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Unlocking Complex Vector Calculus Concepts For Engineering Students Using Geogebra

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ABSTRACT

There is an increasing drive to exploit the power of technology to improve students mathematical conceptual understanding. This work is motivated by the authors research presented at the SEFI 2022 conference which reported on students experienced difficulties with the double integral, a concept central to vector calculus. Some of the difficulties included visualising and sketching three dimensional surfaces and regions of integration and changing coordinate systems from rectangular to polar. Vector calculus is a crucial subject for engineering students, but its abstract concepts can be challenging to grasp. This curriculum proposal is a response to improve visualisation and conceptual understanding and is part of a larger project to develop an innovative, engaging and effective way for undergraduate engineering students at the University of Cape Town to learn vector calculus concepts supported by GeoGebra. The choice was made in favour of the easy to use, freely downloadable mathematical software, GeoGebra which presents a creative, visual and integrative way to experience and understand mathematical concepts.

Informing this curriculum development initiative is Vygotsky's social constructivist perspectives with an emphasis on inclusivity, diversity and participant interactions. In this paper we discuss the above theoretical underpinnings with case studies on how to teach the double integral concept in GeoGebra for conceptual understanding. Additionally the benefits of using GeoGebra including its ability to engage students, promote critical thinking, and increase motivation will be discussed. This research will be of interest to those intending to use GeoGebra to improve the teaching and learning of vector calculus concepts.

1. INTRODUCTION

Vector calculus is a fundamental subject for engineering students, but its abstract concepts can be challenging to grasp. One of the key concepts that students often struggle with is the double integral, which is central to vector calculus. In a previous study presented at the SEFI 2022 conference, we reported on the difficulties that students experienced with this concept, including visualising and sketching three-dimensional surfaces and regions of integration and changing coordinate systems from rectangular to polar. Students perceive the integration of functions of one or more variables as one of the most challenging calculus topics (Kiat, 2005; Mahir, 2009; Maharaj, 2014; Pino-Fan et al., 2018), because typically it is not enough to apply procedures in calculating integrals. This cognitive extension from single variable calculus to multivariable calculus presents challenges for students and calls on them to develop new skills and strategies to successfully navigate this transition. This makes a strong case for research needed to explore students' understanding of double integration (Larsen et al., 2017, p. 539). Our experience of teaching and tutoring various iterations of a vector calculus course confirms that our students experience difficulty understanding the concept of double integration.

To improve students' conceptual understanding of vector calculus, we propose a curriculum development initiative that uses GeoGebra, a freely downloadable mathematical software. GeoGebra provides an innovative, engaging, and effective way for undergraduate engineering students at the University of Cape Town to learn vector calculus concepts. The objective of this research is to explore the benefits of using GeoGebra to teach vector calculus concepts to undergraduate engineering students in the larger project and more specifically double integrals in this research. In the next round of research, we aim to develop case studies that demonstrate how GeoGebra can be used to teach the double integral concept in a way that promotes students' conceptual understanding.

2. LITERATURE REVIEW

In this section we highlight existing scholarship to give rationale for, illustrate significance of and situate our research. Here we consider the following: using technology for the teaching of mathematics, the challenges of teaching and learning vector calculus with a focus on double integrals, and the use of GeoGebra for teaching and learning mathematics. We include a short discussion on the theoretical framework used in this research, Vygotsky's social constructivism.

In keeping with the rapid advancement of the use of technology in the educational landscape, there is an increase in the body of research on the use of technology in teaching calculus concepts. Research shows that technology can have a significant impact on the teaching and learning of calculus concepts. Erens (2015) found that irrespective of their beliefs about the use of technology, high school teachers found that technology can be effective in teaching calculus. The often criticised approach to teaching calculus as merely computational rather than conceptual was addressed by Thompson (2013) who argues that technology enables a conceptual approach to calculus, which can help students develop connected meanings for calculus concepts. This use of technology for learning calculus may present a way to encourage students to engage with mathematics in a deeper conceptual way rather than a mere surface understanding. Supporting this notion, Cuoco (1996) suggests that technology can be used to help students develop “mathematical habits of mind and construct mathematical ideas”. Research provides evidence that technology is a valuable tool in teaching calculus concepts, but its effectiveness depends importantly on how it is used and integrated into the curriculum. Another important aspect highlighted by Raines (2011) is that incorporating technology in the classroom can enhance student learning and motivate students to become engaged in the learning process and active participants in their own learning. As instructors of calculus it is important for our students to have a good conceptual understanding and to achieve success in the course and with their future studies. Heid (1988) found that using computer programs to perform routine manipulations in an applied calculus course led to better understanding of course concepts and increased performance on a final exam.

Vector calculus is a complex subject that requires a high level of mathematical proficiency. Students have difficulties with vector calculus as it involves concepts and problems that require students to think in terms of three-dimensional space and visualize objects such as curves, surfaces, and volumes, requires students to work with functions of several variables, is typically taught at a more advanced level than single variable calculus, and requires a higher level of mathematical maturity and proficiency. Bollen (2015) found that students struggle with interpreting graphical representations of vector fields and applying vector calculus to physical situations however Lohgheswary et al (2018) suggests that teaching vector calculus using computational tools can help students visualize graphs and understand difficult concepts. Vector calculus is a challenging subject for students, and innovative teaching methods should be explored to help students understand the concepts.

Heckler (2016) found that computer-based training with elaborated feedback can be effective in improving student performance in vector calculus, especially for less prepared and low-performing students. Students learn differently and respond differently to various teaching styles. Hamzah (2022) found that the effectiveness of teaching styles can significantly affect students' achievement in vector calculus. However, Tasman (2021) cautions that the blended learning model may be less effective in improving student learning outcomes in vector calculus subjects compared to conventional learning models. What is certain is that effective teaching methods are necessary and should be explored to improve student performance especially in a challenging vector calculus course.

The focus of this research is on the double integral concept. It is well documented that students have various misconceptions when interpreting double integrals. Students often struggle to visualise and interpret three-dimensional surfaces and regions of integration, which are central to many vector calculus concepts. Additionally, changing coordinate systems from rectangular to polar can be a challenging task. Khemane et al (2022) found that students struggle with graphical representation of surfaces and regions of integration, setting up the double integral given these regions, changing the order of integration and performing the integration process.

Technology such as GeoGebra can help to improve students' understanding of calculus concepts, particularly when it comes to visualisation and interpretation. This is in agreement with Arbain and Shukor (2015), Mathevula and Uwizeyimana (2014), Niyukuri et al. (2020), Ocal (2017) and Uwurukundo et al. (2020), whose studies found that ICT, in general, could improve the way students perform in geometry, and that GeoGebra software in particular is effective in improving students geometric understanding. Importantly, Arbain and Shukor report that GeoGebra increased students' interest, motivation, enthusiasm, visualisation and performance in mathematics.

We are aware of limitations which may exist when using GeoGebra for the teaching and learning of vector calculus concepts for engineering students. These limitations may include technical limitations with regard to device access and technical expertise, learning curve for adjusting to use of new software, limited applicability as it relates to real-world engineering applications and pedagogical limitations-it may not be suitable for all types of learners. For the effective use of this software and to derive optimal educational benefit, it is important to consider these limitations when using GeoGebra to teach vector calculus concepts to engineering students. While GeoGebra can be a useful tool, it must be stressed that it should be used in conjunction with other teaching methods and techniques to ensure a comprehensive and effective learning experience.

The proposed curriculum development initiative is informed by Vygotsky's social constructivist perspective, which emphasises the importance of social interactions in the learning process. This perspective highlights the need for inclusivity and diversity in the classroom and emphasises the role of the teacher and tutor as a facilitator of learning. Additionally, the initiative is informed by constructivist learning theory, which emphasises the importance of students' active engagement in the learning process and the role of technology in supporting this engagement. Attard (2020) examines how exemplary teachers use technology to enhance pedagogical relationships with students and promote student-centred pedagogies, leading to greater student engagement with mathematics.

3. METHODOLOGY

The research question which guides this study is: *How can we use GeoGebra to improve students' visualisation and understanding of three-dimensional surfaces and regions of integration* ? This research study is situated in an engineering support programme at the University of Cape Town. The participants were engineering students enrolled for a second year, semester course in vector calculus. Ethics approval was obtained for this study and all participants willingly gave consent to participate in this research study.

The course activities and data collection reported on in this study were carried over 4 weeks. Week 1 was dedicated to lectures, tutorials, and workshops on double integrals with 130 participants. In week 2 students wrote a pre-test divided into pre-test 1 (54 participants), pre-test 2 (33 participants) according to their tutorial slots, week 3 focused on GeoGebra activities followed by an assignment (122 participants), and a post-test in week 4 (121 participants).Since participation was voluntary there is a difference in the numbers of students writing the pre-test and post-test.

3.1 Pre-test

To inform our understanding of how we could use GeoGebra to improve students' visualisation and understanding of three-dimensional surfaces and regions of integration and their (mis)conceptions of double integrals, a pre-test was given to students during their 2-hour afternoon workshop session. Since they had attended lectures on double integrals, the pre-test was designed to probe students' understanding of double integrals with a focus on their visualisation and understanding of three-dimensional surfaces and regions of integration. The first question of the pre-test identifies students' ability to sketch the region when given algebraic equations of curves which make up the region. The second question identifies students' ability to sketch 3d solids along with the projections onto coordinate planes.

3.2 The GeoGebra Intervention

As reported in our previous work (Khemane et al, 2022), sketching 2d regions and 3d solids is a prevalent challenge students face when learning double integrals. To address this, we implemented one of the suggestions from the SEFI 2022 attendees, and integrated GeoGebra in teaching double integrals. In addition to the normal lectures, students spent a week working on GeoGebra activities aimed at improving visualisation and sketching skills. Usually, students attend a 45-minute lecture followed by an hour-long tutorial aimed at reinforcing the concepts learned during lectures by working through related questions. In week 3, the tutorial sessions were substituted by GeoGebra activities. These activities were formed by a range of questions from the course handbook and student tutorials. Some of these activities are shown in appendix C and they were selected due to their relevance to double integrals. Moreover, the GeoGebra activities included sketching quadric surfaces such as paraboloids, spheres, planes, and determining the intersection of these surfaces. These activities were accompanied by students' hand sketches of the same surfaces and their reflections on the differences between the sketches they produced, and those generated by GeoGebra. Some of the results of the activities in appendix C are shown in figure 1 and 2. Figure 1a) shows the intersection of a cone and a plane, while 1b) shows the intersection of a hemisphere and a paraboloid, as well as their resulting xy projection. Different tools in GeoGebra allow students to explore different ways of visualising surfaces.

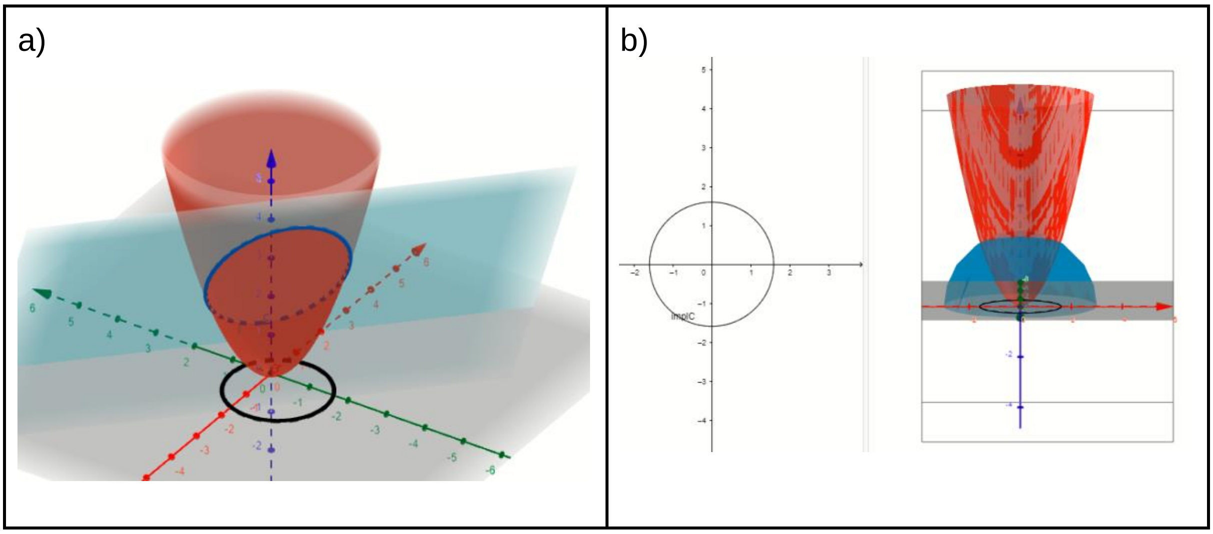

Fig. 1. GeoGebra activities showing intersection of different surfaces

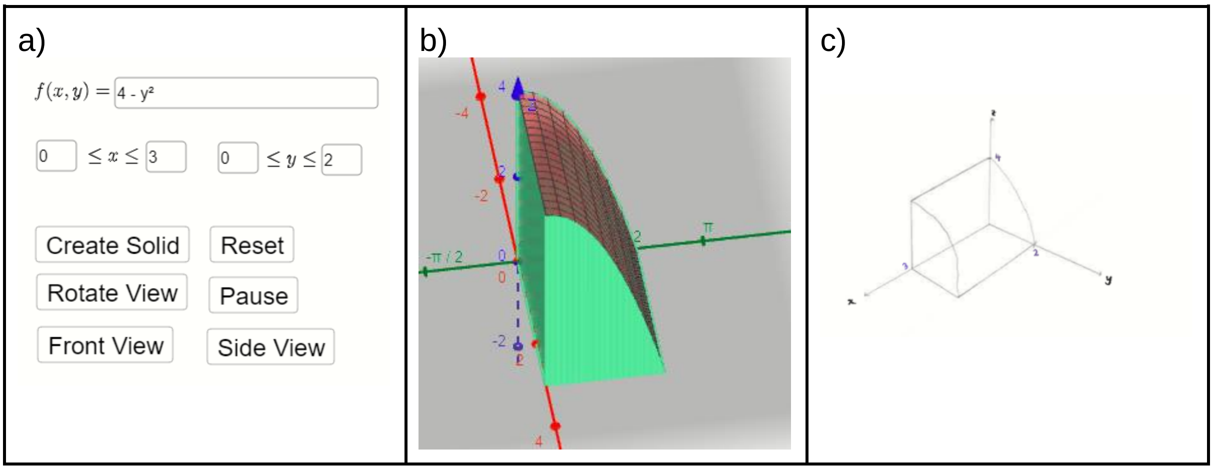

Fig. 2. GeoGebra activities used to help students sketch solids from the pre-test

Although we formally introduced students to GeoGebra in week 3, Some students were observed to be using GeoGebra for at least 4 weeks before it was introduced formally into class. After a week of activities, a GeoGebra assignment was given to students (appendix A). After the intervention with GeoGebra, we gave students a post-test investigating the impact of GeoGebra on their visualisation abilities and their interpretation of the double integral thereof.

3.3 Post test

The post-test required students to perform similar tasks to those of the pre-test. It consisted of a 10 mark question which required students to sketch the solid $\int\int 1 - x^2 dA$ over the triangular region given by $0 \leq y \leq 1 - x, 0 \leq x \leq 1$ and its projections. The test was given to the whole class in the presence of 121 participants.

The quantitative data from pre-test and post-test results were analysed using descriptive statistics. Thematic analysis of the qualitative data from the GeoGebra assignment was performed, and several reflections from students were ranked. Content analysis of the test question was also performed to understand the approach, ability, and presentation of students' sketches. The next section reports on the results and findings of the tests and students' reflections on the process and the importance of visualization tools like GeoGebra in understanding and solving double integrals problems.

4. FINDINGS AND DISCUSSION

In this section, we present the results of the pre-test and post-test, students' activities in GeoGebra as well as their reflections upon using the software. We further draw on the literature and Vygotsky's social constructivist perspectives to discuss our findings.

4.1 Pre-tests 1 and 2 and post test scores

Table 2. Descriptive statistics for tests

	Mean	Median	Mode	Standard Deviation
Pretest 1	5.28	5	10	3.34
Pretest 2	3.7	4	4	2.58
Post Test	5.07	5	7	2.77

The data suggests that the use of GeoGebra has contributed to a more varied performance among students, with the mode score shifting and a more balanced distribution of scores. The use of GeoGebra to teach double integrals may have influenced the results, particularly if the students were not familiar with the software or if the use of technology was not integrated effectively into the course. Perhaps students should have been supported more through their introduction of the software and collaborative peer work should have preceded the individual assignment. We will continue exploring ways to effectively integrate GeoGebra into the teaching and learning of double integrals and to ensure that students receive adequate support and practice in using the tool.

Pre-test 1 is equivalent in content and cognitive level to pre-test 2 however since written on different days little details were changed to preserve the integrity of the test. Table 3 outlines students' results of pre-test 1 and pre-test 2.

Table 3. Pretest results

Questions	*Total marks*	Pretest 1				Pretest 2			
		Students results				*Students results*			
		3	2	1	0	3	2	1	0
1.(a) Sketch region of integration	2		82%	0%	18%		70%	0%	30%
(b) Setup integral with order dxdy	1			54%	46%			24%	76%
(c) Setup integral with order dydx	1			43%	57%			48%	52%
2(a) Sketch the 3D solid	3	39%	4%	5%	52%	12%	3%	58%	27%
(b) xy projection	1			52%	48%			33%	67%
(c) xz projection	1			41%	59%			9%	91%
(d) yz projection	1			46%	54%			15%	85%

Table 3 shows that the majority of students from both tests were able to sketch the region of integration with 82% and 70% getting the correct regions for pre-test 1 and 2 respectively. Their understanding of the relationship between the region sketched in 1(a) and the double integral setup was further probed in the subsequent sub questions 1(b) and 1(c). Despite their success in sketching the region of integration, their success fell short when setting up the limits of integration and this is an indication that not all students were able to interpret the region to sketch the limits of integration. This is usually challenging to students who struggle to interpret inequalities and those who do not understand the geometric interpretation of a double integral. Question 2 of the pre-test gave students a double integral and required them to sketch the 3d solid as well as the projections onto the 3 coordinate planes. 57% and 85% of students from pre-test 1 and pre-test 2 respectively failed to sketch the 3d solid. The results confirmed what we already suspected – the traditional approach to teaching double integrals is not effective in helping students visualise 3d objects.

We then introduced GeoGebra to improve students' visualisation and sketching skills. This introduction was done in a supportive way and students who had not used GeoGebra before were guided through the use of the software. In addition students were encouraged to collaborate with each other in their experience of this new software and new pedagogical approach. In figure 1, the GeoGebra tasks are illustrated.

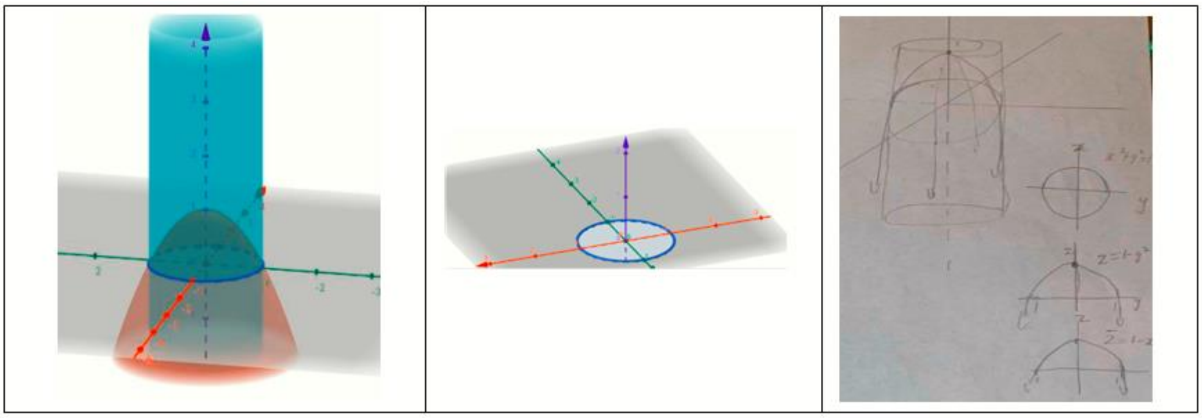

Fig. 3. GeoGebra Activities

Figure 3 is an example of an intersection of two surfaces, a cylinder and a paraboloid. The figure shows the intersection of the two surfaces and a student's sketch of the surfaces as well as the projections onto 3 coordinate planes. It is challenging for students to identify the intersection of these surfaces, and often their resulting projections onto a coordinate plane. These activities allowed students to easily translate the GeoGebra results to sketch the projections and to set up double integrals. Working back and forth between GeoGebra and hand sketches allow students to develop representations of 3d surfaces on a 2d paper. The constructions made during this process enable students to easily draw surfaces in future, visualise projections onto different planes, and to easily isolate intersection curves. This is an illustration of student centred pedagogy with students using technology to enable their own learning.

4.2 Students Reflections

In the qualitative portion of our study, we explored students' reflections on the role and value of visualization tools, such as GeoGebra, in improving their understanding of double integrals. The last task on the GeoGebra assignment required students to discuss the challenges, share insights gained and reflect on their use of GeoGebra. We categorized and tabulated their responses by frequency, as depicted in Table 4.

Table 4: Students reflections on their use of GeoGebra

Description	Frequency
GeoGebra improves visualization	44%
GeoGebra assists in identifying intersections	11%
GeoGebra aids in understanding the region of integration	8%
GeoGebra enhances comprehension of double integrals	8%
GeoGebra facilitates problem-solving	8%
GeoGebra supports determining limits of integration	7%
Grasping GeoGebra's functionality proved difficult	4%
Effective utilization of GeoGebra was learned	3%
GeoGebra improves sketching abilities	3%
GeoGebra enables projection of surfaces onto planes	2%
GeoGebra permits manipulation of graphs	2%

The data suggests that the participants benefited in various ways from the GeoGebra activities. The largest percentage (44%) reported that GeoGebra improves visualisation especially for 3d surfaces and 11% noted that it assists in identifying intersections. Other students indicated that it helps them understand the region of integration, intersection between surfaces, and develops an understanding of double integrals. Some participants noted that even though it was challenging to understand how GeoGebra works for advanced computations, it was a fun exercise that allowed them to manipulate graphs and therefore improved their sketching skills. Some also commented that visualisation tools like GeoGebra aid in visualising formulas that are required to be memorised. They added that they were able to see “what mathematics is doing instead of just merely applying formulas” , and hence the theory of double integrals made sense.

4.3 Post test

The data in table 5 outlines students’ performance in the post-test. Having done activities on GeoGebra to improve visualisation, students were tasked to sketch the 3d solid and its projections onto 3 coordinate-planes, as well as to set up the integral in the order dydx. In sketching a 3D solid, students were awarded 4 marks as opposed to 3 marks from the pre-test because the solid had multiple points of interest in the 3 coordinate planes. This is further verified by students’ poor performance in sketching the yz projection.

Table 5. Post Test results

Questions	*Total marks*	Post test *Students results*				
		4	3	2	1	0
1.(a) Sketch the 3D solid	4	15%	10%	28%	12%	35%
(b) Setup integral with order dxdy	3		75%	0%	0%	25%
(b) xy projection	1				75%	25%
(c) xz projection	1				48%	52%
(d) yz projection	1				0%	100%

Upon examining the post-test results, it is evident that the introduction of the GeoGebra tool has had a varied impact on the students' understanding and performance. In sketching the 3D solid, the percentage of students scoring full marks has significantly decreased in the post-test (15%) compared to the pre-tests. However, the distribution of scores is more balanced, indicating that although students may not have mastered this concept, they have moved away from complete non-understanding. The introduction of GeoGebra also appears to have helped some students to better visualize 3D regions, but additional practice and reinforcement may be required. Although sketching solids remains a challenge, the introduction of GeoGebra seems to have greatly benefited students in setting up the integral with the order of $dxdy$. The percentage of students scoring full marks has risen significantly to 75%. This suggests that the tool has helped deepen students' understanding of determining limits of integration and changing the order of integration. This is further substantiated by students' reflections to that effect.

For projections on the coordinate planes xy, xz, and yz, the xy projection results are very positive, with 75% of students scoring full marks. This indicates that the visualization tool has been effective in helping students understand this concept. However, the results for the xz and particularly the yz projections are less encouraging. The yz projection seems to have been particularly challenging, with all students failing to score any marks. This suggests that the tool may not have been as effective in illustrating these types of projections, or that students need more time to become familiar with using it.

5. CONCLUSION

This study's focus on addressing misconceptions has significant implications for mathematics and engineering educators as well as the professional practice of graduates. Misconceptions in double integration, in particular, also affect students' performance in other sections of vector calculus such as line integrals, surface integrals and Stokes' Theorem, making it equally important for teaching to focus on these misconceptions to improve performance in vector calculus.

The primary objective of this study was to harness the potential of GeoGebra, aiming not only to enhance students' motivation in vector calculus but also to cultivate their critical thinking through the integration of software with other course activities. This approach enabled students to discern meaningful connections between theoretical concepts and applications, facilitating their comprehension of fundamental principles through the effective utilization of graphs. Furthermore, this approach aligned with Vygotsky's perspectives on students independently and collaboratively constructing meaning through engaging in meaningful activities, thus fostering a deeper and more holistic understanding of vector calculus.

Overall, the data suggests that GeoGebra is a useful tool for enhancing visualization, problem-solving, and understanding of complex mathematical concepts. However, the software may have a steep learning curve and may require a significant investment of time and effort to use effectively. It is suggested that vector calculus educators liaise with first year calculus educators to discuss the introduction of this software into first year calculus in a more supported and integrated way and to work together to address misconceptions that develop from students' prerequisite knowledge. Caution should be exercised to carefully integrate the software into the activities of the course and engage all students to participate in all activities of the course. Lauten and Ferrini-Mundy (1994) caution that technology should be used appropriately and not seen as a panacea for all student struggles.

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APPENDIX A: GEOGEBRA ASSIGNMENT

Title: Exploring Double Integrals with GeoGebra
Objectives

- To familiarize students with the concept of double integrals and their applications in finding the volume under a surface in 3D space, using GeoGebra as a visualization tool.
- To deepen students' understanding of double integrals, the process of determining limits of integration, changing the order of integration, and calculating volumes under surfaces. Additionally, students will explore the intersection of quadric surfaces in 3D using GeoGebra.

Task:
Using the GeoGebra software (https://www.geogebra.org/3d?lang=en), complete the following:

1. Consider the following surfaces: $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$.
    a) Graph the functions in GeoGebra.
    b) Determine the curve of intersection in GeoGebra.

2. Given $x^2 + y^2 + z^2 = 4, z \geq 0$.
    a) Graph the assigned function in GeoGebra.
    b) Determine the appropriate domain for the double integral.
    c) Calculate the volume under the surface using double integrals in GeoGebra.
    d) Verify the result by calculating the volume by hand.

3. Consider $z = 1 - x^2 - y^2,\ x^2 + y^2 = 1,\ z \geq 0$,
    a) Graph the assigned functions in GeoGebra.
    b) Determine the appropriate domain for the double integral.
    c) Calculate the volume under the surface using double integrals in GeoGebra.
    d) Rewrite the double integral in c) with the order of integration changed (Determine the new limits of integration and explain the reasoning behind the changes).

4. Present the results of the above and discuss any challenges faced, insights gained, and reflect on the process and the importance of visualization tools like GeoGebra in understanding and solving double integrals problems.

APPENDIX B: PRETEST 2

1. $\mathbb{R}$ is the region bounded by $y = x,\ y = 1 - x$, and $x = 1$.
   (a) Sketch the region $\mathbb{R}$.

   (b) Use your sketch to setup the integral used for finding the area enclosed by $\mathbb{R}$ such that integration with respect to x is first before integration with respect to y.

   (c) Now setup the integral used for finding the area enclosed by $\mathbb{R}$ such that integration with respect to y is first before integration with respect to x.

2. (a) Sketch the solid S whose volume is described by

$$\int_{0}^{\frac{3}{2}} \int_{2x}^{3} (\sqrt{9 - y^2})\,dy\,dx$$

   (b) Sketch the projections of S on the three major planes.

   $\boldsymbol{xy - plane}$

$\boldsymbol{xz - plane}$

$\boldsymbol{yz - plane}$

APPENDIX C: GEOGEBRA ACTIVITIES

Some of the activities from the course handbook and tutorials:

18. Sketch the region in $\mathbf{R}^2$ over which the integral $\int_0^1 \int_{x^2}^{x} xy^2 \, dydx$ is evaluated.

19. Sketch the solid whose volume is described by $\int_0^3 \int_{-\sqrt{y}}^{\sqrt{y}} (3-y) \, dxdy$

20. For each of the following, describe and sketch a solid whose volume is given by the repeated integral:

a) $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (5-x-2y) \, dxdy$ b) $\int_0^1 \int_x^{\sqrt{2-x^2}} (2-x^2-y^2) \, dydx$

20. In each of the cases, sketch the region enclosed by the two given surfaces in $\mathbf{R}^3$, and then express this region in terms of spherical coordinates.

a) $z=\sqrt{x^2+y^2}$, $z=\sqrt{4-x^2-y^2}$

b) $z=\sqrt{x^2+y^2}$, $z=1$

A Student using GeoGebra to sketch Question 20 (a) from the handbook.