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Local Partition Dimension of Grid Graph and Its Application to the Coordinates of Potential Disaster Areas in Jember Regency

Ilham Saifudin[1], Reni Umilasari[2], Nanang Saiful Rizal[3]
[1,2]Informatics Engineering, University of Muhammadiyah Jember, Indonesia
[3]Civil Engineering, Engineering Faculty, University of Muhammadiyah Jember, Indonesia
ilham.saifudin@unmuhjember.ac.id[1], reni.umilasari@unmuhjember.ac.id[2],
nanangsaifulrizal@unmuhjember.ac.id[3]

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ABSTRACT

Partition dimension was introduced as a part of interesting topic in graph theory. It was focus to observe about distance. The local partition dimension is an expansion of the partition dimension by adding certain conditions to the representation of the vertices v to which is Π denoted by $r(v|\ \Pi)$. One of the conditions that must be met for $r(v|\ \Pi)$ is discussed. The minimum value of k so that there is a local distinguishing partition from $V\ (G)$ is the local partition dimension of G or it can can be said that the distance of each neighbor is different. The local partition dimension of a graph G is denoted $pd_l(G)$. In this study, we used an axiomatic deductive methods and pattern recognition. In order to construct the discriminating set on the metric dimension (dim) and the discriminating partition on the partition dimension (pd), the pattern detection method looks for patterns in which the coordinate values are minimum and different. By all observations, the local partition dimensions of $P_n \times P_m$ Grid graph has two condition about the results of resolving partition. The Result of local partition dimension of a Grid graph $pd_l(P_n \times P_m) = 2$, where $n \geq 2$ dan $m \geq 2$. In addition, it will decide how to convert the coordinates of areas in the Jember district that are prone to flooding and landslides into metric dimensions. It was about Coordinates of Flood and Landslide Disaster Locations in Jember Regency. The number of local and minimal partition sets generated for flood-prone areas in Jember Regency is $pd_l(G_{Jember}) = 3$.

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A. INTRODUCTION

Mathematics is a substantial science in every aspect of life. Some of the results of the development of science and technology were born from numbers and calculations. Several sciences are derived from mathematics, including statistics, computer science, engineering, actuarials, construction, astronomy, and others (Badawi, 2017) and (Badawi, 2014). The mathematics field of interest and continues to go through the development process is graph theory. Mathematics has a subfield called graph theory. Discrete Mathematics includes graph theory in computer science (Lestari, Kanedi, & Arliando, 2016). Numerous other fields, including agriculture, forestry, computer security, and others, also utilize graph theory extensively. A graph is defined as a pair of sets (V, E) written with the notation $G = (V, E)$, where V denotes a set of vertices that are not empty and E denotes a set of edges that connect two vertices but could be empty (Chartrand & Lesniak, 1996). The notation G can be used to express a graph without mentioning the set of vertices and edges (Badawi & Rissner, 2020). The partition dimension is one of many types of research in graph theory (Slamin, 2009).

The partition dimension is the result of the development of the metric dimension. As an explanation, a set known as a locating set is often known as a resolving set (Fransiskus Fran, 2020). Furthermore, the concept of a discriminating partition is similar to the resolving set of the graph. Grouping the vertices in graph G into several partition classes and calculating the distance of each vertex in G to all partition classes to represent each vertex in graph G (Riza, 2016), (Saifudin, Umilasari, & Jalil, 2022), (Yi, 2013), and (Budianto & Kusmayadi, 2018). Let $S \subset V(G)$ and the vertex $v \in G$, $d(v, S)$ is the distance between v and S defined as d $d(v, S) = min\{d(v, x)|x \in S\}$. For $k-$partitions of = $\Pi = \{S_1, S_2, S_3, \ldots, S_k\}$ and vertices v of $V(G)$ (Okamoto, Phinezy, & Zhang, 2010), (Rodríguez-Velázquez, García Gómez, & Barragán-Ramírez, 2015), (Rodríguez-Velázquez, Barragán-Ramírez, & García Gómez, 2016), (Kuziak, Rodriguez-Velazquez, & Yero, 2017), and (Rinurwati, Suprajitno, & Slamin, 2017).

The representation of $v \in V(G)$ against Π is $k-$vector $r(v|\Pi) = \{d(v, S_1), d(v, S_2), \ldots, d(v, S_k)\}$. For any two distinct vertices $u, v \in V(G)$ applies r $r(u|\Pi) \neq r(v|\Pi)$, then Π is called the resolving partition of $V(G)$. The resolving partition with minimum cardinality is called the minimum resolving partition of G and is denoted by $pd(G)$ (Rodríguez-Velázquez, González Yero, & Lemańska, 2014), (Cahyabudi & Kusmayadi, 2017), (Klavžar & Tavakoli, 2020) and (Saifudin & Umilasari, 2021).

In previous studies, the partition dimension that has been studied is "Dimensions of Partitions of Several Classes of Trees". This results in the partition dimensions of several tree classes including a maximum of 17 possible representations of $v_{i,1}$, thus the maximum limit is 17 (Arimbawa & Baskoro, 2015). In addition, it was also investigated by Fredlina & Baskoro (2015) with the title "Dimensions of Partitions in Several Tree Graph Families" for example $C(m; n)$ is a homogeneous caterpillar with $m, n \geq 1$. Then, $pd(C(m; n)) = 3$ if and only if ($n =$ 1 and $m \geq 3$) or ($n = 2$ and $m \geq 2$) or ($n = 3$ and $m \leq 3$).

Meanwhile, this research will investigate the resolving set and the local partition dimension number of $P_n \times P_m$ Grid graph that has not been studied before. A Grid graph is chosen as the object to be observe in this research because the characteristic of this graph can represent the geometry of a region. One vertex have 2 or 3 neighbor. Thus, before apply this topic into a real life problem, we focus to determine the local partition dimension number of $P_n \times P_m$. After having some analysis, then we continue to determine the applications related to partition dimensions in the disaster field.

The subject of interest that needs to be implemented recently is related to the increasingly uncertain climate issue. Many disasters hit various regions, and one of them is in Jember Regency. The most recent disaster is flooding. There are several causes for floods, including shallower rivers, less water infiltration, and poor hygiene patterns. There are four locations in Jember Regency where floods happen frequently. The affected locations are Manggisan Village in Tanggul District, Pondok Joyo Village, Pondok Dalem Village in Semboro District, Kramat Sukoharjo Village in Tanggul District, and Bracelet Village in Sumberbaru District (CNN, 2021), (Putri & Sarifuddin, 2019), and (Faizana, Nugraha, & Yuwono, 2015).

As a result, it requires a breakthrough in the form of coordinates for the Jember district's disaster-prone areas. The coordinates of the region or district are represented by each vertex representation, while the center of a region in the Jember Regency that is prone to flooding and landslides is represented by the graph base. The result of the minimum coordinates of a graph representation of the Jember district is the value of the partition dimension. This coordinates show the location to authorized officer quickly and accurately in the process of saving or dealing with floods. From the explanation, this research raises the title "Local Partition Dimension of Grid Graph and Its Application to the Coordinates of Potential Disaster Areas in Jember Regency".

B. METHODS

This section shows all steps to get a local partition dimension of graphs. First, we can decide to observe what kind of familiar or well define graphs. Then, give the vertex label to make it easy for mention every vertex. After that, we can start to constructing the local resolving partition candidates local and choose the local resolving partition with minimum cardinality. So, we can get the value of local partition dimension. At the end continue to analyze and observe the conclusion. Those procedures also can be applied to find the number of local and minimal partition sets generated for flood-prone areas in Jember Regency. To be more clear, see the research flow at the Figure 1.

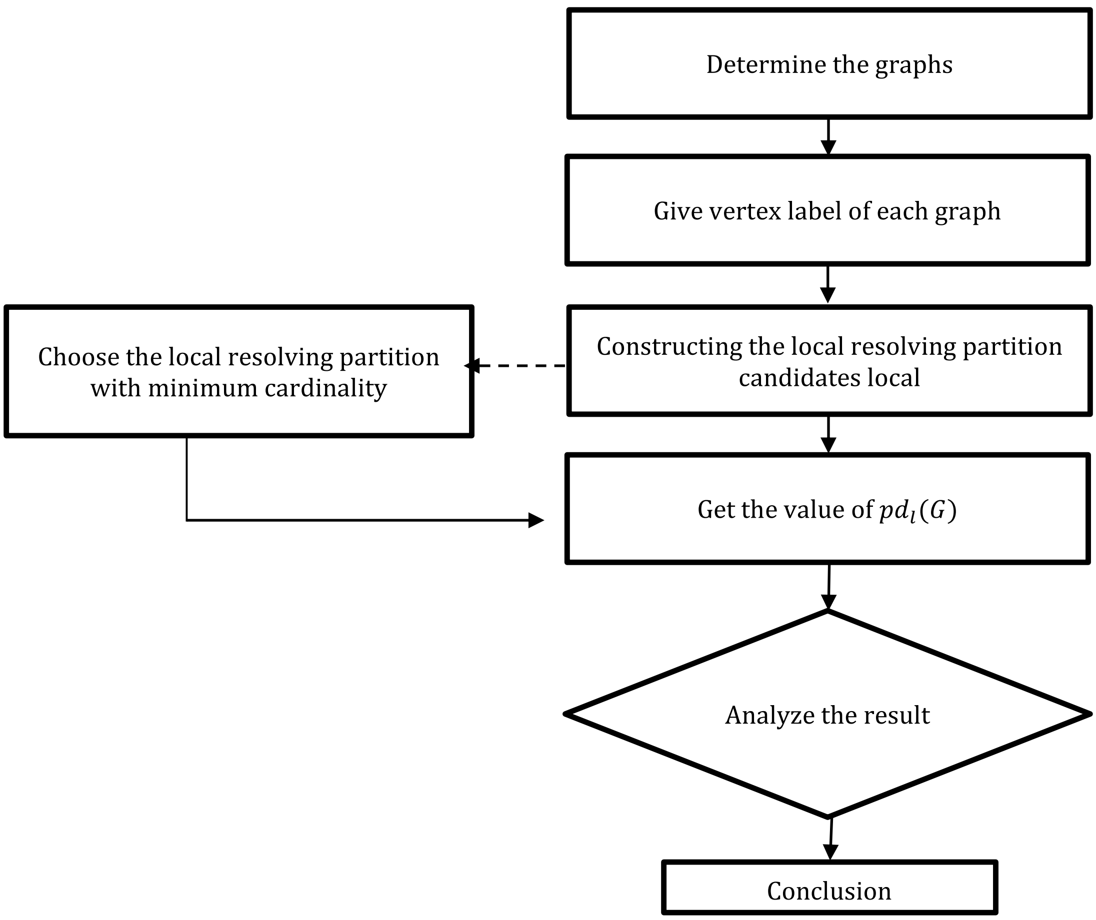

Figure 1. Research Flow

Axiomatic deductive methods and pattern recognition are the tools used in this study. In order to construct the resolving set on the metric dimension (dim) and the resolving partition on the partition dimension (pd) so that the coordinate values are minimum and different, the pattern detection method looks for patterns. In contrast, axiomatic deductive is a research strategy that employs existing axioms or theorems to solve a problem and applies the principles of deductive proof to mathematical logic. The method is then applied to the $P_n \times P_m$ Grid graph's local partition dimensions. In addition, it will decide how to convert the coordinates of areas in the Jember district that are prone to flooding and landslides into metric dimensions. The description of the research design will be explained as follows:

1. determine the graph that will be used to analyze the partition dimensions;
2. determine the label of each vertex so that the vertices are different and get a formulation of the vertex set.
3. determine the local resolving set and local partition dimensions which be denoted by $pd_l(G)$ on the grid graph $P_n \times P_m$;
4. construct the coordinates of $dim_l(G)$ and $pd_l(G)$;
5. analyze the metric dimension values and local partition dimensions of the $P_n \times P_m$ Grid graph;
6. conclude the results from the analysis of the local dimensions of the $P_n \times P_m$Grid graph.

C. RESULT AND DISCUSSION

For the purpose of determining the undirected graph's local resolving partition, we will provide some definitions of the local partition dimension in this section. After that, the results of the local partition dimensions of the $P_n \times P_m$ Grid graph with $n, m \geq 2$ and determine its application to the coordinates of the flood and landslide disaster locations in the Jember Regency area.

1. Local Partition Dimension of Grid Graph

The local partition dimension is the expansion of the partition dimension by adding certain conditions to the representation of v to Π which is denoted by $(v|\ \Pi)$. In this study, one of the conditions that must be met for $(v|\ \Pi)$ is discussed. The minimum value of k such that there is a local resolving partition (the distance of each neighbor is different) from V (G) is the local partition dimension of G or can be denoted by $pd_l(G)$. In the following result, there is a theorem that is explained along with the proof of the local partition dimensions of $P_n \times P_m$ Grid graph for $m, n \geq 2$.

◊ **Theorem 1** Let $P_n \times P_m$ is a grid graph. For $m, n \geq 2, pd_l(P_n \times P_m) = 2$.

Proof.

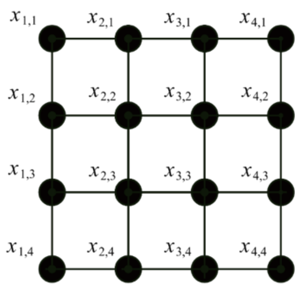

Figure 2. Grid Graph $\boldsymbol{P_4} \times \boldsymbol{P_4}$ with the vertex label

The grid graph $P_n \times P_m$ can be seen at Figure 2 for $n, m = 4$. Let the vertex ser of grid graph is $V(P_n \times P_m) = \{x_{i,j}; 1 \leq i \leq n\ dan\ 1 \leq j \leq m\}$ and $|V| = mn$. While the edge set is $E(P_n \times P_m) = \{x_{i,j}x_{i,j}; 1 \leq i \leq n\ dan\ 1 \leq j \leq m\}$ and $|E| = 2nm - n - m$.

Case 1. For $n, m \geq 2$, it will be shown that the local partition dimensions of the Grid graph $pd_l(P_n \times P_m) \geq 2$ with the local resolving partition $\Pi = \{S_1, S_2\}$, such that $S_k = \{x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}, x_{3,1}, x_{3,2}, x_{4,1}x_{4,2}\}$ for $k = 1$ and
$S_k = \{x_{1,3}, x_{1,4}, x_{2,3}, x_{2,4}, x_{3,3}, x_{3,4}, x_{4,3}x_{4,4}\}$ for $k = 2$.
Where the cardinality of $S_1 = \frac{mn}{2}$ and $S_2 = \frac{mn}{2}$, it means that the position of the local resolving partition is divided into 2 equals to obtain the same or not a different representation for each neighbor on $V(P_n \times P_m)$ with respect to Π. Thus, $pd_l(P_n \times P_m) \ngeq 2$. In the following, we show that each resolving partition has the same coordinates as the neighbors in the vicinity. We give an example of Grid Graph $\boldsymbol{P_4} \times \boldsymbol{P_4}$ with the representation of the local resolving partition at Figure 3.

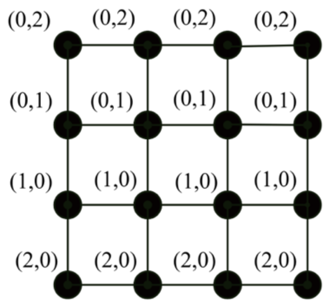

Figure 3. Grid Graph $\boldsymbol{P_4} \times \boldsymbol{P_4}$ with the representation of the local resolving partition

Case 2. The local partition dimension of the Grid graph $pd_l(P_n \times P_m) \geq 2$ with the local resolving partition is $\Pi = \{S_1, S_2\}$, such that $S_k = \{x_{1,1}\}$ for $k = 1$ and $S_k = \{x_{1,1}, x_{i,j}; 2 \leq i \leq n, 1 \leq j \leq m\}$ for $k = 2$. Where $|S_1| = 1$ and $|S_2| = mn - 1$. Then the local resolving partitions are different. Thus, $pd_l(P_n \times P_m) \geq 2$ is proven. Next, to prove that $pd_l(P_n \times P_m) \leq 2$, need a construction that for each vertex of $V(P_n \times P_m)$ has a different representation to Π:
$r(x_{1,1}|\Pi) = (0,1)$ and $r(x_{i,j}|\Pi) = (i, 0)$ for; $2 \leq i \leq n$; $1 \leq j \leq m$.
It can be seen that each vertex of $V(P_n \times P_m)$ has a different local resolving partition concerning Π and has minimal cardinality, then $pd_l(P_n \times P_m) \leq 2$. Since, $pd_l(P_n \times P_m) \geq 2$ and $pd_l(P_n \times P_m) \leq 2$, we get $pd_l(P_n \times P_m) = 2$. The illustration can be seen in Figure 4.

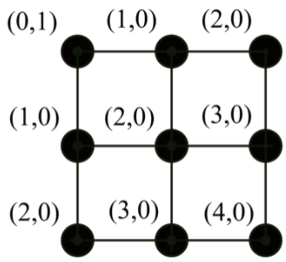

Figure 4. Coordinates of Local Partition Dimensions of Grid Graph $\mathbf{P_3} \times \mathbf{P_3}$

Case 3. The local partition dimension of the Grid graph $pd_l(P_n \times P_m) \geq 2$ with the local resolving partition $\Pi = \{S_1, S_2\}$, such that:

$$S_k = \begin{cases} \{x_{2,1}\}, for\ k = 1 \\ \{x_{1,j};\ 1 \leq j \leq m\} \cup \left\{ \begin{matrix} x_{i,j}; 2 \leq i \leq n\ and \\ 2 \leq j \leq m \end{matrix} \right\}, for\ k = 2 \end{cases}$$

Where the cardinality of $|S_1| = 1$ and $|S_2| = mn - 1$, then the local resolving partition is different. Thus, $pd_l(P_n \times P_m) \geq 2$ is satisfied. For proving $pd_l(P_n \times P_m) \leq 2$, then it can be constructed of each vertex in $V(P_n \times P_m)$ to the local resolving partition Π:

$$r(x_{2,1}|\Pi) = (0,1)$$
$$r(\{x_{1,j};\ 1 \leq j \leq m\}|\Pi) = (i, 0) \text{ for } 1 \leq i \leq n$$
$$r(\{x_{i,j}; 2 \leq i \leq n\ and\ 2 \leq j \leq m\}|\Pi) = (i, 0) \text{ for } 1 \leq i \leq n$$

It can be seen that each vertex of $V(P_n \times P_m)$ has a different local coordinatto concerning Π with minimum cardinality, then $pd_l(P_n \times P_m) \leq 2$. Since $pd_l(P_n \times P_m) \geq 2$ and $pd_l(P_n \times P_m) \leq 2$ then we get $pd_l(P_n \times P_m) = 2$. ■

For more details about the representation of each vertex, we give an example that can be seen in Figure 5 below.

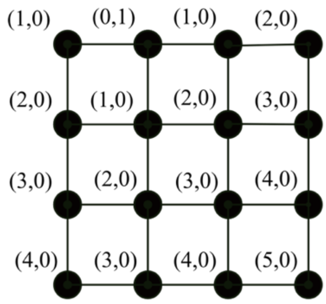

Figure 5. Coordinates of Local Partition Dimensions of Grid Graph $\mathbf{P_4} \times \mathbf{P_4}$

For the grid graph, the local partition dimensions are determined with the position of one partition located between the ends of the grid graph, then we get different local resolving partitions. It can be proved by the explanation of Case 3. On the other side, if one of the partitions is placed at one end of the grid graph, it will obtain a minimal local partition which is different. This proof can be seen in Case 2.

2. The Application of Local Partition Dimension

In this section, the application of local partition dimensions will be explained, namely the coordinates of the location of the flood and landslide disasters in the Jember Regency area. More details can be seen in Figure 6 below.

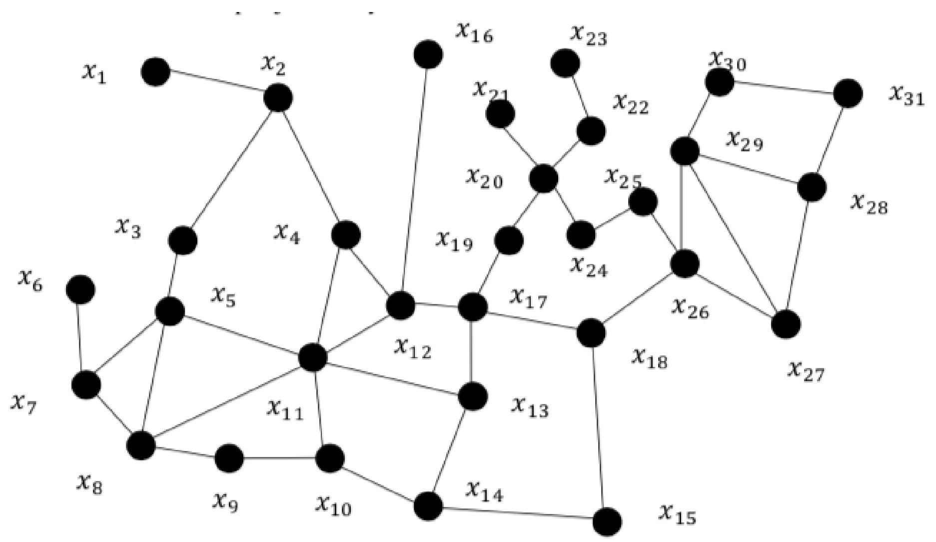

Figure 6. The Representation of Jember District in a Graph

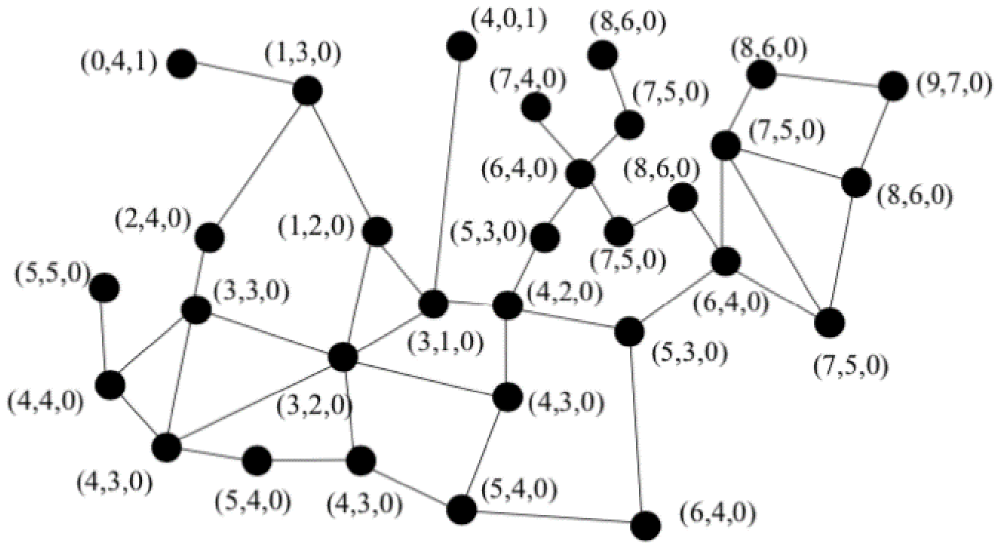

Figure 7. Coordinates of the Flood Location and Landslide Disaster in Jember Regency

Figure 7 above shows the coordinates of the flood location and landslide disaster in Jember Regency. The minimum number of local resolving partitions generated for flood-prone areas in Jember Regency is 3 or we can write $pd_l(G_{Jember}) = 3$. Meanwhile, the number of local coordinates representing the disaster area is 31 districts in Jember Regency. The local resolving partition of G_{Jember} graph can be seen below:

$\Pi = \{S_1, S_2, S_3\}$,
$S_1 = \{x_1\}$;
$S_2 = \{x_{16}\}$;
$$S_3 = \left\{\begin{matrix} x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{17}, x_{18}, x_{19}, \\ x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31} \end{matrix}\right\}$$

D. CONCLUSION AND SUGGESTIONS

Based on the results and previous discussions, the local resolving partition of the grid graph is $\Pi = \{S_1, S_2\}$, such that the formula of S can be devided into two conditions. The local partition dimension of the Grid graph is $pd_l(P_n \times P_m) = 2$, for $n, m \geq 2$. The minimum number of local partition dimensions for generating the flood-prone areas in Jember Regency is $pd_l(G_{Jember}) = 3$. Since it was one of topic development of partition dimension, then we also conclude this research with some open problem below: (1) Find the local resolving partition of some family of graphs; (2) Find the local resolving partition of some product of graphs; and (3) Use this topic to develop/ combine with some other topic in graph theory.

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