CSEs which will help students to acquire better understanding of mathematics. In addition, students who learn mathematics at university level or in secondary school can refer to this booklet to address their misconceptions and can try the Dewis questions several times. Since, in each attempt, Dewis produces questions with random parameters, student can use this facility to correct their misconceptions by practicing the same question but with different parameters.

We anticipate that this book will be useful to identify and address some misconceptions that students have in mathematics. We plan to continue with this research and will update the book if we find new CSEs in the future.

1.1 Common Student Errors (CSEs)

Students arrive at an incorrect answer when answering a mathematical question due to variety of reasons. The reasons can be listed as random errors, calculation errors or misreading the questions. These errors lead to incorrect answers or loss of accuracy marks. Many of these errors are made by just a few students. However, some of these errors are commonly made by a considerable number of students. These commonly made errors are sometimes referred to as common errors (Rushton, 2014).

Researchers express different opinions about the difference between errors and misconceptions in the literature. For Confrey (1990), the reasons for both errors and misconceptions are the rules and beliefs that students hold. They argue that the difference between errors and misconceptions is that misconceptions are attached to particular theoretical positions. However, Nesher (1987) uses the term misconceptions to describe systematic errors without reference to a theoretical position.

Rees and Barr (1984) use the term '*mal-rule*' to refer to an understandable but incorrect implementation of a process resulting from a student's misconception. For example, a classic *mal-rule* students make is to answer $a^2 + b^2$ when asked to expand $(a + b)^2$. The term '*bug*' is used by VanLehn (1982) to refer to a systematic error resulting from wrong steps in the calculation procedure. A *Borrow Across-Zero bug* is a systematic error caused by a student having trouble with borrowing, especially in the presence of zeros (VanLehn, 1982). For example, a student answering 98 when asked to calculate $305 - 117$ would be considered as a *Borrow Across - Zero bug*. In the aforementioned calculation, the student skips the step where the zero changed to nine during borrowing across zero (VanLehn, 1982).

Research has been conducted to identify misconceptions in different areas of mathematics. For example, Brown and Burton (1978) investigated bugs (misconceptions) in high school algebra problems, and Swan (1990) focused on the misconceptions that occur in four operations (addition, subtraction, multiplication and division), and in the interpretation of graphs.

Some Mathematics Education research has explored possible causes and effects of certain mathematical misconceptions and the impact that they have on students' future learning (Booth et al., 2014; Confrey, 1990; Fischbein, 1989; Nesher, 1987; Brown and Burton, 1978). After having investigated bugs (misconceptions) in high school algebra problems, Brown and Burton (1978) discussed possible arithmetic bugs which might lead to some specific algebraic bugs. Booth et al., (2014) conducted a study to assess algebraic misconceptions that algebra students make at school. They concluded that students who make specific persistent errors due to underlying misconceptions in arithmetic may need additional intervention since misconceptions are not corrected through typical instruction. They conclude that these additional interventions can be carried out by targeting individual misconceptions or by improving conceptual understanding throughout the algebra course. The findings of Brown and Burton (1978) and then the findings of Booth et al. (2014) hold the same conclusions, that the arithmetic misconceptions held by students affect their algebraic thinking. Further, Booth et al. (2014) state that these arithmetic misconceptions can obstruct their performance and learning of algebra.

There has been recent research into theorising student errors supported by empirical studies in the topics of natural number bias (Obersteiner et al., 2013), visual saliency (Kirshner and Awtry, 2004) and over-generalisation (Knuth et al., 2006). Rushton (2014) conducted a study of common errors in Mathematics made in certain General Certificate of Secondary Education mathematics papers taken by candidates in England, including an internationally available version, as referenced by examiner reports, and errors were catalogued into themes and subthemes. More recently, Ford et al. (2018) developed a taxonomy of errors made by undergraduate mathematics students. In their study they gathered errors by firstly recalling the most obvious errors that occur and secondly by analysing students' exam scripts to categorise them in a taxonomical manner.

1.2 Dewis e-Assessment System

Dewis is a fully algorithmic open-source e-Assessment system, which was primarily designed and developed for numerate e-Assessments by a team of Mathematicians, Statisticians and Software Engineers at UWE Bristol (Gwynllyw and Henderson, 2009; Gwynllyw and Henderson, 2012). Dewis supports different question input types such as numerical inputs, matrices, vectors, algebraic expressions, multiple-choice, multiple-selection, graphical input, and computer programs. It has a lossless data collection feature and a number of student-friendly features, such as shutdown recovery and pre-processing checks on student input.

Over the past decade, Dewis has been used very successfully to facilitate both formative and summative e-Assessments across a number of modules, delivered to students in a wide range of fields, e.g. Business, Computer Science, Nursing, Software Engineering, Engineering, Mathematics and Statistics. One aim of the CSE project is to enhance the full potential of Dewis, by developing and using additional features allowing Dewis to detect CSEs and to provide instant tailored feedback.

1.3 The Common Student Errors Project at UWE, Bristol

The CSE project at UWE began in 2017 with an aim of developing a technique to detect CSEs and to provide tailored feedback in Dewis e-Assessment questions, used in a first year Engineering Mathematics module (CSE Project at UWE, 2019; Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021). We started the project with the aim of answering the following research questions:

- What CSEs do first year Engineering Mathematics students make in e-Assessment questions?
- How to detect CSEs and improve Dewis feedback to address these CSEs?

There are several benefits to answering these research questions. Even though this research has been done in a particular context using the Dewis e-Assessment system, the research outcomes contribute to the knowledge to inform more general practice in assessment and learning. For example, the collection of mathematical CSEs collected during this research is not only beneficial for first year Engineering mathematics students and lecturers, but also it is equally beneficial for secondary, and first year university level mathematics students and teachers. The CSE collection presented in Sikurajapathi, Henderson and Gwynllyw (2022) can be used to correct students' mathematical misconceptions either in hand-written assessments or e-assessment questions.

Further, this CSE detecting technique will be beneficial to several disciplines and organisations that either use Dewis or any other e-assessment system which has features to give dynamic feedback based on a student answer. The new knowledge raised from this research can be used in any e-assessment system so that it emulates a human marker to provide instant enhanced feedback highlighting possible CSEs. This will help students to correct their mathematical misconceptions. Also, teachers can use the findings to identify areas in which more help is needed in student learning. Integrating the research outcomes from the CSE project into other e-assessment systems will be beneficial to generations to come (Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021; Sikurajapathi, Henderson and Gwynllyw, 2022).

The CSE Project involves five stages (Stage One: Data (CSEs) Collection; Stage Two: CSE code Development; Stage Three: CSE code Trial Phase; Stage Four: Students' Perceptions on CSE Feedback and Stage Five: Impact of CSE Project). Detailed information about these five stages and other findings can be found in CSE Project at UWE Bristol (2019), Sikurajapathi, Henderson and Gwynllyw (2020) and Sikurajapathi, Henderson and Gwynllyw (2021).

In this book, we only focus on Stage One: Data (CSEs) Collection, which provides an answer to the question 'What CSEs do first year Engineering Mathematics students make?

1.4 Common Student Error Collection

The CSEs presented in this booklet were collected by examining the 2017-2018 and 2018-2019 e-examination data on the Dewis e-Assessment system and from students' rough work scripts. These e-examinations were run using the Dewis e-Assessment system and were held under controlled conditions. The e-examinations were held in two sessions (morning and afternoon) to mitigate logistic issues. In each session, all of the students received the same, fixed parameter questions. During the e-examination, students were given booklets to use for their rough work. These booklets were used by students to work through the mathematical questions before submitting their final answers on Dewis.
Altogether 65 CSEs were identified in the following different topics areas of Engineering Mathematics:

- Algebra
- Unit-step function
- Wave forms

- Trigonometric functions
- Differentiation
- Implicit differentiation
- Partial differentiation
- Mean Value Theorem
- Complex numbers
- Geometric series
- Maclaurin Expansion
- Centre of Mass
- Integration by parts
- Volume of revolution
- Dimensions

1.5 Taxonomy of Mathematical Common Student Errors in e-Assessments

All of the CSEs found in the course of the CSE project are documented in a systematic order in the CSE book together with their mathematical taxonomy coding. Here we used the taxonomy coding described in Ford et al. (2018) as a guideline.

The theoretical study of classification, including its bases, principles, procedures and rules is called a taxonomy (Ford et al., 2018; Simpson, 1961, p.11). The entities in a successful taxonomy can be verifiable by observation and will offer both an appropriate and suitable class for each entity (Ford et al., 2018; Bailey, 1994, p.3). The taxonomy of cognitive mechanisms and the phenomenological taxonomy can be considered as the two main styles that can be used to categorise mathematical errors (Senders and Moray, 1991, Ford et al., 2018).

The taxonomy introduced by Ford et al. (2018) was developed to categorise the errors which undergraduate mathematics students make. Ford et al. (2018) identified six main error categories by firstly recalling obvious mathematical errors that occur among mathematics undergraduates and secondly by analysing a selection of students' paper-based exam scripts from first year undergraduate mathematics courses. These main categories were named as Errors of slips of action (S), Errors of understanding (U), Errors in choice of method (CM), Errors in the use of a method (UM), Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C).

The CSEs that we have found during the CSE project only fall into four of the error categories (S, U, CM and UM) from the Ford et al. (2018) taxonomy. Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C) were not found among the CSEs made by the Engineering Mathematics students, due to the nature of the questions asked and the nature of the system used to deliver the questions. None of the e-Assessment questions delivered by Dewis involve mathematical theorems and proofs and hence Errors related to proof (P) were not viable in this CSE collection. Further, the e-examination did not contain questions that required student's communication of their mathematical solutions, correct use of notation or labelling and qualitative judgements on clarity of expression. Therefore, errors in student's communication of their mathematical solutions (C) were not found in this CSE collection. Further, a few of the CSEs found fall into two categories due to the mix of misconceptions made by the students as they arrived at their incorrect answer.

Under the category Errors of slip of action (S), three main errors, namely copying error, careless errors on simple calculations, and incorrect algebraic manipulation were identified. A total of 13 out of 65 CSEs were found to fall into the Errors of slip of action category (S).

Seven main errors were identified under the Errors of understanding (U) category, such as confusing different mathematical structures, incorrect argument, lack of consideration of potential indeterminate forms, proposed solution is not viable, definition/method/theorem not recalled correctly, partial solution given and Incorrect assumptions. In total 45 CSEs are in the Errors of understanding category.

Only one main error was found in each of the Errors in choice of method (CM) and Errors in use of method (UM) categories. Three CSEs were grouped into the main error of applying an inappropriate formula/method/theorem in CM. There were 9 CSEs which fell into Error in use of an appropriate definition/method/theorem in the UM category. All the codes, errors and examples that we found in this CSE collection process are shown in Table 1.

Table 1: Taxonomy of Mathematical Common Student Errors in e-Assessments

Main Category	Code	Error	Examples
Slip of action	S1	Copying error	Incorrect copying of the question
			Mistake copying/ submitting answer into e-assessment
			Incorrect interpretation of the question
	S2	Careless errors on simple calculations	Overlooking negative signs
			Omission of denominator
	S3	Incorrect algebraic manipulation	Incorrect division of two complex numbers
			Sum of product is split as a product of two sums
			Incorrect handling of powers
Errors of understanding	U1	Confusing different mathematical structures	Confusing the structure of completing the square and the quadratic equation
			Stating that a unit step function is a number

	U2	Incorrect argument	Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives
			Taking the integration of the product of two functions as the product of individual integrals
	U3	Lack of consideration of potential indeterminate forms	Taking the square of a negative number to be negative
	U4	Proposed solution is not viable	Angle is not within the given range
	U5	Definition/method/theorem not recalled correctly	Method of completing the square is not recalled correctly
			Definition of waveform properties not recalled correctly
			Method of differentiating a standard function is not recalled correctly
			Method of solving trigonometry equation is not recalled correctly
			Chain rule is not recalled correctly
			Method of Partial differentiation not recalled correctly
			Method of differentiating implicit functions is not recalled correctly
			Mean value theorem is not recalled correctly
			Method of calculating the argument of a complex number is not recalled correctly
			Binomial theorem is incorrectly followed
			Definition of Centre of Mass is not recalled correctly
			Method of finding the principal value of the argument of a complex number is not recalled correctly
			Method of integrating not recalled correctly
			Definition of volume of revolution is not recalled correctly
	U6	Partial solution given	Correct workings but unfinished solution
	U7	Incorrect assumptions	Incorrect assumptions on the mean value theorem

			Taking dimension of velocity is $[v] = [MT^{-1}]$
Errors in choice of method	CM1	Applying an inappropriate formula/method/theorem	Uses a method which is not relevant in the situation
			Uses a formula which is not relevant in the situation
Errors in use of method	UM2	Error in use of an appropriate definition/method/theorem	Error in the use of the chain rule
			Error in use of partial differentiation method
			Incorrect units applied
			Method finding the volume of revolution is incorrectly followed

1.6 Guide to the CSE recording template

Each CSE found to date has been recorded using the template as shown in Figure 1. The template contains seven areas and each area and its contents are described in detail below.

① The link to the online Dewis e-assessment question is available here. The reader may access the online question by clicking the Question hyper-link. By attempting the question and answering with a relevant CSE response, it is possible to see how Dewis detects the CSE and provides instant tailored feedback to address the CSE made in the solution.

② In this area, a screenshot of the Dewis question is given.

③ The correct solution to the question is presented in brief here.

④ The taxonomy code of the CSE, which is presented in ⑤**,** is given here.

⑤ A sample of the CSE and the incorrect answer(s) that led from it is presented here. At the top of this area, the CSE error is summarised by a statement which is presented in red text. Then the detailed steps of the exact way the CSE is made and the solution as written by students in their rough work booklets is presented. We used tilde (~) on the CSE answer to differentiate it from the correct answer. For example, in Figure 1, the CSE answer for this question is denoted as, $\tilde{b} = -6$, in red text.

⑥ In this section, the number of CSE answers made, the total incorrect answers made in the question and the CSE percentage for each year are presented as No. of CSEs /No. incorrect answers **(CSE %)**. For example, in Figure 1, in 2017-18 exam, this particular CSE was made by 28 out of the 56 students who gave an incorrect answer to this question; therefore the CSE percentage is **50%**. This data is presented in this area as 28/56 **(50%)**. Similarly, the data for 2018-19 is presented as 33/57 (**58%**).

⑦ The exam year that data was collected from is presented here. Figure 1 shows that 28/56 **(50%) and** 33/57 (**58%**) presented in ⑥ relate to the years 2017-18 and 2018-19 presented in ⑦ respectively.

Question ①

The expression

$t^2 - 12t + 40$

can be expressed in the form:

$a(t-b)^2 + c$

where a, b and c are constants.

Calculate the values of these constants - note that all these solutions are integers:

Enter the value of a [ ]

Enter the value of b [ ]

Enter the value of c [ ]

②

Correct Solution

$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$

$$= (t-6)^2 + 4$$

$$a = 1, b = 6 \text{ and } c = 4$$

③

CSE 1 related to this question	**CSE Taxonomy Code:**	S1 ④

Give answer $\tilde{b}$ which corresponds to the negative of the correct value of b.

$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$

$$= (t-6)^2 + 4$$

$$\tilde{b} = -6 \text{ and } c = 4$$

⑤

No. of CSEs /No. incorrect answers **(CSE %)**	28/56 (**50%**) 33/57 (**58%**) ⑥	**Date collected**	2017-18 2018-19 ⑦

Figure 1: CSE Template

2 Common Student Errors due to Slip of Action

2.1 Copying Error

2.1.1 Algebra (Completing the Square)

Mistake copying/ submitting answer into e-assessment

Question		
The expression $t^2 - 12t + 40$ can be expressed in the form: $a(t-b)^2 + c$ where a, b and c are constants. Calculate the values of these constants - note that all these solutions are integers: Enter the value of a [ ] Enter the value of b [ ] Enter the value of c [ ]		
Correct Solution		
$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$ $= (t-6)^2 + 4$ $a = 1, b = 6$ and $c = 4$		
CSE 1 related to this question	**CSE Taxonomy Code**:	**S1**
Give answer $\tilde{b}$ which corresponds to the negative of the correct value of b. $t^2 - 12t + 40 = (t-6)^2 - 36 + 40$ $= (t-6)^2 + 4$ $\tilde{b} = -6$ and $c = 4$		
No. of CSEs /No. incorrect answers **(CSE %)**	28/56 (**50%**) 33/57 (**58%**)	**Date collected**: **2017-18** **2018-19**

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2.1.2 Binomial Series

Incorrect copying of the question

Question		
Use the binomial theorem to expand $(2x+3y)^5$ and enter the fourth term (that is the full term involving x^2y^3) as a function of x and y :		
Correct Solution		
When n is a positive integrer, the binomial theorem states that $$(ax+by)^n = \binom{n}{0}(ax)^n + \binom{n}{1}(ax)^{n-1}(by)^1 + \binom{n}{2}(ax)^{n-2}(by)^2 + \cdots + \binom{n}{n}(by)^n$$ So in this case, the fourth term is $$T_4 = \binom{5}{3}(2x)^{5-3}(3y)^{5-2}$$ $$= \frac{5!}{3! \times 2!}(2x)^2(3y)^3$$ $$= 1080x^2y^3$$		
CSE 1 related to this question	**CSE Taxonomy Code**:	**S1**
Submitting terms up, and including, the fourth term $$(2x+3y)^5 = \binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4(3y)^1 + \binom{5}{2}(2x)^3(3y)^2 + \binom{5}{3}(2x)^2(3y)^3 + \binom{5}{4}(2x)^1(3y)^4 + \binom{5}{5}(3y)^5$$ $$\widetilde{T}_4 = 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3$$		

No. of CSEs /No. incorrect answers **(CSE %)**	9/165**(5%)**	**Date collected**	**2018-19**

2.1.3 Maclaurin Expansion

Incorrect interpretation of the question

Question
Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, of $f(x) = e^{4x}$ up to and including the cubic term. Give the values of a_1 and a_2 below. Enter your answer for a_1 (to three decimal places) here: [ ] Enter your answer for a_2 (to three decimal places) here: [ ] Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$ Enter your approximate value for $e^{0.8}$ (to three decimal places) here: [ ]

Correct Solution

Using MacLaurin expansion,

$$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2!} + \frac{(4x)^3}{3!} \ldots$$

$$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$

$$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$

$$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$

CSE 3 related to this question	**CSE Taxonomy Code:**	**S1**

Submitted a_2 value for a_1 and a_3 value for a_2

$$\tilde{a_1} = \frac{4^2}{2!} = 8.000$$

$$\tilde{a_2} = \frac{4^3}{3!} = 10.667$$

No. of CSEs /No. incorrect answers **(CSE %)**	5/53**(9%)**	**Date collected**	**2018-19**

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2.1.4 Dimensions

Incorrect interpretation of question

Question

The quantity $K = \rho^2 r^{-1} v^{1.5}$

where ρ represents density, r represents distance, v represents velocity.

The dimensions of K are given by $[M^a L^b T^c]$. Calculate the values of a, b and c.

Enter a:
Enter b:
Enter c:

Correct Solution

$$K = \rho^2 r^{-1} v^{1.5}$$

$$[K] = [ML^{-3}]^2 [L]^{-1} [LT^{-1}]^{1.5}$$

$$= [M]^2 [L]^{-6-1+1.5} [T]^{-1.5}$$

$$= [M]^2 [L]^{-5.5} [T]^{-1.5}$$

$$a = 2, b = -5.5, c = -1.5$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**S1**

Submitted

a = the power of ρ in the question

b = the power of r in the question

c = the power of v in the question

$$K = \rho^2 r^{-1} v^{1.5}$$

$$a = 2, b = -1, c = 1.5$$

No. of CSEs /No. incorrect answers **(CSE %)**	9/46 **(20%)**	**Date collected**	**2018-19**

2.1.5 Dimensions

Copying error

Question

The quantity $K = \rho^2 r^{-1} v^{1.5}$

where ρ represents density, r represents distance, v represents velocity.

The dimensions of K are given by $[M^a L^b T^c]$. Calculate the values of a, b and c.

Enter a:

Enter b:

Enter c:

Correct Solution

$$K = \rho^2 r^{-1} v^{1.5}$$

$$[K] = [ML^{-3}]^2[L]^{-1}[LT^{-1}]^{1.5}$$

$$= [M]^2[L\ ]^{-6-1+1.5}[T]^{-1.5}$$

$$= [M]^2[L]^{-5.5}[T]^{-1.5}$$

$$a = 2, b = -5.5, c = -1.5$$

CSE 3 related to this question	**CSE Taxonomy Code**:	**S1**

Omitted the powers of ρ, r and v

$$[K] = [ML^{-3}][L][LT^{-1}]$$

$$= [M][L]^{-3+1+1}[T]^{-1}$$

$$= [M]^1[L]^{-1}[T]^{-1}$$

$$\tilde{a} = 1, \tilde{b} = -1, \tilde{c} = -1$$

No. of CSEs /No. incorrect answers **(CSE %)**	8/20 **(40%)**	**Date collected**	**2018-19**

2.2 Careless Errors on Simple Calculations

2.2.1 Complex Numbers (Argument of $z = a - bj$)

Overlooking negative sign

Question			
Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \leq \pi$. Express your answers correct to three decimal places. Enter r: [ ] Enter θ: [ ]			
Correct Solution			
$z = 2 - 3j$ $r = \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{13}$ $r = 5.39$ z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$ $= -0.983$			
CSE 1 related to this question		**CSE Taxonomy Code**:	S2
Taking $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ *when* $z = a - bj$ *(missing the negative sign)* $\tilde{\theta} = \tan^{-1}\left(\frac{3}{2}\right)$ $= 0.983$			
No. of CSEs /No. incorrect answers **(CSE %)**	19/88**(22%)**	**Date collected**	**2018-19**

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2.2.2 Complex Numbers (Argument of $z = a - bj$)

Overlooking negative sign and incorrect units applied

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \leq \pi$.

Express your answers correct to <u>three</u> decimal places.

Enter r: [ ]
Enter θ: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z = 2 - 3j$
$r = \sqrt{(2)^2 + (-3)^2}$
$= \sqrt{13}$
$r = 5.39$
z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$
$= -0.983$</td></tr>
<tr><td>CSE 4 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>S2, UM2</td></tr>
<tr><td colspan="4">Calculating $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ in degrees (and missing a negative sign) when $z = a - bj$

$\tilde{\theta} = \tan^{-1}\left(\frac{3}{2}\right)$
$= 56.31^o$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>11/88(13%)</td><td>Date collected</td><td>2018-19</td></tr>
</table>

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2.2.3 Series (Maclaurin Expansion)

Omission of denominator / correct working and unfinished solution is given

<table>
<tr><td colspan="5">Question</td></tr>
<tr><td colspan="5">Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, of $f(x) = e^{4x}$ up to and including the cubic term.

Give the values of a_1 and a_2 below.

Enter your answer for a_1 (to three decimal places) here: [ ]

Enter your answer for a_2 (to three decimal places) here: [ ]

Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$

Enter your approximate value for $e^{0.8}$ (to three decimal places) here: [ ]</td></tr>
<tr><td colspan="5">Correct Solution</td></tr>
<tr><td colspan="5">Using MacLaurin expansion,

$$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6} \dots$$
$$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$
$$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$
$$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$</td></tr>
<tr><td colspan="2">CSE 2 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>S2, U6</td></tr>
<tr><td colspan="5">Missing division by 2!

$$\widetilde{a_2} = 4^2 = 16.000$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td colspan="2">11/59(19%)
16/80(20%)</td><td>Date collected</td><td>2017-18
2018-19</td></tr>
</table>

2.2.4 Integration ($\int x\cos(ax)\,dx$)

Overlooking negative sign

Question
Evaluate the following: $$\int x\cos(3x)dx$$ as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:

Correct Solution

Use integration by parts

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Take

$$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$

$$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$

$$I = \int x\cos(3x) = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3}dx$$

$$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 2 related to this question	CSE Taxonomy Code:	S2

$$Taking \int sin(3x) = \frac{cos(3x)}{3} \ (missing\ a\ negative\ sign)$$

$$I = \int xcos(3x)dx = x\left(\frac{sin(3x)}{3}\right) - \int \frac{sin(3x)}{3}dx$$

$$\tilde{I} = \frac{sin(3x)}{3} - \frac{cos(3x)}{9} + c$$

No. of CSEs /No. incorrect answers (CSE %)	13/143 **(9%)**	**Date collected**	**2017-18**

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2.3 Incorrect Algebraic Manipulation

2.3.1 Complex Numbers (Real and Imaginary parts of $z = \frac{a+bj}{c+dj}$)

Incorrect division of two complex numbers

<table>
<tr><td colspan="3">Question</td></tr>
<tr><td colspan="3">Find the real and imaginary parts of
$z = \frac{2-7j}{3+5j}$
correct to three decimal places.
Enter $Re(z)$: [ ]
Enter $Im(z)$: [ ]</td></tr>
<tr><td colspan="3">Correct Solution</td></tr>
<tr><td colspan="3">$$z = \frac{2-7j}{3+5j}$$ $$z = \frac{(2-7j)(3-5j)}{(3+5j)(3-5j)}$$ $$z = \frac{-29-31j}{34}$$ $$Re(z) = \frac{-29}{34} = -0.853, \quad Im(z) = \frac{-31}{34} = -0.912$$</td></tr>
<tr><td>CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>S3</td></tr>
<tr><td colspan="3">Consider $z = \frac{a+bj}{c+dj}$
$$Re(z) = \frac{a}{c}, Im(z) = \frac{b}{d}$$ $$\widetilde{Re(z)} = \frac{2}{3} = 0.667, \quad \widetilde{Im(z)} = \frac{-7}{5} = -1.4$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>9/109 (8%)
9/95(9%)</td><td>Date collected — 2017-18
2018-19</td></tr>
</table>

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2.3.2 Centre of Mass

Sum of product is split as a product of two sums

Question

Masses of 4 kg, 6 kg and 10 kg are located at points with co-ordinates (-2,5) , (1,-4) and (3,1) respectively.

Find the co-ordinates of their Centre of Mass,$(\bar{x}, \bar{y})$, correct to one decimal place.

Enter $\bar{x}$: [ ]

Enter $\bar{y}$: [ ]

Correct Solution

$$\bar{x} = \frac{\sum_{i=1}^{3} m_i\, x_i}{\sum_{i=1}^{3} m_i} = \frac{4 \times (-2) + 6 \times 1 + 10 \times 3}{(4 + 6 + 10)} = 1.4$$

$$\bar{y} = \frac{\sum_{i=1}^{3} m_i\, y_i}{\sum_{i=1}^{3} m_i} = \frac{4 \times 5 + 6 \times (-4) + 10 \times 1}{(4 + 6 + 10)} = 0.3$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**S3**

Taking

$$\sum_{i=1}^{n} m_i x_i = \sum_{i=1}^{n} m_i \sum_{i=1}^{n} x_i$$

and $$\sum_{i=1}^{n} m_i y_i = \sum_{i=1}^{n} m_i \sum_{i=1}^{n} y_i$$

$$\tilde{\bar{x}} = \frac{(4 + 6 + 10) \times (-2 + 1 + 3)}{(4 + 6 + 10)} = 2$$

$$\tilde{\bar{y}} = \frac{(4 + 6 + 10) \times (5 - 4 + 1\ )}{(4 + 6 + 10)} = 2$$

No. of CSEs /No. incorrect answers **(CSE %)**	7/28 **(25%)** 12/64**(19%)**	**Date collected**	**2017-18** **2018-19**

2.3.3 Integration Applications (Volume of Revolution)

Incorrect handling of powers

Question

Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]

Correct Solution

The volume of revolution of y over the range $a < x < b$ given by

$$V = \pi \int_a^b y^2 dx$$

For given data,

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$= \pi \int_1^4 0.64\, x^3 dx$$

$$= 128.18$$

CSE 4 related to this question	**CSE Taxonomy Code**:	**S3**

Taking $(x^p)^q = x^{p^q}$

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$\tilde{V} = \pi \int_1^4 0.8^2\, x^{1.5^2} dx$$

$$\tilde{V} = \pi \int_1^4 0.64\, x^{2.25} dx$$

$$\tilde{V} = 0.64\, \pi \left[\frac{x^{3.25}}{3.25}\right]_1^4 = 55.38$$

No. of CSEs /No. incorrect answers **(CSE %)**	7/107 **(7%)**	**Date collected**	**2017-18**

2.3.4 Integration Applications (Volume of Revolution)

Incorrect handling of powers

Question

Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]

Correct Solution

The volume of revolution of y over the range $a < x < b$ given by

$$V = \pi \int_a^b y^2 dx$$

For given data,

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$= \pi \int_1^4 0.64\, x^3 dx$$

$$= 128.18$$

CSE 6 related to this question	**CSE Taxonomy Code**:	**S3**

Taking $(x^p)^q = x^{p+q}$

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$\tilde{V} = \pi \int_1^4 0.8^2\, x^{1.5+2} dx$$

$$\tilde{V} = 0.64\, \pi \left[\frac{x^{4.5}}{4.5}\right]_1^4$$

$$= 228.32$$

No. of CSEs /No. incorrect answers **(CSE %)**	6/135 **(4%)**	**Date collected**	**2018-19**

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3 Common Student Errors due to Errors of Understanding

3.1 Confusing Different Mathematical Structures

3.1.1 Algebra (Completing the Square)

Confusing the structure of completing the square and the quadratic equation

<table>
<tr><td colspan="4"><u>Question</u></td></tr>
<tr><td colspan="4">The expression
$t^2 - 12t + 40$
can be expressed in the form:
$a(t-b)^2 + c$
where a, b and c are constants.
Calculate the values of these constants - note that all these solutions are integers:
Enter the value of a [ ]
Enter the value of b [ ]
Enter the value of c [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$ $$= (t-6)^2 + 4$$ $$a = 1, b = 6 \text{ and } c = 4$$</td></tr>
<tr><td colspan="2">CSE 3 related to this question</td><td>CSE Taxonomy Code:</td><td>U1</td></tr>
<tr><td colspan="4">Incorrectly assign b and c to be the t and constant coefficients of the original expression.
$$t^2 - 12t + 40$$ $$\tilde{b} = 12$$ $$\tilde{c} = 40$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>4/26 (15%)
2/24 (8%)</td><td>Date collected</td><td>2017-18
2018-19</td></tr>
</table>

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3.1.2 Functions (Unit-Step Function)

Stating that a unit step function is a number

Question

The function $f(t) = 7u(t+5) - 3u(t-4)$

where $u(t)$ represents the unit step function.

Calculate the value of $f(2)$.

Enter $f(2)$: [ ]

Correct Solution

$$f(t) = 7u(t+5) - 3u(t-4)$$
$$f(2) = 7u(2+5) - 3u(2-4)$$
$$= 7u(7) - 3u(-2)$$
$$= 7 \times 1 - 3 \times 0$$
$$f(2) = 7$$

CSE 1 related to this question	CSE Taxonomy Code:	U1

Answer was derived by assuming $u = 1$ and not a function.

$$f(t) = 7u(t+5) - 3u(t-4)$$
$$f(2) = 7u(7) - 3u(-2)$$
$$\tilde{f}(2) = 7(7)u - 3(-2)u$$
$$\tilde{f}(2) = 49u + 6u$$
$$\tilde{f}(2) = 55u \quad \text{since } u = 1$$
$$\tilde{f}(2) = 55$$

No. of CSEs /No. incorrect answers (CSE %)	35/86(**41%**) 32/100(**32%**)	**Date collected**	**2017-18** **2018-19**

3.2 Incorrect Argument

3.2.1 Differentiation ($e^{ax}\ sin(bx)$)

Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives

Question

Select the most appropriate method to use in order to find the derivative of $f(x) = e^{2x}\sin(5x)$.

Select

Hence find $\dfrac{df}{dx}$ as a function of x.

Enter the answer as a function of x:

Correct Solution

Use the product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$$u(x) = e^{2x} \text{ and } v(x) = \sin(5x)$$
$$u'(x) = 2e^{2x} \text{ and } v'(x) = 5\cos(5x)$$
$$f'(x) = e^{2x} \times 5\cos(5x) + \sin(5x) \times 2e^{2x}$$
$$f'(x) = e^{2x}(2\sin(5x) + 5\cos(5x))$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U2**

Taking $f'(x) = u'v'\ when\ f(x) = uv$

$$f(x) = e^{2x}\sin(5x)$$
$$u(x) = e^{2x} \text{ and } v(x) = \sin(5x)$$
$$u'(x) = 2e^{2x} \text{ and } v'(x) = 5\cos(5x)$$
$$\tilde{f}'(x) = 2e^{2x} \times 5\cos(5x)$$
$$\tilde{f}'(x) = 10e^{2x}\cos(5x)$$

No. of CSEs /No. incorrect answers **(CSE %)**	13/73**(18%)**	**Date collected**	**2018-19**

Question			
Given $x^4+2x^2y^3=5y$, find the derivative $\frac{dy}{dx}$ as a function of x and y. $\frac{dy}{dx}=$			
Correct Solution			
$x^4+2x^2y^3=5y$ $4x^3+2x^2\times 3y^2\frac{dy}{dx}+4x\times y^3=5\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{4x^3+4xy^3}{5-6x^2y^2}$			
CSE 1 related to this question	**CSE Taxonomy Code**:	**U2**	
Taking $\frac{d}{dx}(f(x)g(y))=\frac{d}{dx}(f(x))\times\frac{d}{dx}(g(y))$ $x^4+2x^2y^3=5y$ $4x^3+4x\times 3y^2\frac{dy}{dx}=5\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{4x^3}{5-12xy^2}$			
No. of CSEs /No. incorrect answers **(CSE %)**	18/158 **(11%)** 15/180 **(8%)**	**Date collected**	**2017-18** **2018-19**

3.2.3 Integration ($\int xe^{ax^n} dx$)

Taking the integration of the product of two functions as the product of individual integrals

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Evaluate the following:
$\int xe^{4x^2} dx$
as a function of x, to within an additive constant (i.e. do not put a "+c" in your answer). Enter the answer as a function of x:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">Use the method of substitution and put $u = 4x^2$, then $\frac{du}{dx} = 8x$
$I = \int xe^{4x^2} dx = \int \frac{1}{8} e^u du$
$= \frac{1}{8} e^u + c$
$= \frac{1}{8} e^{4x^2} + c$</td></tr>
<tr><td>CSE 3 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>U2</td></tr>
<tr><td colspan="4">Taking $\int u(x)v(x)dx = \int u(x)dx \times \int v(x)dx$ and $\int e^{ax^2} dx = \frac{e^{ax^2}}{a}$
$\tilde{I} = \int xe^{4x^2} dx = \int x\,dx \times \int e^{4x^2} dx$
$= \frac{x^2}{2} \times \frac{e^{4x^2}}{4}$
$= \frac{x^2 e^{4x^2}}{8}$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>10/192(5%)</td><td>Date collected</td><td>2018-19</td></tr>
</table>

3.2.4 Integration ($\int x \cos(ax)\, dx$)

Taking the integration of the product of two functions as the product of individual integrals

Question

Evaluate the following:

$$\int x \cos(3x) dx$$

as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:

Correct Solution

Use integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Take

$$u(x) = x, \quad \frac{dv}{dx} = \cos(3x)$$

$$\frac{du}{dx} = 1, \quad v(x) = \frac{\sin(3x)}{3}$$

$$I = \int x \cos(3x) = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3} dx$$

$$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 4 related to this question	**CSE Taxonomy Code**:	**U2**

Integrating x and the trigonometric function separately, and then multiplying them together to get the answer

$$I = \int x \cos(3x) = \left(\frac{x^2}{2}\right) \times \left(\frac{\sin(3x)}{3}\right) + c$$

$$\tilde{I} = \frac{x^2 \sin(3x)}{6} + c$$

No. of CSEs /No. incorrect answers **(CSE %)**	8/143 **(6%)**	**Date collected**	**2017-18**

3.3 Lack of Consideration of Potential Indeterminate Forms

3.3.1 Complex Numbers (Modulus of $z = -a + bj$)

Taking the square of a negative number to be negative

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Find the modulus $|z|$ of the complex number $z = -2 + 5j$, correct to two decimal places.

Enter $|z|$ correct to 2 decimal places: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z = -2 + 5j$
$|z| = \sqrt{(-2)^2 + 5^2}$
$= \sqrt{4 + 25}$
$= \sqrt{29}$
$|z| = 5.39$</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>U3</td></tr>
<tr><td colspan="4">Taking $(-n)^2 = -n^2$
$z = -2 + 5j$
$|z| = \sqrt{(-2)^2 + 5^2}$
$|z| = \sqrt{-4 + 25}$
$= \sqrt{21}$
$|z| = 4.58$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>40/57(70%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

3.4 Proposed Solution is not Viable

3.4.1 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Correct working but unfinished solutions and angles is not in the given range

Question		
Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of $z = \frac{(w_1)^3}{(w_2)^2}$, where $-\pi < \theta \leq \pi$. Enter your answers correct to three decimal places. Enter $r = \lvert z \rvert$: ☐ Enter $\theta = arg(z)$: ☐		
Correct Solution		
$z = \frac{(w_1)^3}{(w_2)^2}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times0.8-2\times-0.4)}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{3.2j}$ $\lvert z \rvert = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\arg(z) = 3.2 - 2\pi = -3.083$		
CSE 1 related to this question	**CSE Taxonomy Code:**	**U4, U6**
Neglecting the required range of $arg(z)$ $z = \frac{(1.8)^3}{(1.2)^2} e^{3.2j}$ $\lvert z \rvert = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\widetilde{arg(z)} = 3.2$		
No. of CSEs /No. incorrect answers **(CSE %)**	66/ 197**(34%)**	**Date collected** — **2017-18**

3.5 Definition/Method/Theorem not Recalled Correctly

3.5.1 Algebra (Completing the Square

Method of completing the square is not recalled correctly

Question
The expression $t^2 - 12t + 40$ can be expressed in the form: $a(t-b)^2 + c$ where a, b and c are constants. Calculate the values of these constants - note that all these solutions are integers: Enter the value of a [ ] Enter the value of b [ ] Enter the value of c [ ]
Correct Solution
$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$ $= (t-6)^2 + 4$ $a = 1, b = 6$ and $c = 4$

CSE 2 related to this question	**CSE Taxonomy Code:**	**U5**
Incorrectly add b^2 instead of subtracting b^2 when completing the square. $t^2 - 12t + 40 = (t-6)^2 + 36 + 40$ $= (t-6)^2 + 76$ $\tilde{c} = 76$		

No. of CSEs /No. incorrect answers **(CSE %)**	6/45 (**13%**) 14/51(**27%**)	**Date collected**	**2017-18** **2018-19**

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3.5.2 Wave Forms $y = A\sin(\alpha t)$

Definition of waveform properties not recalled correctly

Question			
Consider the waveform $y = 14\sin(8t)$. State the amplitude of this waveform and the number of cycles in a interval of length 2π of the waveform. Enter the amplitude: [ ] Enter the number of cycles in a 2π interval: [ ]			
Correct Solution			
$y = 14\sin(8t)$ The amplitude $A = 14$ The number of cycles $\omega = 8$			
CSE 1 related to this question		**CSE Taxonomy Code:**	**U5**
Swapped the correct answers for the amplitude and the number of cycles. $y = 14\sin(8t)$ The amplitude $\tilde{A} = 8$ The number of cycles $\tilde{\omega} = 14$			
No. of CSEs /No. incorrect answers **(CSE %)**	5/13(**38%**) 9/19(**47%**)	**Date collected**	**2017-18** **2018-19**

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3.5.3 Wave Forms $y = A\sin(\alpha t)$

Definition of waveform properties not recalled correctly

Question			
Consider the waveform $y = 14\sin(8t)$. State the amplitude of this waveform and the number of cycles in a interval of length 2π of the waveform. Enter the amplitude: [ ] Enter the number of cycles in a 2π interval: [ ]			
Correct Solution			
$y = 14\sin(8t)$ The amplitude $A = 14$ The number of cycles $\omega = 8$			
CSE 2 related to this question	**CSE Taxonomy Code:**	**U5**	
Taking the number of cycles in a π interval to be ω The number of cycles in a π interval = 8 Therefore, the number of cycles in 2π interval $\tilde{\omega}$ = 16			
No. of CSEs /No. incorrect answers **(CSE %)**	7/29(**24%**)	**Date collected**	**2018-19**

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3.5.4 Trigonometric Function $\sin(\theta) = A$

Method of solving a trigonometric equation not recalled correctly

Question
Find the two values of θ in the interval $-180^\circ < \theta \leq 180^\circ$ such that $\sin(\theta) = -0.55$. Enter your answers to θ_1 and θ_2 **in degrees**, correct to **two decimal places**. Enter θ_1 : [ ] to two decimal places Enter θ_2 : [ ] to two decimal places *(Please note that the order in which you enter your answers does not matter.)*
Correct Solution
$\sin(\theta) = -0.55$ $\theta_1 = \sin^{-1}(-0.55)$ $\theta_1 = -33.37^0$ $\theta_2 = -180^0 + 33.37^0$ $\theta_2 = -146.63^0$

CSE 1 related to this question	**CSE Taxonomy Code:**	**U5**
Add 180^0 to θ_1 to find θ_2 $\sin(\theta) = -0.55$ $\theta_1 = \sin^{-1}(-0.55)$ $\theta_1 = -33.37^0$ $\tilde{\theta}_2 = -33.37^0 + 180^0$ $\tilde{\theta}_2 = 146.63^0$		

No. of CSEs /No. incorrect answers **(CSE %)**	37/61 **(61 %)**	**Date collected**	**2017-18**

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3.5.5 Trigonometric Function $\sin(2\theta) = A$

Method of solving a trigonometric equation not recalled correctly

Question			
Solve the equation $\sin(2t) = 0.45$ for t, where $0 \leq t \leq \pi$. Enter your two values of t **in radians**, correct to **two decimal places**. Enter t_1 : [ ] to two decimal places Enter t_2 : [ ] to two decimal places			
Correct Solution			
Let $z = 2t$, then the problem is equivalent to solving the equation: $\sin(z) = 0.45$ for z, where $0 \leq z \leq 2\pi$ $z_1 = \sin^{-1}(0.45)$ and $z_2 = \pi - \sin^{-1}(0.45)$ $z_1 = 0.44477$ and $z_2 = 2.67483$ $t_1 = 0.23$ and $t_2 = 1.34$			
CSE 1 related to this question	**CSE Taxonomy Code:**	**U5**	
Taking $t_2 = \pi - t_1$ $\sin(2t) = 0.45$ $t_1 = \frac{\sin^{-1}(0.45)}{2} = 0.23$ $\tilde{t_2} = \pi - t_1 = \pi - 0.23 = 2.91$			
No. of CSEs /No. incorrect answers **(CSE %)**	47/81 **(58 %)**	**Date collected**	**2018-19**

3.5.6 Differentiation ($ln(ax)$)

Method of differentiating a standard function is not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Obtain the derivative of the function
$f(x) = 3\ln(5x)$
Enter the answer as a function of x:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$f(x) = 3\ln(5x)$$ $$f'^{(x)} = 3 \times \frac{1}{5x} \times 5$$ $$f'(x) = \frac{3}{x}$$</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>U5</td></tr>
<tr><td colspan="4">Taking the differential of $ln(ax)$ to be $\frac{1}{ax}$
$$f(x) = 3\ln(5x)$$ $$f'(x) = 3 \times \frac{1}{5x}$$ $$f'(x) = \frac{3}{5x}$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>30/59 (51%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

3.5.7 Differentiation ($ln(ax)$)

Method of differentiating a standard function is not recalled correctly

Question

Obtain the derivative of the function

$f(x) = 8\ \ln(4x) - 3x^2$

and find the value of this derivative when $x = 0.6$.

Enter $f'(0.6)$, correct to two decimal places: [ ]

Correct Solution

$$f(x) = 8\ln(4x) - 3x^2$$

$$f'(x) = 8 \times \frac{1}{4x} \times 4 - 6x$$

$$f'(x) = \frac{8}{x} - 6x$$

$$f'(0.6) = \frac{8}{0.6} - 6(0.6)$$

$$f'(0.6) = 9.73$$

CSE 1 related to this question | **CSE Taxonomy Code:** | **U5**

Taking the derivative of $ln(ax)$ to be $\frac{1}{ax}$

$$f(x) = 8\ln(4x) - 3x^2$$

$$\tilde{f}'(x) = 8 \times \frac{1}{4x} - 6x$$

$$\tilde{f}'(x) = \frac{2}{x} - 6x$$

$$\tilde{f}'(x) = \frac{2}{0.6} - 6(0.6)$$

$$\tilde{f}'(x) = -0.27$$

No. of CSEs /No. incorrect answers (CSE %)	30/59 (51%) 7/33 (21%)	Date collected	2017-18[1] 2018-19

[1] In 2017/18 the question asked for an algebraic entry (as opposed) to numerical but CSE was the same.

3.5.8 Differentiation ($ln(ax)$)

Method of differentiating a standard function not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Obtain the derivative of the function
$f(x) = 8\ \ln(4x) - 3x^2$
and find the value of this derivative when $x = 0.6$.
Enter $f'(0.6)$, correct to two decimal places: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$f(x) = 8\ln(4x) - 3x^2$
$f'(x) = 8 \times \frac{1}{4x} \times 4 - 6x$
$f'(x) = \frac{8}{x} - 6x$
$f'(0.6) = \frac{8}{0.6} - 6(0.6)$
$f'(0.6) = 9.73$</td></tr>
<tr><td colspan="2">CSE 2 related to this question</td><td>CSE Taxonomy Code:</td><td>U5</td></tr>
<tr><td colspan="4">Taking the derivative of $\ln(ax)$ to be $\frac{a}{x}$
$f(x) = 8\ln(4x) - 3x^2$
$f'(x) = 8 \times \frac{4}{x} - 6x$
$f'(x) = \frac{32}{x} - 6x$
$f'(x) = \frac{32}{0.6} - 6(0.6)$
$f'(x) = 49.73$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>5/33 (15%)</td><td>Date collected</td><td>2018-19</td></tr>
</table>

41

3.5.9 Differentiation ($cos(ax^n)$)

Chain rule is not recalled correctly

Question
Select the most appropriate method to use in order to find the derivative of $f(x) = \cos(2x^7)$. Select ▾ Hence find $\dfrac{df}{dx}$ as a function of x. Enter the answer as a function of x:
Correct Solution
$$f(x) = \cos(2x^7)$$ $$f'(x) = -\sin(2x^7) \times 14x^6$$ $$f'(x) = -14x^6 \sin(2x^7)$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**

Taking $\frac{d}{dx}(cos(ax^n)) = -a\ sin(ax^n)$ *;*

overgeneralising $\frac{d}{dx}(cos(ax)) = -a\ sin(ax)$

$$f(x) = \cos(2x^7)$$

$$f'(x) = -2\sin(2x^7)$$

No. of CSEs /No. incorrect answers **(CSE %)**	7/56 **(13%)**	**Date collected**	**2017-18**

3.5.10 Differentiation ($cos^n(ax)$)

Chain rule is not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Select the most appropriate method to use in order to find the derivative of $f(x) = \cos^4(3x)$.

Select

Hence find $\frac{df}{dx}$ as a function of x.

Enter the answer as a function of x:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$f(x) = cos^4(3x)$
$f'(x) = -4 \times \cos^3(3x) \times \sin(3x) \times 3$
$f'(x) = -12\sin(3x)\cos^3(3x)$</td></tr>
<tr><td>CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td colspan="2">U5</td></tr>
<tr><td colspan="4">Taking $\frac{d}{dx}(cos^n(ax)) = -a\ sin^n(ax)$;
overgeneralising $\frac{d}{dx}(cos(ax)) = -a\ sin(ax)$

$f(x) = cos^4(3x)$
$f'(x) = -\sin^4(3x) \times 3$
$f'(x) = -3\sin^4(3x)$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>6/73(8%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

43

3.5.11 Implicit Differentiation

Method of differentiating implicit functions is not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Given
$x^4 + 2x^2y^3 = 5y,$
find the derivative $\frac{dy}{dx}$ as a function of x and y.
$\frac{dy}{dx} =$ [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$x^4 + 2x^2y^3 = 5y$$
$$4x^3 + 2x^2 \times 3y^2\frac{dy}{dx} + 4x \times y^3 = 5\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{4x^3 + 4xy^3}{5 - 6x^2y^2}$$</td></tr>
<tr><td>CSE 2 related to this question</td><td>CSE Taxonomy Code:</td><td colspan="2">U5</td></tr>
<tr><td colspan="4">Differentiating functions of x with respect to x and functions of y with respect to y, separately. Equating the answer to $\frac{dy}{dx}$
$$x^4 + 2x^2y^3 = 5y$$
$$4x^3 + 4x \times 3y^2 = 5$$
$$\widetilde{\frac{dy}{dx}} = 4x^3 + 4x \times 3y^2 - 5$$
$$\widetilde{\frac{dy}{dx}} = 4x^3 + 12xy^2 - 5$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>12/158 (8%)
5/180(3%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

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3.5.12 Implicit Differentiation

Method of differentiating implicit functions is not recalled correctly

Question		
Given $x^4 + 2x^2y^3 = 5y,$ find the derivative $\frac{dy}{dx}$ as a function of x and y. $\frac{dy}{dx} =$		
Correct Solution		
$x^4 + 2x^2y^3 = 5y$ $4x^3 + 2x^2 \times 3y^2\frac{dy}{dx} + 4x \times y^3 = 5\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{4x^3 + 4xy^3}{5 - 6x^2y^2}$		
CSE 3 related to this question	**CSE Taxonomy Code**:	**U5**
Taking y to be a constant when differenting with respect to x. *Equating the answer to* $\frac{dy}{dx}$ $x^4 + 2x^2y^3 = 5y$ $4x^3 + 4x \times y^3$ Therefore, $\frac{dy}{dx} = 4x^3 + 4xy^3$		

No. of CSEs /No. incorrect answers **(CSE %)**	10/158 **(6%)** 2/180 **(1%)**	**Date collected**	**2017-18** **2018/19**

45

3.5.13 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x, y) = acos(xy^n)$)

Method of partial differentiation not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Given
$z(x, y) = 3\cos(xy^4)$
find $\frac{\partial z}{\partial x}$.
Enter your answer below as a function of x and y :</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z(x, y) = 3\cos(xy^4)$
$\frac{\partial z}{\partial x} = -3\sin(xy^4) \times y^4$
$\frac{\partial z}{\partial x} = -3y^4\sin(xy^4)$</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>U5</td></tr>
<tr><td colspan="4">Missing the partial derivative of xy^n with respect to x
$z(x, y) = 3\cos(xy^4)$
$\frac{\partial z}{\partial x} = -3\sin(xy^4)$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>17/ 103 (17%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

46

3.5.14 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x,y) = a\cos(xy^n)$)

Method of partial differentiation not recalled correctly

Question

Given

$z(x,y) = 3\cos(xy^4)$

find $\dfrac{\partial z}{\partial x}$.

Enter your answer below as a function of x and y :

Correct Solution

$$z(x,y) = 3\cos(xy^4)$$

$$\frac{\partial z}{\partial x} = -3\sin(xy^4) \times y^4$$

$$\frac{\partial z}{\partial x} = -3y^4 \sin(xy^4)$$

CSE 2 related to this question	CSE Taxonomy Code:	U5

Taking $\frac{\partial}{\partial x}(a\cos(xy^n)) = -a\sin(xy^n) \times ny^{n-1}$

$$z(x,y) = 3\cos(xy^4)$$

$$\frac{\widetilde{\partial z}}{\partial x} = -3\sin(xy^4) \times 4y^3$$

$$\frac{\widetilde{\partial z}}{\partial x} = -12y^3 \sin(xy^4)$$

No. of CSEs /No. incorrect answers (CSE %)	7/ 103 (7%)	Date collected	2017-18

3.5.15 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x, y) = acos(xy^n)$)

Method of partial differentiation not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Given
$z(x, y) = 3\cos(xy^4)$
find $\frac{\partial z}{\partial x}$.
Enter your answer below as a function of x and y :</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$z(x, y) = 3\cos(xy^4)$$ $$\frac{\partial z}{\partial x} = -3\sin(xy^4) \times y^4$$ $$\frac{\partial z}{\partial x} = -3y^4\sin(xy^4)$$</td></tr>
<tr><td>CSE 3 related to this question</td><td>CSE Taxonomy Code:</td><td colspan="2">U5</td></tr>
<tr><td colspan="4">Taking $\frac{\partial}{\partial x}(a\cos(xy^n)) = -a\sin(y^n)$
$$z(x, y) = 3\cos(xy^4)$$ $$\frac{\partial z}{\partial x} = -3\sin(y^4)$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>6/ 103 (6%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

48

3.5.16 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x, y) = acos(x^n y^m)$)

Method of partial differentiation not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Given
$z(x,y) = 3\cos(x^2y^4)$
find $\frac{\partial z}{\partial x}$.
Enter your answer below as a function of x and y :</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z(x,y) = 3\cos(x^2y^4)$
$\frac{\partial z}{\partial x} = -3\sin(x^2y^4) \times 2xy^4$
$= -6xy^4\sin(x^2y^4)$</td></tr>
<tr><td>CSE 1 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>U5</td></tr>
<tr><td colspan="4">Taking $\frac{\partial(a\cos(x^n y^m))}{\partial x} = -a\sin(x^n y^m) \times nx^{n-1} = -anx^{n-1}\sin(x^n y^m)$
$z(x,y) = 3\,cos(x^2y^4)$
$\frac{\partial z}{\partial x} = -3 \times sin(x^2y^4) \times 2x$
$= -6x\,sin(x^2y^4)$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>32/ 145 (22%)</td><td>Date collected</td><td>2018-19</td></tr>
</table>

3.5.17 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x,y) = acos(xy^n)$)

Method of partial differentiation not recalled correctly

Question			
Given $z(x,y) = 3\cos(x^2y^4)$ find $\frac{\partial z}{\partial x}$. Enter your answer below as a function of x and y :			
Correct Solution			
$z(x,y) = 3\cos(x^2y^4)$ $\frac{\partial z}{\partial x} = -3\sin(x^2y^4) \times 2xy^4$ $= -6xy^4 \sin(x^2y^4)$			
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**	
Missing the partial differential of x^ny^m with respect to x $z(x,y) = 3\cos(x^2y^4)$ $\frac{\partial z}{\partial x} = -3\sin(x^2y^4)$			
No. of CSEs /No. incorrect answers **(CSE %)**	11/ 145 **(8%)**	**Date collected**	**2018-19**

50

3.5.18 Integration Application (Mean Value of $(at^n + b)$)

Mean value theorem is not recalled correctly

Question			
Given that $f(t) = 3t^5 + 4$, find the mean value of $f(t)$ in the interval $1 < t < 3$, correct to two decimal places. Enter correct to two decimal places: [ ]			
Correct Solution			
The mean value, m, of $f(t)$ in the interval $a < t < b$ is given by $\frac{1}{b-a}\int_a^b f(t)dt$ $m = \frac{1}{(3-1)}\int_1^3 (3t^5 + 4)dt$ $= \frac{1}{2}\left[\frac{t^6}{2} + 4t\right]_1^3$ $= 186.00$			
CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**	
Taking the mean value as $\int_a^b f(t)dt$ *instead of* $\frac{1}{b-a}\int_a^b f(t)dt$ $\tilde{m} = \int_1^3 (3t^5 + 4)dt$ $= \left[\frac{t^6}{2} + 4t\right]_1^3$ $= 372$			
No. of CSEs /No. incorrect answers **(CSE %)**	10/121 **(8%)**	**Date collected**	**2018-19**

51

3.5.19 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Method of finding the argument of a complex number is not recalled correctly

Question			
Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of $z = \frac{(w_1)^3}{(w_2)^2}$, where $-\pi < \theta \le \pi$. Enter your answers correct to three decimal places. Enter $r = \lvert z \rvert$: [ ] Enter $\theta = arg(z)$: [ ]			
Correct Solution			
$z = \frac{(w_1)^3}{(w_2)^2}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times 0.8 - 2\times -0.4)}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{3.2j}$ $\lvert z \rvert = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\arg(z) = 3.2 - 2\pi = -3.083$			
CSE 2 related to this question		**CSE Taxonomy Code**:	**U5**
Taking $arg(z) = \theta_1^{\,n} - \theta_2^{\,m}$ $\widetilde{arg(z)} = (0.8)^3 - (-0.4)^2 = 0.352$			
No. of CSEs /No. incorrect answers (CSE %)	9/197(5%)	**Date collected**	**2017-18**

52

3.5.20 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Method of finding the principle value of the argument of a complex number is not recalled correctly

Question			
Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of $z = \frac{(w_1)^3}{(w_2)^2}$, where $-\pi < \theta \leq \pi$. Enter your answers correct to three decimal places. Enter $r = \lvert z \rvert$: [ ] Enter $\theta = arg(z)$: [ ]			
Correct Solution			
$z = \frac{(w_1)^3}{(w_2)^2}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times0.8-2\times-0.4)}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{3.2j}$ $\lvert z \rvert = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\arg(z) = 3.2 - 2\pi = -3.083$			
CSE 3 related to this question	**CSE Taxonomy Code:**	**U5**	
Taking $arg(z) = \theta - \pi$ $\widetilde{arg(z)} = 3.2 - \pi = 0.058$			
No. of CSEs /No. incorrect answers **(CSE %)**	**17/197(9%)**	**Date collected**	**2017-18**

53

3.5.21 Complex Numbers (Argument of $z = a - bj$)

Method of finding the principle value of the argument of a complex number is not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \leq \pi$.

Express your answers correct to three decimal places.

Enter r: [ ]
Enter θ: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z = 2 - 3j$
$r = \sqrt{(2)^2 + (-3)^2}$
$= \sqrt{13}$
$r = 5.39$
z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$
$= -0.983$</td></tr>
<tr><td>CSE 2 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>U5</td></tr>
<tr><td colspan="4">Taking $\theta = \pi + \tan^{-1}\left(\frac{-b}{a}\right)$ when $z = a - bj$

$\tilde{\theta} = \pi + \tan^{-1}\left(\frac{-3}{2}\right)$
$= \pi - 0.983$
$= 2.159$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>13/88(15%)</td><td>Date collected</td><td>2018-19</td></tr>
</table>

54

3.5.22 Binomial Series

Binomial theorem is incorrectly followed

Question

Use the binomial theorem to expand $(2x+3y)^5$ and enter the fourth term
(that is the full term involving x^2y^3) as a function of x and y :

Correct Solution

When n is a positive integrer, the binomial theorem states that

$$(ax+by)^n = \binom{n}{0}(ax)^n + \binom{n}{1}(ax)^{n-1}(by)^1 + \binom{n}{2}(ax)^{n-2}(by)^2 + \cdots + \binom{n}{n}(by)^n$$

So in this case, the fourthterm is

$$T_4 = \binom{5}{3}(2x)^{5-3}(3y)^{5-2}$$

$$= \frac{5!}{3! \times 2!}(2x)^2(3y)^3$$

$$= 1080x^2y^3$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**

Taking the r^{th} term as $\binom{n}{r-1}a(x)^{n-r+1}b(y)^{r-1}$ *instead of* $\binom{n}{r-1}(ax)^{n-r+1}(by)^{r-1}$

$$(2x+3y)^5 = \binom{5}{0}2(x)^5 + \binom{5}{1}2(x)^4 3(y)^1 + \binom{5}{2}2(x)^3 3(y)^2 + \binom{5}{3}2(x)^2 3(y)^3$$

$$+ \binom{5}{4}2(x)^1 3(y)^4 + \binom{5}{5}3(y)^5$$

$$T_4 = \binom{5}{3}2(x)^2 3(y)^3$$

$$= 60x^2y^3$$

No. of CSEs /No. incorrect answers **(CSE %)**	8/165**(5%)**	**Date collected**	**2018-19**

3.5.23 Centre of Mass

Definition of Centre of Mass is not recalled correctly

Question

Masses of 4 kg, 6 kg and 10 kg are located at points with co-ordinates (-2,5) , (1,-4) and (3,1) respectively.

Find the co-ordinates of their Centre of Mass, $(\overline{x}, \overline{y})$, correct to one decimal place.

Enter $\overline{x}$: [ ]

Enter $\overline{y}$: [ ]

Correct Solution

$$\bar{x} = \frac{\sum_{i=1}^{3} m_i\, x_i}{\sum_{i=1}^{3} m_i} = \frac{4 \times (-2) + 6 \times 1 + 10 \times 3}{(4 + 6 + 10)} = 1.4$$

$$\bar{y} = \frac{\sum_{i=1}^{3} m_i\, y_i}{\sum_{i=1}^{3} m_i} = \frac{4 \times 5 + 6 \times (-4) + 10 \times 1}{(4 + 6 + 10)} = 0.3$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**

Taking $\bar{x} = \sum_{i=1}^{n} m_i x_i$

and $\bar{y} = \sum_{i=1}^{n} m_i y_i$

$$\tilde{\bar{x}} = 4 \times (-2) + 6 \times 1 + 10 \times 3 = 28$$

$$\tilde{\bar{y}} = 4 \times 5 + 6 \times (-4) + 10 \times 1 = 6$$

No. of CSEs /No. incorrect answers **(CSE %)**	11/64**(17%)**	**Date collected**	**2018-19**

56

3.5.24 Integration ($\int xe^{ax^n}dx$)

Method of integrating not recalled correctly

Question			
Evaluate the following: $$\int xe^{4x^2}dx$$ as a function of x, to within an additive constant (i.e. do not put a "+c" in your answer). Enter the answer as a function of x:			
Correct Solution			
Use the method of substitution and take $u = 4x^2$, then $\frac{du}{dx} = 8x$ $$I = \int xe^{4x^2}dx = \int \frac{1}{8}e^u du$$ $$= \frac{1}{8}e^u + c$$ $$= \frac{1}{8}e^{4x^2} + c$$			
CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**	
Use integration by parts and take $\int e^{ax^2}dx = \frac{e^{ax^2}}{4a}$ Let $u = x$, *and* $\frac{dv}{dx} = e^{4x^2}$, *then* $\frac{du}{dx} = 1$ *and* $v = \frac{e^{4x^2}}{4}$ Using integration by parts, $$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$ $$I = \int xe^{4x^2}dx = \frac{xe^{4x^2}}{4} - \int \frac{1}{4}e^{4x^2}dx$$ $$= \frac{xe^{4x^2}}{4} - \frac{e^{4x^2}}{16}$$			
No. of CSEs /No. incorrect answers **(CSE %)**	19/192**(10%)**	**Date collected**	**2018-19**

57

3.5.25 Integration ($\int xe^{ax^n}dx$)

Method of integrating not recalled correctly

Question		
Evaluate the following: $\int xe^{4x^2}dx$ as a function of x, to within an additive constant (i.e. do not put a "+c" in your answer). Enter the answer as a function of x:		
Correct Solution		
Use the method of substitution and put $u = 4x^2$, then $\frac{du}{dx} = 8x$ $I = \int xe^{4x^2}dx = \int \frac{1}{8}e^u du$ $= \frac{1}{8}e^u + c$ $= \frac{1}{8}e^{4x^2} + c$		
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**
Use integration by parts and take $\int e^{ax^2}dx = \frac{e^{ax^2}}{2ax}$ Let $u = x$, *and* $\frac{dv}{dx} = e^{4x^2}$, *then* $\frac{du}{dx} = 1$ *and* $v = \frac{e^{4x^2}}{8x}$ Using integration by parts, $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ $\tilde{I} = \int xe^{4x^2}dx = \frac{xe^{4x^2}}{8x} - \int \frac{e^{4x^2}}{8x}dx$ $= \frac{xe^{4x^2}}{8} - \frac{e^{4x^2}}{8x} \times \frac{1}{8x}$ $= \frac{e^{4x^2}}{8} - \frac{e^{4x^2}}{64x^2}$		

No. of CSEs /No. incorrect answers **(CSE %)**	14/192**(7%)**	**Date collected**	**2018-19**

58

3.5.26 Integration ($\int x\cos(ax)\,dx$)

Method of integrating not recalled correctly

Question

Evaluate the following:

$$\int x\cos(3x)dx$$

as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:

Correct Solution

Use integration by parts

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Take

$$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$

Then

$$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$

$$I = \int x\cos(3x)\,dx = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3}dx$$

$$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**

Treating the x in front of the trigonometric function as a constant

$$I = \int x\cos(3x)dx = x\left(\frac{\sin(3x)}{3}\right)$$

No. of CSEs /No. incorrect answers **(CSE %)**	13/143 **(9%)**	**Date collected**	**2017-18**

3.5.27 Integration ($\int x\cos(ax)\,dx$)

Method of integrating is not recalled correctly

Question

Evaluate the following:

$$\int x\cos(3x)dx$$

as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:

Correct Solution

Use integration by parts

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Take

$$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$

$$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$

$$I = \int x\cos(3x) = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3}dx$$

$$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 3 related to this question	**CSE Taxonomy Code**:	**U5**

Missed out x in front of the trigonometric function

$$I = \int \mathrm{xcos(3x)dx} = \left(\frac{\sin(3x)}{3}\right)$$

No. of CSEs /No. incorrect answers **(CSE %)**	**11/143 (8%)**	**Date collected**	**2017-18**

3.5.28 Integration Applications (Volume of Revolution)

Definition of volume of revolution is not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">The volume of revolution of y over the range $a < x < b$ given by
$$V = \pi \int_a^b y^2 dx$$
For given data,
$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$
$$= \pi \int_1^4 0.64\, x^3 dx$$
$$= 128.18$$</td></tr>
<tr><td>CSE 1 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>U5</td></tr>
<tr><td colspan="4">Missing π
$$\tilde{V} = \int_a^b y^2 dx$$
$$\tilde{V} = \int_1^4 (0.8x^{1.5})^2 dx = \int_1^4 0.64\, x^3 dx = 40.80$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>9/107 (8%)
3/135(2%)</td><td>Date collected</td><td>2017-18
2018-19</td></tr>
</table>

3.5.29 Integration Applications (Volume of Revolution)

Definition of volume of revolution is not recalled correctly

Question
Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$. Give your answer correct to two decimal places. The volume of the solid is: [ ]
Correct Solution
The volume of revolution of y over the range $a < x < b$ given by $$V = \pi \int_a^b y^2 dx$$ For given data, $$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$ $$= \pi \int_1^4 0.64\, x^3 dx$$ $$= 128.18$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**
Taking $V = \int_a^b y\, dx$ $$\tilde{V} = \int_1^4 0.8\, x^{1.5} dx$$ $$= 0.8 \left[\frac{x^{2.5}}{2.5}\right]_1^4$$ $$= 9.92$$		
No. of CSEs /No. incorrect answers **(CSE %)**	9/107 **(8%)** 13/135 **(10%)**	**Date collected**	**2017-18** **2018-19**

3.5.30 Integration Applications (Volume of Revolution)

Definition of volume of revolution is not recalled correctly

Question

Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]

Correct Solution

The volume of revolution of y over the range $a < x < b$ given by

$$V = \pi \int_a^b y^2 dx$$

For given data,

$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$

$$= \pi \int_1^4 0.64\, x^3 dx$$

$$= 128.18$$

CSE 3 related to this question	**CSE Taxonomy Code:**	**U5**

Taking $V = \pi \int_a^b y\, dx$

$$\tilde{V} = \pi \int_1^4 0.8\, x^{1.5} dx$$

$$= 0.8\, \pi \left[\frac{x^{2.5}}{2.5}\right]_1^4$$

$$= 31.16$$

No. of CSEs /No. incorrect answers **(CSE %)**	9/107 **(8%)** 8/135 **(6%)**	**Date collected**	**2017-18** **2018-19**

3.6 Partial Solution Given

3.6.1 Trigonometric Function $\sin(2\theta) = A$

Correct working but unfinished solutions

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Solve the equation

$\sin(2t) = 0.45$

for t, where $0 \leq t \leq \pi$.

Enter your two values of t in radians, correct to two decimal places.

Enter t_1 : [ ] to two decimal places
Enter t_2 : [ ] to two decimal places

(Please note that the order in which you enter your answers does not matter.)</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">Let $z = 2t$, then the problem is equivalent to solving the equation:

$\sin(z) = 0.45$ for z, where $0 \leq z \leq 2\pi$

$z_1 = \sin^{-1}(0.45)$ and $z_2 = \pi - \sin^{-1}(0.45)$

$z_1 = 0.44477$ and $z_2 = 2.67483$

$t_1 = 0.23$ and $t_2 = 1.34$</td></tr>
<tr><td colspan="2">CSE 2 related to this question</td><td>CSE Taxonomy Code:</td><td>U6</td></tr>
<tr><td colspan="4">Submitted 2t values for t

$\sin(2t) = 0.45$

$\tilde{t_1} = \sin^{-1}(0.45)$ and $\tilde{t_2} = \pi - \sin^{-1}(0.45)$

$\tilde{t_1} = 0.44477$ and $\tilde{t_2} = 2.67483$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>9/84 (11 %)</td><td>Date collected</td><td>2018-19</td></tr>
</table>

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3.6.2 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Correct working but unfinished solutions and angles is not in the given range

<table>
<tr><td colspan="3">Question</td></tr>
<tr><td colspan="3">Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of

$z = \frac{(w_1)^3}{(w_2)^2}$,

where $-\pi < \theta \leq \pi$.

Enter your answers correct to three decimal places.

Enter $r = |z|$:
Enter $\theta = arg(z)$:</td></tr>
<tr><td colspan="3">Correct Solution</td></tr>
<tr><td colspan="3">$$z = \frac{(w_1)^3}{(w_2)^2}$$ $$z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times0.8-2\times-0.4)}$$ $$z = \frac{(1.8)^3}{(1.2)^2} e^{3.2j}$$ $$|z| = \frac{(1.8)^3}{(1.2)^2} = 4.050 \text{ and } \arg(z) = 3.2 - 2\pi = -3.083$$</td></tr>
<tr><td>CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>U4, U6</td></tr>
<tr><td colspan="3">Neglecting the required range of $arg(z)$ $$z = \frac{(1.8)^3}{(1.2)^2} e^{3.2j}$$ $$|z| = \frac{(1.8)^3}{(1.2)^2} = 4.050 \text{ and } \widehat{arg(z)} = 3.2$$</td></tr>
</table>

No. of CSEs /No. incorrect answers **(CSE %)**	66/ 197(**34%**)	**Date collected**	**2017-18**

3.6.3 Series (Maclaurin Expansion)

Omission of denominator / correct working and unfinished solution is given

Question

Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, of $f(x) = e^{4x}$ up to and including the cubic term.

Give the values of a_1 and a_2 below.

Enter your answer for a_1 (to three decimal places) here: [ ]

Enter your answer for a_2 (to three decimal places) here: [ ]

Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$

Enter your approximate value for $e^{0.8}$ (to three decimal places) here: [ ]

Correct Solution

Using MacLaurin expansion,

$$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6} \dots$$

$$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$

$$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$

$$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**S2, U6**

Missing division by 2!

$$\widetilde{a_2} = 4^2 = 16.000$$

No. of CSEs /No. incorrect answers **(CSE %)**	11/59**(19%)** 16/80**(20%)**	**Date collected**	**2017-18** **2018-19**

3.7 Incorrect Assumptions

3.7.1 Integration Application (Mean Value of $a\ \sin(bt)$)

Incorrect assumptions on the mean value theorem and incorrect units applied

Question

Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places.

Enter correct to two decimal places: [ ]

Correct Solution

$$f(t) = 4\ \sin(10t)$$

Let the mean value of $f(t) = m$

$$m = \frac{1}{(6-2)}\int_2^6 4\ \sin(10t)dt$$

$$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$

$$= -\frac{1}{10}((\cos(60) - \cos(20))$$

$$= 0.14$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**U7, UM2**

Directly substituting the midpoint of the range of t in degrees into the given function.

$$f(t) = 4\ \sin(10t)$$

Let the mean value of $f(t) = m$. The middle point of the range of $t\ = 4$

$$\tilde{m} = 4\sin(10 \times 4^o) = 4\sin(40^o)$$

$$\tilde{m} = 2.57$$

No. of CSEs /No. incorrect answers **(CSE %)**	44/183 **(24%)**	**Date collected**	**2017-18**

3.7.2 Integration Application (Mean Value of $a\sin(bt)$)

Incorrect assumptions on the mean value theorem and incorrect units applied

Question			
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places:			
Correct Solution			
$$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$			
CSE 3 related to this question	**CSE Taxonomy Code:**	**U7, UM2**	
Substituting the end values of the range t into the given function in degrees and then taking the average. $f(t) = 4\sin(10t)$, let the mean value of $f(t) = m$ $$\tilde{m} = \frac{4\sin(10 \times 2^o) + 4\sin(10 \times 6^o)}{2}$$ $$= 2.42$$			
No. of CSEs /No. incorrect answers **(CSE %)**	8/183 **(4%)**	**Date collected**	**2017-18**

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3.7.3 Integration Application (Mean Value of $(at^n + b)$)

Incorrect assumptions on the mean value of a function

Question			
Given that $f(t) = 3t^5 + 4$, find the mean value of $f(t)$ in the interval $1 < t < 3$, correct to two decimal places. Enter correct to two decimal places: [ ]			
Correct Solution			
The mean value, m, of $f(t)$ in the interval $a < t < b$ is given by $$\frac{1}{b-a}\int_a^b f(t)dt$$ $$m = \frac{1}{(3-1)}\int_1^3 (3t^5 + 4)dt$$ $$= \frac{1}{2}\left[\frac{t^6}{2} + 4t\right]_1^3$$ $$= 186.00$$			
CSE 2 related to this question	**CSE Taxonomy Code**:	**U7**	
Directly substitute the midpoint of the range of t into the given function. $$f(t) = 3t^5 + 4$$ Let the mean value of $f(t) = m$. The midpoint of the range is $t = 2$ $$\tilde{m} = 3 \times 2^5 + 4$$ $$= 100$$			
No. of CSEs /No. incorrect answers **(CSE %)**	10/121 **(8%)**	**Date collected**	**2018-19**

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3.7.4 Dimensions

Taking dimension of velocity as $[v] = [MT^{-1}]$)

Question

The quantity $K = \rho^2 r^{-1} v^{1.5}$

where ρ represents density, r represents distance, v represents velocity.

The dimensions of K are given by $[M^a L^b T^c]$. Calculate the values of a, b and c.

Enter a: [ ]
Enter b: [ ]
Enter c: [ ]

Correct Solution

$$K = \rho^2 r^{-1} v^{1.5}$$

$$[K] = [ML^{-3}]^2[L]^{-1}[LT^{-1}]^{1.5}$$

$$= [M]^2[L]^{-6-1+1.5}[T]^{-1.5}$$

$$= [M]^2[L]^{-5.5}[T]^{-1.5}$$

$$a = 2, b = -5.5, c = -1.5$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**U7**

Taking dimensions of v to be $[v] = [MT^{-1}]$
(confusion with Dimentions and Units of velocity

$$K = \rho^2 r^{-1} v^{1.5}$$

$$[K] = [ML^{-3}]^2[L]^{-1}[MT^{-1}]^{1.5}$$

$$= [M]^{3.5}[L]^{-7}[T]^{-1.5}$$

$$\tilde{a} = 3.5, \tilde{b} = -7, c = -1.5$$

No. of CSEs /No. incorrect answers (CSE %)	6/13 (46%)	**Date collected**	2018-19

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4 Common Student Errors due to Errors in Choice of Method

4.1 Applying an Inappropriate Formula/Method/Theorem

4.1.1 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Uses the method which is not relevant in the situation

Question

Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of

$$z = \frac{(w_1)^3}{(w_2)^2},$$

where $-\pi < \theta \leq \pi$.

Enter your answers correct to three decimal places.

Enter $r = |z|$:

Enter $\theta = arg(z)$:

Correct Solution

$$z = \frac{(w_1)^3}{(w_2)^2}$$

$$z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times 0.8 - 2\times -0.4)}$$

$$z = \frac{(1.8)^3}{(1.2)^2} e^{3.2j}$$

$$|z| = \frac{(1.8)^3}{(1.2)^2} = 4.050 \text{ and } \arg(z) = 3.2 - 2\pi = -3.083$$

CSE 4 related to this question	CSE Taxonomy Code:	CM1

Taking $|z| = \sqrt{r_1^2 + r_2^2}$

$$|z| = \sqrt{1.8^2 + 1.2^2}$$
$$= 2.163$$

No. of CSEs /No. incorrect answers (CSE %)	11/73(15%) 14/92(15%)	Date collected	2017-18 2018-19

4.1.2 Infinite Geometric Series

Uses a formula which is not relevant in the situation

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Consider the following geometric series, S, where:
$S = 2 + 2(0.7) + 2(0.7)^2 + 2(0.7)^3 \cdots$.
Write down the first term, a and the common ratio, r in the boxes below.
Enter a: [ ]
Enter r: [ ]
Hence calculate the sum, S and enter your result in the box below.
Enter S (to three decimal places) here: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">The first term $a = 2$
The common ratio $r = 0.7$
The sum of an infinite series (S) exists, provided $|r| < 1$
$S = \frac{a}{1-r}$
$= 6.667$</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>CM1</td></tr>
<tr><td colspan="4">Finding the sum of first four terms instead of the sum of the infinite series.
$\tilde{S} = \frac{a(1-r^n)}{1-r}$
$\tilde{S} = \frac{2(1-0.7^4)}{1-0.7}$
$\tilde{S} = 5.066$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>34/67(51%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

4.1.3 Series (Maclaurin Expansion)

Uses a method which is not valid in the situation

Question

Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, of $f(x) = e^{4x}$ up to and including the cubic term.

Give the values of a_1 and a_2 below.

Enter your answer for a_1 (to three decimal places) here: [ ]

Enter your answer for a_2 (to three decimal places) here: [ ]

Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$

Enter your approximate value for $e^{0.8}$ (to three decimal places) here: [ ]

Correct Solution

Using MacLaurin expansion,

$$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6} \dots$$

$$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$

$$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$

$$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**CM1**

Giving the exact value of e^n instead of the approximate value.

$$e^{0.8} = 2.226$$

No. of CSEs /No. incorrect answers **(CSE %)**	28/116 **(24%)** 29/122**(24%)**	**Date collected**	**2017-18** **2018-19**

5 Common Student Errors due to Errors in Use of Method

5.1 Error in use of an Appropriate Definition/ Method/ Theorem

5.1.1 Differentiation ($cos(ax^n)$)

Error in the use of the chain rule

Question

Select the most appropriate method to use in order to find the derivative of $f(x) = \cos(2x^7)$.

Select

Hence find $\frac{df}{dx}$ as a function of x.

Enter the answer as a function of x:

Correct Solution

$$f(x) = \cos(2x^7)$$

$$f'(x) = -\sin(2x^7) \times 14x^6$$

$$f'(x) = -14x^6 \sin(2x^7)$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**UM2**

Taking $\frac{d}{dx}(cos(ax^n)) = -a\ sin(ax^n) \times anx^{n-1} = -a^2nx^{n-1}sin(2x^n)$

$$f(x) = \cos(2x^7)$$

$$\tilde{f}'(x) = -2\sin(2x^7) \times 2 \times 7x^6$$

$$\tilde{f}'(x) = -28x^6 \sin(2x^7)$$

No. of CSEs /No. incorrect answers **(CSE %)**	10/56 **(18%)**	**Date collected**	**2017-18**

5.1.2 Differentiation ($cos^n(ax)$)

Error in the use of the chain rule

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Select the most appropriate method to use in order to find the derivative of $f(x) = \cos^4(3x)$.

Select ▾

Hence find $\frac{df}{dx}$ as a function of x.

Enter the answer as a function of x:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$f(x) = cos^4(3x)$
$f'(x) = -4 \times \cos^3(3x) \times \sin(3x) \times 3$
$f'(x) = -12\sin(3x)\cos^3(3x)$</td></tr>
<tr><td>CSE 2 related to this question</td><td>CSE Taxonomy Code:</td><td colspan="2">UM2</td></tr>
<tr><td colspan="4">Taking $\frac{d}{dx}(cos^a(bx)) = -a \times sin^{a-1}(bx) \times b = -ab\, sin^{a-1}(bx)$
$f(x) = cos^4(3x)$
$f'(x) = -4 \times \sin^3(3x) \times 3$
$f'(x) = -12\sin^3(3x)$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>22/73(30%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

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5.1.3 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x, y) = acos(xy^n)$)

Error in use of partial differentiation method

Question			
Given $z(x,y) = 3\cos(xy^4)$ find $\frac{\partial z}{\partial x}$. Enter your answer below as a function of x and y :			
Correct Solution			
$$z(x,y) = 3\cos(xy^4)$$ $$\frac{\partial z}{\partial x} = -3\sin(xy^4) \times y^4$$ $$\frac{\partial z}{\partial x} = -3y^4\sin(xy^4)$$			
CSE 4 related to this question	**CSE Taxonomy Code:**	**UM2**	
Taking $\frac{\partial}{\partial x}(a\cos(xy^n)) = -a\sin(xy^n) \times x \times ny^{n-1}$ $$z(x,y) = 3\cos(xy^4)$$ $$\widetilde{\frac{\partial z}{\partial x}} = -3\sin(xy^4) \times x \times 4y^3$$ $$\widetilde{\frac{\partial z}{\partial x}} = -12xy^3\sin(xy^4)$$			
No. of CSEs /No. incorrect answers **(CSE %)**	5/103**(5%)**	**Date collected**	**2017-18**

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5.1.4 Integration Application (Mean Value of $a\sin(bt)$)

Incorrect units applied

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places.

Enter correct to two decimal places: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$f(t) = 4\sin(10t)$$
Let the mean value of $f(t) = m$
$$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$
$$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$
$$= -\frac{1}{10}((\cos(60) - \cos(20))$$
$$= 0.14$$</td></tr>
<tr><td>CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td colspan="2">UM2</td></tr>
<tr><td colspan="4">Substituting for t in degrees
$$f(t) = 4\sin(10t)$$
Let the mean value of $f(t) = m$,
$$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$
$$\tilde{m} = 0.04$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>52/183 (28%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

5.1.5 Integration Application (Mean Value of $a\sin(bt)$)

Incorrect assumptions on the mean value theorem and incorrect units applied

Question			
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places: [ ]			
Correct Solution			
$$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$			
CSE 2 related to this question	**CSE Taxonomy Code:**	**U7, UM2**	
Directly substituting the midpoint of the range of t in degrees into the given function. $$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$. The middle point of the range of $t = 4$ $$\tilde{m} = 4\sin(10\times 4^o) = 4\sin(40^o)$$ $$\tilde{m} = 2.57$$			
No. of CSEs /No. incorrect answers **(CSE %)**	44/183 **(24%)**	**Date collected**	**2017-18**

5.1.6 Integration Application (Mean Value of $a\sin(bt)$)

Incorrect assumptions on the mean value theorem and incorrect units applied

Question
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places:
Correct Solution
$$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$

CSE 3 related to this question	**CSE Taxonomy Code**:	**U7, UM2**
Substituting the end values of the range t into the given function in degrees and then taking the average. $f(t) = 4\sin(10t)$, let the mean value of $f(t) = m$ $$\tilde{m} = \frac{4\sin(10\times 2^o) + 4\sin(10\times 6^o)}{2}$$ $$= 2.42$$		
No. of CSEs /No. incorrect answers **(CSE %)**	8/183 **(4%)**	**Date collected**
		2017-18

79

5.1.7 Complex Numbers (Argument of $z = a - bj$)

Incorrect units applied

Question			
Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \leq \pi$. Express your answers correct to three decimal places. Enter r: [ ] Enter θ: [ ]			
Correct Solution			
$z = 2 - 3j$ $r = \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{13}$ $r = 5.39$ z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$ $= -0.983$			
CSE 3 related to this question		**CSE Taxonomy Code**:	**UM2**
Calculating $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$ *in degrees (not in radians) when* $z = a - bj$ $\tilde{\theta} = \tan^{-1}\left(\frac{-3}{2}\right)$ $= -56.31^{o}$			
No. of CSEs /No. incorrect answers **(CSE %)**	14/88**(16%)**	**Date collected**	**2018-19**

80

5.1.8 Complex Numbers (Argument of $z = a - bj$)

Overlooking negative sign and incorrect units applied

<table>
<tr><td colspan="4"><u>Question</u></td></tr>
<tr><td colspan="4">Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \leq \pi$.

Express your answers correct to <u>three</u> decimal places.

Enter r: [ ]
Enter θ: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z = 2 - 3j$
$r = \sqrt{(2)^2 + (-3)^2}$
$= \sqrt{13}$
$r = 5.39$
z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$
$= -0.983$</td></tr>
<tr><td>CSE 4 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>S2, UM2</td></tr>
<tr><td colspan="4">Calculating $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ in degrees (and missing a negative sign) when $z = a - bj$

$\tilde{\theta} = \tan^{-1}\left(\frac{3}{2}\right)$
$= 56.31^{o}$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>11/88(13%)</td><td>Date collected</td><td>2018-19</td></tr>
</table>

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5.1.9 Integration Applications (Volume of Revolution)

Method finding the volume of revolution is incorrectly followed

Question		
Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$. Give your answer correct to two decimal places. The volume of the solid is: [ ]		
Correct Solution		
The volume of revolution of y over the range $a < x < b$ given by $$V = \pi \int_a^b y^2 dx$$ For given data, $$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$ $$= \pi \int_1^4 0.64\, x^3 dx$$ $$= 128.18$$		
CSE 5 related to this question	**CSE Taxonomy Code**:	**UM2**
Substitute upper and lower limits without integrating $$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$ $$\tilde{V} = 0.64\, \pi \left[\frac{x^3}{3}\right]_1^4$$ $$= 126.67$$		

No. of CSEs /No. incorrect answers **(CSE %)**	5/107 **(5%)** 7/135 **(5%)**	**Date collected**	**2017-18** **2018-19**

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References

Bailey, K.D. (1994) *Typologies and Taxonomies: An Introduction to Classification Techniques.* Thousand Oaks, CA: Sage Publications.

Booth, J.L., Barbieri, C., Eyer, F. and Pare-Blagoev, E.J. (2014) Persistent and Pernicious Errors in Algebraic Problem Solving. *Journal of Problem Solving*. 7, pp. 10-23.

Brown, J. S., & Burton, R. R. (1978) Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science.* 2 (2), 155–192. https://doi.org/10.1207/s15516709cog0202_4

Confrey, J. (1990) Chapter 1: A Review of the Research on Student Conceptions in Mathematics, Science, and Programming. *Review of Research in Education*, 16(1), pp. 3–56. doi: 10.3102/0091732X016001003.

CSE Project at UWE Bristol (2019) *Diagnosing and Remediating Mathematical Common Student Errors in e-Assessment Questions: A Case Study.* Available from: https://fetstudy.uwe.ac.uk/~bin-sikurajapa/dewis/cseproject/ [Accessed 24 February 2022].

Fischbein, E. (1989) Tacit Models and Mathematical Reasoning. For the Learning of Mathematics. *For the Learning of Mathematics*. 9 (2), pp. 9-14.

Ford, S., Gillard, J. and Pugh, M. (2018) Creating a Taxonomy of Mathematical Errors For Undergraduate Mathematics. *MSOR Connections*. 18 (1), pp. 37-45.

Gwynllyw, R. and Henderson, K. (2009) Dewis: A Computer Aided Assessment System for Mathematics and Statistics. *CETL-MSOR 2008 Conference Proceedings*. pp. 38-44.

Gwynllyw, R. and Henderson, K. (2012) Intelligent Marking in Summative E-assessment. *In: Proc. HEA STEM Learning and Teaching Conference.*

Kirshner, D. and Awtry, T. (2004) Visual Salience of Algebraic Transformations. *Journal for Research in Mathematics Education*. 35, pp. 224-257.

Knuth, E., Stephens, A., McNeil, N. and Alibali, M. (2006) Does Understanding the Equal Sign Matter? Evidence from Solving Equations. *Journal for Research in Mathematics Education*. 37, pp. 297-312.

Nesher, P. (1987). Towards an instructional theory: The role of student's misconceptions, *For the Learning of Mathematics*, vol. 7, pp. 33-40,

Obersteiner, A., Dooren, W., Hoof, J. and Verschaffel, L. (2013) The Natural Number Bias and Magnitude Representation in Fraction Comparison by Expert Mathematicians. *Learning and Instruction*. 28, pp. 64-72.

Rees, R. and Barr, G., (1984) *Diagnosis and Prescription in the Classroom: Some Common Maths Problems*. London: Harper & Row.

Rushton, N. (2014) Common Errors in Mathematics. *Research Matters: A Cambridge Assessment Publication*. 17, pp. 8-17.

Senders, J. and Moray, N. (1991) *Human Error: Causes, Prediction and Reduction*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R., (2020) Using E-Assessment to Address Mathematical Misconceptions in Engineering Students. *International Journal of Information and Education Technology*. 10(5), pp.356–361.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2021) Students' Perceptions of Enhanced E-assessment Feedback Addressing Common Student Errors in Mathematics. *MSOR Connections*. 19 (2), pp. 10-27.

Simpson, G.G. (1961) *Principles of Animal Taxonomy*. New York: University Press.

Swan, M. (1990) Becoming numerate: developing conceptual structures. In S. Willis (Ed) *Being numerate: what counts?* Victoria: Australian Council for Educational Research.

VanLehn, K. (1982) Bugs are not enough: Empirical studies of bugs, impasses and repairs in procedural skills. *Journal of Mathematical Behavior*, 3(2) pp. 3-71.

D Appendix: Peer Reviewed Published Journal Papers

D.1 Sikurajapathi, I., Henderson, K. and Gwynllyw, R. (2020) Using e-Assessment to Address Mathematical Misconceptions in Engineering Students. *International Journal of Information and Education Technology*. 10 (5), pp. 356-361.

(Published Version)

Declaration:

Indunil Sikurajapathi conducted the research, analysed the data and wrote the first draft of the paper. Karen Henderson took responsibility for the final version of the paper. Karen Henderson and Rhys Gwynllyw organised and supervised the research.

Using e-Assessment to Address Mathematical Misconceptions in Engineering Students

Indunil Sikurajapathi, Karen Henderson, and Rhys Gwynllyw

***Abstract*—Students, when answering a mathematical question, may make a mistake in their answer for a variety of reasons. For example, not reading the question properly, making a mistake due to carelessness or due to a mathematical misconception. It is this latter category, which is of particular interest to us in this paper. When such mistakes occur in handwritten work then, in general, the teacher is able to identify the mistake(s) during the marking process and give written detailed feedback on the student's script. The disadvantage of this approach is the time and effort it takes to mark and to get feedback back to the student. As a result, e-assessment is becoming a standard means of providing formative and summative assessment of mathematical techniques. The research problem that we have identified is how to detect mathematical misconceptions when students answer e-assessment questions incorrectly, and how to improve the feedback provided to the student in such cases. By analyzing students' rough paper-based workings for an e-examination, we have captured mathematical misconceptions made by first year engineering students. This has enabled us to catalogue common student errors made by students. By amending the e-assessment feedback code, students who make these errors will subsequently benefit from enhanced, tailored feedback, highlighting the mathematical misconception/error made. In addition, detailed guidance on how to improve their knowledge related to the topic will be given. The aim of our work is to improve the e-assessment experience for students as well as addressing and tackling misconceptions in a timely fashion.**

***Index Terms*—Common student errors, Dewis, e-assessment, engineering mathematics.**

I. Introduction

An understandable but incorrect implementation of a process resulting from a student's misconception is called a mal-rule [1]. Mal-rules can be classified as manipulative, parsing, execution/clerical and random [2]. In this paper, we focus on mal-rules or, in other words, common student errors (CSEs) in Engineering Mathematics - a subject in which students tend to make CSEs due to misconceptions in mathematics. For example, a typical CSE students make is to answer a^2+b^2 when asked to expand $(a+b)^2$. Booth [3] states that "Students hold many misconceptions as they transition from arithmetic to algebraic thinking, and these misconceptions can hinder their performance and learning in the subject." This is particularly the case in Engineering, which is a subject that requires a strong mathematics foundation.

Mathematics Education research; see for example [4]-[6], has explored possible causes and effects of certain mathematical misconceptions and the impact that they have on students' future learning. As an example, there has been recent research into theorizing student errors supported by empirical studies in the topics of natural number bias [7], visual saliency [8] and over-generalization [9]. More recently, Rushton [10] conducted a study of common errors in Mathematics made in certain General Certificate of Secondary Education mathematics papers taken by candidates in England, including an internationally available version, as referenced by examiner reports, and errors were catalogued into themes and sub-themes. Khiat [11] looked specifically at the mathematics learning of engineering students at undergraduate level and the focus of the work was on conceptions of understanding using grounded theory methodology.

E-assessment has become a standard method to provide formative and summative assessments in many universities all around the world [12]. A few of the advantages of e-assessment are that it can provide instant tailored feedback to help students to improve their knowledge and performance, they can access it in different geographical locations at different times, and undertake online tests many times to assess and refine their knowledge. Moreover, it allows educators to identify areas in which more help is needed and then to take necessary action to address difficult areas in the subject. Research has found that students learn from e-assessment feedback and enhance their technical knowledge by using it [13]. Therefore, e-assessments that provide effective feedback and select questions based on pedagogic principles should be promoted as a learning resource [14].

Research [15] shows that feedback has to be quick to be effective, while students still remember clearly the work they were engaged in and using e-assessment is one way of achieving this. A computer cannot act flexibly like a human marker when faced with ill-posed or unanticipated student responses [14]. However, if an e-assessment system could detect and report CSEs, it would behave more like a human marker and provide very effective and tailored feedback instantly for the students by pointing out their mal-rule [16]. Providing such tailored feedback will help students to learn from their misconceptions. The CalculEng system [17] has been developed to address this need for Calculus based problems that engineering students encounter, but the development still requires expert teachers with mathematical knowledge to anticipate the errors that students might make.

In this paper, we demonstrate how we have built up a

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collection of CSEs made by Level 1 engineering students in their mathematics module and give an overview of our findings to date. We have achieved this by scrutinizing students' answers to e-assessment questions and by looking at their rough workings to a mid-module e-examination.

II. Methodology

A. Module Overview

Engineering Mathematics (EM) is a 30-credit module making up a quarter of the credit for Level 1 and is delivered to a large and diverse student cohort at the University of the West of England, Bristol (UWE). Students learn mathematical techniques that will support their engineering studies, including learning to program in Matlab. As well as the Matlab weekly PC sessions in Semester 1, all students receive two hours of lectures, supported by a one-hour tutorial each week. In addition, all students have a scheduled weekly two-hour Peer Assisted Learning (PAL) session [18] run by Level 2 PAL tutors and which offer whole course support, not just help with EM. The module is assessed through coursework (25%) and examination (75%). The coursework is designed to encourage engagement in the module. The Matlab assignment comprises 50% of the coursework mark, whilst e-assessments delivered throughout the year comprise the remaining 50% coursework mark. Further details of the e-assessment system used is given in Section II B and the e-assessment implementation is expanded on in Section II C.

B. The Dewis e-Assessment System

The Dewis system was used to deliver all the e-assessments on this module [19], [20]. Dewis is a fully algorithmic open-source web-based e-assessment system that was designed and developed at UWE. It was primarily designed for the assessment of mathematics and statistics and supports a range of inputs, such as numeric entry, algebraic entry, matrix entry, computer programs, multiple choice and multiple selection. Using an algorithmic approach enables the separate solution, marking and feedback algorithms to respond dynamically to a student's input. The question parameters are randomized and generated at the point of delivery; therefore, no two students receive exactly the same question. Students can practice the same question several times with different parameters in order to gain mastery. All Dewis questions have full feedback bespoke to that question and its specific randomly generated parameters. The feedback not only supplies the correct answer but a fully worked solution showing how that the correct answer was obtained. An example of an e-assessment question used for EM is illustrated in Fig. 1 together with the full feedback received.

All data relating to every assessment attempt is recorded on the Dewis server. This enables the academic to track efficiently how a student or cohort of students has performed on a particular e-assessment [16]. The highly developed reporting system enables tracking at module cohort level, tutorial group level and individual student level. Fig. 2 shows a reporting session for a particular e-assessment, in this case viewing the mark awarded for each individual question in the test. Each mark is a web link, which contains the realization of a particular question as delivered to that student, the student's answer and the resulting feedback given to them.

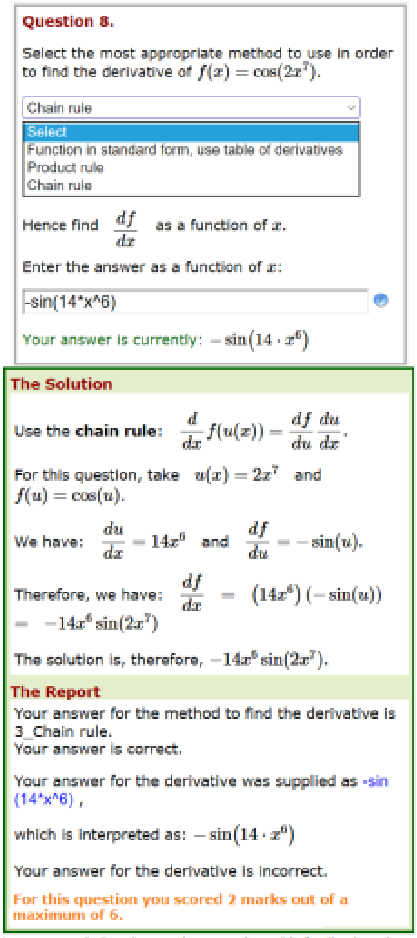

Fig. 1. An example Dewis question, together with feedback and marking bespoke to the random parameters used in this question. This question illustrates partial marking; the method of solution is correct, but the implementation was not.

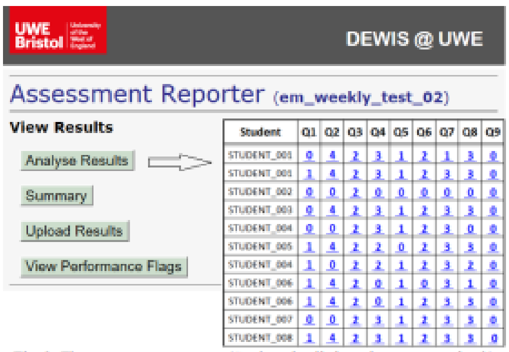

Fig. 2. The assessment reporter. (Student details have been anonymized.)

C. E-Assessment Delivery Implementation

We have used e-assessment at UWE since 2000 and migrated to the Dewis e-assessment system in EM in 2009. Over that time, we have built a substantial library of Dewis questions to support the teaching of engineering mathematics. The question library resource has enabled us to try out different delivery patterns of e-assessment in order to improve year-long student engagement with the module and hence improve attainment levels.

Since the 2015/2016 academic year, the module has used 22 weekly e-assessments and students are given access to these e-assessments throughout the year and are allowed unlimited attempts. The e-assessment coursework mark is calculated from the top 20 marks from the 22 weekly tests (twelve tests in Semester 1 and ten in Semester 2). All weekly tests are open from the start of the module. Each test can contribute two marks to the coursework mark, comprising one engagement mark and one attainment mark.

At the end of the first semester, students are required to take a two-hour e-examination, sat under controlled conditions and questions on this e-examination are based on the questions students have already encountered in their weekly e-assessments [21]. Due to the lack of available computers, this January e-examination was delivered in two sessions. Approximately half the students were timetabled for the morning session and the other half for the afternoon. For each separate run of the e-examination, we fixed the parameters of the questions in order to ensure fairness. This approach also meant that, at the start of the exam, students were given a hardcopy of the specific questions that they were attempting. Students valued this, as some found it easier to work from a paper copy than from the screen. In this paper, we have focused on the January 2018 e-examination. Each version (morning and afternoon) contained 19 questions. Both exam versions contained a mixture of input types: numerical, algebraic and dropdown. The question structure and subject content were the same for both papers but different numeric parameters were used in each case to make the two tests different but of comparable difficulty. A total of 298 students sat the e-examination, 148 in the morning and 150 in the afternoon. The official submission was electronic but students were given exam booklets in order to write their rough workings to questions and these booklets were collected at the end of the e-examination.

D. Detection of Common Student Errors

In terms of detecting CSEs, it was natural to start by analyzing the submissions from the January e-examination. This was because all morning/afternoon students sat the same paper and provided written solutions in booklets, as well as submitting their final answer electronically. We examined these written answer scripts along with the corresponding Dewis answers for all instances in which the students had inputted an incorrect answer in the e-examination. Firstly, the Dewis Reporter output was used to select the most common incorrect answers to each question. Secondly, the written answer scripts of the students who inputted the same mistake were carefully examined. The aim of this process was to understand what kind of mistake had led the students to arrive at that common wrong answer. Having access to the students' workings was invaluable for this process.

III. Results

We analyzed all 19 questions from both the morning and afternoon versions of the e-examination and 17 questions were found to exhibit CSEs. We found several of the questions to have more than one CSE associated with them and we catalogued 40 CSEs in total.

For each question, we designated the principal CSE to be the one that was triggered by the largest proportion of students. This quantity was measured as a percentage of the number of students who made the CSE compared to the total number of students who answered that question incorrectly. The results of these principal CSEs are illustrated in Fig. 3. In this chart, the height of the rectangle represents the number of students, aggregated over the two sittings, who answered the question incorrectly whilst the height of the shaded rectangle represents the number of students who triggered the principal CSE for that question. Please note that there is no shaded box for Questions 1 and 4 because no CSE was found for either question. We can see that Question 14 was the least well-answered question (197 incorrect responses) and the principal CSE for this question was triggered by 34% of students. Question 5 was the one for which the least number of incorrect responses were submitted (13 in total) and the principal CSE for that question was triggered by 38% of students. The principal CSE that was triggered by the largest proportion of students occurred for Question 12, namely 70%.

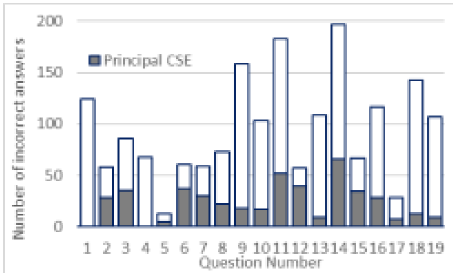

Fig. 3. Total number of incorrect responses to the e-examination questions together with the number of these that are attributable to the principal CSE found (shaded box).

In this paper, we illustrate in detail the principal CSEs found in three questions, (namely questions 3, 11 and 7 of the e-examination) and further details of these are shown in Sections III A-C.

A. CSE Example 1

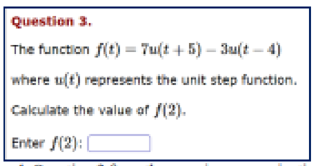

Fig. 4. Question 3 from the morning e-examination.

Fig. 4 shows the morning version of question 3 from the 2018 e-examination paper. This question required students to input a single integer answer. The afternoon paper contained a similar question but with different parameters. We only detected one CSE for this question and it involved students' misunderstanding of the unit step function. Instead of treating $u(t)$ as a function, the detected CSE involved students setting u to take the value of one and misinterpreting the purpose of the brackets. Hence, the student incorrectly evaluated $f(2)$ as $7(2+5)-3(2-4)=55$ whilst the correct answer is $f(2)=7u(7)-3u(-2)=7$. In the morning version, 12 students, out of the 44 who answered this question incorrectly (27%), triggered this CSE whilst in the afternoon 23 from 42 (55%) did. This resulted in an aggregate of 35 students from 86 (41%) making this mistake as confirmed in Fig. 3.

B. CSE Example 2

Fig. 5 shows the morning version of question 11 from the 2018 e-examination paper. This question required students to input a single floating-point answer. The afternoon paper contained a similar question but with different parameters. We detected three CSEs for this question. The principal CSE involved students performing the integration step correctly but incorrectly using the calculator in degree mode when evaluating the antiderivative of the integrand at the two limits. In the morning version, 22 students, out of the 86 who answered this question incorrectly (26%), triggered this CSE whilst in the afternoon 30 from 97 (31%) did. This resulted in an aggregate of 52 students from 183 (41%) making this mistake as confirmed in Fig. 3. The second CSE, which was triggered by 32 students (17%), involved them directly substituting the midpoint of the range of t into the integrand, $4\cos(3t)$, in the morning version of the paper, and using the calculator in degree mode to evaluate their answer. The third CSE, which eight students (4%) triggered, involved them taking the average of the integrand evaluated at the integration limits and using the calculator in degree mode to evaluate their answer.

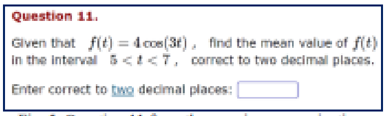

Fig. 5. Question 11 from the morning e-examination.

C. CSE Example 3

Fig. 6 shows the morning version of question 7 from the 2018 e-examination paper. This question required students to input a single answer in algebraic form. The afternoon paper contained a similar question but with different parameters. Only one CSE was detected for this question and involved students' incorrectly differentiating $\ln(ax)$ as $(ax)^{-1}$. So students making this mistake incorrectly inputted $3/(5x)$ instead of $3/x$ as their answer. In the morning version, 15 students, out of the 27 who answered this question incorrectly (56%), triggered this CSE whilst in the afternoon 15 from 32 (47%) did. This resulted in an aggregate of 30 students from 59 (51%) making this mistake as confirmed in Fig. 3.

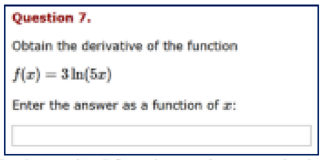

Fig. 6. Question 7 from the morning e-examination.

D. Weekly e-Assessments

All the questions from the 2018 January e-examination had been included in one of the 12 weekly first semester e-assessments taken by the same cohort in the 2017/2018 academic year. For each of the 17 questions on the e-examination paper, for which CSEs were found, we have subsequently altered the e-assessment question to consider each particular CSE. Dewis uses Performance Indicators (PIs) in the Reporter that enable the academic to view the performance of a student on each question attempt [16]. This is particularly useful in order to differentiate between a student scoring zero by not answering the question or by answering the question incorrectly. Additional PIs have been introduced into the altered question code to capture CSEs when they are triggered.

Using the re-mark feature in Dewis [16], academics are able to re-mark e-assessments using the altered question source code. By re-marking the weekly e-assessments with the new question source code, the additional PIs can identify if students made any CSEs in a particular e-assessment, prior to them taking the e-examination. For the three CSEs illustrated in this paper, we found the results as shown in Table I.

We can see that for all three questions the principal CSE percentage in the e-assessment was less than occurred in the e-examination. A possible explanation for it being lower is that students typically attempt the weekly tests with fresh knowledge, that is, soon after or while they are learning the new concept. It could also be due to students being under more pressure in the e-examination due to it being a high-stakes assessment and sat under controlled conditions.

TABLE I: RESULTS OF THE PRINCIPAL CSES FROM RE-MARKING THE WEEKLY E-ASSESSMENTS WHICH INCLUDED QUESTIONS 3, 7, 11 FROM THE E-EXAMINATION

	Principal CSE Occurences	Incorrect Answers	Principal CSE Percentage
Question 3	85	335	25%
Question 7	25	123	20%
Question 11	73	324	23%

IV. DISCUSSION

Re-marking the weekly e-assessments with the CSE software capture included raised some interesting points. Firstly, we found that for some questions, there were particular random parameters for which the correct answer and the CSE answer were the same. This occurs for example, for Question type 3 (as shown in Fig. 4) for the function $f(t)=2u(t+7)-5u(t+1)$ when the value of $f(4)$ is asked for. In this case, both the correct answer and the CSE answer are equal to -3. Therefore, if a student is presented with this realization of the question and entered -3, Dewis would mark them as having answered the question correctly but it could be that the student had erroneously arrived at that answer by performing a CSE instead. This finding shows the importance of awareness of CSEs related to a problem when coding an e-assessment question. In order to mitigate against such scenarios, the random parameters for the question should be selected such that the correct answer differs from the CSE answer(s).

Secondly, we found that it is possible for more than one CSE to be triggered for some questions. This occurred for the question type detailed in Section III B. In the question presented in the morning version of the e-examination (as illustrated in Fig. 5) the second and third CSEs described in that section result in the same incorrect value, namely 3.80. During the CSE collection process, it was straightforward to determine which CSE students had made by examining their written scripts. However, for instances when the same phenomenon occurs in the weekly e-assessments (when no intermediate workings are available) it is not clear how to decide which CSE led the student to obtain that incorrect answer. Again, this finding shows the importance of awareness of CSEs when selecting parameters for a question. Further, when coding a question, if it is difficult to avoid parameters which trigger several CSEs, careful decisions need to be made when providing enhanced feedback.

V. Conclusion

Having catalogued the 40 CSEs found on the 2018 e-examination and introduced Performance Indicators to capture them in the 17 e-assessment questions the next steps are to create detailed feedback based on students' answers. Thus, for future uses of the questions, if one of the pre-coded CSEs is triggered, the student will be provided with information about what could have gone wrong in their calculation together with extra supportive resources for them to work through. In order to assess the impact of the improved feedback, the enhanced e-assessment questions will be integrated into the weekly e-assessments for the Engineering Mathematics module from the 2019/20 academic year. Using the Dewis Reporter, we will be able to see which students triggered CSEs and hence received the enhanced feedback. These students will be asked to fill in a short questionnaire giving information on the tailored feedback that they received and this will allow us to improve the feedback given. The first semester weekly e-Assessments contain over 100 questions in total. As we build a taxonomy of CSEs, our goal is to enhance a significant proportion of these questions. Through the generation of this additional personalized feedback, our aim is to improve the e-assessment experience for students as well as addressing and tackling misconceptions in a timely fashion.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Indunil Sikurajapathi conducted the research, analysed the data and wrote the first draft of the paper. Karen Henderson took responsibility for the final version of the paper. Karen Henderson and Rhys Gwynllyw organised and supervised the research; all authors discussed progress and results of the research and approved the final version.

References

[1] R. Rees and G. Barr, *Diagnosis and Prescription in the Classroom: Some Common Maths Problems*, Harper & Row, London, UK, 1984.
[2] D. Sleeman, "Mis-generalization: An explanation of observed mal-rules," presented at the Annual Conference of the Cognitive Science Society, 1984.
[3] J. L. Booth, C. Barbieri, F. Eyer, and E. J. Pare-Blagoev, "Persistent and pernicious errors in algebraic problem solving," *Journal of Problem Solving*, vol. 7, pp. 10-23, 2014.
[4] P. Nesher, "Towards an instructional theory: The role of student's misconceptions," *For the Learning of Mathematics*, vol. 7, pp. 33-40, 1987.
[5] E. Fischbein, "Tacit models and mathematical reasoning," *For the Learning of Mathematics*, vol. 9, pp. 9-14, 1989.
[6] J. Confrey, "A review of the research on student conceptions in mathematics, science, and programming," *Review of Research in Education*, vol. 16, pp. 3-56, 1990.
[7] A. Obersteiner, W. Dooren, J. Hoof, and L. Verschaffel, "The natural number bias and magnitude representation in fraction comparison by expert mathematicians," *Learning and Instruction*, vol. 28, pp. 64-72, 2013.
[8] D. Kirshner and T. Awtry, "Visual salience of algebraic transformations," *Journal for Research in Mathematics Education*, vol. 35, pp. 224-257, 2004.
[9] E. Knuth, A. Stephens, N. McNeil, and M. Alibali, "Does understanding the equal sign matter? Evidence from solving equations," *Journal for Research in Mathematics Education*, vol. 37, pp. 297-312, 2006.
[10] N. Rushton, "Common errors in mathematics," *Research Matters: A Cambridge Assessment publication*, vol. 17, pp. 8-17, 2014.
[11] H. Khiat, "A grounded theory approach: Conceptions of understanding in engineering mathematics learning," *The Qualitative Report*, vol. 15, pp. 1459-1488, 2010.
[12] C. Sangwin, *Computer Aided Assessment of Mathematics*, Oxford University Press, Oxford, UK, 2013.
[13] M. Gill and M. Greenhow, "How effective is feedback in computer-aided assessments?" *Learning, Media and Technology*, vol. 33, pp. 207-220, 2008.
[14] M. Greenhow, "Effective computer-aided assessment of mathematics; Principles, practice and results," *Teaching Mathematics and Its Applications*, vol. 34, pp. 117-137, 2015.
[15] P. Race, *Making Learning Happen*, Sage Publications, 2014.
[16] P. Walker, R. Gwynllyw, and K. Henderson, "Diagnosing student errors in e-assessment questions," *Teaching mathematics and its Applications*, vol. 34, pp. 160-170, 2015.
[17] M. Davis, G. Hunter, L. Thalaal, V. Ba, and A. Wooding-Olajorin, "Developing smart tutorial tools to assist students learn calculus, taking account of their changing preferred approaches to learning," in *Proc. International Conference on Intelligent Environments Ambient Intelligence and Smart Environments*, ed. A. Muñoz et al., 2019, pp. 227-238.
[18] N. Falchikov, *Learning Together: Peer Tutoring in Higher Education*, Routledge, London, UK, 2001.
[19] R. Gwynllyw and K. Henderson, "Intelligent marking in summative e-assessment," in *Proc. the HEA STEM Learning and Teaching Conference*, 2012.
[20] R. Gwynllyw and K. Henderson, "A computer aided assessment system for mathematics and statistics," in *Proc. the CETL-MSOR 2008 Conference*, 2009, pp. 38-44.
[21] K. Henderson, R. Gwynllyw, and A. Hooper, "Using electronic exams to provide engineering mathematics students with rapid feedback," in *Proc. the 18th SEFI Mathematics Working Group Seminar on Mathematics in Engineering Education*, 2016, pp. 105-111.

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Declaration:

Indunil Sikurajapathi conducted the research, analysed the data and wrote the paper.
Karen Henderson and Rhys Gwynllyw organised and supervised the research.

CASE STUDY

Students' Perceptions of Enhanced e-Assessment Feedback Addressing Common Student Errors in Mathematics

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Abstract

The Common Student Errors Project (CSE Project) has been running at the University of the West of England (UWE, Bristol) since 2017. The main aim of this project is to introduce a method to detect CSEs and to provide tailored feedback in Engineering Mathematics e-Assessment questions. In this case study we briefly describe the process of collecting CSEs related to Engineering Mathematics and delivering personalised enhanced feedback to students who made CSEs on Dewis e-Assessment questions. We then present how we carried out a questionnaire to gather student perceptions on the enhanced feedback they received. Finally, we present the outcomes of the questionnaire data, the conclusions on students' perceptions of the current enhanced feedback and ascertain possible future directions for further development of the enhanced feedback.

Keywords: e-assessment, Dewis e-Assessment system, Common Student Errors, enhanced feedback.

1. Introduction and Background

The main aim of the Common Student Errors Project (CSE Project) is to introduce a method to detect CSEs and to provide tailored feedback in Engineering Mathematics e-Assessment questions. We have focussed on Engineering Mathematics questions using Dewis as the demonstration platform, however, the method could be useful for other e-Assessment systems and in other contexts and disciplines (CSE Project at UWE, 2019).

Developed at UWE Bristol, Dewis is a fully algorithmic open source e-Assessment system which was primarily designed for numerate e-assessments (Gwynllyw and Henderson, 2009; Gwynllyw and Henderson, 2012). Even though Dewis has been used very successfully over the past decade, it is not being used to its full potential. Therefore, one aim of the CSE project is to develop and use additional features, in order to fully realise the benefits of Dewis.

A Common Student Error (CSE) is an understandable error leading to an incorrect answer due to a student's misconception. For example, answering $a^2 + b^2$ when asked to expand $(a + b)^2$ can be considered as a CSE. This kind of an understandable but incorrect implementation of a process is also called a mal-rule (Rees and Barr, 1984; Sleeman, 1984).

Finding students' perceptions on enhanced feedback delivered through Dewis is the fourth stage of the CSE Project. In the first stage of the research, as described by Sikurajapathi, Henderson, and Gwynllyw (2020), we gathered CSEs made in the Engineering Mathematics 2018 January e-examination, by examining students' written answer scripts along with their corresponding Dewis answers. We found 40 CSEs relating to 17 questions (Sikurajapathi, Henderson, and Gwynllyw, 2020).

Having identified CSEs related to the module, in the second stage we altered the original Dewis question code, including additional scripts using the Perl programming language. Performance Indicators (PIs), are a powerful feature of the Dewis administration reporter tool, as they enable the academic to view the performance of a student on each question attempt. This is particularly useful in order to see whether a particular student scored zero by not answering the question or by answering the question incorrectly. For each identified CSE, we introduced additional PIs to the altered Dewis question code, in order to automatically capture each CSE and provide detailed enhanced feedback when they are triggered.

For the third stage of the project, the amended questions were included in the semester 1 weekly e-Assessments used as summative assessments for the 2019-2020 cohort of the Engineering Mathematics module. Further, nine of the amended questions were presented in a revision test at the end of the first semester. The fourth stage of this study, which we are going to discuss in this paper, comprises an online questionnaire given to those students who received enhanced CSE feedback in either the weekly e-assessments or the end of semester revision test.

2. Objective and Research Questions

The primary focus of our research is to design enhanced feedback to address CSEs and underline mathematical misconceptions of engineering students at UWE Bristol. In other words, we want to develop enhanced feedback which promotes students' conceptual change and facilitates student learning. Further, we want the enhanced feedback to be user-friendly with a coherent structure (clear, organised, detailed and yet simple), and to have ergonomic features (user-friendly format, font, font-size, and appropriate labelling and highlighting).

For example, the enhanced feedback given for a question regarding finding the modulus of a given complex number is shown in Figure 1. The CSE related to this problem was to take $(-2)^2$ equal to -4. For the enhanced feedback we used different colours, a step by step method and equation numbering to provide clear and concise feedback to address students' misconceptions.

The aim of the questionnaire study was to gather students' views on the enhanced feedback they received as a result of triggering a CSE. The main research questions were:

- How and to what extent does the current enhanced feedback help students to change their conceptual understanding and facilitate their understanding of the subject?
- What are their views on the user-friendly features of the enhanced feedback?

3. Research Method

The questionnaire was planned to gather students' views on how and to what extent the current enhanced feedback helped them to change their conceptual understanding and facilitate their understanding of the subject. In addition, we wanted to gather students' views on the user-friendly features of the enhanced feedback.

3.1. Ethical Review of the Research

The questionnaire was designed in accordance with policy, procedures and guidance of the Faculty Research Ethics Committee (FREC) at UWE, Bristol. The questionnaire distribution and collection of data for the research was commenced after receiving written approval from the FREC to undertake research involving human participants.

The Question

Find the modulus $|z|$ of the complex number $z = -2 + 5j$, correct to two decimal places.

The Solution

The modulus of $z = a + jb$ is $|z| = \sqrt{(a)^2 + (b)^2}$,
hence when $z = -2 + 5j$ we find $|z| = \sqrt{(-2)^2 + (5)^2} = \sqrt{29}$.

The value of $|z|$ is 5.38516.. which, to two decimal places, is 5.39.

The Report

Your answer for $|z|$ is 4.58.
Your answer is **not** correct.

> Your incorrect answer seems to have been derived by assuming that $(-2)^2$ equals to -4. This is incorrect.
>
> Please note that $(-2)^2 = 4$.

The modulus of the complex number $z = a + jb$ is,

$$|z| = \sqrt{(a)^2 + (b)^2}. \rightarrow Ⓐ$$

To find $|z|$ when $z = -2 + 5j$, we substitute $a = -2$ and $b = 5$ in Ⓐ

$|z| = \sqrt{(-2)^2 + (5)^2}$, [Note that $(-2)^2 = 4$]

$= \sqrt{4 + 25}$

$= \sqrt{29}$

$= 5.38516..$

$= 5.39$ (to two decimal places)

Figure1: An example of the enhanced CSE feedback

3.2. Questionnaire Design and Distribution

The questions in the questionnaire, shown in Figure 2, fell into two groups: Likert-scale and open-ended. Participants received four closed questions, using a 5-point Likert-scale ranging from “Strongly agree" to “Strongly disagree". For each of the three open-ended questions, a comment box was provided for students to input their response.

The questionnaire was administrated via Qualtrics software (Qualtrics, 2005). Qualtrics is a web-based survey software tool which can be used to conduct publicly available surveys, or to give specific users access to a survey. Using online questionnaires has numerous benefits in terms of cost, time, ease of administration, data collation and analysis (Dillman, 2007). Another advantage of using an online questionnaire was that it was easy to reach all of the students who made CSEs by emailing them with a link to the questionnaire. However, the collected responses were anonymous.

UWE Bristol University of the West of England

QUESTIONNAIRE

Evaluating the effectiveness of the enhanced feedback on the Dewis e-Assessment System

This questionnaire has a number of questions asking you for your feedback on the enhanced feedback you received on Engineering Mathematics weekly test (*include assessment number here*) on the Dewis e-assessment system.

Please tick (√) in the appropriate column alongside the question number on the questionnaire.

Do not worry about projecting a good image. Your answers are **CONFIDENTIAL.**
Thank you for your cooperation.

		Strongly disagree	disagree	Neutral	Agree	Strongly agree
1	The enhanced feedback I received on weekly test (*number*) improved my mathematical understanding.					
2	The enhanced feedback makes me feel confident/comfortable with Engineering Mathematics.					
3	The information in the enhanced feedback is relevant to the question asked.					
4	I am satisfied with the overall structure of the enhanced feedback.					

5. What do you **like** about the enhanced feedback you received?

6. What do you **dislike** about the enhanced feedback you received?

7. Do you have any suggestions for improvement?

Figure 2: Example of the questionnaire sent out to students

The questions were designed to avoid long, double-barrelled, technical, ambiguous, leading or double negative questions or statements. In order to make the questionnaire short and clear we avoided lengthy questions and made sure that the questionnaire fitted on one page (Dillman, 2007). Great care was taken to make the questionnaire visually appealing (Frankfort-Nachmias, 1996). The UWE logo was inserted at the top of the questionnaire to make it more professional and institution-related. In the invitation email it was specifically stated how the participants' responses would be used in the future development of Dewis and hence be valued as a whole by the UWE community (Oppenheim, 1992).

As suggested by Dillman (2007), in order to maximise response and completion rates, a clear indication of how long the questionnaire would take to complete was given in the invitation email. Further, clear instructions were included, together with the purpose of the questionnaire and important information related to the research which were available in a separate 'Participant Information Sheet' (PIS). A link to the PIS, which was placed on the CSE project web page (CSE Project at UWE, 2019), was included in the 'Informed Consent' section at the beginning of the questionnaire.

For each assessment, we identified which students had received enhanced feedback on each question by analysing the additional PIs in the Dewis Reporter. At the end of each weekly test we sent a questionnaire to those identified students. There were some students who received CSE enhanced feedback, and hence the questionnaire, in more than one week. The total number of questionnaires sent by the end of the semester was 336 and these were sent to 196 distinct students, who received CSE enhanced feedback in at least one of their weekly tests.

At the end of the revision test, we identified 129 distinct students who had received enhanced feedback for this test. Since we wanted to gather more responses from the students, we decided to send the questionnaire to all of the students who had received enhanced feedback in some form. There were 78 students who received enhanced feedback for both the weekly and revision tests. Therefore, in order to avoid sending the questionnaire to those students twice, we sent the same questionnaire to the 247 distinct students who had received enhanced feedback for either the end of semester revision test or the weekly tests.

4. Data Analysis

In total, we received 33 responses to the 336 weekly questionnaires and 26 responses to the 247 end of semester questionnaires. The 2019-2020 cohort had 330 students and 247 of these students made at least one CSE in either their weekly tests or the revision test. In total, we received 59 responses to all of the questionnaires sent.

4.1. Analysis of the Likert-scale questions

The first four questions of the questionnaire were in Likert-scale format. Therefore, quantitative methods were used to analyse the participant responses. In the following sections we discuss each of these questions in the questionnaire separately and present figures which show the percentages of each Likert-scale response with the agreement percentage for each statement. It should be noted that percentages do not always total to 100% due to rounding. The agreement percentage (AP) is the number that selected "Agree" or "Strongly Agree" divided by the sum of those participants selecting a response on that question.

Q1: The enhanced feedback I received on weekly test [x] improved my mathematical understanding

Figure 3 presents the participants' responses to the statement '*The enhanced feedback I received on weekly test [x] improved my mathematical understanding*' in the weekly questionnaire and/or the end of semester questionnaire.

This shows that the majority of participants either strongly agreed or agreed that the enhanced feedback they received improved their mathematical understanding. The AP of the participants to the statement is 88% and this figure indicates the participants' positive appreciation towards the conceptual change afforded by the enhanced feedback.

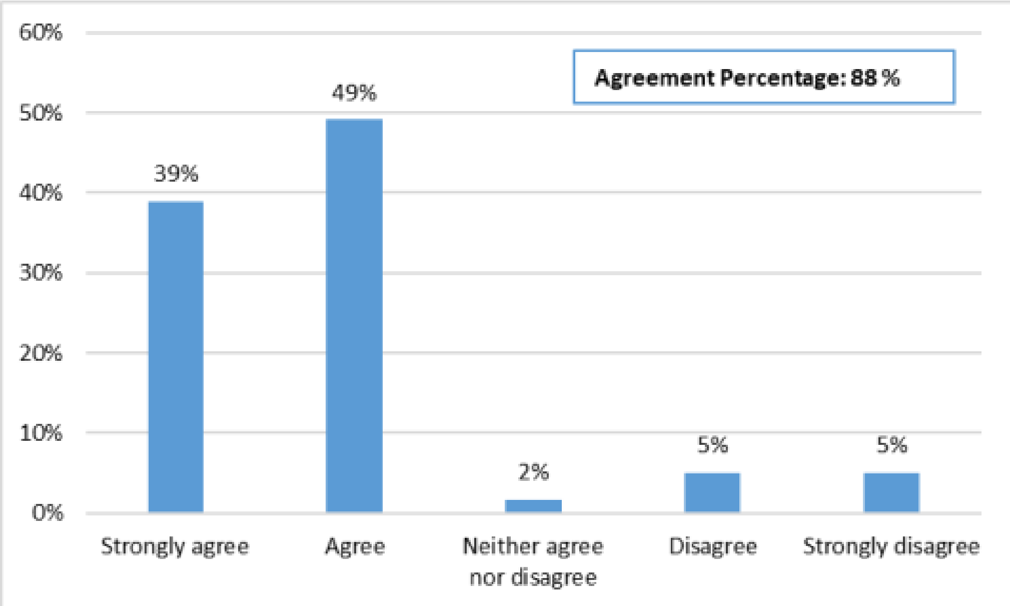

Figure 3: Questionnaire responses to the question "The enhanced feedback I received improved my mathematical understanding"

Q2. The enhanced feedback makes me feel confident/comfortable with Engineering Mathematics

Figure 4 shows the participants' responses to the statement '*The enhanced feedback makes me feel confident/comfortable with Engineering Mathematics*' for the weekly questionnaire and/or the end of semester questionnaire. The results show that the majority of the participants agreed with this statement and the AP of the participants to the statement is 73%.

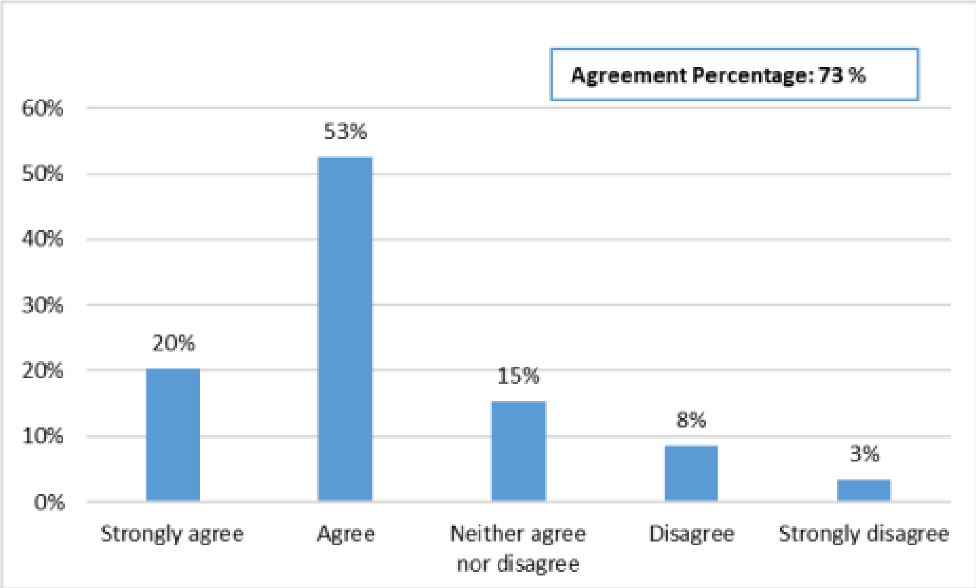

Figure 4: Questionnaire responses to the question "The enhanced feedback makes me feel confident/comfortable with Engineering Mathematics".

Q3. The information in the enhanced feedback is relevant to the question asked

The third statement of the questionnaire is where we are looking for how students feel about the relevance of the enhanced feedback. Figure 5 shows the questionnaire responses to the question *"The information in the enhanced feedback is relevant to the question asked"* for the weekly questionnaire and/or the end of semester questionnaire.

What stands out in Figure 5 is that almost all of the participants agreed or strongly agreed that the information in the enhanced feedback is relevant to the question asked (AP 95%).

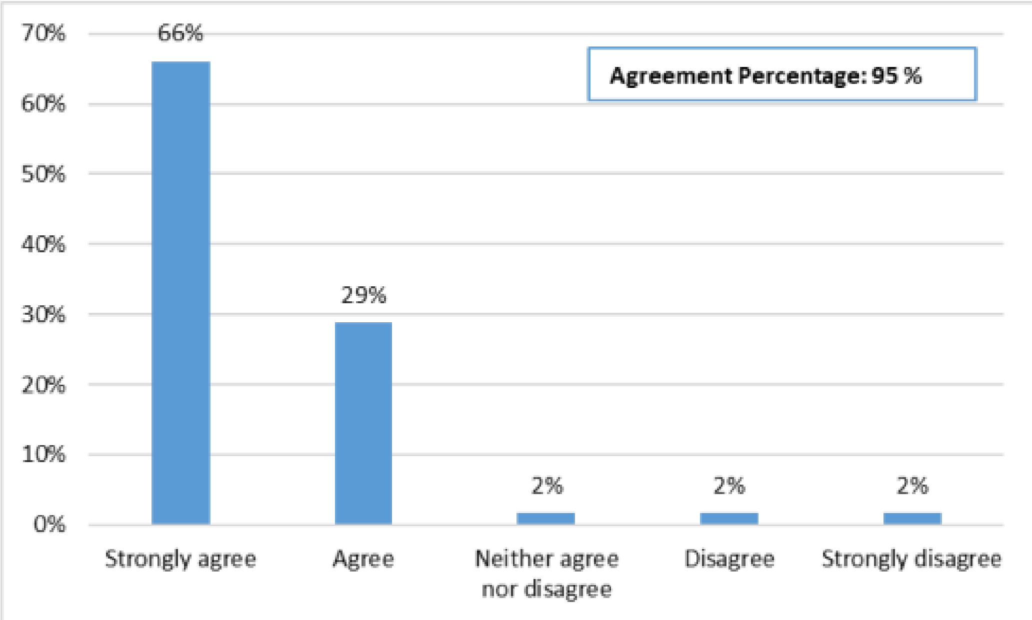

Figure 5: Questionnaire responses to the question "The information in the enhanced feedback is relevant to the question asked"

Q4. I am satisfied with the overall structure of the enhanced feedback

Figure 6 shows the questionnaire responses to the question *"I am satisfied with the overall structure of the enhanced feedback"*. The majority of those who responded to this statement indicated that they were satisfied with the overall structure of the enhanced feedback. The figure shows that the AP for this statement is 87%.

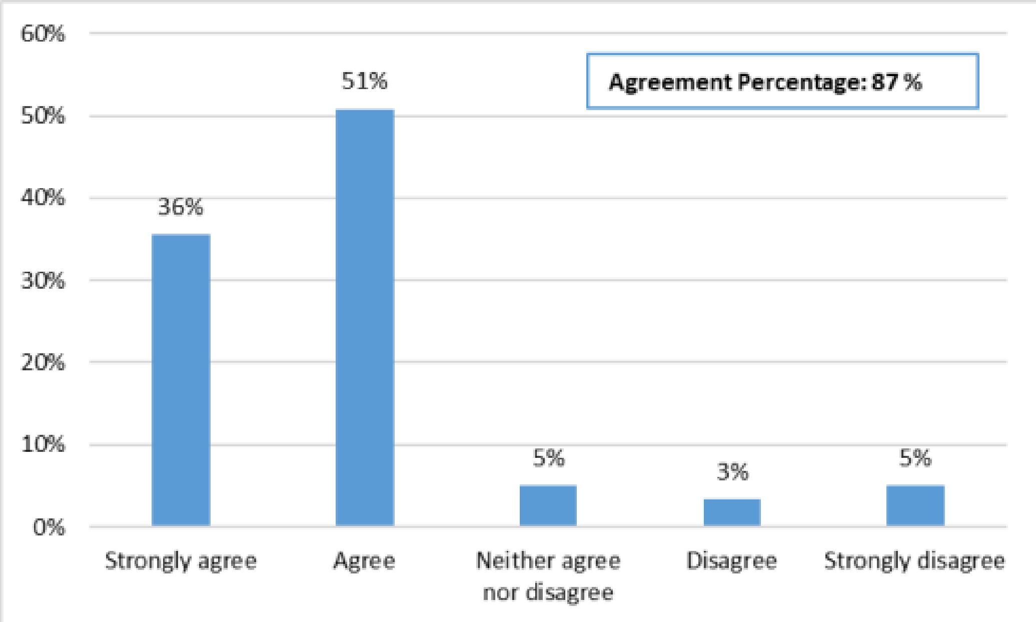

Figure 6: Questionnaire responses to the question "I am satisfied with the overall structure of the enhanced feedback"

4.2. Analysis of the open-ended questions

We used thematic analysis to analyse the three open-ended questions. Thematic analysis is a widely used qualitative method. It is used to analyse qualitative data gathered in the form of open-ended responses to questionnaires (Castleberry and Nolen, 2018).

Thematic analysis is used for identifying, analysing and reporting patterns or themes within data. In their paper, Braun and Clarke (2006) describe how to conduct thematic analysis in six phases (Familiarizing yourself with your data, Generating initial codes, Searching for themes, Reviewing themes, Defining and naming themes, Producing the report). Following the six phases as described by Braun and Clarke (2006), we conducted thematic analyses on the responses to the open-ended questions on the questionnaire for both the weekly and the end of semester questionnaires combined.

Q5. What do you like about the enhanced feedback you received?

From this question, we want to capture what students like about the enhanced feedback so that we can keep those features unchanged when new enhanced feedback is constructed in the future.

Two overarching themes, *Conceptual change* and *User-friendly features*, emerged from a detailed thematic analysis of the texts of students' responses to this question. The sub-themes which emerged from the two aforementioned main themes are summarised in Table 1.

Table 1: Themes resulting from thematic analysis on student responses to the question "What do you like about the enhanced feedback you received?"

Main Themes	Sub-themes
Conceptual change	Correct CSE capture (Correct capture) Facilitate learning (Beneficial) Relevance of the content on CSEs enhanced feedback (Relevance).
User-friendly features	Coherent structure Accessibility

The *Conceptual change* theme highlighted three sub-themes which examined perception on Correct CSE capture (Correct capture), facilitating learning (Beneficial), and Relevance of the content on CSEs enhanced feedback (Relevant).

Under the sub-theme Correct capture, many participants felt that the enhanced feedback they received cleared up their doubts. Further, they claimed that the feedback made them understand why and where they went wrong. The majority of the participants' appreciated the way in which the enhanced feedback helped them to change their misunderstandings/misconceptions of the mathematical concepts and to improve their learning. In the Beneficial sub-theme, a significant number of participants mentioned the benefit they received from the enhanced feedback in improving their understanding. Further, they noted the usefulness and helpfulness of the feedback to their learning and understanding of the subject. In the Relevance sub-theme, a couple of participants mentioned how relevant the received feedback was in their learning. Table 3 in the Appendix contains several examples of such quotes which emerged from the *Conceptual change* theme and three examples of such quotes are given below:

"It makes you feel conscious of errors you made. The fact that it tells you what you've done based on your final input is clever."

"The enhanced feedback got right to the reason the answer was wrong rather than lingering on things already explained above in the solution."

"The Feedback which I received helped me to understand where I was most likely to make errors and showed the correct way of working out solutions."

The *User-friendly features* theme contained two sub-themes: Coherent Structure and Accessibility. Under the sub-theme Coherent Structure, it emerged that many participants liked the structure of the enhanced feedback and particularly highlighted its step-by-step, clear and concise explanations. Several participants appreciated the accessibility features of the enhanced feedback. In particular they commented on its instant availability, quick accessibility, and visibility in different colours. Table 4 in the Appendix shows multiple examples of those quotes which arose from the User-friendly Features Theme. One example from Table 4 is reproduced here:

"The total feedback was overall concise and accessible."

Q6. What do you dislike about the enhanced feedback you received?

From this question we wanted to capture what students disliked about the enhanced feedback so that we can amend and improve the features of future enhanced feedback. Thematic analysis on the responses for this question highlighted four main themes: *Everything is alright*, *Short explanations*, *Less accessibility features* and *Not helpful*.

It was encouraging to see that the majority of the participants said that they were satisfied with the current CSE enhanced feedback and did not indicate any aversion to it. Some comments from the *Everything is alright* theme can be found in Table 5 in the Appendix.

A few participants indicated that the enhanced feedback is very short for some questions and suggested that they would prefer to have more detailed feedback, which would improve it in the future. Two such examples of participants' comments are shown below:

"Some answers can be quite brief so more in depth answers would be great."

"Needs more steps for the student to fully understand what is happening throughout the equation."

The participant who made the above comment also disagreed to all of the Likert-scale statements on the questionnaire except for statement Q3.

Further comments on the *Short explanations* theme can be found in Table 6 in the Appendix.

A few comments related to the *Less accessibility features* theme. Some participants mentioned the issue of visibility of the current enhanced feedback and gave some useful suggestions for increasing its visibility. One participant suggested that moving the enhanced feedback to the general Solution section rather than including it in a separate section (Report section) to avoid scrolling past the enhanced feedback. These inputs were very valuable to us and we will aim to incorporate them and address the issues raised in the future development of the CSE project.

Table 7 in the Appendix shows multiple examples of quotes which arose from this theme. One example of which is given here:

"The incorrect answer could be written right after the correct one rather than right at the very bottom so that it would be easier to understand."

However, only one participant found the enhanced feedback not to be useful and stated that *"It doesn't help me to learn anything."* The same participant strongly disagreed to statements Q1 and Q2 and disagreed to statement Q4. However, the participant agreed with statement Q3.

Q7 Do you have any suggestions for improvement?

The last item of the questionnaire was 'Do you have any suggestions for improvement?' Here we were looking for participants' views on what is lacking in the feedback and for ideas on how to further develop the enhanced feedback in the future. Thematic analysis on the responses for this question revealed three main themes: *Everything is alright*, *Suggestions to improve current features* and *Suggestions for future directions*. Table 2 summarises these themes and all of the sub-themes which emerged from this question.

Table 2: Themes resulting from thematic analysis of student responses to the question *"Do you have any suggestions for improvement?"*

Main Themes	Sub-themes
Everything is alright	Everything is alright
Suggestions to improve current features	Detailed Explanations More Accessibility features
Suggestions for future directions	Enhanced feedback for all the other questions New ideas for further improvement

Most of the participants were satisfied with the enhanced feedback they received and did not give any suggestions for further improvements. A few responses received under the *Everything is alright* theme can be found in Table 8 in the Appendix, and one of these is presented here.

"I think it is as good as it can be. Thank you!"

A few participants echoed the same suggestions that we received for question 6 of the questionnaire. Namely, they suggested providing detailed feedback and making the feedback more accessible, readable and efficient. A few responses received in the *Suggestions to improve current features* theme are detailed in Table 9 in the Appendix and two of these are presented here.

"Include all steps, even if they seem unimportant."

"Provide enhanced feedback not just on hard questions but on easy ones too."

One participant who disagreed to the Likert-scale statements Q1, Q2 and Q4 but agreed to Liker-scale statement Q3 suggested '*To make it more readable and a more efficient design*'.

Some participants provided useful suggestions for future directions, which emerged as a main theme. Within this theme, the comments were categorised into two sub-themes, namely 'Enhanced feedback for all the other questions' and 'New ideas for further improvement'. A few of the comments given in this theme are recorded in Table 10 in the Appendix, and one of these comments is as follows:

"I would also like to know the subject of each question so that I could Google anything that I didn't understand. Another option would be to have a link to the lectures that covered each question, so that if I got a question wrong I could know what lecture covered that topic."

5. Discussion, conclusion and future work

This case study investigated how and to what extent the current enhanced feedback helps students to change their conceptual understanding and facilitate their understanding of the subject, together with their views on the user-friendly features and structure of the enhanced feedback. We were looking for students' views on the effectiveness of the enhanced feedback in correcting their misconceptions and improving their Engineering Mathematics learning. Further, we looked for their satisfaction of the user-friendly features in the enhanced feedback.

The results and the agreement percentages of the Likert-scale questions indicates that the majority of the participants agreed that the enhanced feedback improved their mathematical understanding and made them feel confident/comfortable with Engineering Mathematics. They also indicated that the information in the enhanced feedback is relevant to the question asked and that they are satisfied with the overall structure of the enhanced feedback.

The responses to the Likert-scale questions and the open-ended questions showed that the majority of the participants had positive feelings toward the enhanced feedback. Participants appreciated that the enhanced feedback helped them to address their misunderstanding and to improve their engineering mathematics learning.

The study also gave insight into how students find the user-friendly features of the enhanced feedback. Most of them had positive comments about its coherent structure and ergonomic features. One specific concern that emerged related to improving the visibility of the enhanced feedback. There were some very valuable suggestions of how to improve these features, such as moving it to a more noticeable place on the feedback report, and redesigning the enhanced feedback to have a more efficient and readable structure.

Some other notable suggestions were to include videos within the enhanced feedback and web links to extra materials. The majority of the participants highly valued the effectiveness of the enhanced feedback and suggested/wished to have enhanced feedback for the rest of the questions in the Engineering Mathematics e-assessments.

These suggestions and the highly positive perception of the enhanced feedback suggest that students find the enhanced feedback valuable for their learning. The positive responses on the CSE enhanced feedback have given us the encouragement to continue with the CSE project. We plan to continue our work by searching for further CSEs, providing enhanced feedback on questions delivered through the Dewis e-Assessment system and improving the layout of the enhanced feedback by taking some of the student suggestions on board.

6. Appendix

Table 3: Students' responses of Conceptual Change Theme for the question "*What do you like about the enhanced feedback you received?*"

Sub-Themes in Conceptual Change	**Students' responses**
Correct CSE capture (Correct capture)	*"Told me exactly where I went wrong."* *"It give me a good understanding of what I did."* *"It also explained in detail why I was incorrect."* *"The fact that the feedback tells me where I actually went wrong and if I repeat the test, then I would not make the same mistake."* *"The enhanced feedback got right to the reason the answer was wrong."* *"The Feedback which I received helped me to understand where I was most likely to make errors and showed the correct way of working out solutions."* *"I can clearly see where I went wrong and it gives me a chance to improve."* *"It helps me to make me realize the mistake where I went wrong on some type of questions."* *"It makes you feel conscious of errors you made. The fact that it tells you what you've done based on your final input is clever."* *"I think it is a great model of reinforcing problems of understanding."* *"It made me understand more in depth."*
Facilitate learning (Beneficial)	*"Very useful and well structured. Helps to answer any similar questions."* *"It was certainly useful to receive enhanced feedback alongside the standard feedback."* *"Very useful and helps to further understanding."* *"It helped my understanding."* *"I reckon that the enhanced feedback must be very helpful to those, who struggle with some questions."* *"I have read the feedback and it seemed very helpful and clear to me."*
Relevance of the content on CSEs enhanced feedback (Relevant).	*"Immediate and specific question related instead of a general explanation."* *"Its overall applicability to my work."* *"It's related to the problem."*

Table 4: Students' responses of User-friendly Features Theme for the question "*What do you like about the enhanced feedback you received?*"

Sub-Themes in User-friendly Features	**Students' responses**
Coherent structure	*"Very useful and well structured."* *"The total feedback was overall concise"* *"Step by step method."* *"It also explained in detail why I was incorrect."* *"Short and simple."* *"Clear and concise information."* *"Well detailed with every step explained thoroughly."* *"It shows the correct answer and detailed workings."* *"Clear and concise."* *"It's very well structured so that it is easy to understand."* *"Clear and concise method, made it easier to understand the question."* *"Write all steps of solution."* *"It's well explained."*
Accessibility	*"The total feedback was overall concise and accessible."* *"Its simplicity."* *"That it is instant."* *"It was in a different colour so more visible."* *"Immediate."* *"Accessible feature and introduced to the user."*

Table 5: Students' responses of 'Everything is alright' Theme for the question "What do you dislike about the enhanced feedback you received?"

Main Themes	**Students' responses**
Everything is alright/nothing to dislike	*"Nothing."* *"There is not really much there to dislike, it's just maths feedback."* *"Nothing to dislike."* *"I haven't found any cons regarding the feedback."* *"I find it good enough."* *"No."*

Table 6: Students' responses of 'Short Explanations Theme' for the question "What do you dislike about the enhanced feedback you received?"

Main Theme	**Students' responses**
Short Explanations	*"Some answers can be quite brief so more in depth answers would be great."* *"Some feedback solutions explain steps without showing the working needed for those steps."* *"Sometimes the workings are not easy to understand."* *"Needs more steps for the student to fully understand what is happening throughout the equation."* *"Sometimes it's unclear on how it gets from one step to another."* *"For some questions it is really helpful. For other questions I don't think it goes far enough to explain the workings."* *"I wish the enhanced feedback was more detailed."*

Table 7: Students' responses for 'Less Accessibility Features Theme' for the question "What do you dislike about the enhanced feedback you received?"

Main Theme	**Students' responses**
Less Accessibility features	*"It was below the general feedback and correct answer, so it's easy to just scroll past."* *"It's structure"* *"Needs to be more organised and easier to identify where you made the mistake."* *"The incorrect answer could be written right after the correct one rather than right at the very bottom so that it would be easier to understand."*

Table 8: Students' responses of 'Everything is alright' Theme for the question "Do you have any suggestions for improvement?"

Main Themes	**Students' responses**
Everything is alright/nothing to dislike	*"I think it is as good as it can be. Thank you!"* *"Nothing."* *"It's good enough."* *"No"* *"I think it is as good as it can be. Thank you!"*

Table 9: Students' responses of 'Suggestions to improve current features' Theme for the question "Do you have any suggestions for improvement?"

Sub-Themes	**Students' responses**
Detailed Explanations	*"Include all steps, even if they seem unimportant."* *"Make it a bit clearer to understand."* *"More detailed feedback, especially for integration and differentiation questions."* *"Highlight your mistake, but show other possible common mistakes optionally. That way you can roughly know what to look out for."* *"Include an extra example? Time consuming so understandable if not"*
More Accessibility features	*"To make it more readable and a more efficient design."* *"I would suggest using two columns when designing the layout for the feedback. One should just show my answer. The other shows the right answer with the detailed working."* *"Moving the incorrect answer closer to the correction or right next to it and maybe making it easier to find the questions you got wrong rather than scrolling all the way and having to search for it."*

Table 10: Students' responses of 'Suggestions for future directions' Theme for the question "Do you have any suggestions for improvement?"

Sub-Themes	**Students' responses**
Enhanced feedback for all the other questions	*"I would like more feedback for all question I get wrong, and with a more detailed step by step approach."* *"It doesn't give alternate answers with different questions as an option for more complex questions."* *"Not all questions has enhanced feedback."* *"I would prefer more feedback from Dewis, in particular more steps in how problems are solved."* *"Provide enhanced feedback not just on hard questions but on easy ones too."*
New ideas for further improvement	*"I would also like to know the subject of each question so that I could Google anything that I didn't understand."* *"Another option would be to have a link to the lectures that covered each question, so that if I got a question wrong I could know what lecture covered that topic."* *"Videos of a maths teacher doing each question and talking through each step."*

7. References

Braun, V. and Clarke, V., 2006. Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), pp.77–101.

Castleberry, A. and Nolen, A., 2018. Thematic analysis of qualitative research data: Is it as easy as it sounds? *Currents in Pharmacy Teaching and Learning*. 10, pp. 807–815.

CSE Project at UWE (2019). CSE Project: Participant Information Sheet. Available at: https://fetstudy.uwe.ac.uk/~bin-sikurajapa/dewis/cseproject/docs/Participant_Information_Sheet.pdf [Accessed 24 June 2020].

Dillman, D., 2007. *Mail and Internet surveys: the tailored design method: 2007 update with new internet, visual and mixed-mode guide*. Hoboken, New Jersey: John Wiley & Sons, Inc.

Frankfort-Nachmias, C., 1996. *Research methods in the social sciences*. 5th ed. London: Arnold.

Gwynllyw, R. and Henderson, K., 2009. DEWIS: a computer aided assessment system for mathematics and statistics. *CETL-MSOR 2008 Conference Proceedings*. pp. 38-44.

Gwynllyw, R. and Henderson, K., 2012. "Intelligent marking in summative e-assessment". In: *Proc. HEA STEM Learning and Teaching Conference*.

Oppenheim, A. N., 1992. *Questionnaire design, interviewing and attitude measurement*. 2nd ed. London: Pinter Publications.

Qualtrics, 2005. *Qualtrics* (2019) [computer program]. Available from: https://www.qualtrics.com [Accessed 01 March 2021].

Rees, R. and Barr, G., 1984. *Diagnosis and Prescription in the Classroom: Some Common Maths Problems*. London: Harper & Row.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R., 2020. Using E-Assessment to Address Mathematical Misconceptions in Engineering Students. *International Journal of Information and Education Technology*. 10(5), pp.356–361.

Sleeman, D., 1984. "Mis-generalization: An Explanation of Observed Mal-rules." In: *Proc. The Sixth Annual Conference of the Cognitive Science Society*.

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Indunil Sikurajapathi conducted the research, analysed the data and wrote the paper. Karen Henderson and Rhys Gwynllyw organised and supervised the research.

RESEARCH ARTICLE

Gathering and Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification

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Abstract

This article gives an overview of the interactive book called '*Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)*' which has been produced as a result of the Common Student Errors Project (CSE Project) set at the University of the West of England, (UWE Bristol). The process of creating this CSE Book is discussed in this article, namely, through the systematic collection and compilation of CSEs, and classification of them taxonomically according to a taxonomy presented in existing literature by examining first year Engineering Mathematics students' rough answer scripts, and e-Assessment-stored data. We believe that the CSEs presented in the CSE book would be useful for mathematics teachers when providing feedback to students to correct CSEs. Further, institutions can utilise it in the future development of teaching and support resources to ensure that these CSEs will be addressed to help students to acquire better understanding of mathematics. Moreover, mathematics learners can try these questions online by using the respective hyperlinks given in the CSE Book. If any of the identified CSEs are entered in the solution, then enhanced feedback is provided to correct their misconceptions instantly. Currently, the CSE Book is freely available at UWE Bristol's Repository.

Keywords: Mathematical Common Student Errors, Dewis e-Assessment system, Taxonomy

1. Introduction and Background

1.1 Common Student Errors

Students arrive at an incorrect answer when answering a mathematical question due to variety of reasons. The reasons can be listed as random errors, calculation errors or misreading the questions. These errors lead to incorrect answers or loss of accuracy marks. Many of these errors are made by just a few students. However, some of these errors are commonly made by a considerable number of students. These commonly made errors are sometimes referred to as common errors (Rushton, 2014).

Researchers express different opinions about the difference between errors and misconceptions in the literature. For Confrey (1990), the reasons for both errors and misconceptions are the rules and beliefs that students hold. They argue that the difference between errors and misconceptions is that misconceptions are attached to particular theoretical positions. However, Nesher (1987) uses the term misconceptions to describe systematic errors without reference to a theoretical position.

Rees and Barr (1984) use the term '*mal-rule*' to refer to an understandable but incorrect implementation of a process resulting from a student's misconception. For example, a classic *mal-rule* students make is to answer $a^2 + b^2$ when asked to expand $(a + b)^2$. The term '*bug*' is used by

VanLehn (1982) to refer to a systematic error resulting from wrong steps in the calculation procedure. A *Borrow Across-Zero bug* is a systematic error caused by a student having trouble with borrowing, especially in the presence of zeros (VanLehn, 1982). For example, a student answering 98 when asked to calculate $305 - 117$ would be considered as a *Borrow Across - Zero bug*. In the aforementioned calculation, the student skips the step where the zero changed to nine during borrowing across zero (VanLehn, 1982).

Research has been conducted to identify misconceptions in different areas of mathematics. For example, Brown and Burton (1978) investigated bugs (misconceptions) in high school algebra problems, and Swan (1990) focused on the misconceptions that occur in four operations (addition, subtraction, multiplication and division), and in the interpretation of graphs.

Some Mathematics Education research has explored possible causes and effects of certain mathematical misconceptions and the impact that they have on students' future learning (Booth et al., 2014; Confrey, 1990; Fischbein, 1989; Nesher, 1987; Brown and Burton, 1978). After having investigated bugs (misconceptions) in high school algebra problems, Brown and Burton (1978) discussed possible arithmetic bugs which might lead to some specific algebraic bugs. Booth et al., (2014) conducted a study to assess algebraic misconceptions that algebra students make at school. They concluded that students who make specific persistent errors due to underlying misconceptions in arithmetic may need additional intervention since misconceptions are not corrected through typical instruction. They conclude that these additional interventions can be carried out by targeting individual misconceptions or by improving conceptual understanding throughout the algebra course. The findings of Brown and Burton (1978) and then the findings of Booth et al. (2014) hold the same conclusions, that the arithmetic misconceptions held by students affect their algebraic thinking. Further, Booth et al. (2014) state that these arithmetic misconceptions can obstruct their performance and learning of algebra.

There has been recent research into theorising student errors supported by empirical studies in the topics of natural number bias (Obersteiner et al., 2013), visual saliency (Kirshner and Awtry, 2004) and over-generalisation (Knuth et al., 2006). Rushton (2014) conducted a study of common errors in Mathematics made in certain General Certificate of Secondary Education mathematics papers taken by candidates in England, including an internationally available version, as referenced by examiner reports, and errors were catalogued into themes and subthemes. More recently, Ford et al. (2018) developed a taxonomy of errors made by undergraduate mathematics students. In their study they gathered errors by firstly recalling the most obvious errors that occur and secondly by analysing students' exam scripts to categorise them in a taxonomical manner.

1.2 Assessments, e-Assessments and Feedback in Higher Education

Assessment plays a vital role in higher education. It determines the extent of students' skill and knowledge in order to ensure that they have achieved the desired learning outcomes (Stödberg, 2012). Assessment is considered an integral parts of students' learning. Not only does it promote student learning but it also allows them to receive support in order to improve their learning (JISC, 2010). Preparation and marking of traditional paper-based assessments is an expensive and long process and it also requires a significant amount of time and effort by teachers. To mitigate this situation, the use of information technologies to conduct assessment has significantly risen in higher education (Stödberg, 2012; Rolim and Isaias, 2019).

Over the past years, several e-Assessment systems, such as STACK (Sangwin, 2004), Dewis (Gwynllyw and Henderson, 2009), Math e.g. (Greenhow and Kamavi, 2012), and Numbas (Foster at al., 2012) have been developed at several universities in the UK. Easy accessibility and advantages of e-Assessment systems have led mathematics departments in many universities to conduct formative and summative assessments in the form of e-Assessments (Sangwin, 2013).

Properly performing e-Assessments are hugely beneficial for both teachers and students. Some benefits of using e-Assessment are its capability to provide instant and tailored feedback, that it can be accessed in different geographical locations at any time, and that students can undertake online tests several times to improve their learning (Sikurajapathi, Henderson and Gwynllyw, 2020; Gwynllyw and Henderson, 2009).

Dermo (2009) and Gikandi et al (2011) posit that high quality and accurate feedback delivered in a timely manner plays an important role in students' learning. In addition, by reviewing and studying this feedback, students can identify their weakness as well as their strengths in order to achieve continuous improvement in their learning. Gill and Greenhow (2008) conducted a study to find out the effectiveness of e-Assessment feedback and found that students improve their performance by engaging with the feedback provided in e-Assessments. Therefore, Greenhow (2015) suggests that e-Assessments which select questions based on pedagogic principles should be promoted as a learning tool due to its capability of providing effective feedback.

E-Assessments cannot act very flexibly like a human marker when faced with ill-posed or unanticipated student responses (Greenhow 2015). Detecting CSEs on traditional paper-based assignments compared to e-Assessments is more straightforward since the human marker has access to the students' intermediate workings and thus can spot when a CSE has been made. E-Assessment systems cannot easily point out CSEs on student answers since typically few intermediate working steps are submitted. Also, each student attempts a different but equivalent version of the question due to the use of random parameters (Walker et al, 2015).

In their paper, Walker et al (2015) states that an e-Assessment would act more like a human marker, if it could detect and report CSEs, and provide effective and tailored feedback instantly by correcting students' misconceptions. Sikurajapathi, Henderson and Gwynllyw, (2021) developed a method to detect CSEs and to provide tailored feedback in Engineering Mathematics e-Assessment questions. Sikurajapathi, Henderson and Gwynllyw, (2021) then conducted a questionnaire to find out the effectiveness of addressing CSEs in e-assessments through enhanced feedback. The questionnaire findings reveal that the majority of participants had positive feelings towards the CSE enhanced feedback. Students appreciated that the CSE enhanced feedback helped them to correct their misunderstandings and to improve their engineering mathematics learning. The highly positive perception of the enhanced feedback suggests that students find the CSE enhanced feedback valuable and that it helped them to correct conceptual understanding while improving their learning (Sikurajapathi, Henderson and Gwynllyw, 2021).

1.3 Dewis e-Assessment System

Dewis is a fully algorithmic open-source e-Assessment system, which was primarily designed and developed for numerate e-Assessments by a team of Mathematicians, Statisticians and Software Engineers at UWE Bristol (Gwynllyw and Henderson, 2009; Gwynllyw and Henderson, 2012). Dewis supports different question input types such as numerical inputs, matrices, vectors, algebraic expressions, multiple-choice, multiple-selection, graphical input, and computer programs. It has a lossless data collection feature and a number of student-friendly features, such as shutdown recovery and pre-processing checks on student input.

Over the past decade, Dewis has been used very successfully to facilitate both formative and summative e-Assessments across a number of modules, delivered to students in a wide range of fields, e.g. Business, Computer Science, Nursing, Software Engineering, Engineering, Mathematics and Statistics. One aim of the CSE project is to enhance the full potential of Dewis, by developing and using additional features allowing Dewis to detect CSEs and to provide instant tailored feedback.

1.4 The Common Student Errors Project at UWE Bristol

The CSE project at UWE began in 2017 with an aim of developing a technique to detect CSEs and to provide tailored feedback in Dewis e-Assessment questions, used in a first year Engineering Mathematics module (CSE Project at UWE, 2019; Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021). We started the project with the aim of answering the following research questions:

- What CSEs do first year Engineering Mathematics students make in e-Assessment questions?
- How to detect CSEs and improve Dewis feedback to address these CSEs?

There are several benefits to answering these research questions. Even though this research has been done in a particular context using the Dewis e-Assessment system, the research outcomes contribute to the knowledge to inform more general practice in assessment and learning. For example, the collection of mathematical CSEs collected during this research is not only beneficial for first year Engineering mathematics students and lecturers, but also it is equally beneficial for secondary, and first year university level mathematics students and teachers. The CSE collection presented in Sikurajapathi, Henderson and Gwynllyw (2022) can be used to correct students' mathematical misconceptions either in hand-written assessments or e-assessment questions.

Further, this CSE detecting technique will be beneficial to several disciplines and organisations that either use Dewis or any other e-assessment system which has features to give dynamic feedback based on a student answer. The new knowledge raised from this research can be used in any e-assessment system so that it emulates a human marker to provide instant enhanced feedback highlighting possible CSEs. This will help students to correct their mathematical misconceptions. Also, teachers can use the findings to identify areas in which more help is needed in student learning. Integrating the research outcomes from the CSE project into other e-assessment systems will be beneficial to generations to come (Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021; Sikurajapathi, Henderson and Gwynllyw, 2022).

The CSE Project involves five stages (Stage One: Data (CSEs) Collection; Stage Two: CSE code Development; Stage Three: CSE code Trial Phase; Stage Four: Students' Perceptions on CSE Feedback and Stage Five: Impact of CSE Project). Detailed information about these five stages and other findings can be found in CSE Project at UWE Bristol (2019), Sikurajapathi, Henderson and Gwynllyw (2020) and Sikurajapathi, Henderson and Gwynllyw (2021).

2. Creating Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)

2.1. Gathering Mathematical CSEs in in e-Assessments Questions

The main aim of the CSE Project at UWE Bristol was to identify CSEs made in First Year Engineering Mathematics e-Assessment questions. The CSEs presented in the CSE Book were collected by examining the 2017-2018 and 2018-2019 e-examination data on the Dewis e-Assessment system and from students' rough work scripts. These e-examinations were run using the Dewis e-Assessment system and were held under controlled conditions. The e-examinations were held in two sessions (morning and afternoon) to mitigate logistic issues. In each session, all of the students received the same, fixed parameter questions. During the e-examination, students were given booklets to use for their rough work. These booklets were used by students to work through the mathematical questions before submitting their final answers on Dewis.

All of the CSEs that students made are documented in the CSE Book, regardless of whether they are mal-rules, bugs, slips, misconceptions, systematic errors etc. The reason for this is that all of these CSEs can be useful for educators, institutions, assessment makers, and most importantly for mathematics learners. Altogether 65 CSEs were identified in the following different topics areas of Engineering Mathematics: Algebra, Unit-step functions, Wave forms, Trigonometric functions, Differentiation, Implicit differentiation, Partial differentiation, Mean Value Theorem, Complex numbers, Geometric series, Maclaurin Expansion, Centre of Mass, Integration by parts, Volume of revolution and Dimensions.

This CSE Book (Sikurajapathi, Henderson and Gwynllyw, 2022) can be freely access at UWE Bristol's Repository on Public URL: https://uwe-repository.worktribe.com/output/9303961

2.2. Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification

All of the CSEs found in the course of the CSE project are documented in a systematic order in the CSE book together with their mathematical taxonomy coding. Here we adapted the general taxonomy proposed by Ford et al. (2018) to select and categorise only those CSEs which are relevant to e-assessment.

The theoretical study of classification, including its bases, principles, procedures and rules is called a taxonomy (Ford et al., 2018; Simpson, 1961, p.11). The entities in a successful taxonomy can be verifiable by observation and will offer both an appropriate and suitable class for each entity (Ford et al., 2018; Bailey, 1994, p.3). The taxonomy of cognitive mechanisms and the phenomenological taxonomy can be considered as the two main styles that can be used to categorise mathematical errors (Senders and Moray, 1991, Ford et al., 2018).

The taxonomy introduced by Ford et al. (2018) was developed to categorise the errors which undergraduate mathematics students make. Ford et al. (2018) identified six main error categories by firstly recalling obvious mathematical errors that occur among mathematics undergraduates and secondly by analysing a selection of students' paper-based exam scripts from first year undergraduate mathematics courses. These main categories were named as Errors of slips of action (S), Errors of understanding (U), Errors in choice of method (CM), Errors in the use of a method (UM), Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C). Here we sought to use the same Main Categories, Codes and Errors given in the taxonomy introduced by Ford et al. (2018) to categorise mathematical CSEs in the e-Assessment questions.

The CSEs that we have found during the CSE project only fall into four of the error categories (S, U, CM and UM) from the Ford et al. (2018) taxonomy. Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C) were not found among the CSEs made by the Engineering Mathematics students, due to the nature of the questions asked and the nature of the system used to deliver the questions. None of the e-Assessment questions delivered by Dewis involve mathematical theorems and proofs and hence Errors related to proof (P) were not viable in this CSE collection. Further, the e-examination did not contain questions that required student's communication of their mathematical solutions, correct use of notation or labelling and qualitative judgements on clarity of expression. Therefore, errors in student's communication of their mathematical solutions (C) were not found in this CSE collection. Further, a few of the CSEs found fall into two categories due to the mix of misconceptions made by the students as they arrived at their incorrect answer.

Under the category Errors of slip of action (S), three main errors, namely copying error, careless errors on simple calculations, and incorrect algebraic manipulation were identified. A total of 13 out of 65 CSEs were found to fall into the Errors of slip of action category (S).

Seven main errors were identified under the Errors of understanding (U) category, such as confusing different mathematical structures, incorrect argument, lack of consideration of potential indeterminate forms, proposed solution is not viable, definition/method/theorem not recalled correctly, partial solution given and Incorrect assumptions. In total 43 CSEs are in the Errors of understanding category.

Only one main error was found in each of the Errors in choice of method (CM) and Errors in use of method (UM) categories. Five CSEs were grouped into the main error of applying an inappropriate formula/method/theorem in CM. There were 9 CSEs which fell into Error in use of an appropriate definition/method/theorem in the UM category. Table 1 shows how we categorised the CSEs we found related to e-Assessment questions into Main Categories, relevant Codes and Errors using the taxonomy introduced in Ford et al. (2018) together with examples from an e-Assessment context.

Table 1: Taxonomy of Mathematical Common Student Errors in e-Assessments

Main Category	**Code**	**Error**	**Examples**
Slip of action	S1	Copying error	Incorrect copying of the question
			Mistake copying/ submitting answer into e-assessment
			Incorrect interpretation of the question
	S2	Careless errors on simple calculations	Overlooking negative signs
			Omission of denominator
	S3	Incorrect algebraic manipulation	Incorrect division of two complex numbers
			Sum of product is split as a product of two sums
			Incorrect handling of powers

Errors of understanding	U1	Confusing different mathematical structures	Confusing the structure of completing the square and the quadratic equation
			Stating that a unit step function is a number
	U2	Incorrect argument	Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives
			Taking the integration of the product of two functions as the product of individual integrals
	U3	Lack of consideration of potential indeterminate forms	Taking the square of a negative number to be negative
	U4	Proposed solution is not viable	Angle is not within the given range
	U5	Definition/method/ theorem not recalled correctly	Method of completing the square is not recalled correctly
			Definition of waveform properties not recalled correctly
			Method of differentiating a standard function is not recalled correctly
			Method of solving trigonometry equation is not recalled correctly
			Chain rule is not recalled correctly
			Method of Partial differentiation not recalled correctly
			Method of differentiating implicit functions is not recalled correctly
			Mean value theorem is not recalled correctly
			Method of calculating the argument of a complex number is not recalled correctly
			Binomial theorem is incorrectly followed
			Definition of Centre of Mass is not recalled correctly

			Method of finding the principal value of the argument of a complex number is not recalled correctly
			Method of integrating not recalled correctly
			Definition of volume of revolution is not recalled correctly
	U6	Partial solution given	Correct workings but unfinished solution
	U7	Incorrect assumptions	Incorrect assumptions on the mean value theorem
			Taking dimension of velocity is $[v] = [MT^{-1}]$
Errors in choice of method	CM1	Applying an inappropriate formula/ method/ theorem	Uses a method which is not relevant in the situation
			Uses a formula which is not relevant in the situation
Errors in use of method	UM2	Error in use of an appropriate definition/ method/ theorem	Error in the use of the chain rule
			Error in use of partial differentiation method
			Incorrect units applied
			Method finding the volume of revolution is incorrectly followed

2.3. Guide to the CSE Recording Template

Each CSE found to date has been recorded using the template as shown in Table 2 below. The template contains seven areas and each area and its contents are described in detail below.

① The link to the online Dewis e-assessment question is available here. The reader may access the online question by clicking the Question hyper-link. By attempting the question and answering with a relevant CSE response, it is possible to see how Dewis detects the CSE and provides instant tailored feedback to address the CSE made in the solution.

② In this area, a screenshot of the Dewis question is given.

③ The correct solution to the question is presented in brief here.

④ The taxonomy code of the CSE, which is presented in ⑤ , is given here.

⑤ A sample of the CSE and the incorrect answer(s) that led from it is presented here. At the top of this area, the CSE error is summarised by a statement which is presented in red text. Then the detailed steps of the exact way the CSE is made and the solution as written by students in their rough work booklets is presented. We use tilde (~) on the CSE answer to differentiate it from the correct answer. For example, in Table 2, the CSE answer for this question is denoted as, $\tilde{f}(2) = 55$ in red text.

⑥ In this section, the number of CSE answers made, the total incorrect answers made in the question and the CSE percentage for each year are presented as No. of CSEs /No. incorrect answers **(CSE %)**. For example, in Table 2, in the 2017-18 exam, this particular CSE was made by 35 out of the 86 students who gave an incorrect answer to this question; therefore, the CSE percentage is **41%**. This data is presented in this area as 35/86 (**41%**). Similarly, the data for 2018-19 is presented as 32/100 (**32%**).

⑦ The exam year that data was collected from is presented here. Table 2 shows that 35/86 (**41%**) and 32/100 (**32%**) presented in ⑥ relate to the years 2017-18 and 2018-19 presented in ⑦ respectively.

Table 2: CSE Recording Template

Question ①			
The function $f(t) = 7u(t+5) - 3u(t-4)$ where $u(t)$ represents the unit step function. Calculate the value of $f(2)$. Enter $f(2)$: ②			
Correct Solution			
$f(t) = 7u(t+5) - 3u(t-4)$ $f(2) = 7u(2+5) - 3u(2-4)$ $= 7u(7) - 3u(-2)$ $= 7 \times 1 - 3 \times 0$ $f(2) = 7$ ③			
CSE 1 related to this question	**CSE Taxonomy Code:**	**U1** ④	
Answer was derived by assuming $u = 1$ and not a function. $f(t) = 7u(t+5) - 3u(t-4)$ $f(2) = 7u(7) - 3u(-2)$ $\bar{f}(2) = 7(7)u - 3(-2)u$ $\bar{f}(2) = 49u + 6u$ $\bar{f}(2) = 55u$ since $u = 1$ $\bar{f}(2) = 55$ ⑤			
No. of CSEs /No. incorrect answers (CSE %)	35/86 (**41%**) 32/100 (**32%**) ⑥	**Date collected**	**2017-18** **2018-19** ⑦

3. Common Student Error Examples

In this section we present examples of CSEs in each taxonomical category (Slip of action, Errors of understanding, Errors in choice of method, and Errors in use of method). These and the rest of the CSEs we found in the CSE Project can be found in UWE Bristol's Repository (Sikurajapathi, Henderson and Gwynllyw, 2022).

3.1. Common Student Errors due to Slip of Action

Table 3 shows a CSE related to a question in Algebra (Completing the Square) (see Section 2.1.1. Sikurajapathi, Henderson and Gwynllyw, (2022)). Students' answer scripts indicated that even though students had solved the question correctly, they submitted incorrect answers for b which corresponded to the negative of the correct value of b. Therefore, this CSE can be considered as copying error when submitting answer into e-assessment. In 2017-2018, 28 students, out of the 56 who answered this question incorrectly (50%) made this CSE. In 2018-2019, 33 students from 57 who answered this question incorrectly (58%) made the same mistake.

Table 3: CSE in Algebra (Completing the Square) Question due to Slip of action in algebra

Question			
The expression $t^2 - 12t + 40$ can be expressed in the form: $a(t-b)^2 + c$ where a, b and c are constants. Calculate the values of these constants - note that all these solutions are integers: Enter the value of a [ ] Enter the value of b [ ] Enter the value of c [ ]			
Correct Solution			
$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$ $= (t-6)^2 + 4$ $a = 1, \quad b = 6$ and $c = 4$			
CSE 1 related to this question	**CSE Taxonomy Code:**	**S1**	
Give answer $\tilde{b}$ which corresponds to the negative of the correct value of b. $t^2 - 12t + 40 = (t-6)^2 - 36 + 40$ $= (t-6)^2 + 4$ $\tilde{b} = -6$ and $c = 4$			
No. of CSEs /No. incorrect answers (CSE %)	28/56 (**50%**) 33/57 (**58%**)	**Date collected**	**2017-18** **2018-19**

3.2 Common Student Errors due to Errors of Understanding

Table 4 shows a CSE related to a question on complex numbers (rectangular form) (see Section 3.3.1. Sikurajapathi, Henderson and Gwynllyw, (2022)) Students' answer scripts indicated that the square of a negative number was taken to be negative. Therefore, this CSE can be considered as lack of consideration of potential indeterminate forms. In 2017-2018, 40 students, out of the 57 who answered this question incorrectly **(70%)** triggered this CSE.

Table 4: CSE in Complex Number (Rectangular Form) Question due to Error of understanding

Question			
Find the modulus $\lvert z\rvert$ of the complex number $z=-2+5j$, correct to <u>two</u> decimal places. Enter $\lvert z\rvert$ correct to 2 decimal places: [ ]			
Correct Solution			
$z=-2+5j$ $\lvert z\rvert=\sqrt{(-2)^2+5^2}$ $=\sqrt{4+25}$ $=\sqrt{29}$ $\lvert z\rvert=5.39$			
CSE 1 related to this question	**CSE Taxonomy Code:**	**U3**	
Taking $(-n)^2=-n^2$ $z=-2+5j$ $\lvert z\rvert=\sqrt{(-2)^2+5^2}$ $\widetilde{\lvert z\rvert}=\sqrt{-4+25}$ $=\sqrt{21}$ $\widetilde{\lvert z\rvert}=4.58$			
No. of CSEs /No. incorrect answers (CSE %)	40/57**(70%)**	**Date collected**	**2017-18**

3.3. Common Student Errors due to Errors in Choice of Method

Table 5 shows a CSE related to a question on infinite geometric series (see Section 4.1.2. Sikurajapathi, Henderson and Gwynllyw, (2022)). Students' answer scripts indicated that 34 students out of 67 who answered this question incorrect (51%) just summed the first four terms instead of using the formula to find the sum of the infinite series. Therefore, this CSE can be considered as applying an inappropriate formula in Error in Choice of Method.

Table 5: CSE in Infinite Geometric Series Question due to Errors in Choice of Method

Question			
Consider the following geometic series, S, where: $S = 2 + 2(0.7) + 2(0.7)^2 + 2(0.7)^3 \cdots$. Write down the first term, a and the common ratio, r in the boxes below. Enter a: [ ] Enter r: [ ] Hence calculate the sum, S and enter your result in the box below. Enter S (to three decimal places) here: [ ]			
Correct Solution			
The first term $a = 2$. The common ratio $r = 0.7$. The sum of an infinite series (S) exists, provided $\lvert r \rvert < 1$. $S = \frac{a}{1-r} = 6.667$			
CSE 1 related to this question	**CSE Taxonomy Code:**	**CM1**	
Finding the sum of first four terms instead of the sum of the infinite series. $\tilde{S} = \frac{a(1-r^n)}{1-r}$, $\tilde{S} = \frac{2(1-0.7^4)}{1-0.7}$, $\tilde{S} = 5.066$			
No. of CSEs /No. incorrect answers (CSE %)	34/67(**51%**)	**Date collected**	**2017-18**

3.4. Common Student Errors due to Errors in Use of Method

Table 6 shows a CSE related to differentiating $f(x) = cos^4(3x)$ (See Section 5.1.2. Sikurajapathi, Henderson and Gwynllyw, (2022)). 22 students out of 73 **(30%)** incorrectly answered that the differentiation of $f(x)$ is $-12\sin^3(3x)$ due to an error in the use of the Chain Rule. Therefore, this CSE can be considered as an error in use of an appropriate method.

Table 6: CSE in Differentiation (Chain Rule) Question due to Errors in Use of method

Question			
Select the most appropriate method to use in order to find the derivative of $f(x) = \cos^4(3x)$. Select Hence find $\frac{df}{dx}$ as a function of x. Enter the answer as a function of x:			
Correct Solution			
$f(x) = cos^4(3x)$ $f'(x) = -4 \times \cos^3(3x) \times \sin(3x) \times 3$ $f'(x) = -12\sin(3x)\cos^3(3x)$			
CSE 2 related to this question	**CSE Taxonomy Code:**	**UM2**	
Taking $\frac{d}{dx}(cos^n(ax)) = -a \times sin^{n-1}(ax) \times a = -a^2 sin^{n-1}(ax)$ $f(x) = cos^4(3x)$ $f'(x) = -4 \times \sin^3(3x) \times 3$ $f'(x) = -12\sin^3(3x)$			
No. of CSEs /No. incorrect answers (CSE %)	22/73**(30%)**	**Date collected**	**2017-18**

4. Discussion, conclusion and future work

This article presents an overview of the CSE Book created by collecting and compiling CSEs systematically by examining First Year Engineering Mathematics students' rough answer scripts, and Dewis e-Assessment-stored data. All of the CSEs found in this process have been categorised taxonomically. One of the special features of this book is that it provides hyperlinks to each question on the Dewis e-Assessment system in order to facilitate the reader to try these questions online. If any of the identified CSEs are submitted as answers, then enhanced feedback will be provided, which aims to correct any misconceptions in a timely manner.

The information in this book may be used to inform teachers so that they can provide students with a better understanding of the mathematical skills and knowledge while teaching the subject. It may also be useful for institutions as they can utilise it in the future development of teaching materials to ensure that these CSEs will be addressed. Further, the content of this book can be used to develop support materials and resources to address CSEs which will help students to acquire better understanding of mathematics. In addition, students who learn mathematics at university level or in secondary school can refer to this booklet to address their misconceptions and can try the Dewis questions several times. Since, in each attempt, Dewis produces questions with random parameters, student can use this facility to correct their misconceptions by practicing the same question but with different parameters.

We anticipate that this book will be useful to identify and address some misconceptions that students have in mathematics. We plan to continue with this research and will update the book if we find new CSEs in the future. Currently, the CSE Book is freely available at UWE Bristol's Repository.

5. References

Bailey, K.D. (1994) *Typologies and Taxonomies: An Introduction to Classification Techniques*. Thousand Oaks, CA: Sage Publications.

Booth, J.L., Barbieri, C., Eyer, F. and Pare-Blagoev, E.J. (2014) Persistent and Pernicious Errors in Algebraic Problem Solving. *Journal of Problem Solving*. 7, pp. 10-23.

Brown, J. S., & Burton, R. R. (1978) Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science*. *2*(2), 155–192. https://doi.org/10.1207/s15516709cog0202_4

Confrey, J. (1990) Chapter 1: A Review of the Research on Student Conceptions in Mathematics, Science, and Programming. *Review of Research in Education*, 16(1), pp. 3–56. doi: 10.3102/0091732X016001003.

CSE Project at UWE Bristol (2019) *Diagnosing and Remediating Mathematical Common Student Errors in e-Assessment Questions: A Case Study*. Available from: https://fetstudy.uwe.ac.uk/~bin-sikurajapa/dewis/cseproject/ [Accessed 24 February 2022].

Dermo, J. (2009) E-Assessment and the student learning experience: a survey of student perceptions of e-assessment. *British Journal of Educational Technology*, 40(2), 203–214.

Fischbein, E. (1989) Tacit Models and Mathematical Reasoning. For the Learning of Mathematics. *For the Learning of Mathematics*. 9 (2), pp. 9-14.

Ford, S., Gillard, J. and Pugh, M. (2018) Creating a Taxonomy of Mathematical Errors For Undergraduate Mathematics. *MSOR Connections*. 18 (1), pp. 37-45.

Foster, B., Perfect, C. and Youd, A. (2012) A Completely Client-side Approach to E-assessment and E-learning of Mathematics and Statistics. *International Journal of E-assessment*. 2 (2), pp. 1-12.

Gikandi, J. W., Morrow, D., & Davis, N. E. (2011) Online formative assessment in higher education: a review of the literature. *Computers & education*, 57(4), 2333–2351.

Gill, G. and Greenhow, M. (2008) How Effective Is Feedback in Computer-aided Assessments?. *Learning, Media and Technology*. 33 (3), pp. 207-220.

Greenhow, M. (2015) Effective computer-aided assessment of mathematics; Principles, practice and results," *Teaching Mathematics and Its Applications*, 34, pp. 117-137.

Greenhow, M. and Kamavi, K. (2012) Maths e.g. - a Web Assessment Application for Stem and Beyond. *Proceedings of the HEA Stem Learning and Teaching Conference*. DOI: 10.11120/stem.hea.2012.062

Gwynllyw, R. and Henderson, K. (2009) Dewis: A Computer Aided Assessment System for Mathematics and Statistics. *CETL-MSOR 2008 Conference Proceedings*. pp. 38-44.

Gwynllyw, R. and Henderson, K. (2012) Intelligent Marking in Summative E-assessment. *In: Proc. HEA STEM Learning and Teaching Conference.*

JISC. (2010) Effective assessment in a digital age: A guide to technology-enhanced assessment and feedback. *Technology enhanced Assessment*, 26–28.

Kirshner, D. and Awtry, T. (2004) Visual Salience of Algebraic Transformations. *Journal for Research in Mathematics Education*. 35, pp. 224-257.

Knuth, E., Stephens, A., McNeil, N. and Alibali, M. (2006) Does Understanding the Equal Sign Matter? Evidence from Solving Equations. *Journal for Research in Mathematics Education*. 37, pp. 297-312.

Nesher, P. (1987). Towards an instructional theory: The role of student's misconceptions, *For the Learning of Mathematics*, vol. 7, pp. 33-40,

Obersteiner, A., Dooren, W., Hoof, J. and Verschaffel, L. (2013) The Natural Number Bias and Magnitude Representation in Fraction Comparison by Expert Mathematicians. *Learning and Instruction*. 28, pp. 64-72.

Rees, R. and Barr, G., (1984) *Diagnosis and Prescription in the Classroom: Some Common Maths Problems*. London: Harper & Row.

Rolim, C. and Isaias, P. (2019) Examining the Use of E-assessment in Higher Education: Teachers and Students' Viewpoints. *British Journal of Educational Technology*. 50 (4), pp. 1785-1800.

Rushton, N. (2014). Common errors in Mathematics. *Research Matters: A Cambridge Assessment publication*, *17,* 8-17.

Sangwin, C. (2004) Assessing Mathematics Automatically Using Computer Algebra and the Internet. *Teaching Mathematics and Its Applications*. 23 (1), pp. 1-14.

Sangwin, (2013) *Computer Aided Assessment of Mathematics*. Oxford, UK: Oxford University Press.

Senders, J. and Moray, N. (1991) *Human Error: Causes, Prediction and Reduction*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R., (2020) Using E-Assessment to Address Mathematical Misconceptions in Engineering Students. *International Journal of Information and Education Technology*. 10(5), pp.356–361.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2021) Students' Perceptions of Enhanced E-assessment Feedback Addressing Common Student Errors in Mathematics. *MSOR Connections*. 19 (2), pp. 10-27.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2022) *Collection of Taxonomically Classified Mathematical Common Student Errors in E Assessments – (CSE Book)*. [online]. University of the West of England Bristol, United Kingdom. Available from: https://uwe-repository.worktribe.com/output/9303961 [Accessed 08 April 2022].

Simpson, G.G. (1961) *Principles of Animal Taxonomy*. New York: University Press.

Stödberg, U. (2012) A research review of e-assessment. *Assessment & Evaluation in Higher Education*, 37(5), 591–604.

Swan, M. (1990) Becoming numerate: developing conceptual structures. In S. Willis (Ed) *Being numerate: what counts?* Victoria: Australian Council for Educational Research.

VanLehn, K. (1982) Bugs are not enough: Empirical studies of bugs, impasses and repairs in procedural skills. *Journal of Mathematical Behavior*, 3(2) pp. 3-71.

Walker, P., Gwynllyw, R., and Henderson, K., (2015) Diagnosing Student Errors in E-assessment Questions. *Teaching Mathematics and Its Applications*. 34, pp. 160-170.

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Declaration:

Indunil Sikurajapathi conducted the research, analysed the data and wrote the paper. Karen Henderson and Rhys Gwynllyw organised and supervised the research.

RESEARCH ARTICLE

Correct for the wrong reason: why we should know more about Mathematical Common Student Errors in e-Assessment questions

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Abstract

Students may arrive at an incorrect answer when answering a mathematical question due to several reasons, such as random errors, calculation errors or misreading the question. Such errors are sometimes referred to as Common Student Errors (CSEs). This article explains why it is important to know more about Mathematical CSEs in e-Assessment questions, using several examples encountered while conducting the CSE Project at the University of the West of England (UWE Bristol). The CSE Project at UWE Bristol began with an aim of developing a technique to detect CSEs and provide tailored feedback in e-Assessment questions delivered via Dewis, UWE Bristol's in-house e-Assessment system. In this research article, we present one important finding of this project that is related to the parameter selection(s) of e-Assessment questions which have at least one CSE. We highlight why, in this digital era, it is more vital than ever to know more about mathematical CSEs.

Keywords: Mathematical Common Student Errors, Dewis e-Assessment system, e-Assessment Parameters

1. Introduction and Background

Students may make a mistake when answering a mathematical question for a variety of reasons. For example making a mistake in their calculation, misconceptions or misreading the question. When the same error is made by several students, those errors are sometimes referred to as common errors (Rushton, 2014).

Several different terms are used in the literature to refer to either mathematical errors or misconceptions. VanLehn (1982) use the term '*bug*' to refer to a systematic error resulting from wrong steps in the calculation procedure. The term, '*mal-rule*' is used by Rees and Barr (1984) to refer to an understandable but incorrect implementation of a process resulting from a student's misconception. For example, a classic *mal-rule* students make is to answer $a^2 + b^2$ when asked to expand $(a + b)^2$. In this article we use the term Common Student Error (CSE) to refer to an error made by several students.

This article is concerned with CSEs in e-Assessments. Assessment is a key element of teaching and learning and is used widely in higher education. It enables educators to assess the extent of students' skill and knowledge and to ascertain whether students have achieved the desired learning outcomes (Stödberg, 2012). Assessments also give students the opportunity to receive feedback on their work. Race (2014) suggests that, in order for feedback to be effective, it should be available while students still remember clearly the work they were engaged in. Using e-Assessments is one way of achieving this. A comprehensive review of the advantages of e-Assessment to the student, teacher, institution and education aims can be found in Alruwais et al (2018).

The use of E-Assessment for the formative and summative assessment of procedural mathematical techniques has become standard practice in many UK higher education institutions (Sangwin, 2013). Several e-Assessment systems allow the creation of equivalent but different assessments through the use of random variables. One disadvantage is that, typically, in answering an e-Assessment question the student does not enter their intermediate workings, as would be the case for a paper-based assessments. This, together with the fact that each student takes an equivalent but different assessment, makes detecting CSEs in e-Assessment questions harder than for traditional paper-based submissions.

A technique for detecting CSEs and providing tailored feedback in e-Assessment questions has been developed for several Dewis e-Assessment questions used in a first year Engineering Mathematics module (Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021; Sikurajapathi, Henderson and Gwynllyw, 2022a; Sikurajapathi, Henderson and Gwynllyw, 2022b). This research forms part of The CSE Project at UWE Bristol (2019) and further details of the methodology used can be found in the next section.

2. Methodology

2.1. CSE data collection

For the work presented in this article we use Dewis as the e-Assessment system and a first year Engineering Mathematics (EM) Module for the data collection. Dewis (2012) is well-established, was developed at UWE Bristol by a team of mathematicians, statistics and software engineers and uses an algorithmic approach to question generation, marking and feedback. Dewis is lossless, this means that the data for every assessment attempt is recorded and stored on the Dewis server (Gwynllyw and Henderson, 2009). The EM module has used Dewis to deliver e-Assessments since 2009 and as such a huge amount of e-Assessment data is available. This, together with the fact that between 2017 and 2020 the assessment of the EM module included a controlled conditions e-examination were two of the reasons it was selected for the collection of CSEs.

The e-Assessment profile for the mathematical techniques learnt in EM, for the period of interest for the CSE Project, is as follows:

- 22 weekly e-Assessments available throughout the year, with students being allowed unlimited attempts. The e-Assessment coursework mark was calculated from the top 20 marks from these 22 weekly tests;
- A two-hour mid-module e-examination, sat under controlled conditions in January. All of the questions in this e-examination were based on questions students had already encountered in their weekly e-Assessments;
- Formative revision e-Assessments, made available to students a few weeks before the e-examination. Students were allowed unlimited attempts.

Due to a lack of computer rooms, each January e-examination was delivered to a morning and afternoon cohort of students. For each cohort, the parameters of the e-examination questions were fixed, so each cohort sat the same test. Although the official submission was via Dewis, each student was given an examination booklet for their rough workings and these were collected at the end of each e-examination.

A total of 298 and 321 students sat the January e-examination in 2018 and 2019 respectively. Output from the Dewis Reporter was scrutinised in order to select the most common incorrect answers (MCIAs) to each question on the 2018 and 2019 January e-examinations. Once the MCIAs were identified, the rough workings booklets of those students who submitted each of the MCIAs were carefully examined. Having access to the students workings allowed us to work out what mistake(s) had been made by students resulting in each MCIA.

For each MCIA, the CSE percentage is calculated as follows:

$$CSE\ percentage = \frac{Number\ of\ CSE\ answers}{Number\ of\ incorrect\ answers}\ \%$$

If the CSE percentage is 4% or more, then that MCIA is considered as a CSE in this study.

Through this process, a bank of CSEs has been found and further details of the data collection process and results can be found in Sikurajapathi et al. (2020). Furthermore, this collection of CSEs has been taxonomically classified by Sikurajapathi et al. (2022a) using the taxonomy coding described in Ford et al. (2018) as a guideline.

2.2. CSE capture

In Dewis, the marking of each e-Assessment question, populates performance indicators (PIs). These PIs contain information on how a student has answered a question and are used to allocate marks, report outcomes and provide feedback. For example, for a question that requires one integer input, the three possible PI values would be 1 (correct), 0 (incorrect) and -1 (not answered). In order to capture the identified CSEs within Dewis, each e-Assessment question was amended and an additional PI was introduced, typically taking the value of 1 if the CSE was triggered and 0 if not. This not only allowed Dewis to provide enhanced feedback to the student to address the potential CSE (Sikurajapathi et al., 2021) but also allows the academic, through the Dewis Reporter, to identify all of the students in a cohort that made that CSE.

Since the data for every assessment attempt is recorded and stored on the Dewis server, it is possible to re-mark an assessment, for example, using an amended marking or feedback algorithm for one or more questions in that assessment. The amended CSE capture code for each question was validated by re-marking the e-examinations for the 2017-2018 cohort. This was done by checking that the additional PIs were populated for those students who had already been identified as making CSEs on the e-examination. Once this process had been completed satisfactorily, the weekly e-Assessments and revision tests were also re-marked, using the amended question code. In this research article, we present one important finding from this process, which is related to the parameter selections of e-Assessment questions which have at least one CSE. Details of the prevalence of CSEs made by EM students in e-examinations is available from Sikurajapathi et al. (2022a).

3. Results

During the re-marking of the weekly assessments for the 2017-2018 cohort, some restrictions related to the random parameter selections of the questions which have CSEs were found. Specifically, for some questions, there were particular parameters for which the correct answer and the CSE answer were the same. In these cases, in the marking of the e-Assessment, some students may have been awarded full marks and hence thought that they had answered the question correctly when in fact they had made a CSE.

In this section, several cases in which this happened are presented. For each case, we present a generic form of the question, an example of the parameter selections that lead to the correct and CSE answer being the same, the correct method of solution and the CSE. We use tilde (~) on the CSE answer to differentiate it from the correct answer.

3.1. Case 1

An instance of the first question considered is shown in Figure 1, which requires the student to find the value of the difference between two Unit Step functions at a given point (It should be noted that, the Unit Step function, $u(t)$ is equal to 1 for $t \geq 0$ and 0 for $t < 0$). The generic form of this question involves the function, $f(t) = a\,u(t+b) - c\,u(t+d)$ and the value of $f(p)$ is asked for, where parameters a, b, c, d, p are all integers, and created randomly for each instance of the question.

> The function $f(t) = 2u(t+4) - u(t+2)$
>
> where $u(t)$ represents the unit step function.
>
> Calculate the value of $f(0)$.
>
> Enter $f(0)$: [ ]

Figure 1: An instance of a question on the difference of two Unit Step functions

One CSE has been identified related to this question. This CSE occurs by assuming that the unit step function, u, is equal to 1 and is not a function. Whilst re-marking the weekly tests, it was noted that for some parameter values, the correct answer and the CSE answer for this question were the same. This occurs for example, when $a = 2,\ b = 7,\ c = 5,\ d = 1$ and $p = 4$. For this particular parameter selection, the correct answer and the CSE answer can be calculated as shown in Figure 2 and both are equal to -3.

Correct Answer	CSE Answer *CSE: taking the unit step function, u, to be equal to 1 and not a function.*
$f(4) = 2u(4+7) - 5u(4+1)$ $= 2u(11) - 5u(5)$ $= 2 \times 1 - 5 \times 1$ $= -3$	$\tilde{f}(4) = 2u(4+7) - 5u(4+1)$ $= 2u(11) - 5u(5)$ $= 2 \times u \times 11 - 5 \times u \times 5$ $= 22u - 25u$ $= -3u$ $= -3$

Figure 2: Workings showing the correct answer and the CSE answer of Case 1

3.2. Case 2

The second question considered here is related to the Geometric Series. Students were presented with an infinite geometric series of the form: $+a(r)+a(r)^2 + a(r)^3 + \cdots$, where parameters a and r are generated randomly for each instance of the question. The question requires students to calculate the sum S correct to three decimal places. One CSE was identified with this question and it occurs by finding the sum of the first four terms instead of the sum of the infinite series. An example in which the CSE answer is equal to the question's answer was found during the re-marking process and occurs when the sum of the infinite series, $S = 2 + 2(0.1) + 2(0.1)^2 + 2(0.1)^3 + \cdots$ is asked for. This is illustrated in Figure 3.

Correct Answer	CSE Answer *CSE: finding the sum of the first four terms instead of the sum of the infinite series*
$S = \frac{a}{(1-r)} = \frac{2}{(1-0.1)}$ $= 2.22222\ldots$ $= 2.222$ (correct to 3 dp)	$\tilde{S} = \frac{a(1-r^n)}{(1-r)} = \frac{2(1-0.1^4)}{(1-0.1)}$ $= 2.222$

Figure 3: Workings showing the correct answer and the CSE answer of Case 2

As shown in Figure 3, it can be seen that, to three decimal places, both the correct answer and the CSE answer are the same in this case.

3.3. Case 3

For this case, students were asked to find the power series expansion, $P_3(x)$, of $f(x) = e^{ax}$, up to and including the cubic term, and to use $P_3(x)$, to calculate an approximate value for $f(x)$ at $x = c$, correct to three decimal places. The parameters a and c are generated randomly for each instance of the question. One of the identified CSEs of this question is to give the exact value of e^{ax} instead of the approximate value of e^{ax} at $x = c$.

It was found that when $a = 2$ and $c = -0.1$, the correct answer and the CSE answer of this question are the same, to three decimal places, namely 0.819, as shown in Figure 4.

Correct Answer	**CSE Answer** ***CSE:*** *finding the exact value of* e^{ax} *instead of the approximate value of* e^{ax} *at* $x = c$.
$P_3(x) = 1 + 2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{6}$ $P_3(-0.1) = 1 + (-0.2) + \frac{(-0.2)^2}{2} + \frac{(-0.2)^3}{6}$ $= 0.818667 \ldots$ $= 0.819 \text{ (correct to 3 dp)}$	$P_3\widetilde{(-0.1)} = e^{-0.2} = 0.818731 \ldots$ $= 0.819 \text{ (correct to 3 dp)}$

Figure 4: Workings showing the correct answer and the CSE answer of Case 3

3.4. Case 4

The question for this case, required the student to find the mean value of $f(t) = a\sin(bt)$ in the interval $p < t < q$ correct to two decimal places, where the parameters a, b, p and q are generated randomly for each instance of the question. One of the identified CSEs of this question is to evaluate the mean value of $f(t)$ using degrees instead of radians in the calculation.

During the re-marking process, it was found that when $f(t) = -3\sin(5t)$ and the interval is $3 < t < 7$, the value of the mean, which is $m = -0.02$, is the same as the CSE answer, $\tilde{m}$, correct to two decimal places as shown in Figure 5.

Correct Answer	CSE Answer *CSE: evaluating the mean value of $f(t)$ using degrees instead of radians.*
$m = \frac{1}{(7-3)}\int_3^7 -3\sin(5t)\ dt$ $= \frac{1}{4}\left[\frac{3}{5}\cos(5t)\right]_3^7$ $= \frac{3}{20}[\cos(35) - \cos(15)]$ $= \frac{3}{20}[-0.9037 + 0.7596]$ $= -0.021615 \ldots$ $= -0.02$ (correct to 2 dp)	$\tilde{m} = \frac{1}{(7-3)}\int_3^7 -3\sin(5t)\ dt$ $= \frac{1}{4}\left[\frac{3}{5}\cos(5t)\right]_3^7$ $= \frac{3}{20}[\cos(35^o) - \cos(15^o)]$ $= \frac{3}{20}[0.8192 + 0.9659]$ $= -0.022005 \ldots$ $= -0.02$ (correct to 2 dp)

Figure 5: Workings showing the correct answer and the CSE answer of Case 4

3.5. Case 5

The question in this case involves finding the volume, V, of the solid formed when the part of the curve $y = a\,x^b$ is rotated about the x –axis between $x = f$ and $x = g$ and quoting the answer to two decimal places. The parameters a, f and g are generated randomly for each instance of the question and b is selected randomly from a pre-determined list of possible values. One of the identified CSEs of this question was to calculate V without integrating the required expression, but instead substituting the upper and lower limits directly into the integrand.

During the re-marking process, it was found that for some question parameters the correct answer and the CSE answer of this question were the same. For example, this occurs when $a = 6$, $b = 1$, $f = 0$ and $g = 3$. In this case, the correct answer and the CSE answer ($\tilde{V}$) can be calculated as shown in Figure 6.

Correct Answer	CSE Answer *CSE: finding the volume of revolution by substituting for the upper and lower limits without integrating.*
$V = \pi \int_0^3 (6x)^2\, dx$ $= \pi \int_0^3 36x^2\, dx$ $= 36\,\pi \left[\frac{x^3}{3}\right]_0^3$ $= 36\,\pi\,[3^2 - 0]$ $= 1017.88$ (correct to 2 dp)	$\bar{V} = \pi\,[(6x)^2]_0^3$ $= 36\,\pi\,[x^2\,]_0^3$ $= 36\,\pi[3^2 - 0]$ $= 1017.88$ (correct to 2 dp)

Figure 6: Workings showing the correct answer and the CSE answer of Case 5

3.6. Case 6

Another identified CSE of the question presented in Case 5 was to find the volume of revolution by taking $(x^p)^q$ to be x^{p^q}. The correct answer and the aforementioned second CSE answer of this question are the same when $a = 0.6$, $b = 2$, $f = 1$ and $g = 4$. In fact, this would be the case when $b = 2$ no matter the values of a, f, g since in this case $\left(x^b\right)^2 = (x^2)^2 = x^{(2^2)} = x^4$ and from there on the workings for the CSE answer would be exactly the same as for the correct answer.

4. Resolution

Without rough workings, for the examples presented in Section 3, there is no way of ascertaining whether the student arrived at the final answer by following the correct approach or by making the identified CSE. We have resolved this issue by ensuring that, for future instances of the question, the random parameters are selected so as the correct answer and the CSE answer(s) are different. This was achieved by further amending the CSE question code. For Cases 1-5, at the parameter selection stage of the code, the correct answer and the CSE answer(s) were calculated for each set of parameters. A while loop was then used to re-select the parameters until the correct answer and the CSE answer(s) were all different to each other.

In the original question code for Case 5, b was selected randomly from the following list of values: $[0.25, 0.5, 1, 1.25, 1.5, 2]$. In order to avoid the correct answer being equal to the CSE answer identified in Case 6, the value 2 was removed from the list of possible values for b in the amended code. In addition, for Case 4, a further CSE was identified in which students neglected to divide the integral by the interval $q - p$. To avoid the correct answer being equal to this CSE answer, in the amended code q is randomly selected so that q does not equal $1 + p$.

After finding these cases, all of the other CSE question codes were amended to avoid parameter selections for which the correct answers were equal to the CSE answers. As a further precaution, the question codes were amended so that CSE enhanced feedback is provided only when the PI value of the correct answer is zero and the PI of the CSE answer is one. Thus, the respective CSE enhanced feedback is only given to students making a CSE when their answer is incorrect.

5. Discussion and Conclusion

In this article we have shown why it is important to know more about Mathematical CSEs in e-Assessment questions, using several examples. These examples were discovered while conducting the CSE Project at UWE Bristol. We have shown how a correct answer can take the same value as a CSE answer for certain e-Assessment question parameters. In such cases, there may have been instances where some students were awarded full marks and hence thought that they had answered the question correctly when in fact they had made a CSE. We have described, how we addressed this issue by amending the original question code for all identified CSEs.

There has been a significant increase in usage of e-Assessments in higher education in this millennium. Even before the Covid 19 pandemic (World Health Organization 2020), a JISC report (2020) concluded that the archaic pen and paper assessment process is in need of a technological overhaul by 2025. We believe that, in this digital era, the work presented in our research article demonstrates why it is more important than ever to know more about mathematical CSEs.

6. References

Alruwais, N., Wills, G. and Wald, M. (2018) Advantages and Challenges of Using e-Assessment. *International Journal of Information and Education Technology* 8(1), 34-37. https://doi.org/10.18178/ijiet.2018.8.1.1008.

CSE Project at UWE Bristol (2019) *Diagnosing and Remediating Mathematical Common Student Errors in e-Assessment Questions: A Case Study*. Available from: https://fetstudy.uwe.ac.uk/~bin-sikurajapa/dewis/cseproject/ [Accessed 24 February 2022].

Dewis, 2012. Dewis welcome page. Available from: http://dewis.uwe.ac.uk [Accessed 02 November 2022].

Ford, S., Gillard, J. and Pugh, M. (2018) Creating a Taxonomy of Mathematical Errors For Undergraduate Mathematics. *MSOR Connections*. 18 (1), pp. 37-45.

Gwynllyw, R. and Henderson, K. (2009) Dewis: A Computer Aided Assessment System for Mathematics and Statistics. *CETL-MSOR 2008 Conference Proceedings*. pp. 38-44.

JISC. (2020) The future of assessment: five principles, five targets for 2025. Available from: https://www.jisc.ac.uk/reports/the-future-of-assessment [Accessed 02 November 2022].

Race, P. (2014) *Making Learning Happen*, Sage Publications.

Rees, R. and Barr, G., (1984) *Diagnosis and Prescription in the Classroom: Some Common Maths Problems*. London: Harper & Row.

Rushton, N. (2014). Common errors in Mathematics. *Research Matters: A Cambridge Assessment publication*, *17*, 8-17.

Sangwin, (2013) *Computer Aided Assessment of Mathematics*. Oxford, UK: Oxford University Press.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R., (2020) Using E-Assessment to Address Mathematical Misconceptions in Engineering Students. *International Journal of Information and Education Technology*. 10(5), pp.356–361.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2021) Students' Perceptions of Enhanced E-assessment Feedback Addressing Common Student Errors in Mathematics. *MSOR Connections*. 19 (2), pp. 10-27.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. (2022a) *Collection of Taxonomically Classified Mathematical Common Student Errors in E Assessments – (CSE Book)*. [online]. University of the West of England Bristol, United Kingdom. Available from: https://uwe-repository.worktribe.com/OutputFile/9305581 [Accessed 08 April 2022].

Sikurajapathi, I., Henderson, K., & Gwynllyw, R. (2022b). Gathering and Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification. *MSOR Connections*, 20 (3), pp. 55-71

Stödberg, U. (2012) A research review of e-assessment. *Assessment & Evaluation in Higher Education*, 37(5), 591–604.

VanLehn, K. (1982) Bugs are not enough: Empirical studies of bugs, impasses and repairs in procedural skills. *Journal of Mathematical Behavior*, 3(2) pp. 3-71.

World Health Organization 2020, Coronavirus disease (COVID-19) Pandemic. Available from: https://www.who.int/emergencies/diseases/novel-coronavirus-2019 [Accessed 02 November 2022].

E Glossary

AP	Agreement Percentage
CSE	Common Student Error
CSE ID	CSE Identification number
CSE %	CSE percentage
EFB	Enhanced Feedback
EM	Engineering Mathematics
ID	Identification number
MCIAs	Most Common Incorrect Answers
PIs	Performance Indicators
PIS	Participant Information Sheet