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Transformation of Special Relativity into Differential Equation by Means of Power Series Method

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***Abstract:** Partial differential equations such as those involving Bessel differential function, Hermite's polynomial, and Legendre polynomial are widely used during the separation of the wave equation in cylindrical and spherical coordinates. Such functions are quite applicable to solve the wide variety of physical problems in mathematical physics and quantum mechanics, but until now, there has been no differential equation capable for handling the problems involved in the realm of special relativity. In order to avert such trouble in physics, this article presents a new kind of differential equation of the form:* $(c^2 - x^2)\, d^2y/dx^2 - 3x\, dy/dx - y = 0$, *where c is the speed of light in a vacuum. In this work, the solution of this equation has been developed via the power series method, which generates a formula that is completely compatible with relativistic phenomena happening in nature. In this highly exciting topic, the particular purpose of this paper is to define entirely a new differential equation to handle physical problems happening in the realm of special relativity.*

***Keywords:** Bessel Differential Equation, Hermite's Polynomial, Legendre Polynomial, Mass Variation, Special Relativity, Time Dilation Etc.*

I. INTRODUCTION

Differential equations such as Bessel's differential equation [1], Legendre's differential equation [2] and Hermite's differential equation [3] are fundamental mathematical tools to solve a wide variety of physical problems. Separation of wave equation in circular cylindrical coordinates and separation of Helmholtz's equation in spherical coordinates leads to Bessel's differential equation solutions, which are defined as Bessel functions. Legendre's differential equations occur in many areas of applied mathematics, physics, and chemistry in a physical situation with spherical geometry, such as the flow of an ideal fluid past a sphere. Hermite's polynomials are relevant for the analysis of the quantum harmonic oscillator [4]. Also, there are publications in which numerous space coordinates and time transformations in special relativity [5,6] are presented. The transformation obtained in article [7] is equivalent to the Lorentz transformation. It is only the Lorentz transformation differently written down after the change in the manner of time measurement in the inertial frames of reference. In work [8], the author obtained transformation equations through the synchronization of clocks in inertial frames by the geometric method. The author, in [9], outlined an innovative method to derive infinitely many dynamics in relativistic mechanics. In work [10], based on the direction perpendicular to the direction of the body's velocity in relation to the universal reference system, the whole class of time and position transformation was derived. Ref. [11] presents the original definition of acceleration in the special theory of relativity, while Ref. [12] develops the formalism for three-vector and four-vector relative velocity. Ref. [13] presents an analysis of various problems related to special theory of relativity, while Ref. [14] is investigating the subject of relativistic velocity addition. There are many papers on relativistic mechanics with significant theoretical results. Articles [15], [16], [17] presents research that relate the special theory of relativity with De-Broglie wavelength of a particle and electric permittivity and magnetic permeability of electromagnetic wave, while work [18] shows the variation of mass in gravitational field with the use of formula $E = mc^2$. Paper [19] presents an original derivation of Lorentz transformation in three-dimensional space. There are numerous publications conducted on theory of relativity with a universal frame of reference [20], [21], [22] and all possible experiments to falsify these theories were conducted in [23]. There are many published documents in special theory of relativity [24, 25] with important theoretical results, but all such publications have been written without the use of differential equations. Differential form of special relativity involving variation of mass, time dilation, space contraction is yet to be investigated. Hence, this paper investigates new concepts of special relativity to develop its all solutions by means of power series method. The remainder of this paper is as follows. Section 2 presents a new differential equation similar to that Bessel's differential equation, Legendre's differential equation and Hermite's differential equation and also derives the solution by means of power series method. Section 3 develops the formula of length contraction, time dilation and mass variation by using obtained solution of proposed differential equation. Finally, conclusions are summarized in section 4.

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II. METHODS

Power series method is an important mathematical tool to derive the solution of a differential equation. In order to deal with the mathematical complexity of special relativity, let us introduce a differential equation of the form:

$$(c^2 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0 \tag{1}$$

Where c is the speed of light in a vacuum.

This equation has a non-essential singularity at the point $x = c$. The equation can be integrated in a series of ascending powers of x. Let the solution be a form of

$$y = \sum_{r=0}^{\infty} a_r\, x^{k+r}, \tag{2}$$

Where $a_0 \neq 0$

Now, differentiating with respect to x, we get

$$\frac{dy}{dx} = \sum_{r=0}^{\infty} a_r\,(k+r)x^{k+r-1}$$

$$\frac{d^2y}{dx^2} = \sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2}$$

Now, substituting the value of y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ on equation (1) we get,

$$\text{or,}\quad (c^2 - x^2)\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - 3x\sum_{r=0}^{\infty} a_r\,(k+r)x^{k+r-1} - \sum_{r=0}^{\infty} a_r\, x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r} - 3\sum_{r=0}^{\infty} a_r\,(k+r)\,x^{k+r} - \sum_{r=0}^{\infty} a_r\, x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r} - \sum_{r=0}^{\infty}[3(k+r)+1]a_r\, x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,[(k+r)(k+r-1) + 3(k+r) + 1]x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,[(k+r)^2 - (k+r) + 3(k+r) + 1]x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,[(k+r)^2 + 2(k+r) + 1]x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,[(k+r)^2 - 1^2 - 1^2 + 2(k+r) + 1]x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,[(k+r-1)(k+r+1) + 2(k+r) + 2]x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,[(k+r-1)(k+r+1) + 2(k+r+1)]x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,[(k+r+1)(k+r-1+2)x^{k+r} = 0$$

$$\text{or,}\quad c^2\sum_{r=0}^{\infty} a_r\,(k+r)(k+r-1)x^{k+r-2} - \sum_{r=0}^{\infty} a_r\,[(k+r+1)(k+r+1)x^{k+r} = 0$$

$$\text{or,}\quad c^2[(k(k-1)a_0x^{k-2} + k(k+1)a_1x^{k-1} + c^2\sum_{r=0}^{\infty} a_{r+2}\,[(k+r+2)(k+r+2-1)x^{k+r+2-2}$$

$$-\sum_{r=0}^{\infty}\,[(k+r+1)\,(k+r+1)]\,a_r\,x^{k+r} = 0$$

$$\text{or, } c^2[(k(k-1)a_0x^{k-2} + k(k+1)a_1x^{k-1} + c^2\sum_{r=0}^{\infty} a_{r+2}\,[(k+r+2)(k+r+1)x^{k+r} - \sum_{r=0}^{\infty}\,[(k+r+1)^2]a_r\,x^{k+r} = 0 \tag{3}$$

Equating the coefficient of x^{k-2} to zero we get,

$c^2k(k-1)a_0 = 0$

This is the indicial equation and it gives the value of indicial constant as follow,

$k = 0, \quad k = 1$

Since $a_0 \neq 0$, being the coefficient of first term in solution. It is of some interest to examine the coefficient of x^{k-1} also. Equating this coefficient to zero, we get

$c^2k(k+1)a_1 - (k-1+1)^2a_{-1} = 0$

$c^2k(k+1)a_1 = 0 \qquad$ Since $a_{-1} = 0$

$k(k+1)a_1 = 0$

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Case (I): If $k = 0$ $then$ $a_1 \neq 0$

Case (II): $k = 1$ $then$ $a_1 = 0$

From equation (3), equating coefficient of x^{k+r} to zero we get,

$$c^2 a_{r+2}(k+r+2(k+r+1) = (k+r+1)^2 a_r$$

$$a_{r+2} = \frac{(k+r+1)^2 \quad a_r}{(k+r+2)(k+r+1)c^2}$$

$$a_{r+2} = \frac{(k+r+1) \quad a_r}{(k+r+2)c^2} \tag{4}$$

A. Solution for $k = 0$ and $a_0 \neq 0$

In previous section we show that when $k = 0$ $then$ $a_0 \neq 0$. Now, analyzing equation (4) under this condition we get,

$$a_{r+2} = \frac{(r+1) \quad a_r}{(r+2)c^2}$$

Put $r = 0$, $\quad a_2 = \frac{1}{2c^2} a_0$

Put $r = 2$,, $\quad a_4 = \frac{3}{4c^2} a_2$

$$a_4 = \frac{3.1}{4.2.1.(c^2)^2} a_0$$

Put $r = 4$, $\quad a_6 = \frac{5}{6c^2} a_4$

$$a_6 = \frac{5.3.1}{6.4.2.(c^2)^3} a_0$$

$$a_{2j} = \frac{1.3.5.7 \ldots..(2j-1)}{2.4.6.8 \ldots.2j\,(c^2)^j} a_0$$

$$a_{2j} = \frac{1.3.5.7 \ldots..(2j-1)}{2.4.6.8 \ldots.2j} \times \frac{2.4.6.8 \ldots.2j}{2.4.6.8 \ldots.2j} \frac{a_0}{(c^2)^j}$$

$$a_{2j} = \frac{1.2.3.4.5 \ldots \ldots.(2j-1).2j}{2^r(1.2.3.4 \ldots..j)2j(1.2.3.4 \ldots \ldots j)} \frac{a_0}{(c^2)^j}$$

$$a_{2j} = \frac{(2j)!}{2^{j+j}\,j!\,j!} \frac{a_0}{(c^2)^j}$$

$$a_{2j} = \frac{(2j)!}{4^j\,(j!)^2} \frac{a_0}{(c^2)^j}$$

Therefore, the real solution is

$$y_1 = \sum_{j=0}^{\infty} a_{2j}\,x^{2j}$$

$$y_1 = \sum_{j=0}^{\infty} \frac{(2j)!}{(c^2)^j\,4^j\,(j!)^2}\,a_0\ x^{2j}$$

$$y_1 = \sum_{j=0}^{\infty} \frac{(2j)!}{(c^2)^j\,4^j\,(j!)^2}\,a_0\ x^{2j} \tag{5}$$

This is one of the solutions of equation (1).

B. Solution for $k = 0$ and $a_1 \neq 0$

From case (I) obtained in previous section just above equation (4), we have $k = 0$ $then$ $a_1 \neq 0$. Now, analyzing equation (4) under this condition we get,

$$a_{r+2} = \frac{(r+1)}{(r+2)\,c^2} a_r$$

Put $r = 1$, $\quad a_3 = \frac{2}{3c^2} a_1$

Put $r = 3$, $\quad a_5 = \frac{4}{5c^2} a_3$

$$a_5 = \frac{4.2}{5.3.(c^2)^2} a_1$$

Put $r = 5$, $\quad a_7 = \frac{6}{7c^2} a_5$

$$a_7 = \frac{6.4.2}{7.5.3.(c^2)^3} a_1$$

$$a_{2j+1} = \frac{2.4.6 \ldots.2j}{3.5.7 \ldots \ldots.(2j+1)\,c^{2j}} a_1$$

$$a_{2j+1} = \frac{2.4.6 \ldots.2j}{3.5.7 \ldots \ldots.(2j+1)} \times \frac{2.4.6 \ldots.2j}{2.4.6 \ldots..2j} \frac{a_1}{c^{2j}}$$

$$a_{2j+1} = \frac{2^j(1.2.3.4 \ldots \ldots j)2^j(1.2.3.4 \ldots \ldots j)}{2.3.4.5.6.7 \ldots \ldots 2j.(2j+1)} \frac{a_1}{c^{2j}}$$

$$a_{2j+1} = \frac{2^{2j}\,j!\,j!}{(2j+1)!} \frac{a_1}{c^{2j}}$$

$$a_{2j+1} = \frac{4^j\,(j!)^2}{(2j+1)!} \frac{a_1}{c^{2j}}$$

Therefore, the real solution is

$$y_2 = \sum_{j=0}^{\infty} a_{2j+1}\,x^{2j+1}$$

$$y_2 = \sum_{j=0}^{\infty} \frac{4^j\,(j!)^2}{(2j+1)!} \frac{a_1}{c^{2j}} x^{2j+1} \tag{6}$$

The is another one solution of equation (1).

C. Solution for $k = 1$

From case (II) obtained in previous section just above equation (4), we have $k = 1$ $then$ $a_0 \neq 0$ but $a_1 = 0$. Now, analyzing equation (4) under this condition we get,

$$a_{r+2} = \frac{r+2}{r+3} a_r$$

Put $r = 0$, $\quad a_2 = \frac{2}{3c^2} a_0$

Put $r = 1$, $\quad a_3 = \frac{3}{4} a_1 = 0$

Put $r = 2$, $\quad a_4 = \frac{4}{5} a_2 = \frac{4.2. \quad a_0}{5.3 \ldots.(c^2)^2}$

Put $r = 3$, $\quad a_5 = \frac{5}{6} a_3 = 0$

Put $r = 4$, $\quad a_6 = \frac{6}{7} a_4 = \frac{6.4.2. \quad a_0}{7.5.3 \ldots.(c^2)^3}$

$$a_{2j} = \frac{2.4.6 \ldots.2j}{3.5.7 \ldots \ldots.(2j+1)} \frac{a_0}{c^{2j}}$$

$$a_{2j} = \frac{2.4.6 \ldots.2j}{3.5.7 \ldots \ldots.(2j+1)} \times \frac{2.4.6 \ldots.2j}{2.4.6 \ldots.2j} \frac{a_0}{c^{2j}}$$

$$a_{2j} = \frac{2^j(1.2.3.4 \ldots \ldots j) \times 2^j(1.2.3.4 \ldots \ldots j)}{2.3.4.5.6.7 \ldots \ldots.(2j+1)} \frac{a_0}{c^{2j}}$$

$$a_{2j} = \frac{2^{2j}\,j!\,j!}{(2j+1)!} \frac{a_0}{c^{2j}}$$

$$a_{2j} = \frac{4^j\,(j!)^2}{(2j+1)!} \frac{a_0}{c^{2j}}$$

The real solution is

$$y_3 = \sum_{j=0}^{\infty} a_{2j}\,x^{2j+1}$$

$$y_3 = \sum_{j=0}^{\infty} \frac{4^j\,(j!)^2}{(2j+1)!} \frac{a_0}{c^{2j}} x^{2j+1} \tag{7}$$

This is the solution of equation (1) for $k = 1$.In previous section we show that when $k = 0$ $then$ $a_0 \neq 0$. Now, analyzing equation (4) under this condition we get,

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$$a_{r+2} = \frac{(r+1)\quad a_r}{(r+2)c^2}$$

Put $r = 0$, $\quad a_2 = \frac{1}{2\,c^2} a_0$

Put $r = 2$,, $\quad a_4 = \frac{3}{4\,c^2} a_2$

$$a_4 = \frac{3.1}{4.2.1.(c^2)^2} a_0$$

Put $r = 4$, $\quad a_6 = \frac{5}{6\,c^2} a_4$

$$a_6 = \frac{5.3.1}{6.4.2.(c^2)^3} a_0$$

$$a_{2j} = \frac{1.3.5.7\ldots..(2j-1)}{2.4.6.8\ldots.2j\ (c^2)^j} a_0$$

$$a_{2j} = \frac{1.3.5.7\ldots..(2j-1)}{2.4.6.8\ldots.2j} \times \frac{2.4.6.8\ldots.2j}{2.4.6.8\ldots.2j} \frac{a_0}{(c^2)^j}$$

$$a_{2j} = \frac{1.2.3.4.5\ldots\ldots.(2j-1).2j}{2^r(1.2.3.4\ldots..j)2j(1.2.3.4\ldots\ldots j)} \frac{a_0}{(c^2)^j}$$

$$a_{2j} = \frac{(2j)!}{2^{j+j}\ j!\ j!} \frac{a_0}{(c^2)^j}$$

$$a_{2j} = \frac{(2j)!}{4^j\ (j!)^2} \frac{a_0}{(c^2)^j}$$

Therefore, the real solution is

$$y_1 = \sum_{j=0}^{\infty} a_{2j}\, x^{2j}$$

$$y_1 = \sum_{j=0}^{\infty} \frac{(2j)!}{(c^2)^j\, 4^j\, (j!)^2} a_0\quad x^{2j} \tag{5}$$

This is one of the solutions of equation (1).

III. RESULT AND DISCUSSION

The use of the power series method on differential equation (1) generates different solutions, namely equations (5), (6), and (7). These equations play a crucial role for interpreting different ground-breaking results in physics. For convenience, let us consider the solution to equation (5).

$$y_1 = \sum_{j=0}^{\infty} \frac{(2j)!}{(c^2)^j\, 4^j\, (j!)^2} a_0\ x^{2j}$$

$$y_1 = a_0 + \frac{(2)!}{c^2\, 4} a_0 x^2 + \frac{(4)!}{c^4\, 4^2 (2!)^2} a_0 x^4 + \frac{(6)!}{c^6\, 4^3 (3!)^2} a_0 x^6 \ldots$$

$$y_1 = a_0 + \frac{1}{2c^2} a_0 x^2 + \frac{1.2.3.4}{16 \times 4\, c^4} a_0 x^4 + \frac{1.2.3.4.5.6}{64 \times 36\, c^6} a_0 x^6 \ldots$$

$$y_1 = a_0 + \frac{1}{2c^2} a_0 x^2 + \frac{3}{8\,c^4} a_0 x^4 + \frac{5}{16\,c^6} a_0 x^6 + \cdots \tag{8}$$

In this series, y_1 and x are variable parameters, while c is constant.

For convenience, let us assign the following variable:

$y_1 = m$ (body's relativistic mass)

$x = v$ (Velocity of body)

$a_0 = m_0$ (Rest mass of body)

Now equation (8) becomes,

$$m = m_0 + \frac{1}{2c^2} m_0 v^2 + \frac{3}{8} m_0 \frac{v^4}{c^4} + \frac{5}{16} m_0 \frac{v^6}{c^6} + \cdots$$

$$m = m_0 \left(1 + \frac{1}{2c^2} v^2 + \frac{3}{8} \frac{(v^2)^2}{(c^2)^2} + \frac{5}{16} \frac{(v^2)^3}{(c^2)^3} + \cdots \right) \tag{9}$$

As we know,

$$(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \cdots$$

Putting x = $\frac{v^2}{c^2}$ we get $x = \frac{v^2}{c^2}$

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2c^2} v^2 + \frac{3}{8} \frac{(v^2)^2}{(c^2)^2} + \frac{5}{16} \frac{(v^2)^3}{(c^2)^3} + \cdots \tag{10}$$

Using this expression in equation (9)

$$m = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{11}$$

This process reveals that differential equation (1) generates relativistic mass when we substitute $y = m$ and $x = v$. Therefore, equation (1) becomes,

$$(c^2 - v^2)\frac{d^2 m}{dv^2} - 3v\frac{dm}{dv} - m = 0 \tag{12}$$

This equation plays an important role to perceive the relativistic phenomenon in nature because its solution gives the relativistic mass variation formula namely equation (11).

Similarly, from equation (8), we have,

$$y_1 = a_0 + \frac{1}{2c^2} a_0 x^2 + \frac{3}{8\,c^4} a_0 x^4 + \frac{5}{16} a_0 \frac{x^6}{c^6} + \cdots$$

In this series, we assign $y_1 = \frac{1}{L}, a_o = \frac{1}{L_0}$ $and\ x = v, we\ get$

$$\frac{1}{L} = \frac{1}{L_0} + \frac{1}{2c^2} \frac{1}{L_0} v^2 + \frac{3}{8} \frac{1}{L_0} \frac{v^4}{c^4} + \frac{5}{16} \frac{1}{L_0} \frac{v^6}{c^6} + \cdots$$

$$\frac{1}{L} = \frac{1}{L_0}\left(1 + \frac{1}{2c^2} v^2 + \frac{3}{8} \frac{(v^2)^2}{(c^2)^2} + \frac{5}{16} \frac{(v^2)^3}{(c^2)^3} + \cdots \right)$$

Using equation (10) we get,

$$\frac{1}{L} = \frac{1}{L_0}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$\frac{1}{L} = \frac{1}{L_0\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} \tag{13}$$

This process shows that if we substitute $y = \frac{1}{L}$ and $x = v$ in equation (1), we get the corresponding length contraction formula. Therefore, equation (1) becomes,

$$(c^2 - v^2)\frac{d^2}{dv^2}\left(\frac{1}{L}\right) - 3v\frac{d}{dv}\left(\frac{1}{L}\right) - \frac{1}{L} = 0 \tag{14}$$

This equation gives the relativistic length contraction in differential form. Also, let us assign the following variable in equation (8),

$y_1 = T$ (Relativistic time)

$x = v$ (Velocity of frame of reference)

$a_0 = T_0$ (Proper time)

Now equation (8) becomes,

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$$T = T_0 + \frac{1}{2c^2}T_0 v^2 + \frac{3}{8}T_0\frac{v^4}{c^4} + \frac{5}{16}T_0\frac{v^6}{c^6} + \cdots$$

$$T = T_0\left(1 + \frac{1}{2c^2}v^2 + \frac{3}{8}\frac{(v^2)^2}{(c^2)^2} + \frac{5}{16}\frac{(v^2)^3}{(c^2)^3} + \cdots\right) \quad (15)$$

Using equation (10) we get,

$$T = T_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$T = \frac{T_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (16)$$

This process reveals that differential equation (1) generates relativistic time dilation formula when we substitute $y = T$ and $x = v$. Therefore, equation (1) becomes,

$$(c^2 - v^2)\frac{d^2T}{dv^2} - 3v\frac{dT}{dv} - T = 0 \quad (17)$$

This equation plays an important role to perceive the relativistic.

IV. CONCLUSIONS

In this paper, we have presented the relativistic form of length contraction, time dilation, and mass variation by means of a partial differential equation, which is very close to the form of Bessel's differential function, Legendre's polynomial, and Hermite's polynomial. The power series method has been used as the main mathematical tool to achieve relativistic formulas. In order to develop a differential form of relativistic mass variation, we assigned mass as a dependent variable and velocity as an independent variable to get the following formula as explained in equation (12).

$$(c^2 - v^2)\frac{d^2m}{dv^2} - 3v\frac{dm}{dv} - m = 0$$

Similarly, differential equation of length contraction and time dilation can be written from equations (14) and (17) and are of the form:

$$(c^2 - v^2)\frac{d^2}{dv^2}\left(\frac{1}{L}\right) - 3v\frac{d}{dv}\left(\frac{1}{L}\right) - \frac{1}{L} = 0$$

$$(c^2 - v^2)\frac{d^2T}{dv^2} - 3v\frac{dT}{dv} - T = 0$$

The relativistic results generated through such differential equations present a better version of relativistic mass variation, length contraction and time dilation. We hope relativistic equations obtained by such a different way will enrich the scientific literatures connected by relativistic mechanics. And as a final point, we should point out that we restricted our consideration to the case where moment-generating functions and orthogonality solutions exist. These properties of proposed partial differential equation will appear in our future work.

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Publications:

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