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Two-parameters single equation stem taper models of Pinus sibirica in Siberia, Russia

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Abstract. A large number of studies have been devoted to the cedar forests in Russia, but studies of the conicity of trunks remain limited. The main goal of this study was to analyze two-parameter single equation stem tapper models for Siberian cedar trees, which will serve as guidelines for future research. The study tested six two-parameter models of one equation for the conicity of the trunk of Siberian cedar trees. For this, data from 1805 measurements of diameters in 193 felled trees were used. The data were collected in natural pure and mixed Siberian pine stands in Irkutsk region (Southern Siberia) and Tyumen region (Western Siberia), Russia. Our analysis showed that for our data set, the Reed and Green model turned out to be the best model. The model is characterized by the smallest average absolute error percentage, the square root of the root mean square error, and the largest coefficient of determination. The residuals of this model are independent and characterized by constant variance. The analysis performed showed that two-parameter single equation models have a good fit in the central part of the trunk, therefore, forestry management in cedar forests should be used with caution. For the purposes of forestry management in cedar forests, it is recommended to develop and put into practice more complex models.

1 Introduction

The Siberian cedar (*Pinus sibirica*) belongs to the important forest-forming species of Siberia, Russia. Cedar forests are characterized by slow growth, a large number of trees per unit area, large-sized and high-quality wood, and nut production. Siberian cedar is important for humans and forest animals. Cedar forests are of complex economic importance and are important in the forest cover of Siberia [1-2]. Since cedar wood has a high economic value, therefore, an urgent issue is to increase the accuracy of stem tapper models.

In the literature, there are a large number of models of the taper of a tree trunk. Three types of models can be distinguished [3]: i) single equation models, ii) segmented models, and iii) variable-exponent models. The simplest are single-equation models. In them, the stem taper is given by one mathematical function (polynomial, power, trigonometric, etc.). These models can be classified according to the number of estimated parameters: one-parameter,

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two-parameter, three-parameter, etc. In segmented models, the tree trunk is divided into several segments, each of which is included in the taper equation. In variable-exponent models, the exponent changes along the length of the tree trunk, so its individual parts have a shape from neyloid to cone [4]. In addition, there are other types of models [5-6], for example, generalized additive models, mixed-effect models, etc.

A large number of studies have been devoted to the cedar forests in Russia [7-9], but studies of the conicity of trunks remain limited. Thus, the main goal of this study was to analyze two-parameter single equation stem tapper models for Siberian cedar trees, which will serve as guidelines for future research. The results obtained can be used to model the conicity of trunks of other forest-forming species and in forestry management.

2 Materials and methods

2.1 Data

The data were collected in natural pure and mixed Siberian pine stands in Irkutsk region (Southern Siberia) and Tyumen region (Western Siberia). In the selected areas, an inventory of forest stands was carried out with the determination of the average height and diameter of trees, the number of trees per 1 ha, the basal area per 1 ha, site index and the timber stock. Tree samples to study stem taper within a range of diameter variation. The sample was divided into six classes of tree diameters with a gradation of 10 cm (Table 1). Among the felled trees, the most represented are the diameter classes 10-20 cm (67 trees) and 20-30 cm (51 trees). In growing trees, the diameter at breast height was measured. After felling, the tree trunk was cleared of branches, and the length of the trunk was measured. Trunk diameters were measured every 2 m. The sample included trees with good growth and no wood defects. The data set includes 1805 diameter measurements for 193 trees (Figure 1).

Table 1. Summary statistics for *Pinus sibirica* taper dataset for each diameter class.

D-class	Number of trees	Class average diameter. cm	Class minimum diameter. cm	Class average height.	Class minimum height. m
0-10	14	8.5	5.9	8.1	4.9
10-20	67	15.3	10.6	13.1	8.3
20-30	51	24.5	20.0	18.6	13.0
30-40	36	34.6	30.0	21.8	17.5
40-50	23	44.5	40.0	23.2	20.8
50-60	2	51.1	50.2	25.3	25.1

For the entire data set, the mean diameter was 24.7 cm with a standard deviation of 11.4 cm; the average height was 17.1 m with a standard deviation of 5.1 m. In general, the distribution of tree attributes is close to normal. The scatter plot of the relative height (ratio of the height of the off-ground tree stem and total tree height) over relative diameter (ratio of the diameter at the height of the off-ground tree stem to the diameter at breast height) reflects the rate of diameter change along the tree trunk from the root collar to the tip (Figure 1).

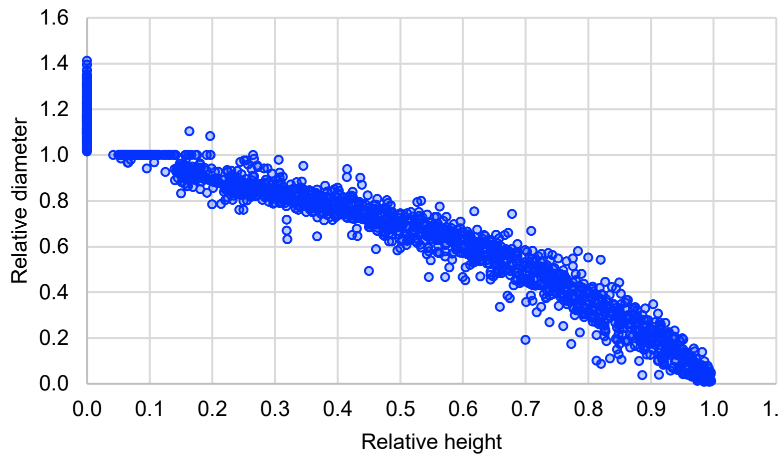


Fig. 1. Scatter plot of the relative height (h / H) over relative diameter (d / D) of 1805 data points for 193 *Pinus sibirica* trees.

2.2 Model selection

Six two-parameter single-equation stem taper models were selected as candidates for this study (Table 2). These include Munro [10] model, Kozak et al. [11] model, Biging [12] model, Reed and Green [13] model, Newberry and Burkhart [14] model and Forslund [15] model. In the equations of these six models: d – diameter at the height of the off-ground tree stem, D – diameter at breast height, h – height of the off-ground tree stem, H – total tree height, b – estimated parameters.

Table 2. Analyzed taper equations in this research.

Model	Equation
Munro [10]	$d = \sqrt{D^2\left(b_1 - b_2\left(\frac{h}{H-1.3}\right)\right)}$
Kozak et al. [11]	$d = \sqrt{D^2\left[b_1\left(\frac{h}{H}\right) + b_2\left(\frac{h^2}{H^2}\right)\right]}$
Biging [12]	$d = D\left[b_1 + b_2 \log\left(1 - \left(\frac{h}{H}\right)^{\frac{1}{3}}\right)\right]\left[1 - \exp\left(-\frac{b_1}{b_2}\right)\right]$
Reed and Green [13]	$d = \sqrt{b_1 D^2\left(1 - \frac{h}{H}\right)^{b_2}}$
Newberry and Burkhart [14]	$d = b_1 D\left(\frac{H-h}{H-1.3}\right)^{b_2}$
Forslund [15]	$d = D\left(1 - \left(\frac{h}{H}\right)^{b_1}\right)^{\frac{1}{b_2}}$

2.3 Model evaluation

The models were compared according to three statistical indicators: coefficient of determination (R^2), root mean square error (RMSE), and mean absolute percentage error (MAPE) [16-17]. Model performance criteria selected for this study shows in Table 3. According to criteria, the models were ranked from best (rank = 1) to worst (rank = 6). All data analyzes in this study were conducted in R 3.6.3 statistical software [18].

Table 3. Model performance criteria selected for this study.

Function name	Equation
Coefficient of determination (R^2)	$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$
Root mean square error (RMSE)	$RMSE = \sqrt{\sum \frac{(y_i - \hat{y}_i)^2}{n}}$
Mean absolute percentage error (MAPE)	$MAPE = 100 \times \sum \left\| \frac{y_i - \hat{y}_i}{y_i} \right\| / n$

Note: n – number of observations, y_i - actual value, $\hat{y}_i$ - model predicted value, $\bar{y}$ – average actual value.

3 Results and discussion

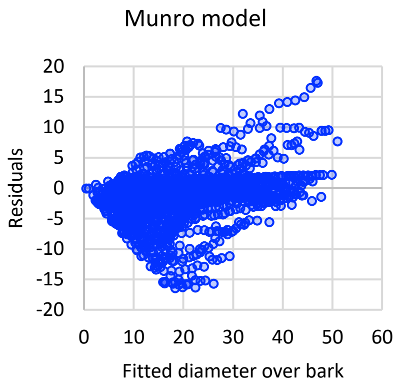
3.1 Selection of Model

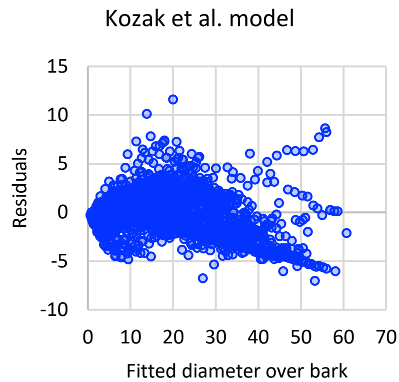
The parameter estimates for the six models and their corresponding performance criteria are presented in Table 4. All estimated parameters are statistically significant at the 5% level. All models in this study performed quite well and explained more than 95% of the total diameter variance, with the exception of some models. Reed and Green [13] model (R^2 = 0.974; RMSE = 1.931; MAPE = 15.183), Newberry and Burkhart [14] model (R^2 = 0.973; RMSE = 1.941; MAPE = 15.611), and Forslund [15] model (R^2 = 0.963; RMSE = 2.229; MAPE = 14.495) has the best performance criteria scores. The worst estimates of performance criteria are characterized by the Biging [12] model (R^2 = 0.947; RMSE = 2.752; MAPE = 49.026) and Munro [10] model (R^2 = 0.869; RMSE = 4.309; MAPE = 68.456). The Reed and Green [13] model received the highest rank.

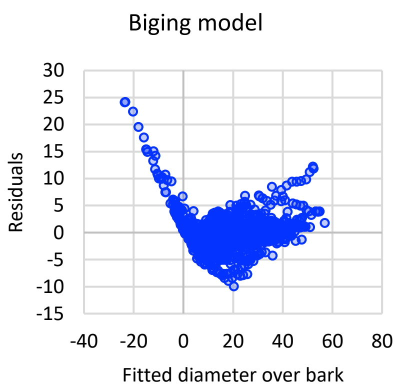
Table 4. Fit parameters and statistics of models.

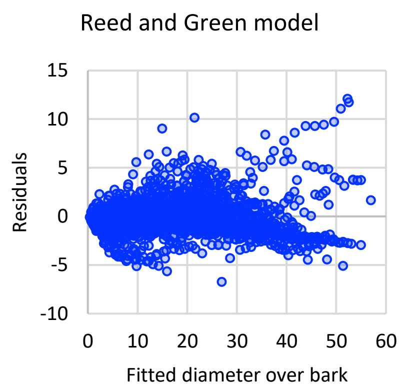
Model	b_1	b_2	R^2	RMSE	MAPE	Rank
Munro [10]	0.961	0.798	0.869	4.309	68.456	6
Kozak et al. [11]	-2.364	1.001	0.965	2.218	18.439	4
Biging [12]	1.104	0.245	0.947	2.752	49.026	5
Reed and Green [13]	1.120	1.368	0.974	1.931	15.183	1
Newberry and Burkhart [14]	1.047	0.682	0.973	1.941	15.611	2
Forslund [15]	1.371	1.328	0.963	2.229	14.495	3

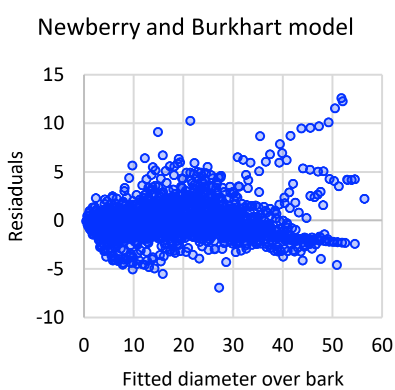
Further comparison of the models was carried out by graphical analysis of the residuals depending on the fitted diameter values. The Munro [10] and Biging [12] models show variability in residual variances. At the same time, the Biging [12] model for small diameters adjusts negative values. Unbiasedness and constant variance characterize the models of Kozak et al. [11], Reed and Green [13], Newberry and Burkhart [14], and Forslund [15]. Based on performance criteria and graphical analysis of residuals, the Reed and Green [13] model was recognized as the best.











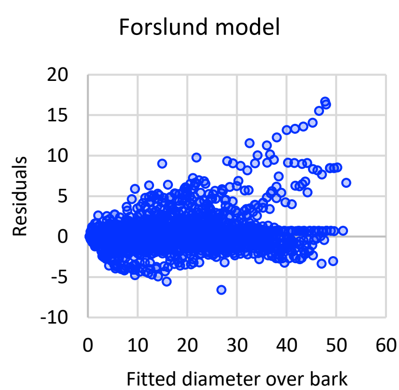


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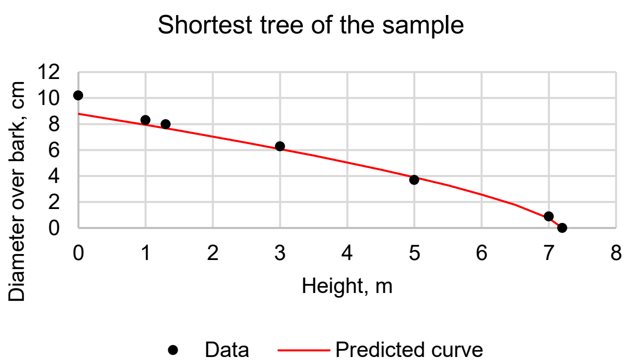
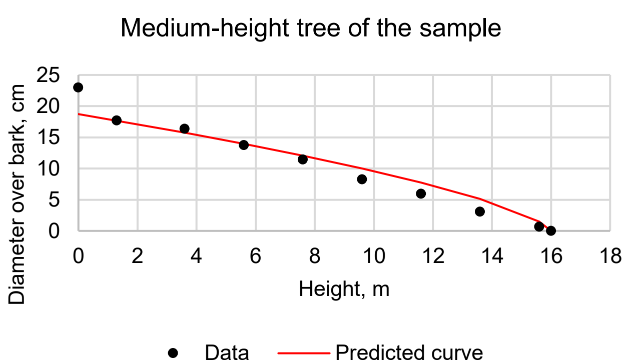
Fig. 2. Residual plots for six two-parameters single equation stem taper models. Fitted diameter over bark is in cm.

3.2 Predicted diameters

Figure 3 shows the fitted diameters for Siberian cedar trees according to the Reed and Green model [13]. From the data set, the shortest tree was selected with a diameter at breast height of 8.3 cm and a height of 7.2 m; tree of medium-height with a diameter at breast height of 17.7 cm and a height of 16.0 m; the tallest tree with a diameter at breast height of 48.0 cm and a height of 25.7 m. The model shows a good fit to the data in the central part of the trunk, but gives large discrepancies in the butt at the level of the stump. It is most expedient to use two-parameter models of one equation not for the entire tree trunk, but for small segments with a slight change in diameter along their length.





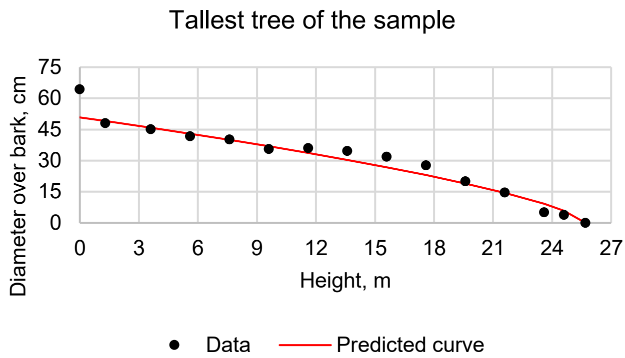


Fig. 3. Prediction curves of Reed and Green [13] stem tapper model over actual measurement points.

3.3 Discussion

Tree taper equations are an important forest management tool. They allow estimating stem volumes without the use of volume tables, output of commercial assortments from the stem without using assortment tables. There are a large number of stem taper models in the literature, but for Siberian pine their development and use are very limited. Among the two-parameter single-equation models, the Reed and Green [13] model showed the best results. But at the same time, all considered two-parameter models showed deviations of the fitted diameters for a height of 1.3 m from the diameter at breast height. Also, all six models show a good fit only for the central part of the trunks. For the purposes of forestry management in cedar forests, it is recommended to develop and put into practice more complex models.

4 Conclusions

The study tested six two-parameter models of one equation for the conicity of the trunk of Siberian cedar trees. For this, data from 1805 measurements of diameters in 193 felled trees were used. Our analysis showed that for our data set, the Reed and Green model turned out to be the best model. The model is characterized by the smallest average absolute error percentage, the square root of the root mean square error, and the largest coefficient of determination. The residuals of this model are independent and characterized by constant variance. The analysis performed showed that two-parameter single equation models have a good fit in the central part of the trunk, therefore, forestry management in cedar forests should be used with caution.

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