International Journal of Evaluation and Research in Education (IJERE)
Vol. 11, No. 2, June 2022, pp. 773~779
ISSN: 2252-8822, DOI: 10.11591/ijere.v11i2.21911

Routines' errors when solving mathematics problems cause cognitive conflict

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Article Info

Article history:

Received Jun 3, 2021
Revised Mar 18, 2022
Accepted Apr 24, 2022

Keywords:

Cognitive conflict
Commognitive
Fraction
Mathematics
Routines

ABSTRACT

Many studies showed that cognitive conflict often occurs in learning and when solving mathematics problems. However, very few studies have looked at cognitive conflicts in solving mathematics problems, incredibly improper fraction problems. This descriptive qualitative study described and analyzed students' errors in solving mathematics problems using a commognitive perspective. The data was collected using a test sheet instrument, where students do the test think-aloud. The answers on the student test sheets were analyzed by adjusting the think-aloud that was carried out, and then the interview process was carried out as a form of triangulation of the method in the study. The data analysis results showed that there was a routine error that causes cognitive conflict when solving the improper fraction problem. The error that occurred indicates that the routine can and cannot resolve the cognitive conflict that occurs. This study's findings indicated the importance of routine procedures to be understood so that their use is appropriate for solving mathematical problems.

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1. INTRODUCTION

Several studies have shown that students still have difficulty solving fraction problems [1]–[6]. The difficulty in solving fractions becomes triggered by misconceptions, including the improper fraction problem [7]. Misconceptions can be one of the essential obstacles when students learn mathematics [8], [9]. Mathematical misconceptions experienced by students are one type of error that occurs continuously as a result of errors in cognitive structures. The errors occur because of the influence of long-established knowledge on students' cognitive structures [10]. However, some researchers confirmed the errors in cognitive structures experienced by these students by using cognitive conflict [11]–[13].

Cognitive conflict is a conflict between existing knowledge or information and new knowledge or information received in a person's cognitive structure. Cognitive conflict is a mismatch that occurs in two things: the differences between components (ideas and beliefs) of a person's cognitive structure [14]. Furthermore, researcher also found four indicators of cognitive conflict, namely: recognition anomalous, interest, anxiety, and cognitive reappraisal [14]. There have been many previous studies that use cognitive conflict conditions as a tool or strategy in learning so that it can correct errors in students' cognitive structures [15]–[18]. Besides, there is research that has developed cognitive conflict-based learning tools [19] as well as developing tools to measure the occurrence of cognitive conflict [20]. Furthermore, the other researchers conducted an assessment of students' cognitive conflict in solving mathematics problems [21].

Previous research descriptions showed that it is interesting to describe more about how cognitive conflict is important in solving mathematical problems by analyzing more deeply. Also, there are studies that analyzed students' mathematical problem-solving using a commognitive perspective [7], [22].

Commognitive is a new term that combines two terms, communication and cognitive, in which the new term is used to indicate that communication and cognition are proceedings which in practice do not separate [23]. Several studies used a commognitive framework in the learning process by analyzing the mathematics discourse used by students [24]–[27]. Moreover, some researchers used commognitive as an analytical method so that it can provide clarity of understanding in students' mathematical thinking [28] and can see the discourse produced by pre-service teachers in completing mathematics tasks [29], [30]. The descriptions of these studies showed that it is possible when using commognitive to describe more clearly how students think in attempting to solve a provided mathematical problem to start generating communication in the form of discourse.

Researchers are interested in using commognitive as a lens to see student errors in solving problems. Commognitive is used to describe the errors of communication throughout the minds of students in order of answers to problems solved. So, these results can describe the causes of student cognitive conflicts that occur in solving fractional mathematics problems, especially improper fraction.

2. RESEARCH METHOD

This was qualitative research with a descriptive approach. The descriptive approach was chosen because it can describe the conditions, they are without giving treatment or manipulation to the variables studied. It can describe the occurrence of cognitive conflict in students solving problems with incorrect fractions. The following is an improper fraction problem given to students.

> "Father bought 2 adjoining land plots (A and B) with the size of each plot of 10 m x 12 m. The plot of land will be given to his two children, Luna and Rohman. Luna got 3/4 plots of land and Rohman got 6/5 land plot. If the picture below is Luna's land plot, then make it picture showing Rohman's plot of land."

There were 25 students asked to solved improper fraction problems with thinking aloud, cognitive conflicts that arise in students were mostly investigated. After students complete the improper fraction problem, the think-aloud results are analyzed to determine participants in the study. Participants are primary school teacher education students who experience errors in solving problems. The two selected participants are further interviewed in-depth semi-structured to find out more about the causes of errors using the four components [24], namely word use, visual mediator, narrative, and routine described in Table 1. The errors arose in solving the improper fraction problem were further analyzed to determine whether it resulted in students' cognitive conflict. The interview process carried out in this study used a triangulation of method.

Table 1. Description and example of the error in the component of commognitive

Component	Description	Example of error
Word use	Through the use of everyday words that have distinct and specific meanings in mathematics, something that is thought in cognitive is then communicated.	Errors in using specific written or spoken words such as numerators, denominators, fractions, equivalent fractions, smaller fractions, larger fractions, and improper fractions to be used in problem-solving
Visual Mediator	Through symbols, graphs, images, and diagrams, something that seems thought of in cognitive terms are then communicated.	Error when creating drawings to solve problems using fraction calculation results
Narrative	Anything that is thought of cognitive terms then communicated by series of speech/text, spoken or written, which is used by the object labelled true or false as a definition of an object, the relationship between objects, or the method.	Error in describing the process for describing fractions with different denominators
Routine	Through repetitive patterns, something that is thought about cognitively is then communicated.	Errors in the problem-solving procedure via the denominator equal

3. RESULTS AND DISCUSSION

3.1. Commognitive analysis in solving mathematics problems in participant 1

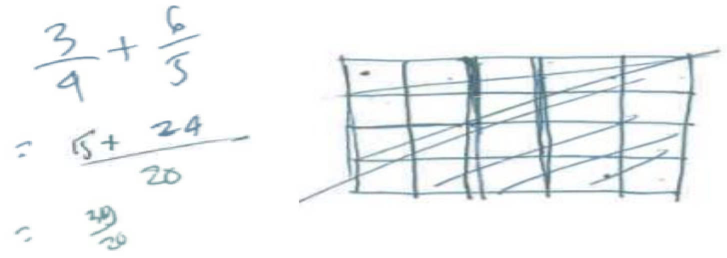
Participant 1 (P1) solves the problem by generating four commognitive components which are discussed further, especially on visual mediator and routine component. P1 understands each part of Father, Mother and Ani based on the problems given. However, when shown in the picture P1 was confused to say it was a form of the improper fraction (recognition anomalous). Confusion and anxiety experienced by P1 are early indicators of cognitive conflict, and this condition can contribute to P1 so that it has a positive or negative impact on the learning process [31].

P1 resolves to start solving the problem by bringing out the visual components of the mediator by depicting a rectangle (interest and cognitive reappraisal). This condition is known through the following think aloud snippets, *"Means the first rectangular image to show the plot of land."* After producing one rectangular image, P1 could not imagine Luna and Rohman's parts. Errors in the resulting image are not the final result in solving the problem. P1 decides to equate two different fractions (narrative and cognitive reappraisal).

P1 can determine the relationship between fractions and improper fractions but in different conditions. This condition made P1 realize that Rohman's picture shows more than one part and this contradicts his initial thought which divided the plot of Luna and Rohman each one part (recognition anomalous and cognitive reappraisal). The contradiction experienced by P1 is an indicator of cognitive conflict [32], which is expressed as the recognition of an anomaly situation. The reinterpretation of the recognition of anomaly experienced will bring up the student's behavior response to what it faces. Suppose students cannot decide on the recognition anomaly they experience. In that case, there is a belief in the anomalous situation they face and causes the circle of cognitive conflict not to end. P1 performs a routine procedure to equalize the two different denominators shown in Figure 1.

After performing the routine procedure as shown in Figure 1 (a), P1 tries to take advantage of the new information in the form of the calculation results to continue completing the previously made image by dividing the rectangular image by twenty equal parts based on the denominator in the results of the calculation. Figure 1 (b) is the result of P1's answer after dividing the rectangle into twenty equal parts. However, the visual mediator produced by P1 causes anomalous recognition which is an indicator of cognitive conflict [32]. Doubts indicate this in P1 regarding the visual mediator produced but still believe that the routine procedures are correct. The doubt is caused by the calculation results showing that the numerator has 39 parts. In comparison, the visual mediator produced only shows twenty parts, so it is impossible to be the correct answer. P1's strong memory for solving fractional problems in the form of images is to produce images based on the results of calculations. The strong memory follows the research results, which states that the ability to perform repeated access to long-term memory can determine how to solve a given problem [33]. This condition is known through the following interview excerpt.

> "*Initially, I divided the whole rectangle into twenty equal parts (visual mediator) based on the results of my calculations (routine). However, after I divided it into 20 equal parts, when I wanted to determine Luna and Rohman's portion, I was confused because their number of parts was 39 (recognition anomalous). My calculation process is correct (I believe the routine), but in my opinion, the resulting image is not suitable (doubt the visual mediator).*"



(a) (b)

Figure 1. Part of P1's work: (a) Routine and (b) Visual mediator

3.2. Commognitive analysis in solving mathematics problems in participant 2

Participant 2 (P2) solves the problem by generating four commognitive components which are discussed further, especially on visual mediator and routine component. In term of word use, P2 knows well each part of Luna and Rohman. However, P2 has doubts about interpreting the two plots of father's land presented in the form of pictures. The confusion condition experienced by P2, apart from being an indicator of cognitive conflict [32], also shows that "knowing" has a different meaning in mathematical and pedagogical discourse. This term cannot be used to describe students' complete understanding [34].

In term of visual mediator, P2 solves the problem by understanding the problem information that the land plot can be shown by drawing a rectangle. However, P2 assumes that Luna's land that has been given in the question does not have a specific size, so it is not easy to describe the land belonging to Rohman. The difficulties experienced by P2 indicate that there is a lack of understanding when information is presented in the form of pictures. Furthermore, the research states that a lack of initial knowledge results in difficulties learning mathematics [35]. However, P2 tries to represent a plot of land with two rectangles (interest). In this condition, P2 realizes that the initial error in producing the image can be represented to find a suitable image. Figure 2 is a visual mediator generated by P2 after determining how to solve the given fraction problem. This condition is known through the following think aloud snippets.

> "*If I want to redraw it, how is this? The size of Luna's part is unknown; what about Rohman? Uhm, if draw two rectangles, then the rectangle is divided into four equal parts, and the second rectangle is divided into six equal parts, can or not? Just try it first.*"

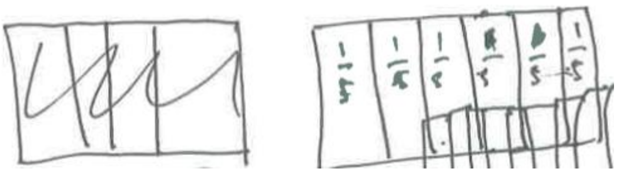


Figure 2. Visual mediator of P2

In term of narrative, P2 finds a relationship between the information provided and what should be described (recognition anomalous). The two plots of Father are the two rectangles in the picture. This condition results in P2 being able to correct errors in the previously generated image. This finding is in line with the results of research which explains that understanding the problem (in the form of text) is an activity of reconstructing cognition to understand the information provided [36].

In term of routine, P2 makes use of the previous information by equating the denominators of the two fractions and redrawing the visual mediator by using the calculation results to show Luna and Rohman's part (anxiety). This condition is known through the following think aloud snippets. Figure 3 is P2's process of equalizing the denominators and redrawing Luna and Rohman's land.

> *"It is necessary to first equate the fraction with these two different denominators."*

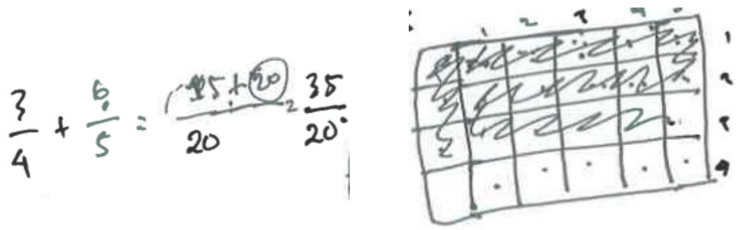


Figure 3. Routine and visual mediator of P2

In the process of solving problems, P2 shows three indicators of cognitive conflict. First, on the interest indicator, namely P2 has an interest in finding out more about the correct answer after producing the wrong visual mediator as in Figure 2. Second, the anomalous recognition indicator, namely P2 is confused because after solving the problem by performing calculations (routines) and redrawing according to the results of these calculations (visual mediator). P2 believes that the resulting visual mediator is not suitable, but P2 believes that the routine procedures it carries out are correct. This condition indicates the next cognitive conflict, namely, anxiety. Therefore, after completing Figure 3, P2 stopped solving the problem for a long time. This condition is known through the following interview excerpt.

"Because earlier I thought it was wrong, I tried to reread the problem (interest). Then I tried to equalize the denominator first by adding up the portion of Luna and Rohman land (narrative). I got the denominator twenty. I redrew it and then divided it into twenty equal parts (visual mediators). When I tried to shade Luna's part, which is fifteen parts, the remaining is only five parts. I am so confused about what kind of recognition (anomalous recognition). If you equate the denominator first, it is even less, even though the method is correct (anxiety)."

Based on data exposure in P1 and P2, an error in interpretation of the routine that is carried out results in error in producing a visual mediator. Besides, the strong memory that affects the routine procedures performed by the participants results in a firm belief as a way to produce a visual mediator for the given problem. This condition shows that the participants call back the determination of the process in solving the previous problem for reuse. However, the recall of determining the process of solving problems gives rise to conditions of cognitive conflict. As found by previous studies, students have different beliefs about learning so that some receive information as it is [37], [38]. For example, some students believed learning consisted of memorizing visible facts, data, and formulas rather than constructing their knowledge, so that students could use the information received without critical consideration. This condition shows that there is no indication that the subject has reached a well-connected understanding of the concept. The absence of these indications indicates that in learning mathematics, the initial routine carried out is an implementation of a ritual and does not change even in a state of cognitive conflict. This condition is different from the results of the study, which stated that the initial routine of bringing up student discourse appeared, which was initially implemented as a ritual [39].

In the long run, this routine is expected to undergo a gradual de-ritualization until it becomes a mature exploration. Likewise, the research results describe a gradual change from visual abilities to more formal discourse, which are characterized by developments in routine abilities [40]. This fact is in line with those that said mathematics learning is enhanced through exploration, not just expecting students to follow routines procedures [39]. So that students can only rely on memorizing procedures and concepts and then apply them in solving the problems given.

4. CONCLUSION

Students made errors in solving improper fraction problems, namely on routine components. Errors in routine procedures affect producing answers in the form of images (visual mediator). Errors in routine procedures carried out are caused by the strong influence of the memory of previous knowledge and experience when studying mathematics, especially in fraction material. Therefore, the memory already possessed cannot experience a change in concepts, and this condition results in the emergence of cognitive conflict circumstances. Cognitive conflict occurs because the memory in the routine procedure performed is believed to be correct, but the resulting visual mediator is considered inappropriate.

This study motivated researchers to conduct further research. They may describe the process of natural cognitive conflict in the learning process of mathematics by using a commognitive perspective. Hopefully, it can provide a detailed description of the causes of cognitive conflict in the learning process to determine the right strategy in implementing mathematics learning, especially in fraction material.

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