Conformal Bulk Fields, Dark Energy and Brane Dynamics

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Abstract

In the Randall-Sundrum scenario we analyze the dynamics of a spherically symmetric 3-brane when the bulk is filled with matter fields. Considering a global conformal transformation whose factor is the Z_2 symmetric warp we find a new set of exact dynamical solutions for which gravity is bound to the brane. The set corresponds to a certain class of conformal bulk fields. We discuss the geometries which describe the dynamics on the brane of polytropic dark energy.

1 Introduction

In the Randall-Sundrum (RS) brane world scenario the visible Universe is a 3-brane boundary of a Z_2 symmetric 5-dimensional anti-de Sitter (AdS) orbifold [1, 2]. There are two basic settings. On one hand the RS1 model [1] with two branes and a compactified fifth dimension and on the other the RS2 model [2] with a single brane which may be associated with an infinite fifth dimension. With two branes the hierarchy problem is reformulated introducing an exponential warp in the fifth dimension. The gravitational field is localized on the hidden positive tension brane and decays towards the visible negative tension brane thus producing an exponential hierarchy between the Planck and weak energy scales. In the RS2 model the same warping of the fifth dimension ensures that the graviton is bound to the brane.

In the RS models the classical field dynamics is defined by 5-dimensional Einstein equations with a negative bulk cosmological constant Λ_B, Dirac delta sources representing the branes and a stress-energy tensor describing other bulk field modes [1]-[3]. In the RS2 model a set of vaccum solutions is given by

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$$d\tilde{s}_5^2 = dy^2 + e^{-2|y|/l} ds_4^2 \ , \tag{1}$$

where the 4-dimensional line element ds_4^2 is Ricci flat, l is the AdS radius given by $l = 1/\sqrt{-\Lambda_B \kappa_5^2/6}$ with $\kappa_5^2 = 8\pi/M_5^3$ and M_5 the fundamental 5-dimensional Planck mass. The brane cosmological constant is fine-tuned to be zero giving $\Lambda_B = -\kappa_5^2 \lambda^2/6$ where $\lambda > 0$ denotes the brane tension. Due to the periodicity and the Z_2 symmetry of the RS orbifold these solutions also hold for the RS1 model. Then the two branes have opposite tensions and $\lambda > 0$ is the tension of the hidden Planck brane.

The low energy theory of gravity on the brane is 4-dimensional general relativity and the cosmology may be Friedmann-Robertson-Walker [1]-[10]. However, it should be noted that in the RS1 model this has only been achieved if a scalar field is introduced in the bulk to stabilize the size of the fifth dimension [3, 6, 9, 10]. The problem of the gravitational collapse of matter was also investigated in the RS scenario [11]-[16]. It was found that a black string solution first discussed in a different context by Myers and Perry [17] induced the Schwarzschild metric on the brane [11]. However for such a solution the Kretschmann scalar diverged both at the AdS horizon and at the black string singularity [11]. The solution is thus expected to be unstable [11, 18]. It may decay to a black cylinder localized near the brane which is free from naked singularities [11]. So far this continues to be a conjecture. Indeed, while exact solutions interpreted as static black holes localized on a brane have been found for a 2-brane embedded in a 4-dimensional AdS space [12], a static black hole localized on a 3-brane remains unknown. The difficulty lies in the simultaneous localization of gravity and matter near the brane without the creation of naked singularities in the bulk [11], [13]-[16]. This has inspired another conjecture stating that $D+1$-dimensional black hole solutions localized on a $D-1$-brane should correspond to quantum corrected D-dimensional black holes on the brane [15]. Related to the AdS/CFT correspondence [19] this connection provides an extra motivation to look for 5-dimensional collapse solutions localized on a brane. There are also many braneworld solutions, obained from the effective 4-dimensional point of view by the application of the covariant Gauss-Codazzi formulation [20, 21], that have so far not been associated with exact 5-dimensional spacetimes [22]-[25].

In this proceedings we report on research about the dynamics of a spherically symmetric 3-brane when conformal fields are present in the bulk [16, 26] (see also [27]). We focus on 5-dimensional solutions which describe the dynamics of polytropic dark energy on the brane [26].

2 Einstein Equations

Let us start by introducing (t, r, θ, ϕ, z) as coordinates in the 5-dimensional bulk. The most general metric consistent with the Z_2 symmetry in z and with 4-dimensional spherical symmetry on the brane is given by

$$d\tilde{s}_5^2 = \Omega^2 \left(-e^{2A}dt^2 + e^{2B}dr^2 + R^2 d\Omega_2^2 + dz^2 \right) \ , \tag{2}$$

where $\Omega = \Omega(t, r, z)$, $A = A(t, r, z)$, $B = B(t, r, z)$ and $R = R(t, r, z)$ are Z_2 symmetric functions. Ω is the warp factor and R the physical radius of the 2-spheres. With a single brane the classical dynamics is defined by

$$\tilde{G}_\mu^\nu = -\kappa_5^2 \left[\Lambda_B \delta_\mu^\nu + \frac{\lambda}{\sqrt{\tilde{g}_{55}}} \delta\left(z - z_0\right) \left(\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5 \right) - \tilde{T}_\mu^\nu \right] \ , \tag{3}$$

where the bulk stress-energy tensor is conserved in the bulk

$$\tilde{\nabla}_\mu \tilde{T}_\nu^\mu = 0 \ . \tag{4}$$

For a general 5-dimensional metric $\tilde{g}_{\mu\nu}$ the Einstein equations (3) are extremely complex. To be able to solve them we need simplifying assumptions about the field variables involved in the problem. Let us first assume that under the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ the bulk stress-energy tensor has conformal weight s,

$$\tilde{T}_\mu^\nu = \Omega^{s+2} T_\mu^\nu \ . \tag{5}$$

Then Eq. (3) may be re-written as

$$\begin{aligned} G_\mu^\nu \ = \ & -6\Omega^{-2} \left(\nabla_\mu \Omega\right) g^{\nu\rho} \nabla_\rho \Omega + 3\Omega^{-1} g^{\nu\rho} \nabla_\rho \nabla_\mu \Omega - 3\Omega^{-1} \delta_\mu^\nu g^{\rho\sigma} \nabla_\rho \nabla_\sigma \Omega \\ & -\kappa_5^2 \Omega^2 \left[\Lambda_B \delta_\mu^\nu + \lambda \Omega^{-1} \delta(z - z_0) \left(\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5 \right) - \Omega^{s+2} T_\mu^\nu \right] \ . \end{aligned} \tag{6}$$

Similarly under the conformal transformation the conservation equation becomes

$$\nabla_\mu T_\nu^\mu + \Omega^{-1} \left[(s+7) T_\nu^\mu \partial_\mu \Omega - T_\mu^\mu \partial_\nu \Omega \right] = 0 \ . \tag{7}$$

If in addition it is assumed that $\tilde{T}_\mu^\nu = \Omega^{-2} T_\mu^\nu$ then Eq. (6) may be separated in the following way

$$G_\mu^\nu = \kappa_5^2 T_\mu^\nu \ , \tag{8}$$

$$\begin{aligned} & 6\Omega^{-2} \nabla_\mu \Omega \nabla_\rho \Omega g^{\rho\nu} - 3\Omega^{-1} \nabla_\mu \nabla_\rho \Omega g^{\rho\nu} + 3\Omega^{-1} \nabla_\rho \nabla_\sigma \Omega g^{\rho\sigma} \delta_\mu^\nu = \\ & \quad -\kappa_5^2 \Omega^2 \left[\Lambda_B \delta_\mu^\nu + \lambda \Omega^{-1} \delta(z - z_0) \left(\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5 \right) \right] \ . \end{aligned} \tag{9}$$

Because of the Bianchi identity we must also have

$$\nabla_\mu T_\nu^\mu = 0 \ , \tag{10}$$

$$3 T_\nu^\mu \partial_\mu \Omega - T \partial_\nu \Omega = 0 \ . \tag{11}$$

Note that Eqs. (8) and (10) are 5-dimensional Einstein equations with matter fields present in the bulk but without a brane or bulk cosmological constant. They do not depend on the conformal warp factor which is dynamically defined by Eqs. (9) and (11). The warp is then the only effect reflecting the existence of the brane or of the bulk cosmological constant. We stress that this is only possible for the special class of bulk fields which have a stress-energy tensor with conformal weight $s = -4$.

Thought now partially decoupled the 5-dimensional Einstein equations are still difficult to solve. Note for instance that the warp depends non-linearly on the metric functions A, B and R. So let us further assume that $A = A(t, r)$, $B = B(t, r)$, $R = R(t, r)$ and $\Omega = \Omega(z)$. Then we obtain

$$G_a^b = \kappa_5^2 T_a^b, \quad \nabla_a T_b^a = 0 \ , \tag{12}$$

$$G_z^z = \kappa_5^2 T_z^z \ , \tag{13}$$

$$6\Omega^{-2}(\partial_z \Omega)^2 = -\kappa_5^2 \Omega^2 \Lambda_B \ , \tag{14}$$

$$3\Omega^{-1}\partial_z^2 \Omega = -\kappa_5^2 \Omega^2 \left[\Lambda_B + \lambda \Omega^{-1} \delta(z - z_0)\right] \tag{15}$$

and [5, 28]

$$2T_z^z = T_c^c \ , \tag{16}$$

where the latin indices represent the coordinates t, r, θ and ϕ.

The warp is now independent of A, B and R. It may be chosen to be the RS factor [1, 2, 11]

$$\Omega = \Omega_{\rm RS} \equiv \frac{l}{|z - z_0| + z_0} \ . \tag{17}$$

Naturally, other warp factors depending only on z such as those of thick branes [29] and non-fine-tuned branes [30] may also be considered (see [14]). On the other hand the highly coupled 5-dimensional collapse dynamics has been reduced to 4-dimensional dynamics. Consider a diagonal stress-tensor,

$$T_\mu^\nu = diag\left(-\rho, p_r, p_T, p_T, p_z\right) \ , \tag{18}$$

where ρ, p_r, p_T and p_z denote the bulk matter density and pressures. Then Eq. (16) is re-written as

$$\rho - p_r - 2p_T + 2p_z = 0 \ . \tag{19}$$

The collapse of the conformal bulk matter is in general inhomogeneous and defined by Eq. (12). The matter dynamics generates a pressure p_z along the fifth dimension which must consistently be given by Eqs. (13) and (19).

3 Polytropic Dark Energy

To behave as polytropic dark energy the stress-energy tensor should be of the form (18) where the bulk matter density ρ and pressures p_r, p_T and p_z are given by

$$\rho = \rho_{\rm P},\ p_r = -\eta {\rho_{\rm P}}^{\alpha},\ p_T = p_r,\ p_z = -\frac{1}{2}\left(\rho_{\rm P} + 3\eta {\rho_{\rm P}}^{\alpha}\right) \ . \tag{20}$$

Above $\rho_{\rm P}$ defines the polytropic dark energy density and the parameters $(\alpha,\, \eta)$ characterize different polytropic phases. In what follows we restrict our attention to the generalized Chaplygin phase characterized by $-1 \le \alpha < 0$ [16, 31, 32].

The polytropic energy density $\rho_{\rm P}$ is obtained by solving the conservation equations in Eq. (12) which in this case reads

$$\dot{\rho_{\rm P}} + \left(\dot{B} + 2\frac{\dot{R}}{R}\right)\left(\rho_{\rm P} - \eta {\rho_{\rm P}}^{\alpha}\right) = 0,\ A'\left(\rho_{\rm P} - \eta {\rho_{\rm P}}^{\alpha}\right) - \eta\alpha {\rho_{\rm P}}^{\alpha-1}{\rho_{\rm P}}' = 0 \ . \tag{21}$$

Taking an homogeneous density $\rho_{\rm P} = \rho_{\rm P}(t)$, the metric function $A(t, r)$ may be safely set to zero. Then since the off-diagonal Einstein equation $G_t^r = 0$ has solution $e^B = R'/H$ with $H = H(r)$ an arbitrary function of r we obtain

$$\rho_{\rm P} = \left(\eta + \frac{a}{S^{3-3\alpha}}\right)^{\frac{1}{1-\alpha}} , \tag{22}$$

where a is an integration constant and $S = S(t)$ is the Robertson-Walker scale factor of the brane world which is related to the physical radius by $R = rS$.

At small S the Chaplygin dynamics is dominated by the homogeneous dust phase with $\rho_{\rm P} = a^{1/(1-\alpha)}/S^3$. The Chaplygin gas has an intermediate phase defined by the equation of state $p_{\rm P} = -\alpha\rho_{\rm P}$ which satisfies the dominant energy condition. For large S the dynamics is dominated by an effective cosmological constant term. An evolution of the Chaplygin equation of state may thus describe the change in the dark energy behavior during the expansion of the Universe.

Next consider the diagonal Einstein equations in Eq. (12). Because $p_r = p_T$ it must be $G_r^r = G_\theta^\theta$. As a consequence

$$H^2 = 1 - kr^2 , \tag{23}$$

where the constant k is the Robertson-Walker curvature parameter. Then substituting $R = rS$ in

$$-G_t^t + G_r^r + 2G_\theta^\theta = -2\frac{\ddot{R}'}{R'} - 4\frac{\ddot{R}}{R} = \kappa_5^2\left(\rho_{\rm P} - 3\eta {\rho_{\rm P}}^{\alpha}\right) \tag{24}$$

we obtain

$$\frac{\ddot{S}}{S} = -\frac{\kappa_5^2}{6}\left(\rho_{\rm P} - 3\eta {\rho_{\rm P}}^{\alpha}\right) \ . \tag{25}$$

On the other hand the radial equation $G^r_r = \kappa_5^2 p_r$ leads to

$$\dot{S}^2 = \frac{\kappa_5^2}{3}\rho_{\rm P} S^2 - k \ . \tag{26}$$

Naturally, Eqs. (25) and (26) are related by a derivative. They are consistent when $\rho_{\rm P}$ obeys the conservation Eq. (21). Using Eqs. (26), (25), (23) and the expression for p_z given in Eq. (20) we conclude that $G^z_z = \kappa_5^2 p_z$ is an identity for all the parameters of the model.

With $\Omega = \Omega_{\rm RS}$ given by Eq. (17) we obtain the following 5-dimensional polytropic solutions for which gravity is confined to the vicinity of the brane

$$d\tilde{s}_5^2 = \Omega_{\rm RS}^2 \left[-dt^2 + S^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2 \right) + dz^2 \right] \ . \tag{27}$$

In Eq. (27) S satisfies Eq. (26). Thought obtained for the RS2 model these solutions also hold for the RS1 model due to the periodicity and the Z_2 symmetry of the orbifold. In the RS1 model the two branes are then twin universes with opposite tensions and an identical cosmological evolution.

Let us now verify that if a 4-dimensional observer confined to the brane makes the same assumptions about the bulk degrees of freedom then she deduces exactly the same dynamics [16]. Indeed, the non-zero components of the projected Weyl tensor [21] read

$$\mathcal{E}^t_t = \frac{\kappa_5^2}{4}\left(\rho_{\rm P} - \eta\rho_{\rm P}^{\alpha}\right) , \quad \mathcal{E}^r_r = \mathcal{E}^\theta_\theta = \mathcal{E}^\phi_\phi = -\frac{\mathcal{E}^t_t}{3} \ . \tag{28}$$

Then the effective 4-dimensional dynamics is given by

$$G^t_t = -\kappa_5^2 \rho_{\rm P}, \quad G^r_r = G^\theta_\theta = G^\phi_\phi = -\kappa_5^2 \eta \rho_{\rm P}^{\alpha} \ . \tag{29}$$

The 4-dimensional observer also sees gravity confined to the brane since she measures a negative tidal acceleration [24] given by

$$a_T = \frac{\kappa_5^2 \Lambda_B}{6} \ . \tag{30}$$

This implies that the geodesics just outside the brane converge towards the brane and so for the 4-dimensional observer the conformal bulk matter is effectively trapped inside the brane.

The effective 4-dimensional Chaplygin dynamics on the brane may lead to the formation of a shell focusing singularity at $S = 0$ and of regular rebounce epochs at some $S \neq 0$. This can be analyzed [25, 33] with the following potential $V = V(S)$ defined by

$$V(S) = S\dot{S}^2 = \frac{\kappa_5^2}{3}\left(\eta S^{3-3\alpha} + a\right)^{\frac{1}{1-\alpha}} - kS \ . \tag{31}$$

If for all $S \geq 0$ it turns out that $V > 0$ then a shell focusing singularity forms at $S = 0$. However, if an $S = S_* > 0$ exists such that $V(S_*) = 0$ then there is a regular rebounce point at $S = S_*$.

According to recent experimental bounds [32] the allowed range of values for η and a are $\eta > 0$ and $a > 0$ or $\eta < 0$ and $a < 0$. For $k = 0$ the potential is positive for all $-1 \leq \alpha < 0$ if $\eta > 0$ and $a > 0$. If $\eta < 0$ and $a < 0$ then this only happens for the set $\alpha = -p/q$, $q > p$ with q and p , respectively, even and odd integers. In any of these cases there are only singular solutions without rebouncing epochs. The Chaplygin shells may either expand continously to infinity or collapse to the singularity at $S = 0$ where

$$V(0) = \frac{\kappa_5^2}{3} a^{\frac{1}{1-\alpha}} > 0 \, . \tag{32}$$

It is for $k \neq 0$ that new dynamics appears. With $S = Z^{\frac{1}{1-\alpha}}$ we find

$$V = V(Z) = \frac{\kappa_5^2}{3}\left(\eta Z^3 + a\right)^{\frac{1}{1-\alpha}} - kZ^{\frac{1}{1-\alpha}} \, . \tag{33}$$

Consider $k > 0$, $\eta > 0$ and $a > 0$ (see Fig. 1). The condition $V \geq 0$ is equivalent to $\mathcal{V} = \mathcal{V}(Z) \geq 0$ where

$$\mathcal{V} = \left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} \left(\eta Z^3 + a\right) - k^{1-\alpha} Z \, . \tag{34}$$

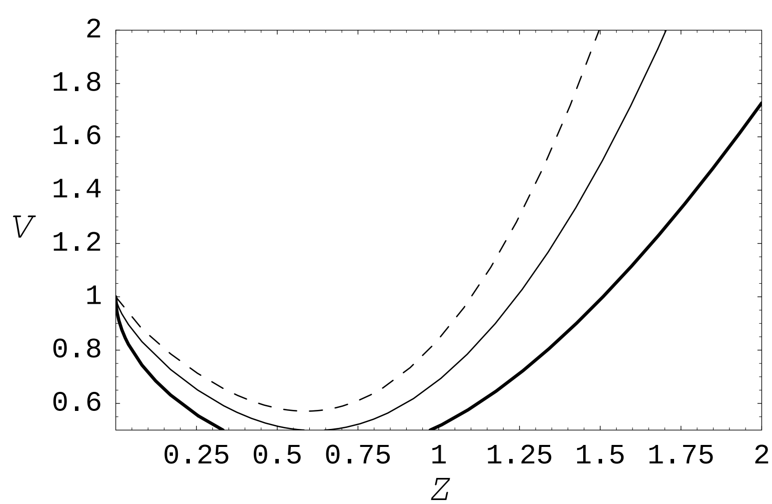

Figure 1: Plots of V for $k > 0$, $\eta > 0$ and $a > 0$. The dashed, thin and thick lines correspond, respectively, to α equal to $-1/4, -1/2$ and -1

Then there are at most two regular rebounce epochs in the allowed dynamical phase space. Since we have

$$\mathcal{V}(0) = \left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} a > 0,\ \mathcal{V}' = 3\left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} \eta Z^2 - k^{1-\alpha} \tag{35}$$

and

$$\mathcal{V}'' = 6\left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} \eta Z \geq 0 \tag{36}$$

this is determined by the sign of $\mathcal{V}$ at its minimum $\mathcal{V}_m = \mathcal{V}(Z_m)$ where $Z_m = \sqrt{(3k/\kappa_5^2)^{1-\alpha}/3\eta}$.

If $\mathcal{V}_m > 0$ then there are no regular rebounce points and the collapsing shells may fall from infinity to the singularity at $S = 0$ where $V(0) > 0$ is given in Eq. (32). For $\mathcal{V}_m = 0$ we have just one regular fixed point $S = S_*$ which divides the phase space into two disconnected regions, a bounded region with the singularity at $S = 0$, $0 \leq S < S_*$, and an infinitely extended region, $S > S_*$, where the shells expand with ever increasing speed to infinity. In this region the solutions are regular. If $\mathcal{V}_m < 0$ then we have two regular rebounce epochs $S = S_-$ and $S = S_+$ such that $S_- < S_+$. For $0 \leq S \leq S_-$ a shell may expand to a maximum radius rS_- and then rebounce to collapse towards the singularity at $S = 0$. For $S \geq S_+$ the collapsing shells shrink to the minimum scale S_+ and then rebounce into accelerated expansion to infinity. For $\eta < 0$ and $a < 0$ we find the same type of dynamics but now only for the special values $\alpha = -p/q$, $q > p$ with q and p , respectively, even and odd integers.

If $k < 0$ then for $\eta > 0$ and $a > 0$ the potential is always positive and so there are only singular solutions without rebouncing points. For $\eta < 0$ and $a < 0$ (see Fig. 2) we must consider $\alpha = -p/q$, $q > p$ with q and p, respectively, odd and even integers to find solutions with rebounce epochs. The condition $V \geq 0$ is still equivalent to $\mathcal{V} \geq 0$ but now

$$\mathcal{V} = -\left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} \Big(|\eta| Z^3 + |a|\Big) + |k|^{1-\alpha} Z. \tag{37}$$

Because we have

$$\mathcal{V}(0) = -\left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} |a| < 0,\ \mathcal{V}' = -3\left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} |\eta| Z^2 + |k|^{1-\alpha} \tag{38}$$

and

$$\mathcal{V}'' = -6\left(\frac{\kappa_5^2}{3}\right)^{1-\alpha} |\eta| Z \leq 0 \tag{39}$$

the sign of $\mathcal{V}$ at its maximum $\mathcal{V}_M = \mathcal{V}(Z_M)$ where $Z_M = \sqrt{(3|k|/\kappa_5^2)^{1-\alpha}/3|\eta|}$ shows that the only possibilities are the existence of one or two regular rebounce epochs.

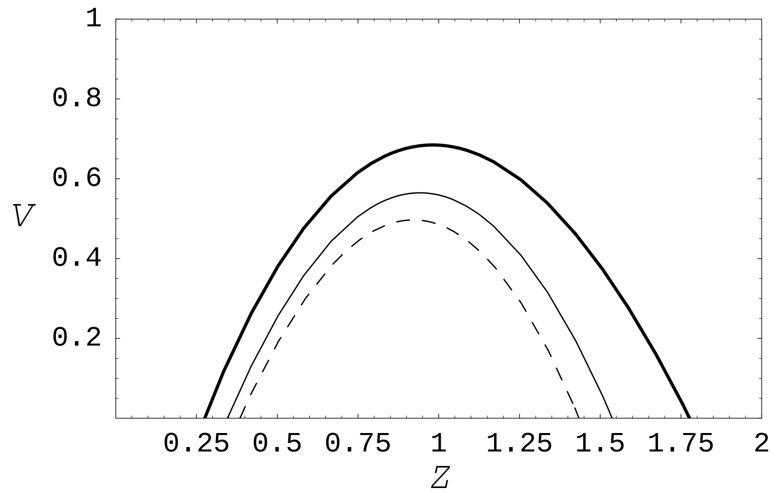

Figure 2: Plots of V for $k < 0$, $\eta < 0$ and $a < 0$. The dashed, thin and thick lines correspond, respectively, to α equal to $-2/7, -2/5$ and $-2/3$

In the former case the classical brane stays forever in the fixed point and in the latter it oscilates back and forth between the two rebounce points.

4 Conclusions

In this work we have presented new exact 5-dimensional solutions for which gravity is localized in the vicinity of the brane and the dynamics of the bulk fields on the brane is that of an homogeneous Chaplygin gas, a possible candidate for the missing dark energy which controls the expansion of the visible Universe. The bulk fields were seen to belong to a special conformal class which has a stress-energy tensor with conformal weight -4. We have seen that the 5-dimensional solutions are valid for the RS1 and the RS2 models and noted that an observer confined to the brane is led to the same localized braneworld dynamics when using an identical description of the field variables. We have analyzed the dynamical phase space describing the evolution of the Chaplygin shells discussing conditions for the formation of shell focusing singularities and of regular rebounce epochs. However, althought gravity is bound to the brane the conformal bulk fields are not localized near the brane. Indeed, the density and pressures increase with z due to the scale factor $\Omega^{-2}(z)$ and diverge at the AdS horizon. This is not a problem in the RS1 model because the space is cut before the AdS horizon is reached. In the RS2 model a solution requires the simultaneous localization of bulk matter and gravity near the brane, an open problem for future research.

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