# Application of Inverted Pendulum in Laplace Transformation of Mathematics Physics 

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#### Abstract

The Laplace transform is a technique used to convert differential equations into algebra, it is often used for the analysis of dynamic systems and inverted pendulum systems. An inverted pendulum is a mechanism that moves objects from one place to another and shows the function of its activity while walking. This system is widely used in various fields, for example in the fields of robotics, industry, technology and organics. In an inverted pendulum there is an inverted pendulum dynamic system with a reading and driving force. The type of research used is pure research with quantitative methods, the aim is to develop science that aims to find new theories and develop existing theories in natural science. The results of the study show that using the Laplace transform can make it easier to find solutions regarding the inverted pendulum system for a variety of conditions, both in the initial conditions and when given an additional force or load, so it is concluded the application of the Laplace transform is useful for understanding how an inverted pendulum system will react to various forces, loads and initial conditions, which can be used to predict how the system will operate in the real world.


Keywords: Inverted pendulum; Laplace transformation; Mathematics physics

## Introduction

Laplace transform is a technique used in converting differential equations into algebra, it is often used for the analysis of dynamical systems and inverse pendulum systems (Arifin et al., 2013; Peker et al., 2011; Wijaya et al., 2015). The reverse pendulum is a mechanism that moves objects from one place to another and shows the function of its activity when walking (Agarana \& Agboola, 2015). This system is widely used in various fields, for example in the fields of robotics, industry, technology and organic. In the reverse pendulum there is a dynamic system of reverse pendulum with damping and driving force (Henner et al., 2023; Tin et al., 2019). On an inverted pendulum or inverted pendulum using a Lagrange multiplier. The Lagrange multiplier becomes an important component in the solution process determined first using the Elzaki transform, then the

Lagrange multiplier obtained through the Elzaki transform matches the result obtained from the Laplace transform (Aranovskiy et al., 2019; Haider et al., 2023; Makkulau Makkulau et al., 2010).

An inverted pendulum is a great benchmark for testing various control algorithms because it is a nonlinear, multivariable, and unstable system (de Jesús Rubio, 2018; Huang et al., 2018). Inverted pendulum systems are a common problem that indicates the use of control systems to stabilize plant systems (Moatimid et al., 2023). It is a mechanical system consisting of a skeleton connected to a fulcrum with elastic straps. (Elzaki et al., 2012). This system can be used to study rotational dynamics and simple harmonic motion, but it can also be analyzed with Laplace transformations (Rizal \& Mantala, 2016).

Equation 1 shows the mathematical definition of Laplace's transformation (Agarana \& Akinlabi, 2019;

[^0]Altland \& von Delft, 2019; Arfken, 2013; Moss, 1992; Yudhi, 2019).

$$
\begin{equation*}
L\{f(t)\}=F(s)=\int_{0}^{\infty} e^{-s t} f(t) d t \tag{1}
\end{equation*}
$$

The description shown in Equation 1 is that $L$ is the laplace transform. $e^{-s t}$ is a kernel function of the transformation and $s$ is the variable of the Laplace transformation.

Laplace's equation and differential equation are inseparable, both have a close relationship (Khan et al., 2023). Applied analytical or numerical methods can solve differential equations to determine the position, velocity, and acceleration of objects at different times. In addition, by using frequency and response analysis to find out how the system will react to the forces acting on the object. The relation to the inverse pendulum can be written in the form of a differential equation as in Equation 2.

$$
\begin{equation*}
m d^{2} x+b d x+k \cdot \sin (x)=F \cos (\omega t) \tag{2}
\end{equation*}
$$

The information shown in Equation $2 m$ is the mass of the object, $b$ is the damping coefficient, $k$ is the tightness coefficient, $x$ is the inverse pendulum angle from the vertical position down, $F$ is the intensity of the driving force, $\omega$ is the frequency of the driving force and $t$ is time.

## Method

The type of research used is pure research with quantitative methods. Pure research is research to develop science that aims to find new theories and develop existing theories in natural science (Arsyam \& Tahir, 2021; Ji et al., 2023). The writing of this article aims to solve the differential equation of the langbanian result on the application of the inverse pendulum in the Laplace transformation in mathematical physics material.

## Result and Discussion

Analyze this inverted pendulum using the laplace transform (Putra \& Agustinah, 2016), The first thing to do is to change the differential equation in the form of an algebraic equation (Fahad et al., 2023).

The following are presented the steps for solving differential equations by applying the Laplace transform (Agarana \& Akinlabi, 2019; Arifin et al., 2013), so that differential equations using the Laplace equation can be
solved by means of both equations carried out Laplace transformations, the results obtained are simplified by algebraic means, then the inverse of the Laplace transform is sought in order to determine the influence of the time function. The solution of a given differential equation is called the inverse laplace transform (Kishimoto \& Ohnuki, 2023).

Systems with conservative forces are needed functions of langrange mechanics in handling work forces at any coordinates by analyzing them energetically (kinetically or potentially) (Agarana \& Agboola, 2015). An illustration of reverse pendulum modeling can be seen in Figure 1, and Figure 2 shows the vertical and horizontal components of the rope contained in the reverse pendulum.


Figure 1. Illustration of the reverse pendulum system


Figure 2. Vertical and horizontal parts of the rope on the reverse pendulum

The definition of lagrangian can be defined in Equation 3, with $T$ defined for kinetics and $V$ for potential (Agarana \& Akinlabi, 2019). The EulerLagrange equation is used to determine the equation of motion (Firdaus et al., 2023; Nasution et al., 2023). which can be seen in equation 4 . Based on Figures 1 and 2 obtained Equation 4.

$$
\begin{align*}
& L=T-V  \tag{3}\\
& \frac{d}{d t}\left(\frac{\partial L}{\partial \theta}\right)=\frac{\partial L}{\partial \theta} \tag{4}
\end{align*}
$$

$$
\begin{equation*}
x=l \sin \theta \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
y=l \cos \theta \tag{6}
\end{equation*}
$$

$$
\frac{d x}{d t}=d x=l(\cos \theta) \theta
$$

$$
\frac{d y}{d t}=d y=-l(\sin \theta) \theta
$$

$$
v^{2}=d x^{2}+d y^{2}
$$

$$
v^{2}=l^{2} d \theta^{2} \cos ^{2} \theta+l^{2} d \theta^{2} \sin ^{2} \theta
$$

$$
v^{2}=l^{2} d \theta^{2}\left(\cos ^{2} \theta+\sin ^{2} \theta\right)
$$

$$
v^{2}=l^{2} d \theta^{2}
$$

$$
L=\frac{1}{2} m l^{2} d \theta^{2}-m g l \cos \theta
$$

$$
\frac{\partial L}{\partial \theta}=m g l \sin d \theta
$$

$$
\frac{\partial L}{\partial \theta} m l^{2} \theta
$$

$$
\begin{equation*}
\frac{d}{d t}\left(m l^{2} d \theta\right)=m g l \sin \theta \tag{16}
\end{equation*}
$$

$m l^{2} d^{2} \theta=m g l \sin \theta$

The reverse pendulum system can be incorporated additional force (Resti, 2017), for Equation 17 it is a mathematical pendulum differential equation with nothing extra (Malik et al., 2022; Yudhi, 2019). An additional force found in the reverse pendulum is the damping force. If the additional damping force lies parallel to the speed, then the formulation of the motion has attenuation, which is shown in Equation 18.

$$
\begin{equation*}
\Upsilon \frac{d \theta}{d t} \tag{18}
\end{equation*}
$$

The equation of 18 is the damping coefficient. The damping coefficient indicates how well a system can dampen or reduce vibration (Wilujeng et al., 2022).

In addition to the presence of damping force, there is also driving force (Li et al., 2023). If there is a driving force then the equation will also have a driving force term, as shown in Equation 19, then from Equation 19 there is a dimensionless form so it is written in Equation 20.

$$
\begin{align*}
& m l^{2} d^{2} \theta+\Upsilon d \theta=m g l \sin \theta+C \cos \omega_{D} t  \tag{19}\\
& d^{2} \theta+\frac{1}{q} d \theta=\sin \theta+a \cos \omega_{D} t  \tag{20}\\
& d^{2} \theta=\sin \theta+a \cos \phi-\frac{1}{q} d \theta  \tag{21}\\
& d \omega=\sin \theta+a \cos \phi-\frac{1}{q} \omega \tag{22}
\end{align*}
$$

$C$ is the amplitodo for driving force and $\omega_{D}=\frac{d \phi}{d t}=$ $d \emptyset$ is driving force. The differential equations above are modeling of pendulums that have additional forces, namely damping forces and driving forces, then for the analysis of reverse pendulums with damping forces and driving forces using the Laplace transform (Susanto, 2023), The initial condition of the inverted pendulum system can be seen in Equation 23, after which an example is given to Equation 23 that is, and so for Equation 22 it becomes Equation 24.

$$
\begin{align*}
& d \omega=1-\frac{1}{2} \omega  \tag{23}\\
& L\{d \omega(t)\}=L\left\{1-\frac{1}{2} \omega\right\}  \tag{24}\\
& s[L\{\omega(t)\}-\omega(0)]=L\{1\}-\frac{1}{2} L\{\omega\}  \tag{25}\\
& s \omega(s)=\frac{1}{s}-\frac{1}{2} \omega(s)  \tag{26}\\
& \left(\frac{2 s+1}{2}\right) \omega(s)=\frac{1}{s}  \tag{27}\\
& \omega(s)=\frac{1}{s}\left(\frac{2}{2 s+1}\right)  \tag{28}\\
& \omega(s)=\frac{1}{s\left(s+\frac{1}{2}\right)}  \tag{29}\\
& \omega(s)=\frac{A}{s}+\left(\frac{B}{s+\frac{1}{2}}\right) \tag{30}
\end{align*}
$$

$$
\begin{equation*}
\omega(s)=\frac{(A+B) s+\frac{1}{2} A}{s\left(s+\frac{1}{2}\right)} \tag{31}
\end{equation*}
$$

Substitution of Equations 30 and 31 will get equation 30 to become Equation 32, then Equation 32 is solved using the inverse of the Laplace transform.

$$
\begin{align*}
& \omega(s)=\frac{2}{s}-\frac{2}{s+\frac{1}{2}}  \tag{32}\\
& L\{\omega(s)\}=2 L\left\{\frac{1}{s+\frac{1}{2}}\right\}  \tag{33}\\
& \omega(t)=2\left(1-e^{-\frac{1}{2} t}\right)
\end{align*}
$$

The values and variations, with the results of the equation, are obtained using the Laplace transform in the same way as above, for the results can be seen in Table 1 below.

Table 1. Variation of Angular Velocity $\theta$ and $\emptyset$

| $\theta$ | $\phi$ | $\omega(t)$ |
| :---: | :---: | ---: |
| 0 | $\frac{\pi}{6}$ | $2\left(1-e^{-\frac{1}{2} t}\right)$ |
|  | $\frac{\pi}{4}$ | $\sqrt{3}\left(1-e^{-\frac{1}{2} t}\right)$ |
|  | $\frac{\pi}{3}$ | $\sqrt{2}\left(1-e^{-\frac{1}{2} t}\right)$ |
| $\frac{\pi}{2}$ | $-2\left(1-e^{-\frac{1}{2} t}\right)$ |  |
| $\frac{\pi}{6}$ | 0 | $3\left(1-e^{-\frac{1}{2} t}\right)$ |



| $\theta$ | $\phi$ | $\omega(t)$ |
| :---: | :---: | ---: |
| $\frac{0}{6}$ | $4\left(1-e^{-\frac{1}{2} t}\right)$ |  |
| $\frac{\pi}{2}$ | $\frac{\pi}{4}$ | $(\sqrt{3}+2)\left(1-e^{-\frac{1}{2} t}\right)$ |
|  | $\left.\frac{\pi}{3}+2\right)\left(1-e^{-\frac{1}{2} t}\right)$ |  |
|  | $3\left(1-e^{-\frac{1}{2} t}\right)$ |  |
|  | $2\left(1-e^{-\frac{1}{2} t}\right)$ |  |

The relationship between the equations in Table 1 can be seen in Figures 3 and 4 below.


Figure 3. Graph of varying angular velocity against time $\emptyset$ on

$$
\theta=\frac{\pi}{6}
$$



Figure 4. Graph of varying angular velocity against time $\theta$
As in Figure 3 shows the angular velocity relationship as a function of time for $\theta=\frac{\pi}{6}$ with borders $0 \leq \phi \leq \pi$. Then in Figure 4 shows the angular velocity
relationship as a function of time for $\varnothing=\frac{\pi}{6}$ with borders
$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

## Conclusion

In the end, it can be concluded that using the Laplace transform can make it easier to find solutions to reverse pendulum systems for various conditions, both in initial conditions and conditions given additional forces or loads. The application of the Laplace transform is useful for understanding how a reverse pendulum system will react to a wide variety of forces, loads and initial conditions, which can be used to predict how the system will operate in the real world.

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## Author Contributions

In this study, the authors made different contributions. Theoretical analysis, data collection, analysis, and article writing were carried out by Trisonia Fitria, while supervision and review of writing were carried out by Wipsar Sunu Brams Dwandaru, Warsono, R. Yosi Aprian Sari, Dian Puspita Eka Putri, Adiella Zakky Juneid.

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## Conflicts of Interest

The authors declare no conflict of interest.

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