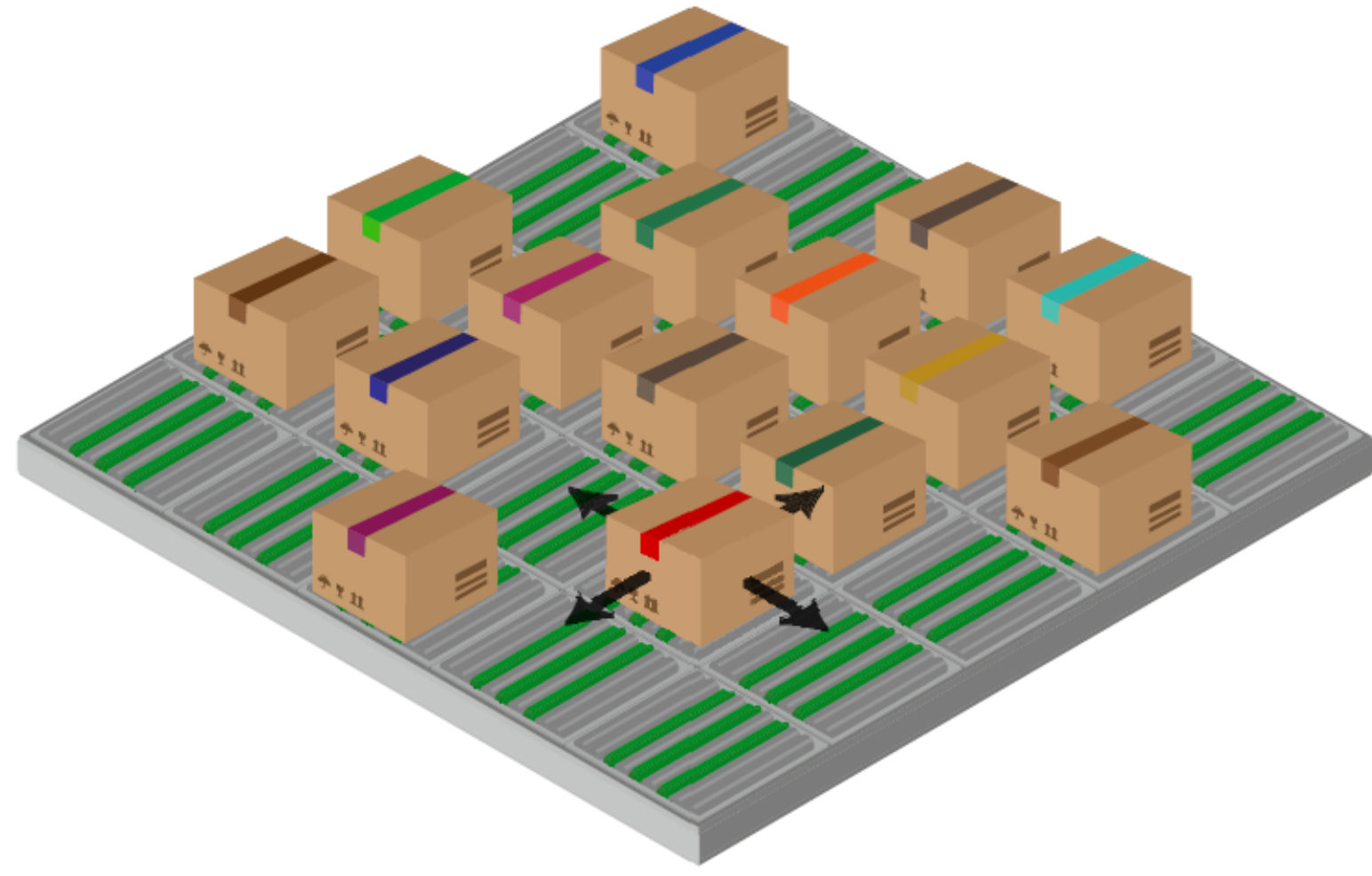


10 An efficient MILP formulation for the parallel-load retrieval in puzzle-based storage systems with simultaneous load movements

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Environment: puzzle-based storage (PBS)



Formulation

$$\min \alpha z + \beta \sum_{t=0}^T \sum_{l \in \mathcal{L}} \sum_{l' \in N'(l)} u_{l,l',t,1} + \gamma \sum_{k=1}^2 \sum_{t=0}^T \sum_{l \in \mathcal{L}} \sum_{l' \in N(l)} u_{l,l',t,k} \quad (1)$$

subject to

$$\sum_{l' \in N'(l)} u_{l,l',0,k} = 1 \quad \forall k \in \{1,2\}, l \in I_k \quad (2)$$

$$\sum_{l' \in N'(l)} u_{l,l',t-1,1} = \sum_{l' \in N'(l)} u_{l,l',t,1} \quad \forall t = 1, \dots, T, l \in \mathcal{L} \setminus O \quad (3)$$

$$\sum_{l' \in N'(l)} u_{l,l',t-1,2} = \sum_{l' \in N'(l)} u_{l,l',t,2} \quad \forall t = 1, \dots, T, l \in \mathcal{L} \quad (4)$$

$$u_{l,l',T,1} = 0 \quad \forall l \in \mathcal{L} \setminus O, l' \in N'(l) \quad (5)$$

$$\sum_{k=1}^2 \sum_{l' \in N(l)} u_{l,l',t,k} \leq 1 \quad \forall l \in \mathcal{L}, t = 0, \dots, T \quad (6)$$

$$\sum_{k=1}^2 \left(\sum_{l' \in N(l)} u_{l,l',t-1,k} + \sum_{l' \in N(l)} u_{l,l',t,k} \right) \leq 1 \quad \forall l \in \mathcal{L}, t = 1, \dots, T \quad (7)$$

$$t \sum_{l' \in N(l)} u_{l,l',t,1} \leq z \quad \forall l \in \mathcal{L}, t = 1, \dots, T \quad (8)$$

$$u_{l,l',t,k} \in \{0,1\} \quad \forall l \in \mathcal{L}, l' \in N(l), t = 0, \dots, T, k \in \{1,2\} \quad (9)$$

Load movement

Main Objective: Optimal retrieval of multiple-items

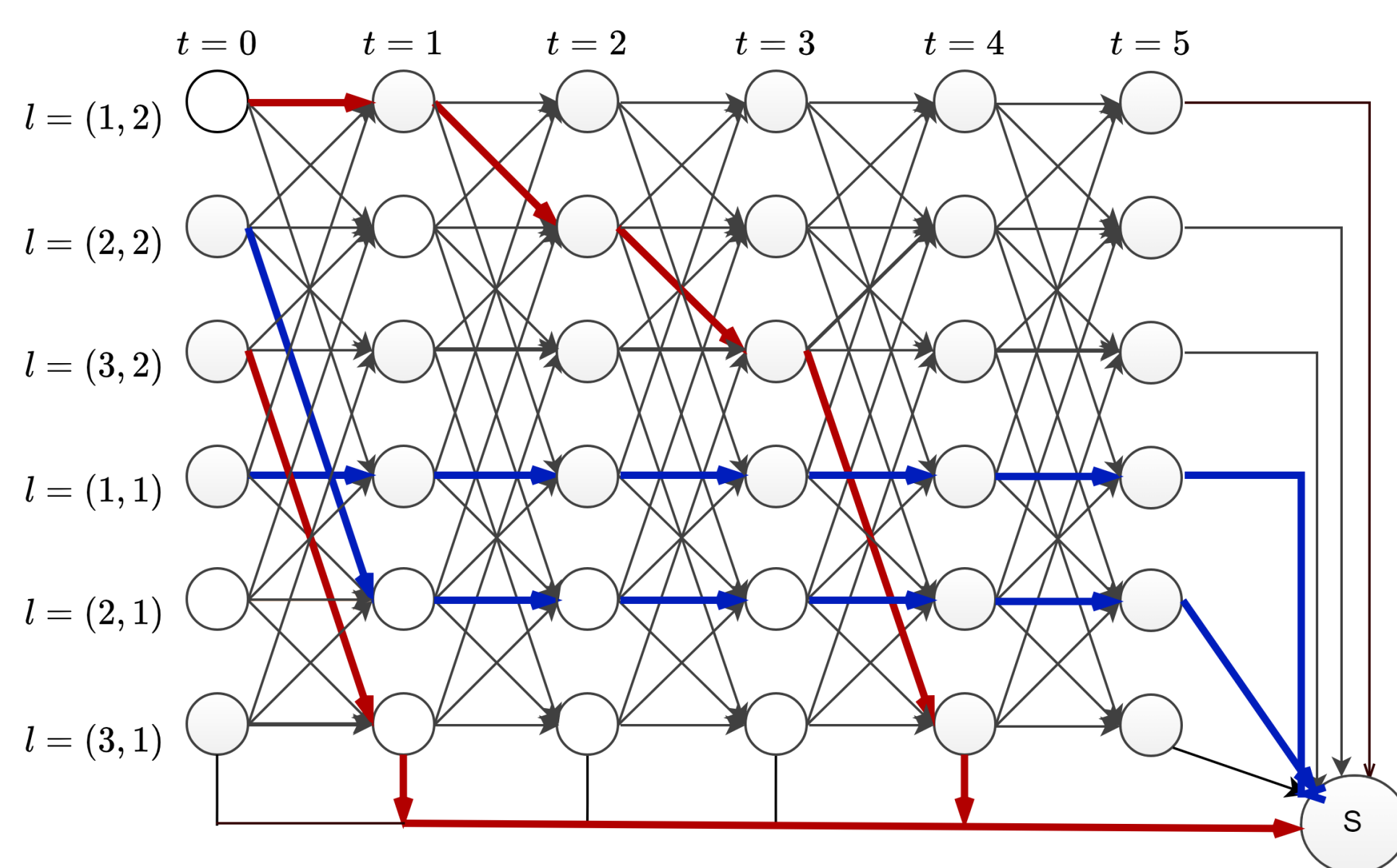
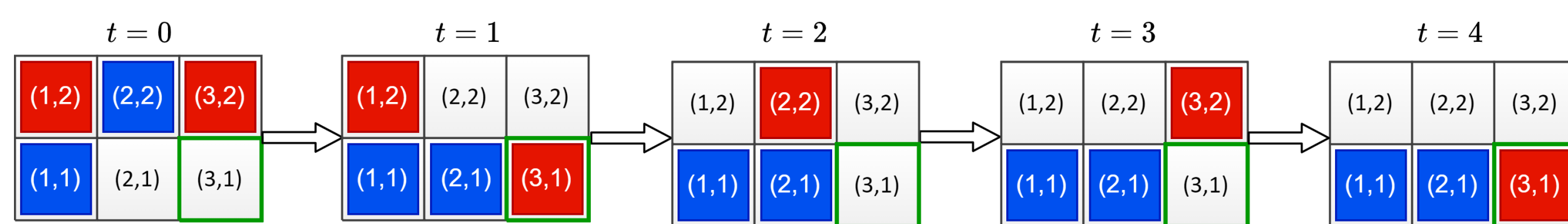
- Minimize (makespan, flowtime, # movements)

Assumptions

- Static problem
- Multiple loads retrieved in parallel
- Simultaneous moves
- Block movements

Methodology

- Time-expanded graph (TEG) based formulation
- 2-commodity (blocking and target loads) network flow model on the TEG

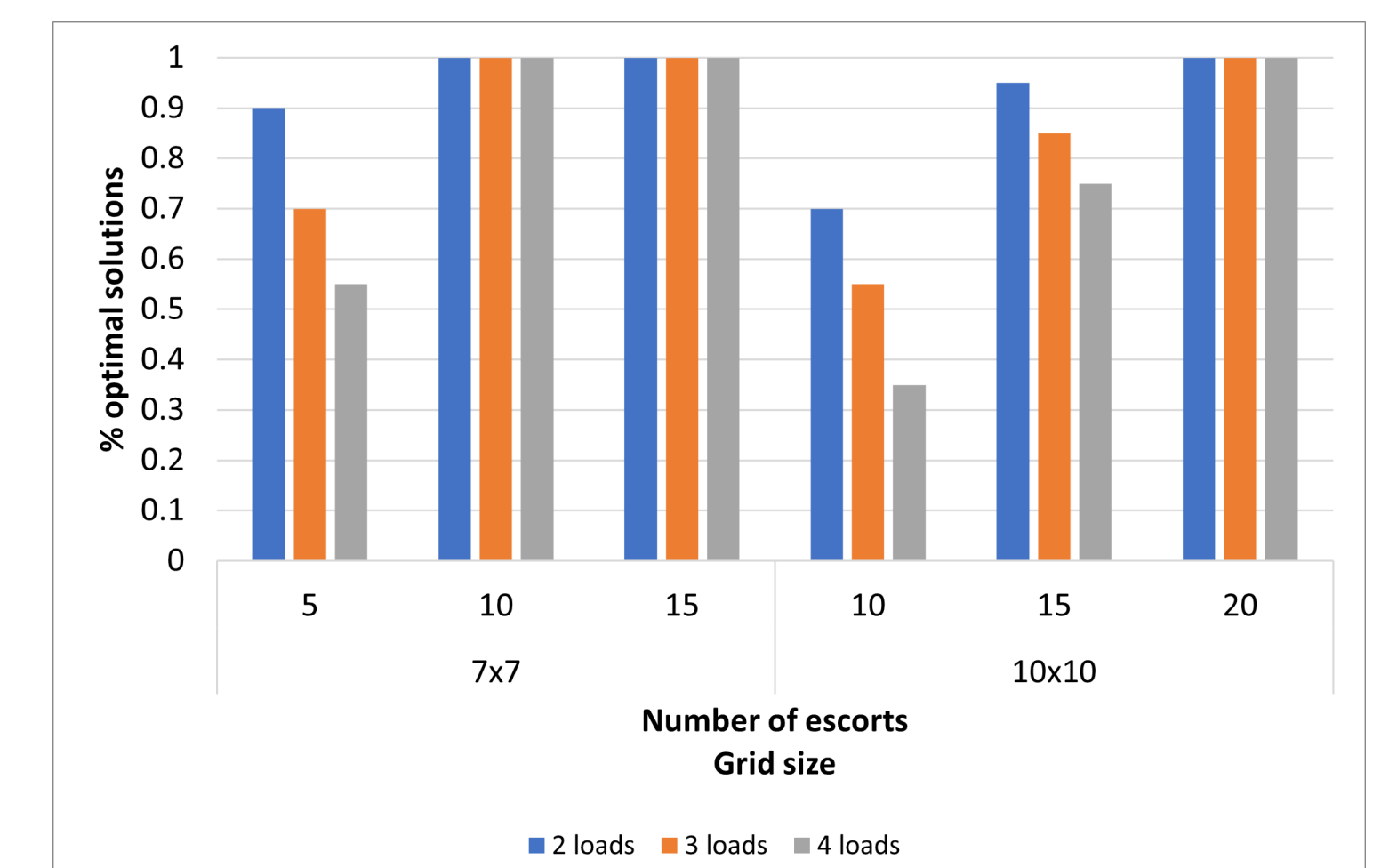
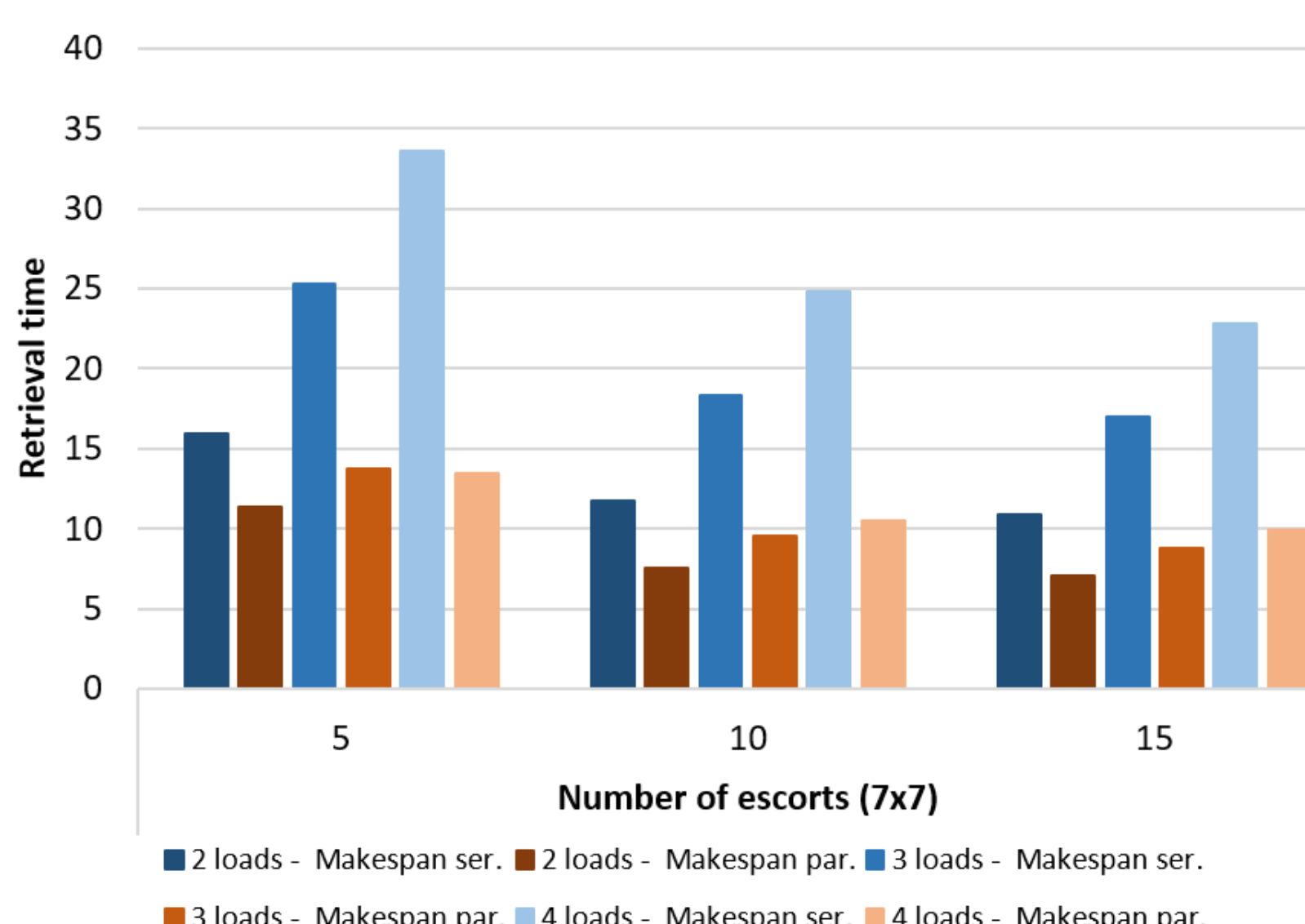
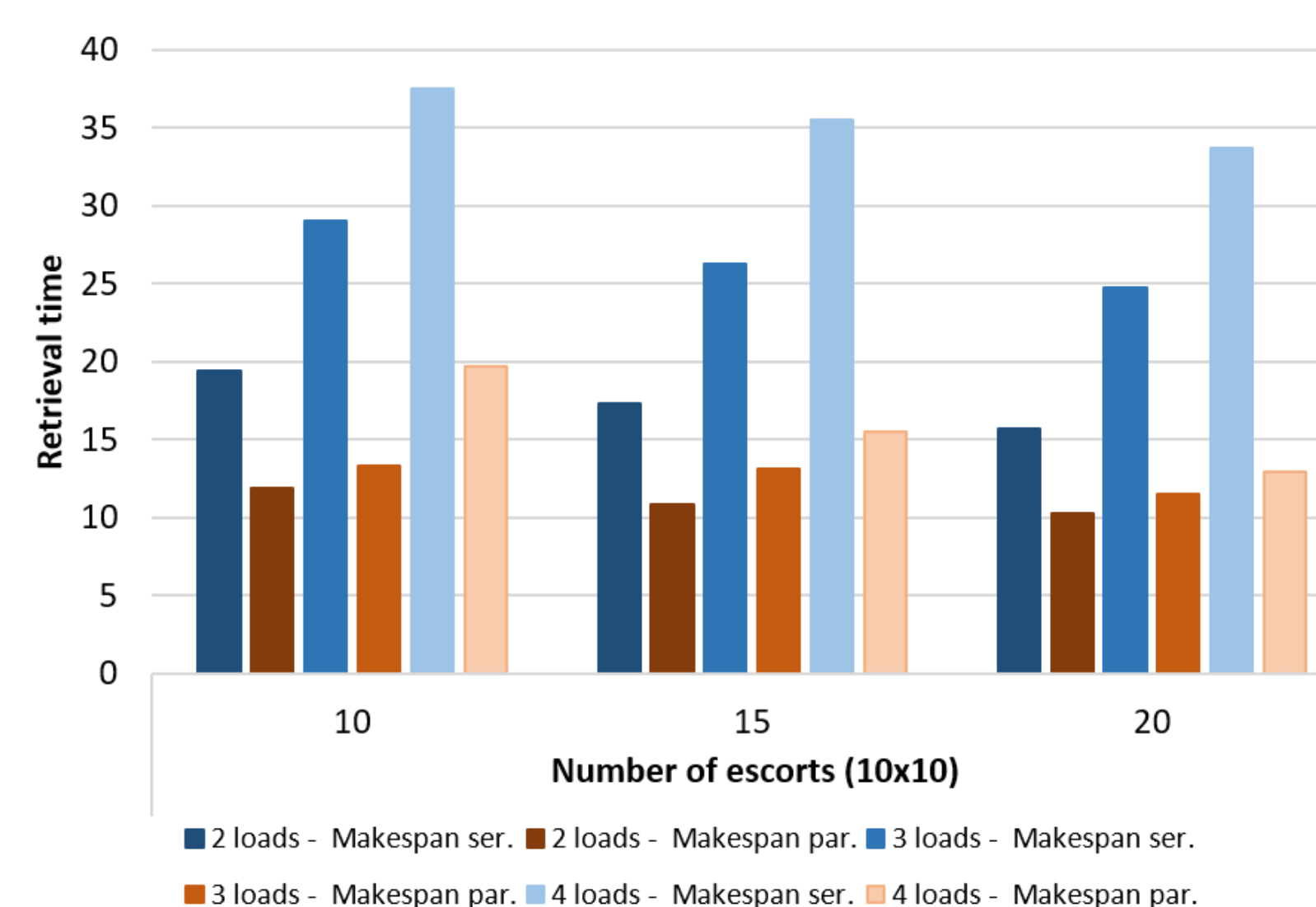


Block Movement [replace (7)]

$$\sum_{k=1}^2 (u_{l,l',t,k} + u_{l',l,t,k}) \leq 1 \quad \forall l \in \mathcal{L}, l' \in N(l), t \in \{0, \dots, T\} \quad (10)$$

$$\sum_{k=1}^2 \sum_{d \in \{-1,1\}} (u_{(x+d,y),(x,y),t,k} + u_{(x,y),(x,y+d),t-1,k}) \leq 1 \quad \forall (x,y) \in \mathcal{L}, t \in \{1, \dots, T\} \quad (11)$$

$$\sum_{k=1}^2 \sum_{d \in \{-1,1\}} (u_{(x,y+d),(x,y),t-1,k} + u_{(x,y),(x+d,y),t,k}) \leq 1 \quad \forall (x,y) \in \mathcal{L}, t \in \{1, \dots, T\} \quad (12)$$



Conclusion & further research

- Fast (close to) optimal solutions for up to 100 cells
- Suitable for static order-picking problems in modular configurations
- Sorting systems?
- Dynamic environments?

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