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Problem-Based Learning of Heuristic Methods for Decision Problems in Mathematics, Computer Science and Industrial Engineering (PRACTICE)

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ABSTRACT

In a digitalized world, most processes can be formalised, measured and described mathematically. The use of analytical methods to optimise such models and decisions constitutes operational research (OR), developing new methods for a specific problem and analysing them are part of discrete optimisation (DO). However, there is limited research on OR and application driven DO in higher education. Furthermore, neither is well integrated into engineering education research.

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In this work, we present a case study of an interdisciplinary Master's course on heuristic methods in the context of OR and DO. We discuss to what extent well-established approaches from engineering education practice, such as Problem-Based Learning, are applicable. Furthermore, we introduce two practical cases and argue that due to its application-oriented nature, OR and DO specifically stimulate independent student work.

Results from evaluations, minute papers and student coursework indicate that the teaching approach successfully contributed to students' achievement of the intended learning outcomes.

To further foster discussion, we not only provide the lecture notes publicly, but also all tutorial and project case data to instructors upon request under a CC BY-NC license.

1 INTRODUCTION

In a digitalized world, most processes as in logistics, health care, education or production can be formalised, measured and described mathematically. The use of analytical methods to optimise such models and decisions constitutes operational research (OR). Developing new methods for specific problems from these fields and analysing them form a rich source of novel discrete optimisation (DO) problems. This designation is not clear cut: analytics, systems engineering, industrial engineering, operations management, management science, discrete and combinatorial optimisation, algorithms and complexity, and operational (operations) research represent closely linked fields that all deal with the use and development of methods to describe, predict and improve processes.

Many problems can be solved exactly within reasonable time, even for large instances. However, there are also numerous problems, e.g. (capacitated) vehicle routing, partitioning or even general integer programming, where finding an exact solution in reasonable time is, as of today, impossible (Sleegers et al. 2020; Peter Cheeseman, Bob Kanefsky, and William M. Taylor 1991). The alternative here is to use heuristics, i.e. algorithms that generate acceptable outcomes in a reasonable time. Today, many real-world problems such as scheduling, assignment, routing and/or logistics require heuristic approaches to solve large instances without a special structure (Gendreau and Potvin 2019; Martí, Pardalos, and Resende 2018).

While there is a lively debate about teaching classical mathematics to engineering students, published research on teaching operational research and discrete optimisation in higher education is sparse to the point of being non-existent. Neither the European Society for Operations Research (EURO) nor various national association have a working group or designated teaching streams as a regular part of their program and conferences. We found two reviews on teaching operations management in Spain, which point out the lack/absence of research into teaching methods (cf. Marin-Garcia 2018; Carmen Medina-López, Alfalla-Luque, and Marin-Garcia 2011). The more recent publication by Marin-Garcia (2018, 612) analyses the research focuses of 25 publications in Spain. He points out that a majority of publications have a research focus (unsuccessfully) aimed at finding a "silver bullet" teaching approach that works equally well for any student and context.

As such, what constitutes appropriate methodology to teach OR largely remains an open question. That is specifically relevant because, while OR draws deeply on discrete mathematics and computer science, it is fundamentally different from much of engineering mathematics teaching in that it does not provide fundamentals for other engineering classes but represents a skillset in itself.

In this work, we present a case study of an interdisciplinary Master's course on heuristic methods in the context of OR. We discuss to what extent well-established approaches from engineering education practice, such as Problem-based Learning (PBL), are applicable. Furthermore, we introduce two project cases and make a point that due to its application-oriented nature, OR specifically stimulates independent student work. To further foster discussion, we not only make the lecture notes are publicly available[2], but also provide all tutorial and project case data available to instructors upon request under a CC BY-NC license.

The paper is organized as follows: In Section 2, we introduce the course design, i.e. the learning outcomes and teaching contents (2.1), the structure of the course (2.2). This is then discussed in the context of active and problem-based learning (2.3). In Section 3, the above is evaluated based on the previously described data. Finally, Section 4 gives a summary and outlines both lessons learned and potential future improvements.

2 COURSE DESIGN

The context of this work is the interdisciplinary Master's course called "Mathematical Heuristics for Discrete Optimisation" (MaHeu) at RWTH Aachen University. The course consists of three main parts: a lecture, which takes place twice a week, a weekly tutorial session, and a practice case that students work on in teams. These three parts are interlinked. The lecture follows a PBL approach, where working sessions and practical problems are used to introduce students to relevant methodology, while relevant software is introduced and practiced in the tutorial. Both serve to prepare students to independently work on the case. In dealing with the case, the students work with data, implement their own algorithms, evaluate these computationally and discuss real-world applicability.

Grading is jointly based on a team grade for the project and individual oral exams. Upon successful participation, students are awarded 9 ETCS. Most participants take MaHeu as a compulsory elective subject in mathematics at either the Bachelor's or Master's level, or as part of their computer science Master's degree. The number of students finishing the course was 12 in 2019, 13 in 2020, 8 in 2021, 16 in 2022 and 21 in 2023.

2.1 Learning outcomes and teaching content

Following constructive alignment, intended learning outcomes (LOs) were formulated at course level (Biggs 1996). First, after successful participation, students *know major principles of both heuristics, metaheuristics and approximation algorithms.* Second, they *evaluate the necessity and suitability of using heuristics to solve given DO/OR problems.* Third, *they apply existing heuristics to established DO/OR problems*. Fourth and fifth, students *model novel, complex real-world problems mathematically* and they *modify and implement existing heuristics to solve those*.

[2] https://combi.rwth-aachen.de/teaching/resources/MaHeu_LectureNotes.pdf

Sixth and finally, they *evaluate the suitability of such methods using both proofs and computational experiments*.

The first three LOs are addressed in both lecture and tutorial, and assessed in an oral exam at the end of term, together with the ability to perform mathematical proofs as asked for in the final LO. As part of the case, the other LOs are developed, and assessed in a presentation and a written report.

Note that single solution-based heuristics are a focus of this specific course, and population-based approaches, e.g. genetic algorithms are not covered in detail. However, an overview is given at the beginning of the term, which includes the optimisation cycle as standard approach to tackle optimisation problems the classification of algorithms and the main components of every heuristic. Then the difference between heuristics and approximation algorithms is discussed and the “no free lunch” theorem is introduced. Based on this, several fundamental paradigms for heuristics are covered in the following sections, as given in Table 1:

Table 1. Mathematical heuristics teaching contents.

Concept	Theory	Problems
Greedy	Different types of approximation ratios, series-parallel graphs	Minimum cost flow, set cover, k-center, Travelling Salesperson (TSP,) Independent sets and matroids
Local Search	Neighbourhoods	Machine scheduling, (Minimum degree) Spanning trees, k-median, Spanning trees with many leaves
Randomisation	Rounding, expected runtime, random approximation, Greedy randomized adaptive search procedure (GRASP)	Max-Satisfiability Problem (Max-SAT), Max Cut
Very Large-Scale Neighbourhood Search	Compound Swaps, DynaSearch, Eject & Reinsert, Lin-Kernighan	Machine Scheduling, TSP, Partitioning, Capacitated Minimum Spanning Trees
Simulated Annealing	Asymptotic Convergence	TSP

There is also a special section in the lecture on evaluation of algorithms that contains both evaluation techniques and practical content for doing computational studies.

2.2 Course structure

As discussed at the beginning of the section, the course consists of three elements: A lecture, a tutorial and a case. These are now covered in more detail.

Lectures take place twice a week. Small algorithm design and programming exercises are interspersed throughout the lecture, e.g. the problem of analysing the practical performance of algorithms and their comparison are introduced on the example of the traveling salesperson problem. The students work in small groups to discuss what questions should be answered by the computational study, and then perform an analysis on a given set of data via R[3] and compare their findings.

The lecture notes are provided digitally via RWTH's Moodle learning management system. Videos of past years are also uploaded and TikZ[4] based animations of all algorithms and concepts covered in the lecture are provided in an extra extension of the lecture notes.

The tutorial sessions take place weekly. Each week students are given a sheet with exercises to solve at home and then present next week. During the week, students send their solutions to an instructor who provides feedback. Some tutorial sessions specifically focus programming with domain specific software. This includes the statistics software R and the modelling language AMPL[5], together with CPLEX[6]/gurobi[7] as solvers. Those software packages constitute standard tools in optimisation/analytics that also offer free academic licenses. Students need to actively participate in the tutorial to gain admission to the exam. Here, active participation consists of presenting one or several solutions, with the number varying based on the number of participants.

Working on the case begins a month after the lecture/tutorial started and goes on for two months. Student groups work together in teams of four to six. At the end, students have to hold a final presentation and hand in a team report of up five pages.

For 2019-2022 the project was on the optimal wiring of heliostats for solar power towers based on previous research work of our group (see Richter et al. 2019). This year we updated the case to an operative surgery scheduling problem. Since real-world surgery data is subject to strict data protection in the European Union, we based the case on publicly available research data (see Leeftink and Hans 2018). Both projects were chosen because they address relevant real-world issues and they allow for the usage of simple heuristics to construct an initial feasible solution. The latter means that every group will be able to present some solution and students can differentiate themselves in terms of solution quality. Moreover, both cases are based on past (heliostats) and ongoing research of our group (surgery scheduling).

Note that the course itself is held in either German or English, depending on student preferences, and lecture notes, case description and tutorial exercises are in English.

2.3 Active and Problem-Based Learning

The course structure is specifically built on established educational practice in the context of engineering education. Active learning has shown to engage students in

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[3] See https://www.r-project.org/ .

[4] See https://tikz.net/

[5] See https://ampl.com/ .

[6] See https://www.ibm.com/products/ilog-cplex-optimization-studio .

[7] See https://www.gurobi.com/ .

the learning process and thus positively affect the acquisition of intended learning outcomes (Freeman et al. 2014; Prince and Felder 2006). Specifically, the combination of activity and variety has been shown to increase student interest, improve attendance and increase learning (Felder and Brent 2016; Prince 2004). This was used as a motivation to change the lecture content away from a standard frontal format towards a more active and student-centred design, and to ensure that the course itself is varied in terms of formats for students.

PBL is a teaching method in the context of active learning. The problem itself is used as context and motivation for learning (Prince 2004; Edström and Kolmos 2014). This is mirrored in Table 1, where each teaching content is interlinked with one or several problems. In each teaching block, these problems are used to motivate the corresponding solution techniques. Furthermore, students experiment on different problems and solution approaches themselves.

Note that allowing students a combination of experimentation, instrumentation, troubleshooting, modelling, self-directed and creative thinking, instead of a fixed sequence of tasks to fulfil, is also an important factor that contribute to the success of practical lab exercises (Felder and Brent 2016). We decided to support this through the real-world case. Specifically, the project description calls for students not just to identify the/one optimal solution but to test out different approaches and compare them based on knowledge acquired during the course.

3 EVALUATIONS

Course evaluation takes place through student coursework, weekly minute feedbacks and a final evaluation. This work is based on five years of teaching, i.e. the spring terms of 2019–2023. In 2020 and 2021, due to the COVID-19 pandemic, lecture and tutorial were held remotely.

All final evaluations are part of RWTH’s quality management system. They consist of a range of items on a 5-point Likert scale ranging from 1 (very good) to 5 (very bad), and two fields for further comments, i.e. notable positive elements and suggestions for improvements. As the course and evaluation were held in German, all comments were translated to English.

Between five and nine students participated in the evaluations. Across all years, the overall grades were between 1 and 2 and no course nor instructor received an evaluation worse than 2 (good). In 2019 and 2020, students repeatedly remarked a lack of summaries as part of the lecture and marked down the corresponding item, those were subsequently added at the end of lecture content. During the COVID-19 pandemic, i.e. the spring terms of 2020 and 2021, multiple students remarked upon changed circumstances due to remote learning. Whereas some students mentioned the advantages of increased flexibility in learning and time saved due to not having to travel to university, others criticised the lack of personal interaction with both peers and instructors. In the exercise, the instructor offered an open digital meet-and-greet session before each exercise, which was received very well by students, although the fundamental criticism remained.

Across all years, students rated the module as providing an appropriate level of challenge and workload. As one student put it:

“As computer science student […] this was the first mathematics module that wasn’t too hard because I lacked prerequisites […] nor to easy […] but demanding in a good way due to the complexity of the content. […]”

Students also remarked positively on both the course teaching: “The interactive nature of the lecture gets you to think for yourself an keeps you attentive.”, and the structure of the whole module: “The structure of the module with exercise, project work and oral exam is a welcome change.”

The weekly minute-feedbacks were divided into two parts: lessons learned, and questions suggestions for improvement. Students could voluntarily fill out the feedbacks. In general, students were diligent in filling out the feedbacks, specifically when it came to listing the topics covered in the last session. Questions frequently focused on formal definitions, e.g. “What precisely is the difference between general and problem specific heuristic?” or “How is an independence oracle defined formally?”, or they focused on follow-up questions regarding extensions of specific algorithms or e.g., general procedures for derandomization. However, most answers simply noted that students were happy with the course and enjoyed both teaching and content.

Generally, students’ reports were well crafted and their solutions made use of a range of different approaches. Frequently, either TSP or MST based heuristics were used as the starting point for the heliostat problem, with local search used for intensification. Similarly, GRASP procedures were frequently employed. Many groups also sliced the heliostat area into parts, making use its geometric structure. For the surgery scheduling problem, all groups started with variations of randomised GREEDY, though sometimes only as a baseline for comparison. Frequently, they extended their approaches with local search (GRASP), in multiple cases using improvement graphs to deal with large search neighbourhoods. Groups also implemented simulated annealing and integer programming based approaches.

In terms of evaluating their algorithms, as showcased in their report and final presentations, students used a range of mathematical tools from the lecture. By determining lower bounds for the best solution quality, they managed to estimate their solution quality. Furthermore, students analysed the run-time and memory requirements of their algorithms.

4 SUMMARY AND OUTLOOK

Based on student feedback, the active learning within the course and the project case were well received. Furthermore, the project reports and presentations showcased that students were able to implement techniques from the lecture and to modify them to suit their needs when dealing with a real-world problem. That indicates that the teaching approach successfully contributed to students achieving the learning outcomes, which is in line with established literature on active learning (Felder and Brent 2016).

It is notable that students remarked as an exceptionally positive fact that MaHeu only requires prerequisites that students had learned before. This would appear obvious, but it apparently is not – an issue for discussion within our faculty.

Furthermore, we find that operational research not only allows for, but indeed is well-suited to PBL and case-based learning. We believe that the OR and DO community would profit from sharing more respective teaching contents. In our case, both cases were based on our own group’s research and both cases can easily be extended to provide follow up work for students interested in a thesis, e.g. by including uncertainty, rostering or bed management in the case of surgery scheduling. This offers opportunities for both sides in terms of recruiting motivated students to be part of ongoing research work.

In the context of RWTH, it would also be interesting to compare teaching approaches. There is another lecture offered on heuristics optimization with comparable LOs for students from business administration and business engineering, but a very different teaching concept based on an inverted classroom paradigm with assessment through a written exam.

A possible extension for our course would be to offer different cases to each group or even set the groups based on case preference. While this does complicate grading and preparation, it also offers students more choices in determining their learning process.

Finally, we would like to point out that the absence of research on higher education OR and DO stands in stark contrast to the evidence-based and optimisation focus mindset of the communities. Closing this gap remains a challenge for both instructors and researcher in the field of OR and DO. Specifically, drawing from established research in engineering education may enable more successful teaching and learning not just generally, but also specifically in the field of OR and DO.

REFERENCES

Biggs, John. 1996. “Enhancing Teaching Through Constructive Alignment.” *Higher Education* 32 (3): 347–64. https://doi.org/10.1007/BF00138871.

Carmen Medina-López, Rafaela, Juan A. Alfalla-Luque, and Juan Marin-Garcia. 2011. “Research in Operations Management Teaching: Trends and Challenges.” *Intangible Capital* 7 (2).

Edström, Kristina, and Anette Kolmos. 2014. “PBL and CDIO: Complementary Models for Engineering Education Development.” *European Journal of Engineering Education* 39 (5): 539–55. https://doi.org/10.1080/03043797.2014.895703.

Felder, Richard M., and Rebecca Brent. 2016. *Teaching and Learning STEM: A Practical Guide*.

Freeman, Scott, Sarah L. Eddy, Miles McDonough, Michelle K. Smith, Nnadozie Okoroafor, Hannah Jordt, and Mary Pat Wenderoth. 2014. “Active Learning Increases Student Performance in Science, Engineering, and Mathematics.” *Proceedings of the National Academy of Sciences of the United States of America* 111 (23): 8410–15. https://doi.org/10.1073/pnas.1319030111.

Gendreau, Michel, and Jean-Yves Potvin. 2019. *Handbook of Metaheuristics* 272. Cham: Springer International Publishing.

Leeftink, Gréanne, and Erwin W. Hans. 2018. “Case Mix Classification and a Benchmark Set for Surgery Scheduling.” *J Sched* 21 (1): 17–33. https://doi.org/10.1007/s10951-017-0539-8.

Marin-Garcia, Juan. 2018. “What Are the Research Focuses Regarding Learning in the Field of Operations Management in Higher Education? The Case of Spain in 2017.” *JIEM* 11 (4): 607. https://doi.org/10.3926/jiem.2550.

Martí, Rafael, Panos M. Pardalos, and Mauricio G. C. Resende. 2018. *Handbook of Heuristics.* Cham: Springer International Publishing.

Peter Cheeseman, Bob Kanefsky, and William M. Taylor. 1991. “Where the Really Hard Problems Are.” In *Proceedings of the 12th International Joint Conference on Artificial Intelligence.* 1 vol, 331–37. Sydney, New South Wales, Australia: Morgan Kaufmann Publishers Inc.

Prince, Michael J. 2004. “Does Active Learning Work? A Review of the Research.” *Journal of Engineering Education* 93 (3): 223–31. https://doi.org/10.1002/j.2168-9830.2004.tb00809.x.

Prince, Michael J., and Richard M. Felder. 2006. “Inductive Teaching and Learning Methods: Definitions, Comparisons, and Research Bases.” *Journal of Engineering Education* 95 (2): 123–38. https://doi.org/10.1002/j.2168-9830.2006.tb00884.x.

Richter, Pascal, Fynn Kepp, Christina Büsing, and Sascha Kuhnke. 2019. “Optimization of Robust Aiming Strategies in Solar Tower Power Plants.” In *SOLARPACES 2018: International Conference on Concentrating Solar Power and Chemical Energy Systems*, 30045. AIP Conference Proceedings: AIP Publishing.

Sleegers, Joeri, Richard Olij, Gijs van Horn, and Daan van den Berg. 2020. “Where the Really Hard Problems Aren’T.” *Operations Research Perspectives* 7:100160. https://doi.org/10.1016/j.orp.2020.100160.