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Research article

Neural networking study of worms in a wireless sensor model in the sense of fractal fractional

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Abstract: We are concerned with the analysis of the neural networks of worms in wireless sensor networks (WSN). The concerned process is considered in the form of a mathematical system in the context of fractal fractional differential operators. In addition, the Banach contraction technique is utilized to achieve the existence and unique outcomes of the given model. Further, the stability of the proposed model is analyzed through functional analysis and the Ulam-Hyers (UH) stability technique. In the last, a numerical scheme is established to check the dynamical behavior of the fractional fractal order WSN model.

Keywords: fractal-fractional operator; neural networking; Mittag-Leffler kernel; Ulam-Hyers stability; Banach contraction; numerical analysis
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1. Introduction

Computer systems and computer networks are the primary engineering devices. Being utilized in the operation, planning, control, and design of all sizes of buildings, businesses, machinery, transportation, and life-supporting technologies. As a result, computer viruses have become one of the most significant causes of uncertainty, lowering the reliability of critical processes. A computer worm is self-replicating software that can spread itself without the assistance of other programs. Worms replicate themselves by exploiting security flaws or policy flaws in the target program or operating system, such as the instinctive automatic receiving and transmitting characteristics common to many gadgets. It will scan and infect the other computer using this device as a host. It has a high rate of self-replication. As a result, the virus' continual multiplication and infection of other software packages can be dangerous to computer users [1,2]. In a short period of time, a worm may control and infect a large number of devices. Furthermore, new types of viruses are produced on a regular basis. Because of its numerous uses in earthquake measures in battlefield, military, and civic arenas, WSN has attracted a lot of interest. Due to a cyber attack by a worm, the computer and WSNs' dependability and unification are at risk. Computer worms are a sort of computer virus, however, there are various characteristics that distinguish them from conventional viruses. The fact that viruses actually spread through human action such as running software, uploading data, installing programs, and opening a file [3,4]. Computer worms, on the other hand, can replicate themselves without human intervention. Internet technology has evolved over the years, in step with technological advancement and demand, to provide a wide range of capabilities and services. The development of Internet technology has posed serious issues, such as the need for a competent cyber defense system to protect the valuable information kept on the system as well as information in transit.

Mathematical expressions are powerful tools that are used in modeling various procedures in the computer sciences to explain the dynamical behavior of complex processes. With the help of mathematical models, researchers can predict the future of a phenomenon. For instance, with the help of mathematical models of infectious or viral diseases, from their simulations, we get plenty of information about the transmission dynamics, their eradication, or their further spreading in society. In the field of computer science, a mathematical model is a tool that forms relationships between variables, constants, functions, and parameters. In addition, the aforesaid tools are increasingly used to solve problems in military operations or business, which makes up a large part of the field of operations research. For the said discussion we refer to [5, 6]. Many natural systems in physics, chemistry, astronomy, biology, manufacturing, and climatology, as well as human systems in psychology, economics, health care, engineering, and social science, have benefited from mathematical modeling. Mathematical models come in a variety of shapes and sizes differential equations, dynamical systems, game theoretic, and statistical models are just a few examples.

Here, the point to be noted is that the model is an abstract structure that has many kinds, including dynamical systems, game theoretical models, statistical and stochastic type models, bioethical models operational research models, etc. All the mentioned models have important applications in computational and information sciences. The idea of the term model was used a few centuries ago. In this regard, Bernoulli [7] created a mathematical system in the case of infectious diseases, and the influence of smallpox immunizations as well as life tables was examined. Following that, a large number of models were systemized for different complex phenomena. It is stated that the first attempt to develop models for the transmission of computer viruses based on their epidemiological equivalents has been made [8]. Using the Markov chains to depict the local patterns of infection, action in a particular node like susceptible-infected-recovered models was introduced to attempt the investigation of long-term patterns of virus transmission (see detail [9]).

The theory of differentiation and integration of arbitrary orders, often known as fractional calculus, is an extension of the classical approaches where orders of differentiating operators can take arbitrary values. Fractional calculus has recently been popular in a variety of fields due to its realistic modeling of certain aspects, such as memory effects, which can be performed using fractional calculus, whereas traditional models with integer orders are unable to leverage this attribute. The development of definitions and fractional derivatives has increased in recent years. For a variety of causes, these acts were carried out. On the one hand, mathematicians wish to extend existing definitions to include a wider variety of kernel functions by generalizing many present definitions beyond basic power functions. A novel type of nonlinear differential and integral equations may be created with these efficient operators, and current chaotic models can be expanded, as seen in [10–14]. However, in order to test the efficacy of such fractal-fractional operators in simulating chaotic attractors, the models must be solved numerically because their precise solutions are difficult to get using present analytical approaches. A few numerical approaches have been employed to discretize such models so far, with some numerical findings for chaotic attractors. Atangana [15] based on combination of power law, modified exponential decay law and Mittag-Leffler law, developed novel differential operators with fractal derivatives. Akgul et al. [16] studied a computer virus model for the existence of solutions, steady state, equilibrium point and stability solutions in the context of the Atangana-Baleanu operator. Abdel-Gawad et al. [17] present a study about the model of antivirus in computer network and endemic strategy, transformation to non-atounmus ODEs from fractional-order system and numerical solutions for the model in the sense of Caputo and Caputo-Fabrizio operators. Singh et al. [18] analyzed a fractional order computer virus epidemiological model for existence results and numerical analysis. Ozdemir et al. [19] provided a qualitative study for the fractional order computer viruses spreading for the existence and uniqueness of results and numerical analysis. Mishra and Keshri [20] present stability, reproduction number, and equilibria for the wireless sensor networks model. Kumar and Singh [21] studied the existence and uniqueness of a fractional epidemiological model for computer viruses in the context of the Mittag Leffler kernel by the fixed point theory and obtained numerical solutions. Since the irregular and complex geometry in the dynamics of the aforesaid model cannot be explained with usual fractional derivatives, we utilize the concept of fractal theory to study the irregular, and complex geometry pattern of the considered networking model. Therefore, we replaced the integer order derivative with the fractal fractional derivative in the context of the Mittage-Leffler kernel. Further, existence results, stability, and numerical analysis for the fractal fractional order WSN model are studied.

The manuscript is structured as follows. In section 2, basic definitions of fractal fractional derivatives are given. In Section 3, the Wireless Sensor Network e-Epidemic Model is addressed. In section 4, the results are given. In section 5, a numerical scheme for the system is given. In section 6, graphical representations are given. In section 7, a brief conclusion is given.

2. Fundamentals concepts

This section is enriched with some definitions and results recollected from [15, 22–25].

Definition 2.1. *[22] The fractal fractional derivative in the sense of Reimann-Liouville for* $\mathcal{Y} \in C(a, b)$

associated with power law is defined as

$$
\begin{aligned}
&{}^{ffp}_{\ a}\mathbf{D}_t^{\epsilon,\sigma}\mathcal{Y}(t) = \frac{1}{\Gamma[n-\epsilon]}\frac{d}{dt^{\sigma}}\int_a^t \mathcal{Y}(\mathrm{v})(t-\mathrm{v})^{n-\epsilon-1}d\mathrm{v},\\
&\textit{where},\ \ n-1<\epsilon\leq n,\ 0<n-1<\sigma\leq n,\\
&\frac{d\mathcal{Y}(\mathrm{v})}{d\mathrm{v}^{\sigma}} = \lim_{t-\mathrm{v}}\frac{\mathcal{Y}(t)-\mathcal{Y}(\mathrm{v})}{t^{\sigma}-\mathrm{v}^{\sigma}},
\end{aligned}
\tag{2.1}
$$

generalized form is

$$
\begin{aligned}
&{}^{ffp}_{\ a}\mathbf{D}_t^{\epsilon,\sigma}\mathcal{Y}(t) = \frac{1}{\Gamma[n-\epsilon]}\frac{d^{\theta}}{dt^{\sigma}}\int_a^t \mathcal{Y}(\mathrm{v})(t-\mathrm{v})^{n-\epsilon-1}d\mathrm{v},\\
&\textit{where}\\
&\frac{d^{\theta}\mathcal{Y}(\mathrm{v})}{d\mathrm{v}^{\sigma}} = \lim_{t-\mathrm{v}}\frac{\mathcal{Y}^{\theta}(t)-\mathcal{Y}^{\theta}(\mathrm{v})}{t^{\sigma}-\mathrm{v}^{\sigma}},\\
&\textit{where},\ \ n-1<\epsilon\leq n,\ 0<n-1<\theta,\sigma\leq n.
\end{aligned}
\tag{2.2}
$$

Definition 2.2. *[22] If $\mathcal{Y}:(a,b)\rightarrow R$, is a continuous function with order σ, then the fractal fractional derivative ϵ in the sense Reiamman-Liouville associated with the exponential kernel is defined as*

$$
\begin{aligned}
&{}^{ffe}_{\ a}\mathbf{D}_t^{\epsilon,\sigma}\mathcal{Y}(t) = \frac{\mathscr{W}(\epsilon)}{[1-\epsilon]}\frac{d}{dt^{\sigma}}\int_a^t \mathcal{Y}(\mathrm{v})\exp\Big[-\frac{\epsilon}{1-\epsilon}(t-\mathrm{v})\Big]d\mathrm{v},\\
&\textit{where},\quad 0<\epsilon,\sigma\leq n,
\end{aligned}
\tag{2.3}
$$

generalized form is

$$
{}^{ffe}_{\ a}\mathbf{D}_t^{\epsilon,\sigma,\theta}\mathcal{Y}(t) = \frac{\mathscr{W}(\epsilon)}{[1-\epsilon]}\frac{d^{\theta}}{dt^{\sigma}}\int_a^t \mathcal{Y}(\mathrm{v})\exp\Big[-\frac{\epsilon}{1-\epsilon}(t-\mathrm{v})\Big]d\mathrm{v},\quad 0<\epsilon,\sigma,\theta\leq 1,
\tag{2.4}
$$

such that $\mathscr{W}(\epsilon)$ is normalization function, such that $\mathscr{W}(0)=1=\mathscr{W}(1)$.

Definition 2.3. *[22] The fractal fractional derivative ϵ in the sense of Reimann-Liouville associated with Mittag-Leffler kernel for a continuous function $\mathcal{Y}:(a,b)\rightarrow R$, with order σ is described as*

$$
\begin{aligned}
&{}^{ffm}_{\ a}\mathbf{D}_t^{\epsilon,\sigma}\mathcal{Y}(t) = \frac{\mathscr{W}(\epsilon)}{[1-\epsilon]}\int_a^t \frac{d\mathcal{Y}(\mathrm{v})}{dt^{\sigma}}E_{\epsilon}\Big[-\frac{\epsilon}{1-\epsilon}(t-\mathrm{v})^{\epsilon}\Big]d\mathrm{v},\\
&\textit{where},\quad 0<\epsilon,\sigma\leq 1,\ \ \mathscr{W}(\epsilon)=1-\epsilon+\epsilon/\Gamma(\epsilon),
\end{aligned}
\tag{2.5}
$$

generalized form is

$$
{}^{ffm}_{\ a}\mathbf{D}_t^{\epsilon,\sigma,\theta}\mathcal{Y}(t) = \frac{\mathscr{W}(\epsilon)}{[1-\epsilon]}\int_a^t \frac{d^{\theta}\mathcal{Y}(t)}{d\mathrm{v}^{\sigma}}E_{\epsilon}\Big[-\frac{\epsilon}{1-\epsilon}(t-\mathrm{v})^{\epsilon}\Big]d\mathrm{v},\quad 0<\theta\leq 1.
\tag{2.6}
$$

Definition 2.4. *[22] Let $0<\epsilon\leq 1$, be fractal and fractional order and $\mathcal{Y}\in L[a,b]$, then integral with power law kernel is given by*

$$
{}^{ffp}_{\ 0}\mathbf{J}_t^{\epsilon}\mathcal{Y}(t) = \frac{1}{\Gamma(\epsilon)}\int_0^t (t-\mathrm{v})^{\epsilon-1}\mathrm{v}^{1-\sigma}\mathcal{Y}(\mathrm{v})d\mathrm{v}.
\tag{2.7}
$$

Definition 2.5. *[22] Let $0 < \epsilon, \sigma \leq 1$, be fractal and fractional order and $\mathcal{Y} \in L[a, b]$, then integral with exponential kernel is given by*

$$ {}_{0}^{ffe}\mathbf{J}_{t}^{\epsilon,\sigma}\mathcal{Y}(t) = \frac{\sigma(1-\epsilon)t^{\sigma-1}\mathcal{Y}(t)}{\mathscr{W}(\epsilon)} + \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)}\int_{0}^{t} \mathrm{v}^{\epsilon-1}\mathcal{Y}(\mathrm{v})d\mathrm{v}. \tag{2.8}$$

Definition 2.6. *[22] Let $0 < \epsilon, \sigma \leq 1$, be fractal and fractional order and $\mathcal{Y} \in L[a, b]$, then integral with Mittag-Leffler kernel is given by*

$$ {}_{0}^{ffm}\mathbf{J}_{t}^{\epsilon,\sigma}\mathcal{Y}(t) = \frac{\sigma(1-\epsilon)t^{\sigma-1}\mathcal{Y}(t)}{\mathscr{W}(\epsilon)} + \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)}\int_{0}^{t} (t-\mathrm{v})^{\epsilon-1}\mathrm{v}^{\epsilon-1}\mathcal{Y}(\mathrm{v})d\mathrm{v}. \tag{2.9}$$

3. Wireless sensor network e-epidemic model

Here, we will study the dynamics of integer order WSN model and for more explanation one can see [16–21], which is formulated for susceptible infected recovered (SIR) system. The variables of the model stands for $\mathbf{S}(t)$, rate of suspectable at time t, $\mathbf{E}(t)$, rat of exposed at time t, $\mathbf{I}(t)$, rat of infectious at time t, $\mathbf{R}(t)$, rat of recovered at time t and $\mathbf{R}(t)$, rate of recovered at time t. The following Wireless Sensor Network e-Epidemic Model

$$\begin{cases} \dot{\mathbf{S}}(t) = \mathcal{A} - \beta\mathbf{S}(t)\mathbf{I}(t) - \mu\mathbf{S}(t) - p\mathbf{S}(t) + \delta\mathbf{R} + \eta\mathbf{V}(t), \\ \dot{\mathbf{E}}(t) = \beta\mathbf{S}(t)\mathbf{I}(t) - (\mu+\alpha)\mathbf{E}(t), \\ \dot{\mathbf{I}}(t) = \alpha\mathbf{E}(t) - (\mu+\lambda+\gamma)\mathbf{I}(t), \\ \dot{\mathbf{R}}(t) = \gamma\mathbf{I}(t) - (\mu+\delta)\mathbf{R}(t), \\ \dot{\mathbf{V}}(t) = p\mathbf{S}(t) - (\mu+\eta)\mathbf{V}(t), \end{cases} \tag{3.1}$$

subject to initial conditions

$$\mathbf{S}(0) = \mathbf{S}_0, \quad \mathbf{E}(0) = \mathbf{E}_0, \quad \mathbf{I}(0) = \mathbf{I}_0, \quad \mathbf{R}(0) = \mathbf{R}_0, \quad \mathbf{V}(0) = \mathbf{V}_0.$$

Where, μ is the crashing rate of the sensor nodes due to software/hardware problem, β is the infectivity contact rate, $\mathbf{A}$ is the inclusion of new sensor nodes to the population, γ is the rate of recovery, α is the rate of transmission from $\mathbf{E}$-class to $\mathbf{I}$-class, δ is the rate of transfer from $\mathbf{R}$-class to $\mathbf{S}$-class, p is the vaccinating rate coefficient for the susceptible nodes. η is the rate of transmission from $\mathbf{V}$-class to $\mathbf{S}$-class, λ is denotes the damaging rate due to attack of worms.

The prominent of this article to establish existence of solutions and numerical outcomes of fractal fractional order WSN model. In the next section, we are going to provide existence of results for the model (4.1) and next step to uniqueness of results is our target for the proposed model. For these, to define a Banach space. Let $\mathscr{K} = \mathcal{I} \times \mathcal{R}^5 \to \mathcal{R}$, be the Banach space such that $\mathscr{I} = [0, \tau]$, for $0 < t < \tau < \infty$, under the norm $\|(\mathbf{S}, \mathbf{E}, \mathbf{I}, \mathbf{R}, \mathbf{V})\| = \max_{t\in\mathscr{I}}\{|\mathbf{S}(t)| + |\mathbf{E}(t)| + |\mathbf{I}(t)| + |\mathbf{R}(t)| + |\mathbf{V}(t)|\}$.

4. Qualitative theory

We re-write the model (3.1) under the fractal fractional derivative as

$$ {}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{S}(t)) \quad = \quad \mathcal{A} - \beta\mathbf{S}(t)\mathbf{I}(t) - \mu\mathbf{S}(t) - p\mathbf{S}(t) + \delta\mathbf{R} + \eta\mathbf{V}(t),$$

$$
\begin{aligned}
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{E}(t)) &= \beta\mathbf{S}(t)\mathbf{I}(t) - (\mu+\alpha)\mathbf{E}(t),\\
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{I}(t)) &= = \alpha\mathbf{E}(t) - (\mu+\lambda+\gamma)\mathbf{I}(t),\\
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{R}(t)) &= \gamma\mathbf{I}(t) - (\mu+\delta)\mathbf{R}(t),\\
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{V}(t)) &= p\mathbf{S}(t) - (\mu+\eta)\mathbf{V}(t),
\end{aligned} \tag{4.1}
$$

subject to initial conditions

$$
\mathbf{S}(0) = \mathbf{S}_0, \quad \mathbf{E}(0) = \mathbf{E}_0, \quad \mathbf{I}(0) = \mathbf{I}_0, \quad \mathbf{R}(0) = \mathbf{R}_0, \quad \mathbf{V}(0) = \mathbf{V}_0. \tag{4.2}
$$

In addition, we can write right sides of

$$
\begin{aligned}
&\Theta_1(z,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V}) = \mathcal{A} - \beta\mathbf{S}(t)\mathbf{I}(t) - \mu\mathbf{S}(t) - p\mathbf{S}(t) + \delta\mathbf{R} + \eta\mathbf{V}(t),\\
&\Theta_2(z,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V}) = \beta\mathbf{S}(t)\mathbf{I}(t) - (\mu+\alpha)\mathbf{E}(t),\\
&\Theta_3(z,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V}) = \alpha\mathbf{E}(t) - (\mu+\lambda+\gamma)\mathbf{I}(t),\\
&\Theta_4(z,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V}) = \gamma\mathbf{I}(t) - (\mu+\delta)\mathbf{R}(t),\\
&\Theta_5(z,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V}) = p\mathbf{S}(t) - (\mu+\eta)\mathbf{V}(t).
\end{aligned} \tag{4.3}
$$

Rearranging the system (4.1) as

$$
{}^{\mathcal{ABR}}\mathbb{D}_{0,t}(\Phi(t)) = \Psi(t,\Phi(t)), \tag{4.4}
$$

such that Φ and Ψ represent

$$
\Phi(t) = \begin{cases} \mathbf{S}(t)\\ \mathbf{E}(t)\\ \mathbf{I}(t)\\ \mathbf{R}(t)\\ \mathbf{V}(t) \end{cases} \quad \mathit{and} \quad \Psi\{t,\Phi(t)\} = \begin{cases} \Theta_1(t,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V})\\ \Theta_2(t,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V})\\ \Theta_3(t,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V})\\ \Theta_4(t,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V})\\ \Theta_5(t,\mathbf{S},\mathbf{E},\mathbf{I},\mathbf{R},\mathbf{V}). \end{cases} \tag{4.5}
$$

In view of Definition 2.3, Eq (4.4) yields

$$
\frac{\mathscr{W}(\epsilon)}{1-\epsilon}\frac{d}{dt}\int_0^t \Psi(\omega,\Phi(\omega))E_\epsilon\Big[-\frac{\epsilon}{1-\epsilon}(t-\omega)^\epsilon\Big]d\omega = \sigma t^{\sigma-1}\Psi(t,\Phi(t)). \tag{4.6}
$$

In view of Definition 2.6, Eq (4.6) yields

$$
\Psi(t) = \Psi(0) + \frac{(1-\epsilon)\sigma t^{\sigma-1}}{\mathscr{W}(\epsilon)}\Psi(t,\Phi(t)) + \frac{\sigma\epsilon}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\Psi(\omega,\Phi(\omega))\omega^{\sigma-1}d\omega. \tag{4.7}
$$

Consider here

$$
\mathscr{B}_a^q = \mathscr{H}_n(t_n)\times\overline{\mathscr{C}_0(\Phi_0)}, \tag{4.8}
$$

such that $\mathscr{H}_n = [t_{n-a},\ \ t_{n+a}]$, and $\overline{\mathscr{C}_b(\Phi_0)} = [t_0 - b, t_0 + b]$. Let $\sup_{t\in\mathscr{B}_a^q}||\Psi|| = \mathcal{P}$, and consider norm as

$$
||\Omega||_\infty = \sup_{t\in\mathscr{B}_a^q}|\Omega(t)|. \tag{4.9}
$$

Let $\mathcal{G}$ be space of continuous functions from $\mathscr{H}_n$ to $\mathscr{C}_b(t_n)$, then define an operator $\mho : \mathcal{G} \to \mathcal{G}$, such that

$$\mho\Phi(t) = \Phi_0 + \frac{(1-\epsilon)\sigma t^{\sigma-1}}{\mathscr{W}(\epsilon)}\Psi(t,\Phi(t)) + \frac{\sigma\epsilon}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\Psi(\omega,\Phi(\omega))\omega^{\sigma-1}d\omega. \tag{4.10}$$

We consider

$$\begin{aligned} \|\mho\Phi(t) - \Phi_0\| &\leq \frac{1-\epsilon}{\mathscr{W}(\epsilon)}\sigma t^{\sigma-1}\|\Psi(t,\Phi(t))\| \\ &+ \frac{\sigma\epsilon}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\|\Psi(\omega,\Phi(\omega))\|\omega^{\sigma-1}d\omega, \\ &\leq \frac{1-\epsilon}{\mathscr{W}(\epsilon)}\sigma t^{\sigma-1}\mathcal{P} + \frac{\sigma\epsilon}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\mathcal{P}\int_0^t (t-\omega)^{\epsilon-1}\omega^{\sigma-1}d\omega. \end{aligned} \tag{4.11}$$

If we replace ω by $= tv$ in Eq (4.11), one has

$$\|\mho\Phi(t) - \Phi_0\| \leq \frac{1-\epsilon}{\mathscr{W}(\epsilon)}\sigma t^{\sigma-1}\mathcal{P} + \frac{\sigma\epsilon\mathcal{P}}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\omega^{\sigma+\epsilon-u}Q(\sigma,\epsilon). \tag{4.12}$$

Equation (4.12) yields

$$\|\mho\Phi(t) - \Phi_0\| \leq q \quad \to \mathcal{P} < \frac{qQ(\sigma,\epsilon)\mathscr{W}(\epsilon)\Gamma(\epsilon)}{(1-\epsilon)\Gamma(\alpha)\sigma t^{\sigma-1} + \epsilon\sigma t^{\sigma+\epsilon-u}}.$$

For Φ_1, Φ_2, one has

$$\begin{aligned} \|\mho\Phi_1 - \mho\Phi_2\| \leq \quad & \frac{1-\epsilon}{\mathscr{W}(\epsilon)}\sigma t^{\sigma-1}\|\Psi(t,\Phi_1(t)) - \Psi(t,\Phi_2(t))\| \\ & + \frac{\sigma\epsilon}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\|\Psi(t,\Phi_1(t)) - \Psi(t,\Phi_2(t))\|\omega^{\sigma-1}d\omega, \end{aligned}$$

as $\mho$ is a contraction, hence

$$\begin{aligned} \|\mho\Phi_1 - \mho\Phi_2\| \leq \quad & \frac{(1-\epsilon)\sigma t^{\sigma-1}}{\mathscr{W}(\epsilon)}\mathcal{X}\|\Phi_1 - \Phi_2\|_\infty \\ & + \frac{\sigma\epsilon\mathcal{X}}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\|\Phi_1 - \Phi_2\|_\infty\omega^{\sigma-1}d\omega. \\ \leq \quad & \frac{1-\epsilon}{\mathscr{W}(\epsilon)}\sigma t^{\sigma-1}\mathcal{X}\|\Phi_1 - \Phi_2\|_\infty \\ & + \frac{\sigma\epsilon\mathcal{X}}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\|\Phi_1 - \Phi_2\|_\infty t^{\epsilon+\sigma-3}Q(\sigma,\epsilon). \end{aligned}$$

$$\|\mho\Phi_1 - \mho\Phi_2\| \leq \quad \Big[\frac{1-\epsilon}{\mathscr{W}(\epsilon)}\sigma t^{\sigma-1}\mathcal{X} + \frac{\sigma\epsilon\mathcal{X}}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}t^{\epsilon+\sigma-3}Q(z,\epsilon)\Big]\|\Phi_1 - \Phi_2\|_\infty. \tag{4.13}$$

Thus, $\mho$ is a contraction if

$$\|\mho\Phi_1 - \mho\Phi_2\|_\infty \leq \|\Phi_1 - \Phi_2\|, \tag{4.14}$$

holds which implies that

$$\mathcal{X} < \frac{1}{\frac{1-\epsilon}{\mathscr{W}(\epsilon)}\sigma t^{\sigma-1} + \frac{\sigma\epsilon}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}t^{\epsilon+\sigma-3}Q(\sigma,\epsilon)}, \tag{4.15}$$

and

$$\mathcal{P} < \frac{1}{\frac{1-\epsilon}{\mathscr{W}(\epsilon)}\sigma t^{\sigma-1} + \frac{\sigma\epsilon}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}t^{\epsilon+\sigma-3}Q(\sigma,\epsilon)}. \tag{4.16}$$

Thus the model (4.1) has a unique solution.

5. Stability analysis

This section is devoted to derive the UH stability for the proposed model (4.1), one can see [26–28].

Definition 5.1. *The fractal fractional WSN model* (4.1) *is UH stable, if for every* $(\mathbf{S}^*, \mathbf{E}^*, \mathbf{I}^*, \mathbf{R}^*, \mathbf{V}^*)$ *there exists* $\hbar_j > 0,\ j \in \mathscr{H}_1^5$, *one has a number* $\psi_j > 0,\ \ j \in \mathscr{H}_1^5$, *such that*

$$\begin{cases} |{}^{ffm}\mathbb{D}_t^{\epsilon,\sigma}\mathbf{S}^*(t) - \Theta_1(t,\mathbf{S}^*)| \le \psi_1, \\ |{}^{ffm}\mathbb{D}_t^{\epsilon,\sigma}\mathbf{E}^*(t) - \Theta_2(t,\mathbf{E}^*)| \le \psi_2, \\ |{}^{ffm}\mathbb{D}_t^{\epsilon,\sigma}\mathbf{I}^*(t) - \Theta_3(t,\mathbf{I}^*)| \le \psi_3, \\ |{}^{ffm}\mathbb{D}_t^{\epsilon,\sigma}\mathbf{R}^*(t) - \Theta_4(t,\mathbf{R}^*)| \le \psi_4, \\ |{}^{ffm}\mathbb{D}_t^{\epsilon,\sigma}\mathbf{V}^*(t) - \Theta_5(t,\mathbf{V}^*)| \le \psi_5, \end{cases} \tag{5.1}$$

such that $(\mathbf{S}, \mathbf{E}, \mathbf{I}, \mathbf{R}, \mathbf{V})$, *for the fractal fractional WSN model* (4.1) *which implies that*

$$\begin{cases} \|\mathbf{S} - \mathbf{S}^*\| \le \hbar_1\psi_1, \\ \|\mathbf{E} - \mathbf{E}^*\| \le \hbar_2\psi_2, \\ \|\mathbf{I} - \mathbf{I}^*\| \le \hbar_3\psi_3, \\ \|\mathbf{R} - \mathbf{R}^*\| \le \hbar_4\psi_4, \\ \|\mathbf{V} - \mathbf{V}^*\| \le \hbar_5\psi_5, \end{cases} \tag{5.2}$$

where $\Theta_j \in \mathscr{H}_1^5$ *is mentioned in Eq* (4.3).

Remark 5.2. *For an mapping* Π *not depend on the solution, one has*

$$\begin{aligned} &(1) \quad |\Pi_1(t)| < \psi_1, \\ &(2) \quad {}^{ffm}\mathbb{D}_t^{\epsilon,\sigma}\mathbf{S}^*(t) = \Theta_1(t,\mathbf{S}^*)| \le \psi_1 + \Pi_1(t). \end{aligned} \tag{5.3}$$

Theorem 5.3. *The model* (4.1) *is UH stable if the condition*

$$\lambda_i[\frac{\epsilon\sigma\Gamma(\sigma) + \sigma(1-\epsilon)\Gamma(\epsilon+\sigma)}{\mathscr{W}(\epsilon)\Gamma(\epsilon+\sigma)}] \le 1, i \in \mathscr{H}_1^5, i = 1,2,3,4,5, \tag{5.4}$$

holds.

Proof. For any $\mathbf{S}^*$ and $\psi > 0$, one has

$$|^{ffm}\mathbb{D}_t^{\epsilon,\sigma}\mathbf{S}^*(t) - \Theta_1(t,\mathbf{S}^*)| \leq \psi_1. \tag{5.5}$$

By the above Remark 5.2, then Eq (5) takes the form

$$^{ffm}\mathbb{D}_t^{\epsilon,\sigma}\mathbf{S}^*(t) = \Theta_1(t,\mathbf{S}^*) \leq \psi_1 + \Pi_1(t). \tag{5.6}$$

From Eq (5.6), the solution is obtained as

$$\begin{aligned} \mathbf{S}^*(t) = \mathbf{S}^0 &+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\omega^{\sigma-1}\Theta_1(\omega,\mathbf{S}^*(\omega))d\omega + \frac{(1-\epsilon)\sigma}{\mathscr{W}(\epsilon)}t^{\sigma-1}\Theta_1(\omega,\mathbf{S}^*(t)) \\ &+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\omega^{\sigma-1}\Pi_1(\omega)d\omega + \frac{(1-\epsilon)\sigma}{\mathscr{W}(\epsilon)}t^{\sigma-1}\Pi_1(t). \end{aligned} \tag{5.7}$$

Let $\mathbf{S}$ be the unique solution, then one has

$$\begin{aligned} |\mathbf{S}^*(t) - \mathbf{S}(t)| &\leq \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\omega^{\sigma-1}|\Theta_1(\omega,\mathbf{S}^*(\omega)) - \Theta_1(\omega,\mathbf{S}(\omega))|d\omega \\ &+ \frac{(1-\epsilon)\sigma}{\mathscr{W}(\epsilon)}t^{\sigma-1}|\Theta_1(\omega,\mathbf{S}^*(t)) - Theta_1(\omega,\mathbf{S}(t))| \qquad (5.8) \\ &+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\omega^{\sigma-1}|\Pi_1(\omega)|d\omega \\ &+ \frac{(1-\epsilon)\sigma}{\mathscr{W}(\epsilon)}t^{\sigma-1}|\Pi_1(t)|, \\ &\leq \lambda_i[\frac{\epsilon\sigma\Gamma(\sigma) + \sigma(1-\epsilon)\Gamma(\epsilon+\sigma)}{\mathscr{W}(\epsilon)\Gamma(\epsilon+\sigma)}]|\mathbf{S}^*(t) - \mathbf{S}(t)| \\ &+ [\frac{\epsilon\sigma\Gamma(\sigma) + \sigma(1-\epsilon)\Gamma(\epsilon+\sigma)}{\mathscr{W}(\epsilon)\Gamma(\epsilon+\sigma)}]\psi_1. \qquad (5.9) \end{aligned}$$

In addition

$$||\mathbf{S}^*(t) - \mathbf{S}(t)|| \leq \frac{[\frac{\epsilon\sigma\Gamma(\sigma)+\sigma(1-\epsilon)\Gamma(\epsilon+\sigma)}{\mathscr{W}(\epsilon)\Gamma(\epsilon+\sigma)}]\psi_1}{1 - [\frac{\epsilon\sigma\Gamma(\sigma)+\sigma(1-\epsilon)\Gamma(\epsilon+\sigma)}{\mathscr{W}(\epsilon)\Gamma(\epsilon+\sigma)}]\lambda_1}. \tag{5.10}$$

Let consider

$$\hbar_1 \leq \frac{[\frac{\epsilon\sigma\Gamma(\sigma)+\sigma(1-\epsilon)\Gamma(\epsilon+\sigma)}{\mathscr{W}(\epsilon)\Gamma(\epsilon+\sigma)}]}{1 - [\frac{\epsilon\sigma\Gamma(\sigma)+\sigma(1-\epsilon)\Gamma(\epsilon+\sigma)}{\mathscr{W}(\epsilon)\Gamma(\epsilon+\sigma)}]\lambda_1}, \tag{5.11}$$

which gives $||\mathbf{S}^* - \mathbf{S}|| \leq \lambda_1\psi_1$. In the same way

$$\begin{aligned} ||\mathbf{E}^* - \mathbf{E}|| \leq \lambda_2\psi_2, &\qquad ||\mathbf{I}^* - \mathbf{I}|| \leq \lambda_3\psi_3 \\ ||\mathbf{R}^* - \mathbf{R}|| \leq \lambda_4\psi_4, &\qquad ||\mathbf{V}^* - \mathbf{V}|| \leq \lambda_5\psi_5. \end{aligned}$$

Therefore, the Eq (4.1) satisfies all the conditions of UH stability. □

6. Scheme for numerical results

We consider the fractal fractional WSN model (4.1) in the context of fractal fractional Mittag-Leffler kernel with using $\mathscr{U} = (\mathbf{S}, \mathbf{E}, \mathbf{I}, \mathbf{R}, \mathbf{V})$ as

$$\begin{aligned}
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{S}(t)) &= \Theta_1(t, \mathscr{U}),\\
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{E}(t)) &= \Theta_2(t, \mathscr{U}),\\
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{I}(t)) &= \Theta_3(t, \mathscr{U}),\\
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{R}(t)) &= \Theta_4(t, \mathscr{U}),\\
{}^{ffm}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{V}(t)) &= \Theta_5(t, \mathscr{U}).
\end{aligned} \tag{6.1}$$

The solution of Eq (6.1) can be obtained as

$$\begin{aligned}
\mathbf{S}(t) &= \mathbf{S}(0) + \frac{\sigma t^{\sigma-1}(1-\epsilon)\Theta_1(t,\mathscr{U})}{\mathscr{W}(\epsilon)}\\
&+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\Theta_1(t,\mathscr{U})\omega^{\sigma-1}d\omega,\\
\mathbf{E}(t) &= \mathbf{E}(0) + \frac{\sigma t^{\sigma-1}(1-\epsilon)\Theta_2(\omega,\mathscr{U})}{\mathscr{W}(\epsilon)}\\
&+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\Theta_2(\omega,\mathscr{U})\omega^{\sigma-1}d\omega,\\
\mathbf{I}(t) &= \mathbf{I}(0) + \frac{\sigma t^{\sigma-1}(1-\epsilon)\Theta_3(t,\mathscr{U})}{\mathscr{W}(\epsilon)}\\
&+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\Theta_3(\omega,\mathscr{U})\omega^{\sigma-1}d\omega,\\
\mathbf{R}(t) &= \mathbf{R}(0) + \frac{\sigma t^{\sigma-1}(1-\epsilon)\Theta_4(t,\mathscr{U})}{\mathscr{W}(\epsilon)}\\
&+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\Theta_4(\omega,\mathscr{U})\omega^{\sigma-1}d\omega,\\
\mathbf{V}(t) &= \mathbf{V}(0) + \frac{\sigma t^{\sigma-1}(1-\epsilon)\Theta_5(t,\mathscr{U})}{\mathscr{W}(\epsilon)}\\
&+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^t (t-\omega)^{\epsilon-1}\Theta_5(\omega,\mathscr{U})\omega^{\sigma-1}d\omega.
\end{aligned} \tag{6.2}$$

Let set $t = t_{n+1}$ in Eq (6.2) yields

$$\begin{aligned}
\mathbf{S}^{n+1}(t) = \mathbf{S}^0 &+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_1(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^{t_{n+1}} (t_{n+1}-\omega)^{\epsilon-1}\Theta_1(t,\mathscr{U})\omega^{\sigma-1}d\omega,\\
\mathbf{E}^{n+1}(t) = \mathbf{E}^0 &+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_2(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}
\end{aligned} \tag{6.3}$$

$$
\begin{aligned}
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^{t_{n+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_2(\omega,\mathscr{U})\omega^{\sigma-1}d\omega,\\
\mathbf{I}^{n+1}(t) = \mathbf{I}^0 \quad &+ \quad \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_3(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^{t_{n+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_3(\omega,\mathscr{U})\omega^{\sigma-1}d\omega,\\
\mathbf{R}^{n+1}(t) = \mathbf{R}^0 \quad &+ \quad \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_4(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^{t_{n+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_4(\omega,\mathscr{U})\omega^{\sigma-1}d\omega,\\
\mathbf{V}^{n+1}(t) = \mathbf{V}^0 \quad &+ \quad \frac{\sigma t^{\sigma-1}(1-\epsilon)\Theta_5(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\int_0^{t_{n+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_5(\omega,\mathscr{U})\omega^{\sigma-1}d\omega.
\end{aligned}
$$

Approximate the integral in Eq (6.3), one has

$$
\begin{aligned}
\mathbf{S}^{n+1}(t) = \mathbf{S}^0 \quad &+ \quad \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_1(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\sum_{r=0}^{\ell}\int_{t_r}^{t_{r+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_1(t,\mathscr{U})\omega^{\sigma-1}d\omega, \qquad (6.4)\\
\mathbf{E}^{n+1}(t) = \mathbf{E}^0 \quad &+ \quad \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_2(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\sum_{r=0}^{\ell}\int_{t_r}^{t_{r+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_2(t,\mathscr{U})\omega^{\sigma-1}d\omega, \qquad (6.5)\\
\mathbf{I}^{n+1}(t) = \mathbf{I}^0 \quad &+ \quad \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_3(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\sum_{r=0}^{\ell}\int_{t_r}^{t_{r+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_3(t,\mathscr{U})\omega^{\sigma-1}d\omega, \qquad (6.6)\\
\mathbf{R}^{n+1}(t) = \mathbf{R}^0 \quad &+ \quad \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_4(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\sum_{r=0}^{\ell}\int_{t_r}^{t_{r+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_4(t,\mathscr{U})\omega^{\sigma-1}d\omega, \qquad (6.7)\\
\mathbf{V}^{n+1}(t) = \mathbf{V}^0 \quad &+ \quad \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_5(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \quad \frac{\epsilon\sigma}{\mathscr{W}(\epsilon)\Gamma(\epsilon)}\sum_{r=0}^{\ell}\int_{t_r}^{t_{r+1}}(t_{n+1}-\omega)^{\epsilon-1}\Theta_5(t,\mathscr{U})\omega^{\sigma-1}d\omega.
\end{aligned}
$$

In view of Lagrangian interpolation for discretization, and using

$\Lambda_{\ell,\mathrm{r},\epsilon,p} = ((\ell+1-\mathrm{r})^{\epsilon}(\ell-\mathrm{r}+2+\epsilon)-(\ell-\mathrm{r})^{\epsilon}(p-\mathrm{r}+2+2\epsilon)),\ \Upsilon_{\ell,\mathrm{r},\epsilon,p} = ((\ell-\mathrm{r}+1)^{\epsilon+1}-(p-\mathrm{r})^{\epsilon}(p-\mathrm{r}+1+\epsilon)),$

one has

$$
\begin{aligned}
\mathbf{S}^{n+1}(t) = \mathbf{S}^0 &+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_1(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \frac{\sigma(\mathrm{h})^{\epsilon}}{\mathscr{W}(\epsilon)\Gamma(\epsilon+2)}\sum_{\mathrm{r}=0}^{\ell}\Big[t_{\mathrm{r}}^{\sigma-1}\Theta_1(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}})\Lambda_{\ell,\mathrm{r},\epsilon,p} - t_{\mathrm{r}-1}^{\sigma-1}\Theta_1(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}-1})\Upsilon_{\ell,\mathrm{r},\epsilon,p}\Big],\\
\mathbf{E}^{n+1}(t) = \mathbf{E}^0 &+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_2(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \frac{\sigma(\mathrm{h})^{\epsilon}}{\mathscr{W}(\epsilon)\Gamma(\epsilon+2)}\sum_{\mathrm{r}=0}^{\ell}\Big[t_{\mathrm{r}}^{\sigma-1}\Theta_2(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}})\Lambda_{\ell,\mathrm{r},\epsilon,p} - t_{\mathrm{r}-1}^{\sigma-1}\Theta_2(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}-1})\Upsilon_{\ell,\mathrm{r},\epsilon,p}\Big],\\
\mathbf{I}^{n+1}(t) = \mathbf{I}^0 &+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_3(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \frac{\sigma(\mathrm{h})^{\epsilon}}{\mathscr{W}(\epsilon)\Gamma(\epsilon+2)}\sum_{\mathrm{r}=0}^{\ell}\Big[t_{\mathrm{r}}^{\sigma-1}\Theta_3(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}})\Lambda_{\ell,\mathrm{r},\epsilon,p} - t_{\mathrm{r}-1}^{\sigma-1}\Theta_3(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}-1})\Upsilon_{\ell,\mathrm{r},\epsilon,p}\Big],\\
\mathbf{R}^{n+1}(t) = \mathbf{R}^0 &+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_4(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \frac{\sigma(\mathrm{h})^{\epsilon}}{\mathscr{W}(\epsilon)\Gamma(\epsilon+2)}\sum_{\mathrm{r}=0}^{\ell}\Big[t_{\mathrm{r}}^{\sigma-1}\Theta_4(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}})\Lambda_{\ell,\mathrm{r},\epsilon,p} - t_{\mathrm{r}-1}^{\sigma-1}\Theta_4(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}-1})\Upsilon_{\ell,\mathrm{r},\epsilon,p}\Big],\\
\mathbf{V}^{n+1}(t) = \mathbf{V}^0 &+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Theta_5(t_n,\mathscr{U}^n)}{\mathscr{W}(\epsilon)}\\
&+ \frac{\sigma(\mathrm{h})^{\epsilon}}{\mathscr{W}(\epsilon)\Gamma(\epsilon+2)}\sum_{\mathrm{r}=0}^{\ell}\Big[t_{\mathrm{r}}^{\sigma-1}\Theta_5(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}})\Lambda_{\ell,\mathrm{r},\epsilon,p} - t_{\mathrm{r}-1}^{\sigma-1}\Theta_5(t_{\mathrm{r}},\mathscr{U}^{\mathrm{r}-1})\Upsilon_{\ell,\mathrm{r},\epsilon,p}\Big].
\end{aligned}
$$

7. Neural network analysis of WSN model

The approximate solutions of the fractal fractional order WSN model have been obtained by constructing a numerical technique. Various scenarios for the fractal fractional order WSN model have been addressed. We utilized the above iterative technique for the numerical outcomes to graphically represent the fractal fractional WSN model. For dynamical observation of the model, we select appropriate constant values for the model's parameters.

Figure 1 Shows a dynamical representation of the constant value of σ, and ϵ for the WSN model. Let $\mathcal{A} = 0.33,\ \beta = 0.1,\ \mu = 0.003,\ p = 0.3,\ \eta = 0.06,\ \alpha = 0.25,\ \lambda = 0.07,\ \gamma = 0.4$ and let $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$ be initial values.

Figure 2 Shows a dynamical representation of susceptible nodes for various σ and ϵ. Assume $\mathcal{A} = 0.33,\ \beta = 0.1,\ \mu = 0.003,\ p = 0.3,\ \eta = 0.06,\ \alpha = 0.25,\ \lambda = 0.07,\ \gamma = 0.4$ and let

$(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0,)$
$\mathbf{V}_0) = (100, 3, 1, 0, 0)$ be initial values.

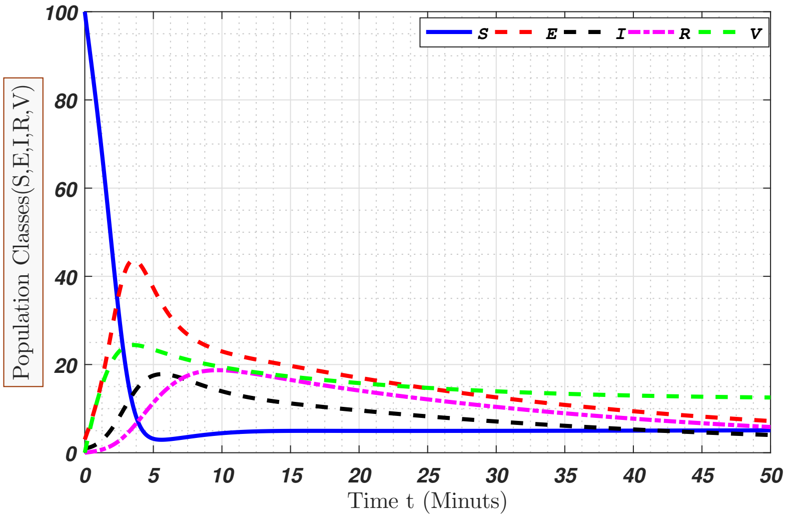

Figure 1. Numerical simulation of the system (3.1) for the classical derivative $\epsilon = \sigma = 1$.

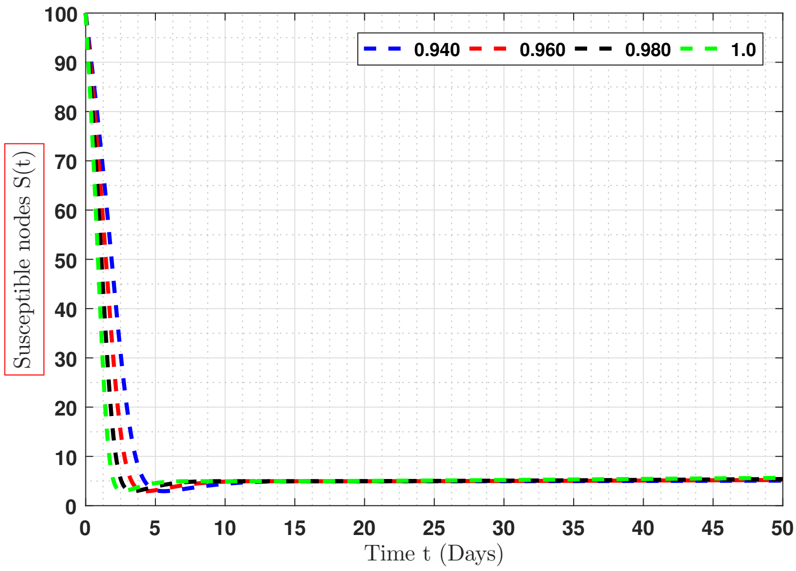

Figure 2. Numerical simulation of susceptible class **S** of (4.1) involving fractal-fractional order derivative with different values of $\epsilon = \sigma = 1, 0.98, 0.96, 0.94$.

Figure 3 Shows a dynamical representation exposed node at various values of σ and ϵ in the model under consideration. Assuming $\mathcal{A} = 0.33$, $\beta = 0.1$, $\mu = 0.003$, $p = 0.3$, $\eta = 0.06$, $\alpha = 0.25$, $\lambda = 0.07$, $\gamma = 0.4$ and $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$ be initial values.

Figure 4 Shows dynamically infected exposed nodes at various values of σ and ϵ. Assuming $\mathcal{A}$ =

0.33, $\beta = 0.1$, $\mu = 0.003$, $p = 0.3$, $\eta = 0.06$, $\alpha = 0.25$, $\lambda = 0.07$, $\gamma = 0.4$ and consider $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$ be initial values.

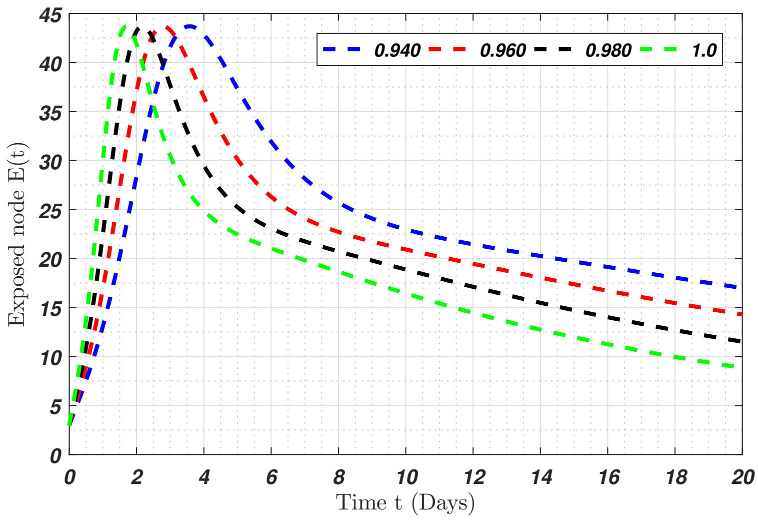

Figure 3. Numerical simulation of exposed node **S** of system (4.1) involving fractal-fractional order derivative with different values of $\epsilon = \sigma = 1, 0.98, 0.96, 0.94$.

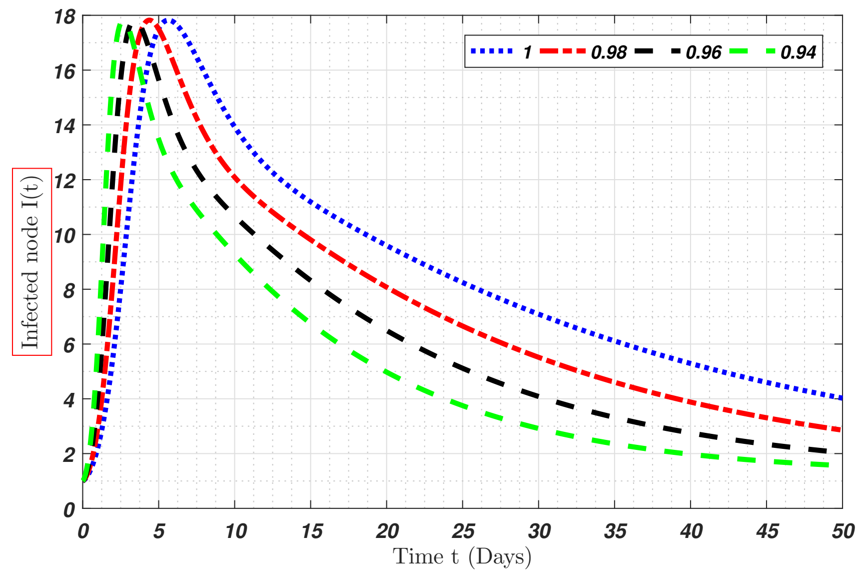

Figure 4. Numerical simulation of infected node **S** of system (4.1) involving fractal-fractional order derivative with different values of $\epsilon = \sigma = 1, 0.98, 0.96, 0.94$.

Figure 5 Shows a dynamical representation recovered node at various values of σ and ϵ. Assuming $\mathcal{A} = 0.33$, $\beta = 0.1$, $\mu = 0.003$, $p = 0.3$, $\eta = 0.06$, $\alpha = 0.25$, $\lambda = 0.07$, $\gamma = 0.4$ and considering $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$ be initial values.

Figure 6 Shows a dynamical representation of vaccinated nodes at various values of σ and ϵ. Assuming $\mathcal{A} = 0.33$, $\beta = 0.1$, $\mu = 0.003$, $p = 0.3$, $\eta = 0.06$, $\alpha = 0.25$, $\lambda = 0.07$, $\gamma = 0.4$ and $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$.

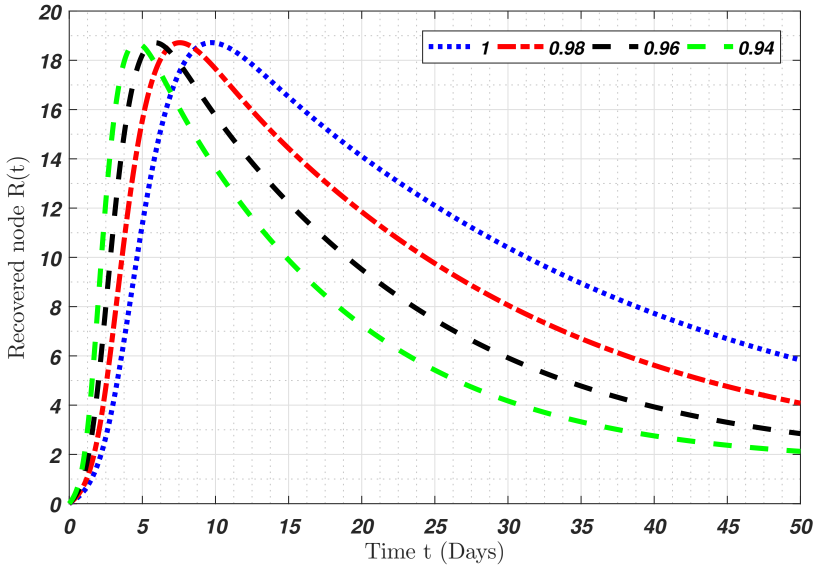

Figure 5. Numerical simulation of recovered node **S** of system (4.1) involving fractal-fractional order derivative with different values of $\epsilon = \sigma = 1, 0.98, 0.96, 0.94$.

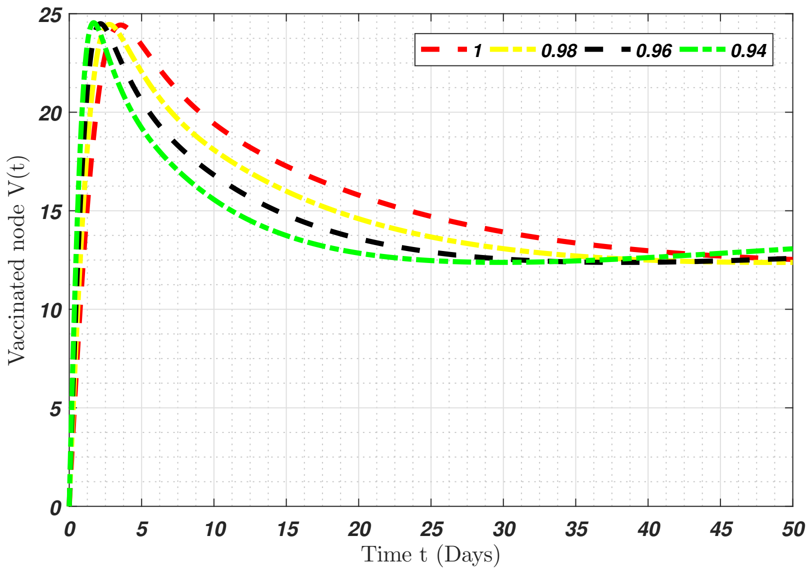

Figure 6. Numerical simulation of vaccinated node **S** of system (4.1) involving fractal-fractional order derivative with different values of $\epsilon = \sigma = 1, 0.98, 0.96, 0.94$.

8. Conclusions and discussion

In this article, we suggest a new WSN model with fractal-fractional order and investigate chaos analysis and numerical model solutions. For the WSN, new models may be derived. By taking into account some of the transmission rates (or particulars) as random or uncertain, it is also feasible to build connections within this WSN model and stochastic models. We come to the conclusion that the model set out in this research yields effective results and might be used for a variety of computer mathematical applications. One of the most essential characteristics of these derivatives is the idea of memory. The development of the events depicted by such systems may be anticipated more precisely with this useful characteristic. The proposed new strategies have been put to the test by applying them to fractional order WSN model (6.1). For the dynamical observation of the considered model, we choose suitable constant values for the model's parameters. Figure 1 shows a dynamical representation of the constant value of σ and ϵ for the WSN model, considering the values of the parameters in the WSN model, including $\mathcal{A} = 0.33,\ \ p = 0.3,\ \ \eta = 0.06, \beta = 0.1,\ \ \mu = 0.003,\quad \alpha = 0.25,\ \ \lambda = 0.07,\ \ \gamma = 0.4$ and considering the initial data as $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$. Figure 2 shows a dynamical representation of susceptible nodes for different values of σ and ϵ in the fractional order WSN model, showing the parameter values in the fractional WSN model, including $\mathcal{A} = 0.33,\ \ p = 0.3,\ \ \eta = 0.06, \beta = 0.1,\ \ \mu = 0.003,\quad \alpha = 0.25,\ \ \lambda = 0.07,\ \ \gamma = 0.4$ and considering the initial data as $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$. Figure 3 shows the dynamical representation of exposed nodes different values of σ and ϵ for the fractional order WSN model, showing the parameter values in the fractional WSN model, including $\mathcal{A} = 0.33,\ \ p = 0.3,\ \ \eta = 0.06, \beta = 0.1,\ \ \mu = 0.003,\quad \alpha = 0.25,\ \ \lambda = 0.07,\ \ \gamma = 0.4$ and considering the initial data as $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$. Figure 4 shows dynamically infected exposed nodes with different values of σ and ϵ for the proposed fractional order WSN model, showing the parameter values in the fractional WSN model, including $\mathcal{A} = 0.33,\ \ p = 0.3,\ \ \eta = 0.06, \beta = 0.1,\ \ \mu = 0.003,\quad \alpha = 0.25,\ \ \lambda = 0.07,\ \ \gamma = 0.4$ and considering the initial data as $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$. Figure 5 shows a dynamical representation of recovered nodes with different values of σ and ϵ for the considered fractional order WSN model, showing the parameter values in the fractional WSN model, including $\mathcal{A} = 0.33,\ \ p = 0.3,\ \ \eta = 0.06, \beta = 0.1,\ \ \mu = 0.003,\quad \alpha = 0.25,\ \ \lambda = 0.07,\ \ \gamma = 0.4$ and considering the initial data as $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$. Figure 6 shows a dynamical representation vaccinated node with different values of σ and ϵ for the suggested fractional order WSN model, showing the parameter values in the fractional WSN model, including $\mathcal{A} = 0.33,\ \ p = 0.3,\ \ \eta = 0.06, \beta = 0.1,\ \ \mu = 0.003,\quad \alpha = 0.25,\ \ \lambda = 0.07,\ \ \gamma = 0.4$ and considering the initial data as $(\mathbf{S}_0, \mathbf{E}_0, \mathbf{I}_0, \mathbf{R}_0, \mathbf{V}_0) = (100, 3, 1, 0, 0)$. The approaches were applied to these challenges and showed some extremely intriguing system behaviors with reasonable explanations. The numerical approaches described in this paper have the potential to be applied to other models. These are essential research possibilities left for future investigation since each new numerical approach should be evaluated in terms of stability, convergence, and consistency.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflicts of interest

The authors declare no conflict of interest.

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References

1. P. S. Tippett, The kinetics of computer virus replication: A theory and preliminary survey, in *Safe Computing: Proceedings of the Fourth Annual Computer Virus and Security Conference*, 1991, 14–15.
2. J. R. Piqueira, F. B. Cesar, Dynamical models for computer viruses propagation, *Math. Probl. Eng.*, **2008** (2008), 1–11. https://doi.org/10.1155/2008/940526
3. J. O. Kephart, S. R. White, Directed-graph epidemiological models of computer viruses, *Comput. Micro Macro View.*, **1992** (1992), 71–102. https://doi.org/10.1142/9789812812438-0004
4. F. Cohen, A cost analysis of typical computer viruses and defenses, *Comput. Secur.*, **10** (1991), 239–250. https://doi.org/10.1016/0167-4048(91)90040-K
5. M. E. J. Newman, S. Forrest, J. Balthrop, Email networks and the spread of computer viruses, *Phys. Rev. E.*, **66** (2002), 1–4. https://doi.org/10.1103/PhysRevE.66.035101
6. B. K. Mishra, D. Saini, Mathematical models on computer viruses, *Appl. Math. Comput.*, **187** (2007), 929–936. https://doi.org/10.1016/j.amc.2006.09.062
7. D. Bernoulli, D. Chapelle, Essai dune nouvelle analyse de la mortalite causee par la petite verole, et des avantages de linoculation pour la prevenir, *Histoire de lAcad, Roy. Sci.(Paris) avec Mem.*, **1760** (1760), 1–45. https://inria.hal.science/hal-04100467
8. J. O. Kephart, S. R. White, D. M. Chess, Computers and epidemiology, *IEEE Spectrum.*, **30** (1993), 20–26. https:// doi.org/10.1109/6.275061
9. L. Billings, W. M. Spears, I. B. Schwartz, A unified prediction of computer virus spread in connected networks, *Phys. Lett. A.*, **297** (2002), 261–266. https://doi.org/10.1016/S0375-9601(02)00152-4
10. L. Zhang, S. Ahmad, A. Ullah, A. Akgãœl, E. K. Akgãœl, Analysis Of Hidden Attractors Of Non-Equilibrium Fractal-Fractional Chaotic System With One Signum Function, *FRACTALS*, **30** (2022), 1–6. https://doi.org/10.1142/S0218348X22401399
11. S. Etemad, B. Tellab, A. Zeb, S. Ahmad, A. Zada, S. Rezapour, et al., A mathematical model of transmission cycle of CC-Hemorrhagic fever via fractal–fractional operators and numerical simulations, *Results Phys.*, **40** (2022), 105800. https://doi.org/10.1016/j.rinp.2022.105800

12. H. Khan, J. Alzabut, D. Baleanu, G. Alobaidi, M. U. Rehman, Existence of solutions and a numerical scheme for a generalized hybrid class of n-coupled modified ABC-fractional differential equations with an application, *AIMS Math.*, **8** (2023), 6609–6625. https://doi.org/10.3934/math.2023334

13. H. Khan, J. Alzabut, A. Shah, Z. Y. He, S. Etemad, S. Rezapour, et al., On fractal-fractional waterborne disease model: A study on theoretical and numerical aspects of solutions via simulations, *Fractals.*, 2023. https://doi.org/10.1142/S0218348X23400558

14. H. Khan, J. Alzabut, H. Gulzar, Existence of solutions for hybrid modified ABC-fractional differential equations with p-Laplacian operator and an application to a waterborne disease model, *Alex. Eng. J.*, **70** (2023), 665–672. https://doi.org/10.1016/j.aej.2023.02.045

15. A. Atangana, J. F. Gómez-Aguilar, Numerical approximation of Riemann-Liouville definition of fractional derivative: From Riemann-Liouville to Atangana-Baleanu, *Numer. Methods Partial Differ, Equations.*, **34** (2018), 1502–1523. https://doi.org/10.1002/num.22195

16. A. Akgül, I. M. Sajid, U. Fatima, N. Ahmed, Z. Iqbal, A. Raza, et al., Optimal existence of fractional order computer virus epidemic model and numerical simulations, *Math. Methods Appl. Sci.*, **44** (2021), 10673–10685. https://doi.org/10.1002/mma.7437

17. H. I. Abdel-Gawad, D. Baleanu, A. H. Abdel-Gawad, Unification of the different fractional time derivatives: An application to the epidemic-antivirus dynamical system in computer networks, *Chaos Soliton. Fract.*, **142** (2021), 110416. https://doi.org/10.1016/j.chaos.2020.110416

18. J. Singh, D. Kumar, Z. Hammouch, A. Atangana, A fractional epidemiological model for computer viruses pertaining to a new fractional derivative, *Appl. Math. Comput.*, **316** (2018), 504–515. https://doi.org/10.1016/j.amc.2017.08.048

19. N. Özdemir, S. Uçar, B. B. Eroglu, Dynamical analysis of fractional order model for computer virus propagation with kill signals, *Int. J. Nonlinear Sci. Numer. Simul.*, **21** (2020), 239–247. https://doi.org/10.1515/ijnsns-2019-0063

20. B. K. Mishra, N. Keshri, Mathematical model on the transmission of worms in wireless sensor network, *Appl. Math. Model.*, **37** (2013), 4103–4111. https://doi.org/10.1016/j.apm.2012.09.025

21. D. Kumar, J. Singh, New aspects of fractional epidemiological model for computer viruses with Mittag-Leffler law, Math. Model. Health Soc. *Appl. Sci.*, **2020** (2020), 283–301. https://doi.org/10.1007/978-981-15-2286-4-9

22. A. Atangana, Fractal-fractional differentiation and integration: connecting fractal calculus and fractional calculus to predict complex system, *Chaos Soliton. Fract.*, **102** (2017), 396–406. https://doi.org/10.1016/j.chaos.2017.04.027

23. A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, Theory and applications of fractional differential equations, *Elsevier Sci. Ltd.*, 2006. https://shop.elsevier.com/books/theory-and-applications-of-fractional-differential-equations/kilbas/978-0-444-51832-3

24. T. Abdeljawad, D. Baleanu, Discrete fractional differences with nonsingular discrete Mittag-Leffler kernels, *Adv. Differ. Equations.*, **2016** (2016), 1–22. https://doi.org/10.1186/s13662-016-0949-5

25. A. Khan, J. F. Gomez-Aguilar, T. S. Khan, H. Khan, Stability analysis and numerical solutions of fractional order HIV/AIDS model, *Chaos Soliton. Fract.*, **122** (2019), 119–128. https://doi.org/10.1016/j.chaos.2019.03.022

26. D. H. Hyers, On the stability of the linear functional equation, *Proc. Natl. Acad. Sci.*, **27** (1941), 222–224. https://doi.org/10.1073/pnas.27.4.222

27. T. M. Rassias, On the stability of the linear mapping in Banach spaces, *Proc. Amer. Math. Soc.*, **72** (1978), 297–300. https://www.ams.org/journals/proc/1978-072-02/S0002-9939-1978-0507327-1

28. D. H. Hyers, T. M. Rassias, Approximate homomorphisms, *Aequat. Math.*, **44** (1992), 125–153. https://doi.org/10.1007/BF01830975

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