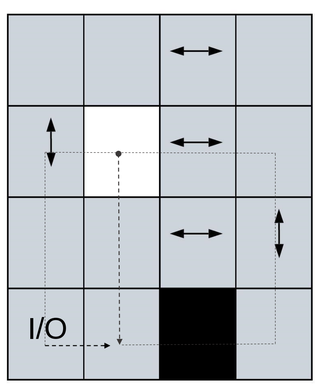
2 Design of Puzzle-Based Storage Systems with Unidimensional Cells

Héctor J. Carlo, Ph.D. and Andrés F. Blanco
University of Puerto Rico - Mayagüez

Research Objective and Question

This research aims to develop insights for designing single-escort *unidimensional Puzzle-Based Storage Systems* (UPBS). UPBS are conveyor-based PBS where some cells are designed without pop-ups so their movement is bidirectional but in one dimension (horizontal or vertical).



This paper seeks to understand:

- How to determine the optimal retrieval path for UPBS?
- What is the added value of a pop-up in each cell location?
- What is the tradeoff in expected retrieval distance if some cells are unidimensional?
- How should UPBS be designed?

Fig. 1. UPBS Representation (escort white, requested load black, gray bidimensional).

Motivation

The main implementation challenge for PBS is investment cost. Reducing the number of pop-up conveyors would simultaneously reduce the capital and operational costs, at the expense of system throughput.

Modeling Assumptions

The following modeling assumptions are made:

- One load is requested with known initial location
- Single escort
- Single I/O point
- The travel distance between locations is known corresponds to rectilinear travel considering unidimensional cells
- Each horizontal or vertical movement is of one distance unit (DU)
- Loads may only move to the escort
- Acceleration/deceleration are not considered

Modeling Assumptions

Sets:

$\boldsymbol{N}$ = set of all (grid) cell locations, $\boldsymbol{N} = \{1, \dots, \|\boldsymbol{N}\|\}$

$\boldsymbol{J_k}$ = set of neighborhood locations for cell s, $\boldsymbol{J_k} = \{1, \dots, \|\boldsymbol{J_k}\|\}, k \in \boldsymbol{N}$

Parameters:

s = initial location of the load to be retrieved, $s \in \boldsymbol{N}$

r = location of the I/O, $r \in \boldsymbol{N}$

d_{js} = minimum distance for the escort to move from its initial location to s through neighborhood location j in DUs

d_{ijk} = minimum distance for the escort to move from location i, using the intermediate location j, to reach the load located at k. This distance cannot go through k to reach j as it would imply that the load is moved in the process.

Decision Variables:

y_j = Binary variable equal to 1 if escort moves from its initial location to s through j, zero otherwise

x_{ijk} = Binary variable equal to 1 if escort moves from location i to the item's location k via intermediate location j, zero otherwise

LP Formulation

$$Min\ z = \sum_{j \in \boldsymbol{J_s}} d_{js}\, y_j + \sum_{i \in \boldsymbol{N}} \sum_{j \in \boldsymbol{J_k}} \sum_{k \in \boldsymbol{N}} d_{ijk}\, x_{ijk}$$

s.t.

$$\sum_{j \in \boldsymbol{J_s}} y_j = 1$$

$$\sum_{h \in \boldsymbol{J_j}} x_{shj} = y_j \ \forall j \in \boldsymbol{J_s}$$

$$\sum_{i \in \boldsymbol{N}} x_{ijk} = \sum_{h \in \boldsymbol{J_k}} x_{khj} + \sum_{i \in \boldsymbol{N}} x_{irk} \ \forall k \in \boldsymbol{N} \backslash \{s\}, j \in \boldsymbol{J_k}$$

$$\sum_{i \in \boldsymbol{N}} \sum_{k \in \boldsymbol{N}} x_{irk} + y_r = 1$$

$$0 \le y_j \le 1 \ \forall j \in \boldsymbol{J_s}$$

$$0 \le x_{ijk} \le 1 \ \forall i, k \in \boldsymbol{N}, j \in \boldsymbol{J_k}$$

Experimental Results

The LP was coded in Python calling Gurobi 9.1.2 for a 4×4 PBS grid. The LP was used recursively to determine the "next best" unidimensional cell based on the retrieval distance considering a random load and escort until no more unidimensional cells may be added. Fig. 2 summarizes the experiments performed. Figs. 3 and 4 summarize the results.

Do until STOP
  $E[\bar{D}_{\min}] = \infty$
  For all feasible unidim. dimensions (1EW, 1NS,..)
    For all load locations
      For all escort locations
        Solve LP(k) and update $E[\bar{D}]$
    if $E[\bar{D}] < E[\bar{D}_{\min}]$
      $E[\bar{D}_{\min}] = E[\bar{D}]$
  if $D_{\min} = \infty$ then STOP
    else add unidim. cell corresponding to $E[\bar{D}_{\min}]$

Fig. 2. Pseudocode of Experiments Performed.

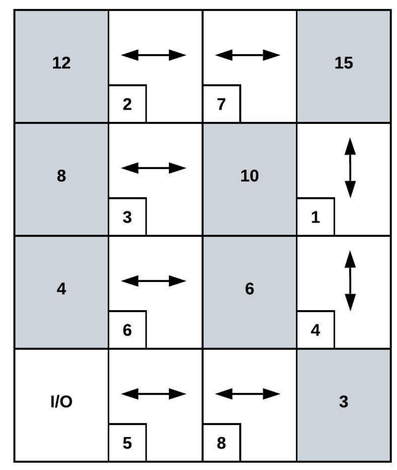


Fig. 3. Optimized UPBS layout.

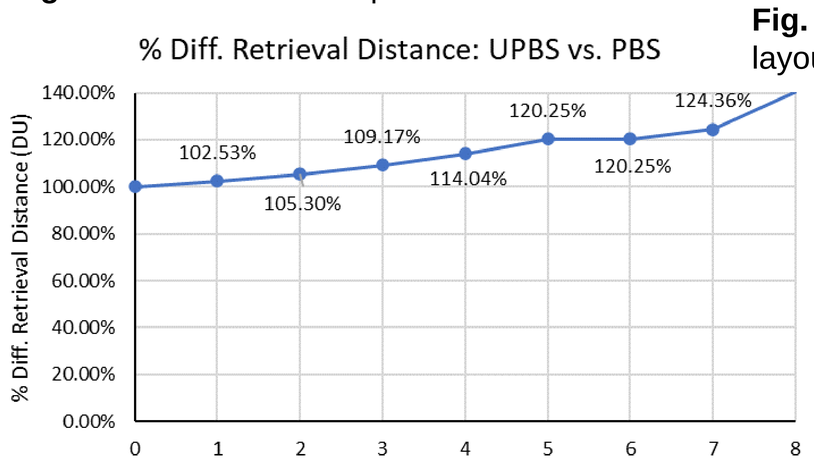


Fig 4. Retrieval Distance % Diff. as a Function of the Unidimensional Cells.

Conclusions

In this study, an LP was designed to find the optimal retrieval distance for a load in a UPBS using a single escort. Experimental results conclude that the maximum number of unidimensional cells that could be implemented in a 4×4 grid were 8 (out of the 12 cells, excluding the corners). It is concluded that a layout including 2 unidimensional cells yields an expected retrieval distance increase of ~5% over a regular PBS (~20% with 6 unidimensional cells).

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