

1996

Regression Analyses for ABAB Designs in Educational Research

T. Mark Beasley
St. John's University

Follow this and additional works at: <https://scholarworks.bgsu.edu/mwer>

[How does access to this work benefit you? Let us know!](#)

Recommended Citation

Beasley, T. Mark (1996) "Regression Analyses for ABAB Designs in Educational Research," *Mid-Western Educational Researcher*. Vol. 9: Iss. 4, Article 6.

Available at: <https://scholarworks.bgsu.edu/mwer/vol9/iss4/6>

This Featured Article is brought to you for free and open access by the Journals at ScholarWorks@BGSU. It has been accepted for inclusion in Mid-Western Educational Researcher by an authorized editor of ScholarWorks@BGSU.

Regression Analyses for ABAB Designs in Educational Research

T. Mark Beasley, St. John's University, New York

Abstract

Too many practitioners interpret ABAB research based on visual inspection rather than statistical analyses. This article illustrates the techniques and importance of regression analyses on a hypothetical single-case study group given Cooperative Learning as an instructional strategy for increasing cooperative behavior.

The ABAB design has become one of the most commonly used designs in single-case research because it is believed to control for the confounding effects of history and maturation (Barlow & Hersen, 1984). Hence, the researcher can be reasonably sure that the results cannot be attributed to extraneous factors or confounding variables. Unfortunately, most practitioners of single-case methodologies take little precaution against the potential of their results being attributed to chance. That is, they overwhelmingly interpret their results based on visual inspection rather than statistical analysis (Busk & Marascuilo, 1992).

Though practically simple, the validity of visual inspection is questionable (e.g., Park, Marascuilo, & Gaylord-Ross, 1990). Many researchers extol the virtues of visual inspection and graphic analysis of data, asserting that an "important" effect will be manifest in an obvious manner, and that in applied settings only marked effects have practical significance (Baer, 1977). However, it has been suggested that the presentation of both visual and statistical analyses gives more credence to research findings (Huitema, 1985; Park et al., 1990). Yet, no consensus on what constitutes an appropriate analysis has been reached. Variants of the repeated-measures analysis of variance (ANOVA) have been suggested (Shine & Bower, 1971; Gentile, Roden, & Klein, 1972). However, based on simulation results (Toothaker et al., 1983), these tests are not recommended because they seriously inflate the Type I error rate when there is nonzero autocorrelation. Furthermore, an important assumption of linear models, however, is independence of error terms for all observations. Therefore, statistical analysis of single-case data becomes problematic when dependency exists in the data (see Busk & Marascuilo, 1992).

To circumvent the violation of the independence of errors assumption (i.e., autocorrelation), *interrupted time-series analysis* has been recommended for the analysis of single-subject data (Jones, Vaught, & Weinrott, 1977). Although this approach has the advantage of accounting for serial dependency, it raises new difficulties. Namely, the

modeling process may become quite complex, to the point that it is difficult to make inferential decisions or even identify the null hypothesis being tested (Gorsuch, 1983). However, many single-case researchers contend that data from behavioral experiments can be analyzed with the simplest of the time-series models and that time-series and least-squares methodologies can be combined (Horne, Yang, & Ware, 1982).

Huitema (1985) contends that little evidence exists to indicate that serial dependency is a major threat to single-case behavioral researchers; however, this contention has met with substantial resistance and rebuttal. Moreover, time-series analyses typically require a minimum of 50 to 60 data points per phase (Jones et al., 1977), yet single-case experiments provide far fewer observations for the entire study. Several nonparametric approaches to time-series data have also been suggested (see Edgington, 1992; Levin, Marascuilo, & Hubert, 1978); however, most of these tests were not developed to assess the effects of treatment on trends. Since the detection and control of dependency or autocorrelation involves an investigation of overall and within-phase trends, discussion will focus on regression-based trend models. For the sake of simplicity and to make these suggestions concrete, an ABAB design will be presented.

Hypothetical Example of an ABAB Design

Based on an experiment using Cooperative Learning groups as an instructional strategy for integrating autistic students into a fourth-grade social studies class (Dugan et al., 1995), hypothetical data for one student from an ABAB reversal design were created. Suppose data were obtained from a three-week initial baseline period in which students received a 40-minute teacher lecture four times per week. Five-minute time-sampling probes were systematically conducted to assess the cooperative behavior of the subject. The number of seconds engaged in appropriate interaction during the probe was used as the dependent measure of the student's ability to cooperate. During this initial baseline, $n_{a1} = 10$ observations were made. A four-week Cooperative

Learning (B) phase, which involved a 10-minute whole class lecture followed by the construction of Cooperative Learning groups of four students, yielded $n_{b1} = 12$ data points for this student. A three-week reversal phase yielded $n_{a2} = 8$ cooperation measures. A five-week reinstatement of Cooperative Learning groups yielded $n_{b2} = 9$ data points. The total time-series of $T = 39$ cooperation measures is displayed in Figure 1. Descriptively, the Cooperative Learning conditions appear to increase cooperation for this student. During the initial baseline, cooperation was at a mean level of 43.40 seconds of interaction (SD = 25.33). After the initial Cooperative Learning intervention, the mean number of seconds engaged in appropriate interaction drastically increased

autocorrelated due to the subject's acclimation to the classroom, which is substantiated by the trends in the first Cooperative Learning phase where cooperation measures are trending upwards as phases are changed. This being the case, we cannot be completely confident that the apparent results are a function of the treatment and not merely a function of cycles or errors that are correlated with these variables. However, because of the small number of observations within each phase, one must consider the low power of investigating within-phase autocorrelation. Therefore, autoregressive analyses would not be considered valid and trend models would be preferred.

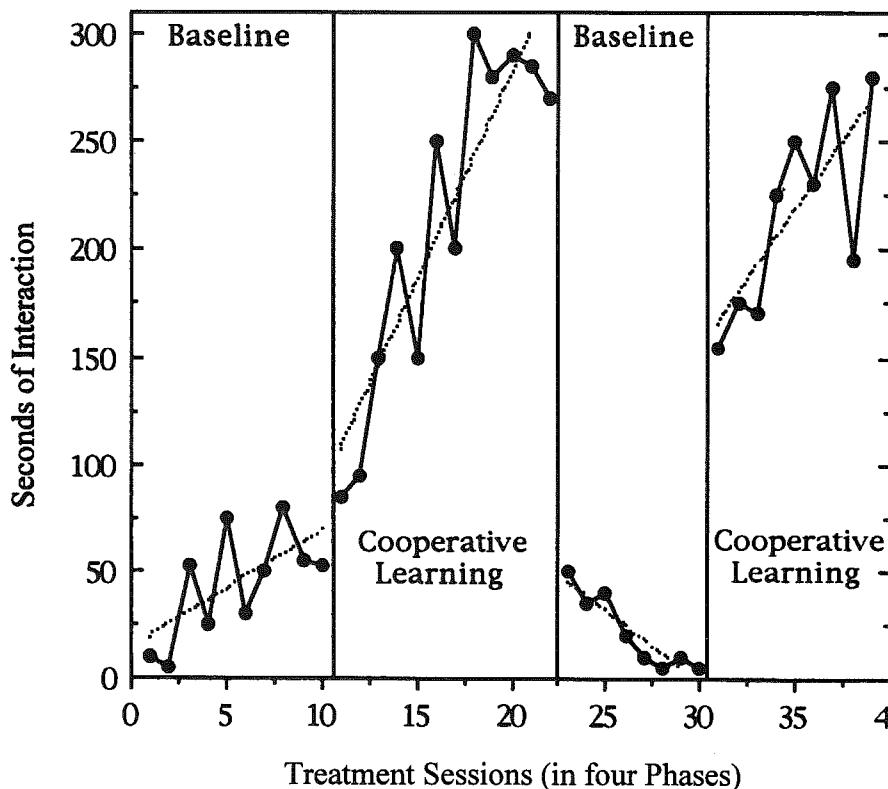


Figure 1. Seconds of Interaction (Cooperation) as a function of Treatment phases. Dotted lines represent the within-phase regressions solved in Equations 2 through 6.

to 212.92 (SD = 77.68). During the reversal phase, cooperation reduced to a mean of 21.88 (SD = 17.51). When Cooperative Learning groups were reinstated, cooperation increased to a mean level of 217.22 (SD = 46.04).

Trend Analysis for Single-Case Data

Similar to changes in level and slope, autocorrelation is frequently difficult to detect through visual inspection alone. In examining the cooperation data in Figure 1, the potential confound of autocorrelation is plausible when we consider the nature of the variables. That is, the reliability of the cooperation may be questionable, which could lead to autocorrelated errors. Furthermore, the effects of cooperative groups could be obscured because during the initial baseline phase, the amount of interaction may be

One cause of autocorrelation in single-case observations is the failure to specify elements in the model which represent all the influences at work. If the statistical model is a regression analysis, these trends can be included as a covariate. The effect of an intervention, however, may result in a change in slope as well as a change in level, or in some cases just a change in slope (Kazdin, 1984).

Kelly, McNeil, and Newman (1973) proposed a comprehensive approach that assesses shifts in level and slope. This approach uses time of observation (t) as a covariate, dummy codes (X_j) to represent the phases, and an interaction term involving the multiplication of t and X_j to estimate the change in slope. However, t and X_j are necessarily correlated in the ABAB design, which often makes the separation of baseline and post-treatment effects difficult. Therefore, the use of a piecewise regression, akin to regression-discontinuity models (Trochim, 1984), has

been recommended. Thus, a researcher interested in statistically controlling autocorrelative trends and assessing changes in slope may use some variant of the following Model:

$$Y_t = b_0 + b_1X_{1t} + b_2X_{2t} + b_3X_{3t} + b_4t^* + b_5X_{1t}t^* + b_6X_{2t}t^* + b_7X_{3t}t^* + c_t \quad (1)$$

where X_{1t} , X_{2t} , and X_{3t} are dummy codes for each phase following the initial baseline. That is, observations belonging to the initial baseline received zeros on all three dummy codes. Based on the idea of centering, t is rescaled such that $t^* = 1$ at the start of each phase thus reducing the correlation of t^* , X_j , and the interaction (change in slope) term. For the entire time-series, $t^* = t$. For observations in the first treatment phase, $t^* = (t - n_{a1})$, $t^* = (t - n_{a1} - n_{b1})$ for the reversal

phase observations, and $t^* = (t - n_{a1} - n_{b1} - n_{a2})$ for the fourth phase, where n_{a1} , n_{b1} , and n_{a2} are the number of data points in each of the first three phases, respectively. It should be noted that other approaches to rescaling t have been suggested (Gorsuch, 1983; Huitema, McKean, & McKnight, 1994; Kelley et al., 1973); however, all versions lead to the same full-model R^2 and test statistics, only the interpretation of the regression coefficients and tests of specific hypotheses differ. For model (1), the initial baseline level of behavior is estimated by b_0 . Also in this model, b_1 , b_2 , and b_3 estimate the respective changes in level as compared to the initial baseline. The three interaction terms (b_5 , b_6 , and b_7) estimate changes in slope as compared to the baseline regression (b_4). For those more familiar with other types of analyses, Model (1) gives parameter estimates and test statistics equivalent to an ANCOVA with time (t) as a covariate. It should be noted that in ANCOVA models, a significant time by phase interaction indicates that slopes significantly change (are not parallel) across treatment which is the regression equivalent to rejecting the following null hypothesis, $H_0: b_5 = b_6 = b_7 = 0$. This significant heterogeneity of regression effect may obscure the interpretation of Phase main effects in single-case designs.

Through the regression approach, pairwise tests of regression coefficients are also available. For example in model (1), the test of $b_4 = 0$ assesses whether the initial baseline regression significantly differs from zero. Tests of $b_5 = 0$, $b_6 = 0$, and $b_7 = 0$, assess whether the slopes of the first treatment phase, removal phase, and second treatment phase significantly differ from the initial baseline regression (b_4), respectively. Furthermore, testing $b_8 = b_9$ assesses whether the slope changes between the first treatment and removal phases. Likewise, testing $b_{10} = b_{11}$ assesses whether slope changes between removal phases and the second treatment phase. It

should be mentioned that if a researcher is interested in multiple comparisons, corrections for the inflation of Type I errors (i.e., Bonferroni adjustment) is strongly suggested. In this case with four phases, six pairwise comparisons are possible. Thus, the Bonferroni adjusted significance level is $\alpha = .05/6 = .0083$.

For the cooperation data in Figure 1, the statistical trend Model (1) was employed to control dependency. The Appendix gives the raw data in Figure 1 and the SAS (1993)

Dependent Variable: COOP		Analysis of Variance				
Source	DF	Sum of Squares	Mean Square	F Value	Prob>F	
Model	7	387087.71324	55298.24475	72.657	0.0001	
Error	31	23593.72266	761.08783			
C Total	38	410681.43590				
Root MSE	27.58782	R-square	0.9425	Dep Mean	131.25641	
C.V.	21.01827	Adj R-sq	0.9296			
Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > T	
INTERCEP	1	13.466667	18.84606907	0.715	0.4802	
X1	1	73.881818	25.36661513	2.913	0.0066	
X2	1	38.140476	28.58781834	1.334	0.1919	
X3	1	139.172222	27.51108071	5.059	0.0001	
TSTAR	1	5.442424	3.03731905	1.792	0.0829	
X1T	1	13.875758	3.81413155	3.638	0.0010	
X2T	1	-12.049567	5.22938292	-2.304	0.0281	
X3T	1	7.474242	4.68082302	1.597	0.1205	
Dependent Variable: COOP						
Test: INTERACT	Numerator:	8549.6306	DF:	3	F value:	11.2334
	Denominator:	761.0878	DF:	31	Prob>F:	0.0001
Test: B5EQB6	Numerator:	21820.3648	DF:	1	F value:	28.6700
	Denominator:	761.0878	DF:	31	Prob>F:	0.0001
Test: B5EQB7	Numerator:	1732.0356	DF:	1	F value:	2.2757
	Denominator:	761.0878	DF:	31	Prob>F:	0.1415
Test: B6EQB7	Numerator:	9417.3669	DF:	1	F value:	12.3736
	Denominator:	761.0878	DF:	31	Prob>F:	0.0014
Test: IMPACTB1	Numerator:	2348.1142	DF:	1	F value:	3.0852
	Denominator:	761.0878	DF:	31	Prob>F:	0.0889
Test: NB1EQ2	Numerator:	5916.7740	DF:	1	F value:	7.7741
	Denominator:	761.0878	DF:	31	Prob>F:	0.0090
Test: NB1EQ3	Numerator:	11648.5775	DF:	1	F value:	15.3052
	Denominator:	761.0878	DF:	31	Prob>F:	0.0005
Test: NB1EQ12	Numerator:	98605.1992	DF:	1	F value:	129.5582
	Denominator:	761.0878	DF:	31	Prob>F:	0.0001
Test: IMPACTA2	Numerator:	74059.4949	DF:	1	F value:	97.3074
	Denominator:	761.0878	DF:	31	Prob>F:	0.0001
Test: NA2EQ8	Numerator:	17396.4649	DF:	1	F value:	22.8574
	Denominator:	761.0878	DF:	31	Prob>F:	0.0001
Test: IMPACTB2	Numerator:	35023.3343	DF:	1	F value:	46.0175
	Denominator:	761.0878	DF:	31	Prob>F:	0.0001
Test: NB2EQ9	Numerator:	91856.6676	DF:	1	F value:	120.6913
	Denominator:	761.0878	DF:	31	Prob>F:	0.0001

Figure 2. Edited SAS Output for Analysis of Data in Figure 1.

commands for performing the following analyses. Figure 2 shows a SAS output for Model (1), which yielded the following regression solution:

$$\hat{Y}_t = 13.467 + 19.458X_{1t} - 81.593X_{2t} - 24.101X_{3t} + 5.442t^* + 13.876X_{1t}t^* - 12.050X_{2t}t^* + 7.474X_{3t}t^* \quad (2)$$

As can be seen in Figure 2, the first two interaction regression coefficients (X_1 and X_2) were significantly different from zero indicating that the slopes in the first treatment phase and the reversal phase changed from the regression slope of the initial baseline, respectively. Also, ANCOVA model with t as the covariate showed a statistically significant interaction of time and phase ($H_0: b_5 = b_6 = b_7 = 0$), $F(3,31) = 11.23, p < .0001$, which indicates that the slopes changed significantly across phases.

With this statistical interaction, one might choose to plot each phase separately, for descriptive purposes. Furthermore, in the presence of significant interactions (i.e., changes in slope), however, one should be cautious in interpretation of changes in level. Since, in the dummy coding process baseline observations were given values of zero across X_1, X_2 , and X_3 , their respective regression coefficients are not weighted. Thus for the baseline data:

$$\hat{Y}_{A1} = 13.467 + 5.442t^* \quad (3)$$

For the data in the first Cooperative Learning phase, only X_1 was assigned values of 1, X_2 and X_3 were assigned 0; thus, the regression equation for the acquisition phase is:

$$\hat{Y}_{B1} = 13.467 + 73.882 + 5.442t^* + 13.876t^*$$

Since, $t^* = (t - n_{a1})$ in the first treatment phase,

$$\hat{Y}_{B1} = 87.348 + 19.318t^* \quad (4)$$

Using this same process, the regression solution for the reversal phase is,

$$\hat{Y}_{A2} = 51.607 - 6.607t^* \quad (5)$$

and for the last treatment phase,

$$\hat{Y}_{B2} = 152.639 + 12.917t^* \quad (6)$$

Other pairwise tests show that the slope in the reversal phase was significantly different than the slopes in the first [$F(1,31) = 28.67, p < .0001$] and second treatment phases, $F(1,31) = 12.37, p = .0014$. The slopes of the two treatment phases did not significantly differ, $F(1,31) = 2.28, p = .1415$. Therefore, it seems that the removal of Cooperative Learning groups changed rate of cooperation to a negative slope. When Cooperative Learning groups were reinstated the rate of cooperation over time again became an increasing function. Thus, cooperative behavior increases more rapidly when the student is part of a Cooperative Learning group.

As in ANCOVA models, when a significant interaction of the independent variable and the covariate is present,

the use of some follow-up analysis such as the Johnson-Neyman technique is suggested. In single-case research, this means that one searches for observation points within a phase where the values of the dependent variable are significantly changed as compared to some other phase, while statistically controlling the effects of the within-phase regression. Rogosa (1980) contends that researchers should select points of theoretical interest, which allows for tests of very specific hypotheses. In single-case research, an experimenter may wish to test all observation points in the phase for statistical significance. Thus, Rogosa's (1980) pick-a-point method was used to investigate at what observation point after changing phases did cooperation significantly change. However, this requires many statistical tests. For this example, a Bonferroni adjustment was used so that $\alpha = .0017$.

To assess the immediate impact of the first Cooperative Learning phase, one must compare the first observation ($t^* = 1$) of the treatment phase (b_1) as compared to the last observation ($t^* = 10$) in the initial baseline phase (see Equations 1 and 2). Thus, a method similar to regression-discontinuity analysis (Trochim, 1984) was employed to test the following null hypothesis:

$$H_0: 10b_4 = b_1 + 1b_4 + 1b_5 \quad (7)$$

The results showed that the first treatment phase did not immediately increase cooperation over the last observation of the baseline, $F(1, 31) = 3.09, p = .0889$. To test whether significant differences occur through any of the n_{b1} points in the first treatment phase, null hypotheses of the following general form can be used:

$$H_0: n_{a1}b_4 = b_1 + n_{b1}b_4 + n_{b1}b_5 \quad (8)$$

Given that $n_{a1} = 10$, testing (8) from $n_{b1} = 2$ to 12, showed that the amount of time engaged in social interaction did not significantly increase by the second observation in the treatment phase, $F(1, 31) = 7.77, p = .0090$. However, cooperation was significantly increased over the last baseline observation by the third treatment observation, $F(1, 31) = 15.31, p = .0005$ (see Figure 2 for results). This increased level of cooperation continued through the rest of the first treatment phase ($ps < .0001$). This indicates that the implementation of Cooperative Learning groups, which has strong effects on the rate of cooperative behavior, had a gradual, but permanent effect on the level of cooperation.

To assess whether a statistically significant reduction in cooperation occurred at any point (n_{a2}) during the removal phase as compared to the last observation of the first treatment phase ($n_{b1} = 12$), null hypotheses of the following general form can be tested:

$$H_0: b_1 + n_{b1}b_4 + n_{b1}b_5 = b_2 + n_{a2}b_4 + n_{a2}b_5 \quad (9)$$

Statistical tests for $n_{a2} = 1$ to 9 showed that the removal of Cooperative Learning groups immediately lowered cooperative behavior as compared to the last observation in the second baseline, $F(1, 31) = 97.31, p < .0001$. This re-

duction in cooperation remained statistically significant throughout the reversal phase (see Figure 2).

To assess whether a statistically significant increase in cooperation occurred during the last treatment phase as compared to the last observation of the removal phase ($n_{a2} = 8$), null hypotheses similar to (9), with n_{a2} substituted for n_{b1} and n_{b2} substituted for n_{a2} , were formed. The results indicate that the re-instatement of Cooperative Learning groups immediately increased cooperative behavior over the last observation in the second baseline, $F(1, 31) = 46.02$, $p < .0001$ (see Figure 2). This significant increase in the number of seconds engaged in social interaction remained through last treatment phase ($ps < .0001$). The reader is referred to Rindskopf (1984) and Rogosa (1980) for more details on testing specific linear hypotheses of this form. Jennings (1988) provides a very usable guide to ANCOVA follow-up procedures and test of linear hypothesis.

In summary, despite the claims that single-case data are not amenable to regression-based methods, the models presented provide a flexible approach to the statistical analysis of single-case data. Furthermore, regression-based models provide a way of statistically controlling the confounding effects of autocorrelation. It should also be noted that if nonlinear processes are suspected within phases they can also be modeled statistically (Kelly et al., 1973; Trochim, 1984). In the presence of autocorrelative or nonlinear processes, as with other analytic models, it would be important to obtain a sufficient number of baseline observation so that the within-phase trends could be modeled. In situations where the baseline series were not of adequate length, nonparametric tests of single case intervention effects may be employed (e.g., Edgington, 1992; Levin et al., 1978). Many studies have shown the benefits of nonparametric and randomization tests, however, least-squares regression approaches to analyzing single case designs have fared well in simulation studies (Huitema et al., 1994). Moreover, regression models can be generalized to many single case designs including multiple baseline, and alternating treatment designs (see Kelly et al., 1973). One caveat that must be addressed, however, is that the assumptions of linear regression apply. Therefore, regression diagnostics such as inspection of residuals and tests/corrections for heteroscedasticity are important. Future simulation studies should focus on comparing the statistical properties of least-squares regression methods to nonparametric and randomization tests under situations that occur most frequently in single-case research.

References

- Baer, D. (1977). "Perhaps it would be better not to know everything." *Journal of Applied Behavior Analysis*, 10, 167-172.
- Barlow, D. H., & Hersen, M. (1984). *Single-case experimental designs: Strategies for studying behavior change*. New York: Pergamon Press.
- Busk, P. L., & Marascuilo, L. A. (1992). Statistical analysis in single-case research: Issues, procedures, and recommendations, with applications to multiple behaviors. In T. R. Kratochwill & J. R. Levin (Eds.), *Single case research design and analysis: New Directions for psychology and education* (pp. 133-158). New York: Academic Press.
- Dugan, E., Kamps, D., Leonard, B., Watkins, N., Rheinberger, A., & Stackhaus, J. (1995). Effects of cooperative learning groups during social studies for students with autism and fourth-grade peers. *Journal of Applied Behavior Analysis*, 28, 175-188.
- Edgington, E. S. (1992). Nonparametric tests for single-case experiments. In T. R. Kratochwill & J. R. Levin (Eds.), *Single case research design and analysis: New Directions for psychology and education* (pp. 133-158). New York: Academic Press.
- Gentile, J. R., Roden, A., & Klein, R. D. (1972). An analysis-of-variance model for the intrasubject replication design. *Journal of Applied Behavior Analysis*, 5, 193-198.
- Gorsuch, R. L. (1983). Three methods for analyzing limited time-series ($N = 1$) data. *Behavioral Assessment*, 5, 141-154.
- Horne, G. P., Yang, M. C. K., & Ware, W. B. (1982). Time series analysis for single-subject designs. *Psychological Bulletin*, 91, 178-189.
- Huitema, B. E. (1985). Autocorrelation in applied behavior analysis: A myth. *Behavioral Assessment*, 7, 107-118.
- Huitema, B. E., McKean, J. W., & McKnight, S. (August, 1994). Small-sample time-series intervention analysis: Problems and solutions. Paper presented at the meeting of the American Psychological Association. Los Angeles, CA.
- Jennings, E. (1988). Analysis of covariance with non-parallel regression lines. *Journal of Experimental Education*, 56, 129-134.
- Jones, Vaught, & Weinrott, (1977). Time-series analysis in operant research. *Journal of Applied Behavior Analysis*, 10, 151-166.
- Kazdin, A. E. (1984). *Single-case research designs: Methods for clinical and applied settings (2nd ed.)*. New York: Oxford University.
- Kelly, F. J., McNeil, K., & Newman, I. (1973). Suggested inferential statistical models for research in behavior modification. *Journal of Experimental Education*, 41(4), 54-63.
- Levin, J. R., Marascuilo, L. A., & Hubert, L. J. (1978). $N =$ nonparametric randomization tests. In T. R. Kratochwill (Ed.), *Single subject research: Strategies for evaluating change*. New York: Academic Press.
- Park, H., Marascuilo, L.A., & Gaylord-Ross, R. (1990). Visual inspection and statistical analysis in single-case designs. *Journal of Experimental Education*, 58, 311-320.

Rindskopf, D. (1984). Linear equality restrictions in regression and loglinear models. *Psychological Bulletin*, 96, 597-603.

Rogosa, D. (1980) Comparing nonparallel regression lines. *Psychological Bulletin*, 88, 307-321

SAS Institute Inc. (1993). *SAS/STAT (Version 7): Users' guide*. Cary, NC: Author.

Shine, L. C., & Bower, S. M. (1971). A one-way analysis of variance for single-subject designs. *Educational and Psychological Measurement*, 31, 105-113.

Toothaker, L. E., Banz, M., Noble, C., Camp, J., & Davis, D. (1983). N = 1 designs: The failure of ANOVA-based tests. *Journal of Educational Statistics*, 8, 289-309.

Trochim, W. M. K. (1984). *Research design for program evaluation: The regression discontinuity approach*. Newbury Park, CA: Sage.

APPENDIX

SAS Commands for Analyzing Data in Figure 1.

```
data one;options ls=73;
input coop @@;
na1=10;nb1=12;na2=8;nb2=9;
t=_n_;
if t <= na1 then phase=1;
if t > na1 & t <= (na1+nb1) then phase=2;
if t > (na1+nb1) & t <= (na1+nb1+na2) then phase=3;
if t > (na1+nb1+na2) then phase=4;
x1=0;x2=0;x3=0;
if phase=2 then x1=1;
if phase=3 then x2=1;
if phase=4 then x3=1;
tstar=t;
if phase=2 then tstar=t-na1;
if phase=3 then tstar=t-na1-nb1;
if phase=4 then tstar=t-na1-nb1-na2;
x1t=x1*tstar;x2t=x2*tstar;x3t=x3*tstar;
cards;
  10  5  52  25  75  30  50  80  55  52
  85  95 150 200 150 250 200 300 280 290 285 270
  50  35  40  20  10  5  10  5
155 175 170 225 250 230 275 195 280
proc reg;model coop= x1 x2 x3 tstar x1t x2t x3t;
  interact: test x1t=0,x2t=0,x3t=0;** Test of interaction, Ho: b5=b6=b7=0 ;
  b5EQb6: test x1t=x2t;** Pairwise test of b5 = b6 ;
  b5EQb7: test x1t=x3t;** Pairwise test of b5 = b7 ;
  b6EQb7: test x2t=x3t;** Pairwise test of b6 = b7 ;
** Introduction of Treatment (Phase 2) Compared to Initial Baseline(Phase 1) ;
  impactb1: test 10*tstar = 1*tstar + x1 + 1*x1t;*Test of Eq. 7, 10b4=b1+b4+b5;
  nb1eq2: test 10*tstar = 2*tstar + x1 + 2*x1t;*Test of Eq. 8, na1=10 nb1=2;
  nb1eq3: test 10*tstar = 3*tstar + x1 + 3*x1t;*Test of Eq. 8, na1=10 nb1=3;
.
.
.
  nb1eq12: test 10*tstar =12*tstar + x1 +12*x1t;
  ** Test of Eq. 8, na1=10 nb1=12;
*** Reversal Effects (Phase 3) Compared to Treatment (Phase 2) ***;
  impacta2: test 12*tstar + x1 + 12*x1t = 1*tstar + x2 + 1*x1t;
  ** Test of Eq. 9, nb1=12 na2=1;
.
.
.
  na2eq8: test 12*tstar + x1 + 12*x1t = 8*tstar + x2 + 8*x1t;
  ** Test of Eq. 9, nb1=12 na2=8;
*** Effect of Treatment (Phase 4) Compared to Reversal )Phase 3) ***;
  impactb2: test 8*tstar + x2 + 8*x2t = 1*tstar + x3 + 1*x3t;**Test of Eq. 9,
  na2 substituted for nb1, nb2 substituted for na2, na2=8 nb2=1;
.
.
.
  nb2eq9: test 8*tstar + x2 + 8*x2t = 9*tstar + x3 + 9*x3t;**Test of Eq. 9,
  na2 substituted for nb1, nb2 substituted for na2, na2=8 nb2=9;

proc glm;class phase;
  model coop= phase tstar phase*tstar/solution;
```