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# A CRITICAL APPROACH TO ENGINEERING MATHEMATICS FOR SUSTAINABLE DEVELOPMENT

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## ABSTRACT

Engineering projects are frequently experienced through the complexity of knowledge co-production between experts and local communities. This involves an ability to work critically and creatively within unfamiliar epistemologies, drawing from quantitative, social and scientific methods to realise high-impact solutions. In this work-in-progress paper, we put forward a prototype for a case-control study aiming to evaluate student buy-in and learning outcomes for a cross-cultural implementation of critical mathematics approaches contextualised by sustainability challenges. We outline and discuss aspects of mathematical modelling activities that can scaffold an environment where human subjectivity amplifies the quality and relevance of quantitative arguments. As proof-of-concept, we analyse exemplary work of first-year engineering students as they design, implement, and evaluate a model of population dynamics towards proposing solutions for the endangerment of a wild species. We then identify critical learning outcomes springing from the social and subjective context that envelops the processes of mathematical modelling, analysis and communication in the real world. Our initial results show that interdisciplinary sustainability-driven mathematics activities have the potential to empower students to adopt a conscious approach to societal and environmental challenges.

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# 1 INTRODUCTION

## 1.1 Knowledge Co-Production and the Connected Curriculum

Modern sustainability challenges are reportedly more effectively addressed when knowledge is co-produced between ‘experts’ and local communities. Research shows that high-impact engineering interventions are distinct for being context-aware, inclusive, goal-oriented, and interactive [1]. Knowledge co-production is built on the principle that the people affected by a certain project are the most suitable to evaluate its value and validity and sees non-experts as partners to judge the impact of an intervention. In practice, this involves modes of work that incorporate several ways of knowing in decision-making, planning and design. Given the fundamental character of mathematics within engineering, this paper aims at exploring the opportunities for knowledge co-production in the teaching and learning of engineering mathematics.

The challenges of uncertainty in engineering projects highlight a need for engineering education that enables students to navigate through unfamiliar epistemologies, drawing from a blend of scientific and social forms of knowledge to solve problems that can be transparently evaluated and continuously improved upon. Towards this end, the connected curriculum framework brings the idea that teaching can join different threads that had previously been considered to be unrelated, embodying public engagement as well as intellectual and ethical positions in education [2]. The aim of this connectivity is to embed an element of human complexity within day-to-day instruction, connecting the classroom to the wider world and communities around it. Similar ideas also underpin the provision of problem/project-based learning (PBL), where students learn through collaborative, self-motivated research and enquiry in solving authentic, open-ended problems. [3].

## 1.2 Sustainability and Critical Mathematics

Over the last three decades, a convincing argument has been made for mathematics education that is student- and community centred. Critical theorists [4,5] maintain that mathematics can be used in subjective ways to propose, sustain, or change ideas about society, economy or the environment. These pedagogies present mathematics as a creative process towards changing precarious realities, aiming to engage students in acting effectively against global challenges and to communicate quantitative ideas in an inclusive way. It is hence necessary to enable students to discover the transformative possibilities of learning mathematics to build a more sustainable world. This idea was first implemented in mathematics education for the empowerment of marginalized communities [4, 5], and has been more recently applied to present numerical evidence for arguments relating generally to real life problems [6-7].

Mathematics plays a hybrid, scientific and social role in engineering projects. Statistical mathematics and modelling can be used as a tool to systematically characterise, optimise, forecast or explain phenomena. Mathematics also plays a social role because it increases the potential of people to influence systems, processes or policies that bear direct impact on their life [5-7]. In this paper, we describe our initial efforts and experiences in designing and delivering activities that give students opportunities to reflect on how we can act more effectively via mathematical modelling.

### 1.3 Research questions

The rationale for the present approach to mathematics activities lies in that mathematical modelling can help students crystallise subjective ideas into "quantitative landmarks" upon which a shared understanding of a complex situation can be built [6]. This is a move away from mathematics education that is exclusively based on factual recall and routine procedures, instead opening up space for mathematics learning to be a dialectic exercise [5]. In this paper, we present mathematical modelling as a creative process, where mathematics can be communicated, evaluated, negotiated and transformed around the uncertain and non-ideal constraints of a real-life challenge. Therefore, the broad questions for this are:

- How can critical approaches to engineering education encourage students to communicate their ideas in the form of mathematics?
    - And more specifically, what are the synergies between a critical approach to sustainability education and the technical or abstract concepts in the mathematics curriculum?
- What are the lessons learnt across borders from exploring student awareness and empowerment in sustainability challenges via mathematics?
    - And more specifically, what are the features of student collaboration and exchange in sustainability-oriented engineering projects?

## 2 METHODOLOGY

### 2.1 Hypothesis-generating data

The hypothesis-generating data that motivates this study was sourced from summative student activities undertaken during the 2021-2022 academic year at a cross-disciplinary first-year engineering mathematics module. This module introduces students to engineering mathematics via collaborative case-studies exemplifying applications of mathematical concepts in science, healthcare, technology, and sustainability. For example, students are introduced to differential calculus by engaging with activities framed around the optimisation of family-run agriculture in the Global South and learn integrals by modelling non-invasive surgery protocols [8]. The course is delivered in a hybrid format, where passive activities such as knowledge acquisition are done online in preparation for staff-led workshops that activate student learning through hands-on mathematical modelling of engineering problems.

### 2.2 Activity Design

The source activity explored herein was themed around the endangerment of wild animals in Sub-Saharan Africa, through mathematical modelling of a finite-difference system predicting the evolution of subpopulations of pup (0 – 1 year), yearling (1 – 2 years) and adult (>2 years) animals [9]. In the activity brief, students were asked to propose solutions towards three important challenges for the survival of the species: (i) shrinking natural habitats, (ii) lethal diseases, and (iii) being hunted by local farmers. Final solutions to this activity were required to contain two discursive elements that should be based on the models and results obtained by students after performing numerical simulations. For this, students were encouraged to test their hypotheses via simulation-based cause-effect comparisons. This activity design intended to prompt students to make connections between mathematics parameters and empirical measurements, as well as mathematics-based action and their practical impact.

The activity design guided students through documenting factual mathematics knowledge such as assembling matrices and vectors, performing matrix-vector and matrix-matrix multiplication, or inverting a matrix. The main activity discussion regarded describing and comparing the effects of different survival probabilities on the total number of animals. The scaffold outlined below guided students as they chose on which subpopulation their proposed solutions should focus. These objective steps served as "quantitative" landmarks where students could validate their mathematical work against previously established criteria.

- The first landmark consisted of modelling empirical timeseries data of a wild species population as vectors that change with respect to time. This allows for assembling and solving linear systems of equations toward calculating survival probabilities of the three subpopulations of wild dogs [9].
- The second quantitative landmark was based on applying matrix multiplication toward deriving a forward/backward predictive model for each of the subpopulations of the species. This task relied on students using induction to assemble a forward system of difference equations with yearly time-steps. For backwards modelling, students need to employ the properties of matrix inversion towards predicting past populations.
- After obtaining a mathematical model that is based on matrix multiplication, inversion, and matrix-vector operations, students were asked to implement computer code to perform a sensitivity analysis towards demonstrating the effects of the survival and reproduction probabilities on the total number of living animals of the species.

### 2.3 Identifying examples of critical thinking in student solutions

Although students were asked to use mathematics towards determining *what* is objectively important in this challenge, such as the survival of wild animals, they were also prompted to discuss *how* this could be done in sustainable, systemic or humanizing ways. As an additional step, students were asked to give examples of realistic and feasible interventions that could result in the preservation or repopulation of the species. This activity component was included intending to steer students away from impersonal engagement with mathematics in favour of a reflection on the impact of mathematics-based creativity and action in the real world [4,5]. Based on existing frameworks for the identification of critical consciousness and mathematical critical thinking [6-7], we discuss three examples of critical thinking in student work:

- Recognising underlying assumptions in modelling and disclosing the possible limitations brought to results, contrasting and comparing different scenarios and evaluating their appropriacy as a mathematical solution (Examples 1 and 2).
- Communicating mathematical information in verbal or graphical form and vice-versa, explaining mathematical relationships and proposing analogies with real-life processes or systems (Example 2).
- Proposing sustainable and humanising solutions that are based on quantitative information obtained via mathematical modelling and that consider diverse value- and belief systems (Examples 3, and 5).

## 3 ANALYSIS OF STUDENT DISCOURSE

The exemplary evidence presented below was extracted from student work conducted as part of a previous study in the activity described in section 2.2 and is reproduced herein with the consent of the authoring students. These solutions were not evaluated based on their real-life feasibility, but rather on whether their underlying discourse was synthesised by a blend ethical and sustainable principles to objective mathematics reasoning. The content of student proposals submitted to this task ranged from a frequent attention to mathematical accuracy to a distinctive life-preserving care for people and the environment.

Example 1. Explicitly stating the underlying assumptions of a model:

> *[…] Firstly, it was assumed that the rates for survival and reproduction are the same across years, without any probabilistic variation. […] Change in the local climate, the presence of food and predators, or humans, can occur over the long term; if any of these events have significant impact on the rates, the model would not be able to take them into account.*

In Example 1, the student chose to focus on the validity of the mathematical model proposed in [9] when it is used under the assumption that the survival and reproduction rates of animals are time-independent. In this case, the real-life application of mathematics facilitated the student's conceptual understanding. Example 1 shows evidence that the student was able to evaluate the model by proposing factors (climate, food, or predators) that cause the underlying assumptions of the model to fail. This example is distinctive in that the student did not assume that survival/reproduction probabilities were smooth functions or time, but rather the result of complex interactions between stochastic factors that can be difficult to account for. Most importantly, this example shows that the student was courageous in challenging the stability of mathematical definitions when applied to real life problems.

Example 2. Using modelling and analysis to identify avenues for transformation:

> *[…]the adult survival rate increasing would have the most positive impact on the final outcome after 50 years. This can also be understood in the light of the fact that adults are the only sub-population that carry on to the year after,*

*whereas pups and yearlings either grow up and become part of another sub-population or die[…]*

*Ultimately, it may be impossible to adjust one rate by a specific amount (as was done in the modelling) while keeping the other parameters the same. The behaviour, survival, and reproduction rates are interlinked in a way that makes such a precise controlled intervention implausible. Nonetheless, the rates provide a target that, if achieved, would bring about the non-extinction (and even the repopulation) of the species.*

In Example 2, the student held onto the realisation that mathematical models are simplified ways of understanding a reality developed in Example 1. Within this context, the student was able to perform numerical simulations with different parameters and concluded that increasing the chances of survival of adult animals would have the most beneficial effect to the total number of the population after fifty years. This conclusion was closely followed by a disclosure of the practical possibility of changing isolated parameters in the model, where the student recognises that it is often challenging to change single elements of survival in multi-parameter population systems.

Example 3. Humanizing intervention:

> *In real life, [the species] usually lives in pack. The older animals are in charge of the daily hunting while the Pups watches and learns [sic]. In the process of hunting, inevitably, sometimes the livestock of farmers in the area would become the targets. And sometimes, in protecting their livestock, farmers would kill the animal.*

In complex scenarios, such as the one explored in this activity, there is a risk that students take on an approach that is either hostile to locals or to the animals. Although logical, such argument would be limited in that it ignores any complexity in the local reality of human-fauna interaction. Example 3 highlights work demonstrating an ability to draw information from social and ecological sources to interpret an otherwise theoretical mathematical system. In this example we highlight evidence of humanizing approaches towards addressing the challenge. The student identified the importance of hunting for the survival of the species, but also noted the possible issues that arise from spontaneous human-fauna interactions. This discourse is distinctive because it does not seek to demonise either part in the interaction, but rather to objectively state that hunting is a survival mechanism for wild animals and that protecting livestock is also a self-preserving action taken on by farmers.

Example 4. Large-scale non-invasive interventions:

> *Rabies is a disease which severely affects the species as there is no cure for this disease once it's contracted and always results in the death of the animal (Student Reference 1, 2015). One possible method by which animals in an area could be protected […] is through the distribution of vaccines through edible baits (Student Reference 2, 2016). This method of vaccinating animals in the wild is efficient as a*

*large area can be targeted and there is a little amount of interaction with humans.[…]*

Finally, the student Example 4 highlighted that lethal diseases are a significant threat to the survival of wild dogs, and proposed methods of non-invasive vaccination against rabies through edible baits. The student argued in favour of their proposed solution by considering that an effective solution covers a large area and involves minimal interaction between humans and wild species, basing their argument on references to the literature that had been validated empirically in the past.

# 4 FUTURE DIRECTIONS

Here, we bring together an envisioned research design to investigate the cross-cultural perceptions of a connected curriculum for critical mathematics from the perspective of student buy-in and motivation when presented with sustainability challenges. This paper does not provide a comprehensive account of a completed study, but ought to rather be viewed as a layout of an envisioned research design, that is initially set forth for the purpose of exchange with the SEFI community.

## 4.1 Proposed research methodology

We envision to adopt a single embedded case-study design methodology [10] that takes place over two consecutive stages in two different universities, one in Europe and one in Africa. Within the overall study that explores student buy-in to an integrated mathematics framework is a comparative dimension that contrasts the two local case studies, each representing one context of instruction. Figure 1 summarises the overall methodology envisioned for adoption for this study.

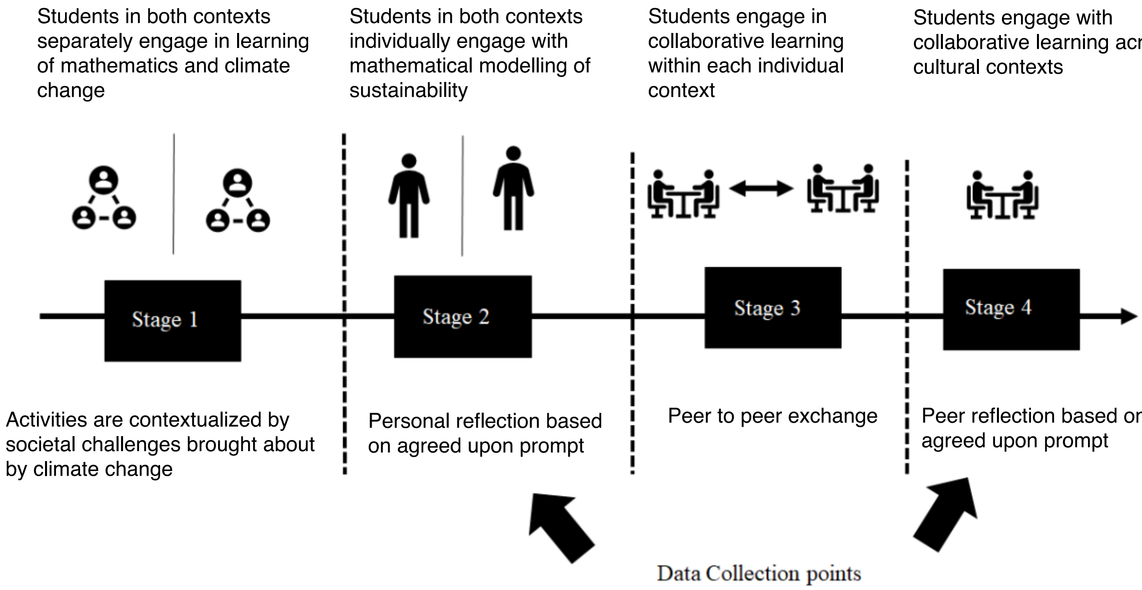


*Figure 1. Data Collection Protocol*

As presented in Figure 1, students from both contexts will take part in a series of sustainability-focused engineering mathematics workshops and activities in Stage 1. These activities aim to introduce students to critical approaches to mathematical modelling [4-7], sustainable development, and the social impact of engineering. Prompted by agreed upon reflection questions, participants are then encouraged to journal their perceptions about integrating questions of climate change into the mathematics curriculum. Students are prompted to reflect from the perspective of the relevance, complexity, and buy-in. In the second phase of the study, students are presented with an exchange platform that allows them to pair with a peer within their context to exchange experiences with. The peers are then prompted to document their reflections and exchange as guided by a pre-designed framework.

### 4.2 Proposed analytical framework

Findings from the first cycle of reflection are triangulated against each other and contrasted across the collaborating teams. Repeating patterns within each context are first recorded. This is followed by a cross matching of patterns across contexts. The final level of investigation also includes a triangulation element and a comparative element. Peer reflections are cross matched and re-occurring patterns are recorded. Peer reflection patterns are then cross matched against the individual reflection patterns. Figure 2 illustrates the analytical framework envisioned for adoption.

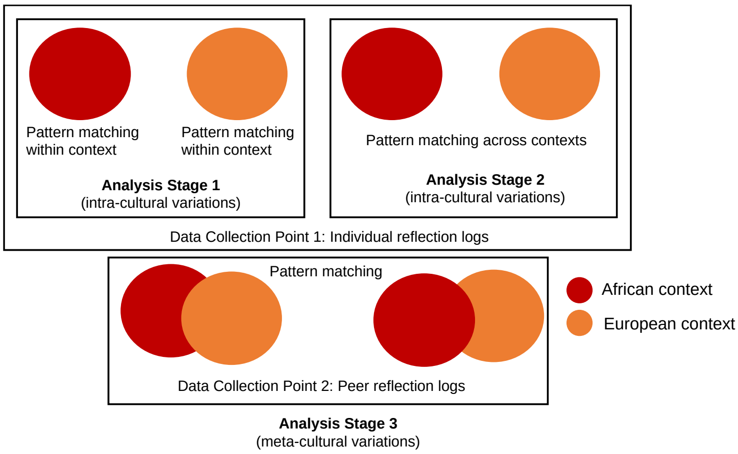


*Figure 2. Envisioned Analytical Framework*

The investigation therefore adopts an intra-cultural lens, exploring variations of patterns for student buy in within a given culture. It also adopts and inter-cultural lens, comparing variations of patterns across cultures. Finally, it adopts a meta-lens, exploring peer reflections and exchange across cultures.

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