Altitude Control of UAV Quadrotor Using PID and Integral State Feedback

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Abstract. Applications of control techniques for stabilizing altitude in a UAV Quadrotor, along with a comprehensive performance comparison, are presented in this paper. The two compared control techniques are: a Proportional Integral Derivative (PID) and Integral State Feedback (ISF) controller. While PID control consists of a Proportional, an Integral and a Derivative Controller, the Integral State Feedback consists of an Integral and a State Feedback Controller. Each part of the control technique provides advantages and drawbacks in the controlled system performance. Numerical simulations in the research were performed on Simulink MATLAB to provide quantitative results in control performance comparison; thus, a quadrotor model was designed prior to the application of control techniques. Based on the numerical results, ISF control resulted in a better settling time with zero overshoot than PID. Meanwhile, the PID control had a better rise time with a big overshoot than ISF in its system response. Therefore, it can be concluded that the ISF Controller was better than PID regarding the settling time and the overshoot response.

1 Introduction

Quadrotor or Quadcopter [1] is a type of Unmanned Aerial Vehicles (UAV) that has four rotors at the end of each frame [2]. It has been used in photography, shipping, delivery, mapping, military, education, hobby, as well as in search and rescue [3][4]. The advantage of a quadrotor is its capability for Vertical Take Off and Landing (VTOL) [5], which enables it to take off and land anywhere in narrow spaces. Besides, it is known for its remarkable potential to perform complex aerial maneuvers and carry a wide array of payloads [6][7].

One of the crucial aspects of quadrotor flight control is maintaining a stable altitude [8]. The proportional Integral Derivative (PID) controller has been known for its wide use in many control systems; hence, it also has been applied to control altitude in quadrotor [9][10]. The other controller is Sliding Mode Control (SMC) [11], Linear Quadratic Regulator (LQR) [12][13], Predictive Control [14], Fuzzy Control [15][16], Neural Network [17][18], Fractional Order PID [19], Feedback Linearization [20], and other control techniques [21].

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This research presented an application of the Integral State Feedback (ISF) controller for altitude control in quadrotor as a practical solution that enables a precise and robust control system performance. Although known as a simple controller with less complex mathematical computations, ISF considers the altitude error, the integral of error, and the system's state feedback in calculating the control signal required by the controlled system, ensuring excellent stability. It compensates for steady-state errors and provides superior altitude accuracy and disturbance rejection [22]. Aside from quadrotors, ISF control has been used in many systems, such as Hub Motor [23], DC Motor System [24][25], Inverted Pendulum [26], Boost Converter [27], Buck Converter [28], Magnetic Levitation [29], Suspension [30], Servo Valve [31], Network Control Systems [32]. According to these results and characteristics, ISF is practically suitable for maintaining a stable altitude in a control system designed for a quadrotor. A comprehensive and numerical-based performance comparison with PID control will also be provided.

This research contributes to designing a quadrotor model specifically for its altitude movement. Besides, it also contributes to designing and applying the controller for altitude control in UAV Quadrotor. Lastly, the research also contributes to assessing a better and more suitable altitude control for quadrotors by determining a comprehensive performance comparison of the two control techniques.

The paper is structured in several sections: introduction, method, result and conclusions. The introduction explains the research background and describes the research problems and objectives. The method section contains quadrotor modeling, PID control, and Integral State Feedback control. Simulation results and discussions are provided in the result section. The research conclusion and future work are stated in the conclusions section.

2 Method

2.1 Quadrotor Model

Quadrotor has four inputs that correspond to its rotor, as shown in Fig. 1. The input of a quadrotor system can be defined as

$$u = [v_1^2 \quad v_2^2 \quad v_3^2 \quad v_4^2]^T \tag{1}$$

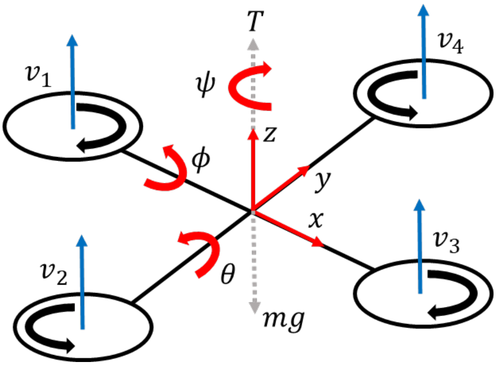

Fig. 1. Quadrotor Model

The position of the quadrotor can be defined as

$$\alpha = [x \quad y \quad z]^T \tag{2}$$

where x is the position in x-axis, y is the position in y-axis, and z is the the position in z-axis.

The orientation of the quadrotor can be defined as

$$\beta = [\phi \quad \theta \quad \psi]^T \tag{3}$$

where ϕ is the roll angle, θ is the pitch angle, and ψ is the yawn angle.

Its translational velocity can be defined as

$$\eta = [\dot{x} \quad \dot{y} \quad \dot{z}]^T \tag{4}$$

while its angular velocity is defined as

$$\delta = [p \quad q \quad r]^T \tag{5}$$

where p is the angular velocity in x-axis, q is the angular velocity in y-axis, and r is the angular velocity in z-axis.

Eventually, the state variables can be written as

$$\boldsymbol{x} = [\alpha^T \quad \beta^T \quad \eta^T \quad \delta^T]^T \tag{6}$$

The equation of motions in quadrotor is described as follows. The forces corresponding to x, y, and z-axis are applied to the quadrotor as a vector $F = [F_x \quad F_y \quad F_z]^T$, which can be written as

$$\boldsymbol{F} = \boldsymbol{R}\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{0}\\ \boldsymbol{T}\end{bmatrix} - \begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{0}\\ \boldsymbol{W}\end{bmatrix} \tag{7}$$

where R is the transformation matrix, T is upward thrust, and W is the quadrotor weight.

The transformation matrix R that transform from Earth frame to body frame is determined as

$$R = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \tag{8}$$

Where C is cos and S is sin.

The upward thrust T that is the total amount of angular speed from rotors can be defined as

$$T = b(v_1^2 + v_2^2 + v_3^2 + v_4^2) \tag{9}$$

Thus, the linear acceleration of the quadrotor is then can be written as

$$\ddot{x} = \frac{T(C_\phi S_\theta C_\psi + S_\phi S_\psi)}{m} \tag{10}$$

$$\ddot{y} = \frac{T(C_\phi S_\theta S_\psi - S_\phi C_\psi)}{m} \tag{11}$$

$$\ddot{z} = \frac{T(C_\phi C_\theta) - W}{m} \tag{12}$$

The weight of the quadrotor is equal to its mass multiplied by the gravity acceleration, or mathematically expressed as $W = mg$. Thus, the last equation can be rewritten as

$$\ddot{z} = \frac{T(C_{\phi}C_{\theta})}{m} - g \qquad (13)$$

where g is the gravity acceleration, and m is the mass of the quadrotor.

Ignoring the gravity acceleration as external disturbance, the Laplace transform of last equation can be obtained as

$$H(s) = \frac{Z(s)}{T(s)} = \frac{C_{\phi}C_{\theta}}{ms^2} \qquad (14)$$

By assuming the roll and pitch angle is 0°, the last equation can be rewritten as

$$H(s) = \frac{1}{ms^2} \qquad (15)$$

2.2 PID Control and Integral State Feedback

Proportional Integral Derivative (PID) Control consists of Proportional, Integral and Derivative controllers [33]. The proportional controller corresponds with the error multiplied by a proportional gain [34]. Similarly, the integral controller corresponds with the total error over time multiplied by the integral gain, and the derivative controller corresponds with the delta error multiplied by the derivative gain [35]. PID controller is widely-known since it is simple, easy to understand, and easy to be applied [36].

The control signal generated by the PID controller can be written as

$$u_{PID} = K_p e(s) + \frac{K_i}{s} e(s) + K_d s e(s) \qquad (16)$$

where e is the error between the pre-determined set point and the feedback value, K_p is the proportional gain, K_i is the integral gain, and K_d is the derivative gain.

Integral State Feedback (ISF) control belongs to modern control techniques with a matrix approach [37]. It consists of an integral and state feedback control [38]. The integral control eliminates the steady state error, while the state feedback control corresponds with system response [39].

The control signal calculated by the ISF control is defined as [40]

$$u_{ISF} = k_I e - \boldsymbol{Kx} \qquad (17)$$

where $\mathbf{x}$ is the state vector of the system, k_I is the integral gain, and $\mathbf{K} = [k_1 \quad \dots \quad k_n]$ is the state feedback gain matrix.

3 Result and discussions

The mass of the actual quadrotor is $m = 0.012$. Therefore, according to (15), the transfer function model of the quadrotor is defined as

$$H(s) = \frac{1}{0.12s^2} \qquad (18)$$

The transfer function model is then inputted for numerical simulations in Simulink MATLAB, followed by an open-loop test. The open-loop response for this quadrotor model is shown in Fig. 2. The open-loop test was done by giving a constant torque, $T = 1N$, as an input to the quadrotor system model. By conducting an open-loop test on a system model, the natural control behavior of a system can be analyzed. Fig. 2 shows that the quadrotor could reach 400 meters in less than 10 seconds. The quadrotor continued to fly higher over time despite the constantly given torque, indicating an unstable system in nature.

Both control techniques, the ISF control and the PID control, were then applied to the quadrotor model in simulations. The two control techniques were supposed to make the quadrotor fly at a precise position as the determined setpoint, which was 10 meters. The closed-loop responses of the quadrotor with controllers are shown in Fig. 3, while detailed parameter values as a performance comparison of the two control techniques are provided in Table 1. In Fig. 3, it can be seen that the quadrotor system controlled by the ISF control showed a critically damped system behavior. In contrast, the quadrotor system controlled by the PID control showed an underdamped system behavior. This finding follows the numerical results. The PID control resulted in a better rise time with a big overshoot than the ISF control. However, the ISF Control resulted in a faster settling time and had no overshoot in its response compared to the PID control.

Overall, ISF control is found to be superior in controlling the altitude of the quadrotor than the PID control since it did not result in any overshoot or undershoot response. ISF control is especially better in its steady-state response. The settling time of the quadrotor system controlled by the ISF was found to be approximately twice faster than that of the PID control. The only drawback of the ISF control is related to its transient time response, especially in its rise time before reaching the peak.

The superiority of the ISF control can be attributed to its structure. As can be comprehended from (16), the structure of the PID only deals with one state variable of a system. In comparison, as seen in (17), all state variables of the controlled system must be stated in the ISF control. This structure allows the ISF control to handle more complex systems, which may not be able to be controlled by the PID control. Thus, for the same system, better system performance can be obtained by the ISF control.

Table 1. Detailed Performance Comparison of PID Control and Integral State Feedback

Systems Response Parameter	PID Control	Integral State Feedback Control
Rise Time	0.3800	0.6274
Settling Time	2.8520	1.2119
Overshoot	16.7338	0.0101
Peak Value	11.6734	10.0010
Peak Time	1.0546	5.5843

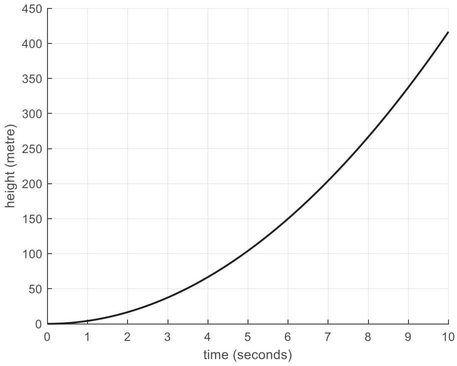

Fig. 2. Quadrotor Open-loop Response

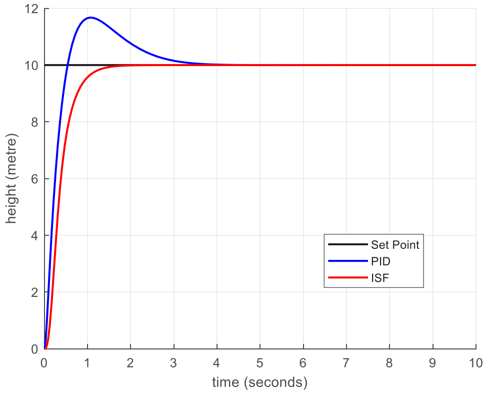

Fig. 3. Quadrotor Closed-loop Responses with PID Control and Integral State Feedback Control

4 Conclusions

This paper presents an altitude control of UAV Quadrotor using PID Control and Integral State Feedback (ISF) control. ISF control resulted in a better settling time with zero overshoot than PID. Meanwhile, the PID control had a better rise time with a big overshoot than ISF in its system response. Therefore, it can be concluded that the ISF Controller was better than PID regarding the settling time and the overshoot response. Eventually, the Integral State Feedback Control can be combined with a Proportional controller based on the PID control technique to overcome the slow rise time in augmented system performance. By combining the ISF control with a simple Proportional controller, the simplicity of the mathematical computations in the controller can still be maintained. Another suggestion for future work is determining the controller gains based on a proper standardized controller tuning method to ensure the best system performance. Lastly, some control parameters, such as the disturbance rejection and the uncertainty, can be explored.

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