Revisiting the phase space of constrained, differentially growing bilayers

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Growth-induced morphological instabilities are ubiquitously observed in biological systems, across various length scales (Li *et al.*, 2012). Indeed, instabilities in the form of wrinkling, folding and creasing are often biologically functional. A material arrangement where morphological instabilities are commonly observed consists of a thin stiff layer mounted on top of a thicker substrate, where the layer and substrate can grow at the same or different rates. Depending on the level of differential growth between film and substrate ($G_\mathrm{f}/G_\mathrm{s}$) and the relative elastic moduli ($E_\mathrm{f}/E_\mathrm{s}$), different patterns such as sinusoidal wrinkling, periodic doubling, period quadrupling, folding or creasing can be observed. An understanding of the morphological phase diagram of growing bilayers relating form to moduli and differential growth ratios may facilitate physical understanding and also contribute to the potential development of new biomedical diagnostic tools.

Current work on growth in bilayers mainly focuses on the initial or intermediate post-critical regime (Wang & Zhao, 2015; Dortdivanlioglu *et al.*, 2017), and often only considers growth in the thin film (Alawiye *et al.*, 2020). Little is known about how the post-critical phase diagram evolves as the elastic moduli ($E_\mathrm{f}/E_\mathrm{s}$) and differential growth ($G_\mathrm{f}/G_\mathrm{s}$) ratios change. To address this gap, we compute morphological phase diagrams of growing bilayers with low elastic moduli ratios as often found in biological systems ($E_\mathrm{f}/E_\mathrm{s} \in [1.5, 50]$), and for a range of film/substrate growth ratios ($G_\mathrm{s}/G_\mathrm{f} \in [0, 10]$). To this end, we used analytical modelling with asymptotic analysis and nonlinear finite element models coupled to continuation algorithms (Groh, 2022). The latter facilitates the comprehensive exploration of the post-critical bifurcation landscape. Phase diagrams constructed in parameter space ($E_\mathrm{f}/E_\mathrm{s}$,$G_\mathrm{s}/G_\mathrm{f}$, $\mathrm{g_p}$)—where g_p is a scalar controlling the overall growth magnitude—reveal that the phase space breaks into flat, sinusoidally wrinkled, period doubled/quadrupled, and creased regimes. We also uncover a relation between $E_\mathrm{f}/E_\mathrm{s}$ and $G_\mathrm{s}/G_\mathrm{f}$ that defines the boundary between supercritical to subcritical wrinkling, which, interestingly, occurs for a periodic wavelength of finite period and not at the transition to a Biot instability.

References

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