

Analysis of Water Vapour Feedback and its Impact on Surface Temperature

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Abstract. The accumulation of greenhouse gases is the main reason for the global warming process. Water vapour, being one of the most abundant and powerful greenhouse gas, strongly influences the warming process in multiple ways. Despite being a greenhouse gas itself, the amount of water vapour in the atmosphere is directly related to the surface temperature of the earth. To understand the global warming process, a deeper look into water vapour and its unique positive feedback mechanism is meaningful. This paper will discuss the mechanism of water vapour feedback, the equilibrium established between the content of water vapour in the air and the surface temperature of the earth. In addition, this study will calculate the time scale and magnitude of the response of water vapour to varying (in real cases increasing) surface temperature, and qualitatively analyse how water vapour feedback would affect the global warming process by serving as an amplifier for the greenhouse effect. These results establish a mathematical model for water vapour feedback's impact on surface temperature rise and could be used as a starting point for lab experiments and more complex analysis.

1 Introduction

Among all greenhouse gases (GHGs), atmospheric water vapour is the most abundant and has a tremendous impact on the Earth's energy budget. It is the main limiting factor of atmospheric transparency in the mm and sub-mm wavelength spectral windows, therefore preventing the globe from losing heat. The atmospheric water plays a crucial role in regional weather processes and global temperature and climate feedback mechanism. Moreover, changes in the hydrological cycle will have widespread consequences for humanity, e.g., through changing precipitation patterns and extremes [1]. To investigate and understand the current global warming process, deep research into various aspects of water vapour behaviour is in need.

Water vapour is recognized as important positive feedback in global warming, as water vapour holds a unique property among all greenhouse gases: its increase mostly attributes to the rising of the surface temperature of the earth rather than the accumulation from human activities, forming a positive feedback on the rising of global surface temperature. To understand this positive feedback and its impact on the water vapour in the atmosphere, both two aspects of this feedback will need to be analyzed: The increase of water vapour content by increasing temperature, and the impact of increased content of water vapour on surface temperature.

The amount of water vapour in the atmosphere has been agreed to have a strong positive relationship with local temperature both in large and small geographical scopes. This relationship has been discussed by many experimental and theoretical researchers such as the one gathering and analysis of the climate profile in northern Chile by the European Southern Observatory [2]. Their work summarizes the temperature profile and local precipitable water vapour (PWV) and using radio telescope tested, they have a strong positive relation in regional areas.

Various research and papers have also discussed the influence on temperature by water vapour feedback: The water vapour change induces significant downward radiative flux perturbation at the tropopause and therefore is hypothesized to substantially amplify the surface warming [3]. Another research has investigated how the observed decrease of water vapour content of about 10% in the atmosphere leads to a slowdown of the global warming process by approximately 25% in 2000 and 2001 [4]. Furthermore, by investigating the Global Cooling After the Eruption of Mount Pinatubo, a team has tested the climate feedback mechanism of water vapour by comparing the simulated climate model with and without the water vapour feedback [5].

The motivation for producing this paper is to further analyze the mechanism and impact of water vapour feedback: how fast the response is, and how it functions

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and to what degree and in what aspect it impacts the global warming process. The methodology part of this paper will provide the models and data used in the discussion and results section of this paper, and the discussion and results section will mainly focus on the theoretical discussion on water vapour feedback and will be separated into three parts. The first part of this paper will qualitatively analyse the speed of response of the content of water vapour with changing surface temperature and argue that the water vapour content in the atmosphere will remain in equilibrium with the global surface temperature at all times. This assumption will then be used as a condition in the second part of this paper, which will discuss theoretically and quantitatively in detail the specific amount of the change in water vapour content with a certain degree of surface warming. Finally, in the last part, this paper will set up a simple mathematical model relating water vapour feedback and surface heating, providing readers with a simple overview of the impact of water vapour feedback on global warming.

2 Methodology

This paper derived the time scale of the water vapour's response to varying surface temperatures. An investigation into the water cycle is conducted. Precipitation and evaporation are two main processes which control and adjust the amount of water vapour in the atmosphere. Therefore, a calculation of the time scale of evaporation and precipitation is conducted. Suppose there is a total amount of water vapour Q in the atmosphere and an additional Q' is put into the atmosphere artificially. E represents the speed of evaporation and R represents the speed of precipitation. Eq. (1) is established as:

$$dQ/dt = E - R \quad (1)$$

As observed from nature, evaporation generally remains in balance with precipitation, so the time scale of evaporation and precipitation are the same. Having that conclusion, the changing rate of Q' to R can be solely expressed as Eq. (2):

$$dQ'/dt \sim R \quad (2)$$

Then, the time scale τ of water vapour response could be estimated by calculating the ratio of total water vapour content and the speed of precipitation as given in Eq. (3):

$$\tau \sim Q/R \quad (3)$$

This timescale will then be used to compare with the timescale of surface temperature growth over years (data collected from NASA [6]) to show the water vapour response is a relatively swift response and the content of water vapour and surface temperature can be seen as remaining in equilibrium in all time.

Following previous studies on the relationship between water vapour content and surface temperature [7], this paper will adopt the fact that they follow Clausius–Clapeyron relation in ideal conditions. As this paper

demonstrated, the water vapour content and the surface temperature remain in equilibrium at all time. Thus, the small change while approaching equilibrium will be neglected from the calculation. The function of the state of equilibrium between saturated water pressure $P_{H_2O}^{sat}$ and surface temperature T_s follows the Clausius–Clapeyron relation given in Eq. (4), in a certain degree of estimation:

$$\frac{1}{P_{H_2O}^{sat}(T)} \frac{dP_{H_2O}^{sat}(T)}{dT} \approx \frac{L_v M_w}{1000RT^2} \quad (4)$$

Following this equation and using the averaged climate data, this paper will try to derive a straightforward equation between water vapour content and surface temperature. Finally, this paper will use a simple leaky greenhouse gas model to estimate the atmosphere. Using this model, a simple relationship between planetary emission temperature and planetary surface temperature can be established in Eq. (5) [8].

$$T_s = \left[\left(\frac{2}{2 - \epsilon_0} \right) \right]^{\frac{1}{4}} T_e \quad (5)$$

Greenhouse gases are the main contributor to the atmospheric absorptivity ϵ . therefore, the absorptivity can be estimated as a function of other greenhouse gases which content are unaffected by surface temperature and water vapour content $\epsilon(\text{other gases}, H_2O)$. However, the quantitative detail of the relationship is beyond the scope of this paper, thus the relationship between ϵ and other gases and H_2O will be modelled by linear Eq. (6) [9]. It is worth mentioning that this paper has demonstrated the strong relationship between δT_s and δH_2O , therefore η represents the effect of water vapour feedback in the process.

$$\delta \epsilon = \mu \delta q_{other} + \eta \delta T_s \quad (6)$$

Having this as a presumption, this paper will derive a simple equation relating water vapour feedback, the amount of other greenhouse gases, and the planetary surface temperature of the earth.

3 Results and discussion

As discussed in the Sec. 2, the time scale of the water vapour content and surface temperature establishment could be estimated by considering the characteristic lifetime of water vapour in the atmosphere, as shown by Eq. (3). As this would only be an approximation in the degree of magnitudes, the requirement for the accuracy of Q and R is relatively low. This paper will adapt the data of the U.S. Geological Survey (USGS) [10]. The total amount of water vapour in the atmosphere is estimated by 37.5 million billion gallons. This is equivalent to 27.8mm if the earth's surface is covered by such water. In addition, the globally averaged annual precipitation is calculated and estimated by NASA and could be estimated 990 millimetres. Thus, with the data above, the time scale of water vapour response is calculated by Eq. (7):

$$\tau = \frac{Q}{R} = 10.25 \text{ days} \quad (7)$$

This result is then compared with the time scale of the global warming process, which is typically represented by the annual averaged surface temperature rise of the earth. This paper adapts a range of data of the yearly averaged land-ocean temperature from 1880 to 2020, as shown in the Fig. 1.

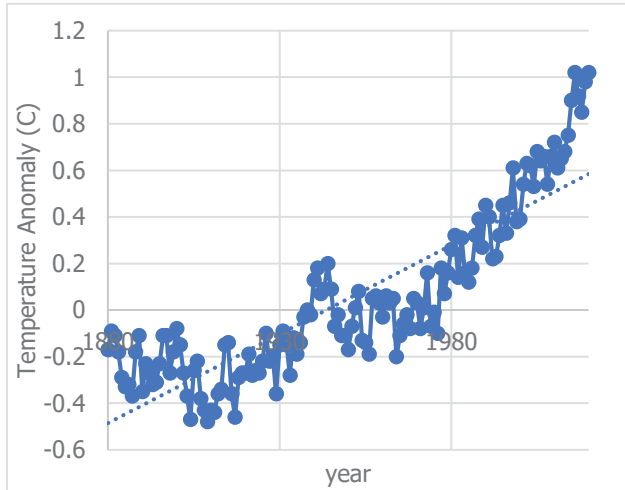


Fig. 1. Land-Ocean Temperature Index. Picture credit: Original

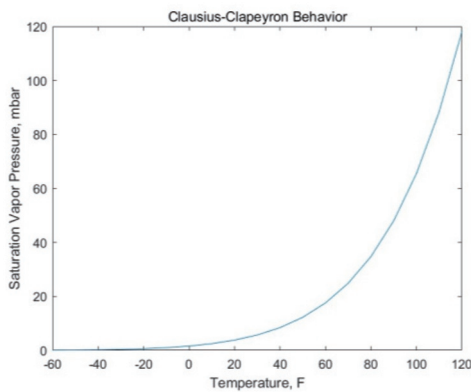


Fig. 2. Clausius–Clapeyron relation. Picture credit: Original

The fluctuations are due to factors such as the ocean’s large heat capacity and solar activities. Overall, the global mean temperature demonstrates a rising trend. From 1980, of annually averaged surface temperature of 272.99K , to 274.17K in 2020. The temperature increases by 0.43% over 40 years, with an annual growth rate of around 0.01%. Therefore, the time scale of significant change in global temperature and carbon dioxide content is measured in decades. The time scale of water vapour response to surface temperature change, τ , therefore, is much shorter than the time scale of global warming. Thus the assumption that water vapour content in the atmosphere will remain in equilibrium with the surface temperature at all times is valid in most calculations. This assumption will serve as a presumption in further reasoning. The water vapour content and the surface temperature remains in equilibrium at all time in reasonable estimation. Thus, the small change while approaching equilibrium will be neglected from the calculation. The function of the state

of equilibrium between saturated water pressure $P_{H_2O}^{sat}$ and surface temperature T_s follows the Clausius–Clapeyron relation (8), in a certain degree of estimation [7].

$$\frac{1}{P_{H_2O}^{sat}(T)} \frac{dP_{H_2O}^{sat}(T)}{dT} \approx \frac{L_v M_w}{1000RT^2} \quad (8)$$

The latent heat of evaporation L_v is known to be $2.5 \times 10^6 \text{ J kg}^{-1}$, the molecular weight of water M_w is estimated to be 18.016, and the universal gas constant R is approximated to be $8.314 \times \text{J K}^{-1} \text{ mol}^{-1}$. Fig. 2 shows this relation with parameters above.

To see numbers of temperature rise in detail, this paper uses current weather data. By rearranging equation Eq. (8), Eq. (9) is derived:

$$\frac{dP_{H_2O}^{sat}(T)}{dT} = \frac{L_v M_w P_{H_2O}^{sat}(T)}{1000RT^2} \quad (9)$$

$P_{H_2O}^{sat}(273 \text{ K})=6.11 \text{ hpa}$ is measured at STD, substituting this into (6), the relationship between saturated water pressure and surface temperature could be estimated as:

$$\delta P_{H_2O}^{sat}(T) \approx 0.44 \delta T \quad (10)$$

The true water vapour pressure at temperature T is then measured to be $uP_{H_2O}^{sat}(T)$, where u is the constant relative humidity in consideration. In this case, u is estimated to be 0.8. In the next step, the predictable water W (water vapour content) is related to water vapour pressure by (11) [4].

$$W = \frac{0.622}{g} \frac{P_{H_2O}(T)}{1+\lambda}, \delta W = \frac{0.622}{g} \frac{\delta P_{H_2O}(T)}{1+\lambda} \quad (11)$$

Here λ is the adjusting factor and is calculated by $\lambda = \frac{\text{scale height of atmosphere}}{\text{scale height of water vapour}}$, which can be estimated by 3.5. Combining Eq. (10) and Eq. (11), substituting $g=9.8$, a simple equation relating precipitable water and temperature is derived

$$\delta W = \frac{0.622}{g} \frac{0.352 \delta T}{1+\lambda} \quad (12)$$

Therefore, using Eq. (9), under a rational and realistic assumption that $T=273\text{K}$ and $u=0.8$ [11], δW is calculated to be 0.00495mm When a 1-degree increase is added to the surface temperature. On the other hand, by substituting $T=273\text{k}$ into Eq. (12), this paper estimates the total precipitable water W to be 0.0688mm at STD [12]. The change in the water vapour content is therefore about 7.2%. This is a very significant change, showing an exponential relationship between water vapour content and surface temperature. In other words, a small fractional change in surface temperature leads to a significant change in water vapour content.

Now previous discussion has demonstrated the strong correlations between water vapour amount and surface temperature, this paper will invoke Eq. (6) to find the relationship between the content of other greenhouse gases with surface temperature T_s . This is, in the end, the same as finding the change of surface temperature concerning the change in the opacity of atmosphere, ϵ ,

$\frac{\delta T_s}{\delta \varepsilon}$. When $\delta \varepsilon$ approaches zero, it equals the derivative of T_s with respect to ε_0 , $\frac{dT_s}{d\varepsilon}$. To find $\frac{dT_s}{d\varepsilon}$, linearising Eq. (10) by taking the natural log on both sides:

$$\ln T_s = \frac{1}{4} [-\ln(2 - \varepsilon) + \ln 2 + 4 \ln T_e] \quad (13)$$

Implicit differentiate with respect to ε leads to Eq. (14),

$$\frac{dT_s}{d\varepsilon} = 2^{(-4/7)} T_e (2 - \varepsilon)^{(-5/4)} \quad (14)$$

Seen from Fig. 3, for $\varepsilon_0=0.1, 0.2, 0.3, 0.4$, with higher value of ε_0 , the same emission temperature will correspond to higher surface temperature.

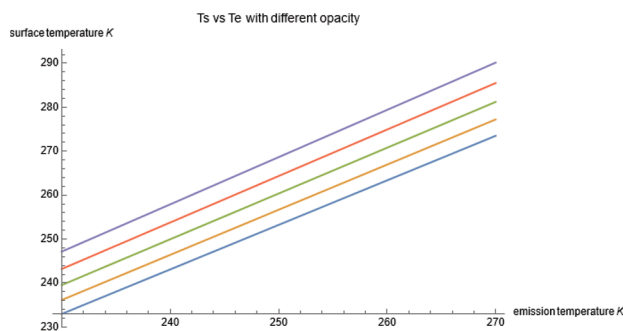


Fig. 3. Emission temperature as a function of surface temperature. Picture credit: Original

By observing Eq. (12), two constants can be abstracted from the formula. Let $\alpha = 2^{(-4/7)}$, and $\beta = (2 - \varepsilon)^{(-5/4)}$, in which way Eq. (14) could be simplified to be Eq. (15).

$$\frac{dT_s}{d\varepsilon} = \alpha \beta T_e \quad (15)$$

Only β is related to the concentration of other greenhouse gases, substituting Eq. (12) with Eq. (15) gives equation (16)

$$dT_s = \left(\frac{\alpha \beta \mu}{1 - \alpha \beta \eta} \right) dq_{\text{other}} \quad (16)$$

η , as discussed before, represents the degree of effect of water vapour feedback. Eq. (16) indicates that water vapour feedback will increase the factor in front of dq_{other} , therefore intensifying the power of warming by other greenhouse gases. The water vapour feedback serves as an amplifier for the greenhouse effect, global warming process.

4 Limitations and prospects

As a matter of fact, this paper only uses an extremely simple model of atmosphere and thermal relations, and is only a theoretical discussion in many aspects. To be specific, this report ignores the multi-layer structure of atmosphere. The real atmosphere is a complex arrangements of different gases and distributes unevenly in multi aspects. In further discussion, the atmosphere will be need to analyse and calculate multi-dimensionally, considering three-dimension distribution and properties of different components. Another important drawback of

this analysis is that this paper neglects wavelength of radiation which have already been fully absorbed. This fact will decrease the effect of our final calculation. Finally, this paper only adapted data from other teams of research, and is only an analysis of macroscopic, globally-averaged view.

Thus, as suggested, a deeper look into different wavelength spectrum is needed. Properties of different existing gas in the atmosphere should be include in the discussion. Moreover, a more sophisticated structure of atmosphere (with ingratiated levels of density and correspondant gas distribution) is needed. If further quantitative discussion will be conducted, a more thorough analysis of the system is required. In the aspect of data, a more throughout investigation and lab experiments of regional scope and more abundant data source will be very useful in future discussions. A discussion of regional sense would be meaningful in the way of interacting with local human, animal and plant, activities.

5 Conclusion

In summary, this paper discussed the process and mechanism of water vapour feedback. In specific, this paper proves the equilibrium established between water vapour content and surface temperature. showed in detail the strong relationship between surface temperature and atmospheric water vapour content, $\delta W = 0.622(0.352 \delta T)/g(1+\lambda)$, and qualitatively demonstrates how the water vapour feedback serves as an strong amplifier on the response of surface temperature (global warming) in response to rising level of other greenhouse gases. Overall, these results shed light on guiding further exploration of water vapour impact on global warming.

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