Compressive Sensing Based on HQS for Image reconstruction

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Abstract: This work solves the image distortion problem caused by the noise generated during the sampling and reconstruction process, a compressive sensing algorithm based on half quadratic splitting (CS-HQS) is proposed to reconstruct images in this paper. For the part dominated by error terms, the regularization term is introduced and the second-order momentum adaptive gradient descent method is used to get the auxiliary variables. For the part dominated by the sparse prior of compressive sensing, the Bayesian maximum posterior inference is used to get the sparse coefficient. The combination of the two methods not only avoids the generation of random noise, but also enhances the stability of the model. The experimental results demonstrate that the strong robustness of the proposed algorithm.

Keywords: Compressive sensing, Bayesian learning; Second-order momentum adaptive gradient descent; Image reconstruction

1. Introduction

As a new sampling technology, compressive sensing has attracted extensive attention in the past few years. The existing compressive sensing methods include greedy iterative algorithm and convex optimization algorithm. In recent years, the Bayesian method has been widely applied in compressive sensing, the signal is usually given a sparse prior, such as Laplacian priori and gaussian inverse gamma prior. Compressive sensing can be understood as the known compressed observed value y, which is used to reconstruct the unknown sparse signal x, it is a process from low dimension to high dimension, in the process of obtaining the observed value, noise may be generated. The following is the mathematical expression Eq.1:

$$y=\Phi x+\varepsilon \tag{1}$$

Where is the measurement matrix, $\Phi=[\phi_1,\phi_2,...,\phi_M]^T, \in R^{M\times N}, M \ll N,$ y is the compressed signal representation $y=[y_1,y_2,...,y_M]^T \in R^M$, x is the original digital signal $x=[x_1,x_2,...,x_N]^T \in R^N$, is the noise, $\varepsilon=[\varepsilon_1,\varepsilon_2,...,\varepsilon_M]^T \in R^M$, As shown in Eq.1, signal acquisition is a process of linear projection. Usually, the signal is not sparse, but it is sparse in a transformation domain, so the signal should be sparsified firstly : is the sparse matrix $\Psi=[\Psi_1,\Psi_2,...,\Psi_N]^T \in R^{N\times N}$. s is the sparsity coefficient and it is k sparse, which is sorted into Eq.2.

$$y=As+\varepsilon \tag{2}$$

Where, A=ΦΨ, Eq.2 is an underdetermined equation, when solving the result of Eq.2, it is a NP-hard problem, But when Φ and Ψ satisfy the restricted isometry property(RIP), can solve the Eq.2. Compressive sensing and relevant knowledge are introduced in , which proved that when Φ and Ψ satisfy the condition of the low correlation, can replace RIP conditions. The correlation between Φ and Ψ can be represented as:

$$\mu(\Phi, \quad) = \sqrt{n}\cdot\max_{1\le k,j\le n} |, <\varphi_k|\psi_j> \tag{3}$$

We can solve the Eq.2 by the following Eq.4:

$$\min_s \|s\| \quad subject\ to\ As=y \tag{4}$$

When we solve the problem Eq.4, It usually includes greedy iterative algorithm, convex optimization algorithm and reconstruction algorithm based on Bayesian framework.The reconstruction problem of compressive sensing is solved from the perspective of Bayesian, which improves the sparsity.A new depth heterogeneous optimization framework is proposed to learn the scale parameters of prior distribution. By combining the regularization term of deep learning with the neighboring operator, introducing the regularization term by the residual regression network. A new normal product prior is introduced to solve the problem of compressive sensing without noise and these two algorithms of NP-0 and NP-1 are derived. New adaptive gradient descent models are proposed in; A new Bayesian algorithm based on Bayesian learning is proposed;A method of combining CS with convolutional neural network (CNN) is proposed, The sparse array design is used to reduce the cost and achieve high resolution imaging;By minimizing cross entropy cost function and learning model parameters, a greedy solver for sparse vector reconstruction is proposed;The layered Laplace prior model is used in the signal coefficients;The AMP-Net model is proposed for image denoising.

However, Bayesian methods are prone to generate Gaussian noise, and adaptive gradient descent methods can avoid local optimization, but they are easy to generate fringe noise during image reconstruction. In this paper, we combine the two and propose a new model:CS-HQS. Our contributions are mainly reflected in the following three aspects:

1. Introduce auxiliary variables and regular terms to avoid the accumulation of errors;
2. Using second-order momentum adaptive gradient descent to solve the problem can avoid local optimum;
3. Considering the sparse prior problem of compressive sensing, the inference can be more accurate by using the prior characteristics of Bayes.

2. Compressive Sensing based on HQS

In this paper, using the idea of half quadratic splitting, the reconstruction process of compressive sensing is divided into two parts. The first part is dominated by the error term and uses the second-order momentum adaptive gradient descent method to obtain auxiliary variables; the second part is dominated by sparse prior and uses Bayesian learning to obtain sparse coefficients.

$$z=\arg\min\|Az-y\|_2^2+\lambda_1\|z-s\|_2^2 \tag{5}$$

$$S=\arg\min\|z-s\|_2^2+\lambda_2\|s\|_1 \tag{6}$$

Where, λ_1 is the penalty, $\|\cdot\|_2$ is the 2-norm.

2.1 Second order momentum Adaptive gragient descent

Adaptive gradient descent is an optimization algorithm for gradient descent method, which can avoid local optimum and make the function converge faster. In this paper, the second-order momentum adaptive gradient descent method is used to solve the auxiliary variables, and the regularization term is introduced to obtain the minimization function Eq.5 in the above equation. Adaptive gradient descent defines second-order momentum as the sum of squares of all gradients in history:

$$\hat{v}_t=\sum_{i=0}^{t}\nabla grad_i^2 \tag{7}$$

Where, *grad* is the function gradient, and the model related parameters update formula is as follows:

$$m_t=m_{t-1}*\beta_1+grad(1-\beta_1) \tag{8}$$

$\lambda_1, \lambda_2 \in [0,1]$ is the penalty term, let m_t be the first-order momentum at time t.

$$v_t=v_{t-1}*\beta_2+\hat{v}_t*(1-\beta_2) \tag{9}$$

$\beta_1, \beta_2 \in [0,1]$ is the momentum factor.

$$z=z_{t-1}-a*\frac{m_t}{\sqrt{v_t}} \tag{10}$$

α is the learning rate.

2.2 Bayesian learning

In compressive sensing, the sparsity coefficients can be regarded as T-distribution or Gaussian distribution. In this paper, we adopt the Gaussian distribution. For Eq.6, it is assumed that the sparsity coefficients and noise parameters obey gamma distribution Eq.11 and Eq.12, whose prior information is :

$$p(\alpha)=\prod_{i=1}^{N}Gamma(\alpha_i\mid a,b) \tag{11}$$

$$p(\beta)=\prod_{i=1}^{N}Gamma(\beta\mid c,d) \tag{12}$$

Where, let the scale parameter set $M=\{a,b,c,d\}$ be the hyperparameter set of the prior distribution, Parameters $a=[a_1,a_2,...a_N]$, $b=[b_1,b_2,...b_N]$, α_i^{-1} and β^{-1} are respectively the variances of Gaussian distributions of sparsity coefficient s_i and noise ε_i,which can be obtained from the maximum likelihood function. The mean and covariance of sparse signal Eq.13 and Eq.14 are:

$$\mu=\beta\sum\Phi^T y \tag{13}$$

$$\sum=(\beta\Phi^T\Phi+A)^{-1} \tag{14}$$

Where,$A=diag(\alpha_1,\alpha_2,...,\alpha_N)$

According to the maximum likelihood estimation method, parameters Eq.15, Eq.16,

$$\hat{\alpha}_i=\frac{1+2\alpha_i}{\sum_{ii}+\mu_i^2+2b} \tag{15}$$

$$\hat{\beta}=\frac{M-\sum_{i=1}^{N}\gamma_i+2c}{\|y-\Phi\mu\|_2^2+2d} \tag{16}$$

Where, $\gamma_i=1-\alpha_i\sum_{ii}$,$\|\bullet\|_2$ is the 2-norm.

2.3 Evaluative criteria

In this paper, two indexes are used to evaluate the effect of image reconstruction, which are peak signal-to-noise ratio (PSNR) and mean square error (MSE).

PSNR is a broad image evaluation index. The larger the PSNR, the more similar two images are,which is defined as Eq.17:

$$PSNR=10\log_{10}\left(\frac{\max^2}{MSE}\right) \tag{17}$$

Where, $\max^2$ represents the maximum possible pixel value of the image, and MSE represents Eq.18 as follows;

$$MSE=\frac{1}{hw}\sum_{i=0}^{h-1}\sum_{j=0}^{w-1}\|K(i,j)-I(i,j)\|^2 \tag{18}$$

3. Experimental Results

Through experimental analysis, this paper shows the algorithm framework, including OMP algorithm, IHT algorithm, compressive sensing based on second-order momentum adaptive gradient descent, Bayesian compressive sensing and algorithm. minist handwriting dataset is taken as the experimental object, and the size of the image is 62×152.

3.1 Selection of observation values and image Sparse

In compressive sensing, observation values need to be obtained, the measurement matrix and sparse matrix need to meet the RIP condition. When the measurement matrix and sparse matrix are not related, they can be equivalent to the RIP condition. Generally, when the measurement matrix and sparse matrix are both random matrices, they meet the irrelevant condition. The measurement matrix is taken as

Gaussian random matrix, and the sparse matrix is obtained through two-dimensional discrete cosine transformation.

3.2 Image reconstruction

The sampling rate needs to be set in advance. We set these sampling rates of 30% - 90% for comparison and analysis. Under each sampling rate, we set the number of iterations to 30. Each algorithm is evaluated by PSNR, MSE and runtime.

When the sampling rate is 70% and the algorithm is iterated for 30 times, the runtime and PSNR of , BCS, SGD, IHT and OMP is shown in Table.1:

Table 1 Comparison results

Algorithm	PSNR(dB)	time(s)
	31.786	11.2897s
BCS	31.590	11.2126
SGD	31.252	1.0214s
IHT	28.703	0.2270s
OMP	27.473	0.2491s

Compared with the above five algorithms, CS-HQS takes the longest relative time; As shown in Fig. 1, compared with CS-HQS, SGD and BCS, MSE gradually decreases with the increase of iteration times, and BCS converges fastest, but MSE is larger than CS-HQS, CS-HQS converges slowly, but with higher accuracy. As shown in Fig 2, shows that with the increase of the sampling rate, the PSNR growth of CS-HQS algorithm is very stable and higher than that of other algorithms. The PSNR growth of BCS and SGD is unstable and has a downward trend. The PSNR growth of OMP and IHT is also unstable and small.

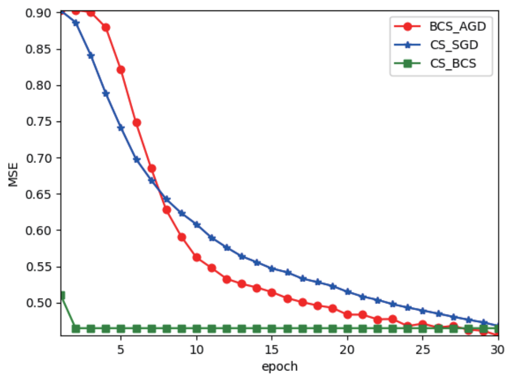


Fig 1 The MSE of algorithms

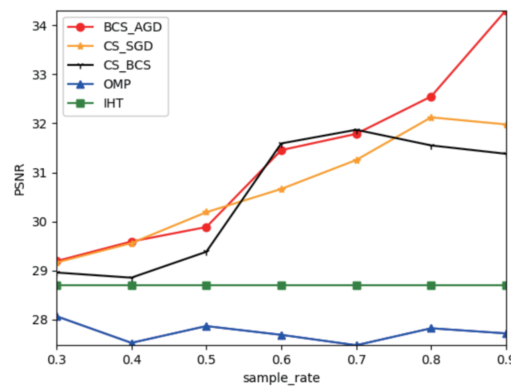
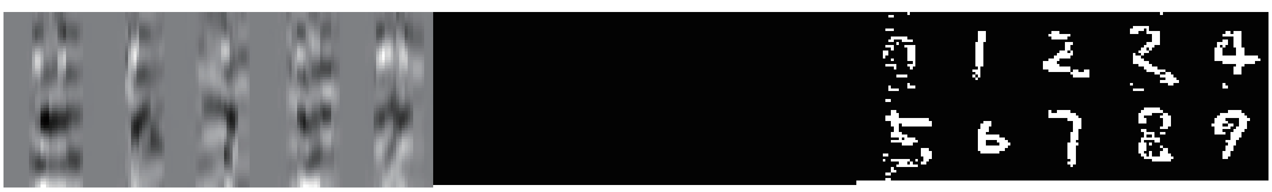
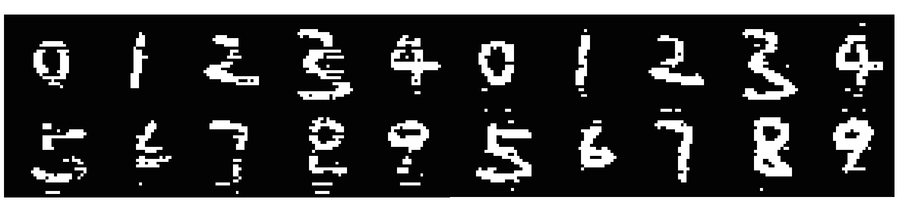


Fig 2 The PSNR of algorithms

As shown in Fig.3, these are reconstructed images of five algorithms. It is not difficult to see that the image reconstructed by CS-HQS is the easiest to identify. The OMP algorithm and IHT algorithm are distorted. Although some BCS figures are well reconstructed, but individual numbers are not easy to distinguish. The image reconstructed by SGD contains stripe noise.



OMP **IHT** **BCS**



SGD **CS-HQS**

Fig 3 Image reconstruction

Conclusion

This paper mainly studies the reconstruction algorithm of CS-HQS to reconstruct the image. CS-HQS splits the solution of model sparse coefficient into two parts, which not only avoids the generation of random noise caused by Bayesian learning, but also prevents the stripe noise caused by adaptive gradient decline, and also enhances the stability of the model. Although this paper does not use many images,but compared with the single minist data set, it can reflect the reconstruction effect of each number.

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