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Research Article

The impact of asymmetry of information and different ambiguity reactions on insurance demand

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Abstract: We extend the model of ex-ante asymmetric information in the insurance market of Stiglitz (1977) by incorporating consumers' reactions to uncertainty. Specifically, we assume that some agents are able to assign a precise probability measure to the event of loss (Savage, 1954), whilst others are not. The behavior of the latter group complies either with proba-bilistic sophistication or ambiguity aversion as modeled by maxmin expected utility (Gilboa and Schmeidler, 1989). When the former class exhibits sufficient pessimism, these two types are virtually indistinguishable when exposed to rare hazards. Thus, pooling constitutes as an optimal solution to the problem of the monopolist.

1. Introduction

The past few decades delivered a large amount of evidence showing that the majority of individuals violate some of the key principles underlying the standard decision model of subjective expected utility (SEU) of Savage (1954). Some of the most pronounced differences between this model's predictions and real-life practice were unraveled in the context of insurance, where this model fails to make provision for reactions to ambiguity. In addition, little is known about the impact ambiguity has on insurance demand in the market offering coverage against rare hazards, such as an earthquake or a food. Thus, in the present study, we adopt an insurance setting anew to analyze how different ways in which ambiguity reactions are modeled affect the monopoly equilibrium in the presence of hidden information. Specifically, we examine the impact that the different forms of ambiguity reactions have in a largely unexplored market for catastrophe insurance, where consumers experience particular difficulties evaluating small probabilities (Cutler and Zeckhauser, 2004). Assuming that consumers have information advantage over the insurance company, we complement a long string of theoretical literature focusing on the impact of ambiguity reactions on insurance demand in a setting with asymmetric information.

Generally, ambiguity refers to uncertainty about the probability of an event (Camerer and Weber, 1992, p.330). However, as pointed out by Knight (1921), there is a difference between risk-uncertain events, which occur with specified probabilities, and uncertainty-uncertain events characterized by unspecified or ambiguous probabilities. This distinction, not accounted for in the standard framework, gave rise to a new class of models, in which ambiguity is represented in a two-fold manner. Some models capture reactions to ambiguity by means of non-additive decision weights, which reflect an individual's probabilistic beliefs. Then, such an individual

is labelled as probabilistically sophisticated (Machina and Schmeidler, 1992)[1]. The other method in which ambiguity reactions are manifested is based on the notion of multiple priors, where the impossibility to assign a unique probability to an event imposes the need for more than one probability measure for that event. This class of models has been successfully pioneered through the maxmin expected utility (MEU) decision criterion of Gilboa and Schmeidler (1989) that has evolved into numerous variants since.

In this study, we employ both conceptualizations of ambiguity reactions to examine the behavior of individuals in the monopoly insurance market. Specifically, whereas some individuals are able to form subjective beliefs concerning the probability of loss, others are not[2]. While the first type of decision makers describes SEU maximizers, the second type encompasses different reactions to ambiguity. Agents of the latter type are either probabilistically sophisticated or uncertain about the probability of loss in a sense of being unable to assign a unique probability distribution to the event of loss.

Probabilistically sophisticated individuals know the objective probability of loss, but due to the lack of experience, they assess it incorrectly. This case is common in the markets providing insurance against rare hazards, where in spite of knowing the relevant probability clients tend to overestimate it, exposing pessimism[3]. To model probabilistic sophistication we adopt the rank-dependent utility (RDU) formulation of Quiggin (1982, 1993) as it rationalizes the disparity between objective probabilities and subjective beliefs[4].

RDU is considered to be one of the most sound descriptive theories of individual decision making under risk (Quiggin, 1982, 1993) and uncertainty (Schmeidler, 1989) (see Starmer, 2000, for details). According to RDU, individual preferences are jointly reflected through an agent's perception of wealth (measured by a utility function) and his perception of probability distribution of loss (captured by a probability transformation function). More importantly, the probability transformation employed by the agent relies on non-additive decision weights that accentuate the importance of extreme (small and large) outcomes and deemphasize the role of outcomes in the intermediate range. Specifically, the probability transformation function is concave for small probabilities and convex for intermediate and large probabilities, resulting in the so called inverse-S shape (Camerer and Ho, 1994; Wu and Gonzales, 1996; Gonzales and Wu, 1999, and Abdellaoui, 2000). This shape of the probability transformation function has received strong empirical and experimental support (for an extensive list of references referring to the evidence for the inverse-S probability weighting see Wakker, 2001, 2010). Consequently, this feature of RDU helps explain common ratio and common consequence effects demonstrated by Allais (1953), and sensitivity to marginal changes in probability as shown by Tversky and Kahneman (1992), Wu and Gonzales (1996), Abdellaoui (2000), Bleichrodt and Pinto (2000) and Abdellaoui et al. (2005). RDU also accommodates optimism and pessimism, where the overweighting of a small probability of the best (worst) event corresponds to the former (the latter). We adopt the RDU notion of pessimism to characterize one possible reaction to uncertainty in the insurance markets in which risk materializes with low probability, as in the case of catastrophes and natural disasters.

The other possible reaction to uncertainty is ambiguity aversion. This attitude is common for individuals who prefer known probabilities but have no or vague information about them - they are ambiguity averse[5]. In

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[1] Most of the non-expected utility preferences comply with probabilistic sophistication.

[2] The heterogeneity in behavior is consistent with the findings by Slovik, (2000), List (2004), and Huck and Muller (2012), who show that the evaluation of the likelihood of an event considerably varies among individuals and it is driven by different determinants.

[3] This result is consistent with the observation of Kunreuther and Hogarth (1989) who establish that propensity to pay above the fair premium for full insurance is especially marked for consumers exposed to low level so frisk.

[4] We employ the RDU model, however, for a binary prospect its representation coincides with most non -expected utility theories and biseparable preferences (Ghirardato. And Marinacci, 2001).

[5] Ambiguity aversion is a well-documented phenomenon. For an excellent survey on experimental literature on Ambiguity aversion see Camerer and Weber(1992), and Camerer(1995).

this study, we adopt the notion of ambiguity aversion consistent with MEU preferences, according to which, an agent evaluates his options with the least favorable probability distribution in mind[6].

The MEU model was proposed by Gilboa and Schmeidler (1989) in response to numerous empirical and experimental inconsistencies of SEU (Savage, 1954) found for decision making under uncertainty. Specifically, additive (subjective) probability measures representing decision makers' beliefs in the SEU framework cannot accommodate a pattern of choice found in Ellsberg Paradox, later coined as ambiguity aversion. The issue arises because decision makers do not hold precise subjective beliefs about the events of interest. For instance, consumers fail to estimate accurately their accident probabilities because of the lack of data at their disposal. Hence, they may consider several probability measures without knowing which of these measures is the correct one. MEU accounts for this 'anomaly' by assuming that a consumer has a set (an interval) of probability beliefs (priors) rather than a single prior, and he orders his preferences based on the minimum expected utility with respect to this set. Since the length of the interval of priors considered by an agent measures his degree of ambiguity, the ordering of choices consistent with MEU reflects the agent's dislike of ambiguous odds, hence, ambiguity aversion. The MEU model and the corresponding notion of ambiguity aversion have been applied in various contexts to model agents' behaviour (e.g., to agency theory by Karni, 2009). In this study, we examine the impact that ambiguity aversion has on insurance choices in the presence of asymmetric information. Thus, the present analysis also contributes to relatively unexplored literature focused on the effects of interactions between ambiguity aversion and hidden information (for more detail refer to Koufopoulos and Kozhan, 2016).

Employing a mix of preference theories to model individual behavior in the insurance market enables us to cast aside the criticism of Wilcox (2006) and Conte et al. (2011), according to which, a unique decision making paradigm is inadequate for modeling purposes. Moreover, as both patterns of ambiguity reactions (pessimism and ambiguity aversion) are particularly pronounced in the context of insurance against rare hazards[7], extending the model of Stiglitz (1977) by incorporating this additional dimension of heterogeneity among consumers has important implications for the equilibrium characterization.

We find that consumers' pessimism and ambiguity aversion have the same qualitative implications for equilibrium in the monopoly market with hidden information. In particular, they lead to an increased propensity to pay for insurance, which implies that pessimistic or ambiguity averse consumers are willing to pay a higher premium than standard utility maximizers for the equivalent level of compensation. Thus, the equilibrium involves pooling at full insurance if the number of high-risk standard utility maximizers in the population is low. Specifically, pessimism of the low-risk group implies that they overweigh their likelihood of suffering loss. If instead of pessimism, low-risk types exhibit ambiguity aversion, then they choose a contract based on the least favorable probability distribution. Consequently, the ambiguity averse types evaluate the odds of suffering loss in a pessimistic manner, too. Therefore, if overweighing the loss probability is sufficiently large, that is, when pessimism and ambiguity aversion imply that the low-risk group is willing to pay for insurance more than the high-risk types, pooling is optimal. The monopolist's profit is maximized by offering an identical contract to all agents, as long as the additional profit earned from low-risk pessimistic or ambiguity averse consumers exceeds the value of information rent paid to high-risk types.

Contrary, when the proportion of high-risk standard utility maximizers is large, separating is preferred. Explicitly, separation requires the incentive compatibility constraint of the high-risk group to be binding, meaning that the insurer must forsake any profit earned on the low-risk consumers to avoid losses on the high-risk types. Since the indifference curve of the low-risk (pessimistic and ambiguity averse alike) agents is always inside that of the high-risks, the only separation involves null insurance for the former group. Consequently, low-risk consumers are either fully insured (when they are sufficiently numerous) or not insured at all (when there are too few of them), in which case the high-risk types receive the symmetric information insurance contract.

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[6] Whenever agents' aversion to ambiguity is infinite, the prediction generated under MEU and smooth ambiguity model of Klibanoff et al.(2005), coincide.

[7] For instance, Hogarth and Kunreuther (1985,1989,and1992) show that consumers' evaluation of insurance is affected by uncertainty with respect to unfavorable events.

The rest of the paper is organized as follows. The next section provides a review of related literature. This section is followed by an introduction of the model in Section 3. The analysis and the proof of our results are given in Section 4. This section also includes the discussion, which points to some limitations of the present study. Finally, Section 5 concludes.

2. Related literature

There is a growing number of studies examining insurance demand in the presence of consumers with non-standard preferences and asymmetry of information. Two studies closely related to ours are Ryan and Vaithianathan (2003, henceforth R&V) and Jeleva and Villeneuve (2004, henceforth J&V). Like us, these authors consider a monopoly insurance market with low- and high-risk agents, whose beliefs might not reflect their objective likelihood of loss. Whereas R&V use the RDU model with non-additive decision weights to determine consumers' beliefs, in J&V's framework, agents' beliefs, and consequently their behavior, are approximated by a range of preference theories, RDU being one of them. In analogy with our results, both R&V and J&V establish optimality of pooling for a small fraction of high-risk consumers in the population. However, a number of other equilibria can also arise in their models: pooling with partial insurance, separating with both agents insured, etc. These equilibria are not feasible in our model. Specifically, unlike R&V or J&V, we are interested in the markets providing coverage against rare hazards, in which pessimism has important implications. Thus, to explore these implications we adopt exclusively a strong notion of pessimism, which is not equivalent to the pessimism characterized in their models. Moreover, R&V and J&V allow for optimism of the high-risk group. Since our purpose is to contrast the behavior of groups whose beliefs are represented not solely via non-additive decision weights, we dispense of optimism. Consequently, the range of equilibria arising in our model is considerably smaller and specific to the setting with natural disasters.

Another study that pays particular attention to pessimism is Werner (2016). This study implicitly allows for overinsurance of pessimistic types. If the pessimistic types are exposed to a low level of risk, they receive a larger coverage than high-risk agents.

Like pessimism, ambiguity aversion plays a key role in consumers' choice of insurance. Koufopoulos and Kozhan (2014, 2016) incorporate aversion to ambiguity, as modeled by MEU preferences, in order to examine equilibria in a market with different configurations of consumers' risk exposures and degrees of ambiguity aversion. They establish the uniqueness of pooling equilibrium in the competitive setting. In contrast to Koufopoulos and Kozhan, separation can be optimal in our framework, where proportions of different types in the population have a large impact on the profit earned by a monopolistic supplier.

A number of other studies incorporate ambiguity aversion into the analysis of insurance demand with asymmetry of information. Huang et al. (2015) employ the notion of ambiguity aversion defined in the smooth ambiguity model of Klibanoff et al. (2005), in order to explain advantageous selection in the competitive insurance market. The purpose of Chassagnon and Villenueve (2005) is different - their goal is to assess the impact of improving agents' risk perception in the Bayesian setting with a monopolistic insurer. Whereas the findings of the latter study provide no clear support for the idea of transmitting information from insurer to an agent, Huang et al. (2015) show that advantageous selection arises when consumers exhibit increasing aversion to ambiguity.

Neither of these studies is explicitly concerned with the implications that the presence of different reactions to ambiguity, as modeled by distinctive decision making criteria, has for equilibria in insurance markets with adverse selection. As highlighted by Bruhin et al. (2010), heterogeneity in agents' behavior may be well due to differences in their objective probabilities and beliefs. By incorporating different ways in which such differences are modeled for known and unknown probabilities, the present work aims to complement a large theoretical literature on the impact that (multi-dimensional) heterogeneity among consumers has on insurance demand in the monopoly market affected by hidden information.

3. The model

3.1 *The consumers*

Consider a simple model of an insurance policy to cover a potential loss. Let there be two states of the world, a good state s_1 and a bad state s_2. The two states occur with fixed probabilities, such that p is the probability of s_2 and $1-p$ is the probability of s_1 for $p \in (0,1)$. All consumers in the insurance market have the same von Neumann and Morgenstern (vNM) utility function, $u : \mathbb{R} \to \mathbb{R}$, which is twice continuously differentiable with $u'(\cdot) > 0$; and $u''(\cdot) < 0$.

An initial wealth of every consumer is given by $\bar{x}_1 = W$ in s_1 and $\bar{x}_2 = W - D > 0$ in s_2, where D is an exogenous parameter denoting monetary loss (damage). A consumer may insure against a loss by purchasing an insurance policy $\psi = (R, C)$, which specifies the premium paid in each state, R, and the compensation received in the bad state, C, where $0 \le C \le D^{\beta}$. Hence, an insurance policy is like a prospect, which gives a final wealth level $x_2 = W - R - D + C$ in the bad state, and $x_1 = W - R$ in the good state, with $x_1, x_2 \in \mathbb{R}_+$. We denote such a prospect by $P = (x_2, p; x_1, 1 - P)$.

Since agents react differently when faced with uncertainty, their evaluations of P differ. In particular, we distinguish three groups of agents in the insurance market: subjective expected utility (SEU) maximizers (Savage, 1954), rank-dependent utility (RDU) maximizers (Quiggin, 1982, 1993), and maxmin expected utility (MEU) maximizers (Gilboa and Schmeidler, 1989). Each group assesses the likelihood of an uncertain event distinctively.

When the subjective probability of an event is known and linear, binary prospect P can be represented using SEU model in Eq. (1). According to the model, an individual forms a precise probability measure when exposed to uncertainty.

$$SEU(P) = u(x_2)q + u(x_1)(1 - q)\#(1) \tag{1}$$

Here, q is the subjective probability of loss. Without loss of generality, we assume that subjective and objective probabilities coincide, $q = p$, so that the insurance contract is assessed by means of a standard expected utility (EU) model (von Neumann and Morgenstern, 1944). The EU representation will be used throughout the rest of the paper.

Unlike SEU types, some consumers in the insurance market are unable to pin down a precise probability distribution to an uncertain event. RDU and MEU types are among those consumers. In some cases subjective probabilities can be formed but they do not obey probability laws. These probabilities are represented by non-additive decision weights which explain ambiguity reactions (e.g., Tversky and Kahneman, 1992). Then, preferences are consistent with the so called probabilistic sophistication and admit a number of representations. One of them is the RDU representation shown in Eq. (2).

$$RDU(P) = \begin{cases} u(x_2)[1 - \omega(1 - p)] + u(x_1)\omega(1 - p), & x_1 \ge x_2 \\ u(x_2)\omega(p) + u(x_1)[1 - \omega(p)], & x_1 < x_2 \end{cases} \tag{2}$$

The central element in Eq. (2) is the probability transformation function ω. This function rationalizes the disparity between objective and subjective probabilities. w is strictly increasing and continuous, it maps from $[0,1] \to [0,1]$ and satisfies $\omega(0) = 0$ and $\omega(1) = 1$. For $\omega(p) = q$, Eq. (1) and (2) coincide. Since in the present framework not having an accident is weakly preferred to an accident, $x_1 \ge x_2$, we are particularly interested in the RDU representation, according to which, an individual evaluates an uncertain prospect P as $u(x_2)[1 - \omega(1 - p)] + u(x_1)\omega(1 - p)$.

Unlike the RDU model, the MEU decision criterion can be applied when different sets of probability measures are required to evaluate the uncertain prospect P, as shown in Eq. (3a) and (3b).

$$MEU(x_1, x_2) = \min_{p \in [\underline{p}, \overline{p}]} EU(P) \tag{3a}$$

$$= \begin{cases} EU\big((x_1, x_2), \underline{p}\big), & x_1 \ge x_2 \\ EU\big((x_1, x_2), \overline{p}\big), & x_1 < x_2 \end{cases} \tag{3b}$$

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[8] The latter assumption implies that $x_1 \ge x_2$; so that overinsuring is not possible.

The MEU decision criterion has the notion of ambiguity aversion built in it[9]. Ambiguity aversion reflects the agent's dislike for uncertainty associated with the occurence of a bad event[10]. Specifically, an MEU type evaluates his choices with the most pessimistic prior in the interval $\left[\underline{p}, \overline{p}\right]$, where $\underline{p}$ and $\overline{p}$ denote the lowest and the highest probabilities of loss considered by the agent, respectively: Whenever $\left[\underline{p} = \overline{p} = p\right]$; the min operator in Eq. (3a) is inutile and representation in Eq. (3a) reduces to Eq. (1). In analogy with the RDU representation, we are only interested in the MEU evaluation of prospect P that corresponds to the case of $x_1 \geq x_2$ in Eq. (3b).

In addition to preferences represented by Eq. (1), (2), or (3b) for $x_1 \geq x_2$, each individual is characterized by a risk exposure, which is either low (low-risk types L) or high (high-risk types H), so that $1 > p_H > p_L > 0$. Thus, the present analysis extends the model of insurance demand of Stiglitz (1977) by incorporating reactions to ambiguity. Together with risk exposure, the reaction to ambiguity fully determine each agent's choice of an insurance contract.

3.2 *The insurer*

A monopolistic risk- and ambiguity-neutral firm supplies insurance to the market. The insurer is unable to observe different risk exposures, H and L, in the population, but she knows their proportions: α and $1 - \alpha$, respectively. Moreover, a large volume of past data on insurance enables the monopolist to infer the consumers' utility function of wealth, the shape of the transformation function w used by RDU consumers, and the probability interval $\left[\underline{p}, \overline{p}\right]$ employed by MEU types. Based on this information, the monopolist offers a pair of policies, $\left(\psi_i, \psi_j\right)$, to a random agent $i, j \in \{L, H\}, i \neq j$, and earns profit π_i upon selling policy $\psi_i = (R_i, C_i)$ to the consumer with risk exposure $\dot{\iota}$:

$$\psi_i = R_i - p_i C_i \qquad (4)$$

The set of contracts, $\Psi^* = (\psi_i^*, \psi_j^*)$; entailing a policy for each agent is the equilibrium, if the following conditions are satisfied:
  (i) Each consumer maximizes the utility of his type from the chosen contract.
  (ii) The contract makes a non-negative expected profit.
  (iii) The expected profit from no other contract exceeds that from Ψ^*.

4. The analysis

Cutler and Zeckhauser (2004) note that people have considerable difficulties making insurance decisions involving small-probability large-consequence events, which may explain why their behavior deviates from the premises of the EUT model. Particularly, Sandroni and Squintani (2013) suggest the tendency of individuals to overestimate the odds of negative events in the context of fires, floods and earthquakes. This has been further confirmed in the context of airtravel, where Eisner and Strotz (1961) find that subjects overinsure. Indeed, Hogarth and Kunreuther (1989) observe that the willingness to pay an actuarially unfair premium for full insurance is especially marked for consumers exposed to low levels of risk. Given this evidence, we assume that the ambiguity reactions that we consider characterize only the low-risk consumers[11]. Since both ambiguity aversion and pessimism have a significant impact on insurance contracting in the market in which risk materializes with a low probability, we consider two relevant cases.
Case 1 $1 - \omega(1 - p_L) > p_H > \omega(p_L)$
Case 2 $\overline{p}_L > p_H > \underline{p}_L$

Case 1 refers to the scenario, in which low-risk agents display pessimism. Hence, they assign a greater likelihood to an unfavorable outcome than the objective likelihood of that outcome. This treatment of probability is consistent with the inverse-S transformation function, which captures attitudes to probabilities in the RDU framework. Inverse-S overweighs a small likelihood of (good and bad) extreme outcomes. In addition,

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[9] Actually, the key axiom in the model of Gilboa and Schmeidler (1989) is uncertainty aversion.

[10] For the review of various notions of ambiguity aversion see Etner et al.(2012).

[11] We discuss the limiting nature of this assumption in more detail sin Section 4.4.

for sufficiently close risk exposures of the RDU and EU types, this function accounts for the reversal in the usual relationship between the low- and high-risk exposure.

In Case 2, the low-risk types' dislike for uncertainty generates a whole range of beliefs about the likelihood of loss, with the worst belief exceeding the objective loss probability of the high-risk types. This behaviour is predicted by Einhorn and Hogarth (1985), who show that ambiguity aversion at low probabilities is common in insurance markets. This observation is further reinforced by the lack of learning opportunities in the context of rare hazards. In addition, we assume that $\overline{p}_L \in [\underline{p}_L, \overline{p}_L]$; so that the low-risk types' beliefs of the likelihood of loss contain the true (objective) loss probability.

Next, we outline the implications of pessimism and ambiguity aversion on the low risks' willingness to pay for insurance using an indifference curve analysis.

4.1 *Indifference curves*

This section demonstrates the implications that consumers' reactions to ambiguity have on their willingness to pay for an insurance asset.

Consider Figure 1 (Hirschleifer-Yaari diagram). The two states with probabilities p and $1 - p$, respectively, have been fixed. The horizontal axis in the diagram measures wealth level x_1, while the vertical axis shows wealth level x_2. The slope of the indifference curve associated with EU maximizers (the dashed line) is equal to $-\frac{1-p}{P}\frac{u'(x_1)}{u'(x_2)}$, and it corresponds to the marginal rate of substitution between these types' final wealth levels in the good and bad states.

In contrast, the indifference curve of pessimistic RDU agents has a kink on the dotted certainty line, which depicts the non-smoothness property of RDU indifference curves induced by the change of the rank of outcome (from x_1 to x_2 or vice versa). Consequently, the slope of the indifference curve (the solid curve) below the certainty line equals $-\frac{w(1-p)}{1-w(1-p)}\frac{u'(x_1)}{u'(x_2)}$, while above that line it equals $-\frac{1-w(p)}{w(p)}\frac{u'(x_1)}{u'(x_2)}$.

In analogy with RDU, the MEU indifference curve has a kink on the certainty line. This kink highlights the agent's uncertainty about the true probability of loss. Particularly, above the certainty line, MEU agents act as if their loss probability were the lowest, $\underline{p}$, while below that line they behave as if this probability were the highest, $\overline{p}$. Thus, the attitude toward ambiguity changes the slope of the indifference curve between the states in a fashion qualitatively similar to that in the RDU framework with optimism and pessimism. The corresponding slopes are equal to $-\frac{1-\underline{p}}{\underline{P}}\frac{u'(x_1)}{u'(x_2)}$, and $-\frac{1-\overline{p}}{\overline{P}}\frac{u'(x_1)}{u'(x_2)}$, respectively (see Figure 1).
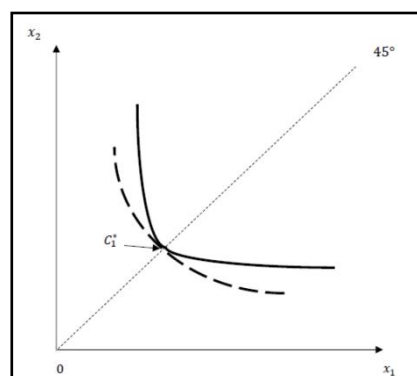

Figure 1: Indifference curves of SEU agents (- - -) and RDU (also MEU) types (-).

The slopes of the indifference curves are important, since they demonstrate how different reactions to ambiguity affect the propensities to pay for insurance. In Case 1, the indifference curve of RDU agents lies inside the indifference curve of EU individuals, which follows on from observing that $-\frac{w(1-p)}{1-w(1-p)}\frac{u'(x_1)}{u'(x_2)} > -\frac{1-p}{P}\frac{u'(x_1)}{u'(x_2)} > -\frac{1-w(p)}{w(p)}\frac{u'(x_1)}{u'(x_2)}$. This property implies that when overinsurance is prohibited, RDU agents are willing to pay a higher premium than EU types for the equivalent level of compensation. Similarly, in Case 2 where $-\frac{1-\overline{p}}{\overline{P}}\frac{u'(x_1)}{u'(x_2)} > -\frac{1-p}{P}\frac{u'(x_1)}{u'(x_2)} > -\frac{1-\underline{p}}{\underline{P}}\frac{u'(x_1)}{u'(x_2)}$, the implication is similar: MEU consumers are prepared to pay more than EU types for a unit of insurance asset. This equivalence between the indifference curves of RDU and MEU

groups is a key to the results of this paper. We use the analysis of the indifference curves for EU, RDU, and MEU types to identify equilibria in the market.

4.2 Case 1

In this section we examine the equilibrium in the monopoly insurance market with high-risk EU maximizers and probabilistically sophisticated low-risks in a sense of RDU. The monopolist is unable to observe different types, thus, her profit maximization problem is described by the expression in Eq. (5).

$$\underset{\{(R_H),(C_H)\} \in \psi}{\max} \alpha \pi_H(R_H, C_H) + (1 - \alpha)\pi_L(R_L, C_L) \tag{4}$$

To induce consumers' participation and to ensure a non-negative profit, the behaviour of each agent must satisfy the following conditions:

$$\mu(\chi_{2i}^m)p_i^m + \mu(\chi_{1i}^m)\left(1 - p_i^m\right) \geq \mu\left(\bar{\chi}_2\right)p_i^m + \mu\left(\bar{\chi}_1\right)(1 - _i^m), \qquad \text{IR}$$

$$\mu\left(\chi_{2i}^m\right)p_i^m + \mu\left(\chi_{1i}^m\right)(1 - p_i^m) \geq \mu\left(\chi_{2j}^m\right)p_i^m + \mu\left(\chi_{1j}^k\right)(1 - p_i^m), \qquad \text{IC}$$

where IR stands for individual rationality, and IC for incentive compatibility. The IR constraint ensures that having insurance yields at least as high utility as the utility from being uninsured, while IC enables the uninformed insurer to screen between RDU and EU types.

$\bar{x}_1$ and $\bar{x}_2$ in inequality IR denote the levels of initial wealth (prior to insuring) in the good and bad states, given by $\bar{x}_1 = W$ and $\bar{x}_2 = W - D$. Moreover, x_{1i}^m in inequalities IR and IC denotes final wealth of an individual with risk type i and ambiguity reaction type m in the good state, where $m, k \in \{EU, RDU\}, m \neq k$, and $i, j \in \{L, H\}, j \neq i$.

Since pessimistic low-risk consumers exhibit a propensity to pay a higher premium than high-risk EU maximizers for the equivalent level of compensation, we establish the following result.

Proposition 3 *The equilibrium contract in the monopoly market for Case 1 involves pooling at full insurance if the fraction of high-risk EU maximizers, α is sufficiently small. Otherwise, only high-risk types are served.*

Proof:

We analyze this scenario using the indifference curves of EU and RDU types illustrated in Figure 1. At C_1^* the indifference curves of both types are tangent. Additionally, the indifference curve of pessimistic low-risk individuals is entirely inside that of high-risk EU maximizers. This implies that it is impossible to separate consumers and at the same time offer a positive level of insurance coverage to both groups. Specifically, the insurer wants to sell complete coverage to low-risk agents, since these types are prepared to pay an actuarially unfair premium price for insurance. However, any positive amount of coverage offered to low-risks will attract high-risk consumers. If the proportion of the latter type in the population is low, the extra profit earned on low-risk agents outweighs the value of information rent paid to high-risk types. As a result, both groups are pooled at C_1^*. Since overinsurance is not permitted, the equilibrium contract provides low-risk consumers with the amount of coverage that coincides with the amount that this group would have obtained had the asymmetry of information been absent.

When the fraction of high-risk EU maximizers is large enough, so that gains obtained from serving pessimistic agents cannot compensate for losses made by pooling both groups, separating is optimal. Since the slope of the indifference curve of low-risk RDU types is always flatter than that of the high-risk group, the only separating contract satisfying incentive compatibility gives no insurance to RDU consumers. In such a case, the equilibrium allocation provides high-risk individuals with full insurance at an actuarially fair price.

The next section characterizes the equilibrium in the market where a fraction of agents are averse to ambiguity.

4.3 Case 2

Failures of some insurance markets occur due to various factors, ambiguity concerning the likelihood of a potential loss as well as asymmetry of information being coined as the key reasons. Therefore, in this section we analyze the impact that aversion to ambiguity has on equilibrium in a monopoly insurance market with asymmetry of information. Specifically, we consider Case 2, where low-risk consumers are ambiguity averse in a sense of abiding to the MEU decision criterion. The remaining proportion of agents in the population behave according to the standard EU model.

In the MEU model (recall Eq. (3a).and (3b)), the degree of aversion to ambiguity is reflected by the size of the set of priors used by an agent, who is uncertain of the likelihood of loss. The MEU decision maker then takes the minimum with respect to that set of priors. Hence, the MEU type in Case 2 acts in a pessimistic manner, equivalent to the RDU type in Case 1, who overweighs his probability of loss. Moreover, both types, low-risk RDU and low-risk MEU consumers, believe that their risk exposure is greater than that of the high-risk EU group (although their beliefs are founded on different grounds). As a result, the impact of pessimism in Case 1 and aversion to ambiguity in Case 2 are indistinguishable.

The equivalence between Cases 1 and 2 carry on to the equilibrium characterization. In particular, when the insurer pursues profit maximization as defined in Eq. (5), subject to the usual individual rationality and incentive compatibility constraints (recall IR and IC, with $m, k \in \{MEU, EU\}$ and $m \neq k$ for Case 2), the resulting equilibrium is equivalent to that characterized in Proposition 3. Henceforth, the monopolist pools both groups of individuals (high-risk EUs and low-risk MEUs) by insuring them fully, since overinsurance (which would be preferred by ambiguity averse low-risk agents) is not permitted.

A convention in the monopoly market is that the proportion of different types of consumers matter for the optimality of contracts. Indeed, in analogy with Proposition 3, pooling arises only if the number of high-risk types in the population is sufficiently low. If this condition is not satisfied, the insurer is better of offering separate contracts. Then, the only contract that generates a non-negative profit subject to the incentive compatibility constraint provides null insurance to low-risk individuals. We summarize the results.

Summary 4 *The equilibrium in the monopoly insurance market is qualitatively unchanged if one replaces pessimism defined in Case 1 with ambiguity aversion defined in Case 2.*

Both pessimism and ambiguity aversion of low-risk agents lead to an increment in aversion to risk[12], which emerges as a propensity to pay an actuarially unfair premium price for insurance. Hence, given a sufficient number of these types in the population, monopolist's gains exceed losses, resulting in pooling. However, pooling is no longer optimal when this proportion is low. Separation is preferred because it eliminates the risk of making a loss on the high-risk EU group. As a result, the monopolist is forced to drop the low-risk clientele from the market, in which case high-risk types receive full insurance.

The next section outlines strenghts but also potential limitations of our analysis with recommendations for future research.

4.4 *Discussion and limitations*

The main result of this paper shows that pessimism and ambiguity aversion impose the same requirements on the behavior of a decision maker exposed to an unlikely loss. In the following, we interpret this result in the present framework and highlight potential issues and limitations with its generalization.

First, our results are derived assuming that only the low-risk types exhibit ambiguity reactions, while the high-risk types comply with the principles of (subjective) expected utility maximization. Recall that the present analysis focuses on an insurance market providing coverage against rare hazards. This implies that the likelihood of suffering loss is relatively small for all consumers in this market. Using the arguments given in Section 4, one could then argue that even the high-risk types may exhibit ambiguity reactions. Indeed, such an assumption is feasible, and it could lead to quite different characterization of equilibrium contracts obtained by both types (the ultimate outcome would again depend on the combination of the agents' ambiguity reactions and

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[12] For instance, Klibanoff et al.(2005) and more recently Ahn et al.(2014) show that ambiguity aversion reinforces risk aversion.

their proportions). However, Sydnor (2010) and Cohen and Einav (2007) provide evidence on the choice of deductibles, which suggests that agents' dislike for negative events is significant when the loss associated with these events arises with the probability of about five percent or less. Thus, to defend our assumption, we could argue that while low-risk types are those with the likelihood of loss smaller than five percent, the high-risk types are those characterized by the probability of loss exceeding this threshold, hence, estimating probabilities better, and in turn, not exhibiting ambiguity reactions. The subtle difference in the likelihood of an uncertain event could be due to the geographical location.

Second, the concept of pessimism employed in the RDU framework (Case 1) is stronger than generally, implying not only overweighing a small probability of a bad event, but also overweighing this probability beyond the likelihood of the high-risk type. While such a configuration can be easily justified by a relatively small difference in the true probabilities of loss of both low- and high-risk types, it is important to clarify that this type of behavior is not common in all types of insurance markets. For instance, Bhattacharya et al. (2004) find that people underestimate their likelihood of death and are unwilling to hold life insurance.

Finally, as explained above, we employed a stronger notion of pessimism, which qualitatively coincides with the MEU conceptualization of ambiguity aversion. However, in general, pessimism and ambiguity aversion are not equivalent. For instance, Gollier (2011) notes that an ambiguity averse individual generally acts more pessimistically than a probabilistically sophisticated agent, which an RDU type is. In fact, in their recent work, Abdellaoui et al. (2011) emphasize the importance of distinguishing between pessimism and ambiguity aversion by means of the so called source functions. Such functions assign subjective probabilities to decision weights, but unlike other transformation functions they depend not only on the individual but also on the source (being either risk or uncertainty). Specifically, ambiguity aversion is established as the difference in pessimism for uncertainty and for risk. Then, the main outcome of Abdellaoui et al. (2011)' paper confirms that people dislike unknown probabilities more than the known ones, hence providing evidence for ambiguity aversion. Thus, the analysis of equilibria in insurance markets with a wider range of accident probabilities, in which pessimism and ambiguity aversion are unequal, provides an interesting avenue for future research.

5. Conclusions

In this work, we have extended the analysis of insurance demand in the presence of asymmetry of information by Stiglitz (1977) to incorporate for different reactions of the consumers in the insurance market to ambiguity. For a high number of individuals exposed to low-probability risks, we establish optimality of pooling equilibrium. When pooling is not profitable, incentive compatibility enforces on the monopolist to offer contracts only to the high-risk agents. More interestingly we find that this solution is derived with either pessimism or ambiguity aversion in the market. We argue that for rare hazards, the two attitudes to uncertainty can be applied interchangeably.

Finally, our analysis highlights the role that the disparity between objective probabilities and beliefs plays in consumers' choice of insurance contracts. Particularly, we demonstrate that probability-based attitudes to uncertainty may exert a greater impact on market equilibrium than attitudes solely embedded in the shape of the utility function. Therefore, our results strenghten the need for further comparison among models incorporating such attitudes.

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