Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(11): 25845–25862.
DOI: 10.3934/math.20231318
Received: 07 August 2023
Revised: 24 August 2023
Accepted: 28 August 2023
Published: 07 September 2023

Research article

Fractional comparative analysis of Camassa-Holm and Degasperis-Procesi equations

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Abstract: This paper focuses on novel approaches to finding solitary wave (SW) solutions for the modified Degasperis-Procesi and fractionally modified Camassa-Holm equations. The study presents two innovative methodologies: the Yang transformation decomposition technique and the homotopy perturbation transformation method. These methods use the Caputo sense fractional order derivative, the Yang transformation, the adomian decomposition technique, and the homotopy perturbation method. The inquiry effectively solves the fractional Camassa-Holm and Degasperis-Procesi equations, which also provides a detailed numerical and graphical comparison of the solutions found. The results, which include accurate solutions, derived solutions, and absolute error displayed in tabular style, demonstrate the effectiveness of the suggested procedures. These procedures are iterative, which results in several answers. The estimated absolute error attests to the correctness and simplicity of these solutions. Especially in plasma physics, these approaches may be expanded to handle various linear and nonlinear physical issues, including the evolution equations controlling nonlinear waves.

Keywords: Yang transform; Degasperis-Procesi and Camassa-Holm equations; series solution; adomian decomposition method; homotopy perturbation method
Mathematics Subject Classification: 33B15, 34A34, 35A20, 35A22, 44A10

Nomenclature

υ : Independent variable
$\imath$: Time
$\zeta(\imath, \upsilon)$: Dependent function representing the physical quantity
ς : Fractional order
Y : Shehu transform
Y^{-1} : Inverse Shehu transform
ϵ : Perturbation parameter

1. Introduction

Fractional calculus (FC) offers a practical and straightforward approach to obtaining specific information from different equations. This dynamic field of mathematics generalizes the standard integer order to a fractional order, resulting in a vast array of mathematical models [1–6]. In recent years, fractional differential equations (FDEs) have become a popular area of research for various science and engineering applications. There are several fundamental fractional derivative concepts, such as the Liouville-Caputo, Riemann-Liouville, Caputo-Fabrizio, Atangana-Baleanu, and Hadamard derivatives [7–11]. The fractional Riemann-Liouville derivative allows the inclusion of initial sources expressed through fractional integrals and derivatives, while the fractional Caputo derivatives only accept conventional boundaries and initial conditions. Nonlinear problems have been applied in different fields, including (non)linear waves in plasmas, hydrology, nuclear engineering, astrophysics, and meteorology [12–18].

Mathematicians have become increasingly interested in fractional partial differential equations (FPDEs) in recent years due to their numerous applications in engineering, applied sciences, biology, mathematical physics, solid-state materials science, neurology, geophysics, plasma physics, electrochemistry, physiological scaling laws, chemical physics, dielectric behavior, quantum systems, financial mathematics, fractional dynamics, electromagnetics and quantum computing. Few of the other implementations of fractional partial differential equations can be discovered in viscoplastic and viscoelastic flows [19], spherical flames [20], continuum mechanics [21], image processing [22], wave propagation [23], entropy [24], anomalous diffusion [25], groundwater containment transport [26], turbulent flow [27] and so on [28–30].

FC has piqued the interest of academics because of its practical applicability in various real-world challenges. Mathematicians have worked on discovering numerical and analytical solutions to FPDEs and similar systems to investigate the problem better. To address FPDEs, researchers have expended significant effort in inventing different approaches, resulting in a well-studied field [31–35]. The findings of these investigations contribute to a better understanding of the dynamics of natural systems. Scholars have refined their approaches to solving FPDEs over time, yielding a variety of effective methods such as the Sine-Gordon expansion technique, Yang transformation decomposition technique, finite element method, variational iteration method, natural transform decomposition method, first integral method, finite volume methods, generalized Kudryashov method, and many others [36–45]. A general modified *kappa*-equation with the following type, i.e., a family that includes significant physical equations, is considered in this paper [46]:

$$\frac{\partial^{\varsigma}\zeta(\beta,\upsilon)}{\partial\upsilon^{\varsigma}}-\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+(\kappa+1)\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}-\kappa\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}=0. \quad (1.1)$$

For $\kappa = 3$, the modified Degasperis-Procesi (mDP) equation is obtained

$$\frac{\partial^{\varsigma}\zeta(\beta,\upsilon)}{\partial\upsilon^{\varsigma}}-\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+4\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}-3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}=0. \quad (1.2)$$

In Eq (1.1) for $\kappa = 2$, the result is the manifestation of the modified Camassa-Holm (mCH) equation

$$\frac{\partial^{\varsigma}\zeta(\beta,\upsilon)}{\partial\upsilon^{\varsigma}}-\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+3\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}-2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}=0. \quad (1.3)$$

The fractional modified Camassa-Holm (mCH) and modified Degasperis-Procesi (mDP) equations are fascinating advances in the realm of nonlinear partial differential equations (PDEs) that help us comprehend wave dynamics and soliton behavior. These equations have sparked substantial attention because of their ability to account for anomalous diffusion and dispersive effects, which classical models usually overlook. The fractional mCH equation, a refinement of the conventional Camassa-Holm equation, uses fractional derivatives to simulate wave propagation more precisely. The features of this equation, such as its well-posedness, conservation laws and the behavior of solitary wave solutions, have been widely researched in the scientific literature. This anomalous diffusion feature has been found in a variety of physical systems, including turbulent fluid flow and transport processes [47].

To capture complicated dispersive effects, the modified mDP equation, a version of the classical Degasperis-Procesi equation, additionally incorporates fractional derivatives. This equation's integrability, symmetries and wave propagation features have all been examined. In wave dynamics, nonlinearity, dispersion and fractional calculus interact extensively. The appearance of these fractional alterations has opened up new areas for research into this relationship. Researchers have used a variety of analytical and computational approaches to examine the characteristics of solutions, the integrability of these equations, and their application in simulating real-world occurrences.

The study of fractional PDEs, as exemplified by the fractional mCH and modified mDP equations, demonstrates the profound impact fractional calculus has on enhancing the predictive ability of mathematical models in numerous scientific fields. By considering fractional derivatives, researchers obtain a deeper understanding of the complex behaviors exhibited by waves and solitons in various physical systems [48, 49].

The Yang transformation, a significant concept in soliton theory and integrable systems, is a potent mathematical tool to derive solutions for certain nonlinear partial differential equations (PDEs). It establishes a connection between various remedies, frequently transforming one into another. This transformation is particularly relevant in the context of soliton equations, where it can systematically produce multi-soliton solutions by relating them to simpler seed solutions. The Yang transformation has applications in numerous disciplines, such as fluid dynamics, plasma physics, and mathematical physics, contributing to a greater comprehension of complex nonlinear phenomena and their mathematical representations [50–52]. In this study, two novel schemes, the YTDM and HPTM, are combined with the YT to approximate the fractional mDP and mCH problems. The proposed methods offer high accuracy while requiring less computational work than other techniques. This work's structure is as follows: We give a few basic aspects of calculus theory in Section 2. Sections 3 and 4 provide the HPTM and YTDM formulations for obtaining the general solution. In Section 5, using a few numerical examples and comparisons to the exact solution, we show the viability and effectiveness of both approaches. Finally, Section 6 contains the conclusion.

2. Basic definitions

Definition 1.
The fractional Caputo operator of order ς is written as [53]

$$D_{\upsilon}^{\varsigma}\zeta(\beta,\upsilon)=\frac{1}{\Gamma(k-\varsigma)}\int_{0}^{\upsilon}(\upsilon-\jmath)^{k-\varsigma-1}\zeta^{(k)}(\beta,\jmath)d\jmath,\quad k-1<\varsigma\leq k,\quad k\in N. \tag{2.1}$$

Definition 2.
The YT for $\zeta(\upsilon)$ is defined as [53]

$$Y\{\zeta(\upsilon)\}=M(u)=\int_{0}^{\infty}e^{\frac{-\upsilon}{u}}\zeta(\upsilon)d\upsilon,\ \ \upsilon>0, \tag{2.2}$$

provided the integral exists for some "u".
And the inverse of Yang transform is

$$Y^{-1}\{M(u)\}=\zeta(\upsilon). \tag{2.3}$$

Definition 3.
The inverse of YT is given by [53]

$$Y^{-1}[Y(u)]=\zeta(\upsilon)=\frac{1}{2\pi\iota}\int_{\varsigma-\iota\infty}^{\varsigma+\iota\infty}\zeta\left(\frac{1}{u}\right)e^{u\upsilon}udu=\Sigma\ \ residues\ \ of\ \ \zeta\left(\frac{1}{u}\right)e^{u\upsilon}u\quad\varsigma>0.$$

Definition 4.
The YT of the fractional derivative is given by [53]

$$Y\{\zeta^{\varsigma}(\upsilon)\}=\frac{M(u)}{u^{\varsigma}}-\sum_{k=0}^{n-1}\frac{\zeta^{k}(0)}{u^{\varsigma-(k+1)}},\ \ n-1<\varsigma\leq n,\ \ \forall\ \ n=1,2,3,\cdots. \tag{2.4}$$

3. General implementation of homotopy perturbation transformation method

Consider the general fractional partial differential equation

$$D_{\upsilon}^{\varsigma}\zeta(\beta,\upsilon)=\mathcal{P}_{1}[\beta]\zeta(\beta,\upsilon)+\mathcal{R}_{1}[\beta]\zeta(\beta,\upsilon),\ \ 0<\varsigma\leq 1, \tag{3.1}$$

the initial condition (IC)

$$\zeta(0,\upsilon)=\xi(\upsilon).$$

Let $D_{\upsilon}^{\varsigma}\zeta(\beta,\upsilon)$ denote the Caputo fractional derivative of order ς with respect to υ, and let $\mathcal{P}_1$ and $\mathcal{R}_1$ represent the linear and nonlinear terms, respectively.

Using Yang transform in the above equation

$$Y[D_\upsilon^\varsigma \zeta(\beta,\upsilon)] = Y[\mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon)], \tag{3.2}$$

$$\frac{1}{u^\varsigma}\{M(u) - u\zeta(\beta,0)\} = Y[\mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon)]. \tag{3.3}$$

By utilizing the differential characteristic of the YT, we obtain

$$M(u) = u\zeta(\beta,0) + u^\varsigma Y[\mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon)]. \tag{3.4}$$

By utilizing the inverse of the Yang transform, we get

$$\zeta(\beta,\upsilon) = \zeta(\beta,0) + Y^{-1}[u^\varsigma Y[\mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon)]]. \tag{3.5}$$

Construct a homotopy operator that contains a small parameter ϵ and $\epsilon \in [0,1]$ to create a family of related equations, starting from a simple linear equation and gradually transitioning to the transformed nonlinear equation. This operator effectively bridges the gap between linear and nonlinear problem.

$$\zeta(\beta,\upsilon) = \sum_{k=0}^{\infty} \epsilon^k \zeta_k(\beta,\upsilon). \tag{3.6}$$

The nonlinear components in Eq (3.1) is defined as

$$\mathcal{R}_1[\beta]\zeta(\beta,\upsilon) = \sum_{k=0}^{\infty} \epsilon^k H_k(\zeta), \tag{3.7}$$

The technique obtain polynomial is described in Ref [53]

$$H_k(\zeta_0,\zeta_1,...,\zeta_n) = \frac{1}{\Gamma(n+1)} D_\epsilon^k \left[\mathcal{R}_1\left(\sum_{k=0}^{\infty} \epsilon^i \zeta_i\right)\right]_{\epsilon=0}, \tag{3.8}$$

where $D_\epsilon^k = \frac{\partial^k}{\partial \epsilon^k}$.

By combining Eqs (3.6) and (3.8) with Eq (3.5), we obtain

$$\sum_{k=0}^{\infty} \epsilon^k \zeta_k(\beta,\upsilon) = \zeta(\beta,0) + \epsilon \times \left(Y^{-1}\left[u^\varsigma Y\{\mathcal{P}_1 \sum_{k=0}^{\infty} \epsilon^k \zeta_k(\beta,\upsilon) + \sum_{k=0}^{\infty} \epsilon^k H_k(\zeta)\}\right]\right). \tag{3.9}$$

By analyzing the correlation of the coefficient of ϵ, we get

$$\begin{aligned}
&\epsilon^0 : \zeta_0(\beta,\upsilon) = \zeta(\beta,0),\\
&\epsilon^1 : \zeta_1(\beta,\upsilon) = Y^{-1}\left[u^\varsigma Y(\mathcal{P}_1[\beta]\zeta_0(\beta,\upsilon) + H_0(\zeta))\right],\\
&\epsilon^2 : \zeta_2(\beta,\upsilon) = Y^{-1}\left[u^\varsigma Y(\mathcal{P}_1[\beta]\zeta_1(\beta,\upsilon) + H_1(\zeta))\right],\\
&.\\
&.\\
&.\\
&\epsilon^k : \zeta_k(\beta,\upsilon) = Y^{-1}\left[u^\varsigma Y(\mathcal{P}_1[\beta]\zeta_{k-1}(\beta,\upsilon) + H_{k-1}(\zeta))\right], \quad k>0, k\in N.
\end{aligned} \tag{3.10}$$

Therefore, the approximation of Eq (3.1) takes the following series form

$$\zeta(\beta,\upsilon) = \lim_{M\to\infty} \sum_{k=1}^{M} \zeta_k(\beta,\upsilon). \tag{3.11}$$

4. Implementation of YTDM

Consider the general fractional partial differential equation

$$D_{\upsilon}^{\varsigma}\zeta(\beta,\upsilon) = \mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon), \quad 0 < \varsigma \leq 1, \tag{4.1}$$

with the IC

$$\zeta(0,\upsilon) = \xi(\upsilon).$$

By performing the YT, we get

$$\begin{aligned} Y[D_{\upsilon}^{\varsigma}\zeta(\beta,\upsilon)] &= Y[\mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon)], \\ \frac{1}{u^{\varsigma}}\{M(u) - u\zeta(\beta,0)\} &= Y[\mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon)]. \end{aligned} \tag{4.2}$$

Utilizing the distinguishing characteristic of the YT, we acquire

$$M(u) = u\zeta(\beta,0) + u^{\varsigma}Y[\mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon)], \tag{4.3}$$

By utilizing the inverse of the Yang transform method, we have

$$\zeta(\beta,\upsilon) = \zeta(\beta,0) + Y^{-1}[u^{\varsigma}Y[\mathcal{P}_1[\beta]\zeta(\beta,\upsilon) + \mathcal{R}_1[\beta]\zeta(\beta,\upsilon)]. \tag{4.4}$$

The solution of $\zeta(\beta,\upsilon)$ is given as

$$\zeta(\beta,\upsilon) = \sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon). \tag{4.5}$$

The nonlinear components in Eq (4.1) is expressed as

$$\mathcal{R}_1(\beta,\upsilon) = \sum_{m=0}^{\infty}\mathcal{A}_m(\zeta), \tag{4.6}$$

with

$$\mathcal{A}_m\left(\zeta_0,\zeta_1,\zeta_2,\cdots,\zeta_m\right) = \frac{1}{m!}\left[\frac{\partial^m}{\partial\ell^m}\left\{\mathcal{R}_1\left(\sum_{m=0}^{\infty}\ell^m\zeta_m\right)\right\}\right]_{\ell=0}, \quad m = 0,1,2,\cdots \tag{4.7}$$

By combine source of Eqs (4.5) and (4.7) in Eq (4.4), we obtain as

$$\sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon) = \zeta(\beta,0) + Y^{-1}u^{\varsigma}\left[Y\left\{\mathcal{P}_1\left(\sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon)\right) + \sum_{m=0}^{\infty}\mathcal{A}_m(\zeta)\right\}\right]. \tag{4.8}$$

Thus, we get

$$\zeta_0(\beta,\upsilon) = \zeta(\beta,0), \tag{4.9}$$

$$\zeta_1(\beta,\upsilon) = Y^{-1}\left[u^{\varsigma}Y\{\mathcal{P}_1(\zeta_0) + \mathcal{A}_0\}\right],$$

$$\zeta_{m+1}(\beta,\upsilon) = Y^{-1}\left[u^{\varsigma}Y\{\mathcal{P}_1(\zeta_m) + \mathcal{A}_m\}\right].$$

5. Application

Example 5.1.

Considering the following fractional mDP equation

$$\frac{\partial^{\varsigma}\zeta(\beta,\upsilon)}{\partial\upsilon^{\varsigma}}-\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+4\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}-3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}=0, \quad (5.1)$$

with the IC

$$\zeta(\beta,0)=-\frac{15}{8}\,\mathrm{sech}^{2}\left(\frac{\beta}{2}\right).$$

Applying the YT, we get

$$Y\left(\frac{\partial^{\varsigma}\zeta}{\partial\upsilon^{\varsigma}}\right)=Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-4\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}\right], \quad (5.2)$$

By utilizing the differentiating aspect of the YT, we have

$$\frac{1}{u^{\varsigma}}\{M(u)-u\zeta(\beta,0)\}=Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-4\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}\right], \quad (5.3)$$

$$M(u)=u\zeta(\beta,0)+u^{\varsigma}Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-4\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}\right]. \quad (5.4)$$

By utilizing the inverse of the YT method, we have

$$\begin{aligned}&\zeta(\beta,\upsilon)=\zeta(\beta,0)+Y^{-1}\Bigg[u^{\varsigma}\Bigg\{Y\Bigg[\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-4\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+\\&\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}\Bigg]\Bigg\}\Bigg],\\&\zeta(\beta,\upsilon)=(-\frac{15}{8}\,\mathrm{sech}^{2}\left(\frac{\beta}{2}\right))+Y^{-1}\Bigg[u^{\varsigma}\Bigg\{Y\Bigg[\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})-4\zeta^{2}(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^{2}\zeta(\beta,\upsilon)}{\partial\beta^{2}})+\\&\zeta(\beta,\upsilon)\frac{\partial^{3}\zeta(\beta,\upsilon)}{\partial\beta^{3}}\Bigg]\Bigg\}\Bigg].\end{aligned} \quad (5.5)$$

Using of the HPM, we have

$$\begin{aligned}\sum_{k=0}^{\infty}\epsilon^{k}\zeta_{k}(\beta,\upsilon)=(-\frac{15}{8}\,\mathrm{sech}^{2}\left(\frac{\beta}{2}\right))+\Bigg(Y^{-1}\Bigg[u^{\varsigma}Y\Bigg[\Bigg(\sum_{k=0}^{\infty}\epsilon^{k}\frac{\partial}{\partial\upsilon}(\frac{\partial^{2}\zeta_{k}(\beta,\upsilon)}{\partial\beta^{2}})-4\sum_{k=0}^{\infty}\epsilon^{k}\zeta_{k}^{2}(\beta,\upsilon)\sum_{k=0}^{\infty}\epsilon^{k}\frac{\partial\zeta_{k}(\beta,\upsilon)}{\partial\beta}\Bigg)\\+3\sum_{k=0}^{\infty}\epsilon^{k}\frac{\partial\zeta_{k}(\beta,\upsilon)}{\partial\beta}\sum_{k=0}^{\infty}\epsilon^{k}\frac{\partial^{2}\zeta_{k}(\beta,\upsilon)}{\partial\beta^{2}}+\sum_{k=0}^{\infty}\epsilon^{k}\zeta_{k}(\beta,\upsilon)\sum_{k=0}^{\infty}\epsilon^{k}\frac{\partial^{3}\zeta_{k}(\beta,\upsilon)}{\partial\beta^{3}}\Bigg]\Bigg]\Bigg).\end{aligned} \quad (5.6)$$

By comparing coefficient of ϵ on both sides of Eq (5.6), we get

$$\epsilon^0 : \zeta_0(\beta,\upsilon) = -\frac{15}{8}\operatorname{sech}^2\left(\frac{\beta}{2}\right),$$

$$\epsilon^1 : \zeta_1(\beta,\upsilon) = Y^{-1}\left(u^{\varsigma}Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta_0(\beta,\upsilon)}{\partial\beta^2}) - 4\zeta_0^2(\beta,\upsilon)\frac{\partial\zeta_0(\beta,\upsilon)}{\partial\beta} + 3\frac{\partial\zeta_0(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta_0(\beta,\upsilon)}{\partial\beta^2}) + \zeta_0(\beta,\upsilon)\frac{\partial^3\zeta_0(\beta,\upsilon)}{\partial\beta^3}\right]\right)$$

$$= -450\operatorname{csch}^5(\beta)\sinh^6\left(\frac{\beta}{2}\right)\frac{\upsilon^{\varsigma}}{\Gamma(\varsigma+1)}$$

$$\vdots$$

Finally, the series form solution is given as follows:

$$\zeta(\beta,\upsilon) = \zeta_0(\beta,\upsilon) + \zeta_1(\beta,\upsilon) + \cdots$$

$$\zeta(\beta,\upsilon) = -\frac{15}{8}\operatorname{sech}^2\left(\frac{\beta}{2}\right) - 450\operatorname{csch}^5(\beta)\sinh^6\left(\frac{\beta}{2}\right)\frac{\upsilon^{\varsigma}}{\Gamma(\varsigma+1)} + \cdots$$

A.Implementation of decomposition method

Applying the YT, we get

$$Y\left\{\frac{\partial^{\varsigma}\zeta}{\partial\upsilon^{\varsigma}}\right\} = Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 4\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} + 3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) + \zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]. \quad (5.7)$$

By utilizing the differentiating property of the Yang transform is given as

$$\frac{1}{u^{\varsigma}}\{M(u) - u\zeta(\beta,0)\} = Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 4\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} + 3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) + \zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right], \quad (5.8)$$

$$M(u) = u\zeta(\beta,0) + u^{\varsigma}Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 4\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} + 3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) + \zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]. \quad (5.9)$$

Utilizing the inverse of the Yang transform method, we possess

$$\zeta(\beta,\upsilon) = \zeta(\beta,0) + Y^{-1}\left[u^{\varsigma}\left\{Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 4\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} + 3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) + \zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]\right\}\right],$$

$$\zeta(\beta,\upsilon) = (-\frac{15}{8}\operatorname{sech}^2\left(\frac{\beta}{2}\right)) + Y^{-1}\left[u^{\varsigma}\left\{Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 4\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} + 3\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) + \zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]\right\}\right]. \quad (5.10)$$

The series form solution is defined as

$$\zeta(\beta,\upsilon) = \sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon), \quad (5.11)$$

where $\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} = \sum_{m=0}^{\infty}\mathcal{A}_m$, $\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) = \sum_{m=0}^{\infty}\mathcal{B}_m$ and $\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3} = \sum_{m=0}^{\infty}C_m$ are the Adomian polynomials, which shows the nonlinear terms, and

$$\sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon) = \zeta(\beta,0) - Y^{-1}\left[u^{\varsigma}\left\{Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 4\sum_{m=0}^{\infty}\mathcal{A}_m + 3\sum_{m=0}^{\infty}\mathcal{B}_m + \sum_{m=0}^{\infty}C_m\right]\right\}\right],$$
$$\sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon) = (-\frac{15}{8}\operatorname{sech}^2\left(\frac{\beta}{2}\right)) - Y^{-1}\left[u^{\varsigma}\left\{Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 4\sum_{m=0}^{\infty}\mathcal{A}_m + 3\sum_{m=0}^{\infty}\mathcal{B}_m + \sum_{m=0}^{\infty}C_m\right]\right\}\right]. \tag{5.12}$$

Comparing both sides of Eq (5.12) is defined as

$$\zeta_0(\beta,\upsilon) = -\frac{15}{8}\operatorname{sech}^2\left(\frac{\beta}{2}\right),$$

put $m = 0$ in Eq (5.12)

$$\zeta_1(\beta,\upsilon) = -450\operatorname{csch}^5(\beta)\sinh^6\left(\frac{\beta}{2}\right)\frac{\upsilon^{\varsigma}}{\Gamma(\varsigma+1)}.$$

Finally, the series form solution is defined as

$$\zeta(\beta,\upsilon) = \sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon) = \zeta_0(\beta,\upsilon) + \zeta_1(\beta,\upsilon) + \cdots.$$

$$\zeta(\beta,\upsilon) = -\frac{15}{8}\operatorname{sech}^2\left(\frac{\beta}{2}\right) - 450\operatorname{csch}^5(\beta)\sinh^6\left(\frac{\beta}{2}\right)\frac{\upsilon^{\varsigma}}{\Gamma(\varsigma+1)} + \cdots.$$

Then, ς equal to 1, the solution is obtained as

$$\zeta(\beta,\upsilon) = -\frac{15}{8}\left[\operatorname{sech}^2\frac{1}{2}\left(\beta - \frac{5}{2}\upsilon\right)\right]. \tag{5.13}$$

Example 5.2.

Fractional modified Camassa-Holm (mCH) equation can be expressed as follows:

$$\frac{\partial^{\varsigma}\zeta(\beta,\upsilon)}{\partial\upsilon^{\varsigma}} - \frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) + 3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} - 2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - \zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3} = 0 \tag{5.14}$$

with the IC

$$\zeta(\beta,0) = -2\operatorname{sech}^2\left(\frac{\beta}{2}\right).$$

By applying the YT, we get

$$Y\left(\frac{\partial^{\varsigma}\zeta}{\partial\upsilon^{\varsigma}}\right) = Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} + 2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) + \zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]. \tag{5.15}$$

By using the differential property of the Yang transform, given as

$$\frac{1}{u^{\varsigma}}\{M(u) - u\zeta(\beta,0)\} = Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) - 3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta} + 2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2}) + \zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right], \tag{5.16}$$

$$M(u)=u\zeta(\beta,0)+u^{\varsigma}Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})+\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]. \tag{5.17}$$

Applying inverse Yang transform is defined

$$\begin{aligned}&\zeta(\beta,\upsilon)=\zeta(\beta,0)+Y^{-1}\Bigg[u^{\varsigma}\Bigg\{Y\Bigg[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})+\\&\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\Bigg]\Bigg\}\Bigg],\\&\zeta(\beta,\upsilon)=\left(-2\operatorname{sech}^2\left(\frac{\beta}{2}\right)\right)+Y^{-1}\Bigg[u^{\varsigma}\Bigg\{Y\Bigg[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})+\\&\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\Bigg]\Bigg\}\Bigg].\end{aligned} \tag{5.18}$$

With the help of HPM technique, is expressed

$$\begin{aligned}&\sum_{k=0}^{\infty}\epsilon^k\zeta_k(\beta,\upsilon)=\left(-2\operatorname{sech}^2\left(\frac{\beta}{2}\right)\right)+\Bigg(Y^{-1}\Bigg[u^{\varsigma}Y\Bigg[\Bigg(\sum_{k=0}^{\infty}\epsilon^k\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta_k(\beta,\upsilon)}{\partial\beta^2})-3\sum_{k=0}^{\infty}\epsilon^k\zeta_k^2(\beta,\upsilon)\sum_{k=0}^{\infty}\epsilon^k\frac{\partial\zeta_k(\beta,\upsilon)}{\partial\beta}\Bigg)\\&+2\sum_{k=0}^{\infty}\epsilon^k\frac{\partial\zeta_k(\beta,\upsilon)}{\partial\beta}\sum_{k=0}^{\infty}\epsilon^k\frac{\partial^2\zeta_k(\beta,\upsilon)}{\partial\beta^2}+\sum_{k=0}^{\infty}\epsilon^k\zeta_k(\beta,\upsilon)\sum_{k=0}^{\infty}\epsilon^k\frac{\partial^3\zeta_k(\beta,\upsilon)}{\partial\beta^3}\Bigg]\Bigg]\Bigg).\end{aligned} \tag{5.19}$$

By examining the relationship between the coefficient of ϵ and other factors, we can determine the correlation.

$$\begin{aligned}&\epsilon^0:\zeta_0(\beta,\upsilon)=-2\operatorname{sech}^2\left(\frac{\beta}{2}\right),\\&\epsilon^1:\zeta_1(\beta,\upsilon)=Y^{-1}\left(u^{\varsigma}Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta_0(\beta,\upsilon)}{\partial\beta^2})-3\zeta_0^2(\beta,\upsilon)\frac{\partial\zeta_0(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta_0(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta_0(\beta,\upsilon)}{\partial\beta^2})+\zeta_0(\beta,\upsilon)\frac{\partial^3\zeta_0(\beta,\upsilon)}{\partial\beta^3}\right]\right)\\&=-384\operatorname{csch}^5(\beta)\sinh^6\left(\frac{\beta}{2}\right)\frac{\upsilon^{\varsigma}}{\Gamma(\varsigma+1)}.\\&\vdots\end{aligned}$$

The series form solution is achieved as

$$\begin{aligned}\zeta(\beta,\upsilon)&=\zeta_0(\beta,\upsilon)+\zeta_1(\beta,\upsilon)+\cdots\\\zeta(\beta,\upsilon)&=-2\operatorname{sech}^2\left(\frac{\beta}{2}\right)-384\operatorname{csch}^5(\beta)\sinh^6\left(\frac{\beta}{2}\right)\frac{\upsilon^{\varsigma}}{\Gamma(\varsigma+1)}+\cdots\end{aligned}$$

B. Application of decomposition method

By using the YT, we get

$$Y\left\{\frac{\partial^{\varsigma}\zeta}{\partial\upsilon^{\varsigma}}\right\}=Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})+\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right], \tag{5.20}$$

By utilizing the distinct characteristic of the YT, the following result is obtained

$$\frac{1}{u^{\varsigma}}\{M(u)-u\zeta(\beta,0)\}=Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})+\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right], \tag{5.21}$$

$$M(u)=u\zeta(\beta,0)+u^{\varsigma}Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})+\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]. \tag{5.22}$$

By utilizing inverse of the YT, we get

$$\begin{aligned}&\zeta(\beta,\upsilon)=\zeta(\beta,0)+Y^{-1}\left[u^{\varsigma}\left\{Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})+\right.\right.\right.\\&\left.\left.\left.\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]\right\}\right],\\&\zeta(\beta,\upsilon)=\left(-2\operatorname{sech}^2\left(\frac{\beta}{2}\right)\right)+Y^{-1}\left[u^{\varsigma}\left\{Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}+2\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})+\right.\right.\right.\\&\left.\left.\left.\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}\right]\right\}\right].\end{aligned} \tag{5.23}$$

The series form solution is given as

$$\zeta(\beta,\upsilon)=\sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon) \tag{5.24}$$

with $\zeta^2(\beta,\upsilon)\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}=\sum_{m=0}^{\infty}\mathcal{A}_m$, $\frac{\partial\zeta(\beta,\upsilon)}{\partial\beta}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})=\sum_{m=0}^{\infty}\mathcal{B}_m$ and $\zeta(\beta,\upsilon)\frac{\partial^3\zeta(\beta,\upsilon)}{\partial\beta^3}=\sum_{m=0}^{\infty}C_m$ are the Adomian polynomials which shows the nonlinear terms, and

$$\begin{aligned}&\sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon)=\zeta(\beta,0)-Y^{-1}\left[u^{\varsigma}\left\{Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\sum_{m=0}^{\infty}\mathcal{A}_m+2\sum_{m=0}^{\infty}\mathcal{B}_m+\sum_{m=0}^{\infty}C_m\right]\right\}\right],\\&\sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon)=\left(-2\operatorname{sech}^2\left(\frac{\beta}{2}\right)\right)-Y^{-1}\left[u^{\varsigma}\left\{Y\left[\frac{\partial}{\partial\upsilon}(\frac{\partial^2\zeta(\beta,\upsilon)}{\partial\beta^2})-3\sum_{m=0}^{\infty}\mathcal{A}_m+2\sum_{m=0}^{\infty}\mathcal{B}_m+\sum_{m=0}^{\infty}C_m\right]\right\}\right].\end{aligned} \tag{5.25}$$

$$\zeta_0(\beta,\upsilon)=-2\operatorname{sech}^2\left(\frac{\beta}{2}\right),$$

put $m=0$ in the above equation

$$\zeta_1(\beta,\upsilon)=-384\operatorname{csch}^5(\beta)\sinh^6\left(\frac{\beta}{2}\right)\frac{\upsilon^{\varsigma}}{\Gamma(\varsigma+1)}.$$

Finally, the series form result is defined as

$$\zeta(\beta,\upsilon)=\sum_{m=0}^{\infty}\zeta_m(\beta,\upsilon)=\zeta_0(\beta,\upsilon)+\zeta_1(\beta,\upsilon)+\cdots.$$

$$\zeta(\beta,\upsilon) = -2\,\mathrm{sech}^2\left(\frac{\beta}{2}\right) - 384\,\mathrm{csch}^5(\beta)\sinh^6\left(\frac{\beta}{2}\right)\frac{\upsilon^{\varsigma}}{\Gamma(\varsigma+1)} + \cdots .$$

The exact result is

$$\zeta(\beta,\upsilon) = -2\,\mathrm{sech}^2\left(\frac{\beta-\upsilon}{2}\right). \tag{5.26}$$

6. Results and discussion

In Figure 1, we present the graphical depiction of the homotopy perturbation transformation method (HPTM) and Yang transform-decomposition method (YTDM) for the solution $\zeta(\beta,\upsilon)$ in Example 5.1. The plot showcases the precision achieved at $\varsigma = 1$, revealing the accurate resolution achieved through the combined methodology. Moving to Figure 2, we illustrate the outcomes of the HPTM/YTDM solutions for $\zeta(\beta,\upsilon)$ across various values of ς in Example 5.1 of the Degasperis-Procesi equation. The graph provides a comprehensive view of how the solutions evolve and adapt as ς changes, offering insights into the behavior of the equation.

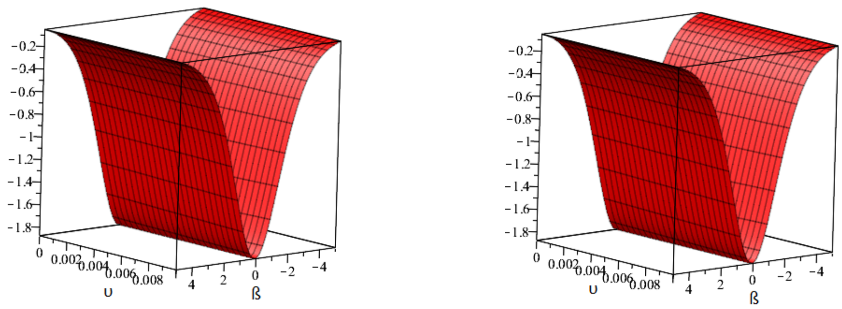

Figure 1. The graphical representation of the HPTM/YTDM and the precise resolution at $\varsigma = 1$ for $\zeta(\beta,\upsilon)$ of Example 5.1.

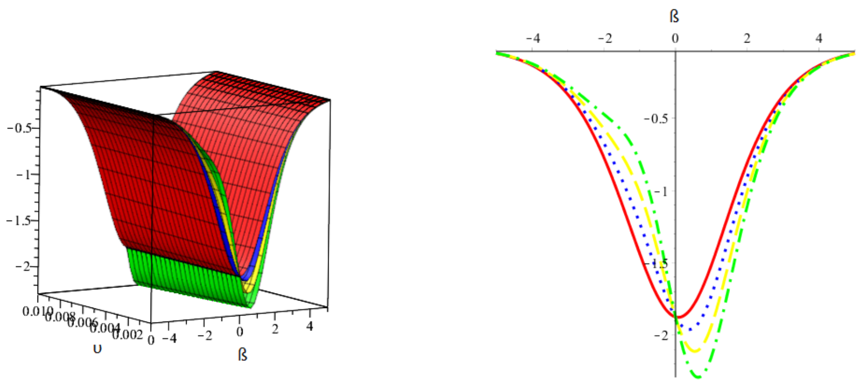

Figure 2. The graphical representation of the the HPTM/YTDM solutions for $\zeta(\beta,\upsilon)$ at different ς of Example 5.1.

In Figure 3, we delve into the graphical representation of the HPTM/YTDM approach and its precise resolution at $\varsigma = 1$ for $\zeta(\beta,\upsilon)$ in Example 5.2. The visualization underscores the accuracy achieved through the combined method, emphasizing its effectiveness. Lastly, in Figure 4, we explore the HPTM/YTDM solutions for $\zeta(\beta, \upsilon)$ in Example 5.2 of the Camassa-Holm equation. The graph showcases the diverse solutions obtained at different values of ς, providing a visual insight into the equation's response to varying parameters.

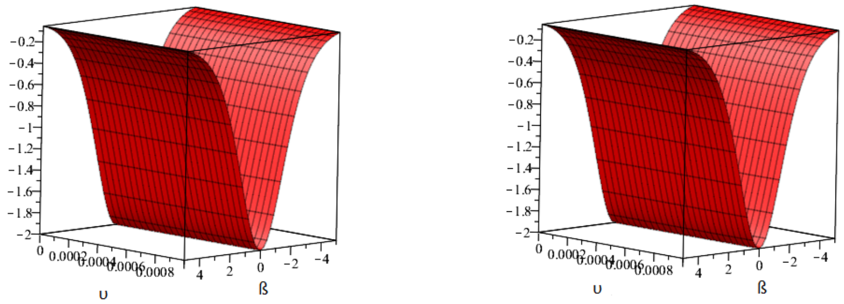

Figure 3. The graphical representation of the HPTM/YTDM and the precise resolution at $\varsigma = 1$ for $\zeta(\beta, \upsilon)$ of Example 5.2.

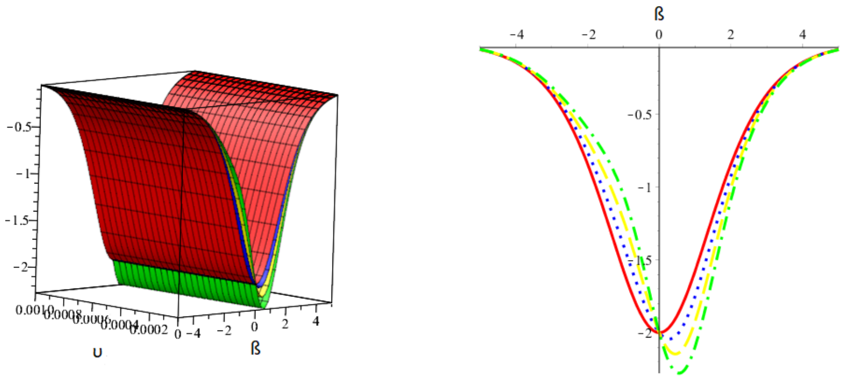

Figure 4. The graphical representation of the HPTM/YTDM solutions for $\zeta(\beta, \upsilon)$ at different ς of Example 5.2.

These graphical representations collectively demonstrate the efficacy of the homotopy perturbation transformation method (HPTM) and Yang transform-decomposition method (YTDM) in solving fractional partial differential equations. They offer a visual narrative of how the methodologies handle different examples of Degasperis-Procesi and Camassa-Holm equations, shedding light on the intricate dynamics of the equations and the effectiveness of the proposed approach.

7. Conclusions

In conclusion, the study delved into the intricate dynamics of the Degasperis-Procesi and Camassa-Holm equations, two fundamental nonlinear partial differential equations with wide-ranging applications. The utilization of the Yang transform emerged as a pivotal mathematical tool, facilitating the transformation between solutions and shedding light on the behavior of these equations. Furthermore, the investigation incorporated the Adomian decomposition method, a versatile technique for solving nonlinear differential equations, as well as series solutions to provide insightful approximations. Notably, the homotopy perturbation method was harnessed to tackle the challenges posed by these complex equations, offering a systematic approach to obtaining analytical solutions. Through the exploration of these methodologies, a deeper understanding of the intricate interplay between mathematical techniques and the physical phenomena described by these equations was achieved, paving the way for further advancements in the realm of nonlinear dynamics and integrable systems.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Funding

This work was supported by the Deanship of Scientific Research, the Vice Presidency for Graduate Studies and Scientific Research, King Faisal University, Saudi Arabia (Grant No. 4021).

Acknowledgements

This work was supported by the Deanship of Scientific Research, the Vice Presidency for Graduate Studies and Scientific Research, King Faisal University, Saudi Arabia (Grant No. 4021).

Conflict of interest

The authors declare that they have no competing interest.

References

1. I. Podlubny, *Fractional differential equations: An introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications*, Elsevier, 1998. https://doi.org/10.1016/s0076-5392(99)x8001-5
2. B. L. Guo, X. K. Pu, F. H. Huang, *Fractional partial differential equations and their numerical solutions*, World Scientific, 2015. https://doi.org/10.1142/9543
3. J. G. Liu, X. J. Yang, L. L. Geng, X. J. Yu, On fractional symmetry group scheme to the higher-dimensional space and time fractional dissipative Burgers equation, *Int. J. Geom. Methods M.*, **19** (2022), 2250173. https://doi.org/10.1142/S0219887822501730
4. M. I. Abbas, Controllability and Hyers-Ulam stability results of initial value problems for fractional differential equations via generalized proportional-Caputo fractional derivative, *Miskolc Math. Notes*, **22** (2021), 491–502. https://doi.org/10.18514/MMN.2021.3470
5. J. G. Liu, Y. F. Zhang, J. J. Wang, Investigation of the time fractional generalized (2+1)-dimensional Zakharov-Kuznetsov equation with single-power law nonlinearity, *Fractals*, **31** (2023), 2350033. https://doi.org/10.1142/S0218348X23500330

6. S. Y. Lu, M. Z. Zhe, L. R. Yin, Z. T. Yin, X. Liu, W. F. Zheng, The multi-modal fusion in visual question answering: A review of attention mechanisms, *PeerJ Comput. Sci.*, **9** (2023), e1440. https://doi.org/10.7717/peerj-cs.1400

7. J. G. Liu, X. J. Yang, Symmetry group analysis of several coupled fractional partial differential equations, *Chaos Soliton. Fract.*, **173** (2023), 113603. https://doi.org/10.1016/j.chaos.2023.113603

8. A. Atangana, D. Baleanu, Caputo-Fabrizio derivative applied to groundwater flow within confined aquifer, *J. Eng. Mech.*, **143** (2017), D4016005. https://doi.org/10.1061/(ASCE)EM.1943-7889.0001091

9. H. Fu, G. C. Wu, G. Yang, L. L. Huang, Continuous time random walk to a general fractional Fokker-Planck equation on fractal media, *Eur. Phys. J-Spec. Top.*, **230** (2021), 3927–3933. https://doi.org/10.1140/epjs/s11734-021-00323-6

10. A. Shafee, Y. Alkhezi, R. Shah, Efficient solution of fractional system partial differential equations using laplace residual power series method, *Fractal Fract.*, **7** (2023), 429. https://doi.org/10.3390/fractalfract7060429

11. H. Yasmin, A. S. Alshehry, A. M. Saeed, R. Shah, K. Nonlaopon, Application of the q-homotopy analysis transform method to fractional-order kolmogorov and Rosenau-Hyman models within the Atangana-Baleanu operator, *Symmetry*, **15** (2023), 671. https://doi.org/10.3390/sym15030671

12. C. Yang, J. S. Zhang, Z. W. Huang, Numerical study on cavitation-vortex-noise correlation mechanism and dynamic mode decomposition of a hydrofoil, *Phys. Fluids*, **34** (2022), 125105. https://doi.org/10.1063/5.0128169

13. A. Akgül, S. A. Khoshnaw, Application of fractional derivative on nonlinear biochemical reaction models, *Int. J. Intell. Netw.*, **1** (2020), 52–58. https://doi.org/10.1016/j.ijin.2020.05.0019

14. J. Song, A. Mingotti, J. H. Zhang, L. Peretto, H. Wen, Accurate damping factor and frequency estimation for damped real-valued sinusoidal signals, *IEEE T. Tnstrum. Meas.*, **71** (2022), 6503504. https://doi.org/10.1109/TIM.2022.3220300

15. T. A. A. Ali, Z. Xiao, H. B. Jiang, B. Li, A class of digital integrators based on trigonometric quadrature rules, *IEEE T. Ind. Electron.*, 2023, 1–11. https://doi.org/10.1109/TIE.2023.3290247

16. C. Q. Guo, J. P. Hu, J. S. Hao, S. Celikovsky, X. M. Hu, Fixed-time safe tracking control of uncertain high-order nonlinear pure-feedback systems via unified transformation functions, *Kybernetika*, **59** (2023), 342–364. https://doi.org/10.14736/kyb-2023-3-0342

17. C. Q. Guo, J. P. Hu, Y. Z. Wu, S. Celikovsky, Non-singular fixed-time tracking control of uncertain nonlinear Pure-Feedback systems with practical state constraints, *IEEE T. Circuits-I*, **70** (2023), 3746–3758. https://doi.org/10.1109/TCSI.2023.3291700

18. Q. T. Meng, Q. Ma, Y. Shi, Adaptive fixed-time stabilization for a class of uncertain nonlinear systems, *IEEE T. Automat. Contr.*, 2023, 1–8. https://doi.org/10.1109/TAC.2023.3244151

19. K. Diethelm, A. D. Freed, On the solution of nonlinear fractional-order differential equations used in the modeling of viscoplasticity, In: *Scientific computing in chemical engineering II*, Springer, 1999, 217–224. https://doi.org/10.1007/978-3-642-60185-9_24

20. M. D. Aloko, O. J. Fenuga, S. A. Okunuga, Solutions of some non-linear Volterra integro-differential equations of the second kind using modified variational iteration method, *FUW Trends Sci. Technol. J.*, **4** (2019), 298–303.

21. F. Mainardi, Fractional calculus, In: *Fractals and fractional calculus in continuum mechanics*, Springer, 1997, 291–348.

22. J. Bai, X. C. Feng, Fractional-order anisotropic diffusion for image denoising, *IEEE Trans. Image Process.*, **16** (2007), 2492–2502. https://doi.org/10.1109/TIP.2007.904971

23. P. L. Butzer, U. Westphal, An introduction to fractional calculus, In: *Applications of fractional calculus in physics*, World Scientific, 2010, 1–86. https://doi.org/10.1142/9789812817747-0001

24. D. Kumar, F. Tchier, J. Singh, D. Baleanu, An efficient computational technique for fractal vehicular traffic flow, *Entropy*, **20** (2018), 259. https://doi.org/10.3390/e20040259

25. J. R. Loh, A. Isah, C. Phang, Y. T. Toh, On the new properties of Caputo-Fabrizio operator and its application in deriving shifted Legendre operational matrix, *Appl. Numer. Math.*, **132** (2018), 138–153. https://doi.org/10.1016/j.apnum.2018.05.016

26. D. A. Benson, S. W. Wheatcraft, M. M. Meerschaert, The fractional-order governing equation of Lévy motion, *Water Resour. Res.*, **36** (2018), 1413–1423. https://doi.org/10.1029/2000WR900032

27. B. A. Carreras, V. E. Lynch, G. M. Zaslavsky, Anomalous diffusion and exit time distribution of particle tracers in plasma turbulence model, *Phys. Plasmas*, **8** (2001), 5096–5103. https://doi.org/10.1063/1.1416180

28. H. Yasmin, N. H. Aljahdaly, A. M. Saeed, R. Shah, Probing families of optical soliton solutions in fractional perturbed Radhakrishnan-Kundu-Lakshmanan model with improved versions of extended direct algebraic method, *Fractal Fract.*, **7** (2023), 512. https://doi.org/10.3390/fractalfract7070512

29. H. Yasmin, N. H. Aljahdaly, A. M. Saeed, R. Shah, Investigating families of soliton solutions for the complex structured coupled fractional Biswas-Arshed model in birefringent fibers using a novel analytical technique, *Fractal Fract.*, **7** (2023), 491. https://doi.org/10.3390/fractalfract7070491

30. H. Yasmin, A. S. Alshehry, A. Khan, R. Shah, K. Nonlaopon, Numerical analysis of the fractional-order Belousov-Zhabotinsky system, *Symmetry*, **15** (2023), 834. https://doi.org/10.3390/sym15040834

31. H. C. Li, R. Peng, Z. A. Wang, On a diffusive susceptible-infected-susceptible epidemic model with mass action mechanism and birth-death effect: Analysis, simulations, and comparison with other mechanisms, *SIAM J. Appl. Math.*, **78** (2018), 2129–2153. https://doi.org/10.1137/18M1167863

32. H. Y. Jin, Z. A. Wang, Boundedness, blowup and critical mass phenomenon in competing chemotaxis, *J. Differ. Equ.*, **260** (2016), 162–196. https://doi.org/10.1016/j.jde.2015.08.040

33. W. B. Lyu, Z. A. Wang, Logistic damping effect in chemotaxis models with density-suppressed motility, *Adv. Nonlinear Anal.*, **12** (2023), 336–355. https://doi.org/10.1515/anona-2022-0263

34. Q. K. Li, H. Lin, X. Tan, S. L. Du, H ∞ Consensus for multiagent-based supply chain systems under switching topology and uncertain demands, *IEEE T. Syst. Man Cy-S.*, **50** (2020), 4905–4918. https://doi:10.1109/TSMC.2018.2884510.

35. B. Wang, Y. M. Zhang, W. Zhang, A composite adaptive fault-tolerant attitude control for a ouadrotor UAV with multiple uncertainties, *J. Syst. Sci. Complex.*, **35** (2022), 81–104. https://doi.org/10.1007/s11424-022-1030-y

36. G. Yel, H. M. Baskonus, H. Bulut, Novel archetypes of new coupled Konno-Oono equation by using sine-Gordon expansion method, *Opt. Quant. Electron.*, **49** (2017), 1–10. https://doi.org/10.1007/s11082-017-1127-z

37. A. M. Zidan, A. Khan, R. Shah, M. K. Alaoui, W. Weera, Evaluation of time-fractional Fisher's equations with the help of analytical methods, *AIMS Mathematics*, **7** (2022), 18746–18766. https://doi:10.3934/math.20221031

38. S. Alyobi, R. Shah, A. Khan, N. A. Shah, K. Nonlaopon, Fractional analysis of nonlinear boussinesq equation under Atangana-Baleanu-Caputo operator, *Symmetry*, **14** (2022), 2417. https://doi.org/10.3390/sym14112417

39. A. A. Alderremy, S. Aly, R. Fayyaz, A. Khan, R. Shah, N. Wyal, The analysis of fractional-order nonlinear systems of third order KdV and Burgers equations via a novel transform, *Complexity*, **2022** (2022), 4935809. https://doi.org/10.1155/2022/4935809

40. N. J. Ford, J. Y. Xiao, Y. B. Yan, A finite element method for time fractional partial differential equations, *Fract. Calc. Appl. Anal.*, **14** (2011), 454–474. https://doi.org/10.2478/s13540-011-0028-2

41. M. Eslami, B. F. Vajargah, M. Mirzazadeh, A. Biswas, Application of first integral method to fractional partial differential equations, *Indian J. Phys.*, **88** (2014), 177–184. https://doi.org/10.1007/s12648-013-0401-6

42. N. A. Shah, Y. S. Hamed, K. M. Abualnaja, J. D. Chung, R. Shah, A. Khan, A comparative analysis of fractional-order kaup-kupershmidt equation within different operators, *Symmetry*, **14** (2022), 986. https://doi.org/10.3390/sym14050986

43. M. K. Alaoui, K. Nonlaopon, A. M. Zidan, A. Khan, R. Shah, Analytical investigation of fractional-order cahn-hilliard and gardner equations using two novel techniques, *Mathematics*, **10** (2022), 1643. https://doi.org/10.3390/math10101643

44. S. Y. Lu, Y. M. Ding, M. Z. Liu, Z. T. Yin, L. R. Yin, W. F. Zheng, Multiscale feature extraction and fusion of image and text in VQA, *Int. J. Comput. Int. Sys.*, **16** (2023), 54. https://doi.org/10.1007/s44196-023-00233-6

45. F. Liu, P. Zhuang, I. Turner, K. Burrage, V. Anh, A new fractional finite volume method for solving the fractional diffusion equation, *Appl. Math. Model.*, **38** (2014), 3871–3878. https://doi.org/10.1016/j.apm.2013.10.007

46. A. M. Wazwaz, Solitary wave solutions for modified forms of Degasperis-Procesi and Camassa-Holm equations, *Phys. Lett. A*, **352** (2006), 500–504. https://doi.org/10.1016/j.physleta.2005.12.036

47. J. S. Kamdem, Z. J. Qiao, Decomposition method for the Camassa-Holm equation, *Chaos Soliton. Fract.*, **31** (2007), 437–447. https://doi.org/10.1016/j.chaos.2005.09.071

48. R. Liu, Several new types of solitary wave solutions for the generalized Camassa-Holm-Degasperis-Procesi equation, *Commun. Pur. Appl. Anal.*, **9** (2010), 77–90. https://doi.org/10.3934/cpaa.2010.9.77

49. G. Omel'yanov, J. N. Rodriguez, Solitary wave solutions to a generalization of the mKdV equation, *Russ. J. Math. Phys.*, **30** (2023), 246–256. https://doi.org/10.1134/S1061920823020103

50. P. Sunthrayuth, R. Ullah, A. Khan, R. Shah, J. Kafle, I. Mahariq, et al., Numerical analysis of the fractional-order nonlinear system of Volterra integro-differential equations, *J. Funct. Space.*, **2021** (2021), 1537958. https://doi.org/10.1155/2021/1537958

51. M. Areshi, A. Khan, R. Shah, K. Nonlaopon, Analytical investigation of fractional-order Newell-Whitehead-Segel equations via a novel transform, *AIMS Mathematics*, **7** (2022), 6936–6958. https://doi.org/10.3934/math.2022385

52. M. K. Alaoui, R. Fayyaz, A. Khan, R. Shah, M. S. Abdo, Analytical investigation of Noyes-Field model for time-fractional Belousov-Zhabotinsky reaction, *Complexity*, **2021** (2021), 3248376. https://doi.org/10.1155/2021/3248376

53. X. J. Yang, A new integral transform method for solving steady heat-transfer problem, *Therm. Sci.*, **20** (2016), 639–642. https://doi.org/10.2298/TSCI16S3639Y

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