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Limitations on High-Frequency Permeability of Magnetic Materials

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Abstract—**Engineering of magnetic composites with high microwave permeability is important for EMC/EMI and other applications. One of the challenging problems is to determine constraints on the achievable high-frequency permeability of magnetic composites. The objective of the present paper is to analyze the data available from the literature on the microwave permeability constraints and derive a generalized constraint condition. It is well known that in many practical occasions the actual magnetic frequency dispersion curves differ from the Lorentzian behavior. For these cases, an integral form of the constraint may be useful. Possible applications of the integral constraint are discussed for the cases of large damping, pronounced effect of eddy currents, and inhomogeneous materials. It is shown that the derived constraint can be successfully used in these cases for both estimating microwave performance of devices containing magnetic materials and obtaining additional data on the magnetic structure of materials.**

I. Introduction

Magnetic materials are widely used in many high-frequency applications, including those related to the problems of electromagnetic compatibility (EMC) and electromagnetic immunity (EMI) [1]. For many technical uses, including microwave absorbers, inductors, magnetic field sensors, patch antennas on magnetic substrates, *etc*., it is of importance to have high permeability over a wide microwave frequency range. For microwave absorbers, high magnetic loss over a desirable, often as wide as possible, frequency range is also imperative.

However, both permeability and magnetic loss have high-frequency physical limitations. Ferromagnetic resonance is mainly responsible for the microwave magnetic behavior of materials. In the first-order approximation, the complex permeability, $\mu=\mu'-j\mu''$, of a material is close to the static permeability μ_s at frequencies below the ferromagnetic resonance frequency f_{res}, and is close to unity at frequencies above the resonance. The microwave permeability is large when both μ_s and f_{res} are high. However, there is well-known Snoek's limit [2],

$$\left(\mu_s-1\right)\cdot f_{res}=\kappa\left(4\pi M_s\,\gamma\right), \qquad (1)$$

which is a consequence of the Landau−Lifshitz equation for the susceptibility of a uniformly magnetized spherical particle, and is applicable to the majority of isotropic polycrystalline magnetic materials. In (1), $\gamma\approx3$ GHz/kOe is the gyromagnetic ratio, $4\pi M_S$ is the saturation magnetization (in Gauss), and κ is a randomization factor. For Snoek's law, κ=2/3 is conventional.

For magnetic particles with significant shape anisotropy, Snoek's law is no longer valid. For obtaining as high as possible microwave permeability, the most interesting case is when two of three effective demagnetization fields involved in the Landau−Lifshitz equation are close to zero. One of these two fields determines the orientation of permanent magnetic moment in the particle, and the permeability along the other field is limited by [3]

$$\left(\mu_s-1\right)\cdot f_{res}^2=\left(4\pi M_s\gamma\right)^2, \qquad (2)$$

whereas the permeability along the two other principal axes of the ellipsoid (including such particular cases as disk, platelet, fiber, *etc.*) is close to unity.

Equation (2) may result in permeability values significantly higher than those limited by Snoek's law (1). Therefore, (2) is considered as an ultimate constraint for the microwave permeability of magnetic materials. It is seen from the well-known summation rule for the Kramers–Kronig relations,

$$\mu_S=\frac{2}{\pi}\int_0^\infty\frac{\mu''(f)}{f}df\,, \qquad (3)$$

where f is the frequency (in Hz), that (2) limits also opportunities of engineering materials with high microwave magnetic loss.

Equation (2) has been proposed for thin ferromagnetic films in [3] and, independently, for platelet inclusions in composites in [4]. It is also known that this equation governs hexagonal ferrites, which are the magnetic materials with the pronounced crystallographic magnetic anisotropy [5], as well as magnetic microwires with circumferential magnetization [6]. These magnets are considered as advantageous from the viewpoint of developing of materials with high microwave permeability.

In some cases, the right-hand part of (2) involves a factor depending on the magnetic structure of the material [7]. The exact value in this factor may vary due to a distribution of magnetic moments in the material [8], demagnetization and the presence of domain structure in thin films [9], [10], the presence of out-of-plane anisotropy field in hexagonal ferrites [5], or non-uniform magnetization in magnetic microwires [6]. However, the quadratic dependence on the saturation magnetization of the material, which provides large permeability at high frequencies, is kept all these cases.

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The right-hand part of (1) is conventionally called Snoek's constant of a material ($S=\kappa\, 4\pi M_s\gamma$). Analogously, the right-hand part of (2) can be referred to as Acher's constant.

Knowledge of constraints for microwave performance of materials is of high importance for modeling of microwave performance of electromagnetic structures and devices incorporating magnetic materials. Examples are an additional limitation on the ultimate bandwidth of radar absorbers arising from constraints on the microwave permeability [11] and estimations of the effect of permeability on the bandwidth of patch antennas with magneto-dielectric substrates [12]. Other applications of the constraints are obtaining additional data on the magnetic structure of materials [13] and consistency check of broadband permeability measurements and calculations [7].

Equation (2) is useful when the magnetic spectrum under consideration consists of a single narrow loss peak related to the ferromagnetic resonance, with both damping of the resonance and inhomogeneity of the material being small. In this case, the frequency dispersion of permeability, $\mu(f)$, in the material is governed by the Lorentzian dispersion law,

$$\mu(f)=1+\frac{\mu_{\mathrm{s}}-1}{1+j\beta f/f_{\mathrm{res}}-(f/f_{\mathrm{res}})^2}, \tag{4}$$

where the parameters μ_{s}, f_{res}, and β are the static permeability, resonance frequency, and damping factor of the Lorentzian resonance, respectively. It has been shown in [14] that the Lorertzian dispersion law describes accurately magnetic frequency dispersion is those materials where high values of high-frequency permeability are feasible.

In real materials, the shape of magnetic loss peak is frequently distorted relatively to the Lorentzian peak because the Landau–Lifshitz–Gilbert (LLG) equation gives a solution differing from the Lorentzian dispersion law in the case of non-zero damping [15]. Two other possible reasons for the magnetic loss peak distortion are the effect of eddy currents and the inhomogeneity of the material [14]. In the case of wide and distorted magnetic spectra, true resonance frequency is often difficult to determine, and, for Snoek's and Acher's constant to be estimated, this frequency is substituted by the frequency where magnetic loss has a peak (mostly, in estimations of ultimate performance of magnetic materials) or by the frequency where the susceptibility is equal to zero (mostly, for the analysis of magnetic structure and consistency check of data). Such estimations can often be not accurate enough.

To obtain more accurate estimations, an integral constraint for microwave permeability is conventionally employed. A generalization of (2) to the cases of an arbitrary dispersion law and inhomogeneous materials has been suggested in [9] as

$$\frac{2}{\pi}\int_0^\infty \mu''(f)\,f\,df \le p\kappa\left(4\pi M_s\gamma\right)^2, \tag{5}$$

where p is the volume fraction of the magnetic constituent, $0<p<1$. For 3D isotropic materials, $\kappa=1/3$ in (5).

Rigorous derivation of (5) for any form of magnetic dispersion law that can be represented as a sum of Lorentzian terms has been given independently in [16] and [17]. The derivation is analogous to that of the Kramers–Kronig relations and is based on the treatment of complex permeability as an analytic function of complex frequency. The same, (5) follows from the Cauchy theorem, where an arbitrary magnetic dispersion law is represented as a sum of Lorentzian terms, high-frequency asymptotes of which are considered. The volume fraction, p, involved in the right-hand part of (5) as a factor arises because the high-frequency asymptote in composites is governed by the Landau–Lifshitz–Looyenga mixing rule, see [8] for details.

For the analysis of measured data for agreement with (5), it is conventional to consider a frequency dependence of the integral

$$I_A(f)=\frac{2}{\pi}\int_0^f \mu''(x)\,x\,dx \tag{6}$$

which is calculated from the measured data. Integral (6) is called Acher's integral [18]. Sometimes, a frequency-dependent analog of M_s, which is referred to as effective dynamic magnetization [7], is introduced by equating the right-hand parts of (5) and (6) and solving the resulting equation for M_s.

The validity of (5) has been checked by plotting the frequency dependence of Acher's integral (6) in many papers, *e.g.*, [7], [9], [19]–[21]. No violation of inequality (5) has ever been observed. On the other hand, most of the measured data obtained in the frequency range up to 20 GHz do not reveal the I_A value saturation (asymptotic approaching a limiting value as frequency increases) that can be attributed to Acher's integral determined by the right-hand part of (5). It should also be mentioned that the validity of (5) is limited [8], and the right-hand part of (6) may diverge with frequency tending to infinity.

Therefore, an application of (2) and (5) to real materials still needs investigation and justification. The objective of this paper is to study the feasibility of estimating Acher's integral from data measured in a limited frequency range and for various types of permeability frequency dependences.

II. The Lorentzian and Debye Frequency Dispersions

Consider the case when the frequency dependence of permeability is governed by Lorentzian frequency dispersion law (4). Then Acher's integral (6) is rewritten as

$$I_A(f)=\frac{2}{\pi}\frac{(\mu_s-1)\tilde{f}}{f_1}\mathrm{Im}\left[\tilde{f}\tanh^{-1}\frac{f}{\tilde{f}}\right], \tag{7}$$

with resonance frequency f_{res} and Lorentzian damping parameter β being substituted by the complex resonance frequency $\tilde{f}=f_1+jf_2$ so that

$$f_{\mathrm{res}}^2=f_1^2+f_2^2,\quad \beta=2f_2/f_{\mathrm{res}}. \tag{8}$$

Normalized Acher's integral,

$$i_A(f)=I_A(f)\Big/\left[(\mu_s-1)\,f_{\mathrm{res}}^2\right], \tag{9}$$

calculated from the Lorentzian dispersion law and plotted against frequency is shown in Fig. 1 for various ratios of f_1/f_2. The imaginary permeability is also given in Fig. 1 for these cases as a function of frequency. As is seen in the figure, Acher's constant can be estimated with sufficient accuracy from a measured frequency dependence of permeability only if the loss peak is comparatively narrow, i.e., $f_1>>f_2$, or $\beta<<1$. Higher damping causes slower convergence of integral (6), as was firstly noticed in [9]. A simple estimate of the magnetic

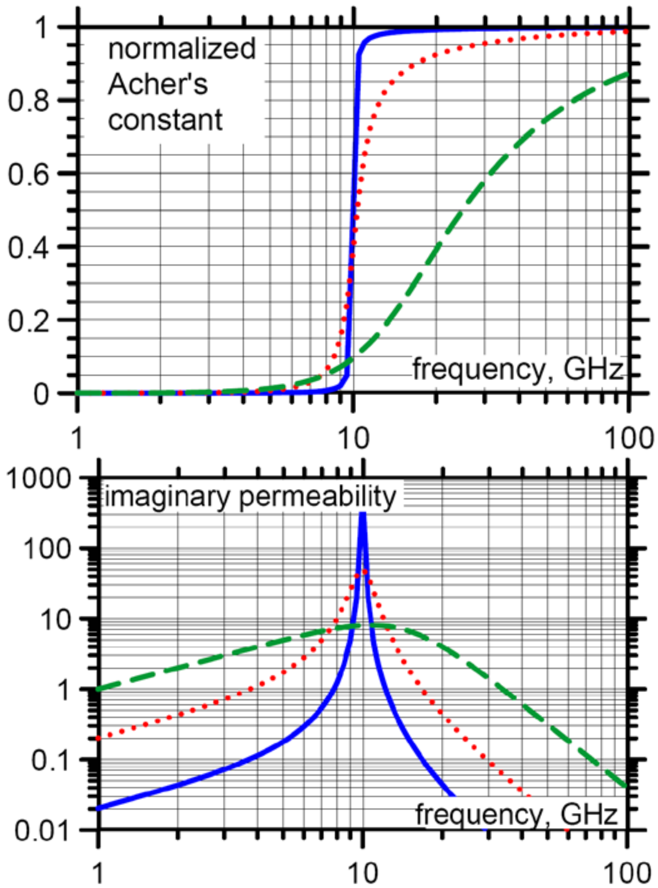

Fig. 1. Above: the calculated frequency dependences of Acher's constraint for the case of Lorentzian dispersion law, f_1=10 GHz, and f_2 =0.1 GHz (solid blue line); f_2 =1 GHz (dotted red line); f_2 =10 GHz (dashed green line). Below: corresponding frequency dependences of imaginary permeability.

damping effect on the convergence of Acher's integral is readily derived from (7). With frequency tending to infinity, normalized Acher's integral is:

$$i_A(f)=1-\frac{4}{\pi}\frac{f_2}{f}. \quad (10)$$

Equation (10) is useful for estimations when $f_2 \approx f_{res}$ and, therefore, β is large. In this occasion, the asymptotic behavior (10) may be used for obtaining more accurate value of I_A from measured data. However, if the magnetic frequency dispersion is governed by the Lorentzian law (4), then Acher's constant may be estimated accurately from measured data by fitting these with the Lorentzian law, see, *e.g.*, [18]. While using such fitting, it is important to check that the FMR frequency (at which the real part of magnetic susceptibility passes through zero) lies within the frequency range of measurements. If this is not so, a significant error can be incurred in the extracted Acher's constant.

From the point of view of the frequency dispersion, the case when the resonance frequency is beyond the measured frequency range would correspond to the Debye dependence, which is obtained from (4) when $f_{res}\to\infty$:

$$\mu(f)=1+\frac{\mu_S-1}{1+jf/f_D}, \quad (11)$$

where f_D is the characteristic Debye frequency. For the Debye dispersion law (11), integral in the left-hand part of (5) diverges. Another integral constraint involving a constant that depends only on the gyromagnetic factor, saturation magnetization, and the volume factor of magnetic constituent in an inhomogeneous material, has been proposed in [8]:

$$\frac{2}{\pi}\int_0^\infty\left(\mu'(f)-1\right)df = p\cdot\frac{2}{3}4\pi M_s\gamma \quad (12)$$

This is an integral analog of Snoek's law. For the Lorentzian dispersion, the integral (12) is zero. For this reason, it is rarely employed in scientific practice, since it is conventional that magnetic properties are always associated with some resonance frequency, and hence the Debye dependence for permeability (or magnetic susceptibility) itself is non-physical.

The further consideration is devoted to the case when the magnetic dispersion curve is non-Lorentzian.

III. The LLG Dispersion Law

Rigorously, the ferromagnetic resonance theory yields the shape of magnetic dispersion curve differing from the Lorentzian law. Kittel's solution of the LLG equation for the ferromagnetic resonance in a single-domain ellipsoidal magnetic particle results in the frequency dependence of the permeability given by [15]

$$\mu(f)=1+\frac{4\pi\gamma M_s\left(f_x+j\alpha f\right)}{\left(f_x+j\alpha f\right)\left(f_y+j\alpha f\right)-f^2}, \quad (13)$$

where $f_x=\gamma(H_k+4\pi M_s(n_x-n_z))$, $f_y=\gamma(H_k+4\pi M_s(n_y-n_z))$, α is the Gilbert damping parameter, H_k is the magnetic anisotropy field, n_x, n_y, n_z are the principal demagnetization factors of the ellipsoidal particle. Equation (13) will be referred below to as the LLG dispersion law. The LLG dispersion law is consistent with the Lorentzian dispersion law only in the case of zero damping, α=0 and β=0.

The difference between the Lorentzian and LLG frequency dispersions arising with non-zero damping has been studied in detail in [14]. Neglecting the frequency dependence of the numerator in (13), the dependences (13) and (4) agree when

$$\mu_s-1=4\pi M_s\gamma/f_y,\ \beta=\frac{\alpha}{1+\alpha^2}\left(\sqrt{f_x/f_y}+\sqrt{f_y/f_x}\right), f_{res}^2=f_xf_y/\left(1+\alpha^2\right), \quad (14)$$

Based on these relations, it was concluded in [20, 21] that Gilbert damping factor α decreases Acher's constant by a factor of $(1+\alpha^2)^{1/2}$.

As is shown in [14], the best agreement between shapes of magnetic loss peaks in the LLG and Lorentzian dispersion laws is achieved with the Lorentzian parameters given by

$$\mu_s-1=4\pi M_s\gamma/f_y,\quad \beta=\alpha\left(\sqrt{f_x/f_y}+\sqrt{f_y/f_x}\right),\quad f_{res}^2=f_xf_y. \quad (15)$$

The condition of consistency between the shapes of magnetic loss peaks yielded by (4) and (13) can be obtained as follows. For the consistency, the contribution from the second term in the numerator of (13) must be negligible in the vicinity of the resonance frequency as compared to the contribution from the first term. For rough estimations, it may be accepted that the contribution from the first term is close to the static permeability in the order of amplitude. From this, by finding the asymptotic value of the imaginary permeability at

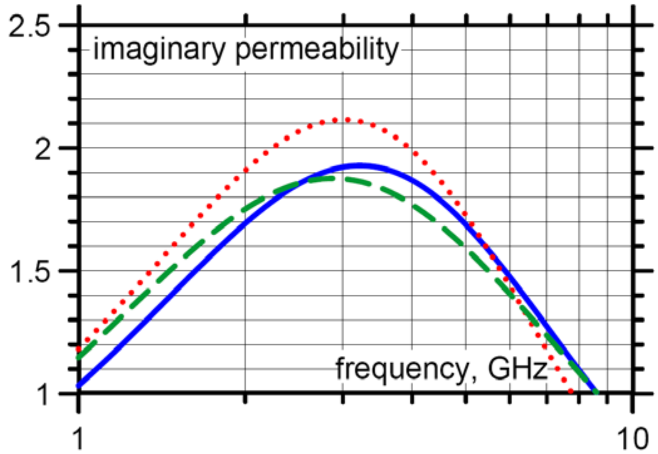

Fig. 2. The calculated frequency dependence of the imaginary part of permeability (13) for f_x=20 GHz, f_y=3 GHz, α=1, $4\pi\gamma M_s$=10 GHz (solid blue line) and the result of the Lorentzian fitting of this dependence with parameters in (14) (dotted red line) and in (15) (dashed green line)

$f\to\infty$ from (13), it is readily derived that the shapes of the LLG and Lorentzian loss peaks coincide when

$$\alpha/\left(1+\alpha^2\right) << \sqrt{f_x/f_y}. \qquad (16)$$

Hence, the Lorentzian dispersion law fits closely a LLG dispersion curve at any value of the Gilbert damping parameter provided that $\sqrt{f_x/f_y} >> 1$. This is illustrated by Fig. 2, where a comparison of shapes of magnetic loss peak yielded by (14) and (15) is given. Parameters (15) provide better fit of the LLG loss peak as compared to (15), although some disagreement is still seen: the LLG loss peak has steeper rise at the low-frequency side of the resonance line, and more gradual descend at the high-frequency side. With low values of $\sqrt{f_x/f_y}$, the agreement is achieved only if α is small enough.

It is readily obtained that the integral (5) for the frequency dispersion (13) diverges due to the presence of the second term in the numerator of (13) that is a linear function of frequency. In the similar way, the expression for I_A can be obtained from the LLG equation (13) in the same way as (7) is obtained from (4). However, the closed-form expression is quite cumbersome in this case and is not presented here.

Therefore, for using of Eq. (6) for the analysis of the measured data, it is important to find a frequency range where the value of the integral (6) does not change with frequency. For such frequency range to exist, two conditions must be satisfied. Firstly, the contribution to (6) from the second tem in the numerator of (13) must be small compared to the limiting value provided by the first term; this means that the frequency is not very high. Secondly, the frequency must be not very low, so that the limiting value will be achieved.

The first condition is

$$f >> \alpha\left(f_x + f_y\right). \qquad (17)$$

It is found from the asymptotic value of the imaginary permeability at $f\to\infty$ according to (13) with the account of (15) compared with the limiting value of Acher's integral.

The second condition,

$$f << A\left(1+\alpha^2\right)/\alpha\ , \qquad (18)$$

is derived from (8), (10), and (15).

In both (17) and (18), factors close to unity on the order of magnitude are omitted. Combining (17) and (18), one can get that integral (6) has a constant value in a certain frequency range only if

$$\frac{\alpha^2}{1+\alpha^2} << \frac{f_x}{f_x+f_y}. \qquad (19)$$

Therefore, Acher's integral (6) is useful for the analysis of the data only if α<<1, independently of the value of f_x/f_y. This is illustrated by Fig. 3 that plots the frequency dependence of the normalized Acher's integral (9) for various values of α. In the top of the figure, the frequency dependence of (6) is shown for the case of $f_x >> f_y$, which produces the magnetic loss peak close to the Lorentzian dispersion law. As is shown in Fig. 3, Acher's constant can be estimated for α=0.03 and, probably, for α=0.1. In the latter case, at least the order of magnitude of Acher's constant can be found. However, at higher loss, *e.g.*, for α=0.5, no conclusions can be made on Acher's constant from integral (6).

If $f_x << f_y$, see bottom of Fig. 3, the accuracy of determination of integral (6) is worse, which is consistent with

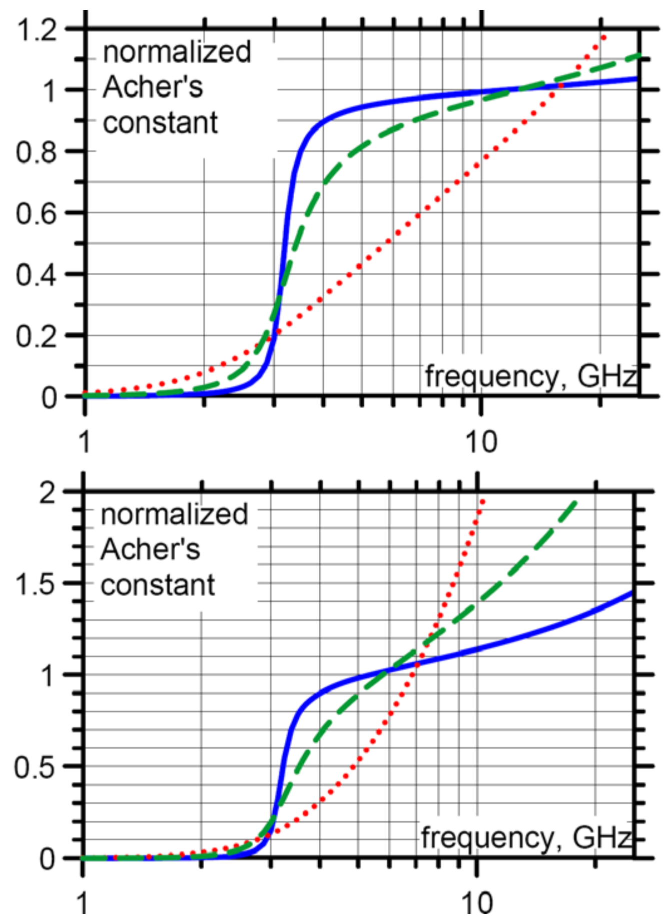

Fig. 3. Normalized Acher's integral (9) for the LLG dispersion law with $4\pi\gamma M_s$=10 GHz; f_x=10 GHz, f_y=1 GHz (above) and f_x=1 GHz, f_y=10 GHz (below); α=0.03 (solid blue line), α=0.1(dashed green line), and α=0.5 (dotted red line)

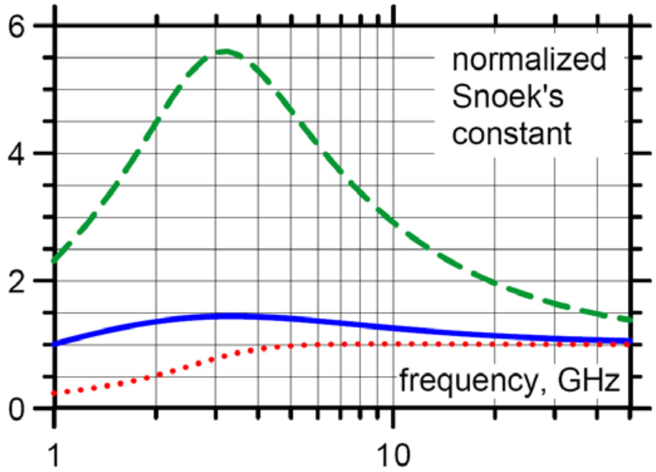

Fig. 4. Normalized Snoek's integral (22) for the LLG dispersion law with f_x=10 GHz, f_y=1 GHz, α=1 (solid blue line); f_x=10 GHz, f_y=1 GHz, α=0.3 (dashed green line); f_x=1 GHz, f_y=10 GHz, α=0.3 (dotted red line). $4\pi\gamma M_s$=10 GHz for all the curves.

(19). In this case, a qualitative estimation of Acher's constant is feasible at α=0.03, while no conclusions can be made in the other two cases involving larger damping.

It is worth noting that in the case of the LLG dispersion law with high damping, Acher's constant can not be found from (2) as well. In the Lorentzian dispersion law, the resonance frequency is found as the frequency where the real permeability is equal to unity. For the LLG dispersion law, the frequency where the susceptibility is zero is found as:

$$f\Big|_{\mu'=1} = 1 + f_x \sqrt{f_y} \Big/ \sqrt{f_x - f_y \alpha^2} \,. \tag{20}$$

Therefore, if $f_x/f_y<\alpha^2$, then the real permeability in the LLG dispersion law is higher than unity at any frequency.

In this case, for analysis of the frequency dependence of permeability, Snoek's integral law (12) may be useful. It is readily obtained that, for the LLG dispersion law,

$$\frac{2}{\pi}\int_0^\infty \left(\mu'(x)-1\right)dx = p\cdot\frac{4\pi\,\alpha\,M_s\gamma}{1+\alpha^2} \tag{21}$$

By analogy to (6), normalized Snoek's integral with varied upper limit,

$$i_S(f) = \frac{1+\alpha^2}{4\pi M_s\alpha}\,\frac{2}{\pi}\int_0^f \left(\mu'(x)-1\right)dx\,, \tag{22}$$

may be introduced. If $f_x/f_y<\alpha^2$, integral (22) arises monotonically with frequency up to its limiting value provided by the right-hand part of (21). If the above inequality does not hold, $I_s(f)$ has a peak at the frequency where the susceptibility is zero but tends to the limiting value with further increase of frequency. These cases are illustrated by Fig. 4.

Therefore, Snoek's integral can be used to evaluate the limiting cases of microwave permeability in the case when Acher's integral diverges.

IV. The Effect of Eddy Currents

In this section, distortions of the magnetic dispersion law due to the effect of eddy currents are considered under the assumption that the intrinsic permeability of inclusions μ is governed by the Lorentzian law. Eddy currents are conventionally accounted for by the renormalization of μ to the apparent permeability μ^*. For the sake of simplicity, the renormalization is written below for the case of flake-shaped particle as [22]

$$\mu^* = \mu\,\frac{\tan\left((1+j)\pi d\sqrt{\mu\sigma f}\,/c\right)}{(1+j)\pi d\sqrt{\mu\sigma f}\,/c}\,, \tag{23}$$

where c is the free-space light velocity; σ and d are the flake conductivity and thickness, respectively. The frequency-dependent magnetic behavior is due to complex poles of the Lorentzian dispersion curve, $\tilde{f}$, as is given by (8). Similarly, the frequency dependence of the renormalized permeability is determined by the poles of the right-hand part of (23). The eddy current phenomenon transforms each Lorentzian pole of the intrinsic permeability μ to an infinite set of poles of the apparent permeability μ^*. From (23), the poses of the apparent permeability, $\tilde{f}_i^* = f_{1,i}^* + j\,f_{2,i}^*$, i=1, … , ∞, are found as

$$f_{i,1}^* = f_1;\quad f_{i,2}^* = f_2 + 4\frac{4\pi\chi_{s,i}\,d^2\sigma}{(2i-1)^2c^2}f_1^2\,, \tag{24}$$

where the intrinsic permeability is assumed to possess a narrow magnetic absorption band, f_1>>f_2. The partial static susceptibilities $\chi_{s,i}$ are derived from the residues in these poles.

Since the effect of eddy currents affects neither the resonance frequency, as it follows from (24), nor the static permeability, then, formally, it does not affect (2). However, eddy currents result in broadening and low-frequency shift of magnetic absorption peak, which may be considered in the same way as in the previous section, by applying an integral similar to (5). If the skin effect is well pronounced, then the magnetic absorption peak is formed by eddy currents solely, with negligible contribution from the ferromagnetic resonance. The magnetic absorption peak is located at the frequency, where the skin depth is equal to the lowest dimension of the particles. The decrease in the cutoff frequency restricts opportunities for the microwave applications. Again, the constraint on the high-frequency magnetic behavior of the material is given by Snoek's law rather than by (2).

In addition to poles (24) related to the ferromagnetic resonance, eddy currents are associated with another set of poles corresponding to the "optical limit" permeability of the material. The parameters of the lines corresponding to this set of poles are also found from (23) by setting μ=1. The resonance frequency corresponding to these lines is infinity, and this results in a divergence of the integral in the left-hand part of (5).

This set of poles appears in non-magnetic conductors as well, where these are responsible for the effective permeability arising due to the eddy current effect.

Therefore, integral (5) is divergent in conductive magnets due to the eddy current effect, even if the intrinsic permeability is of the Lorentzian type. However, the spectrum also includes a Debye line, see [9] for details.

For this Debye frequency dependence, one can get

$$I_A(f) = \frac{2}{\pi}\int_0^f \mu'' f\,df \approx \frac{c^2 f}{4\pi\,\mu_s d^2\sigma} \tag{25}$$

This contribution to the integral (6) increases with frequency and becomes equal to that originating from ferromagnetic resonance at $f \approx 4\pi\mu_s^2 f_{res}^2 d^2 \sigma / c^2$, which is typically about a few hundred gigahertz. Below this frequency, the left-hand part of (6) is related only to the magnetostatic properties of the magnetic material, and is independent of frequency. Therefore, this integral is suitable for estimating microwave performance of magnetic materials.

V. Conclusion

The paper considers application of constraints on high-frequency permeability for various types of frequency dispersion of permeability. It is shown that Acher's integral constraint is feasible in the case of low damping only. With higher damping, the no conclusions on the limiting value of the integral can be drawn form measured frequency dependence of permeability. In this case, another integral constraint based on Snoek's law may be employed. However, the case, when the magnetic frequency dispersion in a material is described by the Lorentzian dispersion law with low damping, is the most promising from the standpoint of obtaining high microwave permeability values.

However, the frequency involved in the left part of (2) is the true resonance frequency, *i.e.*, the frequency, above which the real permeability takes values below unity. From the standpoint of technical applications, this frequency is of minor importance, since the magnetic absorption peak for the case of high damping is located at the lower frequencies. Therefore, the frequency of maximum magnetic absorption f_a must be considered as the high-frequency cut-off frequency of magnetic behavior. For this maximum magnetic absorption frequency, the following equality is readily obtained

$$\mu_s f_a = \frac{\sqrt{1+\alpha^2}}{\alpha} S, \qquad (26)$$

where $S = \mu_s f_{res}$ is Snoek's constant. Therefore microwave permeability at high damping levels cannot be estimated accurately using just (2). Properties of a magnetic material must obey Snoek's law rather than Acher's law.

For the purposes of analyzing of magnetic structure of materials and checking of consistency of the data, the resonance frequency is of much importance than the magnetic loss peak frequency, and application of Acher's constrain is more appropriate.

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References

[1] M. Koledintseva, V.V. Khilkevich, A.G. Razmadze, A.Y. Gafarov, S. De, and J.L. Drewniak, "Evaluation of absorptive properties and permeability of thin sheet magneto-dielectric materials", *J. Magn. Magn. Mater.,* vol. 324, pp. 3389–3392, Oct. 2012.

[2] J.L. Snoek, "Dispersion and absorption in magnetic ferrites at frequencies above 1 Mc/s", *Physica*, vol. 14, pp. 207–217, May 1948.

[3] G. Perrin, O. Acher, J.C. Peuzin, and N. Vucadinovic, "Sum rules for gyromagnetic permeability of ferromagnetic thin films: theoretical and experimental results," *J. Magn. Magn. Mater.*, vols. 157–158, pp. 289–291, May 1996.

[4] R.K. Walser, W. Win, and P.M. Valanju, "Shape-optimized ferromagnetic particles with maximum theoretical microwave susceptibility," *IEEE Trans. Magn.*, vol. 34, pp. 1390–1392, Jul 1998.

[5] A.L. Adenot, O. Acher, T. Taffary, and L. Longuet, "Sum rules on the dynamic permeability of hexagonal ferrites," *J. Appl. Phys.*, vol. 91, pp. 7601–7603, May 2002.

[6] J. Torrejon, A. Thiaville, A.-L. Adenot-Engelvin, and M. Vazquez, "Cylindrical magnetization model for glass-coated microwires with circumferential anisotropy: Dynamics", *J. Magn. Magn. Mater.*, vol. 333, pp. 144–151, May 2013.

[7] A. Thiaville, N. Vukadinovic, and O. Acher, "Sum rule for the magnetic permeability of arbitrary textures," *Phys. Rev. B*, vol. 86, art. no. 214404, Dec. 2012.

[8] A.N. Lagarkov and K.N. Rozanov, "High-frequency behavior of magnetic composites," *J. Magn. Magn. Mater.*, vol. 321, pp. 2082–2092, Jul. 2009.

[9] O. Acher and A.L. Adenot, "Bounds on the dynamic properties of magnetic materials," *Phys. Rev. B*, vol. 62, p. 11324, Nov. 2000.

[10] N.A.Buznikov and K.N.Rozanov, "The effect of stripe domain structure on dynamic permeability of thin ferromagnetic films with out-of-plane uniaxial anisotropy," *J. Magn. Magn. Mater.*, vol. 285, pp. 314–326, Jan. 2005.

[11] K.N. Rozanov and S.N. Starostenko, "Influence of permeability dispersion on the bandwidth of magnetic radio absorbers*", J. Communic. Technol. Electronics*, vol. 48, pp. 652–659, June 2003.

[12] P.M.T. Ikonen, K.N. Rozanov, A.V. Osipov, P. Alitalo, and S.A. Tretyakov, "Magnetodielectric substrates in antenna miniaturization: potential and limitations," *IEEE Trans. Antennas Propag.*, vol. 54, pp. 3391–3399, Nov. 2006.

[13] I.T. Iakubov, A.N. Lagarkov, S.A. Maklakov, A.V. Osipov, D.A. Petrov, K.N. Rozanov, and I.A. Ryzhikov, "Laminates of multi-layered Fe films for microwave applications," *J. Magn. Magn. Mater*, vol.316, pp. e813–e815, Sept. 2007.

[14] K.N. Rozanov and M.Y. Koledintseva, "Analytical representations for frequency dependences of microwave permeability", *Proc. 2012 IEEE Int. Symp. Electromag. Compat.*, Aug. 5−10, 2012, Pittsburgh, USA, p. 422.

[15] T.L. Gilbert, "A phenomenological theory of damping in ferromagnetic materials," *IEEE Trans. Mag.,* vol. 40, no. 6, pp. 3443–3449, Nov. 2004.

[16] O. Acher and S. Dubourg, "Generalization of Snoek's law to ferromagnetic films and composites," *Phys. Rev. B*, **77**, art. no. 104440, March 2008.

[17] A.N. Lagarkov, A.V. Osipov, K.N. Rozanov, and S.N. Starostenko, "Microwave composites filled with thin ferromagnetic films. Part I. Theory," *Proc. Symp. R: Electromagn. Materials, 3rd Int. Conf. on Mater. for Adv. Technol. (ICMAT 2005)*, July 3–8, 2005, Singapore, pp. 74–77.

[18] I.T. Iakubov, A.N. Lagarkov, S.A. Maklakov, A.V. Osipov, K.N. Rozanov, I.A. Ryzhikov, and S.N. Starostenko, "Microwave permeability of composites filled with thin Fe films," *J. Magn. Magn. Mater.,* vol. 300, pp. e74–e77, May 2006.

[19] A. Bonneau-Brault, S. Dubourg, V. Dubuget, D. Plessis, and F. Duverger, "Efficient dynamic magnetization measurement in a Py/Ru/Py trilayer," *J. Phys.: Conf. Ser.*, vol. 303, art. no. 012088, July 2011.

[20] P.H. Zhou, T. Liu, J.L. Xie, and L.J. Deng, "Damping effect on resonance bounds relationship of nanostructured ferromagnets and composites", *J. Appl. Phys.*, vol. 111, art. no. 113912, June 2012.

[21] M. Han, D. Liang, K.N. Rozanov, and L. Deng, "Microwave permeability and Mössbauer spectra of flaky Fe-Si-Al particles," *IEEE Trans. Magn.*, vol. 49, no. 3, pp. 982–985, March 2013.

[22] S. Chikazumi, *Physics of Ferromagnetism*, 2nd ed., Oxford University Press, 1997. 668 p.