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COMPLEX INTUITIONISTIC FUZZY DOMBI PRIORITIZED AGGREGATION OPERATORS AND THEIR APPLICATION FOR RESILIENT GREEN SUPPLIER SELECTION

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Abstract. *One of the main problems faced by resilient supply chain management is how to solve the problem of supplier selection, which is a typical multi-attribute decision-making (MADM) problem. Given the complexity of the current decision-making environment, the primary influence of this paper is to propose the theory of Dombi operational laws based on complex intuitionistic fuzzy (CIF) information. Moreover, we examined the theory of CIF Dombi prioritized averaging (CIFDPA) and CIF weighted Dombi prioritized averaging (CIFWDPA), where these operators are the modified version of the prioritized aggregation operators and Dombi aggregation operators for fuzzy, intuitionistic fuzzy, complex fuzzy and complex intuitionistic fuzzy information. Some reliable properties for the above operators are also established. Furthermore, to state the art of the proposed operators, an application example in the presence of the invented operators is evaluated for managing resilient green supplier selection problems. Finally, through comparative analysis with mainstream technologies, we provide some mechanism explanations for the proposed method to show the supremacy and worth of the invented theory.*

Key words: *Complex intuitionistic fuzzy sets, Prioritized aggregation operators, multi-attribute decision-making problem, Dombi t-norm and t-conorm, Resilient Green Supplier Selection*

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1. INTRODUCTION

Supply chain activity refers to planning, controlling material, data, logistics management, and internal and external financial flow among institutions to meet the investor's needs. Valuable and dominant business strategies or policies are needed to cope with internal and external problems and challenges in supply chains and their valuable and essential stages. In the last few years, especially during COVID-19, the supply chain has faced many pressures for sustainable business evaluation, including global sourcing, environment, short time to market, and demand ambiguity or uncertainties. The lifetime or long-term theme or goal to survive during the new COVID-19 outbreak is to thrive in the "new normal". Moreover, the virus of COVID-19 is not limited to control in the supply chain. Therefore, coping with all stakeholders in the supply chain is very awkward and challenging. To manage the above problems, the companies must select the right supplier to help reduce costs and remain competitive by meeting stakeholders' expectations effectively and accurately. Various well-known supplier selection techniques have been employed, including the Markov chain, Bayesian network, and deterministic and stochastic optimization. However, the past existing information rarely evaluates sustainable supplier considerations from a resilience and sustainability point of view. Therefore, this information aims to study the best supply chain selection in the presence of some suitable supplier chain management, which is a typical multi-attribute decision-making (MADM) problem.

Decision-making procedures, clustering analysis, and many other valuable techniques are used for evaluating the finest optimal under particular criteria. The MADM technique is one of the most valuable and reliable techniques and subparts of the decision-making procedure, which is also used for selecting the optimal alternative from the family of preferences based on some classical set theory. Moreover, in the case of classical set theory, we have only two options, such as zero or one; in many real-life cases, we need more options accepted as zero and one. Therefore, to accommodating the above problem, the fuzzy set (FS) was proposed by Zadeh [1], where the range of FS is more extended than the crisp set because the function defined in FS is defined from universal set to unit interval instead of $\{0, 1\}$. Many generalizations of the FS and some applications have been proposed by different scholars, such as fuzzy superior Mandelbrot set [2], fuzzy N-soft information [3], multi-fuzzy N-soft set theory [4].

The main idea of FS is very famous and reliable because they have received a lot of attention from different scholars. Furthermore, the FS has only talked about the grade of membership but has not considered the theory of falsity information, where the grade of non-membership also plays a very critical and valuable role in decision-making analysis. In many situations, we not only face positive information about anyone in real life because many people have given negative information about their enemies. Therefore, the theory of intuitionistic fuzzy set (IFS) was invented by Atanassov [5]. IFS theory handles two types of information, truth and falsity information, with a valuable and effective option, such as the sum of the duplet, which will be covered in the unit interval. The traditional theory of FS is a special case of the IFS theory because if we remove the falsity information from IFS, then the theory of IFS will be converted to FS theory, where the range of both grades is the same, such as unit interval. Many applications and different generalizations of the IFS have been proposed by scholars, such as Schweizer-Sklar power aggregation operators [6], Maclaurin symmetric aggregation operators [7], Einstein interactive aggregation operators [8], MAIRCA technique [9], twin support vector machine [10], analysis and application of evolutionary procedure [11].

The term "periodic" was critical and essential in complex set theory. In various cases, we failed to cope with the problem without periodic function because of the two-dimensional information in every field of life. In the presence of the FS theory, we can only cope with one-dimensional information at a time. Therefore, the concept of complex fuzzy set (CFS) theory was invented by Ramot et al. [12] by including the periodic term in the structure of truth grade and given a new shape in the form of the complex number, where the range of the real and unreal parts are unit interval. After the construction of CFS, various applications and generalizations of CFS have been proposed by different scholars, such as distance and entropy measures in the presence of the CFSs [13], complex fuzzy N-soft set theory [14], complex multi-fuzzy soft sets [15], and another overview of complex fuzzy logic and their application in decision-making problems [16]. Furthermore, the CFS has only talked about the grade of membership but has not considered the theory of falsity information, where the grade of non-membership also plays a very critical and valuable role in decision-making analysis. In many situations, we not only face positive information about anyone in real life because many people have given negative information about their enemies. Therefore, the complex intuitionistic fuzzy set (CIFS) was invented by Alkouri and Salleh [17], which covers two types of information, truth and falsity information, with a valuable and effective option such as the sum of the real part (same rule for the imaginary part) of the duplet, which will be covered in the unit interval. The traditional theory of CFS is a special case of the CIFS theory because if we remove the falsity of information from CIFS, then the theory of CIFS will be converted to CFS theory. Numerous applications and generalizations of the CIFS have been proposed by different scholars, such as prioritized aggregation operators [18], simple aggregation operators [19], novel aggregation operators [20], generalized geometric aggregation operators [21], and robust averaging\geometric aggregation operators [22]. However, most of the existing aggregation operators are based on CIFS's algebraic operation rules, which lack some flexibility in modeling logic operations, such as dealing with information aggregation and decision-making problems in complex situations.

The main idea of Dombi t-norm and t-conorm was invented by Dombi [23] in 1982. Dombi t-norm and t-conorm have been used in constructing various aggregation operators. Because of parameter λ, the theory of Hamacher (algebraic) t-norm and t-conorm are the special cases of the Dombi t-norm and t-conorm. Furthermore, evaluating or aggregating information collection into a singleton set is a very challenging task for scholars. Hence, the prioritized aggregation operators were established by Yager [24] in 2008, which is the modified version of the averaging and geometric aggregation operators. The prioritized aggregation operator is used for aggregating information collection into a singleton set by considering the priority relationship between the aggregated variables. Thus, Yu and Xu [25] presented a novel theory of prioritized aggregation operators for IFSs. Seikh and Mandal [26] exposed the Dombi aggregation operators for IFS in the presence of the Dombi t-norm and t-conorm. To our knowledge, no scholars have derived the theories of Dombi aggregation operators, prioritized aggregation operators, and Dombi aggregation operators for CIFS theory. However, such operators can effectively improve the flexibility of information integration and the adaptability of decision scenarios.

Given the unique advantages of CIFS in representing decision information and the flexibility brought by the Dombi t-norm and t-conorm for aggregation operations, how to propose a set of flexible operation rules around advanced CIF information and then develop a class of powerful information aggregation operators to solve decision-making problems in complex reality is a topic worth studying. Thus, out of the motives mentioned above, this paper mainly focuses on carrying out the following innovative work:

(1) It proposes the theory of Dombi operational laws based on CIF information and improves further the flexibility of CIF information aggregation modeling.

(2) It presents the CIFDPA and CIFWDPA operators, which aggregate information flexibly and consider the priority relationship between integrated parameters. Meanwhile, some reliable properties for the above mentioned operators are also established.

(3) It constructs an MADM technique based on the proposed operators and applies it to the resilient green supplier selection problem.

(4) It implements a comparative analysis with various prevailing operators and discusses the operating mechanism and advantages of the proposed method.

2. PRELIMINARIES

In this section, we collected the basic information about Dtn, Dtcn, and CIFSs and their valuable information, such as SV and AV.

Definition 1: [23] For evaluating the theory of Dtn and Dtcn, we have used any two real numbers $\dot{x}_u$ and $\dot{y}_u$ ($\dot{x}_u, \dot{y}_u \in [0,1]$), such that

$$D_{Dtn}\left(\dot{x}_u, \dot{y}_u\right) = \frac{1}{1+\left\{\left(\frac{1-\dot{x}_u}{\dot{x}_u}\right)^{\lambda} + \left(\frac{1-\dot{y}_u}{\dot{y}_u}\right)^{\lambda}\right\}^{1/\lambda}}, \lambda > 0 \tag{1}$$

$$D_{Dtcn}\left(\dot{x}_u, \dot{y}_u\right) = 1 - \frac{1}{1+\left\{\left(\frac{\dot{x}_u}{1-\dot{x}_u}\right)^{\lambda} + \left(\frac{\dot{y}_u}{1-\dot{y}_u}\right)^{\lambda}\right\}^{1/\lambda}}, \lambda > 0 \tag{2}$$

Definition 2: [17] Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then the model of CIFS is characterized by:

$$\dot{\nabla}^{CIF} = \left\{\left(\Re_m\left(\dot{x}_u\right), \mathrm{I}_n\left(\dot{x}_u\right)\right) : \dot{x}_u \in \dot{\mathcal{V}}_u\right\} \tag{3}$$

Furthermore, we explained the complex form of truthful information and the complex form of false information, such as $\Re_m\left(\dot{x}_u\right) = \left(\Re_m^{rp}\left(\dot{x}_u\right), \Re_m^{ip}\left(\dot{x}_u\right)\right)$ and $\mathrm{I}_n\left(\dot{x}_u\right) = \left(\mathrm{I}_n^{rp}\left(\dot{x}_u\right), \mathrm{I}_n^{ip}\left(\dot{x}_u\right)\right)$ with a specific condition: $0 \le \Re_m^{rp}\left(\dot{x}_u\right) + \mathrm{I}_n^{rp}\left(\dot{x}_u\right) \le 1$ for real parts and $0 \le \Re_m^{ip}\left(\dot{x}_u\right) + \mathrm{I}_n^{ip}\left(\dot{x}_u\right) \le 1$ for imaginary parts. Moreover, the neutral information is stated by: $\Gamma_m\left(\dot{x}_u\right) = \left(\Gamma_m^{rp}\left(\dot{x}_u\right), \Gamma_m^{ip}\left(\dot{x}_u\right)\right) = \left(1-\left(\Re_m^{rp}\left(\dot{x}_u\right) + \mathrm{I}_n^{rp}\left(\dot{x}_u\right)\right), 1-\left(\Re_m^{ip}\left(\dot{x}_u\right) + \mathrm{I}_n^{ip}\left(\dot{x}_u\right)\right)\right)$ and the final and simplest form of CIF value (CIFV) is stated by: $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right)$, $j = 1, 2, 3, \ldots, z$. For example, there exists a CIFV whose real part is evaluated as an intuitionistic fuzzy form of (0.3, 0.4) and whose imaginary part is evaluated as an intuitionistic fuzzy form of (0.5, 0.1), then its presentation form ((0.3, 0.5), (0.4, 0.1)).

Definition 3: [19] Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any two CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right)$, $j = 1, 2$, we have:

$$\dot{\nabla}_1^{CIF} \oplus \dot{\nabla}_2^{CIF} = \left(\left(\Re_{m_1}^{rp} + \Re_{m_2}^{rp} - \Re_{m_1}^{rp}\Re_{m_2}^{rp}, \Re_{m_1}^{ip} + \Re_{m_2}^{ip} - \Re_{m_1}^{ip}\Re_{m_2}^{ip}\right), \left(\mathrm{I}_{n_1}^{rp}\mathrm{I}_{n_2}^{rp}, \mathrm{I}_{n_1}^{ip}\mathrm{I}_{n_2}^{ip}\right)\right) \tag{4}$$

$$\dot{\nabla}_1^{CIF} \otimes \dot{\nabla}_2^{CIF} = \left(\left(\Re_{m_1}^{rp}\Re_{m_2}^{rp}, \Re_{m_1}^{ip}\Re_{m_2}^{ip}\right), \left(\mathrm{I}_{n_1}^{rp} + \mathrm{I}_{n_2}^{rp} - \mathrm{I}_{n_1}^{rp}\mathrm{I}_{n_2}^{rp}, \mathrm{I}_{n_1}^{ip} + \mathrm{I}_{n_2}^{ip} - \mathrm{I}_{n_1}^{ip}\mathrm{I}_{n_2}^{ip}\right)\right) \tag{5}$$

$$\ell\dot{\nabla}_j^{CIF} = \left(\left(1 - \left(1 - \Re_{m_j}^{rp}\right)^{\ell}, 1 - \left(1 - \Re_{m_j}^{ip}\right)^{\ell}\right), \left(\left(\mathrm{I}_{n_j}^{rp}\right)^{\ell}, \left(\mathrm{I}_{n_j}^{ip}\right)^{\ell}\right)\right), \ell > 0 \tag{6}$$

$$\left(\dot{\nabla}_j^{CIF}\right)^{\ell} = \left(\left(\left(\Re_{m_j}^{rp}\right)^{\ell}, \left(\Re_{m_j}^{ip}\right)^{\ell}\right), \left(1 - \left(1 - \mathrm{I}_{n_j}^{rp}\right)^{\ell}, 1 - \left(1 - \mathrm{I}_{n_j}^{ip}\right)^{\ell}\right)\right), \ell > 0 \tag{7}$$

Definition 4: [19] Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right)$, $j = 1, 2$, we have:

$$SV\left(\dot{\nabla}_j^{CIF}\right) = \frac{1}{2}\left(\left(\Re_{m_j}^{rp} + \Re_{m_j}^{ip}\right) - \left(\mathrm{I}_{n_j}^{rp} + \mathrm{I}_{n_j}^{ip}\right)\right) \in [-1, 1] \tag{8}$$

$$AV\left(\dot{\nabla}_j^{CIF}\right) = \frac{1}{2}\left(\left(\Re_{m_j}^{rp} + \Re_{m_j}^{ip}\right) + \left(\mathrm{I}_{n_j}^{rp} + \mathrm{I}_{n_j}^{ip}\right)\right) \in [0, 1] \tag{9}$$

To evaluate any two CIFVs, we described some characteristics in the presence of the information in Eq. (8) and Eq. (9); we have if $SV\left(\dot{\nabla}_1^{CIF}\right) > SV\left(\dot{\nabla}_2^{CIF}\right)$, thus $\dot{\nabla}_1^{CIF} > \dot{\nabla}_2^{CIF}$; if $SV\left(\dot{\nabla}_1^{CIF}\right) < SV\left(\dot{\nabla}_2^{CIF}\right)$, thus $\dot{\nabla}_1^{CIF} < \dot{\nabla}_2^{CIF}$; if $SV\left(\dot{\nabla}_1^{CIF}\right) = SV\left(\dot{\nabla}_2^{CIF}\right)$, then if $AV\left(\dot{\nabla}_1^{CIF}\right) > AV\left(\dot{\nabla}_2^{CIF}\right)$, thus $\dot{\nabla}_1^{CIF} > \dot{\nabla}_2^{CIF}$; if $AV\left(\dot{\nabla}_1^{CIF}\right) < AV\left(\dot{\nabla}_2^{CIF}\right)$, thus $\dot{\nabla}_1^{CIF} < \dot{\nabla}_2^{CIF}$.

3. Dombi Prioritized Aggregation Operators for CIF Information

Before building the Dombi prioritized aggregation operators, we must present the basic Dombi operational laws in the presence of the CIFVs. Then, by using the Dombi operational laws for CIF values, we aim to derive the theory of CIFDPA and CIFWDPA operators. Moreover, we must examine the proposed operators' basic properties, such as idempotency, monotonicity, and boundedness.

Definition 5: Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, in the presence of any two CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right)$, $j = 1,2$, we have:

$$\dot{\nabla}_1^{CIF} \oplus \dot{\nabla}_2^{CIF} = \left(\begin{array}{l}\left(1-\frac{1}{1+(a_1+a_2)^{1/\lambda}}, 1-\frac{1}{1+(b_1+b_2)^{1/\lambda}}\right),\\ \left(\frac{1}{1+(c_1+c_2)^{1/\lambda}}, \frac{1}{1+(d_1+d_2)^{1/\lambda}}\right)\end{array}\right) \tag{10}$$

$$\dot{\nabla}_1^{CIF} \otimes \dot{\nabla}_2^{CIF} = \left(\begin{array}{l}\left(\frac{1}{1+(a_1+a_2)^{1/\lambda}}, \frac{1}{1+(b_1+b_2)^{1/\lambda}}\right),\\ \left(1-\frac{1}{1+(c_1+c_2)^{1/\lambda}}, 1-\frac{1}{1+(d_1+d_2)^{1/\lambda}}\right)\end{array}\right) \tag{11}$$

$$\varphi\dot{\nabla}_1^{CIF} = \left(\left(1-\frac{1}{1+(\varphi a_1)^{1/\lambda}}, 1-\frac{1}{1+(\varphi b_1)^{1/\lambda}}\right), \left(\frac{1}{1+(\varphi c_1)^{1/\lambda}}, \frac{1}{1+(\varphi d_1)^{1/\lambda}}\right)\right) \tag{12}$$

$$\left(\dot{\nabla}_1^{CIF}\right)^{\varphi} = \left(\left(\frac{1}{1+(\varphi a_1)^{1/\lambda}}, \frac{1}{1+(\varphi b_1)^{1/\lambda}}\right), \left(1-\frac{1}{1+(\varphi c_1)^{1/\lambda}}, 1-\frac{1}{1+(\varphi d_1)^{1/\lambda}}\right)\right) \tag{13}$$

where φ and λ are constants greater than 0; let $a_j = \left(\frac{\Re_{m_j}^{rp}}{1-\Re_{m_j}^{rp}}\right)^{\lambda}$, $b_j = \left(\frac{\Re_{m_j}^{ip}}{1-\Re_{m_j}^{ip}}\right)^{\lambda}$, $c_j = \left(\frac{1-\mathrm{I}_{n_j}^{rp}}{\mathrm{I}_{n_j}^{rp}}\right)^{\lambda}$, and $d_j = \left(\frac{1-\mathrm{I}_{n_j}^{ip}}{\mathrm{I}_{n_j}^{ip}}\right)^{\lambda}$, $j = 1,2$, and these symbols also apply to the following text.

Definition 6: Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any two CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right)$, $j = 1,2,\ldots,z$, then:

$$\begin{aligned} IFDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_z^{CIF}\right) &= \frac{\overline{\overline{\mathrm{H}^1}}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}^j}}}\dot{\nabla}_1^{CIF} \oplus \frac{\overline{\overline{\mathrm{H}^2}}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}^j}}}\dot{\nabla}_2^{CIF} \oplus \ldots \oplus \frac{\overline{\overline{\mathrm{H}^z}}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}^j}}}\dot{\nabla}_z^{CIF} \\ &= \oplus_{j=1}^{z}\left(\frac{\overline{\overline{\mathrm{H}^j}}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}^j}}}\dot{\nabla}_j^{CIF}\right) \end{aligned} \tag{14}$$

where, $\overline{\overline{\mathrm{H}^1}} = 1$, and $\overline{\overline{\mathrm{H}^j}} = \prod_{k=1}^{j-1} SV\left(\dot{\nabla}_k^{CIF}\right)$.

Theorem 1: Assume that $\dot{\mathcal{V}}_u$ is a family of fixed set with generic elements, then in the presence of any two CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right), j = 1, 2, \ldots, z$, and in the presence of the information in Eq. (14), we derive that the below theory, such as:

$$CIFDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_z^{CIF}\right) = \left(\left(1 - \frac{1}{1 + \left(\sum_{j=1}^{z} A_j\right)^{1/\lambda}}, 1 - \frac{1}{1 + \left(\sum_{j=1}^{z} B_j\right)^{1/\lambda}}\right), \left(\frac{1}{1 + \left(\sum_{j=1}^{z} C_j\right)^{1/\lambda}}, \frac{1}{1 + \left(\sum_{j=1}^{z} D_j\right)^{1/\lambda}}\right)\right) \quad (15)$$

where $A_j = \frac{\overline{\overline{\mathrm{H}}}^j}{\sum_{j=1}^{z} \overline{\overline{\mathrm{H}}}^j} a_j$, $B_j = \frac{\overline{\overline{\mathrm{H}}}^j}{\sum_{j=1}^{z} \overline{\overline{\mathrm{H}}}^j} b_j$, $C_j = \frac{\overline{\overline{\mathrm{H}}}^j}{\sum_{j=1}^{z} \overline{\overline{\mathrm{H}}}^j} c_j$, and $D_j = \frac{\overline{\overline{\mathrm{H}}}^j}{\sum_{j=1}^{z} \overline{\overline{\mathrm{H}}}^j} d_j$, and these symbols also apply to the following text.

Proof: Considering the primary procedure of the mathematical induction, we focus on evaluating the data in Eq. (15). Considering $z = 2$, we have

$$\frac{\overline{\overline{\mathrm{H}}}^1}{\sum_{j=1}^{2} \overline{\overline{\mathrm{H}}}^j} \dot{\nabla}_1^{CIF} = \left(\left(1 - \frac{1}{1 + A_1^{1/\lambda}}, 1 - \frac{1}{1 + B_1^{1/\lambda}}\right), \left(\frac{1}{1 + C_1^{1/\lambda}}, \frac{1}{1 + D_1^{1/\lambda}}\right)\right)$$

$$\frac{\overline{\overline{\mathrm{H}}}^2}{\sum_{j=1}^{2} \overline{\overline{\mathrm{H}}}^j} \dot{\nabla}_2^{CIF} = \left(\left(1 - \frac{1}{1 + A_2^{1/\lambda}}, 1 - \frac{1}{1 + B_2^{1/\lambda}}\right), \left(\frac{1}{1 + C_2^{1/\lambda}}, \frac{1}{1 + D_2^{1/\lambda}}\right)\right)$$

Thus,

$$CIFDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}\right) = \frac{\overline{\overline{\mathrm{H}}}^1}{\sum_{j=1}^{2} \overline{\overline{\mathrm{H}}}^j} \dot{\nabla}_1^{CIF} \oplus \frac{\overline{\overline{\mathrm{H}}}^2}{\sum_{j=1}^{2} \overline{\overline{\mathrm{H}}}^j} \dot{\nabla}_2^{CIF}$$

$$= \left(\left(1 - \frac{1}{1 + A_1^{1/\lambda}}, 1 - \frac{1}{1 + B_1^{1/\lambda}}\right), \left(\frac{1}{1 + C_1^{1/\lambda}}, \frac{1}{1 + D_1^{1/\lambda}}\right)\right)$$

$$\oplus \left(\left(1 - \frac{1}{1 + A_2^{1/\lambda}}, 1 - \frac{1}{1 + B_2^{1/\lambda}}\right), \left(\frac{1}{1 + C_2^{1/\lambda}}, \frac{1}{1 + D_2^{1/\lambda}}\right)\right)$$

$$= \left(\left(1 - \frac{1}{1 + \left(\sum_{j=1}^{2} A_j\right)^{1/\lambda}}, 1 - \frac{1}{1 + \left(\sum_{j=1}^{2} B_j\right)^{1/\lambda}}\right), \left(\frac{1}{1 + \left(\sum_{j=1}^{2} C_j\right)^{1/\lambda}}, \frac{1}{1 + \left(\sum_{j=1}^{2} D_j\right)^{1/\lambda}}\right)\right)$$

For $z = 2$, we hold our required information; furthermore, we assume that the data in Eq. (15) is also held for $n = k$, such as

$$CIFDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_k^{CIF}\right) =$$
$$\left(\left(1 - \frac{1}{1 + \left(\sum_{j=1}^{z} A_j\right)^{1/\lambda}}, 1 - \frac{1}{1 + \left(\sum_{j=1}^{z} B_j\right)^{1/\lambda}}\right), \left(\frac{1}{1 + \left(\sum_{j=1}^{z} C_j\right)^{1/\lambda}}, \frac{1}{1 + \left(\sum_{j=1}^{z} D_j\right)^{1/\lambda}}\right)\right)$$

Then, we derive it for $n = k + 1$, such as

$$CIFDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_{k+1}^{CIF}\right) = \frac{\overline{\overline{\mathrm{H}^1}}}{\sum_{j=1}^{k} \overline{\overline{\mathrm{H}^j}}} \dot{\nabla}_1^{CIF} \oplus \frac{\overline{\overline{\mathrm{H}^2}}}{\sum_{j=1}^{k} \overline{\overline{\mathrm{H}^j}}} \dot{\nabla}_2^{CIF} \oplus \ldots \oplus \frac{\overline{\overline{\mathrm{H}^k}}}{\sum_{j=1}^{k} \overline{\overline{\mathrm{H}^j}}} \dot{\nabla}_k^{CIF}$$
$$\oplus \frac{\overline{\overline{\mathrm{H}^{k+1}}}}{\sum_{j=1}^{k+1} \overline{\overline{\mathrm{H}^j}}} \dot{\nabla}_{k+1}^{CIF} = \oplus_{j=1}^{k} \left(\frac{\overline{\overline{\mathrm{H}^j}}}{\sum_{j=1}^{k} \overline{\overline{\mathrm{H}^j}}} \dot{\nabla}_j^{CIF}\right) \oplus \frac{\overline{\overline{\mathrm{H}^{k+1}}}}{\sum_{j=1}^{k+1} \overline{\overline{\mathrm{H}^j}}} \dot{\nabla}_{k+1}^{CIF} =$$
$$= \left(\left(1 - \frac{1}{1 + \left(\sum_{j=1}^{k} A_j\right)^{1/\lambda}}, 1 - \frac{1}{1 + \left(\sum_{j=1}^{k} B_j\right)^{1/\lambda}}\right), \left(\frac{1}{1 + \left(\sum_{j=1}^{k} C_j\right)^{1/\lambda}}, \frac{1}{1 + \left(\sum_{j=1}^{k} D_j\right)^{1/\lambda}}\right)\right)$$
$$\oplus \left(\left(1 - \frac{1}{1 + A_{k+1}^{1/\lambda}}, 1 - \frac{1}{1 + B_{k+1}^{1/\lambda}}\right), \left(\frac{1}{1 + C_{k+1}^{1/\lambda}}, \frac{1}{1 + D_{k+1}^{1/\lambda}}\right)\right)$$
$$= \left(\left(1 - \frac{1}{1 + \left(\sum_{j=1}^{k+1} A_j\right)^{1/\lambda}}, 1 - \frac{1}{1 + \left(\sum_{j=1}^{k+1} B_j\right)^{1/\lambda}}\right), \left(\frac{1}{1 + \left(\sum_{j=1}^{k+1} C_j\right)^{1/\lambda}}, \frac{1}{1 + \left(\sum_{j=1}^{k+1} D_j\right)^{1/\lambda}}\right)\right)$$

Hence, our final result proved the possible values of z. Moreover, we justify the significant properties of the evaluated theory in Eq. (15), such as idempotency, monotonicity, and boundedness.

Property 1: (***Idempotency***) Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any z CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right)$, $j = 1, 2, \ldots, z$, if $\dot{\nabla}_j^{CIF} = \dot{\nabla}^{CIF} = \left(\left(\Re_m^{rp}, \Re_m^{ip}\right), \left(\mathrm{I}_n^{rp}, \mathrm{I}_n^{ip}\right)\right)$, then

$$CIFDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_z^{CIF}\right) = \dot{\nabla}^{CIF} \tag{16}$$

Proof: When $\dot{\nabla}_j^{CIF} = \dot{\nabla}^{CIF} = \left(\left(\Re_m^{rp}, \Re_m^{ip}\right), \left(\mathrm{I}_n^{rp}, \mathrm{I}_n^{ip}\right)\right)$, then

$$CIFDPA\left(\dot{\nabla}_1^{CIF},\dot{\nabla}_2^{CIF},\ldots,\dot{\nabla}_z^{CIF}\right)$$

$$=\left(\begin{array}{c}\left(1-\dfrac{1}{1+\left\{\sum_{j=1}^{z}\dfrac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}}a_m\right\}^{1/\lambda}},1-\dfrac{1}{1+\left\{\sum_{j=1}^{z}\dfrac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}}b_m\right\}^{1/\lambda}}\right),\\ \left(\dfrac{1}{1+\left\{\sum_{j=1}^{z}\dfrac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}}c_n\right\}^{1/\lambda}},\dfrac{1}{1+\left\{\sum_{j=1}^{z}\dfrac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}}d_n\right\}^{1/\lambda}}\right)\end{array}\right)$$

$$=\left(\left(1-\frac{1}{1+a_m^{1/\lambda}},1-\frac{1}{1+b_m^{1/\lambda}}\right),\left(\frac{1}{1+c_n^{1/\lambda}},\frac{1}{1+d_n^{1/\lambda}}\right)\right),\sum_{j=1}^{z}\frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}}=1$$

$$=\left(\left(1-\frac{1}{\frac{1-\Re_m^{rp}+\Re_m^{rp}}{1-\Re_m^{rp}}},1-\frac{1}{\frac{1-\Re_m^{ip}+\Re_m^{ip}}{1-\Re_m^{ip}}}\right),\left(\frac{1}{\frac{\mathrm{I}_n^{rp}+1-\mathrm{I}_n^{rp}}{\mathrm{I}_n^{rp}}},\frac{1}{\frac{\mathrm{I}_n^{ip}+1-\mathrm{I}_n^{ip}}{\mathrm{I}_n^{ip}}}\right)\right)$$

$$=\left(\left(1-\left(1-\Re_m^{rp}\right),1-\left(1-\Re_m^{ip}\right)\right),\left(\mathrm{I}_n^{rp},\mathrm{I}_n^{ip}\right)\right)$$

$$=\left(\left(\Re_m^{rp},\Re_m^{ip}\right),\left(\mathrm{I}_n^{rp},\mathrm{I}_n^{ip}\right)\right)=\dot{\nabla}^{CIF}.$$

Property: (***Monotonicity***) Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any z CIFVs $\dot{\nabla}_j^{CIF}=\left(\left(\Re_{m_j}^{rp},\Re_{m_j}^{ip}\right),\left(\mathrm{I}_{n_j}^{rp},\mathrm{I}_{n_j}^{ip}\right)\right)$, $j=1,2,\ldots,z$, if $\dot{\nabla}_j^{CIF}\le\dot{\nabla}_j^{*CIF}$, then

$$CIFDPA\left(\dot{\nabla}_1^{CIF},\dot{\nabla}_2^{CIF},\ldots,\dot{\nabla}_z^{CIF}\right)\le CIFDPA\left(\dot{\nabla}_1^{*CIF},\dot{\nabla}_2^{*CIF},\ldots,\dot{\nabla}_z^{*CIF}\right) \tag{17}$$

Proof: When $\dot{\nabla}_j^{CIF}\le\dot{\nabla}_j^{*CIF}$, thus $\Re_{m_j}^{rp}\le\Re_{m_j}^{*rp},\Re_{m_j}^{ip}\le\Re_{m_j}^{*ip}$ and $\mathrm{I}_{n_j}^{rp}\ge\mathrm{I}_{n_j}^{**rp},\mathrm{I}_{n_j}^{ip}\ge\mathrm{I}_{n_j}^{**ip}$, then

$$\Re^{rp}_{m_j} \le \Re^{*rp}_{m_j} \Rightarrow \frac{\Re^{rp}_{m_j}}{1-\Re^{rp}_{m_j}} \le \frac{\Re^{*rp}_{m_j}}{1-\Re^{*rp}_{m_j}} \Rightarrow \left(\frac{\Re^{rp}_{m_j}}{1-\Re^{rp}_{m_j}}\right)^{\lambda} \le \left(\frac{\Re^{*rp}_{m_j}}{1-\Re^{*rp}_{m_j}}\right)^{\lambda}$$

$$\Rightarrow \sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{rp}_{m_j}}{1-\Re^{rp}_{m_j}}\right)^{\lambda} \le \sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{*rp}_{m_j}}{1-\Re^{*rp}_{m_j}}\right)^{\lambda}$$

$$\Rightarrow 1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{rp}_{m_j}}{1-\Re^{rp}_{m_j}}\right)^{\lambda}\right\}^{1/\lambda} \le 1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{*rp}_{m_j}}{1-\Re^{*rp}_{m_j}}\right)^{\lambda}\right\}^{1/\lambda}$$

$$\Rightarrow \frac{1}{1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{rp}_{m_j}}{1-\Re^{rp}_{m_j}}\right)^{\lambda}\right\}^{1/\lambda}} \ge \frac{1}{1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{*rp}_{m_j}}{1-\Re^{*rp}_{m_j}}\right)^{\lambda}\right\}^{1/\lambda}}$$

$$\Rightarrow 1-\frac{1}{1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{rp}_{m_j}}{1-\Re^{rp}_{m_j}}\right)^{\lambda}\right\}^{1/\lambda}} \le 1-\frac{1}{1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{*rp}_{m_j}}{1-\Re^{*rp}_{m_j}}\right)^{\lambda}\right\}^{1/\lambda}}$$

Where, by using the same procedure, we get

$$\Re^{ip}_{m_j} \le \Re^{ip}_{m_j} \Rightarrow 1-\frac{1}{1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{ip}_{m_j}}{1-\Re^{ip}_{m_j}}\right)^{\lambda}\right\}^{1/\lambda}} \le 1-\frac{1}{1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{\Re^{*ip}_{m_j}}{1-\Re^{i*p}_{m_j}}\right)^{\lambda}\right\}^{1/\lambda}}$$

Furthermore, we can derive the information of falsity data, such as:

$$\mathrm{I}^{rp}_{n_j} \ge \mathrm{I}^{*rp}_{n_j} \Rightarrow 1-\mathrm{I}^{rp}_{n_j} \le 1-\mathrm{I}^{*rp}_{n_j}$$

$$\Rightarrow \left(\frac{1-\mathrm{I}^{rp}_{n_j}}{\mathrm{I}^{rp}_{n_j}}\right)^{\lambda} \le \left(\frac{1-\mathrm{I}^{*rp}_{n_j}}{\mathrm{I}^{*rp}_{n_j}}\right)^{\lambda} \Rightarrow \sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{1-\mathrm{I}^{rp}_{n_j}}{\mathrm{I}^{rp}_{n_j}}\right)^{\lambda} \le \sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{1-\mathrm{I}^{*rp}_{n_j}}{\mathrm{I}^{*rp}_{n_j}}\right)^{\lambda}$$

$$\Rightarrow \left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{1-\mathrm{I}^{rp}_{j}}{\mathrm{I}^{rp}_{n_j}}\right)^{\lambda}\right\}^{1/\lambda} \le \left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{1-\mathrm{I}^{*rp}_{n_j}}{\mathrm{I}^{*rp}_{n_j}}\right)^{\lambda}\right\}^{1/\lambda}$$

$$\Rightarrow 1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{1-\mathrm{I}^{rp}_{n_j}}{\mathrm{I}^{rp}_{n_j}}\right)^{\lambda}\right\}^{1/\lambda} \le 1+\left\{\sum_{j=1}^{z} \frac{\overline{\overline{\mathrm{H}}}^{j}}{\sum_{j=1}^{z}\overline{\overline{\mathrm{H}}}^{j}} \left(\frac{1-\mathrm{I}^{*rp}_{n_j}}{\mathrm{I}^{*rp}_{n_j}}\right)^{\lambda}\right\}^{1/\lambda}$$

$$\Rightarrow \frac{1}{1+\left\{\sum_{j=1}^{z}\frac{\overline{\overline{H}}^{j}}{\sum_{j=1}^{z}\overline{\overline{H}}^{j}}\left(\frac{1-I_{n_j}^{rp}}{I_{n_j}^{rp}}\right)^{\lambda}\right\}^{1/\lambda}} \geq \frac{1}{1+\left\{\sum_{j=1}^{z}\frac{\overline{\overline{H}}^{j}}{\sum_{j=1}^{z}\overline{\overline{H}}^{j}}\left(\frac{1-I_{n_j}^{*rp}}{I_{n_j}^{*rp}}\right)^{\lambda}\right\}^{1/\lambda}}$$

Where, by using the same procedure, we get

$$I_{n_j}^{ip} \geq I_{n_j}^{*ip} \Rightarrow \frac{1}{1+\left\{\sum_{j=1}^{z}\frac{\overline{\overline{H}}^{j}}{\sum_{j=1}^{z}\overline{\overline{H}}^{j}}\left(\frac{1-I_{n_j}^{ip}}{I_{n_j}^{ip}}\right)^{\lambda}\right\}^{1/\lambda}} \geq \frac{1}{1+\left\{\sum_{j=1}^{z}\frac{\overline{\overline{H}}^{j}}{\sum_{j=1}^{z}\overline{\overline{H}}^{j}}\left(\frac{1-I_{n_j}^{*ip}}{I_{n_j}^{*ip}}\right)^{\lambda}\right\}^{1/\lambda}}$$

Then, in the presence of the data in Eq. (8) and Eq. (9), we have $CIFDPA\left(\dot{\nabla}_1^{CIF},\dot{\nabla}_2^{CIF},\ldots,\dot{\nabla}_z^{CIF}\right) \leq CIFDPA\left(\dot{\nabla}_1^{*CIF},\dot{\nabla}_2^{*CIF},\ldots,\dot{\nabla}_z^{*CIF}\right)$.

Property 3: (***Boundedness***) Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any z CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp},\Re_{m_j}^{ip}\right),\left(I_{n_j}^{rp},I_{n_j}^{ip}\right)\right), j=1,2,\ldots,z$, if $\dot{\nabla}_j^{-} = \left(\left(\min_j \Re_{m_j}^{rp},\min_j \Re_{m_j}^{ip}\right),\left(\max_j I_{n_j}^{rp},\max_j I_{n_j}^{ip}\right)\right)$ and $\dot{\nabla}_j^{+} = \left(\left(\max_j \Re_{m_j}^{rp},\max_j \Re_{m_j}^{ip}\right),\left(\min_j I_{n_j}^{rp},\min_j I_{n_j}^{ip}\right)\right)$, then

$$\dot{\nabla}_j^{-} \leq CIFDPA\left(\dot{\nabla}_1^{CIF},\dot{\nabla}_2^{CIF},\ldots,\dot{\nabla}_z^{CIF}\right) \leq \dot{\nabla}_j^{+} \tag{18}$$

Proof: Select the theory in Proposition 1 and Proposition 2; we have

$$CIFDPA\left(\dot{\nabla}_1^{CIF},\dot{\nabla}_2^{CIF},\ldots,\dot{\nabla}_z^{CIF}\right) \leq CIFDPA\left(\dot{\nabla}_1^{+},\dot{\nabla}_2^{+},\ldots,\dot{\nabla}_z^{+}\right) = \dot{\nabla}_j^{+}$$
$$CIFDPA\left(\dot{\nabla}_1^{CIF},\dot{\nabla}_2^{CIF},\ldots,\dot{\nabla}_z^{CIF}\right) \geq CIFDPA\left(\dot{\nabla}_1^{-},\dot{\nabla}_2^{-},\ldots,\dot{\nabla}_z^{-}\right) = \dot{\nabla}_j^{-}$$

Thus, we have

$$\dot{\nabla}_j^{-} \leq CIFDPA\left(\dot{\nabla}_1^{CIF},\dot{\nabla}_2^{CIF},\ldots,\dot{\nabla}_z^{CIF}\right) \leq \dot{\nabla}_j^{+}.$$

Definition 7: Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any z CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp},\Re_{m_j}^{ip}\right),\left(I_{n_j}^{rp},I_{n_j}^{ip}\right)\right), j=1,2,\ldots,z$, then

$$\begin{aligned} CIFWDPA\left(\dot{\nabla}_1^{CIF},\dot{\nabla}_2^{CIF},\ldots,\dot{\nabla}_z^{CIF}\right) &= \frac{\varpi_1\overline{\overline{H}}^{1}}{\sum_{j=1}^{z}\varpi_j\overline{\overline{H}}^{j}}\dot{\nabla}_1^{CIF} \oplus \frac{\varpi_2\overline{\overline{H}}^{2}}{\sum_{j=1}^{z}\varpi_j\overline{\overline{H}}^{j}}\dot{\nabla}_2^{CIF} \oplus \ldots \\ \oplus \frac{\varpi_z\overline{\overline{H}}^{z}}{\sum_{j=1}^{z}\varpi_j\overline{\overline{H}}^{j}}\dot{\nabla}_z^{CIF} &= \oplus_{j=1}^{z}\left(\frac{\varpi_j\overline{\overline{H}}^{j}}{\sum_{j=1}^{z}\varpi_j\overline{\overline{H}}^{j}}\dot{\nabla}_j^{CIF}\right) \end{aligned} \tag{19}$$

Here, $\overline{\overline{\mathrm{H}^1}} = 1$ and $\overline{\overline{\mathrm{H}^j}} = \prod_{k=1}^{j-1} \dot{\nabla} sv\left(\dot{\nabla}_k^{CIF}\right)$ with weight vector $\varpi_j \in [0,1]$ such that $\sum_{j=1}^{z} \varpi_j = 1$.

Theorem 2: Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any two CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right), j = 1,2,\ldots,z$, then in the presence of the information in Eq. (19), we derive that the below theory, such as:

$$CIFWDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_z^{CIF}\right) = \left(\begin{array}{c}\left(1 - \dfrac{1}{1 + \left\{\sum_{j=1}^{z} \dfrac{\varpi_j \overline{\overline{\mathrm{H}^j}}}{\sum_{j=1}^{z} \varpi_j \overline{\overline{\mathrm{H}^j}}} \left(\mathbb{R}_{m_j}^{rp}\right)^{\lambda}\right\}^{1/\lambda}}, 1 - \dfrac{1}{1 + \left\{\sum_{j=1}^{z} \dfrac{\varpi_j \overline{\overline{\mathrm{H}^j}}}{\sum_{j=1}^{z} \varpi_j \overline{\overline{\mathrm{H}^j}}} \left(\mathbb{R}_{m_j}^{ip}\right)^{\lambda}\right\}^{1/\lambda}}\right), \\ \left(\dfrac{1}{1 + \left\{\sum_{j=1}^{z} \dfrac{\varpi_j \overline{\overline{\mathrm{H}^j}}}{\sum_{j=1}^{z} \varpi_j \overline{\overline{\mathrm{H}^j}}} \left(\mathrm{II}_{n_j}^{rp}\right)^{\lambda}\right\}^{1/\lambda}}, \dfrac{1}{1 + \left\{\sum_{j=1}^{z} \dfrac{\varpi_j \overline{\overline{\mathrm{H}^j}}}{\sum_{j=1}^{z} \varpi_j \overline{\overline{\mathrm{H}^j}}} \left(\mathrm{II}_{n_j}^{ip}\right)^{\lambda}\right\}^{1/\lambda}}\right)\end{array}\right) \quad (20)$$

Proof: Similar to Theorem 1, omitted.

Property 4: (***Idempotency***) Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any z CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right), j = 1,2,\ldots,z$, if $\dot{\nabla}_j^{CIF} = \dot{\nabla}^{CIF} = \left(\left(\Re_m^{rp}, \Re_m^{ip}\right), \left(\mathrm{I}_n^{rp}, \mathrm{I}_n^{ip}\right)\right)$, then

$$CIFWDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_z^{CIF}\right) = \dot{\nabla}^{CIF} \quad (21)$$

Property 5: (***Monotonicity***) Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any two CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right), j = 1,2,\ldots,z$, if $\dot{\nabla}_j^{CIF} \le \dot{\nabla}_j^{*CIF}$, then

$$CIFWDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_z^{CIF}\right) \le CIFWDPA\left(\dot{\nabla}_1^{*CIF}, \dot{\nabla}_2^{*CIF}, \ldots, \dot{\nabla}_z^{*CIF}\right) \quad (22)$$

Property 6: (***Boundedness***) Assume that $\dot{\mathcal{V}}_u$ be the family of fixed set with generic elements, then in the presence of any z CIFVs $\dot{\nabla}_j^{CIF} = \left(\left(\Re_{m_j}^{rp}, \Re_{m_j}^{ip}\right), \left(\mathrm{I}_{n_j}^{rp}, \mathrm{I}_{n_j}^{ip}\right)\right), j = 1, 2, \ldots, z$, if $\dot{\nabla}_j^{-} = \left(\left(\min_j \Re_{m_j}^{rp}, \min_j \Re_{m_j}^{ip}\right), \left(\max_j \mathrm{I}_{n_j}^{rp}, \max_j \mathrm{I}_{n_j}^{ip}\right)\right)$ and $\dot{\nabla}_j^{+} = \left(\left(\max_j \Re_{m_j}^{rp}, \max_j \Re_{m_j}^{ip}\right), \left(\min_j \mathrm{I}_{n_j}^{rp}, \min_j \mathrm{I}_{n_j}^{ip}\right)\right)$, then

$$\dot{\nabla}_j^{-} \leq CIFWDPA\left(\dot{\nabla}_1^{CIF}, \dot{\nabla}_2^{CIF}, \ldots, \dot{\nabla}_z^{CIF}\right) \leq \dot{\nabla}_j^{+} \tag{23}$$

4. An Approach for Resilient Green Supplier Selection -Based Proposed Theory

In this section, we will construct a MADM method based on the proposed CIFWDPA operator to solve the practical problem of resilient green supplier selection.

4.1 Method Description

To accommodate the above problem, we select the collection of finite values of alternatives, that is $\dot{\nabla}_1^{Al}, \dot{\nabla}_2^{Al}, \ldots, \dot{\nabla}_g^{Al}$. For every alternative, we consider the finite values of attributes such as $\dot{\nabla}_1^{At}, \dot{\nabla}_2^{At}, \ldots, \dot{\nabla}_z^{At}$ with weight vector $\boldsymbol{\varpi}_j \in [0,1]$ such that $\sum_{j=1}^{z} \boldsymbol{\varpi}_j = 1$. In the presence of the above information, such as *g*-alternatives and *z*-attributes, we concentrate on computing the decision matrix by using the theory of information of CIF values; we explained the complex form of truthful information and the complex form of falsity information such as $\Re_m(\dot{x}_u) = \left(\Re_m^{rp}(\dot{x}_u), \Re_m^{ip}(\dot{x}_u)\right)$ and $\mathrm{I}_n(\dot{x}_u) = \left(\mathrm{I}_n^{rp}(\dot{x}_u), \mathrm{I}_n^{ip}(\dot{x}_u)\right)$ with a specific condition: $0 \leq \Re_m^{rp}(\dot{x}_u) + \mathrm{I}_n^{rp}(\dot{x}_u) \leq 1$ for the real part and $0 \leq \Re_m^{ip}(\dot{x}_u) + \mathrm{I}_n^{ip}(\dot{x}_u) \leq 1$ for imaginary parts. Moreover, the neutral information is stated by: $\Gamma_m(\dot{x}_u) = \left(\Gamma_m^{rp}(\dot{x}_u), \Gamma_m^{ip}(\dot{x}_u)\right) = \left(1 - \left(\Re_m^{rp}(\dot{x}_u) + \mathrm{I}_n^{rp}(\dot{x}_u)\right), 1 - \left(\Re_m^{ip}(\dot{x}_u) + \mathrm{I}_n^{ip}(\dot{x}_u)\right)\right)$ and the final and simplest form of CIFV is stated: $\dot{\nabla}_{hj}^{CIF} = \left(\left(\Re_{m_{hj}}^{rp}, \Re_{m_{hj}}^{ip}\right), \left(\mathrm{I}_{n_{hj}}^{rp}, \mathrm{I}_{n_{hj}}^{ip}\right)\right)$, $h = 1, 2, 3, \ldots, g;\ j = 1, 2, 3, \ldots, z$. Therefore, to address some real-life problems, we compute the procedure of decision-making for evaluating the finest optimal form of the collection of preferences, such as:

Step 1: Arrange the CIFVs as a decision matrix.

Step 2: Normalize the decision matrix if possible (in the case of cost of data)

$$\dot{\nabla}_{hj}^{CIF} = \begin{cases} \left(\left(\Re_{m_{hj}}^{rp}, \Re_{m_{hj}}^{ip}\right), \left(\mathrm{I}_{n_{hj}}^{rp}, \mathrm{I}_{n_{hj}}^{ip}\right)\right) & \text{for benfit attribute} \\ \left(\left(\mathrm{I}_{n_{hj}}^{rp}, \mathrm{I}_{n_{hj}}^{ip}\right), \left(\Re_{m_{hj}}^{rp}, \Re_{m_{hj}}^{ip}\right)\right) & \text{for cost attribute} \end{cases}$$

Step 3: Aggregate the data in the decision matrix with the help of the CIFWDPA operator as Eq. (20), and obtain the final aggregated value $\dot{\nabla}_h^{CIF}\,(h=1,2,3,\ldots,g)$ for each alternative.

Step 4: Determine the SVs $SV\left(\dot{\nabla}_h^{CIF}\right)(h=1,2,3,\ldots,g)$ of the aggregated data with the help of data in Eq. (8). If necessary, the AV is also required.

Step 5: Evaluate or derive the ranking values in the presence of the values of the score data and examine the finest optimal from the collection of preferences.

Furthermore, we simplify the above procedure of decision-making technique with the help of numerical examples in the environment of the real-life problem and try to evaluate it in the presence of the presented operators.

4.2 Application Example

Smart green resilient, where the short form is SGR, refers to an emerging planning technique or approach that themes to enable pragmatic concerns to be evaluated continuously with main environmental problems and to compute long-life resilience in urban planning strategies, sustainability, masterplans, and frameworks for an extensive range of contexts. With the global economic recovery in the post-pandemic era, the selection of resilient green suppliers has become particularly important in the enterprise supply chain management, as it can significantly improve the economic benefits of enterprises. This problem is taken from Ref. [27], computed based on sustainable supply chain management. This problem aims to study supplier selection by integrating lean, green agile, and sustainable aspects of the supply chains and resilience. Therefore, in this problem, we consider the finest supplier chain for future consideration. For this, we consider five majors from the supplier chain which are used as the alternatives such as:

$\dot{\nabla}_1^{Al}$: ***Sustainable supply chain:*** The green supply chain is becoming an essential and major part of the strategic thrust in business companies.

$\dot{\nabla}_2^{Al}$: ***Lean supply chain:*** Lean supply chain depended on the lean management principles at Toyota Motor Corporation, Japan, concentrating on waste lessening and attaining higher earnings.

$\dot{\nabla}_3^{Al}$: ***Resilient supply chain:*** The capability of a supply chain to cope with surprising interruption is described as a resilient supply chain.

$\dot{\nabla}_4^{Al}$: ***Green supply chain:*** It includes efforts to improve environmental performance, cut costs and waste in internal supply chains, external supply chains, and reverse logistics to eliminate negative environmental effects

$\dot{\nabla}_5^{Al}$: ***Agile supply chain:*** In this procedure, flexibility and quick reconfigurability are the main concentrates.

To select the best alternative, we give the following features as attribute information: $\dot{\nabla}_1^{At}$: Response rate, $\dot{\nabla}_2^{At}$: Capability for unexpected disruptions, $\dot{\nabla}_3^{At}$: Capability for reverse logistics, $\dot{\nabla}_4^{At}$: Sustainable product design, and $\dot{\nabla}_5^{At}$: Long-term relationship. Therefore, to address the above real-life problem, we use the procedure of decision-making for evaluating the finest optimal form of the collection of preferences, such as:

Step 1: Arrange the CIF values as a decision matrix in Table 1.

Step 2: Normalize the decision matrix. It's worth noting that the data in Table 1 is not required to be normalized.

Table 1 Mathematical representation of CIF decision matrix

	$\dot{\nabla}_1^{At}$	$\dot{\nabla}_2^{At}$	$\dot{\nabla}_3^{At}$	$\dot{\nabla}_4^{At}$	$\dot{\nabla}_5^{At}$
$\dot{\nabla}_1^{Al}$	$\begin{pmatrix}(0.7,0.7),\\(0.1,0.2)\end{pmatrix}$	$\begin{pmatrix}(0.71,0.71),\\(0.11,0.21)\end{pmatrix}$	$\begin{pmatrix}(0.72,0.72),\\(0.12,0.22)\end{pmatrix}$	$\begin{pmatrix}(0.73,0.73),\\(0.13,0.23)\end{pmatrix}$	$\begin{pmatrix}(0.74,0.74),\\(0.14,0.24)\end{pmatrix}$
$\dot{\nabla}_2^{Al}$	$\begin{pmatrix}(0.5,0.4),\\(0.2,0.3)\end{pmatrix}$	$\begin{pmatrix}(0.51,0.41),\\(0.21,0.31)\end{pmatrix}$	$\begin{pmatrix}(0.52,0.42),\\(0.22,0.32)\end{pmatrix}$	$\begin{pmatrix}(0.53,0.43),\\(0.23,0.33)\end{pmatrix}$	$\begin{pmatrix}(0.54,0.44),\\(0.24,0.34)\end{pmatrix}$
$\dot{\nabla}_3^{Al}$	$\begin{pmatrix}(0.5,0.5),\\(0.2,0.4)\end{pmatrix}$	$\begin{pmatrix}(0.51,0.51),\\(0.21,0.41)\end{pmatrix}$	$\begin{pmatrix}(0.52,0.52),\\(0.22,0.42)\end{pmatrix}$	$\begin{pmatrix}(0.53,0.53),\\(0.23,0.43)\end{pmatrix}$	$\begin{pmatrix}(0.54,0.54),\\(0.24,0.44)\end{pmatrix}$
$\dot{\nabla}_4^{Al}$	$\begin{pmatrix}(0.4,0.6),\\(0.3,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.41,0.61),\\(0.31,0.11)\end{pmatrix}$	$\begin{pmatrix}(0.42,0.62),\\(0.32,0.12)\end{pmatrix}$	$\begin{pmatrix}(0.43,0.63),\\(0.33,0.13)\end{pmatrix}$	$\begin{pmatrix}(0.44,0.64),\\(0.34,0.14)\end{pmatrix}$
$\dot{\nabla}_5^{Al}$	$\begin{pmatrix}(0.4,0.3),\\(0.2,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.41,0.31),\\(0.21,0.11)\end{pmatrix}$	$\begin{pmatrix}(0.42,0.32),\\(0.22,0.12)\end{pmatrix}$	$\begin{pmatrix}(0.43,0.33),\\(0.23,0.13)\end{pmatrix}$	$\begin{pmatrix}(0.44,0.34),\\(0.24,0.14)\end{pmatrix}$

Step 3: Aggregate the data in the decision matrix with the help of the Eq. (20) (where $\lambda = 3$), and obtain the aggregated value $\dot{\nabla}_1^{CIF} = \left((0.8161, 0.8161),(0.0012, 0.0431)\right)$, $\dot{\nabla}_2^{CIF} = \left((0.0048, 0.0006),(0.3540, 0.8769)\right)$, $\dot{\nabla}_3^{CIF} = \left((0.0048, 0.0048),(0.3540, 0.9840)\right)$, $\dot{\nabla}_4^{CIF} = \left((0.0020, 0.1040),(0.6965, 0.0045)\right)$, $\dot{\nabla}_5^{CIF} = \left((0.0006, 0.0001),(0.3540, 0.0139)\right)$.

Step 4: Determine the score values of the aggregate data with the help of data in Eq. (8), and get $SV\left(\dot{\nabla}_1^{CIF}\right) = 0.7940$, $SV\left(\dot{\nabla}_2^{CIF}\right) = -0.6127$, $SV\left(\dot{\nabla}_3^{CIF}\right) = -0.6642$, $SV\left(\dot{\nabla}_4^{CIF}\right) = -0.2976$, $SV\left(\dot{\nabla}_5^{CIF}\right) = -0.1836$.

Step 5: Evaluate or derive the ranking values in the presence of the values of the score data and examine the finest optimal from the collection of preferences. Thus, we get the rank result of $\dot{\nabla}_1^{Al} > \dot{\nabla}_5^{Al} > \dot{\nabla}_4^{Al} > \dot{\nabla}_2^{Al} > \dot{\nabla}_3^{Al}$, and $\dot{\nabla}_1^{Al}$ is the best decision.

5. Parametric Analysis and Comparative Analysis

In this section, we compare the proposed operators with some prevailing techniques to show the worth and supremacy of the invented techniques. Before making the comparison, we check the sensitivity of the parameter λ by using its different values in the presence of the data in Table 1. The experimental results are shown in Table 2 and Fig. 1.

Table 2 Stability of the parameters for the CIFWDPA operator

Parameters	Ranking results
ƛ = 1	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_2^{AI} = \dot{\nabla}_3^{AI}$
ƛ = 2	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
ƛ = 3	$\dot{\nabla}_1^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
ƛ = 4	$\dot{\nabla}_1^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
ƛ = 5	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
ƛ = 6	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
ƛ = 7	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
ƛ = 8	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
ƛ = 9	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_3^{AI}$
ƛ = 10	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_3^{AI}$

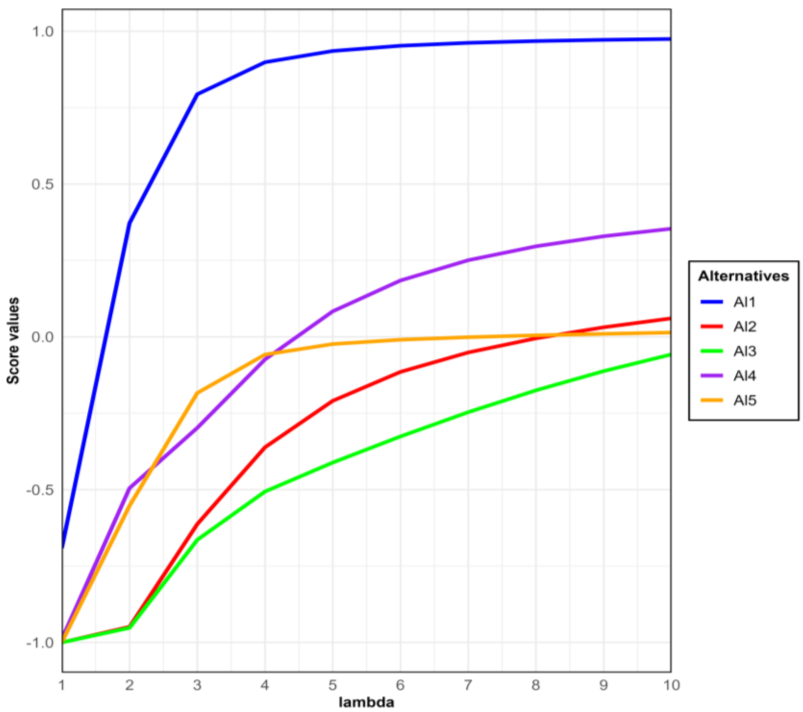

Fig. 1 Score values of alternatives under different ƛ values

According to Table 2, although the ranking results for each parameter are not precisely the same, the optimal results are the same, they are $\dot{\nabla}_1^{AI}$. Meanwhile, as shown in Fig.1, as parameter ƛ increases, the score value of each alternative increases. From the above parameter analysis, it can be seen that the parameter $\hat{\lambda}$ can reflect the preferences of different decision-makers (DMs) so that DMs can choose flexibly according to different decision environments and risk preferences. For example, DMs who are risk-averse can choose a smaller $\hat{\lambda}$ value. Of course, this is also based on the premise that the CIFWDPA operator-based MADM method can maintain a specific stability of the result.

Table 3 Result representation of the comparative analysis

Methods	Ranking results
Yu and Xu's method [25]	$-----Bounded-----$
Seikh and Mandal's method [26]	$-----Bounded-----$
Senapati's method [28]	$-----Bounded-----$
Garg and Rani's method [19] (CIFWA operator)	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} = \dot{\nabla}_2^{AI} = \dot{\nabla}_3^{AI}$
Garg and Rani's method [20] (CIFWA operator)	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} = \dot{\nabla}_2^{AI} = \dot{\nabla}_3^{AI}$
Garg and Rani's method [21] (CIFAWG operator)	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} = \dot{\nabla}_2^{AI} = \dot{\nabla}_3^{AI}$
Garg and Rani's method [22] (CIFWG operator)	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} = \dot{\nabla}_2^{AI} = \dot{\nabla}_3^{AI}$
Our method ($\hat{\lambda}=3$)	$\dot{\nabla}_1^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
Our method ($\hat{\lambda}=5$)	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_3^{AI}$
Our method ($\hat{\lambda}=9$)	$\dot{\nabla}_1^{AI} > \dot{\nabla}_4^{AI} > \dot{\nabla}_2^{AI} > \dot{\nabla}_5^{AI} > \dot{\nabla}_3^{AI}$

Furthermore, to demonstrate the proposed method's superiority, we evaluate the comparison between the proposed work and some existing MADM methods. For this, we consider the following existing operators: Yu and Xu [25] presented a novel theory of prioritized aggregation operators for IFSs. Further, Seikh and Mandal [26] exposed the major information of Dombi aggregation operators for IFS in the presence of the Dombi t-norm and t-conorm. Senapati et al. [28] derived the Aczel-Alsina aggregation operators in the presence of the IFS and their application in decision-making. Furthermore, aggregation operators for CIF values were derived by Garg and Rani [19], novel aggregation operators for CIF information were presented by Garg and Rani [20], geometric aggregation operators for CIFS were exposed by Garg and Rani [21], and robust averaging and geometric aggregation operators for CIFS was presented by Garg and Rani [22]. Based on the data in Table 1, the comparative analysis results are listed in Table 3.

As shown in Table 3, we obtained the same ranking results, where the optimal alternatives are all $\dot{\nabla}_1^{AI}$ according to the theory of the CIFWDPA operator, which is consistent with the results obtained by methods in [19-22]. This first indicates that the method proposed in this paper is effective. The selection results are not exactly the same, mainly because the characteristics of various methods are different. The advantages of the proposed method and the limitations of the existing work are stated below:

The prioritized aggregation operators in [25], the Dombi aggregation operators in [26], and the Aczel-Alsina aggregation operators in [28] are proposed for IFS, and it is clear that the theory of IFS has the special case of the CIFS; therefore, these proposed operators in [25, 26, 28] have failed to evaluate the information in Table 1. However, the method proposed in this paper is based on the CIFS with stronger information representation ability, which can deal with more complex decision-making and evaluation problems and more conveniently simulate human decision-making cognition.

Compared to the theory of Garg and Rani [19-22], the results obtained by their method are significantly different from those obtained by the method proposed in this paper. However, the optimal choice is the same. And the methods in [19-22] cannot effectively distinguish the priority of alternatives $\dot{\nabla}_2^{AI}$, $\dot{\nabla}_3^{AI}$, and $\dot{\nabla}_5^{AI}$. This is because the operators proposed in this paper adopt the Dombi operation rules and consider the priority relationship between the aggregated variables. Therefore, the constructed aggregation operators are more flexible and have more comprehensive functions.

In summary, the characteristics of each method can be summarized in Table 4.

Table 4 Comparison of characteristics of different methods

Characteristics / Methods	Stronger information representation ability	Considering DM's preference attitude	Considering attributes' priority relationship
Yu and Xu's method [25]	No	No	Yes
Seikh and Mandal's method [26]	No	Yes	No
Senapati's method [28]	No	Yes	No
Garg and Rani's method [19]	Yes	No	No
Garg and Rani's method [20]	Yes	No	No
Garg and Rani's method [21]	Yes	Yes	No
Garg and Rani's method [22]	Yes	No	No
Our method	Yes	Yes	Yes

6. CONCLUSIONS

The prioritized aggregation and Dombi norms play a valuable and dominant role in the environment of fuzzy set theory. Inspired by the structure of the CIFS as well as the idea of Dombi prioritized aggregation operators, the significant influence of this paper is listed below:

(1) This paper proposed the theory of Dombi operational laws based on CIF information.

(2) This paper examined the theory of the CIFDPA and CIFWDPA operators, which are the modified versions of the prioritized aggregation operators and Dombi aggregation operators for fuzzy, intuitionistic, complex, and complex intuitionistic fuzzy information. Some reliable properties for the above operators are also established.

(3) This paper derived a MADM method based on the proposed operators and applied it to the resilient green supplier selection problem.

(4) This paper discusses the advantages and characteristics of the proposed method through detailed comparative analysis.

In future research, we will extend CIFS to some novel MADM techniques, such as the RAFSI method [29]. Furthermore, we will apply it to the selection of the group of construction machines [30], evaluation of ecological governance [31] and other fields.

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