# An Integrated Inventory and Order Pick-up Model Considering the Capacities of Vendors, the Limited Number of Vehicles, and the Duration of Vehicle Trips for the Multi-Vendor Single-Buyer (MVSB) System

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***Abstract***
*Collaboration that is mutually beneficial between vendors and buyers to minimize inventory and pick-up costs is the basis for developing an integrated inventory and order pick-up model, known as inbound inventory routing problems (IIRP). Unfortunately, the IIRP model developed so far does not consider several conditions found in the manufacturing industry, including vendors' limited capacities, limited number of vehicles, and duration of vehicle trips. Based on these conditions, this study aims to develop an integrated inventory and order pick-up model that considers vendors' limited capacity, limited number of vehicles, and duration of vehicle trips for the Multi-Vendor Single-Buyer (MVSB) system. The total relevant costs, consisting of set-up, ordering, holding, and pick-up, are kept to a minimum. The model developed in this study is classified as mixed-integer non-linear programming (MINLP), which is hard to solve using an analytic or exact approach. So, a proposed algorithm using ant colony optimization (ACO) was developed to solve the problem. Model testing was carried out by developing three types of numerical examples: small, medium, and large-scale problems. The results obtained show that the proposed algorithm can find the best solution in a realistic amount of time.*

***Keywords***: *inventory, lot-size, milk-run, vehicle routing problem, ant colony optimization*

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## 1. Introduction

In order to produce goods that satisfy consumer demand, manufacturing companies (such as automotive, heavy equipment, machine tools, and electronics) provide a variety of parts either in-house or through outsourcing. Companies tend to outsource components previously produced (Wee, Peng, and Wee, 2010). Muller (2009) said that outsourcing ratios had reached at least 60% in numerous industries. In order to focus on their core competencies, manufacturing industries have outsourced a number of essential components in recent years (Glock and Kim, 2014).

Outsourcing is used in collaborations between suppliers, manufacturers, and other supply chain participants (including freight forwarders). As a result, manufacturers favor JIT procurement over short-term and self-serving purchasing strategies. Both Moura and Botter (2016) and Chen and Sarker (2010) recommended purchasing parts in small amounts as frequently as possible. There were trade-offs between inventory and transportation costs; thus, both had to be controlled simultaneously. The determination of vendor lot size and delivery policy is essential for JIT procurement to offer long-term benefits for all parties (Goyal and Deskmukh, 1991; Beck, Glock, and Kim, 2017).

The joint economic lot size (JELS) is a lot size that integrates all relevant costs between vendors and buyers (Banerjee, 1986; Glock, 2012; Kim and Goyal, 2009). JELS is classified into four types based on the structure of vendor and buyer relationships: single-vendor single-buyer (SVSB), single-vendor multi-buyer (SVMB), multi-vendor single-buyer (MVSB), and multi-vendor multi-buyer (MVMB) (Kim and Goyal, 2009; Nemoto, Kayashi, and Hashimoto, 2010; Stacey, Natarajarathinam, and Sox, 2007). The MVSB system is the focus of this research, since the manufacturing industry frequently collaborates with numerous vendors who offer various parts. The JELS model in the MVSB system is still limited; according to Glock (2012), who conducted a review of vendor-buyer integration that considered 155 articles, he found only 7 papers related to the MVSB system.

In order to minimize pick-up costs, several inventory models in the MVSB system proposed an integrated pick-up consolidation strategy (Glock and Kim, 2014; Beck et al., 2017; Stacey et al., 2007). In line with this strategy, a buyer provides large-capacity vehicles to collect small parts from each vendor and then carry them all to the buyer. The milk-run is one type of pick-up consolidation strategy that has been effectively used by Toyota and several other companies (Nemoto et al., 2010).

Chen and Sarker (2014), Stacey et al. (2007), and Natarajarathinam, Stacey, and Sox (2012) have developed an integrated inventory and pick-up model using a milk-run transportation mode. This model is a variant of the inbound inventory routing problem (IIRP), which uses a homogeneous vehicle for the MVSB system. Marpaung, Aribowo, Suprayogi, and Halim (2020) have proposed an IIRP model for the MVSB system that also uses a homogeneous vehicle that considers vendor capacities. However, all these model do not consider the types of vehicles, the limited number of pick-up vehicles available, and the duration of the vehicle trips.

In reality, in the manufacturing industry, the vehicles that operate are more than one type; the number of vehicles available for each type of vehicle is limited; and the duration of operations for vehicles is restricted by the maximum working hours of the vehicle's crew. So, there is a need to develop an integrated inventory and pick-up model for the MVSB system that considers the capacities of vendors, the limited number of vehicles, and the restriction of the duration of vehicle trips. This research focuses on developing an integrated inventory and pick-up model for the MVSB system that considers the capacities of vendors, the limited number of vehicles, and the duration of vehicle trips with the milk-run transportation mode to minimize the total relevant costs consisting of set-up, ordering, holding, and pick-up costs.

## 2. Method

### 2.1. Mathematical Model

In this study, the MVSB system consists of several vendors act as a manufacturer who provide parts for a single buyer also act as a manufacturer/assembler. The proposed model used some decision variables: the common pick-up cycle time, pick-up frequency, pick-up lot size, production lot size for each vendor, number of vehicles for each type of vehicle, and vehicle routes. The model minimizes the total relevant costs which consist of set-up, ordering, holding, and pick-up costs.

Some assumptions used in the model: (1) Each vendor offers only one kind of part to the buyer. (2) Production rates of parts at all vendors and the demand rate of parts at buyer are constant. (3) Each vendor's production rate is lower than the buyer's demand rate. (4) Shortage and backlog are not allowed. (5) The system is assumed not to produce defective parts. (6) All vendors use a JIT delivery policy with the same delivery cycle time. (7) The pickup of parts from vendors to buyer involves third-party logistics (TPL) services, using a *milk-run* transportation mechanism. (8) One vehicle handles one transportation route. (9) There are several types of limited-capacities vehicles that are available in limited quantities.

The indices, parameters, and variables used in this paper are described below:

***Indices***

$i,j$ = Vendor index, $i,j = 0,,..,n$
$o$ = Part index, $o = 1,..,O$
$k$ = Vehicle type index, $k = 1,..,K$
$l$ = Vehicle number index, $l = 1,..,L$

***Parameters***

$Do$ = Demand rate for Part $o$ ($o = 1,..,O$), (unit/year)
$Pio$ = Production rate of Vendor $i$ for part $o$, ($i = 1,......,n$; $o = 1,...,O$), (unit/year)
$HM_o$ = Holding cost of Parts $o$ at buyer ($/unit/year)
$HV_{io}$ = Holding cost of Parts $o$ at the Vendor $i$ ($/unit/year)
$A_i$ = Ordering cost from Vendor $i$ ($/order)
$S_{io}$ = Set-up cost of Part $o$ at the Vendor $i$, ($i = 1,..,n$), ($/*set-up*)
$F_k^0$ = Fixed transportation cost of type Vehicle $k$ ($/vehicle)
$F_y^o$ = Variable transportation cost per unit distance per unit weight of type Vehicle $k$ ($/kg/km)
$d_{ij}$ = Distance from Vendor $i$ to Vendor $j$, ($i,j = 1,..,n$), (km)
$C_k$ = Transportation capacity of type Vehicle $k$, (ton/truk)
$W_o$ = Weight of unit Part $o$ (kg)
$V_k$ = Number of vehicle available for Vehicle type
$N$ = Big number (in this paper: N = 1,000,000)

***Variables***

$T$ = Common pick-up cycle time (in year)
$x_{ijkl}$ = A binary variable set to 1 if type of Vehicle $k$ Vehicle number $l$ visited Vendor $j$ immediately after Vendor $i$ and 0 otherwise
$m_{io}$ = The number of pick-up of Parts $o$ from Vendor $i$ to buyer in each production cycle
$q_{io}$ = Pick-up lot size Part $o$ from Vendor $i$ in each common pick-up cycle time (unit)
$u_{ikl}$ = Variable for the sub-tour elimination

The mathematical formulation is stated as follows:

$$Min\ TC^C = \frac{1}{T}\left(\sum_{i=1}^{n}\sum_{o=1}^{O}\frac{S_{io}}{m_{io}} + \sum_{i=1}^{n} A_i\right) + \frac{T}{2}\sum_{o=1}^{O} HM_o\ D_o + \frac{1}{2}\sum_{i=0}^{n}\sum_{o=1}^{O} HV_{io}\,(q_{io}.m_{io})^2.\left(\frac{1}{P_{io}} - \frac{(m_{io}-1)}{m_{io}.D_o}\right) + \frac{1}{T}\left(\sum_{o=1}^{O}\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{K}\sum_{l=1}^{L} F_k^0.x_{ijkl}\right) + \frac{1}{T}\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{K}\sum_{l=1}^{L} F_k^y.x_{ijkl}.d_{ij}.CL_{ikl}\right) \quad (1)$$

Subject to:

$$\sum_{i=0}^{n} q_{io} \geq D_o T, for\ \forall\ \mathrm{i,o} \quad (2)$$

$$q_{io} \leq P_{io} T, for\ \forall\ \mathrm{i,o} \quad (3)$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=0}^{n} x_{ijkl} = 1, for\ \forall\ \mathrm{i};\ j \neq i \quad (4)$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{i=0}^{n} x_{ijkl} = 1, for\ \forall\ \mathrm{j};\ \mathrm{i} \neq j \quad (5)$$

$$\sum_{j=0}^{n} x_{ijkl} = 1\ for\ i = 0; \forall\ \mathrm{k,l} \quad (6)$$

$$\sum_{i=0}^{n} x_{ijkl} = 1\ for\ j = 0; \forall\ \mathrm{k,l} \quad (7)$$

$$\sum_{i=0}^{n} x_{ihkl} - \sum_{j=0}^{n} x_{hjkl} = 0\ for\ \forall\ \mathrm{h}; i,j \neq h; \forall\ \mathrm{k,l} \quad (8)$$

$$U_{jkl} \geq U_{jkl} + Wgt_j - N\left(1 - x_{ijkl}\right) for\ \forall\ \mathrm{i,j,k,l} \quad (9)$$

$$U_{jkl} \geq \sum_{i=0}^{n} x_{ijkl}.Wgt_j\ for\ \forall\ \mathrm{i,j,k,l}\ ; i \neq j \quad (10)$$

$$U_{ikl} \leq C_k\ for\ \forall\ \mathrm{i,j,k,l} \quad (11)$$

$$U_{ikl} = 0\ for\ i = 1\ ; \forall\ \mathrm{j,k,l} \quad (12)$$

$$CL_{ikl} = 0\ for\ i = 1; \forall\ \mathrm{j,k,l} \quad (13)$$

$$CL_{jkl} = \sum_{i=0}^{n}\left(Wgt_j + CL_{ikl}\right).x_{ijkl}\ \ for\ i \neq j;\ j = 2,.,n; \forall \mathrm{o,k,l} \quad (14)$$

$$CL_{ikl} \leq C_k for\ \ i = 1; \forall\ \mathrm{o,k,l} \quad (15)$$

$$\sum_{j=0}^{n}\sum_{l=1}^{L} x_{ijkl} \leq V_k\ for\ i = 0; \forall\ \mathrm{j,k} \quad (16)$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} x_{ijkl}.\left(d_{ij} + LT_i + UT_i\right) \leq 365 * 24 * T\ for\ \forall\ \mathrm{i,j,o,k,l} \quad (17)$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} x_{ijkl}.\left(d_{ij} + LT_i + UT_i\right) \leq 8 * 60\ for\ \forall\ \mathrm{i,j,o,k,l} \quad (18)$$

$$x_{ijkl} \in (0,1)\ \text{for}\ \forall\ i,j,k,l;\ \mathrm{T} > 0; \mathrm{q}_{io} \geq 0\ \text{for}\ \forall\ i,o; \mathrm{m}_{io} > 0\ \text{and}\ integer\ \text{for}\ \forall\ i,o \quad (19)$$

Equation (1) is an objective function consisting of set-up, ordering, holding, and pick-up cost. Constraint (2) and (3) show the vendors' capacity limitations. Constraints (4) and (5) ensure each vendor is only served by one vehicle. Constraints (6) and (7) ensure each route ends at the buyer's location. Constraint (8) ensures continuity of route. Constraints (9)-(12) ensure the elimination of sub-tours. Constraint (13) shows that each vehicle is empty when leaving the buyer's locations. Constraint (14) shows that the cumulative load of each vehicle is the sum of the load from the vendor location being visited and the cumulative load of the vehicle from the location visited previously. Constraint (15) shows the cumulative load of each vehicle does not exceed its capacity. Constraint (16) ensure quantity of vehicles operating for each type of vehicle does not exceed the number available. Constraints (17) and (18) ensure the duration of the vehicle trip does not exceed the common pick-up cycle time and maximum duration of pick-up time. Constraint (19) is the decision variable restriction.

The mathematical model mentioned above is classified as mixed-integer non-linear programming (MINLP), which is a combination of two models: the first is an integrated inventory model, which is an MINLP model, and the second is a vehicle-routing problem model (VRP), which is an NP-hard problem. Because the integration of inventory and VRP on the MVSB system is an NP-hard problem, it is difficult to solve using the exact method. As a result, in the next subsection, we construct a heuristic algorithm to solve this problem.

**2.2. Proposed Algorithm**

The proposed algorithm in this study uses a decomposition approach that divides the problem into two sub-problems: the inventory and the VRP sub-problems. The two sub-problems are solved separately, but both have two common decision variables: the common pick-up cycle time ($T$), and pick-up lot size ($q_{io} = \lambda_{io}.D_o.T$). The pick-up frequency ($m_{io}$) is decision variable that only exists in the inventory sub-problem, while for the VRP sub-problem it is vehicle route ($x_{ijkl}$).

Three propositions are developed as the basis for the development of the proposed algorithm. Proposition 1: The global optimum solution of order allocation for the integrated inventory and VRP model on the MVSB system is the same as the global optimum allocation of orders for the inventory model on the MVSB system developed by Park et al. (2006). Proposition 2: The minimum total relevant cost occurs when the utility of vehicles reaches full capacity. Proposition 3: The set of points $T$ that makes vehicle utility 100 percent (full capacity) is the ratio of the total capacity of the pick-up vehicle to the total weight of the load carried in one pick-up cycle. The heuristic algorithm flowchart developed is stated in Figure 1.

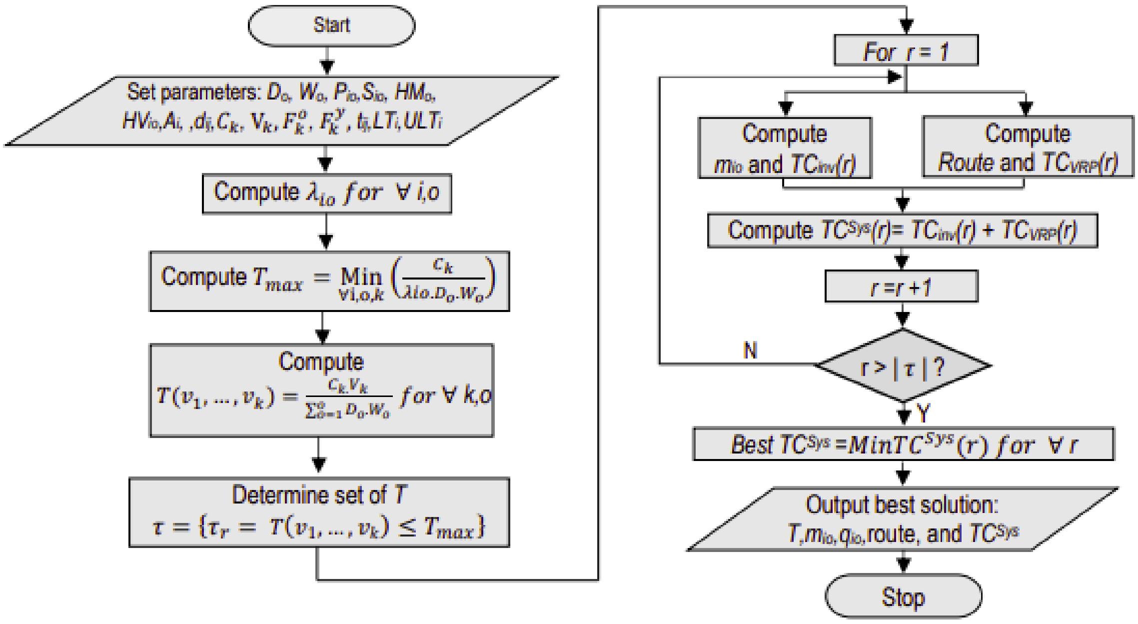



**Figure 1.** Flowchart of proposed algorithm

For the proposed algorithm, using Lingo 18.0, the computation time for the inventory sub-problem is still relatively realistic for various problem scales. Unfortunately, a different situation occurs in the VRP sub-problem, which shows that the computational time increases exponentially as the problem scale increases. Therefore, the VRP sub-problem uses ant colony optimization (ACO) to find the best solution. The ACO parameters used are the number of ant populations ($N$=10), $\alpha$ = 1, $\beta$ = 5, $\rho$ = 0.2, $\phi$ =1, and the maximum iteration number is 1,000. Sub-algorithm for the VRP Sub-problem using ACO is described in Figure 2.

```
Begin: input data and parameters initialization
    Set parameters of ACO
    Set parameters of model
NC = 1; (iterations initialize)
While NC ≤ NCmax; (maximum iteration number)
    For i = 1: N; (number of ant population)
        Route construction considering the capacity of vehicle, the number of
        vehicles available for each type of vehicle, and vehicle duration trip
        Compute f; (objective function)
        Find the best solution
        Record the best solution
    End for
    Entering adaptable control policy of ρ:
    If NC > Nnc (if the solution not improve after Nnc iteration)
        If (f_best (NC) = f_best (NC − Nnc) ;
            (f_best (NC) is the best fitness of iteration NC)
            ρ = 0.80*ρ
        End if
        If ρ ≤ ρmin
            ρ = ρmin
        End if
    End if
    Update pheromone
    NC = NC + 1
    Find the best solution of this iteration
End while
Output best solution
```

**Figure 2.** Pseudo code of the proposed algorithm for the VRP sub-problem

## 3. Result and Discussion

Some trials have been carried out using two small-scale numerical examples that could still be solved using an exact method to check the efficiency and effectiveness of the proposed heuristic algorithm. The first numerical example has a scenario of three vendors that provide one type of part for two type of vehicle (1P_3V_2TV), while the second scenario has four vendors that provide one type of part for three type of vehicle (1P_4V_3TV). For each scenario, as many as five sets of parameter data were developed. For each parameter data set, 10 replications were carried out to obtain the exact and proposed heuristic algorithm solutions using the computer configuration (Intel (R) Core (TM) i5-4200U CPU, 1.60GHz, 4GB memory).

The experiment results are shown in Table 1. AOF is the average of value of the objective function, and ACT is the average of the computational time. Based on the experimental results, it can be seen that the computing time using the exact method increases exponentially as the number of vendors increases. Therefore, it is unrealistic to solve the problem in this model using the exact method, especially for the large number of vendors. On the other hand, the proposed algorithm can find the best solution that is close to the global optimum solution (error below 0.05%) with reasonable computational time.

**Table 1**. The experiment results

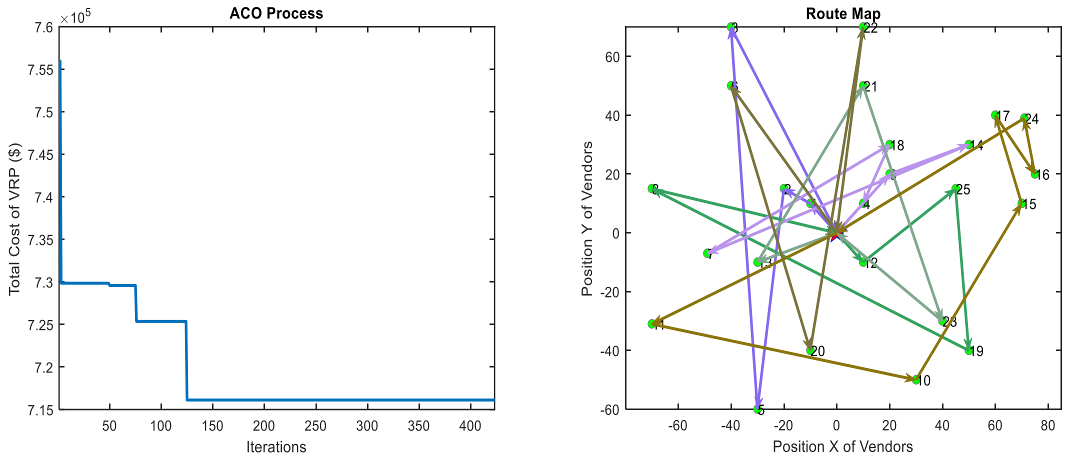
| Dataset name | Exact method | | Proposed heuristic algorithm | | Gap (%) | |
|---|---|---|---|---|---|---|
| | AOF ($) | ACT (second) | AOF ($) | ACT (second) | AOF | ACT |
| 1P_3V_2TV (1) | 242,840.10 | 321.15 | 242,852.20 | 22.14 | 0.00 | -93.11 |
| 1P_3V_2TV (2) | 242,243.40 | 324.11 | 242,353.40 | 21.54 | 0.05 | -93.35 |
| 1P_3V_2TV (3) | 249,441.50 | 344.55 | 249,875.60 | 21.45 | 0.17 | -93.77 |
| 1P_3V_2TV (4) | 315,425.60 | 367.44 | 315,541.10 | 21.55 | 0.04 | -94.14 |
| 1P_3V_2TV (5) | 214,515.60 | 337.65 | 214,765.40 | 21.45 | 0.12 | -93.65 |
| 1P_4V_3TV (1) | 289,765.40 | 25,486.40 | 289,871.50 | 66.42 | 0.04 | -99.74 |
| 1P_4V_3TV (2) | 285,590.40 | 25,165.40 | 285,843.45 | 63.45 | 0.09 | -99.75 |
| 1P_4V_3TV (3) | 201,267.50 | 24,911.20 | 201,341.55 | 73.94 | 0.04 | -99.70 |
| 1P_4V_3TV (4) | 257,765.00 | 25,765.20 | 259,456.45 | 71.15 | 0.66 | -99.72 |
| 1P_4V_3TV (5) | 247,897.00 | 25,986.10 | 247,953.22 | 67.19 | 0.20 | -99.74 |
| Average | 254,675.15 | 12,900.92 | 254,935.39 | 45.03 | 0.12 | -96.67 |

Three numerical examples for small, medium, and large problem scales are developed. For each scenario, five sets of parameter data were developed. For each parameter data set, 10 replications were carried out to obtain the proposed heuristic algorithm solution. The experiment results for proposed heuristic algorithm are shown in Table 2. The computation results show that the proposed algorithm provides stable results, as measured by the coefficient of variation in the objective function value and computation time, which are 1.19% and 1.42%, respectively. So, the proposed algorithm can be used to solve problems in this model at various scales with reasonable computation time.

To provide a simple description of the results, the best solution for the medium-scale problem scenario (5P_25V_3TV) is shown, as stated in Figure 2 and Table 3. The optimal point is 0.0229 years or around 200 hours. For the inventory sub-problem, the optimal pick-up frequency value for each vendor () in one production cycle for each vendor, ordered from vendor 1 to vendor 25, is {6, 11, 5, 6, 6, 5, 5, 6, 14, 5, 6, 5, 6, 6, 14, 6, 15, 5, 5, 5, 7, 4, 4, 5, 5}. The best number of vehicles is 6 units (all of them are type 3 vehicles that are available in as many as 10 units, each with a capacity of 30,000 kg). The trip length of each vehicle is in the range of 41 to 84 km, with vehicle utility varying from 57.15% to 100%. The duration of the vehicle trip (including unloading time at the buyer's location) varies from 5 hours to 8 hours, far below the common pick-up cycle of around 200 hours.

**Table 2**. The computation result for proposed heuristic algorithm

| Dataset name | Total relevant cost ($) | | | Computation time (second) | | |
|---|---|---|---|---|---|---|
| | Average | Standard deviation | Coefficient of variation | Average | Standard deviation | Coefficient of variation |
| 1P_5V_2TV (1) | 354,215.40 | 3,145.40 | 0.0089 | 72.5 | 1.23 | 0.0170 |
| 1P_5V_2TV (2) | 407,160.40 | 4,325.10 | 0.0106 | 77.4 | 1.23 | 0.0159 |
| 1P_5V_2TV (3) | 522,516.20 | 5,215.40 | 0.0100 | 76.5 | 1.34 | 0.0175 |
| 1P_5V_2TV (4) | 731,345.40 | 8,246.60 | 0.0113 | 75.4 | 1.34 | 0.0178 |
| 1P_5V_2TV (5) | 354,534.40 | 4,245.60 | 0.0120 | 75.8 | 1.47 | 0.0194 |
| 5P_25V_3TV (1) | 1,154,456.21 | 14,860.27 | 0.0129 | 145.5 | 1.48 | 0.0102 |
| 5P_25V_3TV (2) | 1,253,634.23 | 13,323.45 | 0.0106 | 127.8 | 1.59 | 0.0124 |
| 5P_25V_3TV (3) | 1,514,472.35 | 24,137.21 | 0.0159 | 128.4 | 1.48 | 0.0115 |
| 5P_25V_3TV (4) | 1,342,342.17 | 17,875.43 | 0.0133 | 135.5 | 2.27 | 0.0168 |
| 5P_25V_3TV (5) | 1,147,565.55 | 18,113.42 | 0.0158 | 147.4 | 2.18 | 0.0148 |
| 10P_50V_5TV (1) | 2,223,414.53 | 32,143.25 | 0.0145 | 342.3 | 3.16 | 0.0092 |
| 10P_50V_5TV (2) | 2,543,627.15 | 32,445.17 | 0.0128 | 356.1 | 3.45 | 0.0097 |
| 10P_50V_5TV (3) | 2,245,334.16 | 30,357.45 | 0.0135 | 323.7 | 4.13 | 0.0128 |
| 10P_50V_5TV (4) | 2,145,411.45 | 33,825.27 | 0.0158 | 345.5 | 4.29 | 0.0124 |
| 10P_50V_5TV (5) | 26,234,442.40 | 36,234.14 | 0.0014 | 346.2 | 5.23 | 0.0151 |
| Average | - | - | 0.0119 | 185.1 | - | 0.0142 |




(a) (b)

**Figure 2**. ACO for the medium-scale problem scenario: (a) ACO process; (b) Route map

**Table 3**. Route vehicle for medium-scale problem scenario

| Route number | Routes of vehicle | Route of length (km) | Duration of vehicle trip (hour) | Vehicle utility (%) |
|---|---|---|---|---|
| 1 | 0-1-2-5-3-0 | 45 | 5.0 | 89.67 |
| 2 | 0-12-25-19-8-0 | 51 | 7.5 | 91.43 |
| 3 | 0-9-14-7-18-4-0 | 63 | 6.0 | 82.75 |
| 4 | 0-13-21-23-0 | 41 | 7.0 | 77.56 |
| 5 | 0-6-20-22-0 | 48 | 8.0 | 100.00 |
| 6 | 0-11-10-15-17-16-24-0 | 84 | 7.0 | 75.97 |

In contrast to the exact approach, which shows that computing time is irrational and tends to increase exponentially as the scale of the problem increases, the proposed algorithm has computing time that tends to increase linearly as the scale of the problem increases but is still rational. The average computing time of the proposed algorithm for the small-scale problem scenario is 75.5 seconds, the medium-scale problem scenario is 137 seconds, and the large-scale problem scenario is 343 seconds. It can be seen that the proposed algorithm's computational time is rational and increases linearly as the problem scale increases.

The results obtained confirm the findings of Chen and Sarker (2014), Stacey et al. (2007), Natarajarathinam et al. (2012), and Marpaung et al. (2020), who recommend pick-up policies as often as possible in small lot sizes. However, the pickup policy depends on the parameter values used in the model; different values of the parameter may change the pickup policy. If the parameters related to the inventory sub-problem increase drastically, and conversely, at the same time, the parameter values related to the VRP sub-problem decrease drastically, then the pick-up policy is potentially different from the findings of this study. The findings of this study also indicate a tendency to use large-capacity vehicles while avoiding small-capacity vehicles. However, these findings certainly cannot be generalized because the results obtained depend on the parameter values used.

## 4. Conclusion

The problem discussed in this paper is modeled as an MINLP, which is hard to solve using an analytic or exact approach. Under these conditions, a proposed heuristic algorithm is developed to find the best solutions with rational computational time. The proposed heuristic algorithm takes a decomposition approach that separates the problem into two sub-problems: the inventory sub-problem and the VRP sub-problem. The inventory sub-problem was solved using Lingo 18.0, while the VRP sub-problem was solved with ACO using MATLAB R2022b. The experimental results indicated that the solution for three scales of problems could be obtained in rational computation time. Thus, the proposed algorithm can be used to solve IIRP problems on MVSB systems that consider the capacities of vendors, the limited number of vehicles, and the duration of vehicle trips.

Accommodating vendor capacity limitations, the limited number of vehicles, and the duration of vehicle trips as found in the manufacturing industry is essential in developing an integrated inventory and order pick-up model on the MVSB system. However, this model still has limitations because it does not consider grouping parts for pick-up, as is commonly found in the manufacturing industry. The model only uses one metaheuristic method, even though using two or more methods makes the solution search technique more reliable. For future research, it is suggested to develop an IIRP model for the MVSB system, considering grouping parts for pick-up and using more than one metaheuristic method to find the optimum solution.

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