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A NUMERICAL FRAMEWORK FOR MODELLING TIRE MECHANICS ACCOUNTING FOR COMPOSITE MATERIALS, LARGE STRAINS AND FRICTIONAL CONTACT

A. Cornejo, L.G. Barbu, P. Wriggers, S. Oller and E. Oñate

CIMNE
EXCELENCIA SEVERO OCHOA

FATIGUE 4LIGHT

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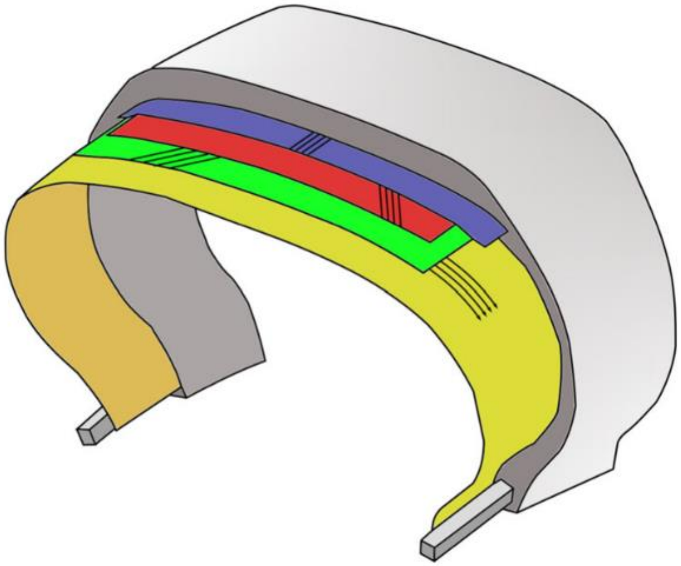
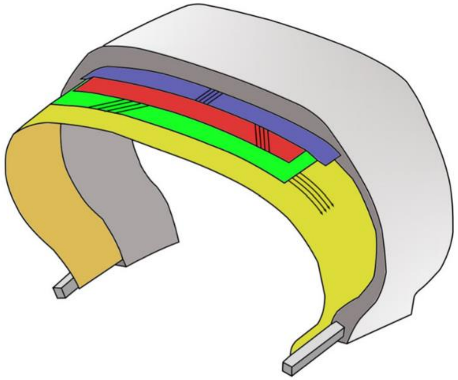
We present a general framework for the analysis and modelling of **tires**:

- Frictional contact (ALM+mortar)
- Finite Strains
- Composite materials
- Incompressible matrix + synthetic/metal fibers (hyper-elasticity)



A numerical framework for modelling tire mechanics accounting for composite materials, large strains and frictional contact, A. Cornejo, V. Mataix, P. Wriggers, L. Barbu and E. Oñate, *Computational Mechanics* (2023) https://doi.org/10.1007/s00466-023-02353-4

To efficiently model **tires** we need:



- Combine incompressible rubber and stiff cords (x10000 stiffer)
- Large displacements and finite strains in composites
- Micro-buckling of the fibers
- Automatic orientation and combination of fibers

Quasi-incompressible rubber (Neo-Hookean)

$$\Psi = \tilde{\Psi} + \Psi_{vol} = C_1(\tilde{I}_C^{(1)} - 3) + \frac{1}{2}\kappa(J-1)^2,$$

where $C_1 = \mu/2$ (Lamé constant) and $\tilde{I}_C^{(1)} = J^{-2/3} I_C^{(1)}$

Differentiating with respect to **C** (Cauchy-Green tensor):

$$\mathbf{S} = 2C_1 J^{-2/3}(\mathbf{I} - \frac{1}{3} I_C^{(1)} \mathbf{C}^{-1}) - pJ\mathbf{C}^{-1}$$

To deal with incompressibility locking: Total Lagrangian mixed *u*-*p* element

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{up} \\ \mathbf{K}_{pu} & K_{pp} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{u} \\ \Delta p \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{ext} \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbf{f}_u^{int} \\ f_p^{int} \end{bmatrix}$$

Since pressure is ctant within the FE we **condensate** DoFs

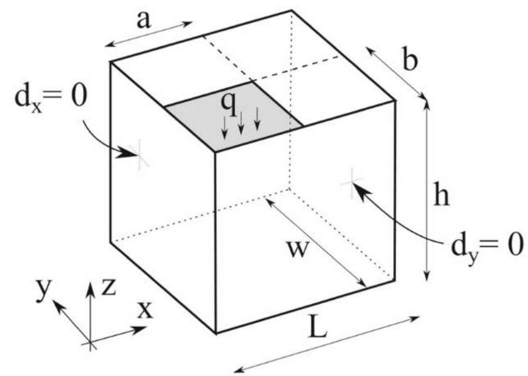
$$\bar{\mathbf{K}} \Delta \mathbf{u} = \mathbf{f}^{ext} - \bar{\mathbf{f}}^{int}$$

$$\bar{\mathbf{K}} = \mathbf{K}_{uu} - \mathbf{K}_{up} K_{pp}^{-1} \mathbf{K}_{up}^{T}$$

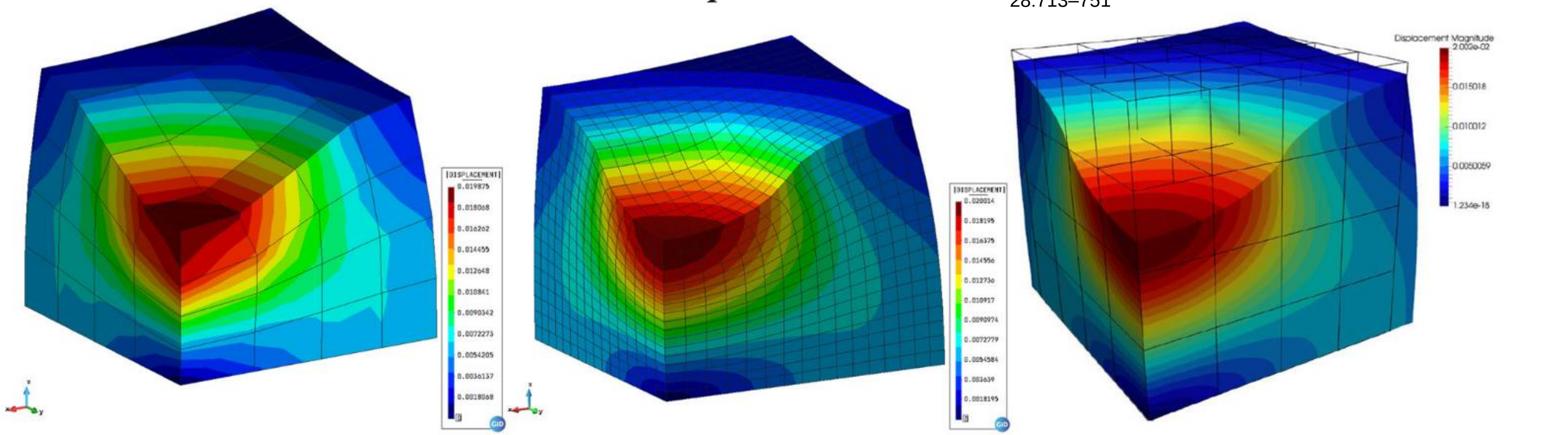
$$\bar{\mathbf{f}}^{int} = \mathbf{f}_u^{int} - \mathbf{K}_{up} K_{pp}^{-1} f_p^{int}$$

Constitutive model for tires

To deal with incompressibility locking: Total Lagrangian mixed *u-p* element



Schroder J et al (2021) A selection of benchmark problems in solid mechanics and applied mathematics. Arch Comput Methods Eng 28:713–751



(a) Implemented FE, coarse mesh (b) Implemented FE, refined mesh (c) Schroder et al.

The dimensions in mm are: h = 50, w = 50, l = 50, a = 25, b = 25 and the load q = 3 MPa. Lamé parameters of the material are λ = 499.92568 MPa and μ = 1.61148 MPa.

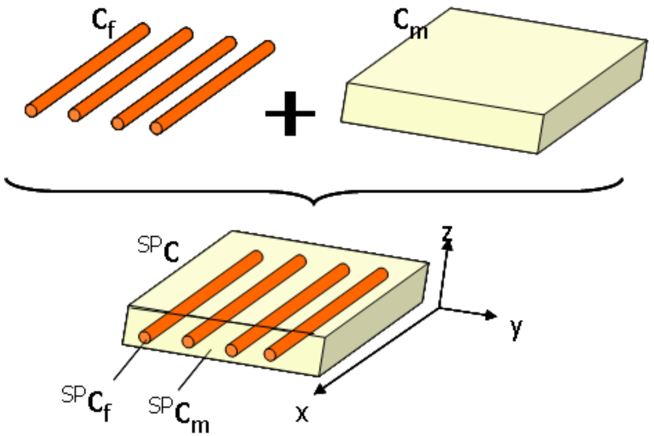
Fabric and steel fibers: Compressible New-Hookean

$$\Psi = C_1(I_C^{(1)} - 3) - C_1\, ln(J) + \frac{C_2}{2}(J - 1)^2,$$

Composite materials modelling: Serial-Parallel Rule of Mixtures

$$\text{Parallel behaviour}: \begin{cases} {}^c\mathbf{E}_P = {}^f\mathbf{E}_P = {}^m\mathbf{E}_P \\ {}^c\mathbf{S}_P = {}^fk\,{}^f\mathbf{S}_P + {}^mk\,{}^m\mathbf{S}_P \end{cases}$$

$$\text{Serial behaviour}: \begin{cases} {}^c\mathbf{E}_S = {}^fk\,{}^f\mathbf{E}_S + {}^mk\,{}^m\mathbf{E}_S \\ {}^c\mathbf{S}_S = {}^f\mathbf{S}_S = {}^m\mathbf{S}_S \end{cases}$$

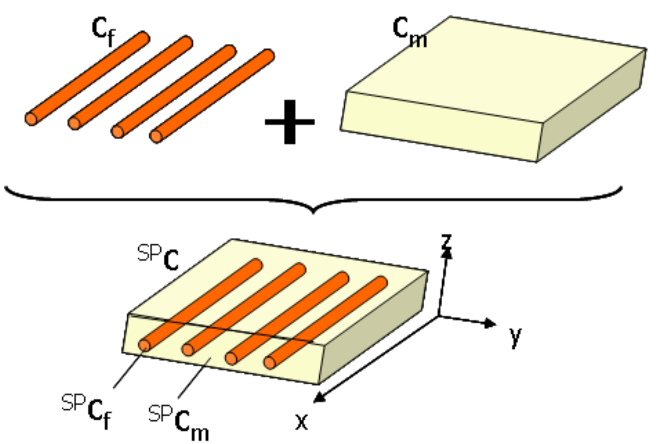


$$\mathbf{E} = \mathbf{E}_p + \mathbf{E}_s\ ,\quad \mathbf{S} = \mathbf{S}_p + \mathbf{S}_s$$

Since the stiffnesses are very different: Unstable

$$\text{Parallel behaviour}: \begin{cases} {}^{c}\mathbf{E}_P = {}^{f}\mathbf{E}_P = {}^{m}\mathbf{E}_P \\ {}^{c}\mathbf{S}_P = {}^{f}k\,{}^{f}\mathbf{S}_P + {}^{m}k\,{}^{m}\mathbf{S}_P \end{cases}$$

$$\text{Serial behaviour}: \begin{cases} {}^{c}\mathbf{E}_S = {}^{f}k\,{}^{f}\mathbf{E}_S + {}^{m}k\,{}^{m}\mathbf{E}_S \\ {}^{c}\mathbf{S}_S = {}^{f}\mathbf{S}_S = {}^{m}\mathbf{S}_S \end{cases}$$



$$ {}^{c}\mathbf{S} = \underbrace{{}^{f}k({}^{f}\mathbf{S}_p) + {}^{m}k({}^{m}\mathbf{S}_p)}_{\text{Parallel behaviour}} + \underbrace{{}^{m}\mathbf{S}_s}_{\text{Serial behaviour}} $$

What if we have several layers?

Algorithm 1 Multi-layered composite material constitutive law integration.

procedure → INSIDE INTEGRATION POINT LOOP WITHIN THE ELEMENT.

Rotate $\mathbf{F}$ and/or strain $\mathbf{E}$ to elemental local axes → ${}^{c}\mathbf{E}_{loc}$

for layer **do**

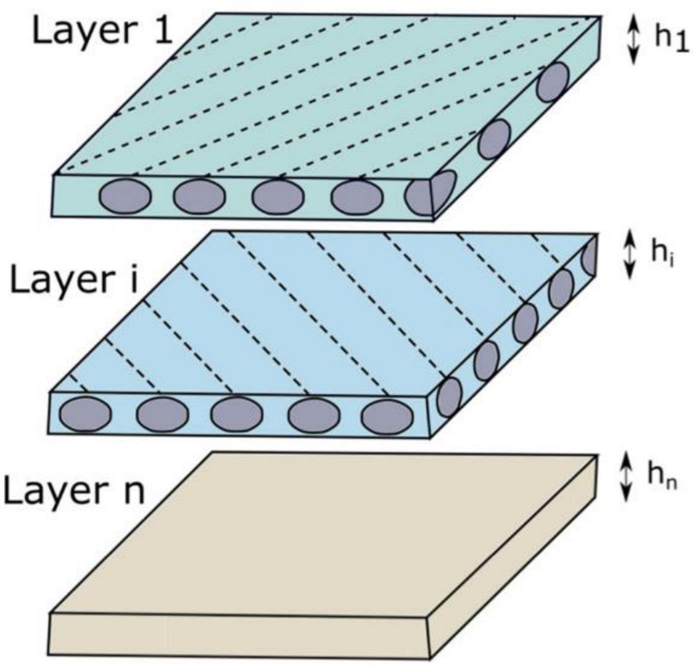
Rotate local elemental strain $\mathbf{E}_{loc}$ to layer local axes → $\mathbf{E}_{lay,loc}$

Integrate the constitutive law of the layer, simple material or SP-RoM algorithm → $\mathbf{S}_{lay,loc}$

Rotate layer strain and stresses to global layer axes, → $\mathbf{S}_{lay,glob}$

Compute composite stress ${}^{c}\mathbf{S} = \sum_{layer=0}^{n} {}^{lay}k\,\mathbf{S}_{lay,glob}$

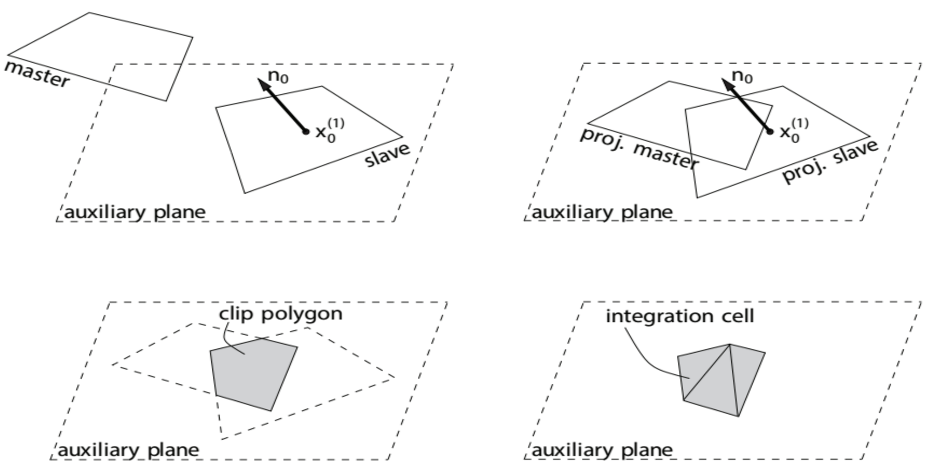
Rotate $\mathbf{F}$, strain and stress to elemental global axes → ${}^{c}\mathbf{S}_{glob}$

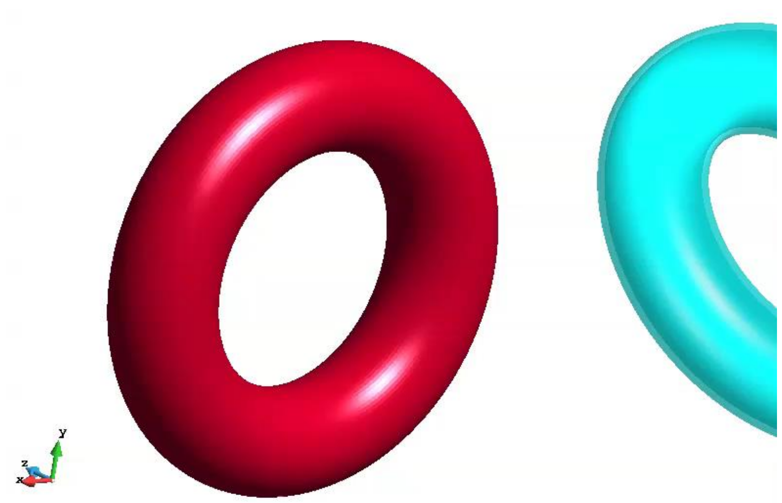


$$ {}^{c}\mathbf{S} = \underbrace{{}^{f}k({}^{f}\mathbf{S}_p) + {}^{m}k({}^{m}\mathbf{S}_p)}_{\text{Parallel behaviour}} + \underbrace{{}^{m}\mathbf{S}_s}_{\text{Serial behaviour}} $$

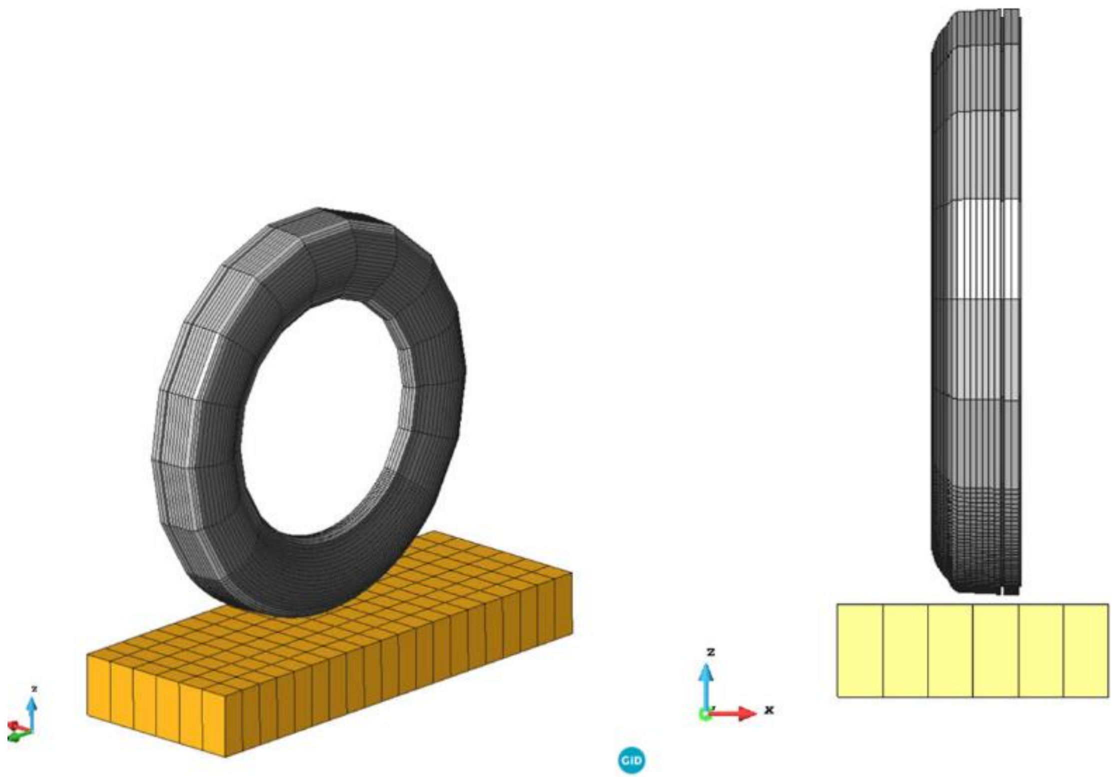
Augmented Lagrangian Multipliers + Mortar gap estimation

$$\delta \mathcal{L}_{co}(\mathbf{u}, \boldsymbol{\lambda}) = \int_{\Gamma_c^1} \begin{cases} \bar{\lambda}_n \cdot \delta g_n + k g_n \delta \lambda_n + \bar{\boldsymbol{\lambda}}_\tau \cdot \delta \mathbf{v}_{\tau,rel} + \mathbf{v}_{\tau,rel} \cdot \delta \bar{\boldsymbol{\lambda}}_\tau & \text{if } \|\bar{\boldsymbol{\lambda}}_\tau\| \leq -\mu \bar{\lambda}_n \text{ (Contact stick zone)} \\ \bar{\lambda}_n \cdot \delta g_n + k g_n \delta \lambda_n - \mu \bar{\lambda}_n \frac{\bar{\boldsymbol{\lambda}}_\tau}{\|\bar{\boldsymbol{\lambda}}_\tau\|} \delta \mathbf{v}_{\tau,rel} - \frac{k \boldsymbol{\lambda}_\tau + \mu \bar{\lambda}_n \frac{\bar{\boldsymbol{\lambda}}_\tau}{\|\bar{\boldsymbol{\lambda}}_\tau\|}}{\varepsilon_\tau} \delta \boldsymbol{\lambda}_\tau & \text{if } \|\bar{\boldsymbol{\lambda}}_\tau\| > -\mu \bar{\lambda}_n \text{ (Contact slip zone)} \\ -\frac{k^2}{\varepsilon_n} \lambda_n \delta \lambda_n - \frac{k^2}{\varepsilon_\tau} \boldsymbol{\lambda}_\tau \delta \boldsymbol{\lambda}_\tau & \text{if } \bar{\lambda}_n > 0 \text{ (Gap zone)} \end{cases} \mathrm{d}\Gamma_{co}^i$$





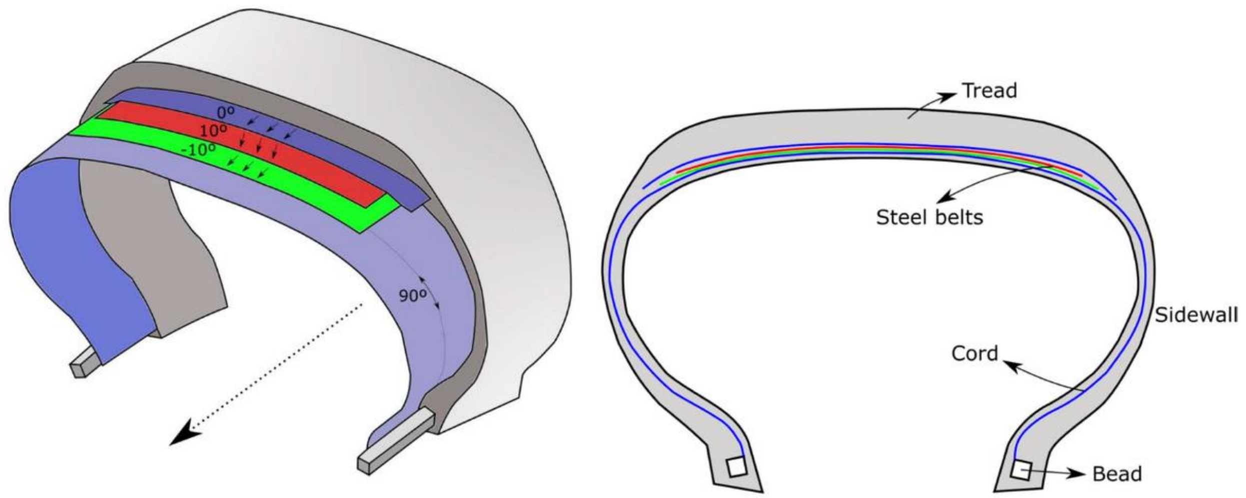
Tire simulation: Goodyear 195/65R15 tire

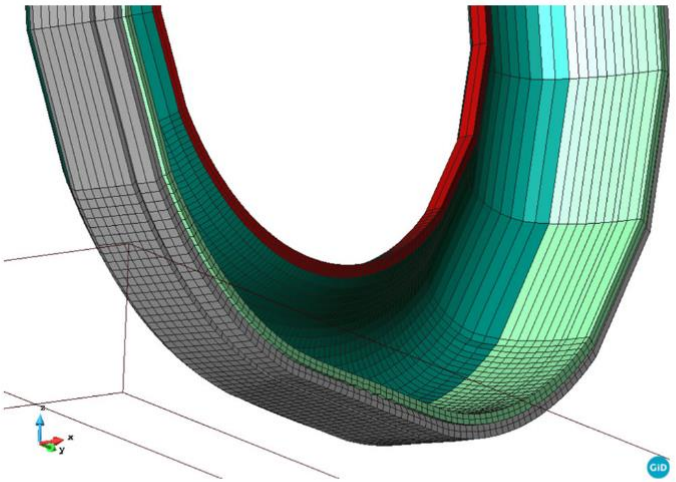


(a) (b)

Holscher H et al (2004) Modeling of pneumatic tires by a finite element model for the development a tire friction remote sensor. In: Center of Advanced European studies and Research

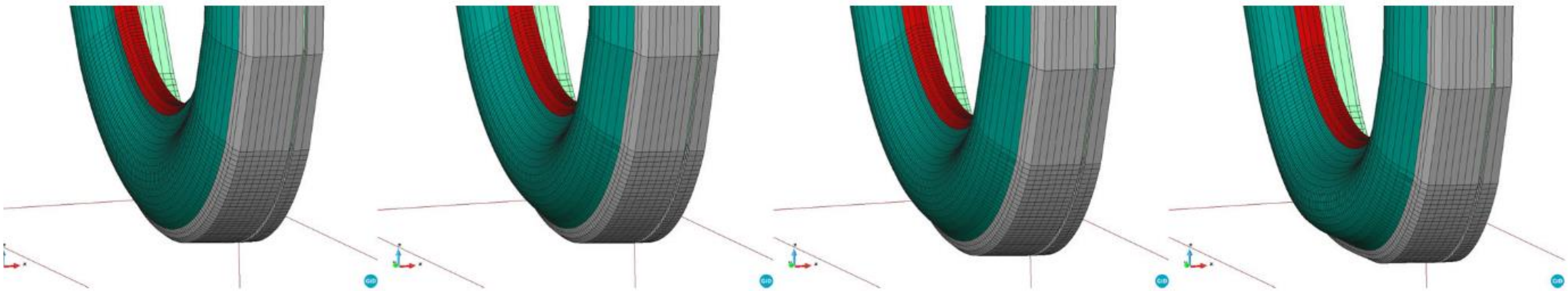
Tire simulation: Goodyear 195/65R15 tire



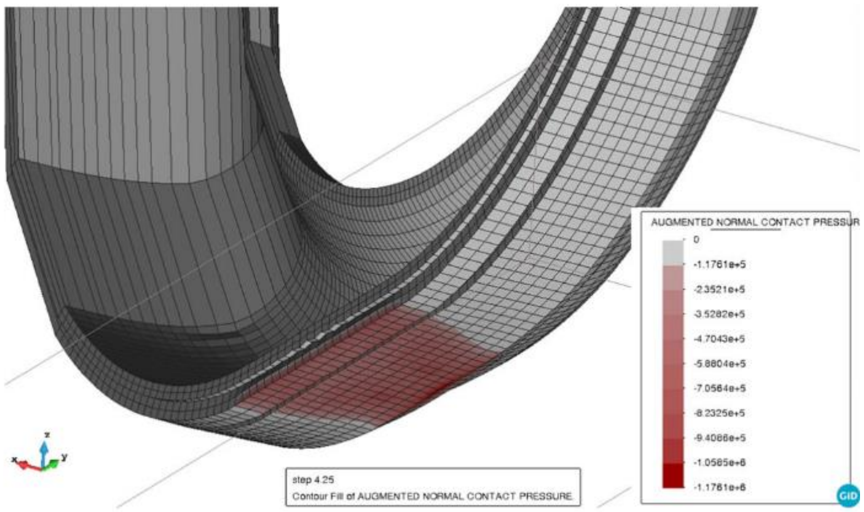


Tread				
Layer Id	Layer volumetric participation	Euler angles	Matrix material, Vol. participation	Fibre material, Vol. participation
1	1.0	(0,0,0)	Rubber (tread), 1.0	-
Steel/fibre composite (tire core)				
Layer Id	Layer volumetric participation	Euler angles	Matrix material, Vol. participation	Fibre material, Vol. participation
1	0.5	(0,0,0)	Rubber (core), 0.84	Fibre cords, 0.16
2	0.25	(0,20,0)	Rubber (core), 0.828	Steel belts, 0.172
3	0.5	(0,-20,0)	Rubber (core), 0.828	Steel belts, 0.172
Sidewall				
Layer Id	Layer volumetric participation	Euler angles	Matrix material, Vol. participation	Fibre material, Vol. participation
1	1.0	(0,0,0)	Rubber (sidewall), 0.62	Fibre cords, 0.38

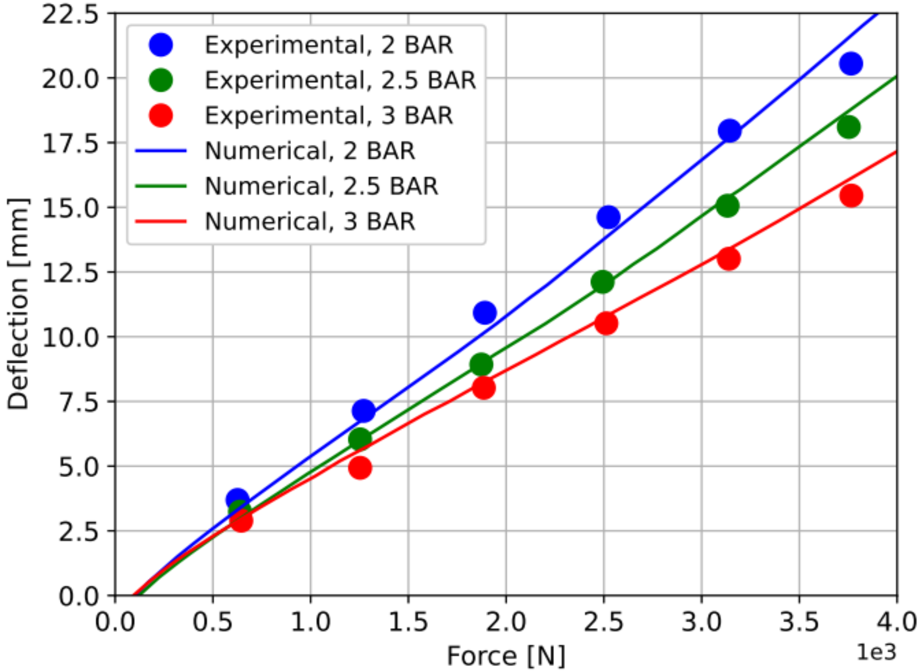
Tire simulation: Goodyear 195/65R15 tire



(a) No internal pressure (b) Internal 2.5 Bar pressure (c) 2000 N force (d) 4000 N force

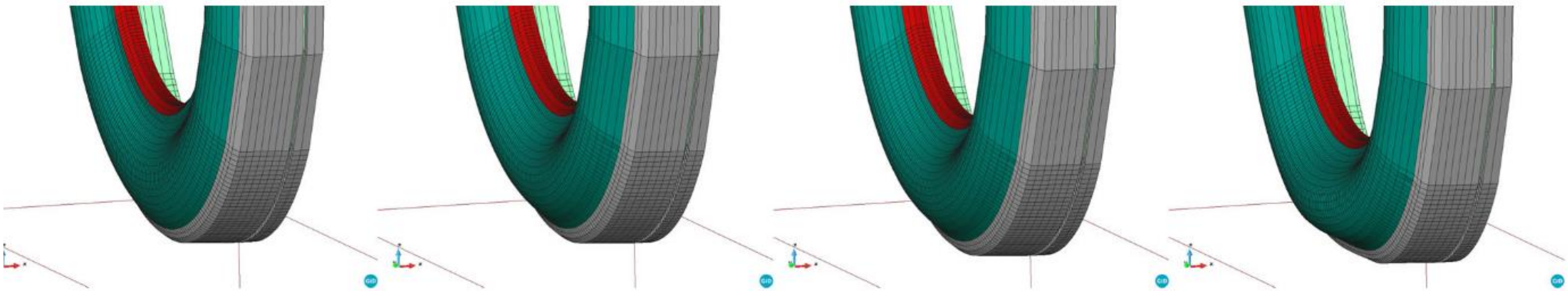


(a) 2000 N reaction force



Numerical example

Tire simulation: Goodyear 195/65R15 tire



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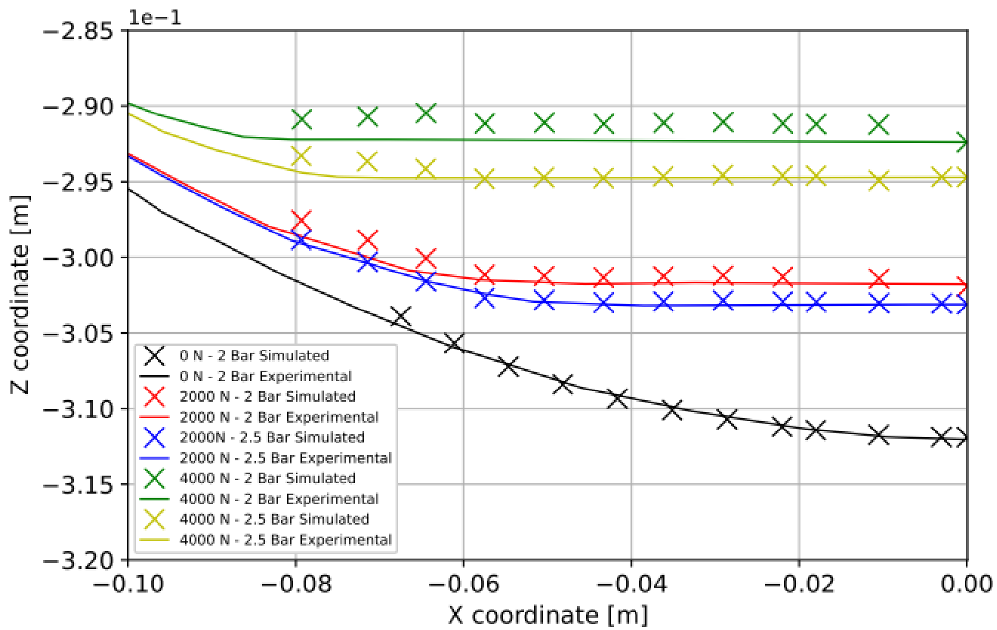

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