

# Braneworlds, Conformal Fields and Dark Energy

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## Abstract

In the Randall-Sundrum scenario we analyze the dynamics of a spherically symmetric 3-brane when matter fields propagate in the bulk. For a well defined class of conformal fields of weight  $-4$  we determine a new set of exact 5-dimensional solutions which localize gravity in the vicinity of the brane and are stable under radion field perturbations. Geometries which describe the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic dark energy on the brane are shown to belong to this set.

## 1 Introduction

In the Randall-Sundrum (RS) scenario [1, 2] the visible Universe is a 3-brane world of a  $Z_2$  symmetric 5-dimensional (5D) anti-de Sitter (AdS) space. In the RS1 model [1] there is a compact fifth dimension and two brane boundaries. The gravitational field is localized near the hidden positive tension brane and decays towards the visible negative tension brane. In this model the hierarchy problem is reformulated as an exponential hierarchy between the weak and Planck scales [1]. In the RS2 model [2] there is a single positive tension brane in an infinite fifth dimension. Then gravity is bound to the positive tension brane now interpreted as the visible brane.

At low energies the theory of gravity on the observable brane is 4-dimensional (4D) general relativity and the cosmology may be Friedmann-Robertson-Walker [1]-[10]. In the RS1 model this is only possible if the radion mode is stabilized using for example a bulk scalar field [3, 6, 9, 10]. Gravitational collapse was also analyzed in the RS scenario [11]-[16]. However, an exact 5D solution representing a stable black hole localized on a 3-brane has not yet been discovered. So far, the only known static black holes localized on a brane remain to be those found for a 2-brane in a

4D AdS space [12]. A solution to this problem requires the non-singular localization of both gravity and matter [11, 13]-[16] and could be connected to quantum black holes on the brane [15]. This is an extra motivation to look for 5D collapse solutions localized on a brane. In addition, the effective covariant Gauss-Codazzi approach [17, 18] has permitted the discovery of many braneworld solutions which have not yet been associated with exact 5D spacetimes [19]-[22].

In this paper we continue the research on the dynamics of a spherically symmetric RS 3-brane when 5D conformal matter fields propagate in the bulk [16, 23] (see also [24]). In our previous work [16, 23] we have found a new class of exact 5D dynamical solutions for which gravity is localized near the brane by the exponential RS warp. These solutions were shown to describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter. However, the density and pressures of the conformal bulk fluid increase with the coordinate of the fifth dimension. In the RS2 model this generates a divergence at the AdS horizon as in the Schwarzschild black string solution [11]. In the RS1 model this is not a problem. However, the solutions turn out to be unstable under radion field perturbations [25]. In this work we report on a new set of exact 5D braneworld solutions which have a stable radion mode and still describe on the observable brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter.

## 2 5D Einstein Equations and Conformal Fields

On the 5D RS orbifold the non-factorizable metric consistent with the  $Z_2$  symmetry in  $z$  and with 4D spherical symmetry on the brane corresponds to the 5D line element  $d\tilde{s}_5^2 = \Omega^2 (dz^2 - e^{2A} dt^2 + e^{2B} dr^2 + R^2 d\Omega_2^2)$ , where  $\Omega = \Omega(t, r, z)$ ,  $A = A(t, r, z)$ ,  $B = B(t, r, z)$  and  $R = R(t, r, z)$  are  $Z_2$  symmetric functions.  $R(t, r, z)$  represents the physical radius of the 2-spheres and  $\Omega$  is the warp factor which defines a global conformal transformation on the metric.

The classical dynamics is defined by the 5D Einstein equations,

$$\tilde{G}_\mu^\nu = -\kappa_5^2 \left\{ \Lambda_B \delta_\mu^\nu + \frac{1}{\sqrt{\tilde{g}_{55}}} [\lambda \delta(z - z_0) + \lambda' \delta(z - z'_0)] (\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5) - \tilde{T}_\mu^\nu \right\}, \quad (1)$$

where  $\Lambda_B$  is the negative bulk cosmological constant,  $\lambda$  and  $\lambda'$  are the brane tensions,  $\kappa_5^2 = 8\pi/M_5^3$  with  $M_5$  the fundamental 5D Planck mass and  $\tilde{T}_\mu^\nu$  is the stress-energy tensor associated with the matter fields. In 5D  $\tilde{T}_\mu^\nu$  is conserved,  $\tilde{\nabla}_\mu \tilde{T}^\mu = 0$ .

Let us consider the special class of conformal bulk matter defined by  $\tilde{T}_\mu^\nu = \Omega^{-2} T_\mu^\nu$  and assume that  $T_\mu^\nu$  depends only on  $t$  and  $r$ . The conformal stress-energy tensor  $\tilde{T}^{\mu\nu}$  may be separated in two sectors  $\tilde{T}^{\mu\nu}$  and  $\tilde{U}^{\mu\nu}$  with the same weight  $s$ ,  $\tilde{T}^{\mu\nu} = \tilde{T}^{\mu\nu} + \tilde{U}^{\mu\nu}$  where  $\tilde{T}^{\mu\nu} = \Omega^s T^{\mu\nu}$  and  $\tilde{U}^{\mu\nu} = \Omega^s U^{\mu\nu}$ . Assuming that  $T_\mu^\nu$  and  $U_\mu^\nu$  are conserved tensor fields,  $A = A(t, r)$ ,  $B = B(t, r)$ ,  $R = R(t, r)$  and  $\Omega = \Omega(z)$  we obtain

$$G_a^b = \kappa_5^2 T_a^b, \quad G_5^5 = \kappa_5^2 T_5^5, \quad \nabla_a T_b^a = 0, \quad \nabla_a U_b^a = 0, \quad (2)$$

$$6\Omega^{-2}(\partial_z\Omega)^2 + \kappa_5^2\Omega^2\Lambda_B = k_5^2U_5^5 \quad (3)$$

$$\left\{3\Omega^{-1}\partial_z^2\Omega + \kappa_5^2\Omega^2\left\{\Lambda_B + \Omega^{-1}[\lambda\delta(z - z_0) + \lambda'\delta(z - z'_0)]\right\}\right\}\delta_a^b = k_5^2U_a^b, \quad (4)$$

where the latin indices represents the 4D coordinates  $t, r, \theta$  and  $\phi$ . On the other hand we also find the following equations of state  $2T_5^5 = T_c^c, 2U_5^5 = U_c^c$ . Note that  $U_\mu^\nu$  must be a diagonal tensor field,  $U_\mu^\nu = \text{diag}(-\bar{\rho}, \bar{p}_r, \bar{p}_T, \bar{p}_5)$  with constant density and pressures satisfying  $\bar{\rho} = -\bar{p}_r = -\bar{p}_T$  and  $\bar{p}_5 = -2\bar{\rho}$ . On the other hand if  $T_\mu^\nu = \text{diag}(-\rho, p_r, p_T, p_5)$  where  $\rho, p_r, p_T$  and  $p_5$  denote bulk matter density and pressures then its equation of state is re-written as

$$\rho - p_r - 2p_T + 2p_5 = 0, \quad (5)$$

where  $\rho, p_r, p_T$  and  $p_5$  must be independent of  $z$  but may be functions of  $t$  and  $r$ . The bulk matter is, however, inhomogeneously distributed along the fifth dimension because the physical energy density,  $\tilde{\rho}(t, r, z)$ , and pressures,  $\tilde{p}(t, r, z)$ , are related to  $\rho(t, r)$  and  $p(t, r)$  by the scale factor  $\Omega^{-2}(z)$ . Note also that the warp depends on the conformal bulk fields only through  $U_\mu^\nu$ . So the role of  $U_\mu^\nu$  is to influence how the gravitational field is warped around the branes. On the other hand  $T_\mu^\nu$  determines the dynamics on the branes. In the RS1 model the two branes have identical cosmological evolutions and gravity will be localized on the Planck brane and not on the visible one.

### 3 Exact 5D Warped Solutions

The dynamics of the  $AdS_5$  braneworlds is defined by the solutions of equations (2) to (5). Let us first solve the warp equations (3) and (4). If  $\bar{p}_5 = 0$  then  $U_\mu^\nu = 0$  and we obtain the usual RS warp equations. A solution is the exponential RS warp [1, 2]. Using the coordinate  $y$  related to  $z$  by  $z = le^{y/l}$  for  $y > 0$  we find

$$\Omega(y) = \Omega_{\text{RS}}(y) = e^{-|y|/l}, \quad (6)$$

where  $l$  is the AdS radius given by  $l = 1/\sqrt{-\Lambda_B\kappa_5^2/6}$ . If  $\bar{p}_5$  is non-zero then there is a new set of warp solutions to be considered. Integrating Eq. (3) and taking into account the  $Z_2$  symmetry we find

$$\Omega(y) = e^{-|y|/l} \left(1 + \frac{\bar{p}_5}{4\Lambda_B} e^{2|y|/l}\right). \quad (7)$$

This set of solutions which depends on the 5D pressure  $\bar{p}_5$  must also satisfy Eq. (4) which contains the Israel conditions. This may only happen if the brane tensions  $\lambda$  and  $\lambda'$  are given by

$$\lambda = \frac{6}{l\kappa_5^2} \frac{1 - \frac{\bar{p}_5}{4\Lambda_B}}{1 + \frac{\bar{p}_5}{4\Lambda_B}}, \quad \lambda' = -\frac{6}{l\kappa_5^2} \frac{1 - \frac{\bar{p}_5}{4\Lambda_B} e^{2\pi r_c/l}}{1 + \frac{\bar{p}_5}{4\Lambda_B} e^{2\pi r_c/l}}, \quad (8)$$

where  $r_c$  is the RS compactification scale.

The conformal factor  $\Omega(y)$  defines how the gravitational field is warped around the brane. To find the dynamics on the brane we need to consider solutions of Eq. (2) when the diagonal bulk matter  $T_\mu^\nu$  satisfies Eq. (5). For inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter such solutions were determined in Refs. [16] and [23]. The latter describes the dynamics on the brane of dark energy in the form of a polytropic fluid. The diagonal conformal matter may be defined by  $\rho = \rho_P, p_r + \eta\rho_P^\alpha = 0, p_T = p_r, p_5 = -(\rho_P + 3\eta\rho_P^\alpha)/2$ , where  $\rho_P$  defines the polytropic energy density and the parameters  $(\alpha, \eta)$  characterize different polytropic phases. For  $-1 \leq \alpha < 0$  the fluid is in its generalized Chaplygin phase (see also [26]). The 5D polytropic solutions are [23]

$$d\tilde{s}_5^2 = \Omega^2 \left[ -dt^2 + S^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right) \right] + dy^2, \quad (9)$$

where the Robertson-Walker brane scale factor  $S$  satisfies

$$\dot{S}^2 = -k + \frac{\kappa_5^2}{3} S^2 \left( \eta + \frac{a}{S^{3-3\alpha}} \right)^{\frac{1}{1-\alpha}}. \quad (10)$$

## 4 Radion Stability

To analyze how these solutions behave under radion field perturbations we apply a saddle point expansion procedure based on the action [27, 28]. Let us write the most general metric consistent with the  $Z_2$  symmetry in  $y$  and with 4D spherical symmetry on the brane in the form  $d\tilde{s}^2 = a^2 ds_4^2 + b^2 dy^2$  with  $ds_4^2 = -dt^2 + e^{2B} dr^2 + R^2 d\Omega_2^2$ . The metric functions  $a = a(t, r, y)$ ,  $B = B(t, r, y)$ ,  $R = R(t, r, y)$  and  $b = b(t, r, y)$  are  $Z_2$  symmetric. Now  $a$  is the warp factor and  $b$  is related to the radion field. The 5D dynamical RS action is given by

$$\tilde{S} = \int d^4x dy \sqrt{-\tilde{g}} \left\{ \frac{\tilde{R}}{2\kappa_5^2} - \Lambda_B - \frac{1}{\sqrt{g_{55}}} [\lambda\delta(y) + \lambda'\delta(y - \pi r_c)] + \tilde{L}_B \right\}. \quad (11)$$

Our braneworld backgrounds correspond to the metric functions  $b = 1$ ,  $B = B(t, r)$ ,  $R = R(t, r)$  and  $a = \Omega(y)$ .

To calculate the radion potential we consider the dimensional reduction of (11). Using the metric with  $a(t, r, y) = \Omega(y)e^{-\beta(t,r)}$  and  $b(t, r) = e^{\beta(t,r)}$  we obtain in the Einstein frame [25]

$$\tilde{S} = \int d^4x \sqrt{-g_4} \left( \frac{R_4}{2\kappa_4^2} - \frac{1}{2} \nabla_c \gamma \nabla_d \gamma g_4^{cd} - \tilde{V} \right), \quad (12)$$

where  $\gamma = \beta/(\kappa_4\sqrt{2/3})$  is the canonically normalized radion field. The function  $\tilde{V} = \tilde{V}(\gamma)$  is the radion potential and it is given by

$$\tilde{V} = \frac{2}{\kappa_5^2} \chi^3 \left[ 3 \int dy \Omega^2 (\partial_y \Omega)^2 + 2 \int dy \Omega^3 \partial_y^2 \Omega \right] + \chi \int dy \Omega^4 (\Lambda_B - \tilde{L}_B)$$

$$+ \chi^2 \int dy \Omega^4 [\lambda \delta(y) + \lambda' \delta(y - \pi r_c)], \quad (13)$$

where the field  $\chi$  is defined as  $\chi = e^{-\sqrt{(2/3)} \kappa_4 \gamma}$ . The integration in the fifth dimension is performed in the interval  $[-\pi r_c, \pi r_c]$  and that we have chosen  $\int dy \Omega^2 = \kappa_5^2 / \kappa_4^2$ .

To analyze the stability of the  $AdS_5$  braneworld solutions we consider a saddle point expansion of the radion field potential  $V$ . If  $\bar{p}_5 = 0$  then  $\Omega = \Omega_{RS}$ . The critical extremum corresponding to our braneworlds is  $\chi = 1$  [25]. Stable solutions must be associated with a positive second variation of the radion potential. If the equation of state of the conformal bulk fields is independent of the radion perturbation then for  $\chi = 1$  the second variation is negative and so the corresponding braneworlds are unstable [25]. If the equation of state is kept invariant under the radion perturbations it is possible to find stable solutions at  $\chi = 1$  if the warp is changed. Indeed, the new relevant warp functions are given in Eq. (7) and stability exists for a range of the parameters if  $\bar{p}_5 < 0$ . As an example consider the interval  $4\Lambda_B e^{-2\pi r_c/l} < \bar{p}_5 < 0$  which corresponds to a brane configuration with  $\lambda > 0$  and  $\lambda' < 0$ . Then the stability conditions are  $l > 3r_c$ ,  $4\Lambda_B e^{-2\pi r_c/l} < \bar{p}_5 < \bar{p}_5^*(l)$  and  $\bar{p}_5^*(l) < \Lambda_B/2$ .

## 5 Conclusions

In this paper we have analyzed exact 5D dynamical solutions with gravity localized near the brane which are associated with conformal bulk fields of weight -4 and describe the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter on the brane. We have discussed their behaviour under radion field perturbations and shown that they are extrema of the radion potential. We have also shown that if the equation of state characterizing the conformal fluid is independent of the perturbation then the radion may be stabilized by a sector of the conformal fields while another sector generates the dynamics on the brane. Stabilization requires a bulk fluid with a constant negative pressure and involves new warp functions. On the brane these solutions also describe the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter. Whether gravity is sufficiently localized on the brane is an open problem for future research.

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