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Optimal Pilot Placement for Frequency Offset Estimation and Data Detection in Burst Transmission Systems

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***Abstract*— In this letter, we address the problem of pilot design for Carrier Frequency Offset (CFO) and data detection in digital burst transmission systems. We consider a quasi-static flat-fading channel. We find that placing half of the pilot symbols at the beginning of the burst and the other half at the end of the burst is optimal for both CFO estimation and data detection. Our findings are based on the Cramér-Rao bound and on empirical evaluations of the bit error rate for different pilot designs. The equal-preamble-postamble pilot design is shown to provide a significant gain in performance over the conventional preamble-only pilot design.**

I. Introduction

PILOT Symbol Assisted Transmission (PSAT) is a practical technique in wireless communication systems. Pilot symbols, which are usually time-division multiplexed with the information bearing symbols, are used for carrier recovery and channel estimation. In a burst-mode transmission, the way the pilot symbols are placed in each burst often affects the performance of the system. Recent papers have derived optimal pilot designs for channel estimation and data detection in the context of time/frequency-selective channels e.g. [1], [4]. The issue of optimal pilot design when there is a mismatch between the transmitter and receiver carrier frequencies has received little attention in the literature. This frequency mismatch or Carrier Frequency Offset (CFO), which is caused by oscillator instability and Doppler effect in mobile communications, needs to be accurately estimated and compensated for, particularly in the case of large size constellations (see e.g. [2], [3] and references therein). In [5], a white training sequence was shown to be asymptotically optimal for joint CFO and channel estimation using the *worst-case asymptotic* CRB. However, the pilot symbols were grouped into a single cluster and optimal pilot placement was not investigated. In [6], for OFDM systems, the CFO was estimated using null-subcarriers and optimal placement of these null-subcarriers was derived.

In this letter, we consider a single carrier burst transmission system and a quasi-static flat fading channel, i.e. the channel is a complex scalar which is fixed over each burst but changes across the bursts. The system is also affected by a CFO which may vary across the bursts. This might occur in mobile satellite communications where the frequency mismatch can significantly vary over time. Here, we derive the pilot placement that minimizes the Bit Error Rate (BER). We achieve this by minimizing the CRB on the CFO and channel estimations and through Monte-Carlo simulations.

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II. Signal Model and Preliminaries

Consider a single carrier burst transmission system operating over a quasi-static flat fading channel. Assuming that timing is perfectly synchronized, the received burst signal after matched filtering and sampling can be modelled as [2]

$$x(n) = \rho e^{j(\omega n+\phi)} s(n) + v(n) \quad n \in [0, 1, \dots, N-1] \quad (1)$$

where N is the burst length, ρ and ϕ are respectively the magnitude and phase of the channel, ω $(-\pi < \omega \leq \pi)$ is the normalized CFO, $\{s(n)\}$ is the transmitted symbol sequence which encompasses both pilot and information-bearing (or data) symbols, and $v(n)$ is an additive noise with variance σ^2.

In burst-by-burst processing, the CFO and the channel are estimated at each burst by making a number, say P, of the N transmitted symbols known to the receiver. Our objective is to find the optimal placement of these pilot symbols in terms of CFO and channel estimation accuracy and BER. In our pilot design, we make the following assumptions:

(A1) The P pilot symbols have the same magnitude, assumed unity without loss of generality.

(A2) $v(n)$ is a circularly symmetric white Gaussian noise.

Note that (A1) is usually made in PSAT systems, where the pilots are chosen to be either BPSK or QPSK constellations. However, this assumption could be relaxed by allowing the pilot symbols to have different magnitudes. The design problem in this case is outside the scope of this letter.

Let $\mathcal{P}$ and $\mathcal{D}$ denote complementary subsets of $\{0, ..., N-1\}$ which respectively represent the placements of the pilot and data symbols in a burst. In coherent PSAT systems, ω, ϕ and ρ are first estimated using $\{x(n),\ n \in \mathcal{P}\}$; let these estimates be denoted by $\hat{\omega}$, $\hat{\phi}$ and $\hat{\rho}$. These estimates are then used to compensate for the distortions due the CFO and the channel. This provides a soft decision on the data symbols, which is given by

$$\hat{s}(n) = f(\hat{\rho}) e^{-j\hat{\psi}_n} x(n), \quad n \in \mathcal{D} \quad (2)$$

where $\hat{\psi}_n = \hat{\omega} n + \hat{\phi}$ is an estimate of the overall phase to be compensated for, $\psi_n = \omega n + \phi$, and $f(\hat{\rho}) = 1/\hat{\rho}$ in the case of zero-forcing equalization and $f(\hat{\rho}) = \hat{\rho}/(\hat{\rho}^2 + \sigma^2)$ in the case of minimum mean square error equalization. The data detection is then performed on a symbol-by-symbol basis by minimizing the Euclidian distance between $\hat{s}(n)$ and the constellations points used in the symbol mapping stage of data generation. It is therefore imperative to obtain an accurate estimation of the ψ_n's and ρ in order to achieve good detection performance.

III. Optimal Placement of Pilot Symbols

The criterion we use to derive the optimal pilot placement is based on the CRB, which is an algorithm-independent measure of performance. Indeed, the CRB is a lower bound on the variance of any unbiased estimator. The use of the CRB is also motivated by the fact that the performance of maximum likelihood (ML) estimation asymptotically achieves the CRB.

A. Cramér-Rao Bound

Let $\boldsymbol{\theta} = [\rho, \phi, \omega]^T$ be the parameter vector to be estimated. The CRB is obtained from the diagonal elements of the inverse of the Fisher Information Matrix (FIM), which is given by [7]

$$\boldsymbol{F} := -E\left\{\frac{\partial^2 L(\boldsymbol{x}_{\mathcal{P}}|\boldsymbol{\theta})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^T}\right\}$$

where $E\{\cdot\}$ denotes the statistical expectation operator and $L(\boldsymbol{x}_{\mathcal{P}}|\boldsymbol{\theta})$ is the log-likelihood function of $\boldsymbol{x}_{\mathcal{P}} := \{x(n),\ n \in \mathcal{P}\}$ given $\boldsymbol{\theta}$. Under assumptions (A1)-(A2), $L(\boldsymbol{x}_{\mathcal{P}}|\boldsymbol{\theta})$ is, after removing irrelevant terms, given by

$$L(\boldsymbol{x}_{\mathcal{P}}|\boldsymbol{\theta}) = -\frac{1}{\sigma^2}\sum_{n\in\mathcal{P}}[-2\rho\Re\{x(n)s^*(n)e^{-j(\omega n+\phi)}\} + \rho^2]$$

where $\Re\{\cdot\}$ is the real part operator. The FIM is found to be

$$\boldsymbol{F} = \frac{2\rho^2}{\sigma^2}\begin{bmatrix} \frac{P}{\rho^2} & 0 & 0 \\ 0 & P & \sum\limits_{n\in\mathcal{P}} n \\ 0 & \sum\limits_{n\in\mathcal{P}} n & \sum\limits_{n\in\mathcal{P}} n^2 \end{bmatrix}$$

Thus, the CRBs for ρ, ϕ and ω are given by:

$$CRB(\rho) = \frac{\sigma^2}{2P} \tag{3}$$

$$CRB(\phi) = \frac{1}{2\ \text{SNR}}\ \frac{\sum\limits_{n\in\mathcal{P}} n^2}{P\sum\limits_{n\in\mathcal{P}} n^2 - \left(\sum\limits_{n\in\mathcal{P}} n\right)^2} \tag{4}$$

$$CRB(\omega) = \frac{1}{2\ \text{SNR}}\ \frac{P}{P\sum\limits_{n\in\mathcal{P}} n^2 - \left(\sum\limits_{n\in\mathcal{P}} n\right)^2} \tag{5}$$

where $\text{SNR} := \frac{\rho^2}{\sigma^2}$ is the Signal-to-Noise Ratio (SNR).

Using the inverse of the FIM, the CRBs for ψ_m, $m \in \mathcal{D}$, are obtained as

$$CRB(\psi_m) = \frac{1}{2\ \text{SNR}}\ \frac{m^2P - 2m\sum\limits_{n\in\mathcal{P}} n + \sum\limits_{n\in\mathcal{P}} n^2}{P\sum\limits_{n\in\mathcal{P}} n^2 - \left(\sum\limits_{n\in\mathcal{P}} n\right)^2}, \quad m \in \mathcal{D} \tag{6}$$

B. Optimal Pilot Placement

Since the CRB for ρ is independent of $\mathcal{P}$, only the CRBs for $\{\psi_m,\ m \in \mathcal{D}\}$ are relevant in the pilot placement design. We therefore propose the following optimum pilot placement

$$\mathcal{P}^* = \arg\min_{\mathcal{P}} \sum_{m\in\mathcal{D}} CRB(\psi_m) \tag{7}$$

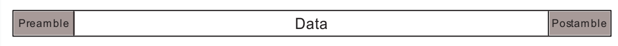

Fig. 1. Optimal pilot placement design.

Solving the above optimization problem analytically seems untractable. Hence, we resort to computer analysis. By carrying out an exhaustive evaluation of the design criterion over all possible pilot placements, $\mathcal{P}^*$ is found to consist of a preamble (i.e. a cluster of symbols at the beginning of the burst) and a postamble (i.e. a cluster of symbols at the end of the burst) of equal size; see Fig. 1. If P is odd, either the preamble or the postamble can have one more symbol than the other without affecting the design criterion. Interestingly, the conventional preamble-only design gives the worst performance, as illustrated in Figs. 2 and 3 (details are given in Section IV). The following results have also been obtained by computer analysis:

- The equal-preamble-postamble (EPP) pilot design also minimizes the CRB for ω.
- The pilot placement that minimizes the CRB for ϕ is also a preamble-postamble placement. However, the size of the postamble is either one or two symbols. Neverthless, the CRB for ϕ when the preamble and postamble are of equal size is quite close to the minimum of $CRB(\phi)$.
- The EPP pilot design also minimizes the $CRB(\psi_m)$'s individually.

We now make the following relevant comments.

1) It is worth recalling that in the absence of CFO, the CRB for ϕ is independent of $\mathcal{P}$. Therefore, it is the presence of an unknown CFO that makes the CRB of ϕ dependent on the pilot placement.

2) Let a_m and b_m denote the amplitude and angle of the data symbol $s(m)$. Thus, from (1), the soft detection output in eq. (2) can be expressed as:

$$\hat{s}(m) = f(\hat{\rho})\rho a_m e^{j(b_m+\Delta\psi_m)} + \tilde{v}(m), \quad m \in \mathcal{D}$$

where $\Delta\psi_m = \psi_m - \hat{\psi}_m$ and $\tilde{v}(m) = f(\hat{\rho})v(m)$. Since the estimation performance for ρ is independent of $\mathcal{P}$, it is reasonable to expect that the probability of detection error for each data symbol, $s(m)$, is minimized when the mean square of $\Delta\psi_m$ is also minimized, which is obtained with the EPP pilot design. Although this statement lacks a rigorously theoretical proof, it is well supported by simulation results (see Section IV.)

3) After some algebra, the CRB for ω in eq. (5) can be re-expressed as

$$CRB(\omega) = \frac{1}{\text{SNR}}\ \frac{P}{\sum\limits_{i=1}^{P}\sum\limits_{j=1;j\neq i}^{P}(n_i - n_j)^2}$$

where n_i, $i = 1, ..., P$, denote the time indexes of the pilot symbols. This expression shows that $CRB(\omega)$ decreases with the sum of square differences $(n_i - n_j)^2$. Using this expression, we can analytically prove for $P = 2, 3, 4$ that the EPP design minimizes $CRB(\omega)$. For $P \geq 5$, the analytical proof becomes tedious. As to why the EPP design is best in terms of CFO estimation, we present the following explanation. For simplicity, assume that $P = 2$ and that the pilot symbols

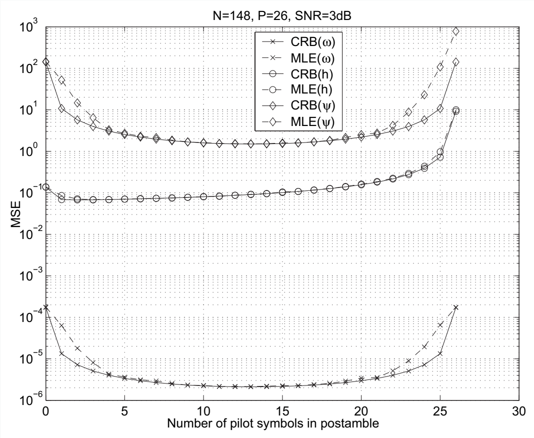

Fig. 2. MSEs and CRBs for different preamble-postamble designs.

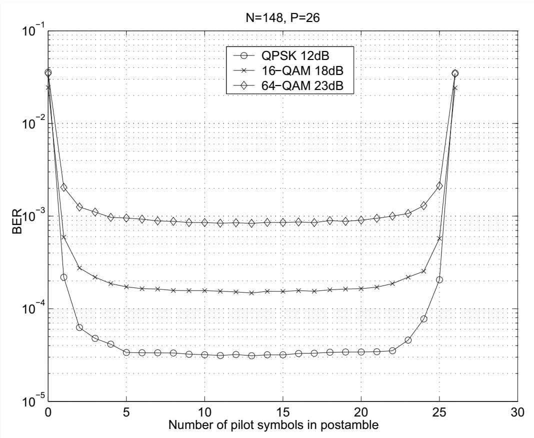

Fig. 3. Bit error rate for different preamble-postamble designs.

are both equal to unity. The received pilot symbols are then $x(n_i) = \rho \exp(j\omega n_i + \phi) + v(n_i)$, $i = 1, 2$, where n_1 and n_2 can take any distinct values from $\{0, ..., N-1\}$. The maximum likelihood estimate of ω can be shown to be $\hat{\omega} = \arg\{x^*(n_1)x(n_2)\}/(n_2 - n_1)$ where $\arg\{.\}$ denotes the argument (or angle) operator. The variance of the numerator is independent of n_1 and n_2. Thus, the variance of $\hat{\omega}$ is inversely proportional to $(n_2 - n_1)^2$. This variance is minimum when $n_1 = 0$ and $n_2 = N - 1$, which is the EPP design. It is however difficult to extend this reasoning to the case where $P > 2$.

4) The EPP pilot design guarantees a full acquisition range provided $P > 2$. Indeed, with $P = 3$, either the preamble or the postamble consists of two consecutive pilot symbols.

5) Using the EPP pilot design and assuming P even, we obtain the following close-form CRB expressions

$$CRB(\omega) = \frac{1}{2\ \text{SNR}} \frac{12}{P[P^2 - 3NP + (3N^2 - 1)]}$$

$$CRB(\phi) = \frac{1}{2\ \text{SNR}} \frac{P^2 - 3NP + 6N^2 - 6N + 2}{P[P^2 - 3NP + (3N^2 - 1)]}$$

IV. Simulation Results

Here, we illustrate the superiority of the proposed EPP design over the conventional preamble-only design using the CRB, the empirical MSE of the maximum likelihood (ML) estimates and the BER. In the evaluation of the latter, we used QPSK, 16-QAM and 64-QAM. The amplitude ρ was set to unity and the phase ϕ was randomly generated using a uniform distribution over $(-\pi, \pi]$. The CFO was set to $\omega = 0.2\pi$. The burst size was $N = 148$ and the total number of pilot symbols was $P = 26$. We evaluate the CRB for ω, $h := \rho \exp(j\phi)$, and the sum of the CRBs for the ψ_m's, $m \in \mathcal{D}$, versus different preamble-postamble designs. Figs. 2 and 3 display the results versus the number of pilot symbols in the postamble. It can be seen that the EPP design provides a significant improvement over the preamble-only design in terms of both estimation accuracy and data detection performance. It is however worth pointing out that if the preamble and postamble are of unequal size, the performance is close to that of the EPP design provided that the preamble or the postamble consists of more than two pilots. We have also run simulations (not shown here because of space limitation) to compare the performance of the EPP design with that of the equally spaced pilot (ESP) design (i.e. the pilots are uniformly scattered across the block), which is traditionally used for channel estimation [4]. Although the EPP design outperforms the ESP design, the difference in performance is not as significant as that between the EPP design and the preamble-only design. However, it is worth pointing out that unlike the EPP design the ESP design does not provide full frequency acquisition range.

V. Conclusions

In burst-mode transmission where the carrier frequency-offset needs to be estimated, we have proven that by splitting the pilot sequence into a preamble and a postamble of equal size, a significant gain in system performance over the preamble-only pilot design can be obtained. Although the gain in performance over the equally spaced pilot design is not significant, the proposed design has the advantage of providing full frequency acquisition range. The proposed pilot design may be particularly useful when the frequency-offset may vary across the bursts as might be the case for some mobile communication systems where a significant Doppler effect is present. The disadvantage of the preamble-postamble scheme is its unsuitability for real-time burst processing. Future work will address the case of frequency selective channels.

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