Improved crest height predictions for nonlinear and breaking waves in large storms

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Abstract

The statistical distribution of zero-crossing crest heights represents a critical design input for a wide range of engineering applications. The present paper describes the development and validation of a new crest height model, suitable for application across a broad range of water depths. The purpose of this model is two-fold: first, to describe the amplifications of the largest crest heights arising due to nonlinear interactions beyond a second-order of wave steepness, and second, to incorporate the dissipative effects of wave breaking. Although these two effects act counter to each other, there is substantial evidence to suggest departures from existing models based upon weakly nonlinear second-order theory; the latter corresponding to current design practice. The proposed model has been developed based on a significant collection of experimental results and a small subset of field measurements. It incorporates effects arising at different orders of nonlinearity as well as wave breaking in a compact formulation and covers a wide range of met--ocean conditions. Importantly, the new model has been independently validated against a very extensive database of experimental and field measurements. Taken together, these include effective water depths ranging from shallow water ($k_p d \sim 0.5$) to deep water ($k_p d > 3$) and sea-state steepnesses covering mild, severe and extreme conditions. The new model is shown to provide a significant improvement in crest height predictions over existing methods. This is particularly evident in the steepest, most severe sea--states which inevitably form the basis of design calculations.

Keywords extreme waves, nonlinearity, wave breaking, wave statistics

1 INTRODUCTION

Traditional oil & gas applications, the rapidly expanding offshore wind energy sector, and different types of marine renewable devices are all examples for which accurate crest height calculations are essential. Importantly, such applications occur in a wide variety of deep, intermediate and shallow water depths. As such, successful crest height predictions must be achievable across the full range of effective water depths.

In this context, the success of a crest height distribution is defined by its potential to incorporate both the amplifications associated with fully nonlinear wave-wave interactions and the dissipative effects of wave breaking. Evidence of the importance of these effects is given in Latheef and Swan (2013) and Karmpadakis et al. (2019); the former relating to deep water and the latter intermediate water depths. Whilst the findings in these studies were largely based upon laboratory data, **Figure 1** provides characteristic examples based upon water surface elevations recorded in the field. In both subplots, the exceedance probability (Q) of the normalised crest heights (η_c / H_s) is compared to the most widely applied models in engineering design. These models include effects arising either at the first (linear) or second--order of wave steepness (Forristall, 2000); the latter representing the recommended practice in most design codes. **Figure 1**(a) shows data recorded at a deep water location ($d = 110$ m), in which the largest crest heights are notably larger than the design (second-order) predictions. In this case, design predictions appear to align with the lower bound of the 95% confidence intervals of the data. In contrast, **Figure 1**(b) shows measurements at a shallow water location $d = 7.7$ m). In this case the largest measured crest heights lie between the linear and second-order predictions; both models clearly misrepresenting the measured field data.

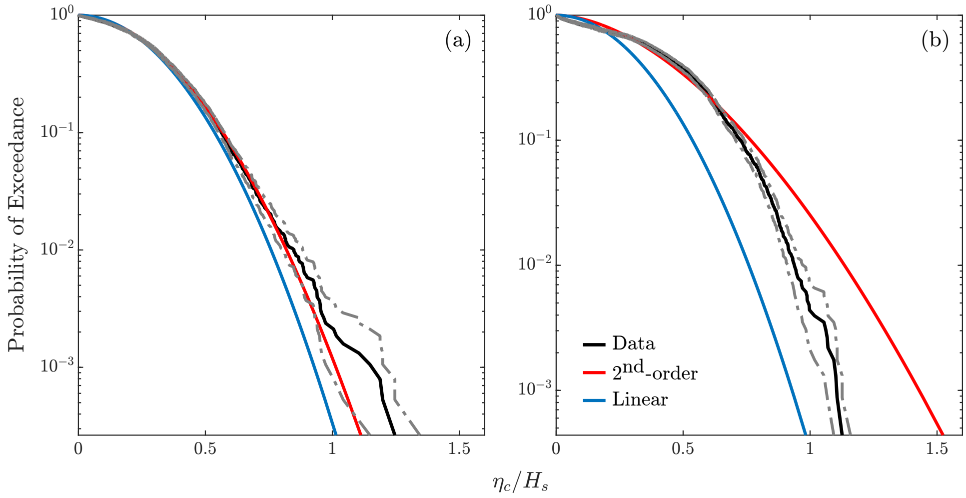

Figure 1: Examples of normalised crest height distributions, η_c/H_s recorded in the field and showing comparisons to linear and second-order predictions. The data relate to: **(a)** $H_s = 8\ m,\ T_p = 10\ s, d = 110\ m$ and **(b)** $H_s = 4.5\ m,\ T_p = 9.5\ s, d = 7.7\ m$. The 95% confidence intervals have been added for reference.

2 BACKGROUND

Having noted the practical importance of crest height predictions, it is not surprising that several studies have addressed their short-term distribution. Among these, the Rayleigh (Longuet-Higgins, 1952), Tayfun (Tayfun, 1980), Forristall (Forristall, 2000) and Tayfun & Fedele (Tayfun and Fedele, 2007) distributions are the most widely applied. These models cover effects arising at increasing orders of nonlinearity; the latter being expressed in terms of effective wave steepness (αk), where α represents the wave amplitude and k the wavenumber. Taken together, these models cover the first three orders of wave steepness and have been extensively validated and assessed using a wide range of wave conditions (Karmpadakis et al., 2019; Latheef and Swan, 2013) The key characteristics of the models are summarised below:

(a) Rayleigh distribution (Longuet-Higgins, 1952)

This is the first-order (linear) model that describes crest heights arising in a Gaussian sea-state. Its functional form is defined as:

$$Q(\eta_c) = \exp\left[-8\left(\frac{\eta_c}{H_s}\right)^2\right], \tag{1}$$

where Q is the exceedance probability, η_c the crest heights and H_s the significant wave height. The significant wave height is calculated using its spectral definition as

$$H_s = 4\sqrt{m_0} = 4\sigma_\eta, \tag{2}$$

where σ_η is the standard deviation of $\eta(t)$ and m_0 is the zeroth spectral moment. Using the variance spectrum, $S_{\eta\eta}(f)$, the spectral moments of order n are described by:

$$m_n = \int_0^\infty f^n S_{\eta\eta}(f)\mathrm{d}f \tag{3}$$

where f represents the frequency of individual wave harmonics. Bound by the assumption of linearity, the Rayleigh distribution is known to significantly under-estimate the largest crests, or those arising at small exceedance probabilities.

(b) Tayfun distribution (Tayfun, 1980)

Considering nonlinear effects arising at a second-order of wave steepness and a narrowband approximation, Tayfun (1980) derived an analytical model to describe the crest height distribution. This is given by:

$$Q(\eta_c) = \exp\left[-\frac{\left(-1+\sqrt{1+8\mu\frac{\eta_c}{H_s}}\right)^2}{2\mu^2}\right], \tag{4}$$

where μ is a measure of wave steepness that accounts for second-order nonlinearities. In its original form, μ was related to the sea-state skewness, λ_3 (Fedele and Tayfun, 2009).

(c) Forristall (3D) distribution (Forristall, 2000)

This model is the most widely applied in engineering practice. It has been derived as a fit to numerical simulations of second-order random wave theory (Sharma and Dean, 1981). Its functional form is a two-parameter Weibull distribution, defined as:

$$Q(\eta_c) = \exp\left[-\left(\frac{\eta_c}{\alpha H_s}\right)^\beta\right] \tag{5}$$

where the scale, α, and shape, β, can be found in Forristall (2000).

(d) Tayfun & Fedele distribution (Tayfun and Fedele, 2007)

This is a third-order model derived on the basis of a GramCharlier series expansion. This method of representation has previously been applied in the description of water surface elevations and related wave statistics by Longuet-Higgins (1963) and Bitner (1980), as well as wave envelopes and phasing by Tayfun and Lo (1990). The functional form of the model is given by:

$$Q(\eta_c) = \exp\left[-\frac{\left(-1+\sqrt{1+8\mu\frac{\eta_c}{H_s}}\right)^2}{2\mu^2}\right]\left\{1+\Lambda\left(\frac{\eta_c}{H_s}\right)^2\left[4\left(\frac{\eta_c}{H_s}\right)^2-1\right]\right\}, \tag{6}$$

where μ and Λ can be found in the original paper.

3 NEW MODEL

A vast dataset of experimental and field measurements has been analysed and employed in the development of a new model. For brevity these are not discussed here, but can be found in Karmpadakis and Swan (2022) The proposed model aims to:

- be applicable across a wide range of effective water depths extending from relatively shallow $(k_p d \approx 0.5)$ to deep $(k_p d > 3)$ conditions;
- capture the amplifications beyond the second-order of wave steepness that have been observed in steep sea-states in both field and experimental measurements; and
- incorporate the effects of wave breaking, limiting the largest crest heights.

To achieve this, the new model has been formulated using the latest understanding of the physical processes that contribute to the formation of the largest crest heights. Any empirical coefficients included in the model have been calibrated using data from 2 experimental campaigns and a small subset of field measurements.

The proposed model is formulated by incrementally including the relevant physical processes that define the largest crest heights. As such, the development of the model begins with the most fundamental assumption of linear wave theory, with additional terms progressively added to represent nonlinear contributions and wave breaking. In adopting this approach, the validity of the model can be assessed at each step and further parametrisations can easily be achieved. The proposed model has the following form:

$$\eta_M = \left(\eta^{(1)} + \eta^{(2)} + \eta^{(NL)}\right) \cdot f_{Br}, \tag{7}$$

where η_M represents the crest heights predicted by the new model, $\eta^{(1)}$ is the linear contribution, $\eta^{(2)}$ is the second order contribution and $\eta^{(NL)}$ is the higher-order nonlinear contribution including all effects above second-order. The additional scaling term f_{Br} incorporates the effects of wave breaking. The parametrisation of the model can be found in Karmpadakis and Swan (2022).

Evidence of the success of the new model is provided in Figure 2. Each plot compares the distribution of measured crest heights with the predictions of the new model, the Forristall and Rayleigh distributions. All the sea-states considered belong to the calibration dataset and have been selected to represent the widest range of effective water depths. Figure 2(a) relates to a deep-water case with effective water depth $k_p d = 3.5$ and sea-state steepness $S_p \approx 0.04$. The observed amplifications above the Forristall model for $Q < 10^{-2}$ are well described by the proposed model. Figure 2(b) shows that the new model is equally successful for an intermediate water depth case with $k_p d = 1.53$ and $S_p = 0.03$. In both cases, the key feature of the sea-states is the increase of the measured crest heights above from the second order model.

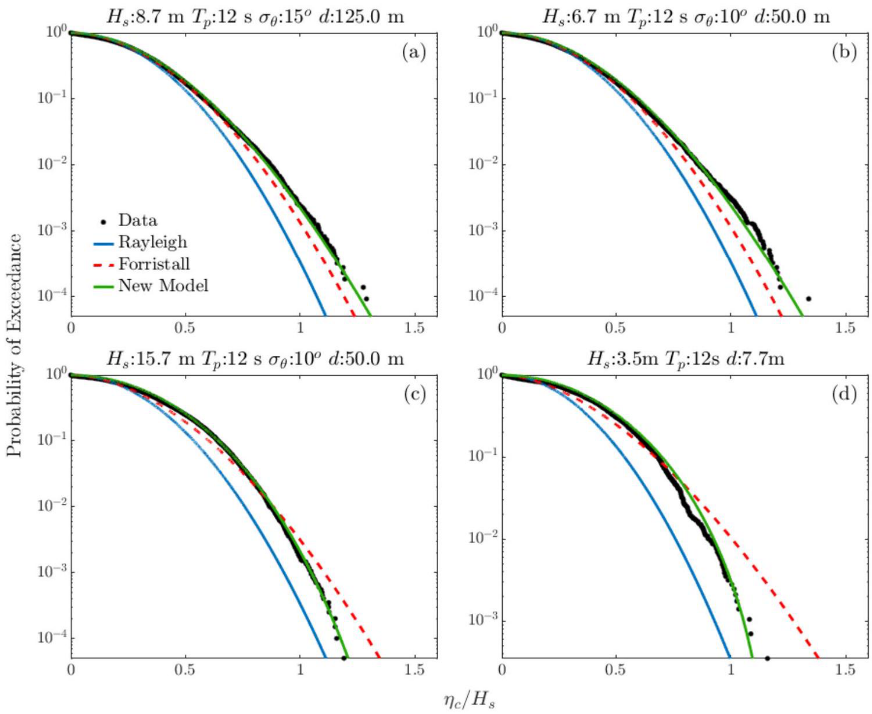

Figure 2: Normalised crest height distributions, η_c/H_s, showing comparisons to linear (Rayleigh), second-order (Forristall) and proposed model predictions. Subplots **(a)-(c)** correspond to experimental cases, while subplot **(d)** to field data.

Considering sea-states that are characterised by extensive wave breaking, Figure 2 (c) concerns an intermediate water depth case with $k_p d = 1.53$ and $S_p = 0.07$. In this case, the observed reduction in the tail of the distribution compared to second-order predictions is well described by the new model. Finally, Figure 2(d) presents results arising in a single storm event recorded at the shallowest measuring location in the field. The data correspond to a sea-state with $k_p d = 1.5$ and $S_p = 0.015$. Considering the shallow depth and the severity (steepness) of this case, it is clear that wave breaking plays a significant role. This becomes apparent by noting the large reductions in crest heights for $Q < 10^{-1}$ when compared to second-order predictions. More importantly, it is clear that the new model is successful in capturing this behaviour.

4 CONCLUSIONS

The new model has been shown to provide considerable improvements over existing models in the vast majority of cases considered. Most importantly, it has been shown to successfully predict the changes in the crest height distribution arising due to the nonlinear amplifications and the dissipative effects of wave breaking. The latter incorporating both steepness-induced wave breaking in deeper water and depth induced wave breaking in shallower water. Importantly, the proposed model is directly relevant to design calculations, being no more difficult to apply than the existing design standard. Moreover, the model is presented in a modular form in the sense that it parametrically describes individual physical processes separately before bringing everything together. In this respect, each component of the model can be assessed individually and improved (if necessary) as more data become available.

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