On the Stability of Randall-Sundrum Braneworlds with Conformal Bulk Fields

Rui Neves

Departamento de Física, Faculdade de Ciências e Tecnologia, Universidade do Algarve

Campus de Gambelas, 8000-117 Faro, Portugal

E-mail: rneves@ualg.pt

Abstract

In the Randall-Sundrum scenario we consider exact 5-dimensional solutions with localized gravity which are associated with a well defined class of conformal bulk fields. We analyze their behaviour under radion field perturbations. We show that if the Randall-Sundrum exponential warp is the localizing metric function and the equation of state of the conformal fields is not changed by the radion perturbation then the 5-dimensional solutions are unstable. We present new stable solutions which describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter.

1 Introduction

In the Randall-Sundrum (RS) scenario [1, 2] the observable Universe is a 3-brane world of a Z_2 symmetric 5-dimensional anti-de Sitter (AdS) space. In the RS1 model [1] the AdS orbifold has a compactified fifth dimension and two brane boundaries. The gravitational field is bound to the hidden positive tension brane and decays towards the visible negative tension brane. In this model the hierarchy problem is reformulated as an exponential hierarchy between the weak and Planck scales [1]. In the RS2 model [2] the AdS orbifold is non-compact with an infinite fifth dimension and a single positive tension brane. Gravity is localized on the positive tension brane now interpreted as the visible brane.

At low energies the theory of gravity on the observable brane is 4-dimensional general relativity and the cosmology may be Friedmann-Robertson-Walker [1]-[10]. In the RS1 model this is only possible if the radion mode is stabilized and this as been achieved using a scalar field in the bulk [3, 6, 9, 10]. The gravitational collapse of matter was also analyzed in the RS scenario [11]-[16]. Using an extended black string solution (first discussed in a different context by Myers and Perry [17]) Chamblin, Hawking and Reall showed that it was possible to induce on the brane the Schwarzschild black hole metric [11]. However, this solution is divergent at the AdS horizon and at the black string singularity. Consequently, it is expected to be unstable [11, 18]. A black cylinder localized near the brane which is free from naked singularities was conjectured to be its stable decay product. This exact 5-dimensional solution has not yet been found. The only known static black holes localized on a brane remain to be those found for a 2-brane in a 4-dimensional AdS space [12]. The problem lies in the simultaneous non-singular localization of gravity and matter in the vicinity of the brane [11], [13]-[16]. This has lead to another conjecture stating that $D+1$-dimensional black hole solutions localized on a $D-1$-brane should correspond to quantum corrected D-dimensional black holes on the brane [15]. This is an extra motivation to look for 5-dimensional collapse solutions localized on a brane. In addition, the effective covariant Gauss-Codazzi approach [19, 20] has permitted the discovery of many braneworld solutions which have not yet been associated with exact 5-dimensional spacetimes [21]-[24].

In this paper we continue the research on the dynamics of a spherically symmetric RS 3-brane when the bulk is filled with conformal matter fields [16, 25] (see also [26]). In our previous work [16, 25] we have found a new class of exact 5-dimensional dynamical solutions for which gravity is localized near the brane by the exponential RS warp. These solutions were shown to be associated with conformal bulk fields characterized by a stress-energy tensor $\tilde{T}_\mu^\nu$ of weight -4 and by the equation of state $\tilde{T}_a^a = 2\tilde{T}_5^5$. They were also shown to describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter. However, the density and pressures of the conformal bulk fluid increase with the coordinate of the fifth dimension. In the RS2 model this generates a divergence at the AdS horizon as in the Schwarzschild black string solution. In the RS1 model this is not a difficulty because the space is cut before the AdS horizon is reached. However, the solutions must be stable under radion field perturbations. In this paper we analyze this problem using a saddle point expansion procedure based on the action [28, 29]. In section 2 we present a brief review of the conformal analysis performed on the 5-dimensional Einstein equations which leads to the set of braneworld backgrounds supported by the class of conformal bulk matter fields with weight -4. In section 3 we consider a dimensional reduction on the RS action and determine the radion effective potential. In section 4 we identify our braneworld solutions as its extrema and show that the exponential RS warp and a radion independent equation of state for the conformal bulk fields lead to unstable solutions. In section 5 we present our conclusions and a new class of stable braneworld solutions which describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter.

2 Einstein Equations and Conformal Bulk Fields

Let (t, r, θ, ϕ, z) be a set of comoving coordinates in the RS orbifold. The most general metric consistent with the Z_2 symmetry in z and with 4-dimensional spherical symmetry on the brane may be written as

$$d\tilde{s}^2 = \Omega^2 \left(-e^{2A}dt^2 + e^{2B}dr^2 + R^2 d\Omega_2^2 + dz^2\right), \tag{1}$$

where the metric functions $\Omega = \Omega(t, r, z)$, $A = A(t, r, z)$, $B = B(t, r, z)$ and $R = R(t, r, z)$ are Z_2 symmetric. Ω is the warp factor and R the physical radius of the 2-spheres.

When the 5-dimensional space is filled with bulk matter fields characterized by a lagrangian $\tilde{L}_B$ the dynamical RS action is given by

$$\tilde{S} = \int d^4x dz \sqrt{-\tilde{g}} \left\{ \frac{\tilde{R}}{2\kappa_5^2} - \Lambda_B - \frac{1}{\sqrt{\tilde{g}_{55}}} \left[\lambda \delta\left(z - z_0\right) + \lambda' \delta\left(z - {z'}_0\right)\right] + \tilde{L}_B \right\}. \tag{2}$$

The Planck brane with tension λ is assumed to be located at $z = z_0$ and the visible brane with tension λ' at ${z'}_0$. Λ_B is the negative bulk cosmological constant. A Noether variation on the action (2) gives the Einstein field equations

$$\tilde{G}_\mu^\nu = -\kappa_5^2 \left\{ \Lambda_B \delta_\mu^\nu + \frac{1}{\sqrt{\tilde{g}_{55}}} \left[\lambda \delta\left(z - z_0\right) + \lambda' \delta\left(z - {z'}_0\right)\right] \left(\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5\right) - \tilde{T}_\mu^\nu \right\}, \tag{3}$$

where the stress-energy tensor associated with the bulk fields is defined by

$$\tilde{T}_\mu^\nu = \tilde{L}_B \delta_\mu^\nu - 2\frac{\delta \tilde{L}_B}{\delta \tilde{g}^{\mu\alpha}} \tilde{g}^{\alpha\nu} \tag{4}$$

and is conserved in the bulk, $\tilde{\nabla}_\mu \tilde{T}_\nu^\mu = 0$.

To find exact solutions of the 5-dimensional Einstein equations (3) we need simplifying assumptions. Let us first consider that under the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ the bulk stress-energy tensor has conformal weight s, $\tilde{T}_\mu^\nu = \Omega^{s+2} T_\mu^\nu$. Then Eq. (3) may be re-written as

$$\begin{aligned} G_\mu^\nu = &-6\Omega^{-2} \left(\nabla_\mu \Omega\right) g^{\nu\rho} \nabla_\rho \Omega + 3\Omega^{-1} g^{\nu\rho} \nabla_\rho \nabla_\mu \Omega - 3\Omega^{-1} \delta_\mu^\nu g^{\rho\sigma} \nabla_\rho \nabla_\sigma \Omega \\ &- \kappa_5^2 \Omega^2 \left\{ \Lambda_B \delta_\mu^\nu + \Omega^{-1} \left[\lambda \delta(z - z_0) + \lambda' \delta(z - {z'}_0)\right] \left(\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5\right) - \Omega^{s+2} T_\mu^\nu \right\}. \end{aligned} \tag{5}$$

Similarly under the conformal transformation the conservation equation becomes

$$\nabla_\mu T_\nu^\mu + \Omega^{-1} \left[(s+7) T_\nu^\mu \partial_\mu \Omega - T_\mu^\mu \partial_\nu \Omega\right] = 0. \tag{6}$$

If in addition it is assumed that $\tilde{T}_\mu^\nu = \Omega^{-2} T_\mu^\nu$ then equation (5) may be separated in the following way

$$G_\mu^\nu = \kappa_5^2 T_\mu^\nu, \tag{7}$$

$$6\Omega^{-2}\nabla_\mu\Omega\nabla_\rho\Omega g^{\rho\nu} - 3\Omega^{-1}\nabla_\mu\nabla_\rho\Omega g^{\rho\nu} + 3\Omega^{-1}\nabla_\rho\nabla_\sigma\Omega g^{\rho\sigma}\delta_\mu^\nu =$$
$$-\kappa_5^2\Omega^2\left\{\Lambda_B\delta_\mu^\nu + \Omega^{-1}\left[\lambda\delta(z-z_0)+\lambda'\delta(z-{z'}_0)\right]\left(\delta_\mu^\nu - \delta_5^\nu\delta_\mu^5\right)\right\}. \tag{8}$$

Because of the Bianchi identity we must also have

$$\nabla_\mu T_\nu^\mu = 0, \tag{9}$$

$$3T_\nu^\mu\partial_\mu\Omega - T\partial_\nu\Omega = 0. \tag{10}$$

Equations (7) and (9) are 5-dimensional Einstein equations with matter fields present in the bulk but without a brane or bulk cosmological constant. They do not depend on the conformal warp factor which is dynamically defined by equations (8) and (10). Consequently, only the warp reflects the existence of the brane or of the bulk cosmological constant. Note that this is only possible for the special class of conformal bulk fields which have a stress-energy tensor with weight -4.

Because the equations still depend non-linearly on the metric functions A, B and R let us further assume that $A = A(t,r)$, $B = B(t,r)$, $R = R(t,r)$ and $\Omega = \Omega(z)$. Then we obtain

$$G_a^b = \kappa_5^2 T_a^b, \quad \nabla_a T_b^a = 0, \tag{11}$$

$$G_5^5 = \kappa_5^2 T_5^5, \tag{12}$$

$$6\Omega^{-2}(\partial_z\Omega)^2 = -\kappa_5^2\Omega^2\Lambda_B, \tag{13}$$

$$3\Omega^{-1}\partial_z^2\Omega = -\kappa_5^2\Omega^2\left\{\Lambda_B + \Omega^{-1}\left[\lambda\delta(z-z_0)+\lambda'\delta(z-{z'}_0)\right]\right\} \tag{14}$$

and (see also [5] and [27])

$$2T_5^5 = T_c^c, \tag{15}$$

where the latin indices represent the coordinates t, r, θ and ϕ. Our braneworld geometries [16, 25] are solutions of equations (11)-(15) when the stress-tensor is diagonal,

$$T_\mu^\nu = diag\left(-\rho, p_r, p_T, p_T, p_5\right), \tag{16}$$

where ρ, p_r, p_T and p_5 denote the bulk matter density and pressures, and Ω is the exponential RS warp,

$$\Omega_{\rm RS}(z) = \frac{l}{|z-z_0|+z_0}, \tag{17}$$

where $z_0 = l$ and l is the AdS radius given by $l = 1/\sqrt{-\Lambda_B\kappa_5^2/6}$ with $\kappa_5^2 = 8\pi/M_5^3$ defined by the fundamental 5-dimensional Planck mass M_5. Then the Planck brane is located at $z_0 = l$ and the observable brane is located at ${z'}_0 = le^{\pi r_c/l}$ where r_c is the RS compactification scale [1]. The former has a positive tension λ and the latter a negative tension $\lambda' = -\lambda$ where $\lambda = -\Lambda_B l$. They are twin Universes with identical collapse or cosmological dynamics.

3 The Radion Potential

To analyze the behaviour of these solutions under radion field perturbations we apply a saddle point expansion procedure based on the action [28, 29]. As a starting point this requires the determination of the radion effective potential. For the calculation it is convinient to work with the coordinate y related to z by $z = le^{y/l}$ for $y > 0$. Let us write the most general metric consistent with the Z_2 symmetry in y and with 4-dimensional spherical symmetry on the brane in the form

$$d\tilde{s}^2 = a^2 ds_4^2 + b^2 dy^2, \quad ds_4^2 = -dt^2 + e^{2B} dr^2 + R^2 d\Omega_2^2, \tag{18}$$

where the metric functions $a = a(t, r, y)$, $B = B(t, r, y)$, $R = R(t, r, y)$ and $b = b(t, r, y)$ are Z_2 symmetric. Now a is the warp factor, R is still the physical radius of the 2-spheres and b is related to the radion field.

The 5-dimensional dynamical RS action is now given by

$$\tilde{S} = \int d^4x dy \sqrt{-\tilde{g}} \left\{ \frac{\tilde{R}}{2\kappa_5^2} - \Lambda_B - \frac{1}{\sqrt{\tilde{g}_{55}}} \left[\lambda \delta(y) + \lambda' \delta(y - \pi r_c) \right] + \tilde{L}_B \right\}. \tag{19}$$

In the new coordinates the Planck brane is located at $y = 0$ and the visible brane at πr_c. Our braneworld backgrounds correspond to the metric functions $b = 1$, $B = B(t, r)$, $R = R(t, r)$ and $a = \Omega_{\rm RS}(y)$ where

$$\Omega_{\rm RS}(y) = e^{-|y|/l}. \tag{20}$$

To calculate the radion potential we consider the dimensional reduction of the action (19). Using the metric (18) we obtain $\sqrt{-\tilde{g}} = a^4 b \sqrt{-g_4}$ and

$$\begin{aligned} \tilde{R} = \frac{1}{a^2} &\left(R_4 - \frac{6}{a} g_4^{cd} \nabla_c \nabla_d a - \frac{2}{b} g_4^{cd} \nabla_c \nabla_d b - \frac{4}{ab} g_4^{cd} \nabla_c a \nabla_d b \right) \\ &- \frac{4}{b^2} \left[3 \left(\frac{\partial_y a}{a} \right)^2 + 2 \frac{\partial_y^2 a}{a} \right], \end{aligned} \tag{21}$$

where g_4^{cd} is the inverse metric associated with the 4-dimensional line element ds_4^2, R_4 is the 4-dimensional Ricci scalar and the covariant derivatives are 4-dimensional. Consider the particular metric setting defined by $a = \Omega e^{-\beta}$ and $b = e^{\beta}$ where $\Omega = \Omega(y)$ and $\beta = \beta(t, r)$. Then in the Einstein frame the dimensional reduction leads to

$$\tilde{S} = \int d^4x \sqrt{-g_4} \left(\frac{R_4}{2\kappa_4^2} - \frac{1}{2} \nabla_c \gamma \nabla_d \gamma g_4^{cd} - \tilde{V} \right), \tag{22}$$

where $\gamma = \beta / (\kappa_4 \sqrt{2/3})$ is the canonically normalized radion field. The function $\tilde{V} = \tilde{V}(\gamma)$ is the radion potential and it may be written in the form

$$\tilde{V} = \frac{2}{\kappa_5^2}\chi^3 \left[3\int dy\Omega^2(\partial_y\Omega)^2 + 2\int dy\Omega^3\partial_y^2\Omega\right] + \chi\int dy\Omega^4\left(\Lambda_B - \tilde{L}_B\right)$$
$$+ \chi^2\int dy\Omega^4\left[\lambda\delta\left(y\right) + \lambda'\delta\left(y - \pi r_c\right)\right], \tag{23}$$

where the field χ is defined as $\chi = e^{-\sqrt{(2/3)}\,\kappa_4\gamma}$. Note that the integration in the fifth dimension is performed in the interval $[-\pi r_c, \pi r_c]$ and that we have chosen

$$\int dy\Omega^2 = \frac{\kappa_5^2}{\kappa_4^2}. \tag{24}$$

4 The Radion Field Instability

To analyze the stability of our braneworld solutions we consider a saddle point expansion of the radion field potential V [28, 29]. This procedure requires the determination of its first and second variations. To do so we consider the integral of the radion potential written in the form

$$\tilde{\mathcal{V}} = \int d^4x\sqrt{-g_4}\tilde{V}(\gamma) = \int d^5x\sqrt{-\tilde{g}}\left\{\frac{2}{\kappa_5^2}\chi^2\left[3\left(\frac{\partial_y\Omega}{\Omega}\right)^2 + 2\frac{\partial_y^2\Omega}{\Omega}\right]\right\}$$
$$+ \int d^5x\sqrt{-\tilde{g}}\left\{\Lambda_B - \tilde{L}_B + \chi\left[\lambda\delta\left(y\right) + \lambda'\delta\left(y - \pi r_c\right)\right]\right\}. \tag{25}$$

Taking into account that an integration by parts and the Z_2 symmetry leads to

$$\int dy\Omega^2(\partial_y\Omega)^2 = -\frac{1}{3}\int dy\Omega^3\partial_y^2\Omega \tag{26}$$

we find that the first variation of the integral of the radion potential is given by

$$\frac{\delta\tilde{\mathcal{V}}}{\delta\gamma} = -\sqrt{\frac{8}{3}}\,\kappa_4\int d^5x\sqrt{-\tilde{g}}\chi\left[\lambda\delta\left(y\right) + \lambda'\delta\left(y - \pi r_c\right)\right] - \sqrt{\frac{2}{3}}\,\kappa_4\int d^5x\sqrt{-\tilde{g}}\Lambda_B$$
$$+ \frac{\kappa_4}{\sqrt{6}}\int d^5x\sqrt{-\tilde{g}}\left(\tilde{T}_a^a - 2\tilde{T}_5^5 - \frac{12}{\kappa_5^2}\chi^2\frac{\partial_y^2\Omega}{\Omega}\right). \tag{27}$$

For our solutions Ω is taken to be the exponential RS warp factor $\Omega_{\rm RS}$ and satisfies the following warp equation in the y coordinate

$$-\frac{6}{\kappa_5^2}\frac{\partial_y^2\Omega}{\Omega} = \Lambda_B + 2\left[\lambda\delta\left(y\right) + \lambda'\delta\left(y - \pi r_c\right)\right]. \tag{28}$$

On the other hand the bulk matter fields which must have a stress-energy tensor of conformal weight -4 must also obey the equation of state (15). Integrating in the fifth dimension with $\Omega = \Omega_{\rm RS}$ we find

$$\frac{\delta \tilde{V}}{\delta \gamma} = \frac{\kappa_4}{\sqrt{6}} \Lambda_B l \left(1 - e^{-4\pi r_c/l}\right) \chi \left(4\chi - 1 - 3\chi^2\right). \tag{29}$$

The critical extrema of the radion potential are the non-zero finite roots of the polynomial equation $\chi(3\chi^2 - 4\chi + 1) = 0$. They are $\chi_1 = 1$ and $\chi_2 = 1/3$. Our braneworld solutions correspond to the first root $\chi_1 = 1$. The same happens if the bulk matter is absent as in the RS vaccum solutions. The other extremum is not a solution of the Einstein equations for the RS1 model. Indeed, a rescaling of the coordinates shows that the point $\chi_2 = 1/3$ corresponds to an exponential warp $\Omega = e^{-|y|/(3l)}$ which is different from Ω_{RS} and does not satisfy the RS warp equations (28) and

$$6\left(\frac{\partial_y \Omega}{\Omega}\right)^2 = -\kappa_5^2 \Lambda_B. \tag{30}$$

The stability of the extrema depends on the sign of the second variation of the radion potential. This variation defines the radion mass. Consequently, stable background solutions must be associated with a positive sign. Consider the first variation of the 4-dimensional integral of V given in Eq. (27). If the equation of state (15) of the conformal bulk fields is independent of the radion perturbation and $\Omega = \Omega_{\mathrm{RS}}$ we find

$$\frac{\delta^2 V}{\delta \gamma^2} = \frac{\kappa_4^2}{3} \Lambda_B l \left(1 - e^{-4\pi r_c/l}\right) \chi \left(9\chi^2 - 8\chi + 1\right). \tag{31}$$

For $\chi = \chi_1 = 1$ we conclude that the second variation of V is negative. This implies that the radion is a negative mass tachyon and that our braneworld solutions are unstable.

5 Conclusions

In this paper we have considered the set of exact 5-dimensional dynamical solutions with gravity localized near the brane which are associated with conformal bulk fields of weight -4 and which describe the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter on the brane. We have studied their behviour under radion field perturbations. We have shown that these solutions are extrema of the radion potential. We have also shown that if the metric function responsible for the localization of gravity is the exponential RS warp and the equation of state characterizing the conformal bulk fluid is independent of the radion field then the braneworld solutions are unstable. We have also found that the radion potential has another extremum which is not a solution of the complete set of Einstein equations. This point is connected with a different warp factor and its existence suggests that stable braneworlds associated with the same state of the conformal bulk matter should correspond to warp functions other than the standard exponential RS warp. This is indeed true. Stable solutions are defined by a new set of warp functions given by

$$\Omega(y) = e^{-|y|/l}\left(1 + \frac{p_5^s}{4\Lambda_B} e^{2|y|/l}\right), \tag{32}$$

where $p_5^s < 0$ is a negative constant fraction of the 5-dimensional pressure of the conformal bulk fields of weight -4. On the brane these solutions also describe the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter. More details will be presented in a forthcoming publication [30].

Acknowledgements

We are grateful for financial support from *Fundação para a Ciência e a Tecnologia* (FCT) and *Fundo Social Europeu* (FSE) under the contracts SFRH/BPD/7182/2001 and POCTI/32694/FIS/2000 (*III Quadro Comunitário de Apoio*) as well as from *Centro Multidisciplinar de Astrofísica* (CENTRA). We would also like to thank Cenalo Vaz for enlighting discussions and to the Physics Department of the University of Cincinnati for kind hospitality during a visit where part of this work was completed.

References

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).

[3] W. D. Goldberger and M. B. Wise, Phys. Rev. D. **60**, 107505 (1999); Phys. Rev. Lett. **83**, 4922 (1999); Phys. Lett. B **475**, 275 (2000).

[4] N. Kaloper, Phys. Rev. D **60**, 123506 (1999);
T. Nihei, Phys. Lett. B **465**, 81 (1999);
C. Csáki, M. Graesser, C. Kolda and J. Terning, Phys. Lett. B **462**, 34 (1999);
J. M. Cline, C. Grojean and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999).

[5] P. Kanti, I. I. Kogan, K. A. Olive and M. Pospelov, Phys. Lett. B **468**, 31 (1999); Phys. Rev. D **61**, 106004 (2000).

[6] O. DeWolf, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D **62**, 046008 (2000).

[7] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000).

[8] S. Giddings, E. Katz and L. Randall, J. High Energy Phys. **03**, 023 (2000).

[9] C. Csáki, M. Graesser, L. Randall and J. Terning, Phys. Rev. D **62**, 045015 (2000).

[10] T. Tanaka and X. Montes, Nucl. Phys. **B582**, 259 (2000).

[11] A. Chamblin, S. W. Hawking and H. S. Reall, Phys. Rev. D **61**, 065007 (2000).

[12] R. Emparan, G. T. Horowitz and R. C. Myers, J. High Energy Phys. **0001**, 007 (2000); J. High Energy Phys. **0001**, 021 (2000).

[13] P. Kanti, K. A. Olive and M. Pospelov, Phys. Lett. B **481**, 386 (2000);
T. Shiromizu and M. Shibata, Phys. Rev. D **62**, 127502 (2000);
H. Lü and C. N. Pope, Nucl. Phys. **B598**, 492 (2001);
A. Chamblin, H. R. Reall, H. a. Shinkai and T. Shiromizu, Phys. Rev. D **63**, 064015 (2001);
M. S. Modgil, S. Panda and G. Sengupta, Mod. Phys. Lett. A **17**, 1479 (2002).

[14] P. Kanti and K. Tamvakis, Phys. Rev. D **65**, 084010 (2002);
P. Kanti, I. Olasagasti and K. Tamvakis, Phys. Rev. D **68**, 124001 (2003).

[15] R. Emparan, A. Fabbri and N. Kaloper, J. High Energy Phys. **0208**, 043 (2002);
R. Emparan, J. Garcia-Bellido and N. Kaloper, J. High Energy Phys. **0301**, 079 (2003).

[16] R. Neves and C. Vaz, Phys. Rev. D **68**, 024007 (2003).

[17] R. C. Myers and M. J. Perry, Ann. Phys. **172**, 304 (1986).

[18] R. Gregory and R. Laflamme, Phys. Rev. Lett. **70**, 2837 (1993); Nucl. Phys. **B428**, 399 (1994).

[19] B. Carter, Phys. Rev. D **48**, 4835 (1993);
R. Capovilla and J. Guven, Phys. Rev. D **51**, 6736 (1995); Phys. Rev. D **52**, 1072 (1995).

[20] T. Shiromizu, K. I. Maeda and M. Sasaki, Phys. Rev. D **62**, 024012 (2000);
M. Sasaki, T. Shiromizu and K. I. Maeda, Phys. Rev. D **62**, 024008 (2000).

[21] J. Garriga and M. Sasaki, Phys. Rev. D **62**, 043523 (2000);
R. Maartens, D. Wands, B. A. Bassett and I. P. C. Heard, Phys. Rev. D **62**, 041301 (2000);
H. Kodama, A. Ishibashi and O. Seto, Phys. Rev. D 62, 064022 (2000);
D. Langlois, Phys. Rev. D **62**, 126012 (2000);
C. van de Bruck, M. Dorca, R. H. Brandenberger and A. Lukas, Phys. Rev. D **62**, 123515 (2000);
K. Koyama and J. Soda, Phys. Rev. D **62**, 123502 (2000);
D. Langlois, R. Maartens, M. Sasaki and D. Wands, Phys. Rev. D **63**, 084009 (2001).

[22] N. Dadhich, R. Maartens, P. Papadopoulos and V. Rezania, Phys. Lett. B **487**, 1 (2000);
N. Dadhich and S. G. Ghosh, Phys. Lett. B **518**, 1 (2001);
C. Germani and R. Maartens, Phys. Rev. D **64**, 124010 (2001);
M. Bruni, C. Germani and R. Maartens, Phys. Rev. Lett. **87**, 231302 (2001)M. Govender and N. Dadhich, Phys. Lett. B **538**, 233 (2002);
R. Casadio, A. Fabbri and L. Mazzacurati, Phys. Rev. D **65**, 084040 (2002).

[23] R. Maartens, Phys. Rev. D **62**, 084023 (2000).

[24] R. Neves and C. Vaz, Phys. Rev. D **66**, 124002 (2002); in Varying Fundamental Constants (VFC): Proceedings of the Workshop, JENAM 2002, The Unsolved Universe: Challenges for the Future, (Kluwer Academic Publishers, Dordrecht, 2003) p. 99, Astrophys. Space Sci. **283** (2003) 537; in New Worlds in Astroparticle Physics: Proceedings of the Fourth International Workshop, edited by A. Krasnitz, A. M. Mourão, M. Pimenta e R. Potting, (World Scientific, Singapore, 2003) p. 82.

[25] R. Neves and C. Vaz, Phys. Lett. B **568**, 153 (2003); in Beyond the Desert 2003, Proceedings of the Fourth Tegernsee International Conference on Particle Physics Beyond the Standard Model, edited by H.-V. Klapdor-Kleingrothaus, (Springer Verlag, Heidelberg, 2004) p. 671.

[26] E. Elizalde, S. D. Odintsov, S. Nojiri and S. Ogushi, Phys. Rev. D **67**, 063515 (2003);
S. Nojiri and S. D. Odintsov, JCAP **06**, 004 (2003).

[27] I. Z. Rothstein, Phys. Rev. D **64**, 084024 (2001).

[28] R. Hofmann, P. Kanti and M. Pospelov, Phys. Rev. D **63**, 124020 (2001);
P. Kanti, K. A. Olive and M. Pospelov, Phys. Lett. B **538**, 146 (2002).

[29] S. M. Carroll, J. Geddes, M. B. Hoffman and R. M. Wald, Phys. Rev. D **66**, 024036 (2002).

[30] R. Neves and C. Vaz, in preparation.