Collective Dynamics

How a Game Theoretic Approach Can Minimize the Cost of Train Passenger Services: An Intermodal Competition between Rail and Road Transport

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Abstract Game theory models provide very powerful tools for evaluating strategies that are beneficial to both rail and road operators competing for passengers on parallel routes. This study examines how game theory can help rail operators who are incurring losses on passenger transport to identify strategies that can minimise costs, using the methodology of dual linear programming to analyse strategies. In identifying the best strategies for minimising costs for the railway operator, the best strategies for maximising profits for the road operators are also identified. The game model is set up between two passenger transport operators (rail and road) and is based on the income earned by the road operators from passengers. This study illustrates the following: how the strategies of the two competitors (rail and road) are determined; the formation of the payoff matrix and the presentation of the mathematical problem for the two competitors; and the results and verification of the best strategies for both competitors. The Leonid Hurwicz criterion was used to verify the optimal strategies.

Keywords Game theory · dual linear programming · hurwicz criterion · minimum costs

1 Introduction

Four decades ago, the railway sector in Africa enjoyed and profited from the business of freight and passenger transport. This was because it was the only reliable and well-connected mode of transport, unlike road transport, which had not been opened up and developed. Unprecedented infrastructure development in Africa has resulted in the construction of roads linking cities and ports. This has led to the shrinking of the railway sector in Africa as freight and passenger business has shifted to road transport. One of the most affected rail services, which often operate at a loss in most African countries and some in other parts of the world, is passenger service. The annual report of the Committee on Parastatal Bodies [1] revealed that the passenger services of the Tanzania Zambia Railway Authority (TAZARA) were operating at a loss, and the little revenue generated from freight transport was channelled into passenger services to keep them running. According to Plumer [2], Amtrak, the national rail passenger company of the United States of America, loses a lot of money every year. It has also been reported that India, Japan and other countries are incurring heavy losses in passenger services [3, 4]. "Game theory examines how incentives affect decisions in strategic environments. An economic environment is said to be strategic if the decisions made by one player affect the opportunities and payoffs available to other players, and the players are mutually aware of this" [5]. In game theory, players are allowed to make independent decisions in the absence of a mechanism to enforce cooperation. In this study, road and rail operators make independent decisions, even though they carry passengers in the same operational areas. Game theory has been used in some studies to analyse inter-modal competition between two modes of transport. This study was particularly needed in the transport sector as it has received little research attention[6–11]. The following section highlights the literature review related to this research study, followed by the discussion section, which consists of two subsections, namely strategies for both competitors and the game theory model, and then the conclusion section.

2 Literature Review

Game theory, combined with other theories and concepts, has been used extensively in many studies to provide and project solutions for decision makers. According to Simon [12], modern game theory is an extensive and energetic search for ways to extend the concept of rational behaviour to situations involving bargaining, fighting and out-guessing. Game theory is used to make decisions in circumstances where there are conflicting interests between the players. It provides powerful tools for analysis in the transport sector.

In a study conducted by Raturi and Verma [13], a game theoretic approach was used to analyse inter-modal competition between high-speed rail and airlines in the Indian context and to assess the impact of speed and passenger characteristics on the equilibrium of the game. The frequency and fares offered by the players (operators) to maximise their profits are the conditions under which the competition was modelled. Roumboutsos and Kapros [14], conducted a study on urban public transport integration policy using the game theory approach. The results highlighted that the game theory model is a strategy guidance tool that can be used to help transport policy makers identify the most cost-effective form of intervention and its timely implementation in public transport integration. A study by Koryagin [15], entitled 'Game theory approach to optimising public transport traffic under conditions of travel mode choice by passengers', shows how decision makers (city authorities) can determine the frequency of minimising the population's time loss and costs associated with transport. Yang et al. [16] conducted a study on a route choice model based on game theory for commuters. The game theory was used to provide reliable route choice for commuters in traffic congestion during peak hours. In measuring the reliability of transport network performance, Bell [17] found that when users are extremely pessimistic about the state of the network, the Nash equilibrium can be used to measure network performance and can indeed be used as a basis for a cautious approach to network design. Škrinjar et al. [18] conducted a study in urban transport planning using game theory. Game theory was found to be an effective tool for decision makers to make optimal decisions in dealing with traffic in large cities.

It is worth noting that most studies of transport systems use non-cooperative games, two-person zero-sum games and perfect games. In a non-cooperative game, each player plays the response based on the strategies played by the opponents. In a two- person zero-sum game, two players are involved and one player wins whatever the other player loses. In a perfect game, each player has complete information about the payoff structure and chooses strategies sequentially. Game theory has been used with other mathematical tools, such as linear programming, to form complex models that attempt to find optimal solutions to real-world problems. Linear programming is a mathematical method for achieving the best outcome, i.e. maximum profit or minimum cost. Ighodae and Ekoko [19] in their study proved that ”every game problem can be calculated by converting it into a related linear programming problem, and every linear programming problem can be artificially converted into a game problem, resulting in a super linear programming problem” (para.1). Thie and Keough [20] assert that ”once our mathematical model for two-person zero-sum games is developed, the problems of existence and of computing a solution to a game will be related to the theory of linear programming, the unifying concept being the notion of duality” (p.8). The use of the Hurwicz criterion cannot go unmentioned in the application of linear programming, as it attempts to strike a balance between the maximax and maximin criteria calculated in linear programming [21].

3 Discussion

Game theory has the potential to address the challenges faced by railway companies in passenger services. It is unfortunate that most passenger train services are loss-making and depend on freight revenues for their operating costs in many railway companies. The losses are attributed to various factors, including low passenger numbers due to road transport running parallel to the railways. One might ask, why continue to operate if you are losing money? The problem is that the state railways cannot allow passenger services to be discontinued, as it is considered a form of service delivery to the vulnerable communities. Therefore, in this scenario, the only option for management and decision makers is to minimise the total cost of transport (the loss minimisation option). Based on the literature review on game theory, this study replicated the conference paper by Stoilova [22] with some modifications, to prove and determine how game theory can be applied to railway companies struggling to minimise the cost of rail passenger services in relation to road operators operating in parallel with rail. This was entirely based on the understanding of concepts, some of which were explained earlier in the introduction, in the literature and in the conference paper by Stoilova [22]. The concepts are game theory, Hurwicz criterion, linear programming, dual programming, perfect information game, non-cooperative game and two-person zero-sum game. In order to best illustrate this application, examples have been elaborated below. Please note that the numbers are fictitious and are used for illustration purposes only. The formulas for the calculations are also well tabulated. The game theory model represents the behaviour of a railway company, which we will call 'QVT', and road operators operating in parallel to the railway, competing only in passenger transport. The model is constructed and based on the revenue generated by the road operators' passengers that the rail operator has lost, applying the two-person zero-sum game principle. The game model would help to identify strategies for maximum revenue for road operators and minimum cost for QVT.

3.1 Strategies for both Competitors

The route under consideration is the route in from point X to Y for both QVT and the road operator. The player of road transport is presented by all road operators that use the very route. The following are QVT (Player Z) strategies:

1. QVT1- transport by express train. Stops only at major stations along the route.
2. QVT2 - transport by the Shuttle train. Like ordinary train but only runs twice a week.
3. QVT3- transport by Ordinary train. This one stops at every station along the route.

The following are the strategies for the road transport player (Player V):

1. RD1 – Transport by large buses (express). These are direct and fast buses with few stations along the route.
2. RD2- Transport by large buses (ordinary). These stop at every station.
3. RD3 – Transport by minibuses (run as ordinary). They stop at every station.

3.2 Method

Game theory is an approach that deals with the problems of conflicting objectives between two opponents. The two opponents are called players in a game conflict, and each has several strategies or alternatives. The game is represented by the decision/payoff $m \times n$ matrix:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots. & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

The rows do present strategies for player A ($i = 1, \ldots, m$), columns present strategies for player B(j = 1, ..., n). For instance,a_{21} , represents the profit of player A, if s/he chooses second strategy and player B chooses first strategy. This game is known as two-person zero-sum games, because a gain for one player implies an equal loss for the other. Whenever two players have more than one optimal strategy, then the game becomes a mixed strategy game. This game can be presented by dual linear programming method. The optimal solution of one problem (player A) automatically provides the optimal solution of the other (player B). Two problems optimize same value of the game. Below is a breakdown of the mixed strategies of the two players. The optimal mixed strategies for player V are:

$$\begin{gathered}\max_x \min \left\{ \sum_{i=1}^m a_{i1}x_i, \sum_{i=1}^m a_{i2}x_i, \ldots, \sum_{i=1}^m a_{in}x_i \right\} \\ x_1 + x_2 + \ldots + x_m = 1, \\ 0 \leq x_i \leq 1, \quad i = 1, 2, \ldots, m \end{gathered} \tag{1}$$

where $x = (x_1, \ldots, x_m)$ represents the respective probabilities for the strategies for player V.

Now, let:

$$\Phi = \min \left\{ \sum_{i=1}^m a_{i1}x_i, \sum_{i=1}^m a_{i2}x_i, \ldots, \sum_{i=1}^m a_{in}x_i \right\} \tag{2}$$

The equation implies that:

$$\sum_{i=1}^m a_{ij}x_i \geq \Phi, \quad j = 1, 2, \ldots, n, \tag{3}$$

where Φ is the value of the game.

The problem of player V can be written:

$$\begin{gathered}\text{Maximize } \Phi \text{ under the constraints:} \\ a_{11}x_1 + a_{12}x_2 + \ldots + a_{m1}x_m \geq \Phi, \\ a_{12}x_1 + a_{22}x_2 + \ldots + a_{m2}x_m \geq \Phi, \\ \vdots \\ a_{1n}x_1 + a_{2n}x_2 + \ldots + a_{mn}x_m \geq \Phi, \\ x_1 + x_2 + \ldots + x_m = 1, \\ 0 \leq x_i \leq 1, \quad i = 1, 2, \ldots, m \end{gathered} \tag{4}$$

For player Z the problem is as follows:

$$\begin{gathered}\min_y \max \left\{ \sum_{j=1}^n a_{j1}y_j, \ldots, \sum_{j=1}^n a_{j2}y_j, \ldots, \sum_{j=1}^n a_{mj}y_j \right\} \\ y_1 + y_2 + \ldots + y_n = 1, \\ 0 \leq y_1 \leq 1, \quad j = 1, 2, \ldots, n \end{gathered} \tag{5}$$

where $y = (y_1, \ldots, y_n)$ represents the respective probabilities for strategies for player Z.

Now let

$$\omega = \max\left\{\sum_{j=1}^{n} a_{1j}y_j, \sum_{j=1}^{n} a_{j2}y_j, \ldots, \sum_{j=1}^{n} a_{mj}y_j\right\} \tag{6}$$

The problem of player Z can be written as follows:

$$\begin{gathered}
\text{Minimise } \omega \text{ under the constraints:}\\
a_{11}y_1 + a_{12}y_2 + \ldots + a_{n1}y_n \leq \omega,\\
a_{12}y_1 + a_{22}y_2 + \ldots + a_{2n}y_n \leq \omega,\\
\vdots\\
a_{m1}y_1 + a_{m2}y_2 + \ldots + a_{mn}y_n \leq \omega,\\
y_1 + y_2 + \ldots + y_n = 1,\\
0 \leq y_1 \leq 1, j = 1, 2, \ldots, n
\end{gathered} \tag{7}$$

The two problems are usually transformed to be solved. The transformations highlighted on Tab. 1 and Tab. 2 below show in the first column the way of transformation and the second column shows the transformed model. The solution of the transformed model provides results for the game model using the dual linear programming.

Transformation for QVT	**Game model for QVT**
Minimise Φ	Maximize $\omega = Y_1 + Y_2 + \ldots + Y_n$
Subject to	Subject to
$a_{11}\frac{y_1}{\Phi} + a_{21}\frac{y_2}{\Phi} + \ldots + a_{m1}\frac{y_n}{\Phi} \geq 1$	$a_{11}Y_1 + a_{21}Y_2 + \ldots + a_{n1}Y_n \leq 1$
$a_{12}\frac{y_2}{\Phi} + a_{22}\frac{y_2}{\Phi} + \ldots + a_{m2}\frac{y_n}{u} \geq 1$	$a_{12}Y_1 + a_{22}Y_2 + \ldots + a_{n2}Y_n \leq 1$
$a_{1m}\frac{y_1}{\Phi} + a_{2m}\frac{y_2}{\Phi} + \ldots + a_{mn}\frac{y_n}{\Phi} \geq 1$	$a_{1m}Y_1 + a_{2m}Y_2 + \ldots + a_{mn}Y_n \leq 1$
$0 \leq y_j \leq 1,\ j = 1, 2, .., n$	$Y_j \geq 0,\ j = 1, 2, .., n$

Table 1 Transformations for QVT (Player Z)

Transformation for Road Operator	**Game model for Road Operator**
Maximise Φ	Minimise $z = X_1 + X_2 + \ldots + X_m$
Subject to	Subject to
$a_{11}\frac{x_1}{\Phi} + a_{21}\frac{x_2}{\Phi} + \ldots + a_{m1}\frac{x_m}{\Phi} \leq 1$	$a_{11}X_1 + a_{21}X_2 + \ldots + a_{m1}X_m \geq 1$
$a_{12}\frac{x_2}{\Phi} + a_{22}\frac{x_2}{\Phi} + \ldots + a_{m2}\frac{x_m}{\Phi} \leq 1$	$a_{12}X_1 + a_{22}X_2 + \ldots + a_{m2}X_m \geq 1$
$a_{1n}\frac{x_1}{\Phi} + a_{2n}\frac{x_2}{\Phi} + \ldots + a_{mn}\frac{x_n}{\Phi} \leq 1$	$a_{1n}X_1 + a_{2n}X_2 + \ldots + a_{mn}X_m \geq 1$
$0 \leq x_i \leq 1, i = 1, 2, .., m$	$X_i \geq 0\ ,\ i = 1, 2, .., m$

Table 2 Transformations for Road Transport Operator (Player V)

3.3 The Model of Game Theory

A payoff matrix indicates the likely value of different alternatives depending on the different possible outcomes associated with each different possible outcome associated with them. The use of a payoff matrix requires that several alternatives are available, that several different events could occur, and that the consequences depend on which alternative is chosen and which event or which event or set of events occurs. An important concept in understanding the payoff matrix is probability. A probability is the likelihood, expressed as a percentage, that a particular event will or will not occur. The payoff matrix in Tab. 3 below shows the revenue earned by the road transport operator from the sale of tickets to passengers, taking into account the differences in ticket prices according to bus category. The payoff matrix for QVT is formed as a transposed matrix of that of the road operator, since the revenue of the road operator is a loss to QVT. In this game model, those passengers who bought tickets from the bus operators are a loss to the rail operator (QVT), applying a two-person zero-sum game principle. This model is not only useful for QVT's decision makers to determine and identify the strategies with minimum costs, but also for the road transport operators who would determine the strategies with maximum profits.

Road Transport (Player V)		QVT (Player Z) — Strategies		
		QVT1	QVT2	QVT3
Strategies	Probabilities	y1	y2	y3
RD1	x1	2500	2000	3500
RD2	x2	5000	4500	5500
RD3	x3	5800	4000	4200

Table 3 Decision matrix (Payoff matrix), USD

The transformation for the player V (Road operators) according to Tab. 3 is as follows:

$$\begin{gathered}\text{Maximize the income, i.e., maximize } \omega = X_1 + X_2 + X_3,\\ \text{subject to}\\ 2500X_1 + 5000X_2 + 5800X_3 \leq 1\\ 2000X_1 + 4500X_2 + 4000X_3 \leq 1\\ 3500X_1 + 5500X_2 + 4200X_3 \leq 1\\ X_i > 0,\ i = 1,2,3\end{gathered} \tag{8}$$

The transformation for QVT according to Tab. 3 is as follows:

$$\begin{gathered}\text{Minimize the cost, i.e., minimize } \omega = Y_1 + Y_2 + Y_3,\\ \text{subject to}\\ 2500Y_1 + 2000Y_2 + 3500Y_3, \geq 1\\ 5000Y_1 + 4500Y_2 + 5500Y_3, \geq 1\\ 5800Y_1 + 4000Y_2 + 4200Y_3, \geq 1\\ Y_j > 0,\ i = 1,2,3\end{gathered} \tag{9}$$

The mathematical models represented by the above formulas are solved by the method of Dual Linear Programming. This means that the optimal (outcome) solution of one problem (road operator) automatically provides the optimal (outcome) solution of the other problem (railway operator).

3.4 Breakdown of Linear Programming Computation

3.4.1 Road Transport Operator (Player V)

The transformation for the player V (Road operators) according to Tab. 3 can be refereed to Eq. 8.

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate: As the constraints are of type '$\leq$', we should add slack variables, After introducing slack variables.

$$\begin{array}{c} \text{Maximize } Z = x_1 + x_2 + x_3 + 0S_1 + 0S_2 + 0S_3 \\ \text{subject to} \\ 2500x_1 + 5000x_2 + 5800x_3 + S_1 = 1 \\ 2000x_1 + 4500x_2 + 4000x_3 + S_2 = 1 \\ 3500x_1 + 5500x_2 + 4200x_3 + S3 = 1 \\ \text{and} \\ x_1, x_2, x_3, S_1, S_2, S_3 \geq 0 \end{array} \tag{10}$$

The optimum is determined iteratively. Please refer to Tab. 7 and 8 in the Appendix B. Based on the results of the Tab. 8 (see appendix B) the optimal solution is obtained with the value of the variables as : $x_1 = 0.0003$, $x_2 = 0$, $x_3 = 0$, and the maximum is $Z = 0.0003$.

3.4.2 QVT (Player Z)

The transformation for the player Z (Rail operator) according to Tab. 3 can be refereed to Eq. 9.

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate: As the constraints of type '$\geq$',we should subtract surplus variables S and add artificial variables A. After introducing surplus, artificial variables

$$\begin{array}{c} \text{Minimize } Z = y_1 + y_2 + y_3 + 0S_1 + 0S_2 + 0S_3 + A_1 + A_2 + A_3 \\ \text{subject to} \\ 2500y_1 + 2000y_2 + 3500y_3 - S_1 + A_1 = 1 \\ 5000y_1 + 4500y_2 + 5500y_3 - S_2 + A_2 = 1 \\ 5800y_1 + 4000y_2 + 4200y_3 - S_3 + A_3 = 1 \\ \text{and} \\ y_1, y_2, y_3, S_1, S_2, S_3, A_1, A_2, A_3 \geq 0 \end{array} \tag{11}$$

The optimum is determined iteratively. Please refer to Tab. 9-13 in the appendix B. Based on the results of the Tab. 13 (please refer to appendix B), the optimal solution is obtained with the value of the variables as : $y_1 = 0$, $y_2 = 0$, $y_3 = 0.0003$ and the minimum is $Z = 0.0003$.

Tab. 4 below presents the results for the respective probabilities for the strategies. Using the mathematical model of linear programming, the optimal solutions for both players has been determined based on the decision matrix above. The table below summarises the results of this model-based dual linear programming method. It shows that the optimal strategy for player V (road operators) is RD_1, i.e. transport by large buses (express buses). The optimal strategy for QVT is QVT_3, i.e. transport by ordinary train. The value game shows that both players are guaranteed a minimum payoff of 4,500 United State Dollars. The results were computed using Excel Solver, a tool built into Microsoft Excel, and later checked using linear programming online calculator (please refer to: https://cbom.atozmath.com/CBOM/Simplex.aspx?q=sm)..

Player road Transport (Player V)		QVT (Player Z)	
Strategies	Probabilities	Strategies	Probabilities
RD1	X1= 0.0003	QVT1	Y1=0
RD2	X2 =0	QVT2	Y2 = 0
RD3	X3=0	QVT3	Y3=0.0003

Table 4 Results for Decision matrix, in USD

3.5 Verifications of results using Leonid Hurwicz Criteria

The Hurwicz criterion allows decision-makers to consider both the worst and best possible outcomes simultaneously. To do this, they choose a ”coefficient of pessimism”, which is alpha (α) and is a number between 0 and 1. This number determines the emphasis on the worst possible outcome. The$(1-\alpha)$ number determines the emphasis to be placed on the outcome. The Hurwitz criterion was used to check the results. The strategies (RD_1,RD_2 and RD_3) for the road operator were aimed at choosing the strategy with the maximum profit (see Tab. 5 in Appendix A for the calculations and formulas) and the strategies(QVT_1, QVT_2 and QVT_3) for the rail operator were aimed at choosing the strategy with the minimum cost (see the Tab. 6 in the appendix A for the calculations and formulas).

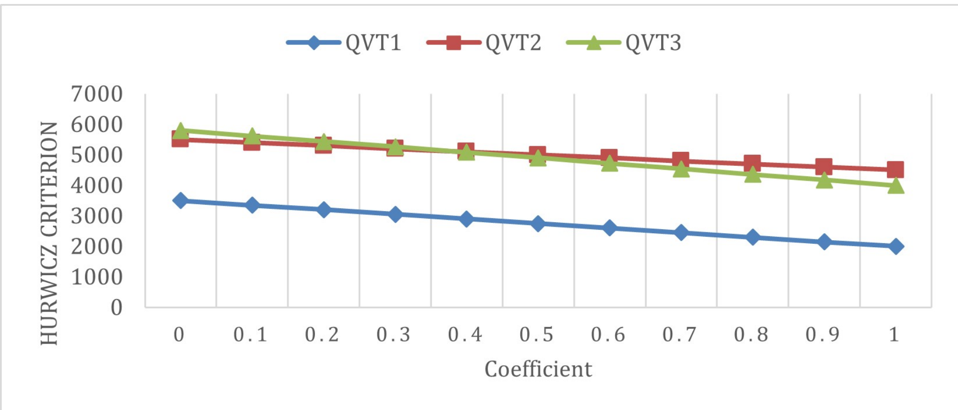

Figure 1 Ranking of QVT Strategies

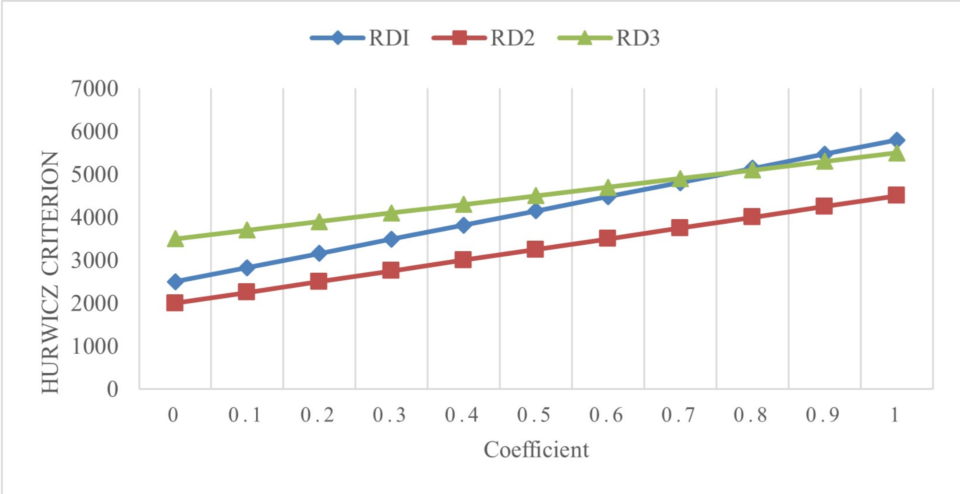

Figure 2 Ranking Strategies for Road Transport Player

The results shown in Tab. 4, Fig. 1 and Fig. 2 are consistent and confirm that the best strategy with the highest profit for road transport is RD_1 - transport by large buses (express), while the best strategy with the lowest cost for QVT is QVT_3 - transport by ordinary train. The application of this model would help decision-makers in the railway sector to identify the best low-cost strategies in rail passenger transport. The model is also essential for road transport operators to determine which strategies have the highest profits. Appendix A presents Leonid Hurwicz's criterion in detail, with both the calculation and the formula for both rail and road operators.

4 Conclusion

Most railway companies, especially the state-owned ones, make heavy losses on passenger services due to many factors. Despite these losses, state-owned railways cannot stop operating because of the need to provide passenger services to the most vulnerable communities, some of whom cannot afford to pay high bus fares and others who have limited access to roads. The managers of these state railways have no choice but to develop low-cost strategies for running passenger trains.

This paper explains how game theory can be used to formulate a model that can help the railway operator to identify the best low-cost strategies for operating passenger trains using the available information from the road operator (competitor) in a perfect game principle. The study also shows how road operators can also identify the best profit maximising strategies. The study has also shown the importance of integrating other tools, i.e. linear programming, Hurwicz criteria with game theory, to come up with the stronger game model for decision making. In particular, this paper also validates the findings of Stoilova [22] on the application of game theory in road and rail transport planning.

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Appendix A: The Leonid Hurwicz 's criterion

The Hurwicz criterion is probably one of the most widely used rules in decision making under uncertainty. Hurwicz's criterion gives each decision a value that is "a weighted sum of its worst and best possible outcomes", represented as (α) and known as the index of pessimism or optimism. It allows the decision maker to simultaneously consider the best and worst possible outcomes by formulating a "coefficient of optimism" that determines the emphasis on the best outcome. The Hurwicz criterion can be seen as a weighted average of the best and worst uncertainty realisations. Thus, it generalises the most optimistic Maximax criterion and the most pessimistic Maximin criterion - both of which are popular alternative rules for decision making under uncertainty - in a unified way. The classical Hurwicz criterion models uncertainty as a random variable governed by a known probability distribution. As such, the decision maker has perfect knowledge to accurately assess the loss under the criterion. However, such perfection is rarely available in practice.

In many real-world applications, the decision maker is typically faced with a data-driven environment with distributional ambiguity: the distribution of uncertainty is ambiguous and only partial knowledge is available, including prior statistical information (such as support and moments) and historical observations of uncertainty. In addition, the classical Hurwicz criterion only considers the best and worst outcomes, which can only occur with small probabilities, while neglecting all the other distributional information mentioned above, which is valuable for characterising the ambiguous distribution of uncertainty. Therefore, the classical Hurwicz criterion needs to be revised to address the challenge of incorporating distributional information under distributional ambiguity in emerging data-driven analytics.

"A Hurwicz weighted average H is calculated for every action strategy as follows: H $(A_1) = (\alpha)$ (row maximum) $+(1-\alpha)$ (row minimum) - for positive flow payoffs of profits or revenues and, H (A_1) = (α) (row minimum) + $(1-\alpha)$(row maximum) - for negative flow payoffs of costs.

Leonid Hurwicz's criteria for rail and road operators are presented in Tab. 5 and 6 below.

Payoff matrix				Leonid Hurwicz 's criterion										
Road	Rail			Coefficient a										
	QVT1	QVT2	QVT3	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
RDI	2500	5000	5800	2500	2830	3160	3490	3820	4150	4480	4810	5140	5470	5800
RD2	2000	4500	4000	2000	2250	2500	2750	3000	3250	3500	3750	4000	4250	4500
RD3	3500	5500	4200	3500	3700	3900	4100	4300	4500	4700	4900	5100	5300	5500

Table 5 Leonid Hurwicz 's criterion for Road Operators

Rail	Road			Coefficient a										
	RD1	RD2	RD3	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
QVTI	2500	2000	3500	3500	3350	3200	3050	2900	2750	2600	2450	2300	2150	2000
QVT2	5000	4500	5500	5500	5400	5300	5200	5100	5000	4900	4800	4700	4600	4500
QVT3	5800	4000	4200	5800	5620	5440	5260	5080	4900	4720	4540	4360	4180	4000

Table 6 Leonid Hurwicz 's criterion for QVT

Appendix B: Numerical Method - iterative equations

It is evident from Tab. 7 that the negative minimum Zj-Cj is -1 and its column index is 1. So, the entering variable is x_1. Minimum ratio is 0.0003 and its row index is 3. So, the leaving basis variable is S3.
The pivot element is 3500.
Entering = x_1, Departing = x_3, Key Element = 3500
$R_3(new) = R_3(old)/3500$
$R_1(new) = R_1(old) - 2500R_3(new)$
$R_2(new) = R_2(old) - 2000R_3(new)$

Iteration-1		Cj	1	1	1	0	0	0	
B	CB	XB	x1	x2	x3	S1	S2	S3	MinRatio $\frac{X_B}{X_1}$
S1	0	1	2500	5000	5800	1	0	0	$\frac{1}{2500} = 0.0004$
S2	0	1	2000	4500	4000	0	1	0	$\frac{1}{2000} = 0.0005$
S3	0	1	(3500)	5500	4200	0	0	1	$\frac{1}{3500} \approx 0.0003$
Z=0		Zj	0	0	0	0	0	0	
		Zj-Cj	-1	-1	-1	0	0	0	

Table 7 Numerical Method - 1st iterative equations

Iteration-2		Cj	1	1	1	0	0	0	
B	CB	XB	x1	x2	x3	S1	S2	S3	MinRatio
S1	0	0.2857	0	1071.4286	2800	1	0	-0.7143	
S2	0	0.4286	0	1357.1429	1600	0	1	-0.5714	
x1	1	0.0003	1	1.5714	1.2	0	0	0.0003	
Z=0.0003		Zj	1	1.5714	1.2	0	0	0.0003	
		Zj-Cj	0	0.5714	0.2	0	0	0.0003	

Table 8 Numerical Method - 2nd iterative equations

Iteration-1		Cj	1	1	1	0	0	0	M	M	M	
B	CB	YB	y1	y2	y3	S1	S2	S3	A1	A2	A3	MinRatio $\frac{Y_B}{Y_1}$
A1	M	1	2500	2000	3500	-1	0	0	1	0	0	$\frac{1}{2500} = 0.0004$
A2	M	1	5000	4500	5500	0	-1	0	0	1	0	$\frac{1}{5000} = 0.0002$
A3	M	1	(5800)	4000	4200	0	0	-1	0	0	1	$\frac{1}{5800} = 0.0002$
Z=3M		Zj	13300M	10500M	13200M	-M	-M	-M	M	M	M	
		Zj-Cj	13300M-1	10500M-1	13200M-1	-M	-M	-M	0	0	0	

Table 9 Numerical Method - 1st iterative equations

It is evident from Tab. 9 above that the positive maximum Z_j - C_j is 13300M-1 and its column index is 1. So, the entering variable is y_1. Minimum ratio is 0.0002 and its row index is 3. So, the leaving basis variable is A_3.
The pivot element is 5800.
Entering = y_1, Departing = A_3, Key Element = 5800
$R_3(new) = R_3(old)/800$
$R_1(new) = R_1(old) - 2500R_3(new)$
$R_2(new) = R_2(old) - 5000R_3(new)$

Iteration-2		Cj	1	1	1	0	0	0	M	M	
B	CB	YB	y1	y2	y3	S1	S2	S3	A1	A2	MinRatio $\frac{Y_B}{Y_3}$
A1	M	0.6	0	275.9	1689.7	-1	0	0.431	1	0	$\frac{0.6}{1689.7} \approx 0.0004$
A2	M	0.1379	0	1051.7241	(1879.3)	0	-1	0.86	0	1	$\frac{0.1379}{1879.3} \approx 0.00007$
y1	1	0.0002	1	0.6897	0.7241	0	0	-0.0002	0	0	$\frac{0.0002}{0.7241} \approx 0.00028$
Z=0.7M+0.0002		Zj	1	1327.59M+0.7	3569M+0.7	-M	-M	1.3M-0.0002	M	M	
		Zj-Cj	0	1328M-0.3	3569M-0.28↑	-M	-M	1.3M-0.0002	0	0	

Table 10 Numerical Method - 2nd iterative equations

The above Tab. 10 shows that the positive maximum Z_j - C_j is 3568.9655M-0.2759 and its column index is 3. So, the entering variable is y_3. Minimum ratio is 0.0001 and its row index is 2. So, the leaving basis variable is A_2.

The pivot element is 1879.3103.
Entering = y_3, Departing = A_2, Key Element = 1879.3103
$R_2(new) = R_2(old)/1879.3103$
$R_1(new) = R1(old) - 1689.6552R_2(new)$
$R_3(new) = R3(old) - 0.7241R_2(new)$

Iteration-3		Cj	1	1	1	0	0	0	M	
B	CB	YB	y1	y2	y3	S1	S2	S3	A1	MinRatio $\frac{Y_B}{S_2}$
A1	M	0.445	0	-669.72	0	-1	0.9	-0.344	1	$\frac{0.445}{0.9} \approx 0.49$
y3	1	0.0001	0	0.56	1	0	-0.0005	0.0005	0	—
y1	1	0.0001	1	0.2844	0	0	(0.0004)	-0.0005	0	$\frac{0.0001}{0.0004} = 0.25$
Z=0.445M+0.0002		Zj	1	-669.8M+0.844	1	-M	0.9M-0.0001	-0.344M+0	M	
		Zj-Cj	0	-669.72M-0.156	0	-M	0.9M-0.0001↑	-0.34M+0	0	

Table 11 Numerical Method - 3rd iterative equations

Tab. 11 above shows that the positive maximum Z_j - C_j is 0.8991M-0.0001 and its column index is 5. So, the entering variable is S_2. Minimum ratio is 0.3095 and its row index is 3. So, the leaving basis variable is y_1. The pivot element is 0.0004.
Entering = S_2, Departing = y_1, Key Element = 0.0004
$R_3(new) = R_3(old)/0.0004$
$R_1(new) = R_1(old) - 0.8991R_3(new)$
$R_2(new) = R_2(old) + 0.0005R_3(new)$

Iteration-4		Cj	1	1	1	0	0	0	M	
B	CB	YB	y1	y2	y3	S1	S2	S3	A1	MinRatio $\frac{Y_B}{S_3}$
A1	M	0.17	-2333	-1333	0	-1	0	(0.8333)	1	$\frac{0.17}{0.8333} = 0.204$
y3	1	0.0002	1.38	0.95	1	0	0	-0.0002	0	—
S2	0	0.31	2595.24	738.1	0	0	1	-1.31	0	—
Z=0.17M+0.0002		Zj	-2333M+1.38	-1333M+0.95	1	-M	0	0.83M-0.0002	M	
		Zj-Cj	-2333M+0.38	-1333M-0.0476	0	-M	0	0.83M-0.0002↑	0	

Table 12 Numerical Method - 4th iterative equations

Tab. 12 shows that a positive maximum Z_j - C_j is 0.8333M-0.0002 and its column index is 6. So, the entering variable is S_3. Minimum ratio is 0.2 and its row index is 1. So, the leaving basis variable is A_1.
The pivot element is 0.8333.
Entering =S_3, Departing =A_1, Key Element = 0.8333
$R_1(new) = R_1(old)/0.8333$
$R_2(new) = R_2(old) + 0.0002R_1(new)$

$$R_3(new) = R_3(old) + 1.3095R_1(new)$$

Iteration-5		Cj	1	1	1	0	0	0	
B	CB	YB	y1	y2	y3	S1	S2	S3	MinRatio
S3	0	0.2	-2800	-1600	0	-1.2	0	1	
y3	1	0.0003	0.7143	0.5714	1	-0.0003	0	0	
S2	0	0.5714	-1071.4286	-1357.1429	0	-1.5714	1	0	
Z=0.0003		Zj	0.7143	0.5714	1	-0.0003	0	0	
		Zj-Cj	-0.2857	-0.4286	0	-0.0003	0	0	

Table 13 Numerical Method - 5th iterative equations