Average Consensus and Gossip Algorithms in Networks with Stochastic Asymmetric Communications

Duarte Antunes, Daniel Silvestre, Carlos Silvestre

***Abstract*— We consider that a set of distributed agents desire to reach consensus on the average of their initial state values, while communicating with neighboring agents through a shared medium. This communication medium allows only one agent to transmit unidirectionally at a given time, which is true, e.g., in wireless networks. We address scenarios where the choice of agents that transmit and receive messages at each transmission time follows a stochastic characterization, and we model the topology of allowable transmissions with asymmetric graphs. In particular, we consider: (i) randomized gossip algorithms in wireless networks, where each agent becomes active at randomly chosen times, transmitting its data to a single neighbor; (ii) broadcast wireless networks, where each agent transmits to all the other agents, and access to the network occurs with the same probability for every node. We propose a solution in terms of a linear distributed algorithm based on a state augmentation technique, and prove that this solution achieves average consensus in a stochastic sense, for the special cases (i) and (ii). Expressions for absolute time convergence rates at which average consensus is achieved are also given.**

I. Introduction

The average consensus problem is a distributed control problem in which a set of agents aims to agree on the average of their initial state values by exchanging messages dictated by a given communication topology. Several multi-disciplinary applications of average consensus algorithms have been reported in the literature. These include distributed optimization [1], [2]; motion coordination tasks, such as flocking, leader following [3], and rendezvous problems [4]; and resource allocation in computer networks [5].

An elegant theory is now available in the literature to solve consensus problems using linear distributed algorithms, in which each agent computes a weighted average between its state value and the state values of the agents to which it can communicate (see, e.g, [6], [7]). Many variations of this problem have been addressed in the literature considering, e.g., stochastic packet drops and link failures [8], [9], quantized data transmissions [10], and time-varying communication connectivity [6].

On the other hand, randomized gossip algorithms have been proposed, e.g., in [11], as a decentralized solution to the average consensus problem that can deal with several features such as the absence of a centralized entity, and the possibly varying topology of the network caused by agents that join and leave the network. The premise of gossip algorithms is that each agent communicates with no more than one neighbor at each transmission time. In [11], it is assumed that at each communication time, the sender and the receiver set their state to the average of their current values, which subsumes that the communication is bidirectional.

In the present work, we consider the average consensus problem in scenarios where communication is unidirectional at each time slot, i.e., at each transmission time a single agent transmits data to one or several agents, but does not receive data. Note that at a different time slot receiver and sender agents may invert their roles, i.e., the word unidirectional refers only to communication at a given transmission time. For concreteness, we consider the two following scenarios: (i) randomized gossip algorithms in wireless networks, where each agent becomes active at randomly chosen times, transmitting its data to a single neighbor; (ii) broadcast wireless networks, where each agent transmits to all the other agents, access to the network occurs with the same probability for every agent, and the intervals between transmissions are independent and identically distributed. As we shall see, the unidirectionality communication constraint precludes in general the existence of a linear distributed algorithm where associated to each agent there is a single scalar state, updated based on the state of the other agents, as in related problems where the communication topology of the network is also time-varying, but satisfies different assumptions (see [6], [7]). We assume a symmetric communication topology, meaning that if an agent a can communicate with an agent b then the agent b can communicate with the agent a, although this does not take place at the same transmission time, i.e., at each transmission time the graphs modeling communications are in general asymmetric. Note that this is typically the case in wireless networks, and therefore this assumption is reasonable to assume in both scenarios (i) and (ii).

The main contribution of the present paper, is to propose a linear distributed algorithm using a state augmentation technique to achieve average consensus when communication constraints impose time-varying unidirectional transmissions, and to prove convergence in expectation to average consensus for scenarios (i) and (ii). Moreover, the stochasticity of the times between transmissions, in both gossip and broadcast scenarios considered herein, render non-trivial the computation of the rates of convergence of the linear distributed algorithms to consensus in absolute time, as opposed to discrete-time, i.e., in terms of the number of transmissions. We provide expressions for the absolute time rates in both scenarios.

A recent work [12] addresses a similar problem, considering gossip algorithms also with asynchronous communica-

Contact the author D. Silvestre through dsilvestre@isr.ist.utl.pt for a free unformatted copy of this article or go to http://ieeexplore.ieee.org/abstract/document/6161444/ for the final formatted version.

tion between the agents. In [12], a method is proposed to achieve average consensus, also using a state augmentation technique, and this method is proved to converge almost surely to consensus. The communication topologies considered in [12] encompass general directed graphs. Hence, [12] does not need to assume a symmetric communication topology , which is crucial to obtain our convergence results. Note, however, that in the method proposed in [12], the state updates depend in a nonlinear way of the current state, while the algorithm that we propose is a linear distributed algorithm, proving a solution that parallels existing algorithms for the standard gossip [11] and linear distributed algorithms [7].

REFERENCES

[1] J. Tsitsiklis, D. Bertsekas, and M. Athans, "Distributed asynchronous deterministic and stochastic gradient optimization algorithms," *Automatic Control, IEEE Transactions on*, vol. 31, no. 9, pp. 803 – 812, Sep. 1986.

[2] B. Johansson, T. Keviczky, M. Johansson, and K. Johansson, "Subgradient methods and consensus algorithms for solving convex optimization problems," in *Decision and Control, 2008. CDC 2008. 47th IEEE Conference on*, 2008, pp. 4185 –4190.

[3] A. Jadbabaie, J. Lin, and A. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," in *Decision and Control, 2002, Proceedings of the 41st IEEE Conference on*, vol. 3, 2002, pp. 2953 – 2958 vol.3.

[4] J. Cortes, S. Martinez, and F. Bullo, "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions," *Automatic Control, IEEE Transactions on*, vol. 51, no. 8, pp. 1289 –1298, 2006.

[5] M. Chiang, S. Low, A. Calderbank, and J. Doyle, "Layering as optimization decomposition: A mathematical theory of network architectures," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 255 –312, 2007.

[6] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *Automatic Control, IEEE Transactions on*, vol. 49, no. 9, pp. 1520 – 1533, 2004.

[7] F. Bullo, J. Cortés, and S. Martínez, *Distributed Control of Robotic Networks*, ser. Applied Mathematics Series. Princeton University Press, 2009, electronically available at http://coordinationbook.info.

[8] S. Patterson, B. Bamieh, and A. El Abbadi, "Convergence rates of distributed average consensus with stochastic link failures," *Automatic Control, IEEE Transactions on*, vol. 55, no. 4, pp. 880 –892, 2010.

[9] F. Fagnani and S. Zampieri, "Average consensus with packet drop communication," *SIAM J. Control Optim*, no. 48, pp. 102–133, 2009.

[10] R. Carli, F. Bullo, and S. Zampieri, "Quantized average consensus via dynamic coding/decoding schemes," *International Journal of Robust and Nonlinear Control*, vol. 20, no. 2, pp. 156–175, 2010. [Online]. Available: http://dx.doi.org/10.1002/rnc.1463

[11] S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah, "Randomized gossip algorithms," *Information Theory, IEEE Transactions on*, vol. 52, no. 6, pp. 2508 – 2530, 2006.

[12] K. Cai and H. Ishii, "Gossip consensus and averaging algorithms with quantization," in *2010 American Control Conference*, 2010, pp. 6306 –6311.