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Abstract

As universities strive to produce the best mathematics teachers possible through both graduate and undergraduate programs, teacher educators must constantly work towards helping teacher candidates create their own conceptual understanding of mathematics. This pilot study examined the effect teaching in a constructivist manner had on teacher candidates' conceptual understandings of the arithmetic mean and on their ability to transfer this knowledge into their instruction. Results indicated that teaching in a constructivist manner can have a positive impact on teacher candidates' understanding of the arithmetic mean, but their abilities to transfer this new knowledge into their own instructional practices was inconsistent.

Having a highly qualified teacher in every classroom is a goal of not only the federal government, but also every parent, school administrator, and teacher educator. Being a highly qualified teacher entails having adequate content knowledge as well as being able to utilize that knowledge to bring students of diverse backgrounds to high levels of learning. Supporting high levels of learning in mathematics requires the teacher to have both a procedural and conceptual understanding of the mathematics being taught. Procedural understandings entail knowledge of the rules and sequences of actions taken in algorithms. Conceptual understandings involve making connections and understanding relationships between discrete pieces of mathematical information (Reys, Suydam, Lindquist, & Smith, 1998). Teachers who have conceptual and procedural understandings of mathematics produce students who "exhibit conceptual understanding, have the ability to learn and reason, and are able to achieve" (Knight, 2001, p. 21).

As colleges and universities strive to produce highly qualified mathematics teachers through both graduate and undergraduate programs, teacher educators must work towards helping teacher candidates create their own conceptual understanding of mathematics. To develop this level of deep understanding, teacher candidates must experience a learning environment that is much different than the one typically found in a traditional college mathematics course. In order to take an active role in constructivist teaching, teacher candidates must first take an active role in constructivist learning. The experiences teachers have as learners will have a tremendous impact on the beliefs and attitudes brought into their own classrooms (Chappell & Thompson, 1994).

The preparation of elementary teachers is of particular importance when trying to establish a conceptually-based foundation of mathematics for teacher candidates. Teachers at the elementary level often are aware of the need to teach algorithms associated with mathematical concepts such as addition, subtraction, multiplication, and division in a conceptually-based manner. However, this is not the case when teaching more advanced concepts such as finding the arithmetic mean of a data set, hereafter referred to as the mean. Teachers typically provide the conventional algorithm yet have no understanding of how the algorithm is derived or why it is useful in solving posed problems. Despite its simple nature, "developing a conceptual underpinning that allows one to use the mean sensibly is surprisingly difficult" (Konold & Higgins, 2003, p. 204).

In this article, we present a pilot study that examines the effect learning in a constructivist manner has on teacher candidates' conceptual understandings of the mean and on their ability to transfer this knowledge into their instruction. Finding methods for empowering teachers to effectively teach mathematical concepts such as the mean is of great importance if the goal of teacher educators is to prepare teachers to teach in line with the National Council of Teachers of Mathematics' (NCTM) *Principal and Standards for School Mathematics* (2000). In addressing this need, the following research questions were posed:

1. Does learning about the mean in a constructivist manner extend teacher candidates' definitions of the mean from a primarily procedurally-based definition to a more conceptually-based definition?
2. Does learning about the mean in a constructivist manner have an impact on teacher candidates' ability to use the algorithm?
3. Are teacher candidates able to connect the algorithm to the equal distribution interpretation of the mean?
4. Does learning about the mean in a constructivist manner have an impact on teacher candidates' ability to design lessons that develop a conceptual understanding of the mean in elementary-aged students?

Sample and Methodology

Subjects

This pilot study was conducted during the spring semester of 2004 and involved 45 teacher candidates enrolled in one of two sections of a probability and statistics course for teachers. This population sample was taken from a southeastern state university which has a current enrollment of over 10,000 students.

Each teacher candidate involved in this pilot study was an early childhood major seeking initial certification. The sample consisted of 43 females and 2 males. Of the 43 females, there were 33 Caucasians and 10 African-Americans. Both males were Caucasians. The sample was comprised of 42 seniors, 2 non-traditional graduate students, and 1 junior. Forty-three of the teacher candidates had successfully completed their mathematics methods course which was designed to develop an understanding of mathematics content, methods, and materials appropriate for the cognitive development of the elementary child. These teacher candidates were in their last semester of coursework prior to their student teaching. The remaining two teacher candidates were taking the statistics course before beginning their junior/senior years of professional coursework. As a result, they had not completed the methods course. This sample was heterogeneous in that the teacher candidates had taken a variety of mathematics courses.

Procedures Used

Prior to the pilot study, teacher candidates completed a pre-survey designed to assess their definition of the mean and their ability to compute the mean and determine numbers that could be added to a data set without affecting the mean. The survey included the following questions.

1. In your own words, explain what the mean (or average) is.
2. Find the mean of the data set: 1, 1, 2, 2, 2, 2, 3, 3, 4, 5
3. Find three numbers that can be added to the given data set and not change the mean. Explain how you chose these three numbers.

The pilot study was conducted over 2 class periods and consisted of three constructivist-based lessons. The lessons were deemed constructivist in that they were aligned to constructivist theory and adhered to the following: the instructor served as a coach or facilitator; a real-world context was employed (Jonassen, 1991); instruction focused on knowledge construction; multiple representations of the content were examined; "collaborative construction of knowledge through social negotiation" was supported (Jonassen, 1994, p. 35); and scaffolding was used to engage students in higher-order thinking. The objective of the first lesson was to understand the mean as equal distribution. The second lesson's objective was to understand the mean as the balance point of the distribution. The final lesson allowed the teacher candidates to utilize technology in exploring the properties of the mean.

Following the three lessons, teacher candidates completed a post-survey that was identical to the pre-survey. In addition to the surveys, teacher candidates were asked to respond to a journal prompt that dealt with their understanding of the mean and to create a lesson plan designed to develop a conceptual understanding of the mean appropriate for fourth graders.

Data Analysis

The researchers used the four research questions previously presented to guide the data analysis process. To investigate research question one, teacher candidates' pre-survey responses to survey question one were compared to their post-survey responses to the same question. For this open-ended question, the researchers utilized codes to represent responses: C represented the ability to conceptually define the mean; P the ability to procedurally define the mean; and W the inability to define the mean correctly. Each researcher independently analyzed the responses using the codes. Together, we reviewed the coding and found the ratings to have inter-rater reliability. Data analysis consisted of a comparison of the percentages of teacher candidates in each of the categories from pre-survey to post-survey.

For further information on teacher candidates' understanding of the mean, we coded the responses to survey question two with a Y representing the ability to correctly calculate the mean or an N representing the inability to calculate the mean. We calculated the percentage of teacher candidates who were able to correctly compute the mean on the pre-survey.

Data for research question two was gathered by comparing teacher candidates' ability to add numbers to a given data set on the pre-survey and the post-survey. We coded the responses with a Y representing the ability to successfully complete the task and an N representing the ability to do so. Percentages of teacher candidates falling into each of the categories were compared.

To investigate research question three, we compared the teacher candidates' responses to the journal prompt which read, "Explain in your own words why it makes sense to sum the numbers in a data set and divide by the quantity of numbers when finding the average." We independently coded the responses to the journal prompt with Y representing the ability to associate the algorithm to the equal distribution interpretation of the mean or an N representing the inability to do so. We verified that the codings had inter-rater reliability. The percentage of teacher candidates in each category was determined.

For research question four, we analyzed the teacher candidates' lesson plans. We independently coded the lesson plans with C representing a conceptually-based lesson or P representing a procedurally-based lesson. Our coding had a high level of inter-rater reliability. In addition to determin-

ing the percentage of teacher candidates in each category, we characterized each teacher candidate according to his or her lesson plan type and journal entry in an effort to explore the relationship between the two.

Results and Limitations

This pilot study examined the effect teaching in a constructivist manner has on teacher candidates' conceptual understandings of the mean. This section presents the results of the data collected and the limitations of the study.

Results

The findings of this pilot study indicate that participating in constructivist-based lessons can have a positive impact on teacher candidates' understanding of the mean. After having completed the 3 constructivist-based lessons, the participants of this study were more capable of conceptually defining the mean. At the onset of the pilot study the pre-survey data showed that only 2.2% (one teacher candidate) of the teacher candidates defined the mean in conceptual terms. The sole conceptually-based definition given was:

> The mean or average is like making all of the numbers in a set even and the outcome is the average. For example, if I have 70, 80, and 90 I would even the numbers out by taking 10 from 90 and giving it to 70 then the average is 80, 80, 80—you add all the # together and divide by how many you have all together.

Twenty percent (nine teacher candidates) failed to provide an accurate definition of the mean, procedurally or conceptually. The following are examples of teacher candidates' inaccurate definitions:

> "The most likely or popular to occur"
>
> "The number that is common among other numbers"
>
> "The mean is number [*sic*] that occurs most when both extremes are combined and divided by two."

The remaining thirty-five teacher candidates (77%) provided an accurate definition for the mean that was exclusively procedural. The following are examples of the procedural definitions provided:

> "The mean is when you take a set of numbers, add them together, and divide by the total number of the numbers in a set. The mean is your answer."
>
> "The number you get when you divide the sum of a set of numbers by the amount of numbers in the set."

Interestingly, regardless of the accuracy of the definition or the type of definition provided, all teacher candidates were able to correctly compute the mean.

After the implementation of the three constructivist-based lessons, a higher percentage (53.3%) of teacher candidates provided a definition that was based on conceptual understandings of what the mean represents. This is a 51.1% increase in the number of teacher candidates who were able to extend their vision of the mean beyond the procedure. The two candidates' responses provided in Table 1 illustrate the shift made in some candidates' definition of the mean from an either inaccurate or procedural definition to one that demonstrates some level of conceptual understanding.

Candidate #7's original definition of the mean was very vague and is not true in general. Candidate #25's original definition of the mean was entirely procedurally-based. She defined how to arrive at the mean versus what the mean is. Following the implementation of the three lessons, both candidates provided a more conceptually-based interpretation of the mean. Candidate #7's definition is based on her understanding of the mean in terms of equal distribution, while Candidate #25's definition is based on her understanding of the mean as the balance point of a distribution. The teacher candidates' references to the mean as the sum of a data set equally distributed and as the balance point of a data set both demonstrate that exposure to conceptually-based lessons can increase teacher candidates' ability to communicate definitions of the mean that extend beyond a procedurally-based definition to a more conceptually-based definition. The example provided by candidate #25 indicates that her new definition was not only an adjustment in language, but also a transformation in her level of understanding.

In examining the candidates' ability to use the algorithm, results indicated that participation in conceptually-based lessons enhanced their ability to use the algorithm for finding the mean. At the onset of the pilot study, 17.7% (8 teacher candidates) were able to correctly identify three numbers

Table 1
Comparison of Pre-lesson Definitions of the Mean with Post-lesson Definitions

Candidates	Pre-Lesson Definitions	Post-Lesson Definitions
#7	It is the typical average found in a group of numerical data.	It is when you find the sum of a data set and you equally distribute the amount you computed to each group/member in the set.
#25	The mean is when you take a set of numbers, add them together, and divide by the total number of the numbers in the set. The mean is your answer.	The mean or average is when you have a set of numbers that balances the set. For example, if you have a 3 and a 5 the mean is 4. There is 1 space on either side of the 4 so the sides are balanced.

that could be added to a given data set without changing the mean. Knowledge of the algorithm should have enabled them to complete this task. However, the non-traditional nature of this task seemed to prevent the candidates from solving the problem.

After the implementation of the three constructivist-based lessons, 82.2% of the teacher candidates were able to add numbers to a data set and maintain the value of the mean. These findings demonstrate that participation in conceptually-based lessons not only increases teacher candidates' ability to understand the mean but also to apply that understanding in a variety of settings.

Journal entries were used to examine teacher candidates' abilities to connect the algorithm to the equal distribution interpretation of the mean. To demonstrate this ability, teacher candidates were asked to respond to the following journal prompt after receiving conceptually-based instruction:

> Explain in your own words why it makes sense to sum the numbers in a data set and then divide by the quantity of numbers when finding the average.

In these writings, 60% of the teacher candidates provided evidence that they could relate the algorithm to the equal distribution interpretation of the mean. Many teacher candidates provided a sample problem similar to those used in the conceptually-based lessons in an effort to support and clarify their explanation. This can be interpreted as evidence that the conceptually-based examples provided in class had an influence on the teacher candidates' ability to understand and solve problems involving the mean. For example, one student wrote this journal entry:

> The average is the one number that every piece of data needs to be moved to so that all of the data is equal. When all of the data is added together and then divided by the quantity of numbers used it gives you that number. For example, if Mom gave Charlie 5 cookies, Jeff 3 cookies, Sarah 3 cookies and Suzy 1 cookie, then there are 12 cookies all together. If 12 is divided by 4 (the number of people with cookies) the quotient is three. If each person gets 3 cookies then everyone would have an equal amount.

Many of the teacher candidates who were unsuccessful in making sense of the algorithm provided reasoning exemplified in the following journal entry:

> To me the definition of average is a number that represents a large group of numbers. It makes sense that if all the numbers in a group are added up and the sum is divided by the total of how many numbers are in the group it will find the middle or average number of that group.

Responses such as this seem to indicate that these teacher candidates have not yet made sense of the algorithm in terms of how it relates to equal distribution. These teacher candidates continue to see the average solely as the number that you obtain when applying the add-and-divide procedure.

In studying the impact of constructively-based lessons, of particular importance was the teacher candidates' ability to transfer this newly acquired knowledge into their own instruction. An examination of teacher candidates' lessons on the mean revealed that 40% of the teacher candidates designed a conceptually-based lesson. These teacher candidates introduced the mean using a problem that involved equal distribution. The remaining 60% of the teacher candidates submitted lessons that focused on the procedure without providing an explanation of the procedure's origin or why the procedure made sense.

Table 2
Relationship between Lesson Plans and Ability to Connect the Algorithm to Equal Distribution

Journal Entry	Number
Connected algorithm to equal distribution	
Conceptual lesson plan	14
Procedural lesson plan	13
Did not connect algorithm to equal distribution	
Conceptual lesson plan	4
Procedural lesson plan	14

Table 2 categorizes the teacher candidates according to their ability to connect the algorithm to equal distribution and the type of lesson plan submitted. As the table indicates, not all teacher candidates who were able to connect the algorithm to equal distribution in their journal writings utilized this understanding in their lesson plans. Eight of these 13 teacher candidates, however, submitted a lesson plan that had been either copied directly or altered slightly from an internet website. Conversely, among teacher candidates' who failed to tie the algorithm to equal distribution, not all submitted procedurally-based lesson plans.

Limitations

Before providing any conclusions from this research, limitations of this pilot study need to be noted. The first limitation of this pilot study is its unique setting and sample. Given that the university is located in the southeast region of the United States and has a current enrollment of primarily in-state students, the results of the pilot study cannot necessarily be generalized to other universities.

The second limitation lies in the teacher candidates' reliance on the state's curriculum resources in providing lesson plans. Had teacher candidates not had access to such a database, the results from the lesson plans may have been different. A total of 15 teacher candidates submitted lesson plans that were acquired from the state's internet-based lesson plan resource website under the fourth grade standard that read, "explores the concepts of mean and median." Teacher candidates may have possibly read the title and sub-

mitted the corresponding lesson without carefully reading the lesson for its conceptual development. Because teacher candidates failed to create original lesson plans, their true ability to develop conceptually-based lesson plans could not be observed.

The third limitation of this pilot study is a result of the journal prompt which did not force the teacher candidates to connect the algorithm to equal distribution. Instead, it instructed them to make sense of the procedure. Perhaps these instructions were not clear for the teacher candidates who had not already connected the algorithm to equal distribution. Teacher candidates would have benefited from a journal prompt that made it clear that they were to connect the two. In aiding these teacher candidates to make sense of the algorithm, they need to be involved in discussions with one another during the constructivist lessons. In these discussions, teacher candidates could share with one another how they have made sense of the mean and its relationship to the algorithm.

The fourth limitation of this pilot study is the lack of field data. At the time of the pilot study, these teacher candidates were not placed in actual classrooms in order to implement their lessons. Had this been the case, more insight would have been given as to their ability to transfer what they had learned from the conceptually-based lessons into their instruction.

Conclusions and Implications

The purpose of this pilot study was to examine the effect teaching in a constructivist manner has on teacher candidates' conceptual understandings of the mean and on their ability to transfer this knowledge into their instruction. Teachers who have a conceptual understanding of mathematics produce students who "exhibit conceptual understanding, have the ability to learn and to reason, and are able to achieve" (Knight, 2001, p. 21). Based on the findings of this pilot study, constructivist-based teaching can have a positive impact on teacher candidates' understandings of the mean. The teacher candidates in this pilot study were able to: extend their understanding of the concept of the mean as a result of participating in constructivist-based lessons; move from a very rigid, narrow interpretation of the mean and its associated algorithm to a more flexible and conceptual interpretation of the algorithm; and associate the algorithm with conceptual definitions such as the equal distribution interpretation.

While increasing teacher candidates' personal content knowledge is important, teacher educators must also be concerned with their ability to transfer this knowledge into instructional practices. As a part of this pilot study, teacher candidates were asked to create a lesson designed to develop conceptual understanding of the mean at the fourth grade level. Sixty percent of the teacher candidates submitted procedurally-based lessons despite having participated in three constructivist-based lessons. This unanticipated result serves as evidence that taking an active role in constructivist learning is not always sufficient for producing teachers who are able to transfer this newly acquired learning into their own instructional practices. This reality is further highlighted by the fact that only half of the teacher candidates who were able to tie the algorithm to a conceptual definition of the mean such as equal distribution submitted a conceptually-based lesson. This finding suggests that in order for teacher candidates to develop a conceptual understanding of the mean in others, they must be required to do more than take an active role in constructivist learning. Teacher candidates need to reflect upon how the constructivist-based lessons enabled them to gain a better understanding of the mean and how such lessons would be beneficial for the children they will teach. They should be engaged in discussions concerning what elements of the lessons could be used with elementary-aged students. To make a more powerful impact on instructional practices, lesson study could be a part of the class as well, enabling teacher candidates to examine lessons on the mean similar to those found on the internet, and contrast these with the lessons they have just experienced.

The goal of all mathematics teacher educators is to produce qualified mathematics teachers. All would agree that the first step to being an effective mathematics teacher is having strong content knowledge which includes both procedural and conceptual understandings. The pilot study reported here has shown that this is clearly possible through constructivist-based teaching. However, if teachers are not able to transfer their own knowledge into their instructional practices, the increase in conceptual understanding has not fully served its purpose. As reported here, participation in a constructivist-based classroom is not necessarily enough to influence instructional practices of all teacher candidates. Future work with teacher candidates should include providing opportunities as part of mathematics courses for them to assess the pedagogical merit of this approach to teaching that they have experienced as learners. Then teacher educators can address how teacher candidates can incorporate these practices into their own teaching.

The findings of this pilot study verify that further research is needed in developing a comprehensive understanding of the connections between the teacher candidates' instructional experiences and the development of their mathematical knowledge as it applies to their instruction. Interaction effects may exist between the instructors of mathematics and mathematics education courses and the conceptual development of teacher candidates. These interactions should be investigated in follow-up studies. In addition, future studies should employ the use of a control group thus enabling statistical comparisons to be made and thereby providing reliable data that supports the positive impact of constructivist-based teaching on teacher candidates' instructional practices. Studies of this nature should involve teacher candidates from multiple settings so that the results will be generalizeable.

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