

## A two-sided logistics matching method considering trading psychology and matching effort under a 4PL

Na Yuan<sup>a,b</sup>, Haiming Liang<sup>c\*</sup>, Min Huang<sup>b</sup>, Qing Wang<sup>b</sup>

<sup>a</sup>*School of Economics and Management, Shenyang Institute of Technology, Shenyang Reform and Innovation Demonstration Zone, Liaoning 113122, China*

<sup>b</sup>*College of Information Science and Engineering, Northeastern University, Shenyang, Liaoning, 110819, China*

<sup>c</sup>*Center for Network Big Data and Decision-Making, Business School, Sichuan University, Chengdu 610065, China*

### Article history:

Received May 4 2023

Received in Revised Format

July 10 2023

Accepted September 3 2023

Available online

September 4 2023

### Keywords:

*Logistics matching*

*Two-sided matching decision making*

*Trading psychology*

*Matching effort*

*Fairness threshold*

As a supply chain integrator, a fourth party logistics (4PL) typically does not have its own logistics facilities, so the 4PL needs to match third party logistics (3PLs) and customers to meet customers' logistics service demands. An effective matching method can not only improve the efficiency of 4PL supply chain management, but also establish more long-term and stable cooperative relationships with customers and 3PLs. Therefore, we propose a novel two-sided logistics matching method considering the trading psychology and matching effort of matching subjects under the 4PL. First, based on considering the trading psychology, the concepts of blocking pair and stable matching are redefined. Then, based on the public values and matching effort of customers and 3PLs, the evaluation values of customers and 3PLs are calculated. And the trading possibilities of customers and 3PLs are calculated by considering the fairness threshold. Next, we consider different stable matching demands of customers and 3PLs and develop a bi-objective matching model to maximize the trading possibilities of both customers and 3PLs. Furthermore, the properties of the proposed method are discussed. Finally, a numerical example and comparison analysis are provided to prove the feasibility and effectiveness of the proposed method.

© 2023 by the authors; licensee Growing Science, Canada

## 1. Introduction

With the continuous development of economic globalization, to improve their competitive advantage, expand market share and improve the satisfaction of logistics service, more and more enterprises focus on their own core business and outsource logistics business to professional third-party logistics (3PLs) (Dong et al., 2022; Özcan & Ahiskali, 2020). And 3PLs gradually became the mainstream at that time.

However, with the continuous improvement of digital technologies, as well as the high development of supply chain management, the disadvantage of 3PLs is increasingly obvious. Hofmann and Osterwalder (2017) point out that the 3PLs, which focus on providing standard logistics services, will lose a lot of market share in the future. In addition, due to the limitations of their capabilities and resources, the 3PLs are unable to integrate all supply chain resources, so the logistics efficiency in the supply chain cannot be further improved. Thus, fourth party logistics (4PL) becomes an ideal solution to the bottleneck problem of 3PLs. The term of 4PL was first proposed by Andersen Consulting in 1998, which defines a 4PL as a supply chain integrator, which provides and implements complete supply chain solutions by integrating the resources and capabilities of 3PLs, leading consulting firms, and technology providers (Gattorna, 1998; Wang et al., 2022; Win & Walters, 2008). Usually, a 4PL has almost no logistics facilities, and as a broker between customers and 3PLs, the 4PL needs to match 3PLs with customers to meet the customers' logistics service demands (Gruchmann et al., 2020). In addition, since a 4PL determine the appropriate matching between two sets of subjects, i.e., customers and 3PLs, this type of matching is called

\* Corresponding author

E-mail: [hmliang@scu.edu.cn](mailto:hmliang@scu.edu.cn) (H. Liang)

ISSN 1923-2934 (Online) - ISSN 1923-2926 (Print)

2023 Growing Science Ltd.

doi: 10.5267/j.ijiec.2023.9.001

two-sided matching which is first studied by Gale and Shapley (1962) to deal with college admission matching problem and marriage matching problem. At present, the two-sided matching is used in the field of logistics, which has become an effective mechanism for a 4PL to select the appropriate 3PL for customers to provide logistics services.

In the actual matching process, the 4PL needs to determine an appropriate matching scheme according to the demand information provided by customers and 3PLs. And the 4PL takes the initiative in the process of determining the optimal matching, which is manifested as follows: (1) In order to enhance the market values of customers and 3PLs, the 4PL tends to appropriately package the information of customers and 3PLs and disclose the packaged matching values to the other party. (2) Different customers and 3PLs have different demands on trading time, which may lead to different trading psychology. The 4PL can improve logistics matching efficiency by evaluating the trading psychology of customers and 3PLs. (3) When they obtain the public value of the other party published by the 4PL, the customers or 3PLs usually inquires with the 4PL about more information of the interested matching subject. The 4PL can determine appropriate matching more pertinently and more efficiently by evaluating the trading possibilities between two-side matching subjects.

Although there are two-sided matching methods which are able to address the logistics matching under the 4PL, there still exists some research gaps.

(1) There are few studies on the active role of brokers in two-sided matching decisions. In fact, in the logistics market dominated by the 4PL, the 4PL, as a broker between customers and 3PLs, has a significant impact on the matching result. For example, the public values provided by the 4PL will affect the matching subjects' evaluation values on the other party, and the trading possibilities evaluated by the 4PL will affect the fairness of the matching. Therefore, the active role of the 4PL as a broker should be considered in two-sided matching under the logistics market dominated by the 4PL.

(2) Few studies have considered the trading psychology of the matching subjects. In fact, in the logistics matching process under the 4PL, customers and 3PLs may have different trading psychology. For example, customers with urgent logistics tasks, who are eager to do a deal with 3PLs. Similarly, when 3PLs encounter some logistics tasks which are highly matched to their own capability, they are eager to transact with the customers with these logistics tasks. And different trading psychology of matching subjects usually make them have different requirements for the matching stability.

(3) The effort level of customers and 3PLs affects the discovery of the real value of the other party. The harder 3PLs and the customers communicate with the 4PL, the more likely they are to obtain more information about the real value of the other party, and the more likely their evaluation value on the other party is to be close to the real value of the other party.

In order to fill the above research gaps, we propose a novel two-sided logistic matching method considering the trading psychology and matching effort of customers and 3PLs under the 4PL. The main contributions of our research are as follows:

(1) We introduce trading psychology of customers and 3PLs into logistics matching under the 4PL. And based on considering the trading psychology of customers and 3PLs, we redefine blocking pair, stable matching and Pareto optimal matching.

(2) We consider the active role of the 4PL in two-sided logistics matching, and propose a novel method to calculate the evaluation values of customers and 3PLs on the other party, which takes into account the public values and effort of customers and 3PLs determined by the 4PL.

(3) We propose a method for calculating the trading possibility based on the trading fairness. And in order to obtain the optimal matching more effectively, we develop a bi-objective matching model considering different demands of customers and 3PLs on stability.

(4) We provide a case study in the background of two-sided logistics matching under the 4PL to demonstrate the feasibility of the proposed two-sided matching method. Meanwhile, we provide a comparison analysis to reveal the effectiveness of the novel method.

The rest of the paper is arranged as follows: The literature review on two-sided matching is presented in Section 2. And Section 3 introduces some preliminaries regarding two-sided matching and stable matching. Section 4 describes the problem background, the redefinition of some concepts and the resolution framework on logistics matching under the 4PL. The novel two-sided logistics matching method under the 4PL is specifically presented in Section 5. And a numerical experiment and comparison analysis are introduced in Section 6. Finally, the research conclusions and future research extensions are summarized in Section 7.

## 2. Literature Review

In this section, we review the existing literature related to our research from two streams respectively, which are the application of two-sided matching and two-sided matching methods.

### (1) The application of two-sided matching

Early applications of two-sided matching mainly focused on marriage, labor, and college admissions and so on (Abraham et al., 2007; Gale & Shapley, 1962; Roth, 1985). With the prosperity of the service economy and the increasingly diversified customer demands, many two-sided matching problems with new characteristics emerge. Jiang et al. (2011) studied the matching problem of one-shot multi-attribute exchanges considering quantity discounts in electronic brokerages, established

a multi-objective model where the objectives are to maximize the matching degree and the trade volume. Chen and Song (2013) studied the matching problem between firms and banks in the loan market, and developed a many-to-one two-sided matching model. Jiang et al. (2018) studied the problem of matching home buyers and sellers in China's real estate market, defined the satisfaction degree functions considering buyers' offer prices and preference orderings, seller' reservation and ideal prices, and developed a linear programming model to obtain optimal matching. Liu et al. (2018) studied the matching problem between surgeons and patients, calculated the satisfactions of both surgeons and patients based on their subjective preferences, and developed a two-objective matching model to maximize the satisfactions of matching subjects. Miao et al. (2019) investigated the two-sided matching between overseas demanders and domestic suppliers in cross-border e-commerce, and based on the integration of language and interval evaluation information, developed an optimal matching model with the maximum satisfaction degrees of both parties. Xiang et al. (2019) studied the matching problem on complex product manufacturing tasks on cloud manufacturing platforms, and proposed a matching model considering dual hesitant fuzzy preference information. For logistics service matching, Peng et al. (2016) investigated matching problems between vessels and cargos in the dry bulk shipping market, developed a stable matching model that maximizes the total surplus of carriers and shippers, and proposed a price game mechanism based on Gale-Shapley. Li, J. et al. (2020) investigated the matching problem between merchants and drivers in a freight O2O platform, and proposed a matching model considering pricing strategy. Ling et al. (2021) investigated the vehicle-cargo matching problem and proposed a two-sided matching model to maximize matching rate and matching profit.

It can be seen that there is more and more research on the application of two-sided matching, but there are few studies on two-sided matching in the logistics market dominated by a 4PL. As a supply chain integrator, the 4PL plays an important active role in the matching process. Therefore, the two-sided logistics matching problem considering the active role of the 4PL is an important research direction.

## (2) Two-sided matching methods

Klumpp (2009) constructed the stable matching model under non-transferable utility and transferable utility to resolve the two-sided matching with spatially differentiated agents. Liu and Ma (2015) designed a new two-matching decision method considering uncertainty of the preference sequences, and developed a matching model to maximize matching number and minimize the total preference distance. Echenique and Galichon (2017) introduced the concept of non-traded stable matching and proposed non-trading stable matching algorithms for solving ordinal matching and cardinal matching. Fan et al. (2017) considered the psychological behavior of agents, and introduced the disappointment theory into calculating the modified preference utilities, and on this basis developed a matching model to maximize the modified preference utilities of both matching parties. Yue et al. (2019) developed a novel two-sided matching method with triangular intuitionistic fuzzy numbers (TIFNs), in which, the similarity measure between TIFNs is refined, and a matching model based on extended similarity measure is developed. Li, P. et al. (2020) proposed a novel two-sided method based on the regret theory for the probabilistic linguistic term sets, in which the lowest acceptability value is considered. Zhang et al. (2020) proposed a novel two-sided matching method with fuzzy preference relation with self-confidence. Zhang et al. (2021a) developed a two-sided matching method with multi-granular hesitant fuzzy linguistic term sets. Pu and Yuan (2023) proposed a matching model where the demands of intermediary and the matching subjects are considered to resolve the matching problem with different preferences and individual bounded rationality.

From the above literature review, we can see that few existing methods consider the trading psychology and matching effort of matching subjects. In the matching process, each matching subject may have different trading psychology, and different effort levels, which affect the final matching result. Thus, it is necessary to propose a two-sided logistics matching method considering different trading psychology and matching effort, to assist the 4PL to realize the logistics matching between customers and 3PLs efficiently and establish long-term cooperative relationships with customers and 3PLs.

## 3. Preliminary

In this section, we introduce the concepts and definitions regarding two-sided matching, and stable matching, which will be useful for the proposed two-sided matching method for the logistics market under the 4PL.

Two-sided matching is an important research method in market formation mechanism. Specifically, the mathematical definition of two-sided matching is given as follows:

Let  $A = \{A_1, A_2, \dots, A_m\}$  and  $B = \{B_1, B_2, \dots, B_n\}$  be two sets of matching subjects, where  $A_i$  represents the  $i$  th subject in  $A$ , and  $B_j$  donates the  $j$  th subject in  $B$ , then the definition of two-sided matching is given as follows.

**Definition 1** (Zhang et al., 2020). A two-sided matching is defined as a mapping  $\mu: A \cup B \rightarrow A \cup B$ , if and only if the following conditions are satisfied for  $\forall A_i \in A, \forall B_j \in B$ .

- (1)  $\mu(A_i) \in B \cup \{A_i\}$ , if  $\mu(A_i) = A_i$ , then  $A_i$  has no matching object in  $B$ .
- (2)  $\mu(B_j) \in A \cup \{B_j\}$ , if  $\mu(B_j) = B_j$ , then  $B_j$  has no matching object in  $A$ .

(3) If  $\mu(A_i) = B_j$ , and  $\mu(B_j) = A_i$ , then  $(A_i, B_j)$  is the matching pair determined by matching  $\mu$ .

(4) If  $\mu(A_i) = B_j$ , then,  $\mu(A_i) \neq B_k$ ,  $B_k \neq B_j$ ,  $B_k \in B$ .

The set of matching pairs determined by a two-sided matching  $\mu$  is called a matching scheme.

Fig.1 shows a two-sided matching  $\mu$ , which includes two sets of matching subjects, namely  $A$  and  $B$ , and a broker. The broker is between  $A$  and  $B$ , and tries to satisfy the demands of both  $A$  and  $B$  to determine the optimal matching. In Fig. 1, the straight lines between the matching subjects  $A$  and  $B$  represent the matching relationship between the two parties, where the thick lines represent that the two parties have formed matching pairs. For example,  $(A_2, B_1)$  and  $(A_3, B_2)$  are two matching pairs determined by  $\mu$ . And the set  $\{(A_2, B_1), (A_3, B_2), \dots, (A_1, B_n)\}$  is a matching scheme.

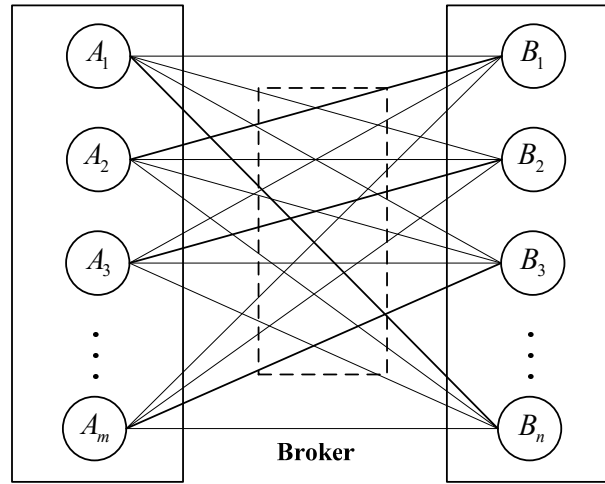


Fig. 1. A two-sided matching  $\mu$

**Definition 2** (Xiang et al., 2019). Let  $s_{ij}^A$  and  $s_{ij}^B$  be the satisfaction degree of  $A_i$  with  $B_j$  and the satisfaction degree of  $B_j$  with  $A_i$ , respectively. The two-sided matching  $\mu$  is stable if and only if none of the following conditions occurs:

- (1)  $\exists A_i, A_k \in A$  and  $\exists B_j, B_l \in B$ ,  $\mu(A_i) = B_l$ ,  $\mu(A_k) = B_j$ , satisfying  $s_{ij}^A > s_{il}^A$  and  $s_{ij}^B > s_{kl}^B$ .
- (2)  $\exists A_i \in A$  and  $\exists B_j, B_l \in B$ ,  $\mu(A_i) = B_l$ ,  $\mu(B_j) = B_j$ , satisfying  $s_{ij}^A > s_{il}^A$ .

#### 4. Problem description and Resolution framework

In this section, we propose a novel solution framework to solve the matching problem between customers and 3PLs in the logistics market dominated by a 4PL. First, we describe the research problem. Second, we redefine matching pairs, stable matching, and Pareto efficient matching. Then, we introduce the notations and sets used in this paper. And finally, we present the novel resolution framework for logistics matching problems.

##### 4.1 Problem description

A logistics market dominated by a 4PL is considered, which consists of a 4PL, a set of customers, and a set of 3PLs. 3PLs can provide professional logistics services. Meanwhile, customers need appropriate 3PLs to provide logistics services for themselves. Customers and 3PLs are two-sided matching subjects. The 4PL as a broker needs to determine the optimal matching between customers and 3PLs. Without loss of generality, we assume that one customer can only match one 3PL at most, and one 3PL can only match one customer at most. In practice, customers and 3PLs both have different trading psychologies in the process of logistics trading. For example, for urgent logistics tasks, customers will be more eager to make transactions, while for non-urgent logistics tasks (such as transportation services procurement for long-term planning), customers will be more patient in the transactions. The same goes for the 3PLs. We assume that the customers and 3PLs have three types of trading psychologies respectively, namely, eager to make a deal, neutral deal, and patient deal.

In the actual matching process, the 4PL can evaluate the real values of the customers and 3PLs according to the relevant information submitted by the customers and 3PLs, as well as the information accumulated in the long-term cooperation process. At the same time, in order to improve the competitiveness of customers and 3PLs in the matching market as much as possible, the 4PL will determine their public values which are slightly greater than the real values and publish them on the matching market. Without loss of generality, each customer and each 3PL has an acceptable matching value threshold. Then,

the challenge for the 4PL is how to determine the optimal matching to facilitate the logistics transaction between customers and 3PLs as much as possible.

#### 4.2. Notations

In this subsection, we introduce the notations used in this paper, which are as follows.

$A$ : set of customers,  $A = \{A_1, A_2, \dots, A_m\}$ , where  $A_i$  denotes the  $i$  th customer,  $i = 1, 2, \dots, m$ ,  $A = Q_1^A \cup Q_2^A \cup Q_3^A$ ,  
 $Q_1^A \cap Q_2^A \cap Q_3^A = \emptyset$

$B$ : set of 3PLs,  $B = \{B_1, B_2, \dots, B_n\}$ , where  $B_j$  denotes the  $j$  th 3PL,  $j = 1, 2, \dots, n$ ,  $B = Q_1^B \cup Q_2^B \cup Q_3^B$ ,  
 $Q_1^B \cap Q_2^B \cap Q_3^B = \emptyset$

$Q_1^A$ : set of customers who belong to eager deal type

$Q_2^A$ : set of customers who belong to neutral deal type

$Q_3^A$ : set of customers who belong to patient deal type

$Q_1^B$ : set of 3PLs who belong to eager deal type

$Q_2^B$ : set of 3PLs who belong to neutral deal type

$Q_3^B$ : set of 3PLs who belong to patient deal type

$h_i^A$ : Real value of customer  $A_i$

$h_i^P$ : Public value of customer  $A_i$  determined by the 4PL, and  $h_i^P \geq h_i^A$

$g_j^B$ : Real value of 3PL  $B_j$

$g_j^P$ : Public value of 3PL  $B_j$  determined by the 4PL, and  $g_j^P \geq g_j^B$

$\alpha_i^c$ : Acceptable matching value threshold of  $A_i$

$\beta_j^c$ : Acceptable matching value threshold of  $B_j$

The purpose of this paper is to determine the optimal matching in logistics market based on the trading psychology of customers and 3PLs, the public values  $h_i^P$  ( $i = 1, 2, \dots, m$ ) and  $g_j^P$  ( $j = 1, 2, \dots, n$ ), and the real values  $h_i^B$  ( $i = 1, 2, \dots, m$ ) and  $g_j^B$  ( $j = 1, 2, \dots, n$ ).

#### 4.3. Redefinition of some concepts based on trading psychology

The introduction of the trading psychologies of matching subjects (i.e., the customers and 3PLs) has a significant effect on the matching result. Therefore, it is necessary to redefine the blocking pair, stable matching and Pareto optimal matching. It is worth pointing out that the above definitions are closely related to the matching values of the matching subjects.

Let  $g_{ij}$  be the evaluation value of customer  $A_i$  on 3PL  $B_j$ . Let  $h_{ij}$  be the evaluation value of 3PL  $B_j$  on customer  $A_i$ . The realistic meanings and calculation methods of  $g_{ij}$  and  $h_{ij}$  are given in detail in Section 5.1.

Based on the definition of two-sided matching, the mathematical definition of individual rational matching is given as follows.

**Definition 3.** Let  $\mu(A_i) = B_j$ ,  $\alpha_i^c$  and  $\beta_j^c$  represent the acceptable matching value thresholds of  $A_i$  and  $B_j$  respectively.

Then,

(1) If  $g_{ij} \geq \alpha_i^c$ , then matching  $\mu$  is the individual rational matching for  $A_i$ .

(2) If  $h_{ij} \geq \beta_j^c$ , then matching  $\mu$  is the individual rational matching for  $B_j$ .

Furthermore, if a matching is the individual rational matching for all customers and 3PLs, the matching  $\mu$  is called an individual rational matching.

Considering that the significant difference between the matching subjects (i.e., customers and 3PLs) with different trading psychologies is their different waiting costs. For example, in the real logistics matching process, if a logistics task has a short time window and needs to be completed as soon as possible, the delay will bring serious breach loss to the customer, then the customer who has the logistics task has a large trading waiting cost. Therefore, we introduce the waiting costs of the matching

subjects into the definition of the blocking pair.

Let  $c_i^A$  and  $c_j^B$  denote the waiting costs of customer  $A_i$  and 3PL  $B_j$  participating in the matching respectively. Then,

(1) If  $A_i \in Q_1^A$  ( $B_j \in Q_1^B$ ), then  $c_i^A = \infty$  ( $c_j^B = \infty$ ), it indicates that the customer  $A_i$  (3PL  $B_j$ ) who belongs to eager deal type is not willing to wait.

(2) If  $A_i \in Q_2^A$  ( $B_j \in Q_2^B$ ), then  $c_i^A > 0$  ( $c_j^B > 0$ ), it indicates that the customer  $A_i$  (3PL  $B_j$ ) who belongs to neutral deal type has a certain waiting cost.

(3) If  $A_i \in Q_3^A$  ( $B_j \in Q_3^B$ ), then  $c_i^A = 0$  ( $c_j^B = 0$ ), it indicates that the customer  $A_i$  (3PL  $B_j$ ) who belongs to patient deal type has no waiting cost because it is not sensitive to the trading time.

In the actual matching process, the 4PL can obtain their matching time requirements (i.e., waiting costs) by directly inquiring the customers and 3PLs.

**Definition 4.** For the matching pair  $(A_i, B_j)$  ( $A_i \in A, B_j \in B$ ), if one of the following conditions is met, then the matching pair  $(A_i, B_j)$  is called a blocking pair.

- (1) If  $\mu(A_i) = A_i$ ,  $\mu(B_j) = B_j$ ,  $g_{ij} \geq \alpha_i^c$ , and  $h_{ij} \geq \beta_j^c$ .
- (2) If  $\mu(A_i) = A_i$ ,  $\mu(B_j) = A_k$ ,  $B_j \in Q_2^B \cup Q_3^B$ ,  $g_{ij} \geq \alpha_i^c$  且  $h_{ij} > h_{kj} + c_j^B$ .
- (3) If  $\mu(A_i) = B_l$ ,  $\mu(B_j) = B_j$ ,  $A_i \in Q_2^A \cup Q_3^A$ ,  $g_{ij} > g_{il} + c_i^A$  且  $h_{ij} \geq \beta_j^c$ .
- (4) If  $\mu(A_i) = B_l$ ,  $\mu(B_j) = A_k$ ,  $A_i \in Q_2^A \cup Q_3^A$ ,  $B_j \in Q_2^B \cup Q_3^B$ ,  $g_{ij} > g_{il} + c_i^A$ , and  $h_{ij} > h_{kj} + c_j^B$ .

Observingly, the definition of blocking pair can be briefly summarized as follows:

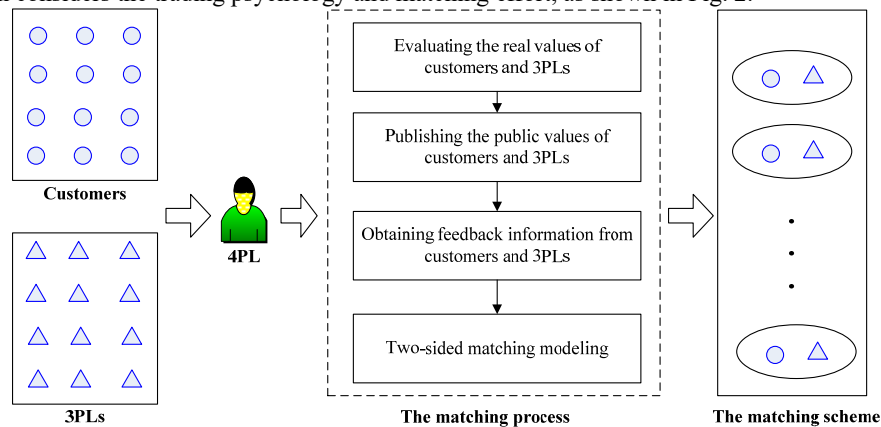
- (1) If neither of the matching subjects is matched, and both parties are each other's individual rational matching.
- (2) If one matching subject has a matching object, but the other matching subject does not be matched, the matched subject is more willing to wait for the unmatched subject to match, and the unmatched subject regards the matched subject as an individual rational matching.
- (3) If two matching subjects already have their respective matching objects, now they prefer to match each other rather than the original matched objects.

**Definition 5.** If a two-sided matching  $\mu : A \cup B \rightarrow A \cup B$  has no blocking pair, the matching  $\mu$  is called a stable matching.

**Definition 6.** For two matchings  $\mu$ , and  $\nu$ , if  $\forall A_i \in A$ , it satisfies  $g_{i\nu(A_i)} \geq g_{i\mu(A_i)}$ . Meanwhile, for  $\forall B_j \in B$ , it satisfies  $h_{\nu(B_j)j} \geq h_{\mu(B_j)j}$ . Then matching  $\nu$  is Pareto dominant over matching  $\mu$ . Particularly, if there is no matching which is Pareto dominant over matching  $\mu$ , then  $\mu$  is called to a Pareto efficient matching.

#### 4.4. Resolution framework

In this subsection, we construct a novel decision framework to solve the matching problem between customers and 3PLs under the 4PL, which considers the trading psychology and matching effort, as shown in Fig. 2.



**Fig. 2.** The matching decision framework under the 4PL

In our proposed framework, customers and 3PLs first make matching requests to the 4PL and provide the corresponding information as required by the 4PL. Then, the 4PL starts the matching process and determines the optimal matching. The matching decision process is described in detail below.

**Step1.** Evaluating the real values of customers and 3PLs. After collecting matching requests from customers and 3PLs, the 4PL will require both matching subjects to provide necessary information (e.g., basic information of enterprises and credit status information, and so on). Subsequently, the 4PL evaluates the real values (that is, the values in the matching market) of the customers and 3PLs according to the expertise, as well as historical information accumulated in the long-term cooperation.

**Step2.** Publishing the public values of customers and 3PLs. The 4PL is always desirable to make as many matching pairs as possible to obtain greater profits. Therefore, the 4PL usually increases the values of each customer and each 3PL based on the real values of both customers and 3PLs, and publishes the increased values (that is, the public values) to the logistics matching platform of the 4PL.

**Step3.** Obtaining the feedback information from customers and 3PLs. When the customers and 3PLs get the public values of the other party published by the 4PL, they usually continue to inquire about the 4PL for the detailed information about the other party they are interested in. And the 4PL further determines evaluation values of customers and 3PLs on other parties according to the inquiry and feedback of the customers and 3PLs. Meanwhile, the customers and 3PLs will give feedback to the 4PL about their eagerness for trading (i.e., trading psychology) according to their own situation.

**Step4.** Two-sided matching modeling. Based on the evaluation values of the customers and 3PLs, the trading possibility between customers and 3PLs can be calculated. The 4PL needs to establish a matching model with the objectives of maximizing the trading possibilities of both customers and 3PLs, based on the different stability requirements of matching subjects, so as to determine the optimal matching.

## 5. Two-sided matching method considering trading psychology and matching effort

In this section, we first propose the method of calculating the evaluation values of customers and 3PLs. Then, a method of calculating the trading possibility of two-sided matching considering matching fairness and trading psychology is provided, subsequently. Next, a bi-objective 0-1 programming model is developed to obtain an optimal matching between customers and 3PLs for the 4PL. Finally, we discuss some properties of our proposed two-sided matching model.

### 5.1. Calculating the evaluation values of customers and 3PLs

After the 4PL has published the public value of one party of matching subjects (e.g., the customers), the other party matching subjects (e.g., 3PLs) usually continue to inquire about the 4PL for more details about the matching subject they are interested in. The above behavior of the matching subjects is called matching effort in this paper. Obviously, the harder the matching effort, the closer the evaluation value of one matching subject on the other party matching subject is to the real value.

Let  $w_i^A$  denote the matching effort of customer  $A_i$ , and  $w_i^A \in [0, 1]$ . If  $w_i^A = 0$ , it indicates that the customer  $A_i$  has not made any matching effort. If  $w_i^A = 1$ , it indicates that the customer  $A_i$  makes complete matching effort. Let  $w_j^B$  denote the matching effort of 3PL  $B_j$ , and  $w_j^B \in [0, 1]$ . If  $w_j^B = 0$ , it indicates that 3PL  $B_j$  has not made any matching effort. If  $w_j^B = 1$ , it indicates that 3PL  $B_j$  makes complete matching effort. In a actual matching process, the 4PL can determine the degree of matching effort based on the fact that one matching subject inquiries it for information about the other matching subject.

The harder the matching effort of the matching subject, the closer the evaluation value of the matching subject on the other party is to the real value of the other party. Meanwhile, because of different matching efforts from different matching subjects, different matching subjects may have different evaluation values on the same matching subject.

Let  $g_{ij}$  be the evaluation value of customer  $A_i$  on 3PL  $B_j$ , which can be calculated by Eq. (1).

$$g_{ij} = w_i^A g_j^B + (1 - w_i^A) g_j^P \quad (1)$$

If  $w_i^A = 0$ , then  $g_{ij} = g_j^P$ , it indicates that the evaluation value of customer  $A_i$  on 3PL  $B_j$  is equal to the public value of 3PL  $B_j$ , when customer  $A_i$  has not made any matching effort. While, if  $w_i^A = 1$ , then  $g_{ij} = g_j^B$ , it indicates that customer  $A_i$  can get the real value of 3PL  $B_j$ , when customer  $A_i$  makes complete matching effort.

Similarly, let  $h_{ij}$  be the evaluation value of 3PL  $B_j$  on customer  $A_i$ , which can be given by Eq. (2).

$$h_{ij} = w_j^B h_i^A + (1 - w_j^B) h_i^P \quad (2)$$

If  $w_j^B = 0$ , then  $h_{ij} = h_i^P$ , it indicates the evaluation value of 3PL  $B_j$  on customer  $A_i$  is equal to the public value of

customer  $A_i$ , when 3PL  $B_j$  has not made any effort. If  $w_j^B = 1$ , then  $h_{ij} = h_i^A$ , it indicates that 3PL  $B_j$  can get the real value of customer  $A_i$ , when 3PL  $B_j$  makes complete matching effort.

5.2. Calculating the trading possibilities of both customers and 3PLs

When there is a big difference between the two parties in the matching pair matched by 4PL in terms of the evaluation values of the other party, the party whose evaluation value is too low will often choose to give up matching with the other party. Only when the gap between the evaluation values of the two-sided matching subjects is within the acceptable range, the two-sided matching subjects have the trading possibility. Therefore, we consider matching fairness as an important basis for calculating the trading possibilities of two-sided matching subjects.

Let  $\varepsilon_1^A$ ,  $\varepsilon_2^A$ , and  $\varepsilon_3^A$  ( $\varepsilon_1^A \geq \varepsilon_2^A \geq \varepsilon_3^A$ ) be respectively the fairness threshold set by the 4PL for deal eager, deal neutral and deal patient customers. Let  $p_{ij}^A$  be the trading possibility that customer  $A_i$  eventually choose to trade with 3PL  $B_j$ . And  $p_{ij}^A$  can be calculated by Eq. (3).

$$p_{ij}^A = \begin{cases} 0.5 + 0.5 \frac{g_{ij} - \alpha_i^c}{\max_j g_{ij} - \alpha_i^c} & 0 \leq h_{ij} - g_{ij} \leq \varepsilon^A, g_{ij} \geq \alpha_i^c \\ 0.5 + 0.5 \frac{g_{ij} - \alpha_i^c}{\max_j g_{ij} - \alpha_i^c} e^{-(g_{ij} - h_{ij})/\varepsilon^A} & 0 \leq g_{ij} - h_{ij} \leq \varepsilon^A, g_{ij} \geq \alpha_i^c \\ 0 & \text{Other} \end{cases} \quad (3)$$

where  $\varepsilon^A$  denotes the fairness threshold set by the 4PL for customers. If  $A_i \in Q_1^A$ , then  $\varepsilon^A = \varepsilon_1^A$ . If  $A_i \in Q_2^A$ , then  $\varepsilon^A = \varepsilon_2^A$ . If  $A_i \in Q_3^A$ , then  $\varepsilon^A = \varepsilon_3^A$ .

Similarly, let  $\varepsilon_1^B$ ,  $\varepsilon_2^B$ , and  $\varepsilon_3^B$  ( $\varepsilon_1^B \geq \varepsilon_2^B \geq \varepsilon_3^B$ ) be respectively the fairness threshold set by the 4PL for deal eager, deal neutral and deal patient 3PLs. Let  $p_{ij}^B$  be the trading possibility that 3PL  $B_j$  eventually choose to trade with customer  $A_i$ . And  $p_{ij}^B$  can be calculated by Eq. (4).

$$p_{ij}^B = \begin{cases} 0.5 + 0.5 \frac{h_{ij} - \beta_j^c}{\max_i h_{ij} - \beta_j^c} & 0 \leq g_{ij} - h_{ij} \leq \varepsilon^B, h_{ij} \geq \beta_j^c \\ 0.5 + 0.5 \frac{h_{ij} - \beta_j^c}{\max_i h_{ij} - \beta_j^c} e^{-(g_{ij} - h_{ij})/\varepsilon^B} & 0 \leq h_{ij} - g_{ij} \leq \varepsilon^B, h_{ij} \geq \beta_j^c \\ 0 & \text{Other} \end{cases} \quad (4)$$

If  $B_j \in Q_1^B$ , then  $\varepsilon^B = \varepsilon_1^B$ . If  $B_j \in Q_2^B$ , then  $\varepsilon^B = \varepsilon_2^B$ . If  $B_j \in Q_3^B$ , then  $\varepsilon^B = \varepsilon_3^B$ .

5.3. The maximum trading possibility matching model

In this subsection, we develop a bi-objective 0-1 programming model, which can assist the 4PL to obtain an optimal matching between customers and 3PLs.

Let  $x_{ij}$  be a 0-1 variable, when  $x_{ij} = 1$ , it implies that  $(A_i, B_j)$  is the matching pair matched by the 4PL, otherwise,  $x_{ij} = 0$ . The matching model P1 is as follows:

$$\max \sum_{i=1}^m \sum_{j=1}^n p_{ij}^A x_{ij} \quad (5)$$



$$\max \sum_{i=1}^m \sum_{j=1}^n p_{ij}^B x_{ij} \tag{6}$$

subject to

$$\sum_{j=1}^n x_{ij} \leq 1 \tag{7}$$

$$\sum_{i=1}^m x_{ij} \leq 1 \tag{8}$$

$$x_{ij} + \sum_{k:g_{ik} \geq \alpha_i^c} x_{ik} + \sum_{k:h_{kj} \geq \beta_j^c} x_{kj} \geq 1 \quad A_i \in Q_1^A, B_j \in Q_1^B \tag{9}$$

$$x_{ij} + \sum_{k:g_{ik} \geq \alpha_i^c} x_{ik} + \sum_{k:h_{kj} > h_{ij} - c_j^B \geq \beta_j^c} x_{kj} \geq 1 \quad A_i \in Q_1^A, B_j \in Q_2^B \cup Q_3^B \tag{10}$$

$$x_{ij} + \sum_{k:g_{ik} > g_{ij} - c_i^A \geq \alpha_i^c} x_{ik} + \sum_{k:h_{kj} \geq \beta_j^c} x_{kj} \geq 1 \quad A_i \in Q_2^A \cup Q_3^A, B_j \in Q_1^B \tag{11}$$

$$x_{ij} + \sum_{k:g_{ik} > g_{ij} - c_i^A \geq \alpha_i^c} x_{ik} + \sum_{k:h_{kj} > h_{ij} - c_j^B \geq \beta_j^c} x_{kj} \geq 1 \quad A_i \in Q_2^A \cup Q_3^A, B_j \in Q_2^B \cup Q_3^B \tag{12}$$

$$x_{ij} \in \{0, 1\} \tag{13}$$

where, objective (1) is to maximize the total trading possibility of customers, objective (2) is to maximize the total trading possibility of 3PLs. Constraint (7) ensures that each customer is matched with at most one 3PL. Constraints (8) ensures that each 3PL is matched with at most one customer. Constraints (9)-(12) is the stability constraint. Constraint (13) indicates that the decision variable is a 0-1 variable.

The model P1 is a bi-objective linear programming model, and there are mature algorithms and optimization software to solve it. In this paper, the variable step size algorithm (Steuer, 1986) is used to solve the two-sided matching model P1.

5.4. Some properties on the proposed two-sided matching method

Let  $X^*$  be the set of optimal matching obtained by solving model P1, where  $X^* = \{(A_i, B_j) \mid x_{ij} = 1, i \in \{1, 2, \dots, m\}, j \in \{1, 2, \dots, n\}\}$ .

**Lemma 1.** If the optimal matching  $X^*$  includes  $(A_i, A_i)$  and  $(B_j, B_j)$ , then one of the following three conditions must be satisfied: (1)  $g_{ij} \geq \alpha_i^c$  and  $h_{ij} < \beta_j^c$ . (2)  $g_{ij} < \alpha_i^c$  and  $h_{ij} \geq \beta_j^c$ . (3)  $g_{ij} < \alpha_i^c$  and  $h_{ij} < \beta_j^c$ .

**Proof of Lemma 1.** Suppose that the optimal matching  $X^*$  includes  $(A_i, A_i)$  and  $(B_j, B_j)$ , and  $g_{ij} \geq \alpha_i^c, h_{ij} \geq \beta_j^c$ . Let  $z_1^*$  and  $z_2^*$  be the two objective values corresponding to the optimal matching  $X^*$ , respectively.

Because  $g_{ij} \geq \alpha_i^c$  and  $h_{ij} \geq \beta_j^c$ , it shows that  $(A_i, B_j)$  is the individual rational matching mutually identified by customer  $A_i$  and 3PL  $B_j$ . Then,  $p_{ij}^A > 0$  and  $p_{ij}^B > 0$ . In addition, when  $x_{ij} = 1$ , it does not affect the stability of the existing matching. Otherwise, either customer  $A_i$  or 3PL  $B_j$  will have a matching scheme, so it is easy to prove that the constraints (7)-(13) are satisfied.

Furthermore, we can obtain that  $z_1^* + p_{ij}^A > z_1^*$  and  $z_2^* + p_{ij}^B > z_2^*$ . Clearly, the matching  $X^* \cup (A_i, B_j)$  outperforms the matching  $X^*$ , contradicting the assumption that  $X^*$  is the optimal matching.

**Theorem 1.** The constraints (9)-(12) can guarantee that there are no blocking pairs in the matching scheme formed.

**Proof of Theorem 1.** According to Definition 4, there are four cases for a blocking pair.

Case1.  $\mu(A_i) = A_i$  and  $\mu(B_j) = B_j$

According to Lemma 1, one of the following three conditions must be satisfied. (1)  $g_{ij} \geq \alpha_i^c$  and  $h_{ij} < \beta_j^c$ , (2)  $g_{ij} < \alpha_i^c$  and  $h_{ij} \geq \beta_j^c$ , (3)  $g_{ij} < \alpha_i^c$  and  $h_{ij} < \beta_j^c$ . Therefore, according to case (1) in Definition 4,  $(A_i, B_j)$  is not a blocking pair.

Case2.  $\mu(A_i) = A_i$  and  $\mu(B_j) \neq B_j$

(1)  $B_j \in Q_1^B$ , then  $\sum_{k:h_{kj} \geq \beta_j^c} x_{kj} \geq 1$ , it shows that customer  $A_k$  is an individual rational matching for 3PL  $B_j$ . According to case (1) in Definition 4,  $(A_i, B_j)$  is not a blocking pair.

(2)  $B_j \in Q_2^B \cup Q_3^B$ , then  $\sum_{k:h_{kj} > h_{ij} - c_j^B \geq \beta_j^c} x_{kj} \geq 1$ , it shows that customer  $A_k$  is superior to customer  $A_i$  for 3PL  $B_j$ . According to case (2) in Definition 4,  $(A_i, B_j)$  is not a blocking pair.

Case3.  $\mu(A_i) \neq A_i$  and  $\mu(B_j) = B_j$

(1)  $A_i \in Q_1^A$ , then  $\sum_{k:g_{ik} \geq \alpha_i^c} x_{ik} \geq 1$ , it indicates that 3PL  $B_k$  is an individual rational matching for customer  $A_i$ . According to case (1) in Definition 4,  $(A_i, B_j)$  is not a blocking pair.

(2)  $A_i \in Q_2^A \cup Q_3^A$ , then  $\sum_{k:g_{ik} > g_{ij} - c_i^A \geq \alpha_i^c} x_{ik} \geq 1$ , it indicates that 3PL  $B_k$  is superior to  $B_j$  for customer  $A_i$ . According to case (3) in Definition 4,  $(A_i, B_j)$  is not a blocking pair.

Case4.  $\mu(A_i) \neq A_i$  and  $\mu(B_j) \neq B_j$

If  $\mu(A_i) \neq A_i$  and  $\mu(B_j) \neq B_j$ , then  $x_{ij} = 0$ . We can obtain that  $\sum_{k:g_{ik} > g_{ij} - c_i^A \geq \alpha_i^c} x_{ik} + \sum_{k:h_{kj} > h_{ij} - c_j^B \geq \beta_j^c} x_{kj} \geq 1$ , which indicates that at least one matching subject of  $A_i$  and  $B_j$  thinks the current matching scheme is better. According to case (4) in Definition 4,  $(A_i, B_j)$  is not a blocking pair.

From the above arguments, we know that constraints (9)-(12) can guarantee that there are no blocking pairs in the matching scheme formed.

**Theorem 2.** If all customers and 3PLs in a two-sided matching are eager to complete the transaction, the model P1 degenerates into a model only including Eqs. (5)-(8) and (13).

**Proof of Theorem 2.** According to Eq. (3), if  $g_{ij} < \alpha_i^c$ , then  $p_{ij}^A = 0$ . It indicates that it is impossible for customer  $A_i$  to match 3PL  $B_j$ . Similarly, according to Eq. (4), if  $h_{ij} < \beta_j^c$ , then  $p_{ij}^B = 0$ . It indicates that it is impossible for 3PL  $B_j$  to match customer  $A_i$ . Therefore, the constraints (7) and (8) can be reduced to  $\sum_{j=1, g_{ij} \geq \alpha_i^c}^n x_{ij} \leq 1$  and  $\sum_{i=1, h_{ij} \geq \beta_j^c}^m x_{ij} \leq 1$ , respectively.

Since we assume that there is at least one individual rational matching for each matching subject, we can obtain: (1) If  $m \leq n$ ,

then any customer can find a matching 3PL, i.e.,  $\sum_{j=1, g_{ij} \geq \alpha_i^c}^n x_{ij} = 1$ . (2) If  $m > n$ , then any 3PL can find a matching customer,

i.e.,  $\sum_{i=1, h_{ij} \geq \beta_j^c}^m x_{ij} = 1$ .  $\min(\sum_{j=1, g_{ij} \geq \alpha_i^c}^n x_{ij} \leq 1, \sum_{i=1, h_{ij} \geq \beta_j^c}^m x_{ij} \leq 1) = 1$ , and  $x_{ij} + \sum_{k:g_{ik} \geq \alpha_i^c} x_{ik} + \sum_{k:h_{kj} \geq \beta_j^c} x_{kj} \geq 1$ , it indicates

that constraint (9) is a redundant constraint. Therefore, if all customers and 3PLs in a two-sided matching are eager to complete the transaction, the model P1 degenerates into the model only including Eqs. (5)-(8) and (13).

**Theorem 3.** If all customers and 3PLs in a two-sided matching are patient transaction types, the model P1 degenerate into an optimization model considering classical stability constraints.

**Proof Theorem 3.** If all customers and 3PLs in a two-sided matching are patient deal types, then,  $c_i^A = 0$ ,  $c_j^B = 0$ . According to constraint (12), we can obtain the following inequality.

$$x_{ij} + \sum_{k:g_{ik} > g_{ij} \geq \alpha_i^c} x_{ik} + \sum_{k:h_{kj} > h_{ij} \geq \beta_j^c} x_{kj} \geq 1 \quad (14)$$

According to theorem 2, because  $\sum_{k:g_{ik}<\alpha_i^c} x_{ik} = 0$ , and  $\sum_{k:h_{kj}<\beta_j^c} x_{kj} = 0$ , then the above inequality can be further simplified as

$$x_{ij} + \sum_{k:g_{ik}>g_{ij}} x_{ik} + \sum_{k:h_{kj}>h_{ij}} x_{kj} \geq 1 \tag{15}$$

Therefore, according to literature (Vate, 1989), the models P1 degenerates into an optimization model considering classical stability constraints.

**Theorem 4.** If all customers and 3PLs in a two-sided matching are neutral deal type, then the model P1 degenerates into an optimization model considering  $\alpha$ -stability constraints.

**Proof of Theorem 4.** If all customers and 3PLs in a two-sided matching are neutral deal type, then,  $c_i^A > 0$ ,  $c_j^B > 0$ .

According to Theorem 3, constraint (12) can be transformed as

$$x_{ij} + \sum_{k:g_{ik}>g_{ij}-c_i^A} x_{ik} + \sum_{k:h_{kj}>h_{ij}-c_j^B} x_{kj} \geq 1 \tag{16}$$

Furthermore, constraint (12) can be sorted out as

$$x_{ij} + \sum_{k:g_{ik}-g_{ij}>-c_i^A} x_{ik} + \sum_{k:h_{kj}-h_{ij}>-c_j^B} x_{kj} \geq 1 \tag{17}$$

Therefore, according to literature (Liang, 2015), the model P1 degenerates into an optimization model considering  $\alpha$ -stability constraints.

**Theorem 5.** The optimal matching  $\mu^*$  is a Pareto efficient matching.

**Proof of Theorem 5.** Suppose that  $\mu^*(A_i) = B_j$ , and  $v_1$  is a stable matching which satisfies  $v_1(A_i) = B_k$ . If  $g_{ij} \geq g_{ik}$ , then it indicates that the matching  $v_1$  is not Pareto dominant to matching  $\mu^*$ . If  $g_{ij} < g_{ik}$ , then we need to consider the following cases.

- i) If  $\mu^*(B_k) = B_k$ , then  $h_{ik} < \beta_k^c$ . Otherwise, it contradicts that the matching  $\mu^*$  is a stable matching.
- ii) If  $\mu^*(A_l) = B_k$  and  $B_k \in Q_1^B$ , then  $h_{lk} \geq \beta_k^c$ . Otherwise, it contradicts that the matching  $\mu^*$  is a stable matching. And when  $h_{lk} \geq \beta_k^c$ , 3PL  $B_k$  who is eager to make a deal, will not give up the customer  $A_l$ .
- iii) If  $\mu^*(A_l) = B_k$  and  $B_k \in Q_2^B \cup Q_3^B$ , then  $h_{lk} + c_k^B \geq h_{ik}$ . Otherwise, it contradicts that the matching  $\mu^*$  is a stable matching. Obviously, the matching  $v_1$  does not Pareto dominate over matching  $\mu^*$ .

From the above arguments, we know that the optimal matching  $\mu^*$  is a Pareto efficient matching.

## 6. Numerical experiment

A numerical example on a 4PL-led two-sided logistics matching is designed to describe the implementation of the proposed matching method. Also, the comparison analysis is conducted to illustrate the effectiveness of the proposed two-sided logistics matching method.

### 6.1. A numerical example

A 4PL providing logistics management services to customers, is responsible to match customers with suitable 3PLs to provide logistics services. We assume that there are 7 customers  $A = \{A_1, A_2, \dots, A_7\}$  who have logistics service demands, and 8 3PLs  $B = \{B_1, B_2, \dots, B_8\}$  who can provide logistics services. The 4PL needs to form the optimal matching between customers and 3PLs.

To provide as good a service as possible, the 4PL immediately evaluates the real values of all customers and 3PLs. And then, based on the real values, the 4PL determines the public values of customers and 3PLs, and publishes them on the logistics matching platform. At the same time, before matching logistics services, the 4PL will investigate the trading psychologies of customers and 3PLs and the historical transaction records of logistics services, and then estimate the trading psychologies and matching efforts of customers and 3PLs. And the trading psychologies of customers and 3PLs are shown in Table 1.

**Table 1**

Trading psychologies of customers and 3PLs

Trading psychology	Customer	Trading psychology	3PL
$Q_1^A$	$A_1, A_5$	$Q_1^B$	$B_1, B_4, B_7$
$Q_2^A$	$A_2, A_3, A_6$	$Q_2^B$	$B_2, B_3, B_5$
$Q_3^A$	$A_4, A_7$	$Q_3^B$	$B_6, B_8$

The real values, public values, matching efforts, lower bounds of acceptable matching value and waiting costs of customers are shown in Table 2.

**Table 2**

Customers' information determined by the 4PL

	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$
$h_i^A$	0.35	0.56	0.67	0.46	0.82	0.75	0.71
$h_i^P$	0.45	0.62	0.74	0.58	0.90	0.83	0.75
$\lambda_i$	0.50	0.66	0.69	0.81	0.52	0.61	0.79
$\alpha_i^c$	0.40	0.45	0.52	0.45	0.60	0.63	0.60
$c_i^A$	-	0.09	0.07	0	-	0.12	0

Meanwhile, the real values, public values, matching efforts, lower bounds of acceptable matching value and waiting costs of 3PLs are shown in Table 3.

**Table 3**

3PLs' information determined by the 4PL

	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7$	$B_8$
$g_j^B$	0.61	0.37	0.55	0.43	0.64	0.82	0.69	0.70
$g_j^P$	0.72	0.45	0.69	0.49	0.77	0.92	0.79	0.81
$w_j$	0.55	0.66	0.61	0.42	0.63	0.84	0.78	0.57
$\beta_j^c$	0.50	0.30	0.40	0.30	0.52	0.62	0.58	0.60
$c_j^B$	-	0.11	0.12	-	0.10	0	-	0

Then, using Eq. (1), the evaluation values of customers on 3PLs are obtained, shown in Table 4.

**Table 4**

Evaluation values of customers on 3PLs

	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7$	$B_8$
$A_1$	0.67	0.41	0.62	0.46	0.71	0.87	0.74	0.76
$A_2$	0.65	0.40	0.60	0.45	0.68	0.85	0.72	0.74
$A_3$	0.64	0.39	0.59	0.45	0.68	0.85	0.72	0.73
$A_4$	0.63	0.39	0.58	0.44	0.66	0.84	0.71	0.72
$A_5$	0.66	0.41	0.62	0.46	0.70	0.87	0.74	0.75
$A_6$	0.65	0.40	0.60	0.45	0.69	0.86	0.73	0.74
$A_7$	0.63	0.39	0.58	0.44	0.67	0.84	0.71	0.72

And using Eq. (2), the evaluation values of 3PLs on customers are calculated, shown in Table 5.

**Table 5**  
Evaluation values of 3PLs on customers

	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7$	$B_8$
$A_1$	0.40	0.42	0.41	0.42	0.40	0.39	0.39	0.39
$A_2$	0.59	0.60	0.60	0.60	0.59	0.58	0.59	0.58
$A_3$	0.71	0.72	0.71	0.72	0.70	0.70	0.70	0.70
$A_4$	0.52	0.54	0.53	0.54	0.52	0.51	0.51	0.51
$A_5$	0.86	0.88	0.87	0.88	0.86	0.85	0.85	0.85
$A_6$	0.79	0.81	0.80	0.81	0.79	0.78	0.78	0.78
$A_7$	0.73	0.74	0.73	0.74	0.73	0.73	0.73	0.73

Based on long-term cooperation experience, the 4PL determines the fairness thresholds of customers  $\epsilon_1^A = 0.30$ ,  $\epsilon_1^A = 0.20$ ,  $\epsilon_3^A = 0.15$  respectively, and the fairness thresholds of 3PLs  $\epsilon_1^B = 0.30$ ,  $\epsilon_2^B = 0.20$ ,  $\epsilon_3^B = 0.15$  respectively. Therefore, according to Eqs. (3)-(4), the matching possibilities of customers and 3PLs can be calculated respectively, as shown in Tables 6-7.

**Table 6**  
Matching possibilities of customers

	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7$	$B_8$
$A_1$	0.65	0.51	0.64	0.56	0	0	0	0
$A_2$	0.72	0	0	0.50	0.74	0.77	0.75	0.75
$A_3$	0.68	0	0.61	0	0.74	0.87	0.79	0.80
$A_4$	0.68	0	0.65	0	0.70	0	0	0
$A_5$	0.61	0	0.54	0	0.69	0.98	0.76	0.78
$A_6$	0.54	0	0	0	0.63	0.94	0.72	0.74
$A_7$	0.56	0	0	0	0.65	0.92	0.73	0.75

**Table 7**  
Matching possibilities of 3PLs

	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$	$B_6$	$B_7$	$B_8$
$A_1$	0	0.60	0	0.60	0	0	0	0
$A_2$	0.63	0.63	0	0.66	0	0	0.52	0
$A_3$	0.75	0	0.74	0.65	0.75	0	0.72	0.70
$A_4$	0.53	0.63	0.64	0.65	0.50	0	0	0
$A_5$	0.84	0	0	0	0.87	1.00	0.91	0.92
$A_6$	0.80	0	0	0	0.83	0.85	0.84	0.84
$A_7$	0.76	0	0.74	0	0.78	0.74	0.77	0.76

Furthermore, according to Eqs. (5)-(13), a two-sided matching model with the goal of maximizing the trading possibilities is established. The preference function proposed by the 4PL is  $P(z_1, z_2) = 0.5(z_1 - z_1^*)^2 + 0.5(z_2 - z_2^*)^2$ , where  $z_1^*$  and  $z_2^*$  denote the maximum possible values of the objective functions  $z_1$  and  $z_2$ . By adopting the variable step size algorithm, the optimal matching is obtained, which is  $\{(A_1, B_2), (A_2, B_4), (A_3, B_7), (A_4, B_3), (A_5, B_6), (A_6, B_8), (A_7, B_1)\}$ .

6.2. Comparison analysis

In order to demonstrate the effectiveness of the proposed method, we compare the proposed method with the existing method. Meanwhile, in the process of comparison analysis, five indexes including matching effort, waiting cost, fairness threshold,

trading psychology and stability are mainly taken into consideration. The comparison results are shown in Table 8.

**Table 8**  
Comparison of different two-sided matching methods

Ref.	Method	Matching effort	Waiting cost	Fairness	Trading psychology	Stability
Azevedo (2014)	Iterative algorithm	×	×	×	×	✓
Echenique and Galichon (2017)	Iterative algorithm	×	×	×	×	✓
Liang et al. (2020)	Iterative algorithm	×	×	×	×	×
Patro et al. (2020)	Iterative algorithm	×	×	✓	×	×
Liu and Li (2017)	Optimization model	×	×	✓	×	✓
Fan et al. (2017)	Optimization model	×	×	✓	×	✓
Zhang et al. (2019)	Optimization model	×	×	×	×	✓
Zhang et al. (2021b)	Optimization model	×	×	×	×	×

From Table 8, we observe:

(1) Existing two-sided matching methods can be roughly divided into two streams, i.e., optimization modeling and iterative algorithm, which do not consider matching effort, waiting cost and trading psychology. However, matching effort, waiting cost and trading psychology have a significant impact on the matching formation.

(2) When the matching effort is not considered, it is difficult to accurately obtain the evaluation value of one matching subject on the other. And when the waiting cost is not considered, it is equivalent to believing that the matching subject has no waiting cost, and then it is easy to mistake the eager matching subject for the patient matching subject. When the trading psychology is not considered, it is easy to cause that all matching subjects have the same stability demand, and then the stable matching scheme determined is not stable. When the fairness threshold is not considered, it is easy to lead to the situation that one party has high value while the other party has low value, which affects the matching efficiency.

(3) Matching effort, waiting cost, fairness, trading psychology and stability are considered in our proposed method. And different from the stability considered by existing two-sided matching decision methods, we allow different matching subjects to put forward personalized stability demands. Meanwhile, the fairness of the proposed two-sided matching decision-making method can not only balance the interests of both matching subjects, but also take into account the fairness of a single matching pair, which provides support for the final matching between the two matching subjects.

## 7. Conclusions

This paper investigates a matching problem between customers and 3PLs in the logistics market dominated by a 4PL. We propose a logistics matching method under the 4PL, in which trading psychologies and matching efforts of customers and 3PLs are considered. The main work of the paper is as follows:

(1) We consider trading psychology and matching efforts of customers and 3PLs in the two-sided matching process under a 4PL and redefine the blocking pair and stable matching considering trading psychology.

(2) On the basis of considering the public values and matching efforts of customers and 3PLs, we calculate the evaluation values of both matching sides, and calculate the trading possibilities of both customers and 3PLs on the basis of considering the fairness threshold. Then, to obtain optimal matching, we develop a bi-objective 0-1 programming model to maximize the trading possibilities of both customers and 3PLs respectively.

(3) We design a numerical example to verify the feasibility of the proposed method and analyze the effectiveness of the method by comparison analysis. Compared with existing studies, this paper considers the dominant role of a 4PL in two-sided logistics matching. Because of the different trading psychology of matching subjects (i.e., customers and 3PLs), the matching process is more in line with reality. Meanwhile, based on the analysis of different stability matching demands of customers and 3PLs, the matching model is established, which makes the proposed matching method more practical. In addition, by considering the fairness threshold and calculating trading possibilities of both customers and 3PLs, we can ensure that the matching result formed can be accepted by as many customers and 3PLs as possible.

In terms of future work, there are two interesting directions, which are as follows.

- (1) In reality, it is more comprehensive to evaluate the matching subjects from different dimensions. Therefore, we can introduce multi-attribute decision making into two-sided logistics matching, which would be an interesting research topic.
- (2) Due to the large number of matching subjects involved in the real logistics matching problem, how to embed the proposed matching model into the recommendation system to solve the large-scale logistics matching problem in 4PL is the focus of the next research work.

### Acknowledgements

This work is supported by the National Key R&D Program of China Grant No. 2021YFB3300900; the NSFC Key Supported Project of the Major Research Plan Grant No. 92267206; the NSFC Grant No. 62032013; the Fundamental Research Funds for State Key Laboratory of Synthetical Automation for Process Industries Grant No. 2013ZCX11; the 111 Project 2.0 (No. B08015).

### References

- Abraham, D. J., Irving, R. W., & Manlove, D. F. (2007). Two algorithms for the Student-Project Allocation problem. *Journal of Discrete Algorithms*, 5(1), 73-90. doi:10.1016/j.jda.2006.03.006
- Azevedo, E. M. (2014). Imperfect competition in two-sided matching markets. *Games and Economic Behavior*, 83, 207-223. doi:10.1016/j.geb.2013.11.009
- Chen, J., & Song, K. (2013). Two-sided matching in the loan market. *International Journal of Industrial Organization*, 31(2), 145-152. doi:10.1016/j.ijindorg.2012.12.002
- Dong, Y., Qian, X., Huang, M., & Ching, W.-K. (2022). Contracts design for serial delivery with connecting time spot: From a perspective of fourth party logistics. *International Journal of Industrial Engineering Computations*, 13(4), 523-542. doi:10.5267/j.ijiec.2022.7.003
- Echenique, F., & Galichon, A. (2017). Ordinal and cardinal solution concepts for two-sided matching. *Games and Economic Behavior*, 101, 63-77. doi:10.1016/j.geb.2015.10.002
- Fan, Z., Li, M., & Zhang, X. (2017). Satisfied two-sided matching: a method considering elation and disappointment of agents. *Soft Computing*, 22(21), 7227-7241. doi:10.1007/s00500-017-2725-1
- Gale, D., & Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *The American Mathematical Monthly*, 69(1), 9-15. doi:10.1080/00029890.1962.11989827
- Gattorna, J. (1998). *Strategic Supply Chain Alignment: Best Practice in Supply Chain Management* (6 ed.): Gower Publishing Company.
- Gruchmann, T., Pratt, N., Eiten, J., & Melkonyan, A. (2020). 4PL Digital Business Models in Sea Freight Logistics: The Case of FreightHub. *Logistics*, 4(2), 10. doi:10.3390/logistics4020010
- Hofmann, E., & Osterwalder, F. (2017). Third-Party Logistics Providers in the Digital Age: Towards a New Competitive Arena? *Logistics*, 1(2), 9. doi:10.3390/logistics1020009
- Jiang, Y., Fan, Z., Liang, H., & Sun, M. (2018). An optimization approach for existing home seller-buyer matching. *Journal of the Operational Research Society*, 70(2), 237-254. doi:10.1080/01605682.2018.1427432
- Jiang, Z., Ip, W. H., Lau, H. C. W., & Fan, Z. (2011). Multi-objective optimization matching for one-shot multi-attribute exchanges with quantity discounts in E-brokerage. *Expert Systems with Applications*, 38(4), 4169-4180. doi:10.1016/j.eswa.2010.09.079
- Klumpp, T. (2009). Two-sided matching with spatially differentiated agents. *Journal of Mathematical Economics*, 45(5-6), 376-390. doi:10.1016/j.jmateco.2009.03.002
- Li, J., Zheng, Y., Dai, B., & Yu, J. (2020). Implications of matching and pricing strategies for multiple-delivery-points service in a freight O2O platform. *Transportation Research Part E: Logistics and Transportation Review*, 136, 101871. doi:10.1016/j.tre.2020.101871
- Li, P., Wang, N., Wei, C., & Zhang, N. (2020). A two-sided matching method considering the lowest value of acceptability with regret theory for probabilistic linguistic term sets. *International Journal of Machine Learning and Cybernetics*, 12(4), 917-930. doi:10.1007/s13042-020-01211-6
- Liang, D., He, X., & Xu, Z. (2020). Multi-attribute dynamic two-sided matching method of talent sharing market in incomplete preference ordinal environment. *Applied Soft Computing*, 93. doi:10.1016/j.asoc.2020.106427
- Liang, H. M., Jiang, Y. P., Kong, D. C. (2015). Decision-making method on multiple targets of satisfied and stable two-sided matching considering the preference ordering. *Systems Engineering-Theory & Practice*, 35(6), 1535-1546.
- Ling, H., Fu, Y., Hua, M., & Lu, A. (2021). An Adaptive Parameter Controlled Ant Colony Optimization Approach for Peer-to-Peer Vehicle and Cargo Matching. *IEEE Access*, 9, 15764-15777. doi:10.1109/access.2020.3045558
- Liu, X., & Ma, H. (2015). A Two-Sided Matching Decision Model Based on Uncertain Preference Sequences. *Mathematical Problems in Engineering*, 2015, 1-10. doi:10.1155/2015/241379
- Liu, Y., Fan, Z., & Jiang, Y. (2018). Satisfied surgeon-patient matching: a model-based method. *Quality & Quantity*, 52(6), 2871-2891. doi:10.1007/s11135-018-0714-9
- Liu, Y., & Li, K. W. (2017). A two-sided matching decision method for supply and demand of technological knowledge. *Journal of Knowledge Management*, 21(3), 592-606. doi:10.1108/jkm-05-2016-0183
- Miao, Y., Du, R., Li, J., & Westland, J. C. (2019). A two-sided matching model in the context of B2B export cross-border e-

- commerce. *Electronic Commerce Research*, 19(4), 841-861. doi:10.1007/s10660-019-09361-8
- ÖZcan, E., & Ahiskali, M. (2020). 3PL Service Provider Selection with a Goal Programming Model Supported with Multicriteria Decision Making Approaches. *Gazi University Journal Of Science*, 33(2), 413-427. doi:10.35378/gujs.552070
- Patro, G. K., Biswas, A., Ganguly, N., Gummadi, K. P., & Chakraborty, A. (2020). *FairRec: Two-Sided Fairness for Personalized Recommendations in Two-Sided Platforms*. Paper presented at the Proceedings of The Web Conference 2020.
- Peng, Z., Shan, W., Guan, F., & Yu, B. (2016). Stable vessel-cargo matching in dry bulk shipping market with price game mechanism. *Transportation Research Part E: Logistics and Transportation Review*, 95, 76-94. doi:10.1016/j.tre.2016.08.007
- Pu, D., & Yuan, G. (2023). Two-sided matching model considering multi-information fusion of stakeholders. *Expert Systems with Applications*, 212, 118784. doi:10.1016/j.eswa.2022.118784
- Roth, A. E. (1985). Common and conflicting interests in two-sided matching market. *European Economic Review*, 27(1), 75-76.
- Steuer, R. E. (1986). *Multiple criteria optimization: Theory, computation, and application*. New York: Wiley.
- Vate, J. H. V. (1989). Linear programming brings marital bliss. *Operations Research Letters*, 8(3), 147-153.
- Wang, H., Huang, M., Wang, H., Feng, X., & Zhou, Y. (2022). Contract design for the fourth party logistics considering tardiness risk. *International Journal of Industrial Engineering Computations*, 13(1), 13-30. doi:10.5267/j.ijiec.2021.9.002
- Win, A., & Walters, D. (2008). The value a 4PL provider can contribute to an organisation. *International Journal of Physical Distribution & Logistics Management*, 38(9), 674-684. doi:10.1108/09600030810925962
- Xiang, J., Zhang, J., Cheng, T. C. E., Sallan, J. M., & Hua, G. (2019). Efficient Multi-Attribute Auctions Considering Supply Disruption. *Asia-Pacific Journal of Operational Research*, 36(03), 1950013. doi:10.1142/s0217595919500131
- Yue, Q., Zhang, L., Yu, B., Zhang, L. J., & Zhang, J. (2019). Two-Sided Matching for Triangular Intuitionistic Fuzzy Numbers in Smart Environmental Protection. *IEEE Access*, 7, 42426-42435. doi:10.1109/access.2019.2906560
- Zhang, Z., Gao, J., Gao, Y., & Yu, W. (2021a). Two-sided matching decision making with multi-granular hesitant fuzzy linguistic term sets and incomplete criteria weight information. *Expert Systems with Applications*, 168, 114311. doi:10.1016/j.eswa.2020.114311
- Zhang, Z., Gao, J., Gao, Y., & Yu, W. (2021b). Two-sided matching decision making with multi-granular hesitant fuzzy linguistic term sets and incomplete criteria weight information. *Expert Systems with Applications*, 168. doi:10.1016/j.eswa.2020.114311
- Zhang, Z., Kou, X., Palomares, I., Yu, W., & Gao, J. (2019). Stable two-sided matching decision making with incomplete fuzzy preference relations: A disappointment theory based approach. *Applied Soft Computing*, 84. doi:10.1016/j.asoc.2019.105730
- Zhang, Z., Kou, X., Yu, W., & Gao, Y. (2020). Consistency improvement for fuzzy preference relations with self-confidence: An application in two-sided matching decision making. *Journal of the Operational Research Society*, 72(8), 1914-1927. doi:10.1080/01605682.2020.1748529



© 2023 by the authors; licensee Growing Science, Canada. This is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license (<http://creativecommons.org/licenses/by/4.0/>).