DISSERTATION

HYDRAULIC AND WATER QUALITY MODEL FOR A RIVER NETWORK

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ABSTRACT

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A river network system consisting of branches and loops is sometimes complicated by downstream effects from tides, lakes, and because of this, management of water quality, sediment control, and floods, in such rivers is a difficult task. Development of tools to aid in the management decision-making process is an important area of research; ultimately resulting in more reliable results. River behavior can be modeled in detail (one-,two-,or three-dimensional models) with a digital computer using numerical methods. Usually the level of detail is determined by the size of the system. Large system models are restricted in size and detail due to the high cost and storage requirements of the computer.

A model was developed to simulate the hydraulic behavior and water quality of a river network on a one-dimensional representation. The two complete St. Venant equations and the transport equation were solved by the finite difference method. The transport equation utilizes the advection, dispersion, and source and sink terms. The system of equations resulting from use of an implicit scheme was solved by a modified Gauss elimination procedure.

The model can simulate biochemical oxygen demand, dissolved oxygen, or any other conservative substance. The basic equations are solved; thus, the simulation of other substances can be added to the model by including the mathematical description of the reaction processes in the source and sink terms of the transport equation.

The hydraulic module of the model was adjusted and verified with data from the Jacui Delta, Brazil. Good agreement between the calculated results and the observed data resulted. The water quality model was tested under hypothetical conditions for the same Delta in order to demonstrate the utility of the mathematical model in making decisions at the management level. This model is a mathematical method that can be used in large systems of variable complexity to help in understanding their processes, controlling data measurements, and reaching sound management decisions.

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LIST OF SYMBOLS

A = the cross-sectional area of the channel

A_f = the flood surface area of the reach

A_s = the surface area at the junction

A_x^y = $(\frac{\partial A}{\partial x})y$, a term which represents a departure from a prismatic channel

C = substance concentration

c^* = celerity of a gravity wave

C_{bod} = BOD concentration

C_{do} = DO concentration

C_ℓ = the substance concentration in the lateral flow

C_s = the saturation dissolved oxygen concentration

D_b = the removal of oxygen by benthal deposits

D_m = the molecular diffusion coefficient

E = longitudinal dispersion coefficient

e_x = turbulent diffusion coefficient in x direction

e_y = turbulent diffusion coefficient in y direction

e_z = turbulent diffusion coefficient in z direction

f = function

g = gravitational acceleration

h_f = energy head loss

K = channel conveyance

K_1 = BOD carbonaceous reaction rate

K_2 = reaeration coefficient

K_3 = rate coefficient for the removal of BOD by sedimentation and adsorption

k_1 = the section in a branch near the confluence

k_2 = the section in a branch near the confluence

k_3 = the section in the main branch near the confluence

L_a = the rate of addition of BOD along the reach

N = the number of branches

n = Manning coefficient

P = wet perimeter

Q = flow

q_ℓ = lateral contribution

q_{ℓ_1} = lateral contribution from the flood plains

q_{ℓ_2} = lateral contribution from the watershed drainage area or other sources

R = hydraulic radius

R_1 = damping ratio

R_2 = velocity ratio

S_i = source and sink

S_f = friction slope

S_o = bottom slope

T = temperature

t = time

U* = shear velocity

V = mean velocity

V_f = storage basin volume

v_ℓ = lateral inflow velocity

V_x = velocity in x direction

V_y = velocity in y direction

x = distance

y = water depth

Z = water level

Z_o = channel bottom level

α = correction factor for energy losses

β = frequency

Δt = time increment

Δx = space increment

ρ = water density

σ = wave number

θ = weighting factor of the time integration

CHAPTER I

INTRODUCTION

A. Water Quality Modeling

Environmental behavior is complex; consequently, superficial analysis of a problem may result in inaccurate solutions that reduce project reliability. A mathematical modeling approach to water quality analysis is necessary for precise answers to intricate problems.

If a mathematical model is designed to use as a predictive tool it must accurately reproduce natural processes. Accuracy is determined by the available data and model formulation. In addition, uncertainty in recording and processing data, and the quantity of available data are constraints that can make the model unreliable. The formulation of the hydrodynamic behavior of rivers or estuaries has been well defined and has produced good results in many models.

The transport processes of a substance in the water body include advection, dispersion and the internal reactions. Reaction processes are not well defined due to their interdisciplinary nature. Iteration of chemical, biological, and physical factors are highly complex and difficult to formulate.

Simulation of hydraulic behavior is achieved through two partial differential equations: the continuity equation, based on mass conservation; and the momentum equation, based on momentum conservation. Solving these equations requires information on river geomorphology, values at the boundaries and at the initial time step for all system sections. The solution gives the discharge, area, velocity, and water surface level at the sections which provide basic information for water resource projects such as flood control, hydropower, and water quality.

Distribution of a substance in a river is simulated by the transport equation that represents these phenomena by advection, dispersion, source, and sink terms. The solution of this equation requires knowledge of flow behavior, concentration distribution at the boundaries, and at the initial time step.

Some models couple the three governing equations into two and solve the system of equations. Other models solve the three equations by first solving the hydraulic equations and then the transport equation for the required parameters.

Simulation of a specific substance is related to the objectives of the study and is also a function of a reliable mathematical formulation for the reaction processes and the available data.

This type of model may include assumptions that minimize computational cost and inefficient calculations. Steady flow is an assumption widely used in systems where a critical constant discharge can be assumed in the analysis. In estuaries, rivers near an estuary, or a river where the pollution source is from runoff, the unsteady flow model is a better simulator. In order for the mathematical model to be accurate, it must have a three-dimensional formulation. In practice, this type of model usually requires much computation, storage, and data generally not available. Two- and one-dimensional models can be used for practical purposes. One-dimensional network models are able to accurately simulate a broad, complex system at minimal cost. The two-dimensional models are commonly used to simulate a specific problem in more detail.

B. Summary

The purpose of this study was to develop a model for unsteady flow conditions that simulate the hydraulic and water quality response of a one-dimensional river network. Such a model is useful for simulating a river near an estuary with a geomorphology system composed of connected branches and loops. It can also be used with rivers that have islands, tributaries, and meanders connected with the main flow.

The model developed in this study was divided into two parts: the hydraulic model and the water quality model. The hydraulic model solved the St. Venant equations by a forward implicit numerical scheme. The confluence condition was defined by use of continuity and momentum equations in steady condition through three sections positioned near the confluence. The system of equations that resulted from use of the implicit scheme were solved in each time step. Since the coefficient matrix is sparse and non-banded, a method was developed to minimize the storage and calculations of the Gauss elimination procedure.

The model utilizes the two complete St. Venant equations, therefore, it has applicability when there are downstream effects. In such a case, the storage and kinematic wave methods are not applicable. Another advantage of the hydraulic model is that it is closer to the physical characteristics of the system.

The water quality model uses the solution of the hydraulic model as input to solve the transport equation in each time step. The one-dimensional transport equation utilizes advection, dispersion, source and sink terms. Water quality parameters that can be simulated are: conservative substance, biochemical oxygen demand, and dissolved oxygen. The model can be modified without major effort to simulate

other parameters. A backward implicit finite difference scheme was used. At the confluence the equations were derived based on conservation of mass.

The convection term of the transport equation can create a numerical dispersion. An accuracy analysis was performed based on a simplified form of this equation. Also, a numerical solution was compared to the analytical solution.

The Jacui Delta, located near Porto Alegre, Brazil was used to test the model. The Delta has a watershed of approximately 100,000 km^2 where four main rivers converge. Two of the rivers are very polluted and the amount of pollution in the other two is expected to increase due to industrial development.

The available data used in the simulation of the hydraulic behavior showed good agreement between observed and calculated values. Since a complete set of data was not available for simulating water quality, tests were performed with a confluence system to demonstrate model capability.

Application of the model to management decision making was demonstrated in the Jacui Delta with a hypothetical critical condition to show how this model could be used in a complex river system. The advantage of this model is that it has the capability to analyze broad complex systems without requiring large amounts of data and calculation. The one-dimensional approach is a limitation for those systems with a wide section, such as lakes and some estuaries.

CHAPTER II

LITERATURE REVIEW

A. Review of Hydraulic Models

A.1 Introduction

Ordinarily, flow in a river system is gradually varied and unsteady. The continuity and momentum equations, also called St. Venant equations, are used to express this type of flow. There are many assumptions for the different uses of the equations, all related to the problems that must be solved. For example, the steady state condition, where flow does not vary with time, is used in backwater calculation and steady state water quality models.

The one-dimensional equations of flow assume the transverse and vertical velocities are low compared to the longitudinal one. In estuaries where the sections are wide, the one-dimensional assumption is usually invalid. In this case two-dimensional models are used to simulate velocities in the transverse and longitudinal directions. A three-dimensional model can be used when vertical stratification is also important.

There are some estuaries with a complex system of branches, loops, and confluences. Here the flow division among the confluences and the tide effect from downstream complicates the problem. A broad river network system is not only expensive to simulate utilizing two- or three-dimensional models, but the data required are difficult and expensive to obtain. The one-dimensional assumption is the best choice if it does not create serious errors in the solution.

Simplifications of the one-dimensional St. Venant equations have been widely used. When the storage effect is the important phenomenon only the continuity equation is used. This model is called Storage Routing. The Kinematic Wave Model considers the storage and friction effects of flow using the continuity equation and the momentum equation with the bottom slope equal to the friction slope. The Diffusion Wave Model uses the continuity equation plus the momentum equation with the pressure, friction, and bed slope terms. When the two complete St. Venant equations are used it is called the Dynamic Wave Model. The first two models are usually used when the bottom slope is much greater than the other terms of the momentum equation (Henderson, 1966). These methods cannot be applied when there are significant backwater effects or inversion flow, as occurs in some rivers near the sea or near the confluence of a tributary of the main river. Ponce et al., (1978a), discussed the applicability of Kinematic and Diffusion Models by comparing the propagation characteristics of sinusoidal perturbations to the steady uniform flow. They concluded that the important physical characteristics in determining the applicability of the approximate models are the bed slope and wave period, and that the Diffusion Model has a wider range of application than the Kinematic Model.

A.2 Review of one-dimensional models

A century ago, St. Venant developed the equations for gradual unsteady river flow, based on the conservation of mass and the conservation of momentum. The derivation of the one-dimensional version of these equations has been described by Chow (1959), Harleman (1971), Chen (1973), Yen (1973), and Liggett (1975).

Valley Authority (TVA) applied a "leap-frog" explicit scheme to problems of flood control and navigation. Ballofet (1969) applied the explicit method for estuaries. A generalized computer program was reported by the U.S. Army Corps of Engineers (1976).

Characteristic methods are usually utilized in solving problems through the use of a characteristic grid or a rectangular grid in explicit or implicit formulation (Wylie, 1970). Amein (1966) tested a characteristic grid for flood routing on the Neuse river in North Carolina. Liggett and Woolhiser (1967) applied this method to overland flow problems. Wylie (1970) used an implicit formulation of a rectangular grid in flood routing. Chen (1973) compared the explicit rectangular grid formulation with other schemes in a hypothetical unit-width open channel.

Implicit shemes have nonlinear or linear formulation. They are also classified by grid distribution. Preissmann (1961), as reported by Liggett and Cunge (1975), used a linear formulation of a forward implicit scheme. Abbott and Ionescu (1967), and Vreugdenhil (1973) used the central implicit scheme. Baltzer and Lai (1968), Amein and Fang (1970), and Fread (1976) used a nonlinear implicit formulation.

Miller and Cunge (1975) summarized some of the applications of one-dimensional models. Liggett and Cunge (1975) also described some guidelines for the use of those schemes.

Analysis of numerical stability and convergence has been discussed in the literature by many authors. Leendertse (1967) introduced the ratios of the damping factor and celerity in examining the accuracy of the numerical solution. Vreugdenhil (1968) applied those ratios to three different schemes. The Von Neumann method, applicable to the stability analysis of linear equations, was used by Abbott and Ionescu (1967), Leendertse (1967), and others. They concluded the solution is unconditionally stable for the weighting factor between 0.5 and 1.0. Since the analysis used a simplified linear version of the equations, some oscillations can still persist.

Fread (1973, 1974) studied the variation of the time step in the implicit schemes and concluded that accuracy decreases as the time increment and the time weighting factor increase. However, when the time weighting factor increases the solution becomes more stable. Price (1974), using four numerical schemes for a flood routing problem, concluded that optimum accuracy is obtained when the finite difference time step is chosen approximately equal to the space step divided by the kinematic wave speed. The time step is an important factor in the computational cost. When routing sediments where the variations are slow in time, the time step can be on the order of days. In the case of flood waves, the time is usually on the order of hours. In estuaries with tidal effects, the time step is usually on the order of minutes.

The literature on confluence boundary condition can be classified according to the following descriptions. Vreugdenhil (1973), Feigner and Harris (1970), and Ballofet (1976) used only the continuity equation at the junctions, which does not consider losses at the confluences. Cunge (1975) suggested the use of both equations in steady state condition at the junctions. Yen and Akan (1976) used an overlapping concept with a four-point implicit scheme for flood routing through the junctions. This concept accounts for the downstream backwater effects only for adjacent branches of the confluence.

Gunaratnam and Perkins (1970), in a broad analysis of the numerical methods applied to the St. Venant equations, described the finite difference schemes and developed a finite element method for the governing equations. In the junctions they used the steady conditions. Dailey and Harleman (1972) used this formulation for the hydrodynamics equations on a one-dimensional model of transient water quality in an estuary network. Keuning (1976) applied the finite element method in combination with Galerkin's principle to the unsteady equations for one-dimensional flow in a channel connected with the sea. Cooley and Moin (1976) also developed a finite element solution for those equations and compared them with other methods. Partridge and Brebbia (1976) used a six-node finite element in implicit and explicit time integration models for coastal engineering problems.

B. Review of Water Quality Models

The difference among the mathematical models developed for water quality are based on the following conditions: dimension of the model, system characteristics, type of flow, numerical methods, type of source, substance, or multiple reaction simulation.

Dimension

To obtain a complete description of the problem, a three-dimensional representation should be used. Pritchard (1971) described the three-dimensional equations for mass and momentum conservation and the transport equation. Leendertse, et al., (1973) described a three-dimensional model of estuaries, bays, and coastal seas in which nonisotropic density conditions exist. This type of model is still in a stage of development due to the basic difficulties of getting large amounts of data, a restricted amount of computer storage, and a vast amount of computations.

The two-dimensional models are averaged over a direction either vertical or transverse. In shallow bays, Leendertse (1971), and Hann and Young (1972) neglected the variation of the vertical direction. Other models use the vertical and longitudinal directions for stratification problems.

The one-dimensional transport equation is a reasonable approximation for the stream which the transverse effect of the non-uniform velocity can be well described by the mean velocity and the longitudinal dispersion coefficient. In this model the transverse and vertical direction are averaged. Harleman (1971) showed that a term involving the cross product of the longitudinal velocities and concentrations deviations about the cross section mean (dispersion) plus the spatial mean value of the turbulent diffusivity is the longitudinal dispersion coefficient. Taylor showed that the former is more than two orders of magnitude larger than the turbulent diffusivity effect.

System Characteristics and Type of Flow

The one-dimensional transport equation for unsteady flow is

$$\frac{\partial (AC)}{\partial t} + \frac{\partial (QC)}{\partial x} = \frac{\partial}{\partial x} (EA \frac{\partial C}{\partial x}) + S_i \tag{2.1}$$

where A is the area, Q the discharge, C the concentration, E the longitudinal dispersion coefficient, x the space, t the time, and S_i is the source and sink term.

When all processes are steady-state and also uniform flow, Equation (2.1) for a first order decay source and sink term becomes

$$v \frac{\partial C}{\partial x} = E \frac{\partial^2 C}{\partial x^2} - K_1 C \tag{2.2}$$

where K_1 is the BOD carbonaceous reaction rate; and v is the mean velocity.

Assuming that dispersion is negligible and since the flow is uniform, dx = v dt, and Equation (2.2) yields

$$\frac{\partial C}{\partial t} = - K_1 C \tag{2.3}$$

which is the first order decay equation used by Streeter and Phelps (1925). They published the first theoretical model of stream waste assimilative capacity using Equation (2.3) for BOD and Equation (2.4) for DO so that

$$\frac{\partial C_{do}}{\partial t} = K_1 C_{bod} - K_2 (C_s - C_{do}) \tag{2.4}$$

where K_2 is the reaeration coefficient; C_s is the saturation dissolved oxygen concentration; C_{do} is the DO concentration; and C_{bod} is the BOD concentration. The assumptions made in this model were steady state conditions, the removal of BOD by the oxidation of the carbonaceous element of the waste, and the supply of oxygen by reaeration through the water surface.

Steady flow models are reported in the literature and are very useful when this flow condition can be assumed in a river. The QUAL I model by the Texas Water Development Board (1971) uses Equation (2.1). However, the partial derivative of time is changed to $A\ \partial C/\partial t$ since the model assumed steady state, nonuniform flow.

In many rivers the critical condition is the low flow during the dry season. The steady state flow condition cannot be used for an accurate solution in estuaries where the flow is continuously changing, some urban watersheds where the flood carries more pollutants, or a flood that can wash the benthal deposit.

Models for unsteady condition were developed with the improvement of the solution of the St. Venant equations and the need to solve an estuary type of problem. Harleman (1971) classified models for estuaries as tidal (real time) and non-tidal. The former uses a small time increment and considers the velocity variation during the tidal period. The latter uses a Δt equal to the tide period. Terms in the transport equation are averaged during this period. O'Connor (1965) proposed a model that considers the pollutant distribution only over a long period. Harleman (1971) pointed out that non-tidal models are forced to exaggerate the effects of dispersion in order to simulate the advection upstream from a discharge point.

The unsteady flow models for water quality use the two St. Venant equations and the transport equations (2.1). The solution of this model in a nonuniform channel has to be obtained by numerical solutions. When the river can be adequately characterized by constant parameters such as area, velocity, longitudinal dispersion, and pollutant input, there are analytical solutions available. Such conditions and solutions were described by O'Connor and Thomann (1971), and Hann and Young (1972).

Numerical Methods

Numerical methods to solve the system of partial differential equations are usually classified as the finite difference method and the finite element method. The finite difference method is the one frequently used. The finite difference methods approximate the functions and partial derivative by the discrete values in the plant x-t (unsteady one dimensional). The finite element methods uses a piecewise continuous approximation for the function in the solution region. This approximation is adjusted to the exact continuous solution by the weight residuals method or other procedures.

The St. Venant equations constitute a hyperbolic system of equations. The transport equation (2.1) is a parabolic type. The numerical scheme is chosen according to the conditions of stability and accuracy of the scheme. The schemes are usually classified as forward, central, and backward in time and space.

Stone and Brian (1963) used weighted coefficients in space to evaluate the term according to the time of the equation, resulting in a six-point implicit scheme (Figure 2.1). The stability of a parabolic partial differential equation was discussed by Keller (1960) using the maximum principle for a central scheme. Lanna and Moretti (1977) extended Keller's procedures for a forward and backward scheme.

Dresnack and Dobbins (1968) described some of those schemes and showed that the numerical solution can create a numerical dispersion in the convective term. Bella and Dobbins (1968) described a multi-step procedure in a finite difference calculating first the convective terms and then the dispersion term, and compared them to the analytical solution.

Leendertse (1967) described a procedure to calculate the accuracy of numerical schemes by the ratio of the numerical and analytical solution of a linear version of the transport equation. The analytical solution was obtained by a Fourier series expansion and the ratios considered the numerical dissipation and dispersion. Siemons (1970) used this procedure in analyzing the weighting of the concentration in time for the equation in the one- and two-dimensional equations. Holly (1975) used this procedure to compare nine numerical schemes for the convection equation. Berhoff (1973), also using those ratios, compared

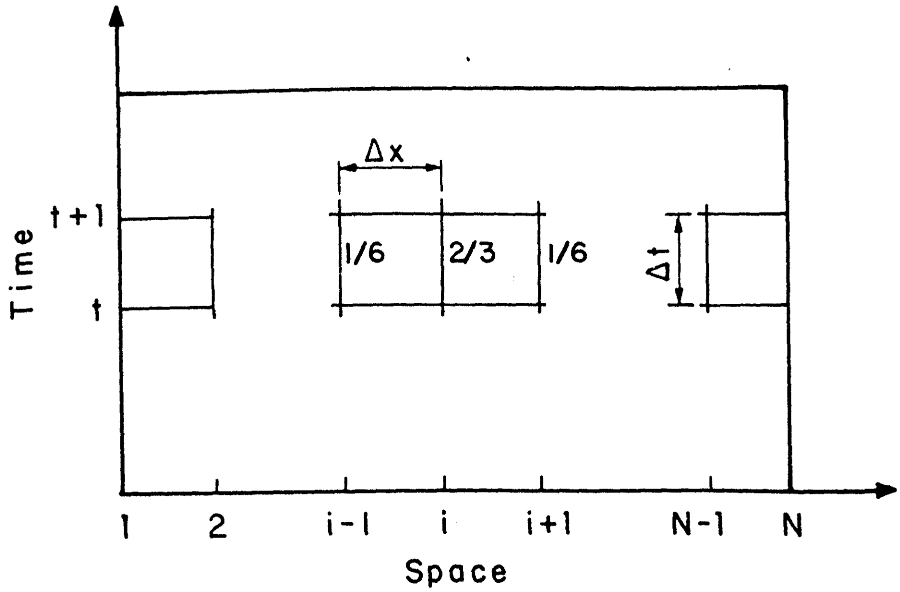


Figure 2.1. Stone and Brian Scheme
for Weighted Coefficients

the explicit, central implicit, and splitting central implicit and concluded the last of the three demonstrated the greatest accuracy.

Hann and Young (1972) made numerical tests for simplified systems and compared them to the analytical solution for an explicit central scheme. Basaran (1976), using numerical experiments, compared the backward and central implicit schemes and concluded that use of the backward scheme was more appropriate.

Lee and Harleman (1971) used the Stone and Brian six-point scheme for the transport equation and the scheme used by Abbott and Ionescu (1967) for the hydraulic equations to solve the system of equations. The FWQA dynamic model described by Feigner and Harris (1971) was developed to handle a complex network of channels, in particular the Sacramento-San Joaquin Delta in California. This model used the momentum equation in the branches and the continuity equation in the confluences in an explicit formulation. The transport processes were simulated by a simplified formulation of the advection and by an eddy diffusion equation. It did not take into account longitudinal dispertion. The reaction processes were assumed to be of the first order.

Berhoff (1973) used the splitting central implicit scheme for the transport equation. In the confluence formulation he used an explicit formulation for the node concentration.

The finite element was used by Holly and Harleman (1972) for a model of transient water quality estuary networks. The hydraulic formulation of this model was based on the work of Gunaratnam and Perkins (1970).

Type of Source

The external sources in a water quality model can be point and non-point. A point source is well identified by the outlet of sewers and tributaries and it is easier to evaluate the discharge and the concentrations that enter the river than the non-point sources.

Non-point sources, such as urban runoff, were reported by Weibel et al. (1964) as a significant source of pollutants. Dornbusch et al. (1974) analyzed the pollution caused by agriculture runoff. Overton and Meadows (1976) classified the methods that evaluate the non-point sources as a concentration-flow rating curve, regression models, and a pollutant removal model.

Substance or Multiple Reaction Simulation

Substances can be conservative and nonconservative in terms of internal reactions. The mathematical formulation for conservative substances uses only the transport equation without the source and sink term for internal sources. Nonconservative substances may have different formulations based on the type of element.

Many models reported in the literature have the capability of simulating different types of substances. The basic difficulty is that chemical, biological, and physical processes that modify the concentration are difficult to evaluate; and in some cases the mathematical formulation is a rough estimate.

In 1925, Streeter and Phelps formulated the source and sink term for the Biochemical Oxygen Demand and Dissolved Oxygen reactions (Equations (2.3) and (2.)) assuming only the carbonaceous stage of BOD and the reaeration on the river surface. Thomas (1948) used a coefficient K_3 to account for the loss of BOD for sedimentation in the river. Dobbins (1964) used the coefficient L_a to account for addition of BOD to the river through scouring of the benthal deposit, and D_b as the net rate of consumption of oxygen by all processes other than the biological oxidation of the flowing BOD load.

The complete decomposition of waste by oxidation has two stages: carbonaceous and nitrification. The nitrification process uses oxygen for oxidation of ammonia to nitrites and the oxidation of nitrites to nitrate. The multistage reaction process is usually simulated by a sequential reaction model or a feedback model. Thomman et al. (1970) used a feedforward multistage reaction for the Delaware estuary. O'Connor, Thomann, and DiToro (1973) combined a model with C-BOD-DO reactions, nitrogen cycle, total DO model, Phytoplankton Dynamics Model, and the organic and inorganic forms of phosphorus. It was used in the Potomac estuary with a hydraulic model in a non-tidal structure. Najarian and Harleman (1975) developed a model for temperature, salt, C-BOD, fecal coliform, the nitrogen cycle, and DO. This model used a unsteady flow hydraulic formulation. Amein and Galler (1978) developed a model for Water Quality Management for the Lower Chowan River in North Carolina. This model simulates the unsteady flow equations by means of a four-point implicit scheme and the multistage reactions of C-BOD-DO, nitrogen , and algae.

Gransrud et al. (1976), in an evaluation of the water quality models, chose 14 models to analyze and classify into six groups: steady-state stream models (DOSAG-I, SNOSCI, SSM), steady state estuary models (ES001, SEM), quasi-dynamic stream models (QUAL I, QUALL II), dynamic estuary and stream models (dynamic estuary model, tidal temperature model, RECEIV, SRMSCI), dynamic lake models (deep reservoir model, LKSCI), and near field models (outfall PLUME). The paper, prepared as a guide for planners, discussed the capabilities, limitations, cost, and availability of each model.

CHAPTER III

DEVELOPMENT OF THE HYDRAULIC MODEL

A. Governing Equations

A.1 Basic equations

The study of the gradually varied unsteady flow in a river is described by two basic partial differential equations; the continuity equation that considers the continuity of the mass flow, and the momentum equation that represents the dynamics effects of the flow.

The one-dimensional continuity and momemtum equations were derived by many authors. The basic assumptions that are made in the derivation are:

a) The river is laterally homogeneous, which means the vertical and transverse velocities are too small and the cross section surface is assumed horizontal. In addition, the river is uniform in the reach.

b) The pressure varies hydrostatically in the vertical.

c) The friction slope of the differential equation is represented by the uniform flow formulas of Chezy or Manning.

The continuity eqaation is derived based on the conservation of mass between two channel sections (Figure 3.1).

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_{\ell} \tag{3.1}$$

where Q is the discharge; A is the cross-section area; q_{ℓ} is the inflow or outflow discharge per unit length of the channel, x is the distance in the longitudinal direction; and t is the time. The lateral flow can be

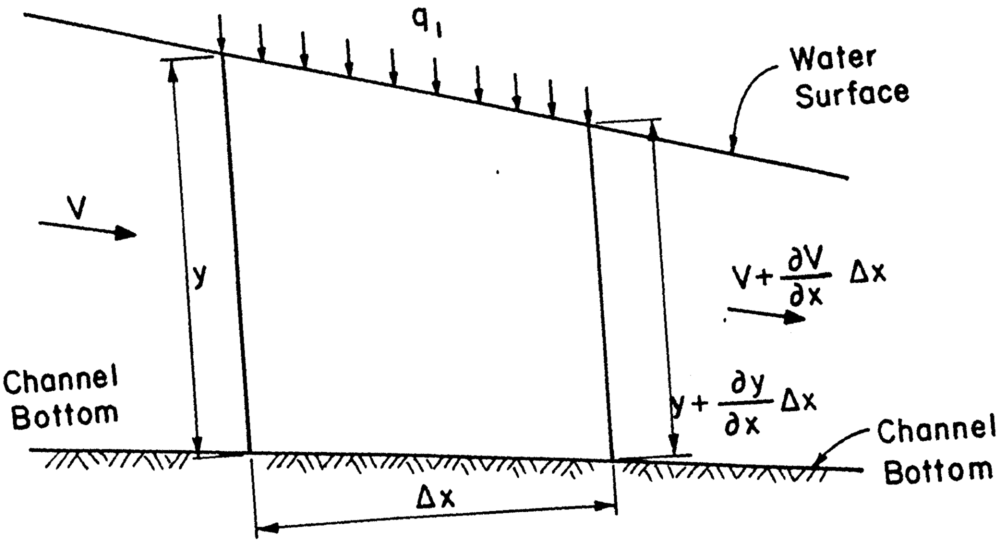


Figure 3.1 Longitudinal Section of Channel

$$q_\ell = q_{\ell 1} + q_{\ell 2} \tag{3.2}$$

where $q_{\ell 2}$ is the lateral contribution from the watershed drainage area or other sources. The term $q_{\ell 1}$ is the contribution from the flood plains (Figure 3.2). This contribution per unit length of channel can be expressed as

$$q_{\ell 1} = - \frac{\partial A_f}{\partial x} \cdot \frac{dy}{dt} \tag{3.3}$$

where A_f is the flood surface area of the reach.

Chen (1973) derived the momentum equation that considers forces that act in the control volume. The resulting momentum equation is

$$\frac{\partial \rho Q}{\partial t} + \frac{v \partial \beta \rho Q}{\partial x} + \beta \rho \frac{v \partial Q}{\partial x} - \beta \rho v^2 T \frac{\partial y}{\partial x} + \frac{gA \partial \rho y}{\partial x} =$$

$$= \rho g A (S_o - S_f + \frac{q_\ell v_\ell}{Ag}) + \beta \rho v^2 A_x^y \tag{3.4}$$

where ρ is the water density; y is the water depth; v is the velocity; S_o is the bottom slope; S_f is the friction slope: v_ℓ is the lateral inflow velocity; β is the momentum coefficient; g is the gravitational acceleration; T is the top width defined as $\partial A/\partial y$; and A_x^y is defined as $(\partial A/\partial x)_{y=\text{constant}}$.

The friction slope is approximated by the equation:

$$S_f = \frac{Q|Q|n^2}{R^{4/3} A^2} \tag{3.5}$$

where n is the Manning coefficient and R is the hydraulic radius. This equation is taken from the steady state formulation since the friction slope for unsteady flow was unavailable. However, this equation gives a good estimate. The above equation is often written as

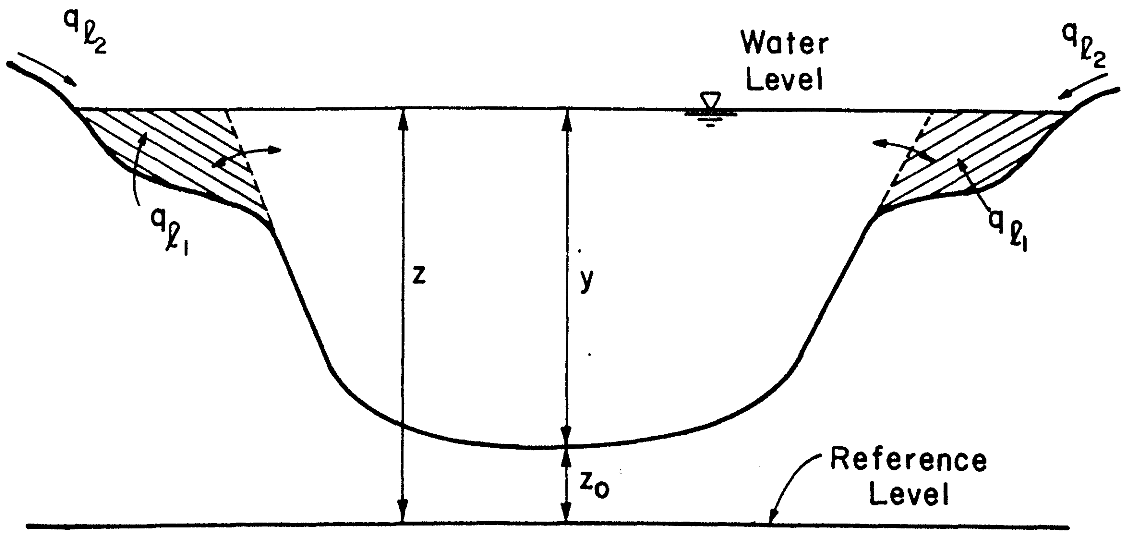


Figure 3.2 Cross Section of Channel

$$S_f = \frac{Q|Q|}{K^2} \tag{3.6}$$

where K is the channel conveyance.

A.2 Specific confluence equations

The confluence is often treated as an internal boundary. This study deals mainly with river networks, therefore, the discussion of confluence equations is presented here.

The following formulations are normally used in the confluence

a) The continuity equation is used to consider the storage in the junction (Feigner and Harris, 1970; and Vreugdenhil, 1973). It is integrated over all branches which converge to the confluence (Figure 3.3) resulting in the equation

$$\frac{\partial(A_s z)}{\partial t} = \sum_{i=1}^{N} Q_i + q_e \tag{3.7}$$

where A_s is the surface area in the junction; N is the number of branches; z is the level at the confluence; and Q_i is the discharge that flows in the branch i. The discharge of the reaches has a positive sign when it enters the junctions and is negative when it comes out of the junction. The lateral contribution or losses are represented by q_e in the above equation.

This type of formulation only uses the continuity equation at the junction; and the momentum equation is used in the branches. The dynamic effects in the junctions were not taken into account. This implies that the momentum is not conserved through these points.

b) A steady state condition is assumed at the junction by using three sections near the confluence in each branch (Figure 3.4). The mass conservation is satisfied by the following equation:

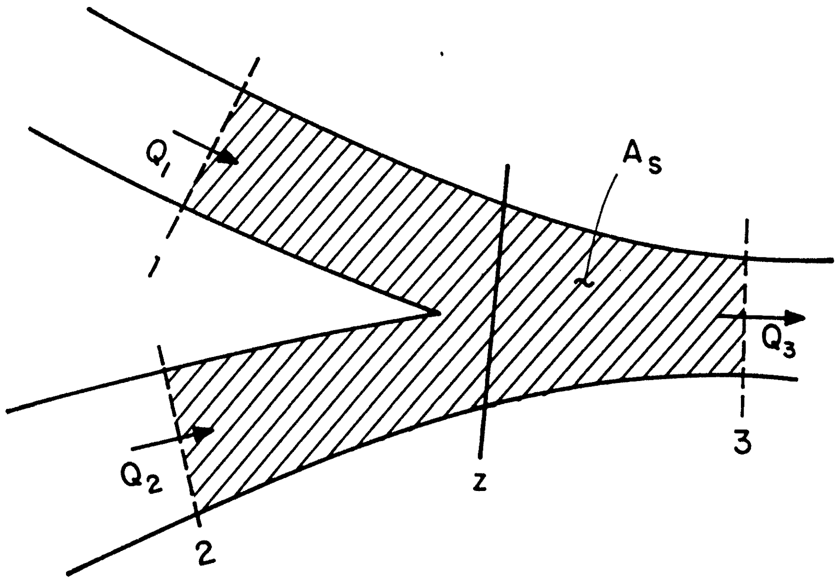


Figure 3.3 Central Scheme at the Junction

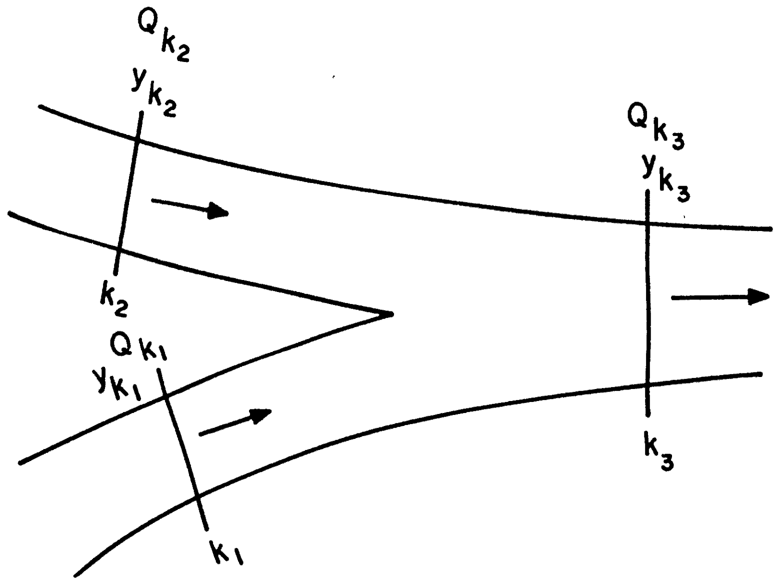


Figure 3.4 Position of the Section at the Junction for the Forward Scheme

$$Q_{k3} = Q_{k2} + Q_{k1} \tag{3.8}$$

and the conservation of energy is represented by the equations

$$z_{o_{k1}} + y_{k1} + \frac{v_{k1}^2}{2g} = z_{o_{k3}} + y_{k3} + \frac{\alpha_{13}\, v_{k3}^2}{2g} + h_{f13}$$

and

$$z_{o_{k2}} + y_{k2} + \frac{v_{k2}^2}{2g} = z_{o_{k3}} + y_{k3} + \frac{\alpha_{23}\, v_{k3}^2}{2g} + h_{f23} \tag{3.9}$$

where α is the correction factor for energy loss and h_f is the energy head loss given by the product of the friction slope and the distance between sections; the α and h_f index for instance, 13 indicates the energy loss is between sections k_1 and k_3; v is the velocity; z_o is the bottom level; and y is the depth.

Equation 3.9 can be simplified as follows:

$$z_{o_{k1}} + y_{k1} = z_{o_{k3}} + y_{k3}$$

$$z_{o_{k2}} + y_{k2} = z_{o_{k3}} + y_{k3} \tag{3.10}$$

This simplified equation (Equation 3.10) is used when the velocity terms and the energy losses are small at the confluence.

B. Numerical Methods

B.1 General formulation

Equations (3.1) and (3.4) form a system of nonlinear hyperbolic partial differential equations that can be solved by analytical methods only in special situations. The numerical methods are usually applied to problems in which the Equations (3.1) and (3.4) are applicable for considerating practical purposes. The basic numerical techniques are the finite difference and finite element methods. The governing equations are the type of partial differential equations that require initial and boundary conditions. The finite difference methods are used to solve those equations and are classified in the explicit scheme, the implicit scheme, and the characteristic method. The basic difference is the explicit scheme uses information from the time t to calculate the variables at the time $t + \Delta t$ (Figure 3.5a) and thus can be solved explicitly. The explicit scheme has the following numerical stability criterion (Courant condition):

$$\Delta t \leq \frac{\Delta x}{|v| + c^*} \tag{3.11}$$

where c^* is the dynamic celerity ($c^* = \sqrt{gy}$) and v is the velocity.

The implicit schemes use the information from the time t and $t + \Delta t$ to calculate the variables at $t + \Delta t$ by the solution of a system of equations (Figure 3.5b). The resulting system is a set of equations with an equal number of unknowns and must be solved simultaneously.

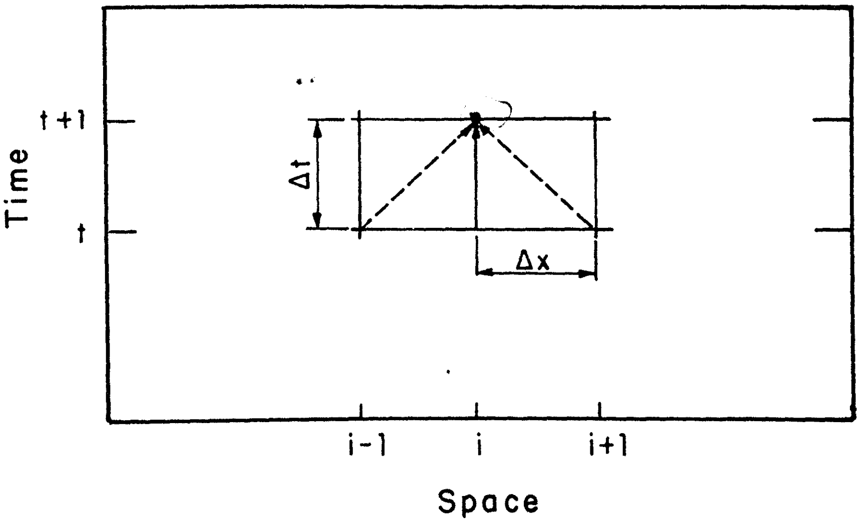
The system of partial differential equations can be transformed into two ordinary differential equations called the characteristics. The characteristic method solves the equations following the characteristic path in the $x - t$ plane.

The general finite-difference approximations of the functions of the partial differential equations are

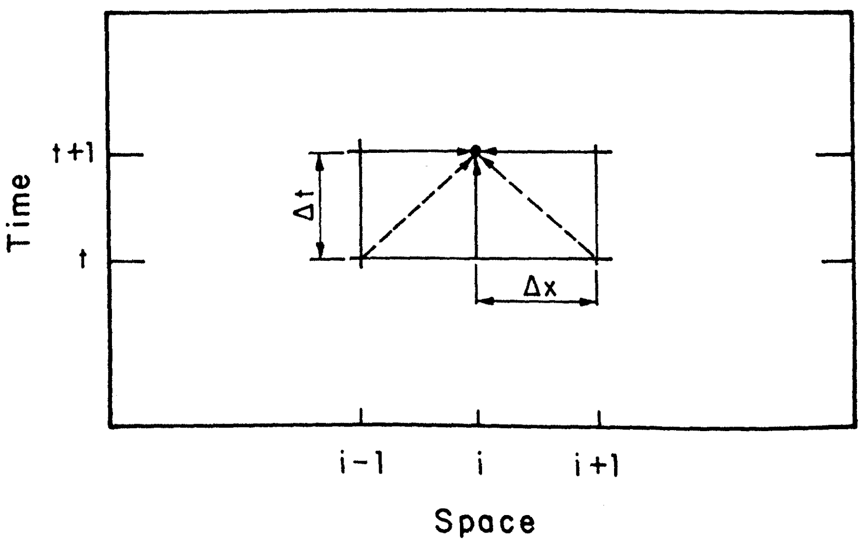
$$f(x,t) \simeq \theta f_j^{t+1} + (1 - \theta) f_j^t$$

$$\frac{\partial f}{\partial x} \approx \frac{\theta(\alpha f_{j+1}^{t+1} - \alpha f_j^{t+1} + \beta f_j^{t+1} - \beta f_{j-1}^{t+1})}{(\alpha + \beta)\Delta x} + (1 - \theta) \frac{(\alpha f_{j+1}^{t} - \alpha f_j^{t} + \beta f_j^{t} - \beta f_{j-1}^{t})}{(\alpha + \beta)\Delta x} \tag{3.12}$$

$$\frac{\partial f}{\partial t} \simeq \frac{f_j^{t+1} - f_j^t}{\Delta t}$$



a. Explicit



b. Implicit

Figure 3.5 Implicit and Explicit Methods for Solution of a Partial Differential Equation where θ is the weighting factor of the time integration, and α and β determines the space distribution. The basic schemes can be categorized utilizing weighting factors as shown in Table 3.1.

Table 3.1 Schemes classification utilizing weighting factors

Weighting factor	Scheme
$\theta = 0$	explicit
$\theta = 0.5$	center, in time, implicit
$\theta = 1.0$	full implicit
$\alpha = 1;\ \beta = 0$	forward
$\alpha = 1;\ \beta = 1$	central
$\alpha = 0;\ \beta = 1$	backward

Chen (1973) and Price (1973) compared some of these numerical methods for flood routing problems. Liggett and Cunge (1974) suggested guidelines for the use of each method. The basic disadvantage of the explicit method is the requirement of a short time interval with high cost needs for computation. In practical problems the grid is also dictated by channel geomorphology that make the solution by the characteristic method more difficult to achieve.

Price (1974) compared four implicit schemes for flood routing using the analytical solution for the monoclinal wave. He concluded that optimum accuracy is reached when the time step is chosen approximately equal to the space step divided by the kinematic wave speed ($c = 1.5v$ when the Chezy equation is used for wide channels) or

$$\Delta t = \frac{\Delta x}{1.5v}$$

This Courant condition for accuracy of implicit schemes is similar to Equation (3.11) of the explicit schemes. The time step of the implicit scheme can be greater than the explicit scheme without loss of accuracy because the condition of Equation (3.11) is based on the celerity of small disturbances. Accuracy of the implicit scheme, however, is based on the velocity of the flood wave celerity (Simons et al., 1977).

The numerical schemes should meet the requirements of stability and convergence. The sources of error in a numerical solution result from rounding the values and discretization. The discretization error is the difference between the numerical solution and the exact solution (Haltiner, 1971). If the discretization error approaches zero when $\Delta x \to 0$ and $\Delta t \to 0$, the finite difference is consistent.

The effect of the numerical errors in the solution of the wave motion in channels changes the amplitude of the wave which is often called numerical damping or numerical dissipation. The amplitude of the wave in the numerical solution can be higher or lower than the real value. The other effect is in the velocity of propagation of the numerical solution. It can be faster or slower than the real value and is called the dispersive effect.

Stability of a implicit scheme is related to the round-off errors. There is no general theory to estimate the numerical stability for a quasilinear partial differential equation. A simple linear version of Equations (3.1) and (3.4) was used to investigate the stability properties of the schemes by the Von Neumann method which uses the Fourier series (Abbott and Ionescu, 1976; and Liggett and Cunge, 1975). The conclusion of this analysis is usually transposable to the complete equation and shows that the difference scheme is numerically stable for $1/2 \leq \theta \leq 1$ and it is unstable for $\theta < 1/2$. Some oscillations can appear in the solution as reported by Liggett and Cunge (1975). They recommend use of θ in the range $0.6 \leq \theta \leq 1.0$ in order to avoid these oscillations. For a greater value of θ the solution is less accurate but more stable.

A complete linear analysis was made by Ponce, et al. (1978b) who performed a theoretical treatment of the convergence of the four-point implicit scheme. They concluded: 1) for kinematic and diffusion waves and inertia-pressure waves the simulation is reasonably good if the numerical dispersion is minimized; accuracy is highly dependent on the value of the weighting factor θ for dynamic waves; 2) when $\theta < 0.5$ there is numerical amplification, $0.5 \leq \theta < 1$ may cause numerical amplification or attenuation, and there is numerical attenuation for $\theta = 1$; 3) accuracy of the simulation is highly dependent on the correct value of the weighting factor θ. In practice an optimum value of that will assure both stability and convergence may be difficult to determine.

B.2 <u>Applied numerical scheme</u>

The numerical method used here is (Chen, 1977)

$$f_{i+\frac{1}{2}} \approx \frac{1}{2} (f_i^t + f_{i+1}^t)$$

$$\frac{\partial f}{\partial x} \approx \frac{(f_{i+1}^{t+1} - f_i^{t+1})}{\Delta x}$$

$$\frac{\partial f}{\partial t} \approx \frac{1}{2\Delta t} [(f_i^{t+1} - f_i^t) + (f_{i+1}^{t+1} - f_{i+1}^t)] \tag{3.13}$$

Using the above numerical scheme in the continuity Equation (3.1) gives

$$\frac{1}{\Delta x_i}(Q_{i+1}^{t+1} - Q_i^{t+1}) + \frac{T_{i+\frac{1}{2}}^{t}}{2\Delta t}(y_i^{t+1} - y_i^{t} + y_{i+1}^{t+1} - y_{i+1}^{t}) = q_{\ell_{i+\frac{1}{2}}}^{t+\frac{1}{2}} \qquad (3.14)$$

Using Equation (3.3) in Equation (3.14) gives

$$\frac{1}{\Delta x_i}(Q_{i+1}^{t+1} - Q_i^{t+1}) + \frac{T_{i+\frac{1}{2}}^{t}}{2\Delta t}(y_i^{t+1} - y_i^{t} + y_{i+1}^{t+1} - y_{i+1}^{t}) =$$

$$= -\frac{(A_f)_{i+\frac{1}{2}}^{t}}{2\Delta x_i} \cdot \frac{1}{\Delta t}(y_i^{t+1} - y_i^{t} + y_{i+1}^{t+1} - y_{i+1}^{t}) + q_{\ell 2_{i+\frac{1}{2}}}^{t+\frac{1}{2}} \qquad (3.15)$$

Equation (3.15) becomes

$$A_i Q_i^{t+1} + B_i y_i^{t+1} + C_i Q_{i+1}^{t+1} + D_i y_{i+1}^{t+1} = E_i \qquad (3.16)$$

where

$$A_i = -\frac{4\Delta t}{\Delta x_i} \quad ; \quad B_i = T_i^t + T_{i+1}^t + \frac{(A_{f_i} + A_{f_{i+1}})^t}{\Delta x_i}$$

$$C_i = \frac{4\Delta t}{\Delta x_i} \quad ; \quad D_i = B_i$$

$$E_i = 4\Delta t\, q_{\ell 2_{i+\frac{1}{2}}}^{t+\frac{1}{2}} + [T_i^t + T_{i+1}^t + \frac{(A_{f_{i+1}} + A_{f_i})^t}{\Delta x_i}]\,(y_i^t + y_{i+1}^t)$$

and A_f is the flood area.

The momentum Equation (3.2) with the numerical approximation (Equation (3.13)) and Equations (3.19) and (3.20) can be written as

$$\frac{1}{2\Delta t}(\rho_i^t Q_i^{t+1} - \rho_i^t Q_i^t + \rho_{i+1}^t Q_{i+1}^{t+1} - \rho_{i+1}^t Q_{i+1}^t) + \frac{1}{\Delta x_i} v_{i+\frac{1}{2}}^t$$

$$[(\beta\rho)_{i+1}^t Q_{i+1}^{t+1} - (\beta\rho)_i^t Q_i^{t+1}] + \frac{1}{\Delta x_i}(\beta\rho v)_{i+\frac{1}{2}}^t (Q_{i+1}^{t+1} - Q_i^{t+1}) -$$

$$- (\beta\rho v^2 T)_{i+\frac{1}{2}}^t \frac{(y_{i+1}^{t+1} - y_i^{t+1})}{\Delta x_i} + gA_{i+\frac{1}{2}}^t \frac{(\rho_{i+1}^t y_{i+1}^{t+1} - \rho_i^t y_i^{t+1})}{\Delta x_i} -$$

$$- g(\rho A)_{i+\frac{1}{2}}^t \frac{(z_{o_i} - z_{o_{i+1}})}{\Delta x_i} + g\{\tfrac{1}{2}[2\rho A \frac{S_f}{Q}(Q^{t+1} - Q^t) -$$

$$- 2\rho A \frac{S_f}{K}\frac{\partial K}{\partial y}(y^{t+1} - y^t) + \rho S_f T (y^{t+1} - y^t)]_i + \frac{1}{2}[2\rho A \frac{S_f}{Q}(Q^{t+1} - Q^t) -$$

$$- 2\rho A \frac{S_f}{K}\frac{\partial K}{\partial y}(y^{t+1} - y^t) + \rho S_f T (y^{t+1} - y^t)]_{i+1} +$$

$$\frac{1}{2}[(\rho A S_f)_i^t + (\rho A S_f)_{i+1}^t]\} - \rho_{i+\frac{1}{2}}^t[- \frac{1}{2}\frac{(A_f)_{i+\frac{1}{2}}^t}{\Delta x_i}\frac{v_{\ell_{i+\frac{1}{2}}}^t}{\Delta t}$$

$$(y_i^{t+1} - y_i^t + y_{i+1}^{t+1} - y_{i+1}^t)] + (\rho q_{\ell 2} v_{\ell 2})_{i+\frac{1}{2}}^{t+\frac{1}{2}} + (\beta \rho v^2 A_x^y)_{i+\frac{1}{2}}^t \quad (3.17)$$

The friction slope, S_f, is a function of Q and y, and from Equation (3.5) using Taylor expansion with first order approximation at time $t + \Delta t$, it becomes

$$S_{f_i}^{t+1} \approx S_{f_i}^t + (\frac{\partial S_f}{\partial Q})_i^t (Q_i^{t+1} - Q_i^t) + (\frac{\partial S_f}{\partial y})_i^t (y_i^{t+1} - y_i^t) \quad (3.18)$$

The partial derivative of S_f with respect to Q, assuming n does not vary with Q, is

$$\frac{\partial S_f}{\partial Q} = \frac{2S_f}{Q} \quad (3.19)$$

The partial derivative of S_f with respect to y is

$$\frac{\partial S_f}{\partial y} = - \frac{2S_f}{K}\frac{\partial K}{\partial y} \quad (3.20)$$

or

$$\frac{\partial S_f}{\partial y} = - 2S_f[\frac{1}{A}(\frac{5T}{3} - \frac{2R}{3}\frac{P}{y}) - \frac{1}{n}\frac{\partial n}{\partial y}] \quad (3.21)$$

where P is the wetted perimeter.

Equation (3.17) results in the following equation

$$A_i^! Q_i^{t+1} + B_i^! y_i^{t+1} + C_i^! Q_{i+1}^{t+1} + D_i^! y_{i+1}^{t+1} = E_i^! \quad (3.22)$$

where

$$A_i^! = \rho_i^t (1 + 2\ CS2_i) - CS1_i$$

$$B_i' = \frac{2\Delta t}{\Delta x_i} [(\beta\rho v^2 T)^t_{i+\frac{1}{2}} - \rho^t_i \, g(A)^t_{i+\frac{1}{2}}] + CS3_i + \rho^t_{i+\frac{1}{2}} \frac{(A_f)^t_{i+\frac{1}{2}}}{\Delta x_i} v_{\ell}{}^t_{i+1}$$

$$C_i' = \rho^t_{i+1} (1 + 2\; CS2_{i+1}) + CS1_{i+1}$$

$$D_i' = \frac{2\Delta t}{\Delta x_i} [\rho^t_{i+1} \, gA^t_{i+\frac{1}{2}} - (\beta\rho v^2 T)^t_{i+\frac{1}{2}}] + CS3_{i+1} + \rho^t_{i+\frac{1}{2}} \frac{(A_f)^t_{i+\frac{1}{2}}}{\Delta x} v_{\ell}{}^t_{i+\frac{1}{2}}$$

$$E_i' = (\rho Q)^t_i (1 + 2CS2_i) + (\rho Q)^t_{i+1} (1 + 2CS2_{i+1}) + \frac{2\Delta t}{\Delta x_i} g(\rho A)^t_{i+\frac{1}{2}}$$

$$(z_{o_i} - z_{o_{i+1}}) + y^t_i \, CS3_i + y^t_{i+1} \, CS3_{i+1} + \rho^t_{i+\frac{1}{2}} \frac{(A_f v_\ell)^t_{i+\frac{1}{2}}}{\Delta x_i}$$

$$(y^t_i + y^t_{i+1}) + 2\Delta t \, \rho^t_{i+\frac{1}{2}} (q_{\ell 2} v_{\ell 2})^{t+\frac{1}{2}}_{i+\frac{1}{2}} + 2\Delta t (\beta\rho v^2 A^y_x)^t_{i+\frac{1}{2}}$$

where

$$CS1_i = \frac{2\Delta t}{\Delta x_i} [(\beta\rho v)^t_{i+\frac{1}{2}} + v^t_{i+\frac{1}{2}} (\beta\rho)^t_i]$$

$$CS2_i = (\frac{S_f}{v})^t_i \, \Delta t g$$

$$CS3_i = \rho^t_i \Delta t g \, \{(S_f T)^t_i - 2 \, (\frac{AS_f}{K})^t_i \, (\frac{\partial K}{\partial y})^t_i\}$$

and z_o is the bottom level of the cross section.

Confluence Equations

The confluence equations used here are Equations (3.8) and (3.9). Equation ~~(3.9)~~ 3.8 is

$$Q^{t+1}_{k1} + Q^{t+1}_{k2} - Q^{t+1}_{k3} = 0 \qquad (3.23)$$

Equation (3.9) can be rewritten for time $t + \Delta t$ as

$$z_{o_{k1}} + y^{t+1}_{k1} + (\frac{v^2_{k1}}{2g})^{t+1} = z_{o_{k3}} + y^{t+1}_{k3} + \alpha_{13} (\frac{v^2_{k3}}{2g})^{t+1} + h^{t+1}_{f_{13}} \qquad (3.24)$$

$$z_{o_{k2}} + y_{k2}^{t+1} + (\frac{v_{k2}^2}{2g})^{t+1} = z_{o_{k2}} + y_{k3}^{t+1} + \alpha_{23} (\frac{v_{k3}^2}{2g})^{t+1} + h_{f_{23}}^{t+1} \quad (3.25)$$

This formulation assumes that the cross section does not change with time within one time step. The notation used here is shown in Figure 3.6. It is used in the computer program to give the positive direction of the flow. The sign in Figure 3.6 indicates the side of the energy equation where the k_3 terms should be placed. Equations (3.24) and (3.25) illustrate Figure 3.6a.

The term h_f is calculated by

$$h_{f_{13}}^{t+1} = \frac{\Delta x_{13}}{2} (S_{f_{k1}} + S_{f_{k3}})^{t+1} \quad (3.26)$$

Using the numerical scheme given by Equation (3.13) in Equations (3.24),

$$z_{o_{k1}} + y_{k1}^{t+1} + \frac{1}{2g} [2(\frac{v}{A})^t Q^{t+1} - 2(\frac{v^2 T}{A})^t y^{t+1} - (v^2)^t + 2(\frac{v^2 Ty}{A})^t]_{k1} =$$

$$= z_{o_{k3}} + y_{k3}^{t+1} + \frac{\alpha_{13}}{2g} [2(\frac{v}{A})^t Q^{t+1} - 2(\frac{v^2 T}{A})^t y^{t+1} - (v^2)^t + 2(\frac{v^2 Ty}{A})^t]_{k3}$$

$$+ \Delta x_{13} \{\frac{1}{2}(S_{f_{k1}}^t + S_{f_{k3}}^t) + [(\frac{S_f}{Q})^t (Q^{t+1} - Q^t) - (\frac{S_f}{K})^t \frac{\partial K}{\partial y} (y^{t+1} - y^t)]_{k1} +$$

$$+ [(\frac{S_f}{Q})^t (Q^{t+1} - Q^t) - (\frac{S_f}{K} \frac{\partial K}{\partial y})^t (y^{t+1} - y^t)]_{k3}\} \quad (3.27)$$

Equation (3.27) results in

$$C_a Q_{k1}^{t+1} + C_b y_{k1}^{t+1} + C_c Q_{k3}^{t+1} + C_d y_{k3}^{t+1} = C_e \quad (3.28)$$

In the same way, Equation (3.25) results in the following equation by the use of the numerical scheme from Equation (3.13)

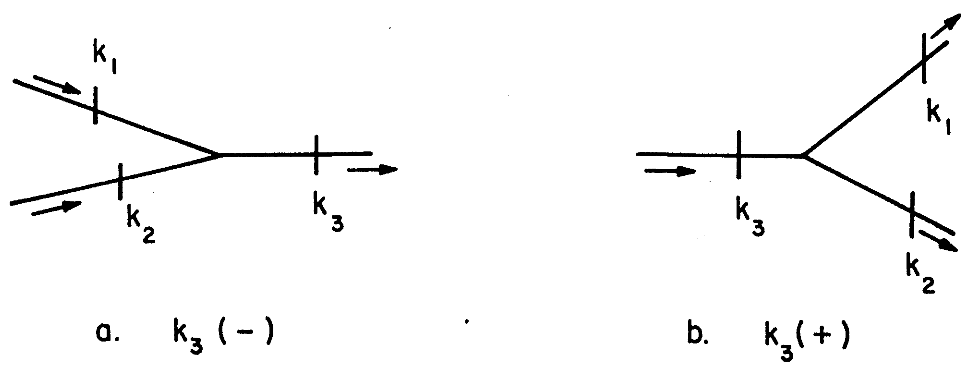


Figure 3.6 Computer Program Notation for Positive Direction of Flow

$$C'_a\, Q_{k2}^{t+1} + C'_b\, y_{k2}^{t+1} + C'_c\, Q_{k3}^{t+1} + C'_d\, y_{k3}^{t+1} = C'_e \tag{3.29}$$

where

$$C_a = \left(\frac{v}{Ag}\right)_{k1}^{t} - \Delta x_{13} \left(\frac{S_f}{Q}\right)_{k1}^{t}$$

$$C_b = 1 + \left(\Delta x_{13}\ \frac{S_f}{K}\ \frac{\partial K}{\partial y} - \frac{v^2 T}{Ag}\right)_{k1}^{t}$$

$$C_c = -\ \alpha_{13}\left(\frac{v}{Ag}\right)_{k3}^{t} - \Delta x_{13} \left(\frac{S_f}{Q}\right)_{k3}^{t}$$

$$C_d = \alpha_{13}\left(\frac{v^2 T}{Ag}\right)_{k3}^{t} + \Delta x_{13} \left(\frac{S_f}{K}\ \frac{\partial K}{\partial y}\right)_{k3}^{t} - 1$$

$$C_e = z_{o_{k3}} - z_{o_{k1}} - \alpha_{13} \left(\frac{v^2}{2g}\right)_{k3}^{t} + \left(\frac{v^2}{2g}\right)_{k1}^{t} + y_{k3}^{t}\ (C_d + 1)\ -$$

$$-\ \frac{1}{2}\left(S_{f_{k1}} + S_{f_{k3}}\right)^{t} \Delta x_{13} + y_{k1}^{t}\ (C_b - 1)$$

The coefficients of the Equation (3.29) are the same as above, only the index changes and k_2 is used instead of k_1 and 23 instead of 13. When the condition exists like that in Figure 3.6b, the equations are the same as above but with an interchange of index (where k_1 is changed to k_3 and k_3 changes to k_1).

B.3 Boundary and initial conditions

External Boundaries

The solution of the momentum and continuity equations by numerical methods requires the specification of the boundary conditions at the upstream and downstream sections and the initial conditions for all sections.

In a river reach with two boundary sections there is an option to specify the level or the discharge in each boundary section during the simulation period. Another type of boundary that can be used is the relationship between Q and y.

When the flow is subcritical ($v < c$), it is necessary to specify one variable for the upstream section and another for the downstream section. In this situation one characteristic of the equation propagates downstream and the other upstream. In the case of supercritical flow, both characteristics propagate downstream, thus, one can specify the two variables in the upstream boundary. The characteristics equations are

$$\frac{dx}{dt} = v + c \qquad \text{(forward)} \qquad (3.30)$$

$$\frac{dx}{dt} = v - c \qquad \text{(backward)} \qquad (3.31)$$

When the flow regimen is supercritical both equations (3.30 and 3.31) are positive on the right hand side and both characteristics are in the forward direction. In the subcritical situation, Equation (3.30) has a negative right hand side (Figure 3.7).

Rating Curve as Boundary

The rating curve can be used as a boundary in some situations. This condition is strictly applicable to kinematic models (Abbott, 1976). When this relationship is used in a dynamic model the solution in the boundary is in conflict with the solution near the boundary which may lead to inaccuracy. This relationship supplies the system of equations with one more equation.

The discharge in a section is a function of the section level or

$$Q^{t+1} = f^{t+1}(y) \qquad (3.32)$$

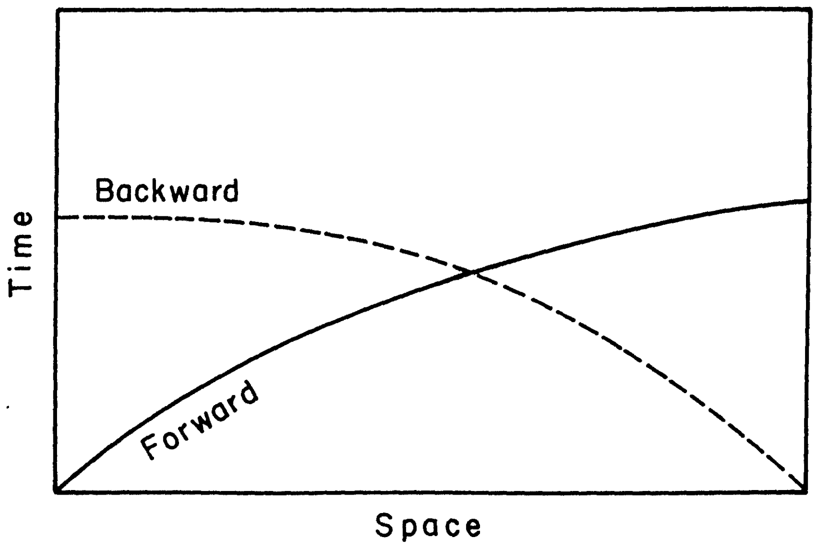


Figure 3.7 Characteristics Lines for a Subcritical Flow

Using the Taylor expansion with first order approximation,

$$Q^{t+1} = f^t(y) + (\frac{\partial f}{\partial y})_t \ (y^{t+1} - y^t) \qquad (3.33)$$

$$Q^{t+1} - (\frac{\partial f}{\partial y})_t \ y^{t+1} = f^t(y) - (\frac{\partial f}{\partial y})_t y^t \qquad (3.34)$$

The use of Equation (3.32) at time t in Equation (3.34) yields

$$Q^{t+1} - (\frac{\partial Q}{\partial y})_t y^{t+1} = Q^t - (\frac{\partial Q}{\partial y})_t y^t \qquad (3.35)$$

If the relationship is tabulated, it is possible to find $(\partial Q/\partial y)_t$ using one more point near y^t. When the function of the Equation (3.32) is known, such as

$$Q = a \ y^b \qquad (3.36)$$

then one can calculate the partial derivative,

$$\frac{\partial Q}{\partial y} = a \ b \ y^{b-1} \qquad (3.37)$$

and use Equation (3.37) in Equation (3.35).

When this curve is unknown, it is possible to estimate this relationship using Manning's equation as

$$Q = \frac{1}{n} A \ R^{2/3} \ S^{1/2} \qquad (3.38)$$

$$\frac{\partial Q}{\partial y} = Q(\frac{2}{3R} \frac{\partial R}{\partial y} + \frac{1}{A}\frac{\partial A}{\partial y} - \frac{1}{n}\frac{\partial n}{\partial y}) \qquad (3.39)$$

The use of Equation (3.39) in Equation (3.35) results in

$$Q^{t+1} - Q^t(\frac{2}{3R} \frac{\partial R}{\partial y} + \frac{1}{A}\frac{\partial A}{\partial y} - \frac{1}{n}\frac{\partial n}{\partial y}) \ y^{t+1} =$$

$$= Q^t[1 - y^t(\frac{2}{3R} \frac{\partial R}{\partial y} + \frac{1}{A}\frac{\partial A}{\partial y} - \frac{1}{n}\frac{\partial n}{\partial y})] \qquad (3.40)$$

If $\underline{n}$ is constant in the cross section and $\frac{\partial A}{\partial y} \simeq T$ and $R \simeq y \simeq \frac{A}{T}$ the above equation results in

$$\frac{5}{y^t} \ y^{t+1} - \frac{3}{Q^t} \ Q^{t+1} = 2 \qquad (3.41)$$

This type of boundary can be used when the downstream effects are small and the loop in the relationship of Q and y can be approximated by a straight line or curve.

In flood problems this condition is often used at the downstream boundary and the hydrograph at the upstream boundary. This boundary condition cannot be used when there are backwater effects or inversion flow.

Interior Boundaries

In a river system the physical characteristics are not uniform. There are uniform changes the basic two equations can determine without much error. There are sudden changes in the river characteristics that should be considered in the solution as the interior boundary condition. Some of these boundaries according to Cunge (1975) are:

1. Junctions of rivers.

2. Flow over weirs (Figure 3.8b). The equations for this condition are

$$Q_1 = Q_2$$

$$Q = f(z_1,\ z_2,\ z_w,\ \text{weir type and size}) \tag{3.42}$$

3. Flow through control gates. The equations are

$$Q_1 = Q_2$$

$$Q = f(z_1,\ z_2,\ z_w,\ \text{gate type and size}) \qquad (3.43$$

4. Storage basin. In some rivers there are storage basins linked to the channel as shown in Figure 3.8b. They contribute only to the storage effect and the continuity equation should account for volume. The volume lost by the river reach in Δt is $\frac{dV}{dt}$, then

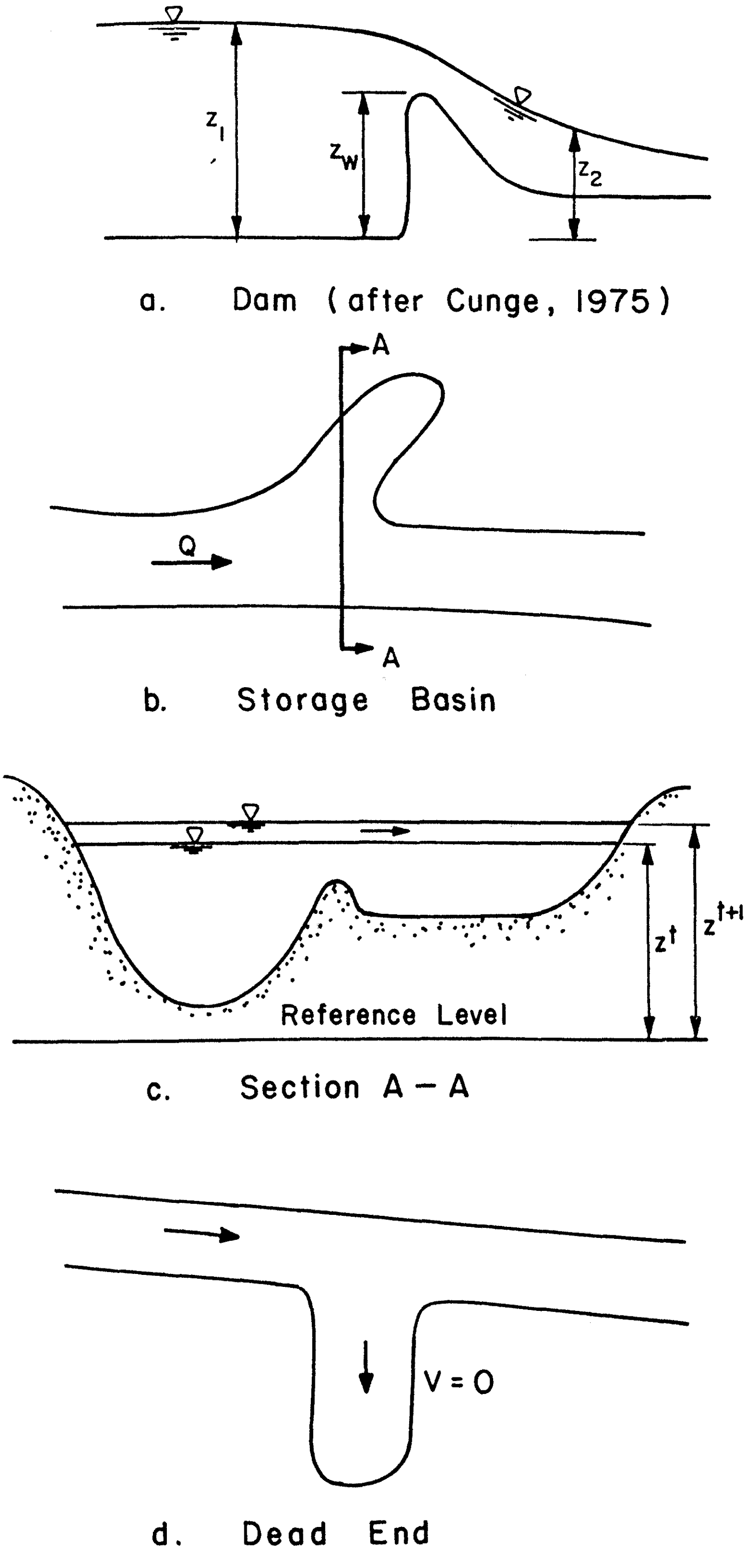


Figure 3.8 Interior Boundary Conditions for a River System

$$\frac{dV_F}{dt} = \frac{\partial V_F}{\partial z} \frac{dz}{dt} \tag{3.44}$$

where V_F is the storage basin volume (Figures 3.8b and 3.8c). Using a numerical approximation

$$\frac{dV_F}{dt} \approx \left(\frac{\partial V_F}{\partial z}\right)_t \left(\frac{z^{t+1} - z^t}{\Delta t}\right) \tag{3.45}$$

The function $V_F = f(z)$ is obtained from the maps. The above term appears with a negative sign in the continuity equation.

When this storage basin has dynamic effects in the flow, it can be assumed to be a flood area with more roughness than the normal river bed. In Equations (3.1) and (3.4) it is included in the term $q_{\ell 1}$.

5. Dead end. When there is a channel with a dead end, the section at that end may have the condition of $V = 0$ at the dead end (Figure 3.8d).

Initial Condition

In order to proceed to the calculations it is necessary to specify the level and discharge at all sections in the initial time step. Usually these values are not known and are estimated. However, the initial values at the boundaries are known and by using the program with about 50 or less time steps, and holding the boundaries constant, the steady state condition for the initial boundaries is reached. Another way is to interpolate the levels making the discharges constant when there are no confluences. In the case of confluences, adequate knowledge of the system is required to specify these values by inspection. A third way is to solve the steady backwater equation. The first case needs an initial condition to start the running that can be obtained by interpolation or inspection.

Normally in this type of equation after some time steps, different initial conditions converge to the same solution. Baltzer and Lai (1968) showed the convergence to the same solution using different values of initial discharge.

C. Systems of Equations

C.1 The equations

Using Equations (3.16) and (3.22) for each reach, and Equations (3.23), (3.28), and (3.29) for each confluence, there will be $2(N - 1)$ equations (if there are only two boundaries), where N is the number of sections. In matrix notation the system of equations is

$$\underline{F}.\underline{X} = \underline{E} \tag{3.46}$$

where

$$\underline{F} = \left| \begin{array}{llllllllllll}
A_1 B_1 C_1 D_1 & & & & & & & & & & & \\
A_1' B_1' C_1' D_1' & & & & & & & & & & & \\
 & A_2 B_2 C_2 D_2 & & & & & & & & & & \\
 & A_2' B_2' C_2' D_2' & & & & & & & & & & \\
 & \cdots\cdots\cdots\cdots & & & & & & & & & & \\
 & & & & 1 & 1 & & -1 & & & & \\
 & & & C_{ai} & C_{bi} & & C_{ci} & C_{di} & & & & \\
 & & & & C_{ai}' & C_{bi}' & C_{ci}' & C_{di}' & & & & \\
 & & & & & \cdots\cdots\cdots\cdots & & & & & & \\
 & & & & & & & & A_{N-1} & B_{N-1} & C_N & D_N \\
 & & & & & & & & A_{N-1}' & B_{N-1}' & C_N' & D_N'
\end{array} \right|$$

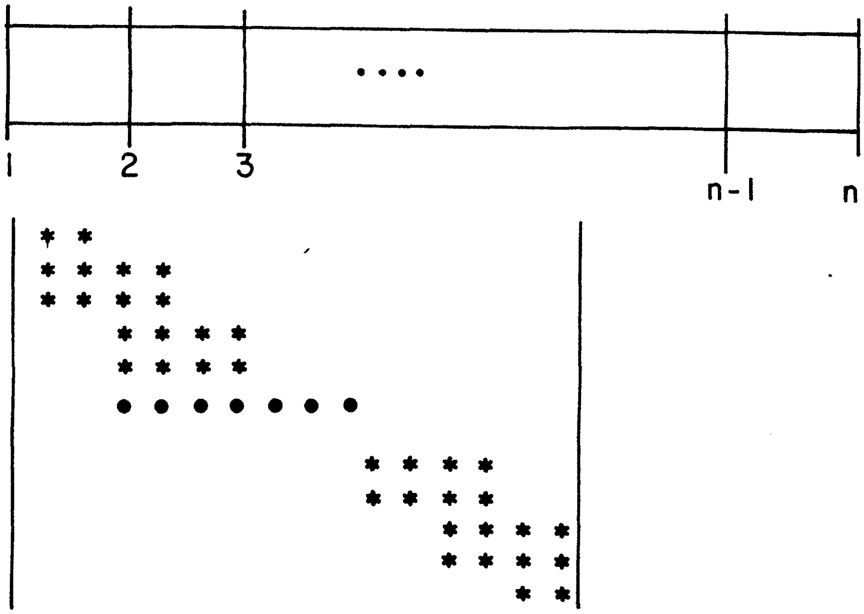
$$\underline{X} = \begin{vmatrix} Q_1^{t+1} \\ y_1^{t+1} \\ \cdot \\ \cdot \\ Q_i^{t+1} \\ y_i^{t+1} \\ \cdot \\ \cdot \\ Q_N^{t+1} \\ y_N^{t+1} \end{vmatrix} ; \qquad \underline{E} = \begin{vmatrix} E_1 \\ E_1' \\ \cdot \\ \cdot \\ 0 \\ C_{ei} \\ C_{ei}' \\ \cdot \\ \cdot \\ E_N \\ E_N' \end{vmatrix}$$

The specification of the boundaries gives two more equations, and the number of equations and unknowns will be the same, thereby permitting the equations to be solved simultaneously. For instance, in the system in Figure 3.9b, there are four reaches that give eight equations, two confluences that give six equations and the boundaries with two more equations. There are sixteen unknowns (8 sections) and sixteen equations.

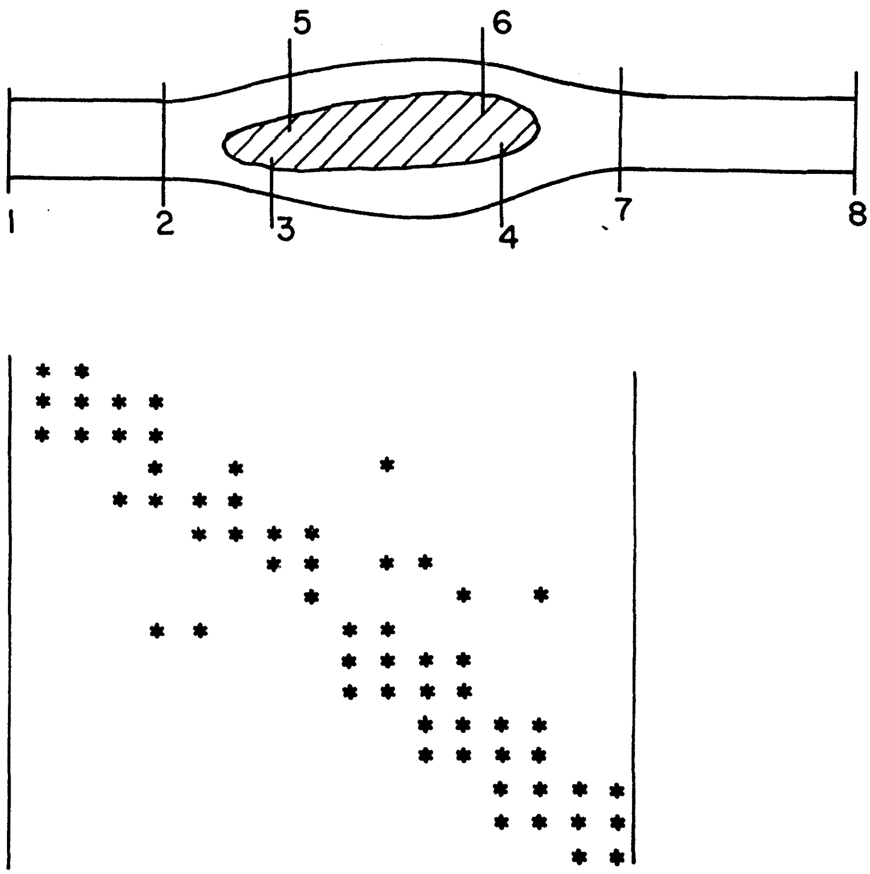
C.2 Solution of the linear system of equations

The linear system of equations (3.46) resulting from the use of the numerical scheme (3.13) in Equations (3.1), (3.4), (3.8), and (3.9) for a river network needs to be solved at each time step.

The coefficient matrix $\underline{F}$ for a river without confluences can be easily transformed into a banded matrix. In this situation the pentadiagonal method or other method that considers only the



a. Matrix F for a River Reach



b. Matrix F for a River with Confluence

* Nonzero Elements

Figure 3.9 Matrix F for a River Reach and for a River with a Confluence non-zero coefficients can be used to solve the system of equations. These methods are more accurate and use less storage and computer time than methods that utilize the full matrix.

In a river network the matrix $\underline{F}$ is a sparse non-banded matrix (Figure 3.9b). In order to solve a system of linear equations when the coefficient matrix is sparse, Vreugdenhil (1973) recommended an iterative method such as Gauss-Seidel. This method requires only the storage of non-zero elements of the matrix and their positions in the matrix. The solution of the time t is used as the initial guess for beginning the iteration of time step $t + \Delta t$. In this way the initial guess is usually good and computer time is saved, as fewer iterations are requested for convergence. The convergence condition for the Gauss-Seidel iterative method is that the matrix $\underline{F}$ should be positive definite. This method was used in some examples and it was found that convergence did not always occur.

A direct method to solve these equations is the Gauss elimination procedure. The following system of equations

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1N-1}x_{N-1} + a_{1N}x_N = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2N-1}x_{N-1} + a_{2N}x_N = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{N1}x_1 + a_{N2}x_2 + \cdots + a_{NN-1}x_{N-1} + a_{NN}x_N = b_N \end{array} \tag{3.47}$$

is solved by transforming the matrix of coefficients in a upper triangle matrix by

$$j = 1,2,\ldots N-1 \;;\quad i = j + 1,\ldots N;$$

$$b_i^j = b_i^{j-1} - b_j^{j-1} \frac{a_{1,j}^{j-1}}{a_{i,i}^{j-1}} \tag{3.48}$$

and

$$k = j+1,\ldots,n$$

$$a^{j}_{i,k} = a^{j-1}_{i,k} - a^{j-1}_{j,k} \frac{a^{j-1}_{i,j}}{a^{j-1}_{j,j}} \qquad (3.49)$$

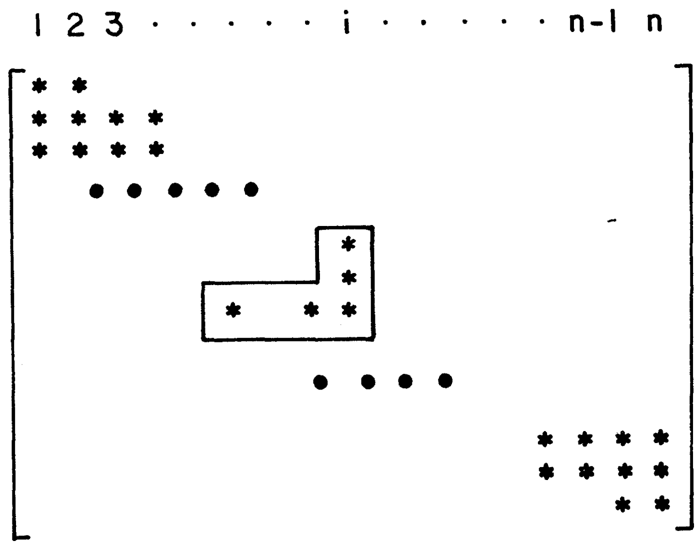
Using backward substitution, the unknown values are calculated by

$$x_i = \frac{(b_i - \sum_{m=n}^{i-1} x_m \, a_{i,m})}{a_{i,i}} \qquad (3.50)$$

The matrix $\underline{F}$ has no more than four non-zero elements in each row, and has many zero elements. If one stores all elements, the solution will be less accurate and too expensive. For instance, 50 sections would use 10,000 words of storage where only about 2% are non-zero elements.

The procedure described here tries to minimize the storage of the Gauss elimination scheme. There is a method called Skyline used in the finite element method (Bathe and Wilson, 1976). It is a storage method for the Gauss elimination procedure in symmetric matrices. Since the matrix $\underline{F}$ is not symmetric a modification is required.

This method used four one-dimensional arrays to store the information contained in matrix $\underline{F}$. A numerical example is shown in Figure 3.10. The coefficients are stored sequentially in a vector. Each diagonal element is followed by all elements in the column of the matrix above that element. Then all elements in the row to the left of the diagonal taking in the inverted L shape ("⅃") shown in Figure 3.11a which extends upwards and to the left as far as the last non-zero element in each direction.



a. Diagonal Element

$$
\begin{bmatrix}
1 & -2 & & & & \\
 & 5 & & 6 & & \\
 & & 7 & & & \\
 & 3 & & 4 & & 4 \\
 & & & 2 & 1 & \\
 & & & & & 6
\end{bmatrix}
$$

b. Example Matrix A

$$
AA(I) = \begin{bmatrix} 1 \\ 5 \\ -2 \\ 7 \\ 4 \\ 0 \\ 6 \\ 0 \\ 3 \\ 1 \\ 2 \\ 6 \\ 0 \\ 4 \end{bmatrix} \quad
IDIAG(I) = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \\ 10 \\ 12 \end{bmatrix} \quad
IHIGH(I) = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \\ 1 \\ 3 \end{bmatrix} \quad
IR(I) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}
$$

c. Definition of the Arrays for the Example in b

Figure 3.10 Storage Scheme for the Gauss Elimination Procedure

In Figure 3.10c vector AA(I) shows the storage sequence of coefficients for matrix A in Figure 3.10b. Vector IDIAG(I) gives the positions of each diagonal element in AA(I). Vector IHIGH(I) gives the number of column elements above each diagonal element including the diagonal element. Vector IR(I) gives the number of row elements to the left of each diagonal element excluding the diagonal element. The method uses Equations (3.49) and (3.50), but with a different index since the coefficients are stored in another way.

The flowchart in Figure 3.11 shows the solution using this storage scheme. This method is useful when the matrix is almost banded with a few sparse elements, as in this case of the river network. For instance, in the Jacui Delta system with 19 confluences and 64 sections, the full matrix would use 16,384 words for storage. This storage scheme uses 1,587 words and in a Cyber 171 computer it takes 1.0 second of Central Processing (CP) time to solve the system of 128 equations by 128 unknowns in each time step.

The section numbering procedure should be done to minimize the storage and calculations. The unknowns are numbered based on the section number. In the reach or confluence equations the section numbers are not continuous integer numbers, zero values will appear among the non-zero values in the coefficient matrix which increases the matrix band and consequently the storage and calculation. The minimization can be done by numbering the sections in a crescent (or decrescent) order and when there are confluences minimize the difference of the section numbers of the reaches and confluences. Some suggestions

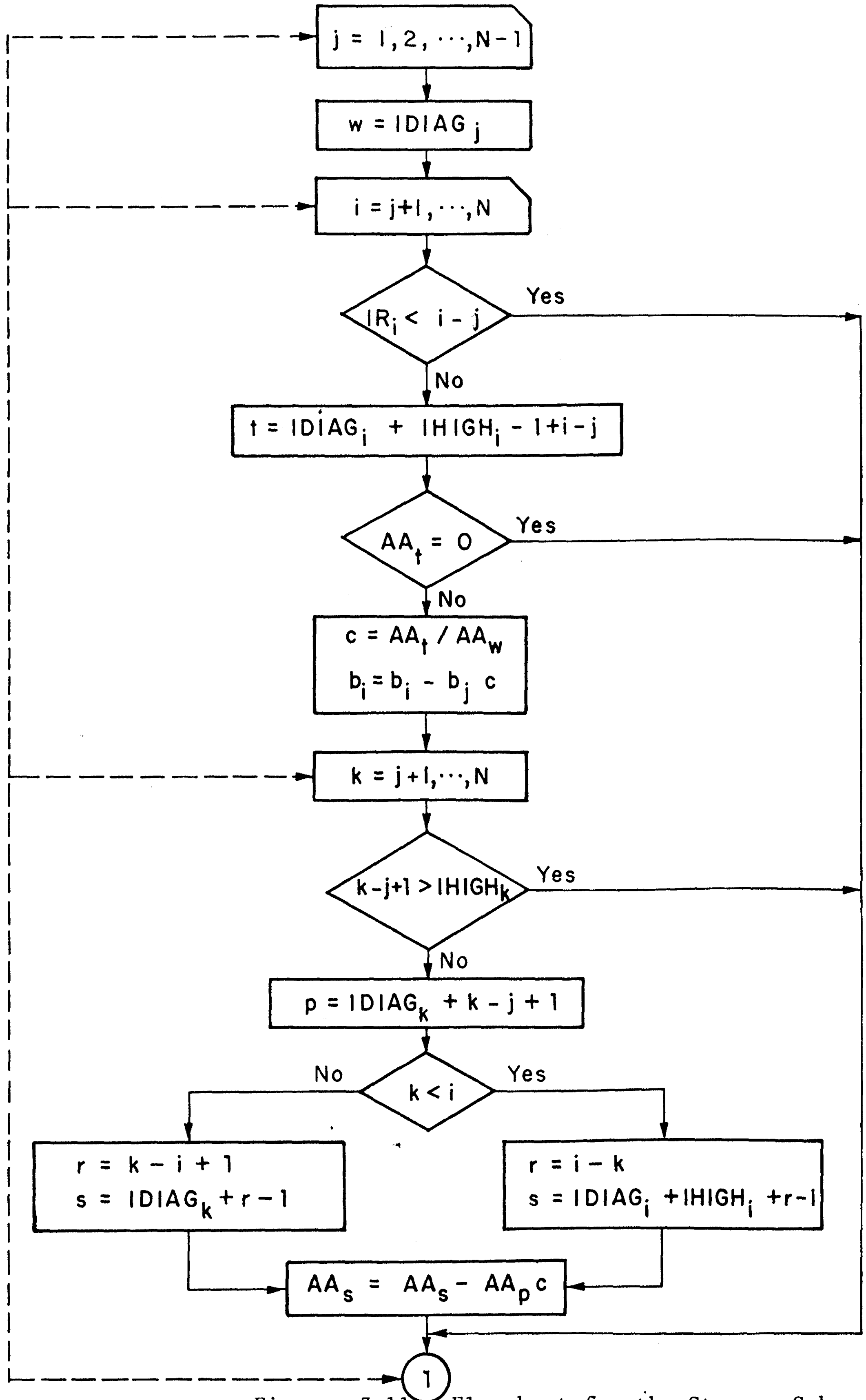


Figure 3.11 Flowchart for the Storage Scheme

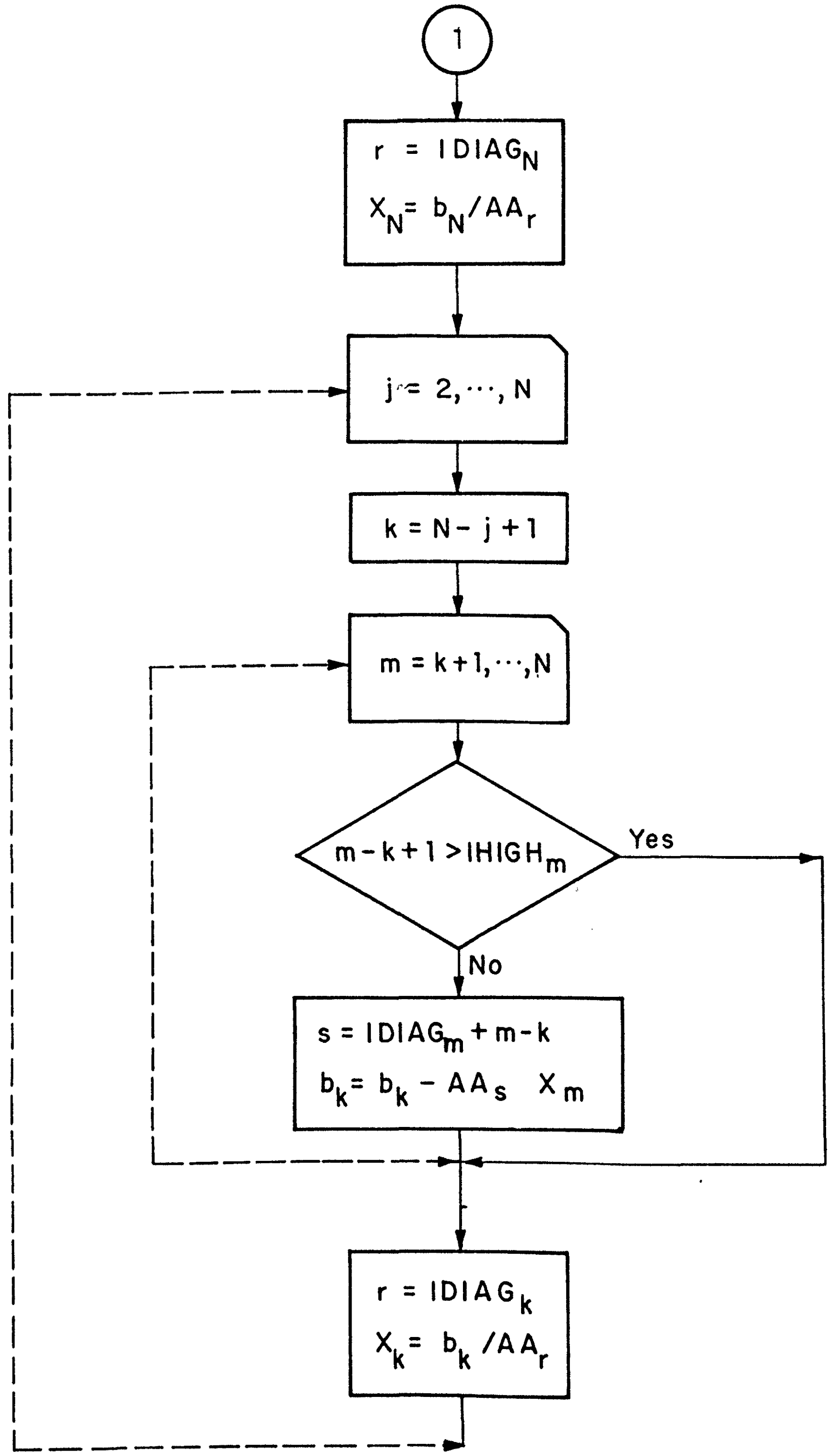


Figure 3.11 (continued)

to execute that procedure include: (i) the number of the sections should be given in crescent order from upstream towards downstream, and (ii) for short loops an alternate numbering is a good procedure.

CHAPTER IV

DEVELOPMENT OF A WATER QUALITY MODEL

A. Governing Equations

A.1 Transport equation

The transport of mass in an environment is due to the advection, diffusion, and dispersion processes. The advection of a concentration element is the transport that results from the flow gradient. This process is described by the equation

$$\frac{\partial C}{\partial t} + \frac{\partial (v_x C)}{\partial x} + \frac{\partial (v_y C)}{\partial y} + \frac{\partial (v_z C)}{\partial z} = 0 \qquad (4.1)$$

where C is the element concentration and v_x, v_y, and v_z are the velocities in the direction x, y, and z. The first term in the equation accounts for the change in time of the concentration and the other term accounts for the variation in space.

Advection is the main process in the streams where velocities are high and diffusion is negligible. In estuaries where velocities are usually low, diffusion and dispersion must be examined.

Through the diffusion process, the concentration of a substance changes due to the element's concentration gradient. Fick's first law states that the rate of mass transport in the i direction is proportional to the concentration gradient in this direction or

$$M_i = - D_m \frac{\partial C}{\partial i} \qquad (4.2)$$

where D_m is the molecular diffusion coefficient.

Using this equation to account for the mass variation in a volumetric element, the diffusion process through this element is represented by the resulting equation:

$$\frac{\partial C}{\partial t} = D_m \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) \tag{4.3}$$

The processes are additive and since the velocities used in Equation (4.1) are time-averaged and associated with turbulent flow, the turbulent diffusion coefficients are used. The three dimensional transport equation for a stream then becomes

$$\frac{\partial C}{\partial t} + \frac{\partial (v_x C)}{\partial x} + \frac{\partial (v_y C)}{\partial y} + \frac{\partial (v_z C)}{\partial z} = \frac{\partial}{\partial x}\left(\frac{e_x \partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{e_y \partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{e_z \partial C}{\partial z}\right) \tag{4.4}$$

where e_x, e_y, and e_z are the turbulent diffusion coefficients.

The one-dimensional form of this equation for a stream has been developed by Holley and Harleman (1965). The longitudinal velocity, concentration, and the turbulent diffusity coefficient were averaged over the cross-section. The unsteady state one-dimensional mass transport equation for a non-conservative substance is

$$\frac{\partial (AC)}{\partial t} + \frac{\partial (QC)}{\partial x} = \frac{\partial}{\partial x}\left(EA\frac{\partial C}{\partial x}\right) + S_i \tag{4.5}$$

where E is the longitudinal dispersion coefficient that examines the non-uniform velocity distribution (dispersion) and the spatial-mean value of the turbulent diffusity. Figure 4.1 shows the spatial variation of the advection and dispersion terms in a isolated channel reach. The term S_i was added to account for the losses and gains of the system. The terms on the left side of the equation are from the advective process and the first term on the right side is the dispersion term.

The basic assumptions made in the derivation of Equation (4.5) are:

1. A mean value may represent the variation of the concentration

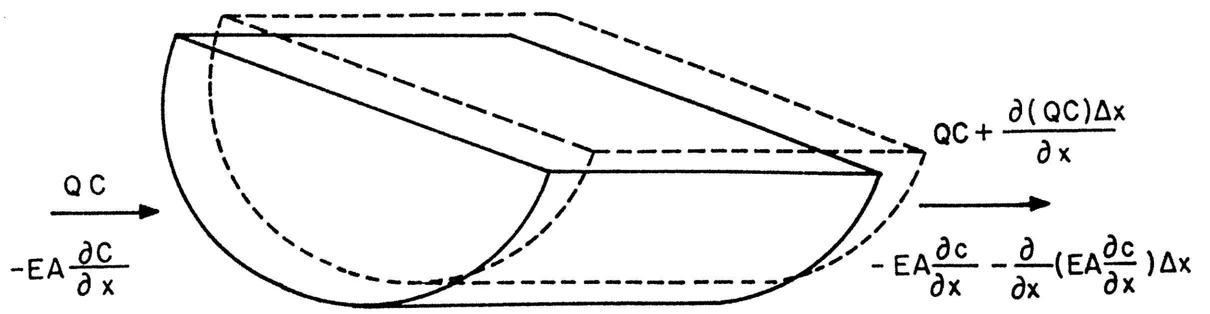


Figure 4.1 Schematic Illustration of Advection and Dispersion in a River Reach

and velocity over the cross-section; consequently, the problem becomes one dimensional in the longitudinal direction.

2. The longitudinal dispersion coefficient represents the cross product of the longitudinal velocities and concentrations about the cross section mean plus the spatial mean value of the turbulent diffusity.

A.2 Source and sink term

A river system can have internal and external sources of pollution. The internal sources result from physical, chemical, and biological reactions of the substances within the water body itself. External sources are inputs into a river system from external sources such as waste disposal, tributaries, and urban runoff.

External sources of pollution are usually classified as point and non-point. Examples of point sources include outlets of industrial and domestic waste, water treatment plant intake, artificial channels, and tributaries. Non-point sources include urban runoff, groundwater flow, and agricultural land runoff.

Velz (1970) classified the type of waste in streams as organic, microbial, radioactive, inorganic,and thermal. Stream water quality is determined through the analysis of substances selected to indicate the level of water quality. The substances to be analyzed are chosen on the basis of the study objectives and the source of pollution. These parameters include temperature, salinity, chlorides, dissolved oxygen, biochemical oxygen demand, nitrogen forms, and coliform concentration.

A conservative substance is defined as one with concentration unchanged by chemical or biological reactions. Salt and other chlorides are examples of conservative substances. The source and sink term for a conservative substance has only the external source term which means that discharge entering or leaving the river has the concentration of the substance.

The nonconservative substance in the water body can react by chemical or biological process thus modifying its concentration. Usually some of these substances such as the Biochemical Oxygen Demand are simulated by first order decay.

The model described here was developed primarily to simulate any conservative substance and the two stage reactions BOD - DO. However, it can be used for any substance in which the first order decay is a good simulator. The model can be modified without major difficulties to simulate other decay processes or consecutive reactions such as nitrification.

Conservative

The source and sink term is

$$S_{\ell} = q_{\ell} \, C_{\ell} \qquad (4.6)$$

where q_{ℓ} is the input or output discharge per unit of length ($m^3/m \cdot s$). The concentration of the substance in this flow (mg/l) is C_{ℓ}.

Biochemical oxygen demand and dissolved oxygen

The waste discharge may have carbonaceous and nitrogenous components. These components are oxided biochemically at different rates and times. The carbonaceous process is usually represented by the first order decay. The nitrogeneous demand is the oxidation of the amonia into nitrates by nitrifying bacteria.

Some of the processes which affect the BOD - DO stage in the river are:

-Oxidation of the carbonaceous component

-Oxidation of the nitrogeneous component

-Sedimentation or adsorption of the substances

-Addition of the substances through the scour of the river bottom increasing the BOD rate.

-Reaeration processes through the water surface

-The increase and loss of oxygen from the phytosynthetic action of plankton and fixed plants.

The following source and sink term is used for the biochemical oxygen demand:

$$S_i = -(K_1 + K_3)\, AC + AL_a + q_\ell C_\ell \qquad (4.7)$$

where K_1 is the BOD carbonaceous reaction rate (per day), K_3 is the rate coefficient for the removal of BOD by sedimentation and adsorption (per day), L_a is the rate of addition of BOD along the reach (ppm per day), q_ℓ is the lateral discharge (m^2/s), A is the cross section area (m^2), and C_ℓ is the concentration of the BOD in the lateral flow (ppm).

The partial differential equation for the BOD is

$$\frac{\partial (AC)}{\partial t} + \frac{\partial (QC)}{\partial x} = \frac{\partial}{\partial x}\left(EA\,\frac{\partial C}{\partial x}\right) - (K_1 + K_3)AC + AL_a + q_\ell C_{\ell(bod)} \qquad (4.8)$$

The source and sink term for the dissolved oxygen used is

$$S_i = -K_1 AC_{bod} + K_2 A(C_s - C) - D_b A + q_\ell C_{\ell(do)} \qquad (4.9)$$

where C_{bod} is the BOD concentration (ppm), K_2 is the reaeration coefficient (per day), C_s is the saturation dissolved oxygen concentration (ppm), and D_b is the removal of oxygen by benthal deposits, plant respiration and the increase in oxygen through photo-synthesis (ppm/day).

The partial differential equation for dissolved oxygen is

$$\frac{\partial (AC)}{\partial t} + \frac{\partial (QC)}{\partial x} = \frac{\partial}{\partial x} (EA \frac{\partial C}{\partial x}) - K_1 AC_{bod} + K_2 A(C_s - C) - D_b A + q_\ell C_{\ell (bod)} \tag{4.10}$$

A.3 Equation coefficients

Longitudinal dispersion coefficient

The longitudinal dispersion coefficient is the result of the effect of the nonuniform distribution of the velocity and concentration over the cross-section and the effect of the turbulent diffusity. The former is usually more important.

In general, when a pollutant enters the river the convective process is initially dominant and the pollutant cloud shows a shape similar to the velocity profile. The concentration curve in the space coordinate has a skewed shape. After this convective period, the cloud disperses due to turbulence and the concentration curve converges to a Gaussian shape. The spreading of the cloud in the convective period is defined by the one-dimensional dispersion coefficient and the mean flow velocity (Fischer, 1967).

Ordinarily, the longitudinal dispersion coefficient depends on the water depth, cross section shape, roughness, and the mean velocity. Taylor (1954) studies the longitudinal dispersion coefficient assuming a steady state version of Equation (4.5) in a long straight pipe. He arrived at the following equation:

$$E = 10.1\ a\ U^* \tag{4.11}$$

where a is the pipe radius, U* is the shear velocity, and E is the dispersion coefficient.

Elder (1959) assumed a logarithmic water velocity distribution in the vertical direction and applied the concept of longitudinal dispersion presented by Taylor (1954) to steady flow in an infinitely wide two dimensional channel. He also assumed the vertical velocity gradient was more important in the dispersion process and arrived at the resulting equation:

$$E = 5.93\ y\ U^* \tag{4.12}$$

where y is the depth of the flow.

Harleman (1971) also developed a modified Taylor's equation for channel flow using the relationship between the shear stress and the resistance parameter giving

$$E = 77\ n\ v\ R^{5/6} \tag{4.13}$$

where E is in square feet per second, n is the Manning roughness coefficient, v the velocity, and R the hydraulic radius in feet.

Fischer (1967, 1968) using a steady flow equation and assuming the lateral velocity distribution has more effect in the longitudinal disperson, presented the equation

$$E = -\frac{1}{A}\int_0^b q\left[\int_0^z \frac{1}{e_z y}\left(\int_0^z q\ dz\right) dz\right]dz \tag{4.14}$$

where q is the flow rate per unit of width, b is the width, A the cross section area, and e_z is the coefficient of lateral turbulent diffusion where $e_z = 0.23\ yU^*$.

Fischer (1969) studied the dispersion coefficient for oscillating flow in the constant density region of an estuary and concluded that the time of transverse mixing was many times greater than for vertical mixing. In this situation the velocity distribution in the vertical direction is more important and the modified Taylor's equation applies (Harleman, 1971).

McQuivey and Keefer (1974) presented a method based on an analogy to the linear version of the momentum equation and a linear dispersion equation. The momentum equation was linearized using a reference steady flow discharge (Q_o) and the related physical characteristics; width, slope of the energy gradient, and Froude number. This analogy resulted in the following linear relationship between the dispersion coefficients and those parameters:

$$E = 0.058 \frac{Q_o}{S_o W_o} \qquad (4.15)$$

This equation can be used when $F < 0.5$ and has an estimated standard error of about 30% according to the results of a comparative study of different conditions in eighteen streams.

Reaeration coefficient

Reaeration is one source of oxygen in water. The reaeration process is mainly a function of water temperature flow velocity and depth.

The reaeration process is represented by the term $K_2(C_s - C)$. The saturation concentration C_s is mainly a function of water temperature and atmospheric pressure. The American Public Health Association (1965) presented the equation

$$C_s = 14.652 - 0.41022T + 7.991.10^{-3}\, T^2 - 7.774.10^{-5}\, T^3 \qquad (4.16)$$

where T is the water temperature in °C. This equation is for a standard atmospheric pressure.

The reaeration coefficient K_2 is related to water depth and velocity. Many researchers have developed empirical equations for this coefficient based on the Streeter and Phelps relation,

$$K_2 = C \frac{v^n}{y^2} \tag{4.17}$$

where C and n are constraints, v is the velocity, and y is the depth. The relationship between K_2 and the temperature is generally expressed by

$$K_2(T) = K_2^* \; \theta^{(T-20)} \tag{4.18}$$

where θ is 1.0238, a constant defined experimentally, and K_2^* is the reaeration coefficient when $T = 20°C$.

Some of the equations developed for the reaeration coefficient are:

1. Churchill, Elmore and Buckingham (1962) developed an empirical equation using data of shallow rivers with high velocity to get

$$K_2 = 11.60 \frac{v^{0.969}}{y^{1.673}} \tag{4.19}$$

where K_2 is in 1/day, the velocity v is in m/s, and the depth y is in m.

2. O'Connor and Dobbins (1958) developed two equations, one for values of the Chezy coefficient less than 17 and the other for values greater than 17. The following equation is the most applicable:

$$K_2 = \frac{(D_m v)^{1/2}}{2.303 \; y^{3/2}} \tag{4.20}$$

where D_m is the molecular diffusion coefficient calculated by

$$D_m = 0.00192 \; [1.04^{(T-20)}]$$

where T is the temperature in °C, D_m in feet/day and K_2 1/day.

3. Owens, Edwards and Gibbs (1964) developed an empirical equation for rivers where the velocity is in the range of 0.1 to 5.0 feet and the depths are from 0.4 to 11.0 feet or

$$K_2 = \frac{9.4\ \bar{v}}{y^{1.85}} \tag{4.21}$$

Kramer (1974) analyzed a number of reaeration formulas and concluded that none could be used to accurately predict this coefficient in the Houston ship channel. Morel-Seytoux and Lau (1975) compared seven formulas and concluded the equations were poor for predicting the reaeration coefficient.

Rathbun (1977) reviewed techniques for measuring and predicting reaeration coefficients in streams and classified the measuring techniques as the DO balance and the disturbed equilibrium and tracer techniques. From the predicting formulas he also concluded that no one equation is best for all streams.

In the model used here the coefficient is either calculated by a subroutine REARE or can be estimated by the user in the input. The O'Connor and Dobbins formula is programmed into REARE but the user can easily exchange it for another equation.

Coefficients K_1 and K_3

The rate of biochemical oxidation of the carbonaceous matter is defined as being proportional to its remaining concentration. The rate at which the oxidation occurs is a constant K_1.

This coefficient depends on the type of organic matter, temperature, and river condition. Calculation of this coefficient is most commonly based on field data. Some of the methods are:

1) least-squares, 2) the slope method, 3) moment method, and 4) logarithm method. For further references see Nemerow (1974).

In literature a coefficient K_R is used to designate the rate of removal of organic material that is the sum of K_1, the removal rate by oxidation of the carbonaceous matter, and K_3 designates the rate of sedimentation or resuspension. The coefficient K_3 can be positive or negative. McKee and Wolf (1963) suggested factors that can make K_3 positive. These include sedimentation, volatilization of the organic material, adsorption, flocculation, and biological growths on the stream bed. The factors that can make it negative are the addition of BOD from sludge banks, scour longitudinal mixing, and short-circuiting across meanders. Table 4.1 shows a sample of the values of those coefficients from Bell (1973).

B. Numerical Method

B.1 Introduction

The solution of Equation (4.5) by analytical methods can be done only in special situations. O'Connor and Thomann (1971) described some of those conditions. When the river geometry does not allow those simplifications and the flow is unsteady the transport equation should be solved by a numerical method.

Equation (4.5) is a parabolic partial differential equation. Solution of this equation requires specification of initial and boundary conditions.

Stability and accuracy are the criteria for deciding on a specific numerical scheme. The backward implicit scheme was used in this study.

B.2 Applied numerical scheme

The backward implicit scheme is stable, accurate, and convenient

Table 4.1 Sample of K_1 and K_3 coefficients, according Bell (1973)

Stream or Estuary	K_1	K_R	K_3	Comments
Elk, Holston, Wabash, & Willamette Rivers	0.15 to 3.0	0.75 to 3.0		
Not Indicated	0.092 to 0.118			Rivers with low pollution
Clinten River	0.138 to 0.274	2.53		
Tittabaussee River	0.035 to 0.100	0.46		
Not Indicated	0.138 to 0.83		-0.83 to +0.83	
Herrisack River	0.138		0.046 to 0.62	
Not Indicated		0.207 to 0.53		Summer values
Not Indicated		0.046 to 0.161		Winter values
Kanauha River	0.041 to 0.296			
Hillstone and Passaic Rivers		0.028 to 1.012		
Hillstone River		0.440		Average K_R value
Passaic River		0.293		Average K_R value
Illinois River		0.0701		Navigational pool
East River		0.25		
Ohio River		0.23 to 0.161		
Iowa River	0.23			
Truckee River	0.288			
Delaware Estuary		0.30		
Grand River		0.80		$K_1 - K_R$
Clinton River		3.50		$K_1 - K_R$
Truckee River		0.49 and 1.30		$K_1 - K_R$
Flint River		0.76 and 0.95		

as it is possible to use the same configuration of sections used for solutions of the hydraulic equations. This scheme is

$$f(x,t) \cong \theta f_i^{t+1} + (1 - \theta) f_i^t$$

$$\frac{\partial f}{\partial x} \cong \theta \frac{(f_i^{t+1} - f_{i-1}^{t+1})}{\Delta x} + (1 - \theta) \frac{(f_i^t - f_{i-1}^t)}{\Delta x}$$

$$\frac{\partial^2 f}{\partial x^2} \cong \frac{1}{\Delta x} \{\theta [\frac{(f_{i+1}^{t+1} - f_i^{t+1})}{\Delta x} - \frac{(f_i^{t+1} - f_{i-1}^{t+1})}{\Delta x}] + (1 - \theta) [\frac{(f_{i+1}^t - f_i^t)}{\Delta x} - \frac{(f_i^t - f_{i-1}^t)}{\Delta x}]\}$$

$$\frac{\partial f}{\partial t} = \frac{1}{\Delta t} (f_i^{t+1} - f_i^t) \quad (4.22)$$

where $0 \leq \theta \leq 1$. Applying the above scheme to Equation (4.5) and using the notation $f^{t+\theta} = \theta f^{t+1} + (1 - \theta) f^t$ yields

$$\frac{1}{\Delta t} [(AC)_i^{t+1} - (AC)_i^t] + \frac{[(QC)_i - (QC)_{i-1}]^{t+\theta}}{\Delta x_{i-1}} = \frac{2}{\Delta x_{i-1} + \Delta x_i} \{\frac{[(AE)_i + (AE)_{i+1}]}{2\Delta x_i} (C_{i+1} - C_i) - \frac{[(AE)_i + (AE)_{i+1}]}{2\Delta x_{i-1}} (C_i - C_{i-1})\}^{t+\theta} \quad (4.23)$$

Equation (4.23) yields

$$L_i C_{i-1}^{t+1} + M_i C_i^{t+1} + N_i C_{i+1}^{t+1} = 0_i \quad (4.24)$$

where

$$L_i = \frac{-\theta}{\Delta x_{i-1}} [Q_{i-1} + \frac{(AE)_i + (AE)_{i-1}}{\Delta x_{i-1} + \Delta x_i}]^{t+1}$$

$$M_i = \frac{A_i^{t+1}}{\Delta t} + \theta \{\frac{1}{\Delta X_{i-1} + \Delta x_i} [\frac{(AE)_i + (AE)_{i-1}}{\Delta x_{i-1}} + \frac{(AE)_i + (AE)_{i+1}}{\Delta x_i}] + \frac{Q_i}{\Delta x_{i-1}}\}^{t+1}$$

$$N_i = -\theta\left[\frac{(AE)_i + (AE)_{i+1}}{\Delta x_i(\Delta x_i + \Delta x_{i-1})}\right]^{t+1}$$

$$O_i = \frac{A_i^t C_i^t}{\Delta t} + (1-\theta)\left\{\frac{1}{\Delta x_i + \Delta x_{i-1}}\left[\frac{(AE)_i + (AE)_{i+1}}{\Delta x_i}(C_{i+1} - C_i) - \frac{(AE)_i + (AE)_{i-1}}{\Delta x_{i-1}}(C_i - C_{i-1})\right] - \frac{[(QC)_i - (QC)_{i-1}]}{\Delta x_{i-1}}^t\right\}$$

The source and sink term was not included in Equation (4.24) because each substance may have its own term. The numerical source and sink term is discussed in B.5.

B.3 Mass conservation

Equation (4.24) is applied to each section resulting in a system of equations to be solved at each time step. Assuming a river without confluences and with constant Δx, the mass conservation of the system, using this numerical scheme, is performed by summing the equations of one time step as

$$\frac{1}{\Delta t}\sum_{i=1}^{N-1}[(AC)_i^{t+1} - (AC)_i^t] + \frac{1}{\Delta x}\sum_{i=2}^{N}[(QC)_i - (QC)_{i-1}]^{t+\theta} =$$

$$= \frac{E}{2\Delta x^2}\left\{\sum_{i=1}^{N-1}[(AE)_i + (AE)_{i+1}](C_{i+1} - C_i) - \sum_{i=2}^{N}[(AE)_i + (AE)_{i-1}](C_i - C_{i-1})\right\}^{t+\theta} + \left[\sum_{i=1}^{N-1} S_i\right]^{t+\theta} \quad (4.25)$$

Assuming $\theta = 1/2$ the above equation yields

$$\Delta x \sum_{i=1}^{N-1}(AC)_i^{t+1} = \Delta x \sum_{i=1}^{N-1}(AC)_i^t - \frac{\Delta t}{2}[(QC)_N^{t+1} + (QC)_N^t] + \Delta t\,\frac{[(QC)_1^{t+1} + (QC)_1^t]}{2} + \left[\sum_{i=1}^{N-1} S_i^{t+1} + \sum_{i=1}^{N-1} S_i^t\right]\frac{\Delta t\ \Delta x}{2} \quad (4.26)$$

This equation shows that the scheme is conserving mass where the terms have the following meaning:

$\Delta x \sum_{i=1}^{N-1} (AC)_i^{t+1}$ is the mass in the channel at the time t+1

$\Delta x \sum_{i=1}^{N-1} (AC)_i^{t}$ is the mass in the channel at time t

$\frac{\Delta t}{2} (Q_N^{t+1} + Q_N^t)$ is the mass that leaves the downstream boundary at Δt

$\Delta t \frac{[(QC)_1^{t+1} + (QC)_1^t]}{2}$ is the mass which enter in the upstream boundary at Δt

$\Delta t \Delta x [\sum_{i=1}^{N-1} S^{t+1} + \sum_{i=1}^{N-1} S_i^t]$ is the variation of the source and sink term at Δt

B.4 Numerical equation at a junction

The numerical scheme at a junction requires some modifications in the representation of mass conservation. The advection term is modified in the section where the equation is applied downstream of the confluence. The dispersion term is modified when the section is downstream or upstream of the confluence. When the section is upstream of the confluence the modification is in term i + 1. When the section is downstream, the modification is in the term i - 1.

Advection term

In Figure 4.2b, for section i, the differential of the advection term of the transport equation over both branches is

$$\frac{\partial (QC)}{\partial x} \approx \left[\frac{Q_k'C_k' - Q_kC_k}{\Delta x_{k,i}} + \frac{Q_j'C_j' - Q_jC_j}{\Delta x_{j,i}} \right]^{t+\theta} \tag{4.27}$$

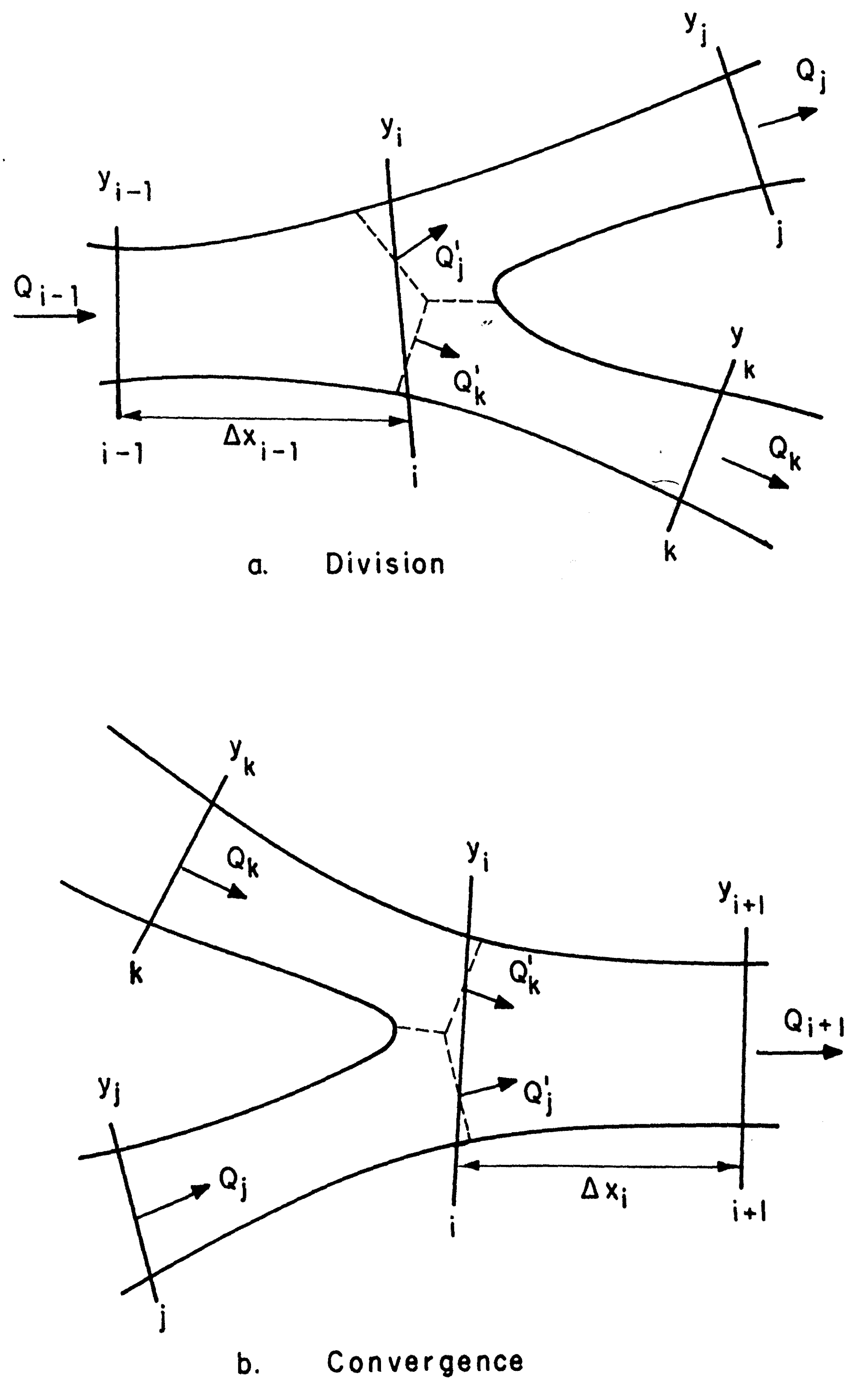


Figure 4.2 Confluence Conditions for the Numerical Scheme

The one-dimensional equation assumes one concentration in each section or a well-mixed pollutant distribution over the cross section that results in $C'_k = C'_j = C_i$. Since the hydraulic model assumes a steady flow condition among the confluence sections, then $Q'_k = Q_k = Q'_j$, and $Q_j + Q_k = Q_i$. The advection results in

$$\frac{\partial(QC)}{\partial x} \cong \left[\frac{Q_k(C_i - C_k)}{\Delta x_{k,i}} + \frac{Q_j(C_i - C_j)}{\Delta x_{j,i}}\right]^{t+\theta} \tag{4.28}$$

Using the same procedure, the advection term in the equation for section k or j in Figure 4.2a is

$$\frac{\partial(QC)}{\partial x} \cong \left[\frac{Q_k(C_k - C_i)}{\Delta x_{k,i}}\right]^{t+\theta} \tag{4.29}$$

<u>Dispersion term</u>

In Figure 4.2b for section i, the differential of the dispersion term over both branches and the reach i, i + 1, yields

$$\frac{\partial}{\partial x}\left(EA\frac{\partial C}{\partial x}\right) \cong \left[\left\{\left[\frac{(EA)_{i+1} + (EA)_i}{2}\right]\frac{(C_{i+1} - C_i)}{\Delta x_i} - (EA)_k\frac{(C_i - C_k)}{\Delta x_{k,i}} - (EA)_j\frac{(C_i - C_j)}{\Delta x_{j,i}}\right\}\frac{2}{\Delta x_i + \frac{(\Delta x_{k,i} + \Delta x_{j,i})}{2}}\right]^{t+\theta} \tag{4.30}$$

Using the same procedure for section i in Figure 4.2a the dispersion term is

$$\frac{\partial}{\partial x}\left(EA\frac{\partial C}{\partial x}\right) \cong \left[\left\{(EA)_k\frac{(C_k - C_i)}{\Delta x_{i,k}} + (EA)_j\frac{(C_j - C_i)}{\Delta x_{j,i}} - \frac{[(EA)_i + (EA)_{i-1}]}{2}\frac{(C_i - C_{i-1})}{\Delta x_{i-1}}\right\}\frac{2}{\Delta x_{i-1} + \frac{(\Delta x_{i,k} + \Delta x_{i,j})}{2}}\right]^{t+\theta} \tag{4.31}$$

There are four equations that can be used to describe the situation at junctions, assuming that not more than three branches are flowing to the junction. The equations are:

1. For the section upstream of the junction in Figure 4.2a, the numerical equation is

$$L_i C_{i-1}^{t+1} + M_i C_i^{t+1} + N_i C_k^{t+1} + O_i C_j^{t+1} = P_i \qquad (4.32)$$

where

$$L_i = -\theta\left[\frac{Q_{i-1}}{\Delta x_{i-1}} + C1_{i-1} \quad C3_{i-1}\right]^{t+1}$$

$$M_i = \frac{A_i^{t+1}}{\Delta t} + \theta\left[C3_{i-1} \; (C1_{i-1} + C2_j + C3_k) + \frac{Q_i}{\Delta x_{i-1}}\right]^{t+1}$$

$$N_i = -\left[\theta C2_k \; C3_{i-1}\right]^{t+1}$$

$$O_i = -\left[\theta C2_j \; C3_{i-1}\right]^{t+1}$$

$$P_i = \frac{(AC)^t}{\Delta t} + (1-\theta)\,\{C3_{i-1}\;[C2_j(C_j - C_i) + C2_k(C_k - C_i) -$$

$$C1_{i-1}\;(C_i - C_{i-1})] - \frac{[(QC)_i - (QC)_{i-1}]}{\Delta x_{i-1}}\}^t$$

and

$$C1_{i-1} = \frac{(AE)_{i-1} + (AE)_i}{\Delta x_{i-1}}$$

$$C2_j = \frac{2(AE)_j}{\Delta x_{i,j}}$$

$$C3_{i-1} = \frac{2}{\Delta x_{i-1} + \dfrac{(\Delta x_{i,j} + \Delta x_{i,k})}{2}}$$

2. For the section downstream of the confluence in Figure 4.2a, the equation is

$$L_k C_i^{t+1} + M_k C_k^{t+1} + N_k C_{k+1}^{t+1} = O_k \qquad (4.33)$$

where

$$L_k = -\theta\left[\frac{Q_k}{\Delta x_{i,k}} + \frac{C2_k}{\Delta x_{k+1} + \Delta x_{i,k}}\right]^{t+1}$$

$$M_k = \frac{A_k^{t+1}}{\Delta t} + \theta\left[\frac{1}{\Delta x_{k+1} + \Delta x_{i,k}} (C1_{k+1} + C2_k) + \frac{Q_k}{\Delta x_{i,k}}\right]^{t+1}$$

$$N_k = -\theta\left[\frac{C1_{k+1}}{\Delta x_{k+1} + \Delta x_{i,k}}\right]^{t+1}$$

$$O_k = \frac{(AC)_k^t}{\Delta t} + (1 - \theta)\ \{\frac{1}{\Delta x_{k+1} + \Delta x_{i,k}} [C1_{k+1}\ (C_{k+1} - C_k) -$$

$$- C2_k\ (C_k - C_i)] - \frac{[(QC)_k - Q_k C_i]}{\Delta x_{i,k}}\}^t$$

3. For the section downstream of the confluence in Figure 4.2b, the equation is

$$L_i C_k^{t+1} + M_i C_j^{t+1} + N_i C_i^{t+1} + O_i C_{i+1}^{t+1} = P_i \qquad (4.34)$$

where

$$L_i = -\theta\left[\frac{Q_k}{\Delta x_{k,i}} + C2_k\ C3_{i+1}\right]^{t+1}$$

$$M_i = -\theta\left[\frac{Q_j}{\Delta x_{j,i}} + C2_j\ C3_{i+1}\right]^{t+1}$$

$$N_i = \frac{A_i^{t+1}}{\Delta t} + \theta\left[C3_{i+1}\ (C1_{i+1} + C2_k + C2_j) + \frac{Q_k}{\Delta x_{k,i}} + \frac{Q_j}{\Delta x_{j,i}}\right]^{t+1}$$

$$O_i = -\theta\left[C1_{i+1}\ C3_{i+1}\right]^{t+1}$$

$$P_i = \frac{(AC)_i^t}{\Delta t} + (1 - \theta) \{C3_{i+1} [C1_{i+1}(C_{i+1} - C_i) - C2_k(C_i - C_k) - C2_j(C_i - C_j)] - [\frac{Q_kC_i - (QC)_k}{\Delta x_{k,i}} + \frac{Q_jC_i - (QC)_j}{\Delta x_{j,i}}]\}^t$$

4. For the section upstream of the confluence in Figure 4.2b, the equation is

$$L_kC_{k-1}^{t+1} + M_kC_k^{t+1} + N_kC_i^{t+1} = O_k \tag{4.35}$$

where

$$L_i = -\theta[\frac{Q_{k-1}}{\Delta x_{k-1}} + \frac{C1_{k-1}}{\Delta x_{k-1} + \Delta x_{k,i}}]^{t+1}$$

$$M_i = \frac{A_k^{t+1}}{\Delta t} + \theta[\frac{1}{\Delta x_{k-1} + \Delta x_{k,i}} (C2_k + C1_{k-1}) + \frac{Q_k}{\Delta x_{k-1}}]^{t+1}$$

$$N_i = -[\frac{\theta C2_k}{\Delta x_{k,i} + \Delta x_{k-1}}]^{t+1}$$

$$O_i = \frac{(AC)_k^t}{\Delta t} + (1 - \theta) \{\frac{1}{\Delta x_{k-1} + \Delta x_{k,i}} [C2_k(C_i - C_k) - C1_{k-1}(C_k - C_{k-1})] - \frac{[(QC)_k - (QC)_{k-1}]}{\Delta x_{k-1}}\}^t$$

Using these equations it is possible to demonstrate that they conserve mass through the junctions.

B.5 Numerical source and sink

For a conservative substance without an intake or outlet from the river, there is no change in the numerical equations. The numerical source and sink term is

$$S_i = \theta S_i^{t+1} + (1 - \theta) S_i^t \tag{4.36}$$

A conservative substance is added to the right hand side of the equation by the term

$$S_i = \theta(qC)_\ell^{t+1} + (1 - \theta)\ (qC)_\ell^t \tag{4.37}$$

For a first order decay equation such as BOD, the term is

$$S_i = -\ \theta(K_1 + K_3)\ A_i^{t+1} C_i^{t+1} - \theta(AL_a + q_\ell C_\ell)_i^{t+1} + (1 - \theta)\ \cdot$$

$$[-\ (K_1 + K_3)\ AC_i + AL_a + q_\ell C_\ell]^t \tag{4.38}$$

The oxygen demand term is

$$S_i = -\ K_2\ A_i^{t+1} C_i^{t+1} + \theta[-(K_1\ AC_{bod})_i + (K_2\ AC_s)_i - (AD_b)_i + (qC)_{\ell_i}]^{t+1}$$

$$+ (1 - \theta)\ [-\ K_1\ AC_{bod} + K_2\ A(C_s - C) - AD_b + (qC)_\ell]_i^t \tag{4.39}$$

Those terms are added to the numerical equations developed in each section and for each substance simulated.

B.6 Stability and accuracy

When a numerical scheme conserves mass there is no guarantee the solution will be stable and accurate. The analysis of accuracy and stability of the complete one-dimensional transport equation is complex, if not impossible to evaluate. The following analysis of stability and accuracy was carried out with a simplified version of the transport equation.

Stability

Keller (1960) used the maximum principle to discuss the stability of the central scheme. Lanna and Moretti (1977), using Keller's work, also presented the conditions for the backward and forward scheme. The transport equation used in those schemes was

$$\frac{\partial C}{\partial t} + v(x,t)\frac{\partial C}{\partial x} = E(x,t)\frac{\partial^2 C}{\partial x^2} - K(x,t)C \tag{4.40}$$

where $E(x,t) > 0$. The stability condition for the backward scheme is

$$\Delta t \leq \frac{\Delta x^2}{(1 - \theta)\ (2E + \Delta x \cdot v + \Delta x^2 K)} \tag{4.41}$$

For the fully implicit scheme ($\theta = 1$), the scheme is unconditionally stable. When the velocity is negative there is one more condition that should be met for this scheme,

$$\Delta x \leq \frac{E}{|v|} \tag{4.42}$$

in which the term $\Delta x \cdot v/E$ is the cell Reynolds number (R_c) (Roache, 1972). The stability conditions when $v < 0$ in the backward scheme are the same conditions for the forward scheme when $v > 0$.

The condition of Equation (4.42) is more difficult to meet when the dispersion coefficient is small. Stability then only occurs for small values of Δx. Those conditions were obtained using a simplified equation, then used as a suggestion in the definition of those numerical variables.

Accuracy

The errors in the numerical computation can create dissipative and dispersive effects on the solution. Leendertse (1970) used the Von Neumann method to compute the ratios of the numerical and analytical solution for a linear version of this equation in order to evaluate these effects.

The general analytical solution of the partial differential Equation (4.40) with constant coefficients expanded following the Fourier series or

$$C(x,t) = \sum_{m=-\infty}^{m=\infty} C_m \, EXP[i(\sigma_m x - \beta_m t)] \qquad (4.43)$$

where C_m is the constant coefficient for the m^{th} component of the series, σ_m is the wave number ($\sigma_m = 2\pi/L_m$), L_m is the wave length, and $i = \sqrt{-1}$.

The discrete form of Equation (4.43) for one component of the series is

$$C_j^n \, (j\Delta x, n\Delta t) = C \, EXP[i(\sigma j\Delta x - \beta n\Delta t)] \qquad (4.44)$$

Using the numerical scheme (4.22) in the transport Equation (4.40) with constant coefficients yields

$$\frac{C_j^{t+1} - C_j^t}{\Delta t} + v\frac{(C_j - C_{j-1})^{t+\theta}}{\Delta x} = \frac{E}{\Delta x^2}\,(C_{j+1} - 2C_j + C_{j-1})^{t+\theta} - K\,C_j^{t+\theta} \qquad (4.45)$$

Using (4.44) in (4.45) yields

$$e^{-i\beta\Delta t} = \frac{\{1 - (1-\theta)[a(1 - e^{-i\sigma\Delta x}) - b(e^{i\sigma\Delta x} - 2 + e^{-i\sigma\Delta x}) + C_k]\}}{1 + \theta[a(1 - e^{-i\sigma\Delta x}) - b(e^{i\sigma\Delta x} - 2 + e^{-i\sigma\Delta x}) + C_k]} \qquad (4.46)$$

where

$$a = \frac{v\Delta t}{\Delta x}$$

$$b = \frac{E\Delta t}{\Delta x^2}$$

$$C_k = K\Delta t$$

Using Euler's identities and trigonometric relations,

$$(i - e^{-i\sigma\Delta x}) = 1 - \cos\sigma\Delta x + i\sin\sigma\Delta x = 2\sin^2\frac{\sigma\Delta x}{2} + i\sin\sigma\Delta x$$

and

$$(e^{i\sigma\Delta x} + e^{-i\sigma\Delta x}) = 2\cos\sigma\Delta x - 2 = -4\sin^2(\sigma\Delta x/2)$$

By substitution into Equation (4.46),

$$e^{-i\beta\Delta t} = \frac{1 - (1 - \theta)[2\sin^2(\sigma\Delta x/2)(2b + a) + ia\sin(\sigma\Delta x) + C_k]}{1 + \theta[2\sin^2(\sigma\Delta x/2)(2b + a) + ia\sin(\sigma\Delta x) + C_k]} = \lambda \qquad (4.47)$$

The frequency β can be a complex number $\beta = \beta_r + i\beta_i$, where β_r is the real part and β_i the imaginary part.

The dissipative effect is usually evaluated by the following ratio

$$R_1 = \frac{\text{Numerical wave damping}}{\text{Analytical wave damping}} \qquad (4.48)$$

The physical damping is due to the diffusion and decay which is $\mathrm{EXP}[-(\sigma^2 E + k)\Delta t]$. The computed wave damping is given by the modulus of λ, then the ratio is

$$R_1 = \left|\frac{1 - (1 - \theta)[2\sin^2(\sigma\Delta x/2)(2b + a) + ia\sin\sigma\Delta x + C_k]}{1 + \theta[2\sin^2(\sigma\Delta x/2)(2b + a) + ia\sin(\sigma\Delta x) + C_k}\right| e^{(\sigma^2 E+K)\Delta t} \qquad (4.49)$$

Calculating the modulus of the above complex number for $\theta = 1/2$, yields

$$R_1 = \frac{\{[1 - \frac{(x^2 + y^2)}{4}]^2 + y^2\}^{1/2}}{(1 + \frac{x}{2})^2 + (\frac{y}{2})^2} e^{(\sigma^2 E+K)\Delta t} \qquad (4.50)$$

where

$$x = \frac{2\Delta t}{\Delta x} \sin^2(\sigma\Delta x/2)(2E/\Delta x + v) + K\Delta t$$

$$y = \frac{v\Delta t}{\Delta x} \sin(\sigma\Delta x)$$

and R_1 is the damping ratio after one time step.

The dispersive ratio is defined by

$$R_2 = \frac{\text{Numerical wave velocity}}{\text{Analytical wave velocity}} \qquad (4.51)$$

If the velocity of one component in the numerical solution is given by β_r/σ, then

$$R_2 = \frac{\beta_r}{\sigma v}$$

To find β_r the following relation is used:

$$e^{i\beta\Delta t} = e^{\beta_i \Delta t} [\cos(-\beta_r\Delta t) + i\sin(-\beta_r\Delta t)] \tag{4.53}$$

Equating the real and imaginary parts of Equation (4.47) with those of Equation (4.53) and dividing the equations, yields

$$\beta_r = \frac{1}{\Delta t} \arctan\left[\frac{y}{1 - \frac{(x^2 + y^2)}{4}}\right]$$

then

$$R_2 = \frac{1}{\Delta t v\sigma} \arctan\left[\frac{y}{1 - \frac{(x^2 + y^2)}{4}}\right] \tag{4.54}$$

The dissipative effect is described by the ratio R_1 that indicates that the numerical damping is smaller than the physical damping for $R_1 > 1$, and the numerical damping is greater than the physical damping for $R_1 < 1$. The ratio R_2 that describes the dispersive effect due to the velocity of the numerical solution indicates that velocity of the numerical wave is slower than the physical velocity for $R_2 < 1$. For $R_2 > 1$ the numerical velocity is faster than the physical velocity.

The term $\sigma\Delta x$ used in Figures 4.3 and 4.4 can be modified to $L/\Delta x$ ($\sigma = 2\pi/L$) as used by Leendertse (1967) where L is the wave length. This last term is the number of discrete points per wave length.

In Figure 4.3, a constant value of b, R_1, and R_2 were plotted for some values of <u>a</u> assuming $\Delta x = 200$ m and $\Delta t = 200$s. This test shows that with the increase of the ratio $v\Delta t/\Delta x$, the solution is less accurate. The greater the number of sections per wave length,

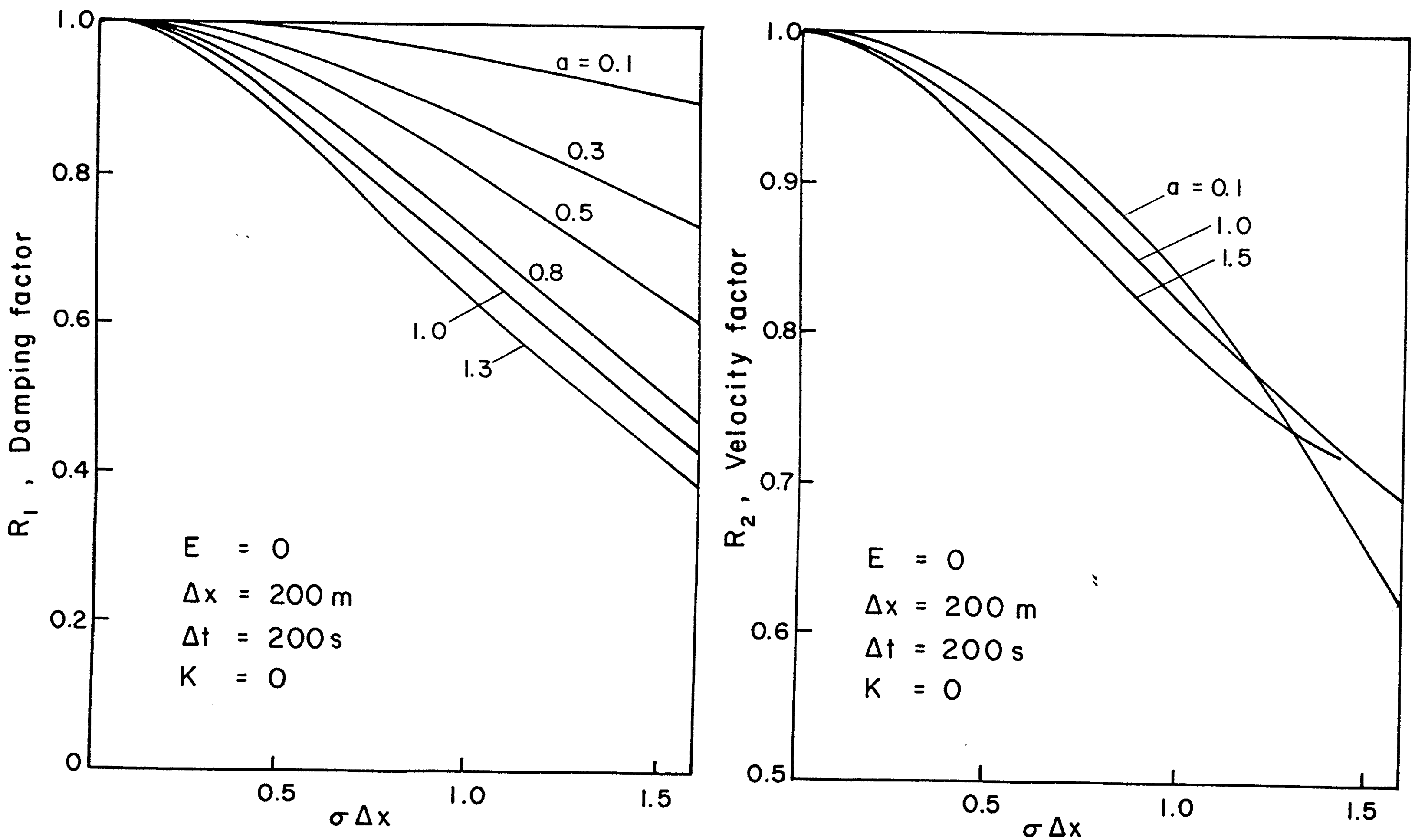


Figure 4.3 Damping and Velocity Factors for Values of a

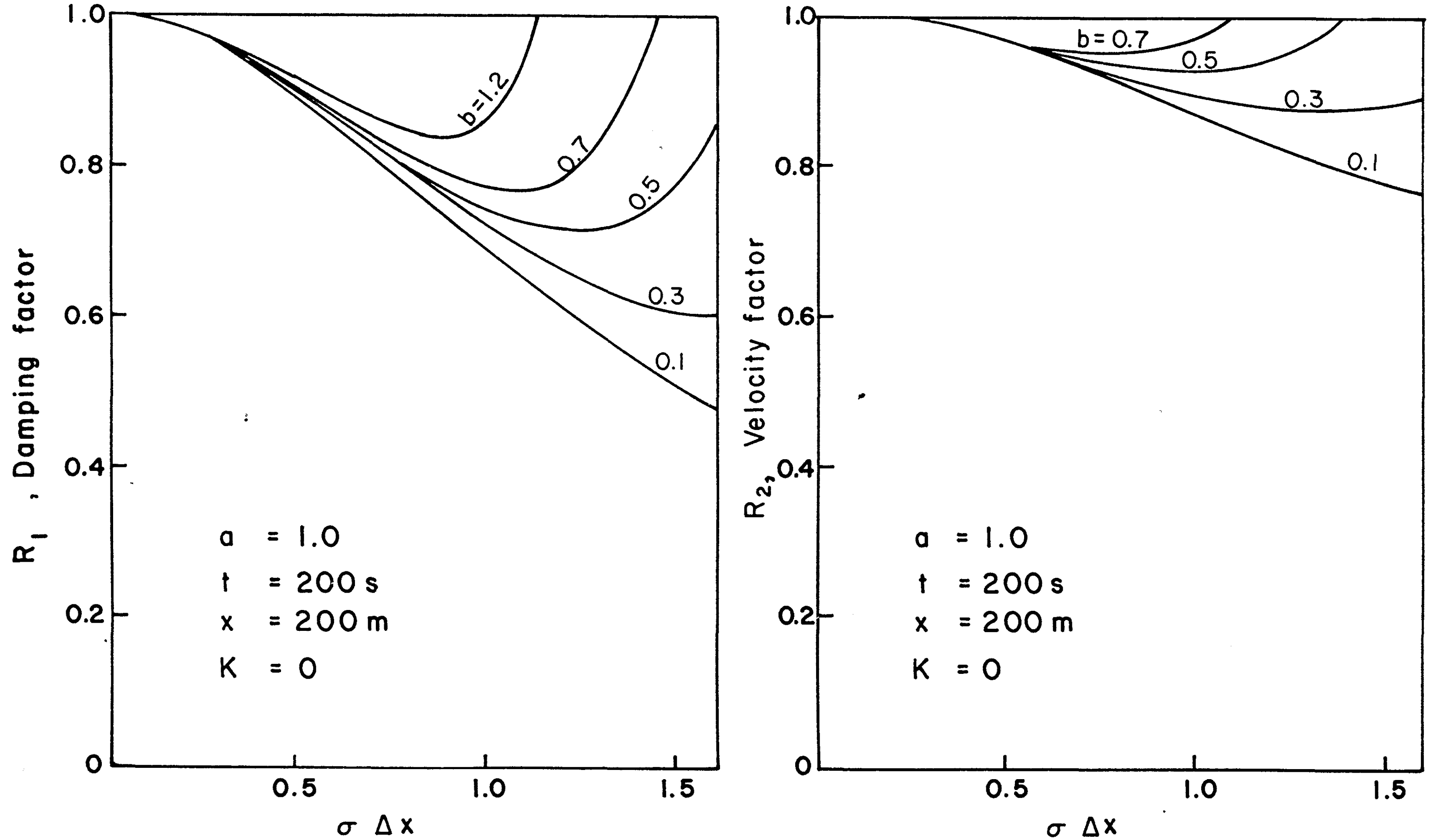


Figure 4.4 Damping and Velocity Factors for Values of b

the more accurate are the solutions. Figure 4.4 shows the R_1 and R_2 values based on the variation of b and $\sigma\Delta x$. The increase of the ratio $E\Delta t/\Delta x^2$ shows that the curve inflexion that creates sudden changes in accuracy moves in the direction of small values of $\sigma\Delta x$.

Conclusions about the accuracy of analysis include the following:

1) The damping and velocity ratios described here were developed based on a linear transport equation. Therefore, these ratios give only a qualitative idea about the accuracy of a difference scheme.

2) Leendertse (1967) computes the modulus of the propagation factor (damping ratio R_1) by the equation

$$R_1^* = R_1^n \tag{4.55}$$

where n is the number of operations performed for the time that the physical wave propagates over its wave length. Then

$$n = \frac{L}{v\Delta t} = \frac{2\pi}{v\Delta t} \tag{4.56}$$

The ratio R_1^* measures the damping after n time steps.

Ponce, et al. (1978b) computed the damping effect based on the logarithmic decrement,

$$R_1^* = \exp(\delta_n - \delta) \tag{4.57}$$

where δ_n is the logarithmic decrement of the numerical solution and δ is the logarithmic decrement of the analytical solution. The logarithmic decrement is defined as

$$\delta = \ell n(a_1) - \ell n(a_o) \tag{4.58}$$

in which a_o and a_1 = the wave amplitude at the beginning and end of the wave period, respectively. The logarithmic decrement can be calculated by

$$\delta = 2\pi \frac{B_i}{|B_r|} \tag{4.59}$$

3) The accuracy analysis of the complete equation can be performed only by numerical experiments. The equations derived here can be used to design the numerical experiments.

Numerical Test

Accuracy can be tested by comparing the numerical solution with the analytical solutions in a simplified system. The analytical solution for a steady-state profile for a nonconservative substance which is continuously released in a river or estuary with a constant cross section at the rate of W pounds per day is the following:

$$C_{bod} = C_o \ \mathrm{EXP}[\frac{vx}{2E} (1 \pm m_1)] \tag{4.60}$$

$$C_{od} = C_s - \frac{K_1 C_o}{(K_2 - K_1)} \ \frac{1}{m_1} \mathrm{EXP}[\frac{vx}{2E} (1 \pm m_1)] - \frac{1}{m_2} \mathrm{EXP}[\frac{vx}{2E} (1 \pm m_2)] \tag{4.61}$$

where

$$m_1 = \sqrt{1 + \frac{4K_1E}{v^2}}$$

$$m_2 = \sqrt{1 + \frac{4K_2E}{v^2}}$$

$$C_o = \frac{W}{Qm_1}$$

where C_s is the oxygen concentration saturation, Q is the flow, x is the distance (when the section is upstream of the point of releast, x is negative, and the sign of m_1 is positive), t is the time, C_o is the concentration at $x = 0$, K_1 is the decay coefficient, K_2 the reaeration coefficient, E is the dispersion coefficient and v is the velocity.

An auxiliary program that solves the transport equation for constant coefficients (described in Appendix C) was used for comparing analytical solutions.

The test was performed by assuming a concentration of 10 ppm of BOD at $x = 0$, $K_1 = 0.25/\text{day}$, $K_2 = 0.5/\text{day}$, $E = 1.5\ \text{km}^2/\text{day}$, $v = 5.0\ \text{km/day}$, $C_s = 9$ ppm, and $\theta = 0.5$.

The stability condition was calculated using Equation (4.41), assuming $\Delta x = 0.5$ km then

$$\Delta t \leq 0.045 \text{ day}$$

The value used was $\Delta t = 0.01/\text{day}$. The boundary condition used downstream was Equation (4.67).

The boundary condition for DO at $x = 0$ was calculated by Equation (4.61). The initial condition for BOD in the numerical solution was $C(x,o) = 0$ for all sections and $C(x,0) = C_s$ for DO. After six days of simulation the numerical solution reached the analytical solution with an error on the order of 10^{-3}. Figure 4.5 shows the solutions.

B.7 Initial and boundary condition

The transport equation with advection and dispersion terms is a parabolic partial differential equation. This type of equation requires the specification of the values in all sections at the beginning of the calculation ($t = 0$) as

$$C(x,0) = C_i \qquad i = 1,2,\ldots n \tag{4.62}$$

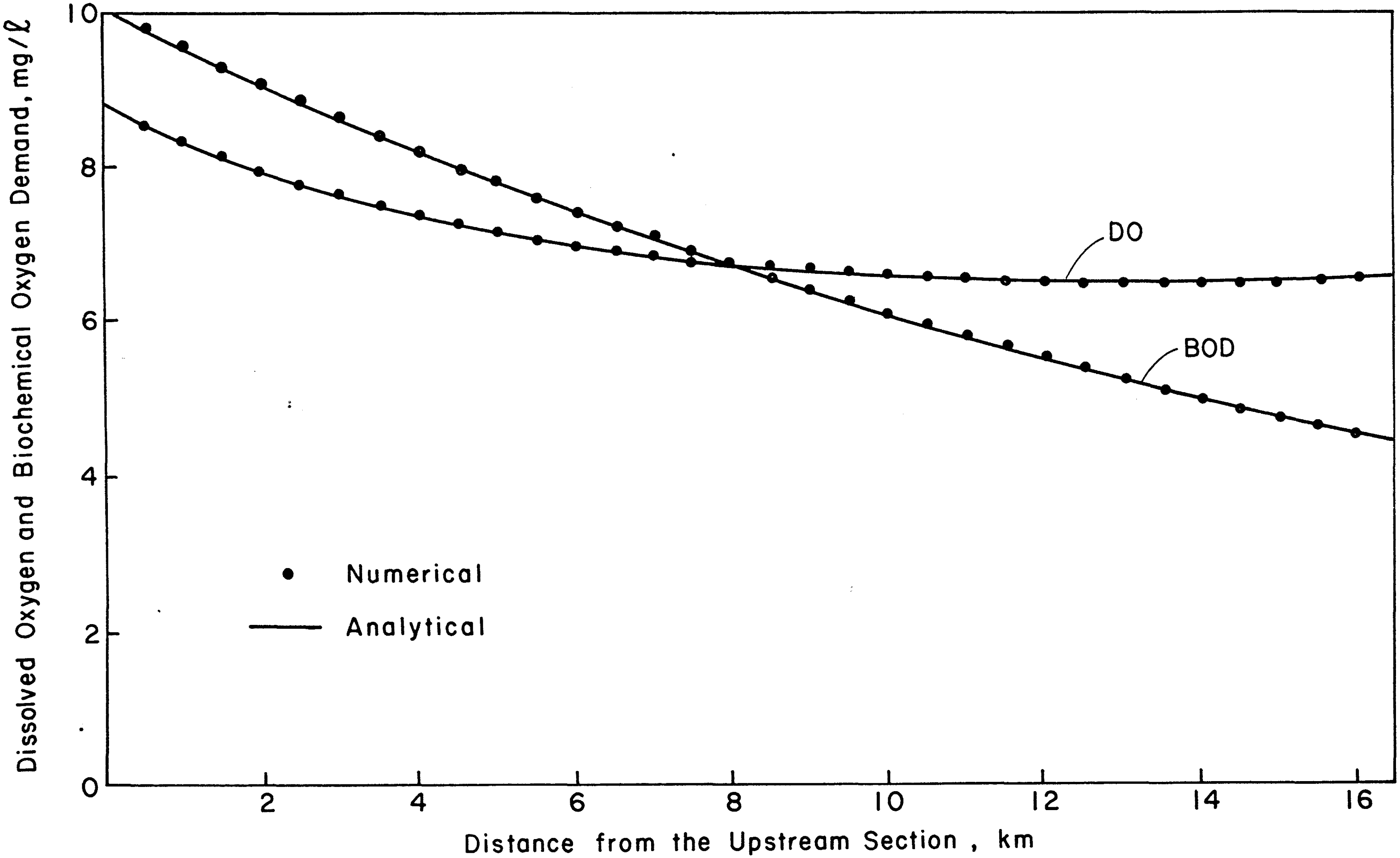


Figure 4.5 Numerical Test for the Backward Scheme

and the two boundary conditions for all time steps. The boundary conditions can be specified in different ways. Some of the more usual procedures are:

1. When the concentration function of time at the boundaries is known,

$$C(0,t) = C_u(t) \qquad \text{(upstream)} \qquad (4.63)$$

$$C(n\Delta x,t) = C_d(t) \qquad \text{(downstream)} \qquad (4.64)$$

2. Assuming the concentration does not change with x at the downstream boundary,

$$\frac{\partial C}{\partial x} = 0 \qquad (4.65)$$

It is implied that $C_n = C_{n-1}$ or $C_n = r$ where r is a constant. This condition can be used when the downstream section is far from a source point because the gradient is steep close to the source.

3. Assuming the second partial derivative of the concentration is equal to zero, which means the concentration has a linear relationship with x at the downstream boundary,

$$\frac{\partial^2 C}{\partial x^2} = 0 \qquad (4.66)$$

then

$$C_n = 2C_{n-1} - C_{n-2} \qquad (4.67)$$

This condition should also be carefully used when the gradient is steep near the boundary.

Condition 2 can be used upstream and condition 3 can be used downstream. The only data required would be the initial condition. This procedure can be used when the boundaries are not sources of pollutants.

The initial condition is usually unknown. When the steady-state solution can be applied as the initial condition, it can be set by running the program for constant boundaries in time.

C. System of Equations

After use of the numerical equation (4.24) for a reach section, and Equations (4.32), (4.33), (4.34), and (4.35) for a confluence section and the boundary conditions, the result is a system of equations to be solved at each time step. The system of equations is

$$\underline{F}_1 \underline{C} = \underline{E} \tag{4.68}$$

where $\underline{F}_1$ is the coefficient matrix, $\underline{C}$ is the concentration at time $t + 1$ matrix, and $\underline{E}$ is the right hand side matrix.

The solution to this system of equations when the system does not have a confluence can easily be performed by the Thomas algorithm (Appendix C). When the system has confluences, the coefficient matrix results in a sparse matrix that is solved by the procedure described in Chapter III, section C.2. When more than one pollutant should be simulated, the program UNSWQ stores the coefficient matrix in another array since they are the same for all pollutants, changes occur only in the diagonal term and in the right hand side term.

CHAPTER V

TESTING OF THE MODELS

A. Case Study

A.1 Description

The system tested in this study was the Jacui Delta, a small delta located in the south of Brazil in the state of Rio Grande do Sul. Four rivers flow into this delta including the Gravatai, the Sinos, the Cai and the Jacui. The total watershed area at section F (Figure 5.1) is about 100,000 km^2, which represents one third of the state. The Jacui River is the main stream; its watershed at section M makes up about 80% of the total watershed. The Jacui Delta is a complex system of branches, confluences, and storage basins, with an area of only 42 km^2 (Figure 5.1). The distance between the confluences is small (small islands), the widths are large (about 1000 m) in the main channels, and the slope is small. Below the downstream section the rivers form a series of large lakes that are linked together until they reach the Atlantic Ocean. The delta is about 250 km away from the ocean. On the eastern side of the delta there is a harbor and Porto Alegre which is the capital of the State, a city of about 1.2 million people.

The water level in this delta shows a cyclic variation with an amplitude of about 30 cm within a 24-hour time period. This cyclic variation is sometimes altered by wind effects and floods (Figure 5.2). In the dry season when the flow is low, a flow inversion can occur due to the backwater effects from the lakes. The flow variation in section F that bounds the delta downstream is illustrated in Figure 5.3. The flow variation for section D, concurrent with section F, is shown in

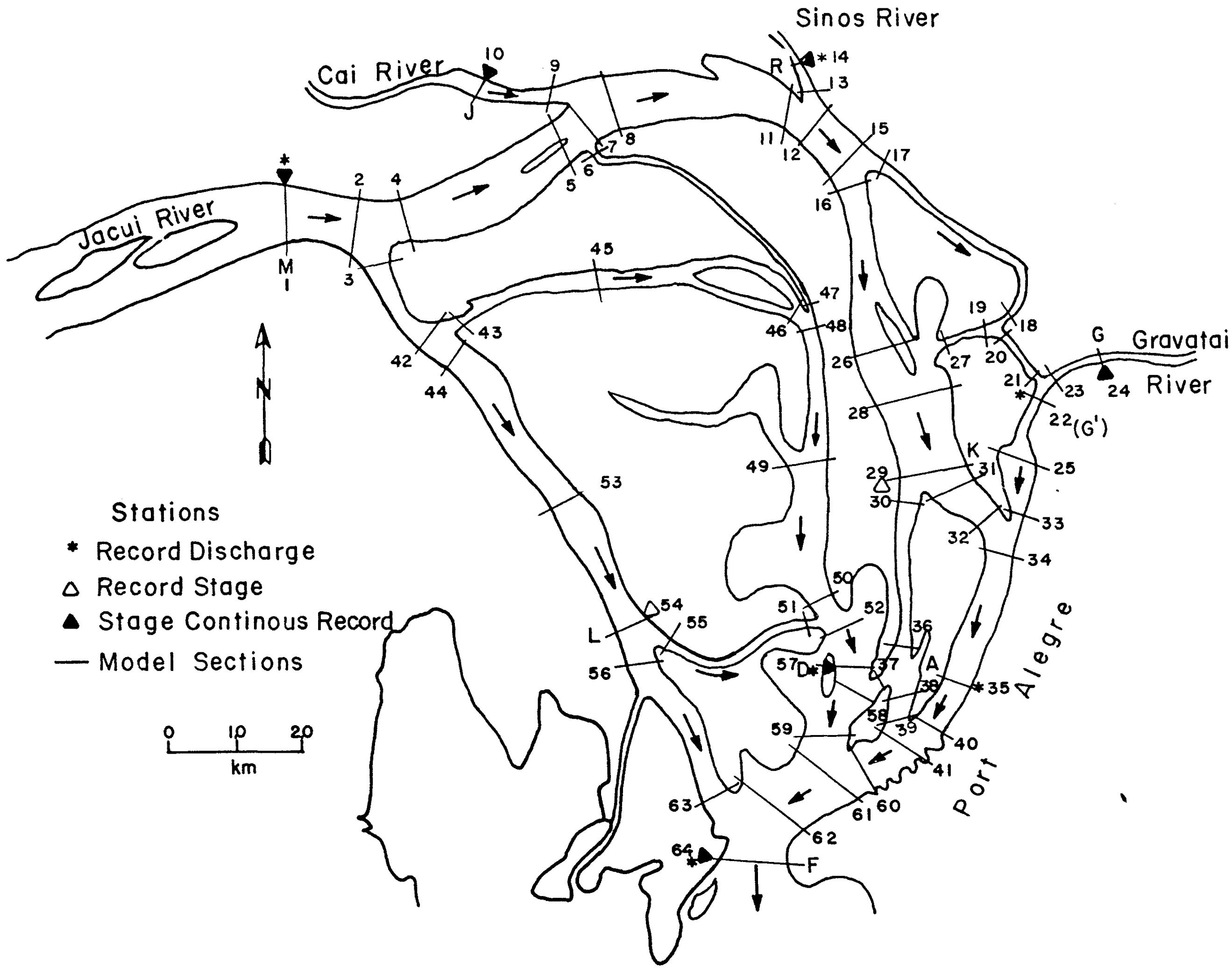


Figure 5.1 Jacui Delta Map Showing Sections Used in the Model

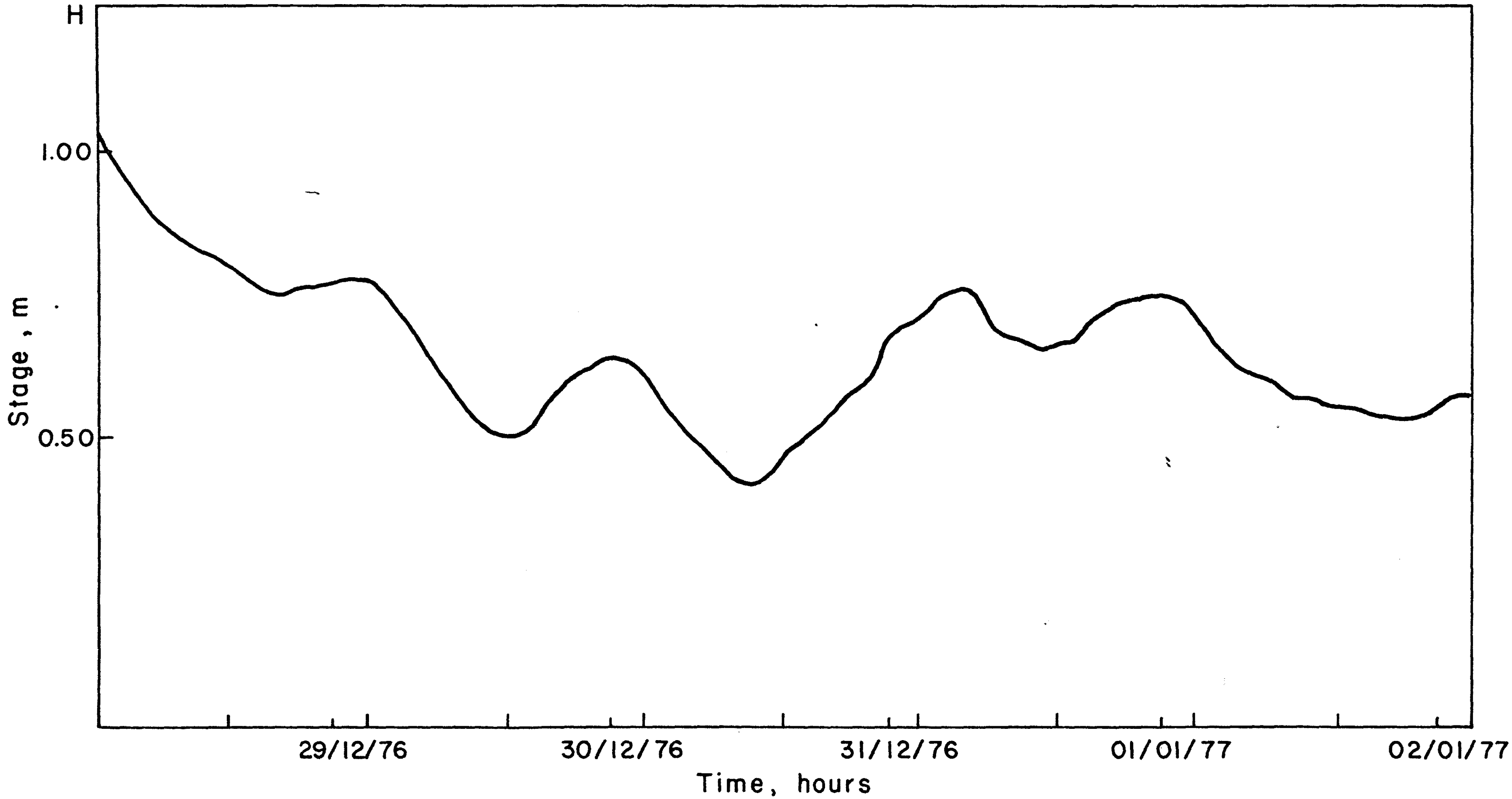


Figure 5.2 Stages at Section F for December 29, 1976 to January 2, 1977

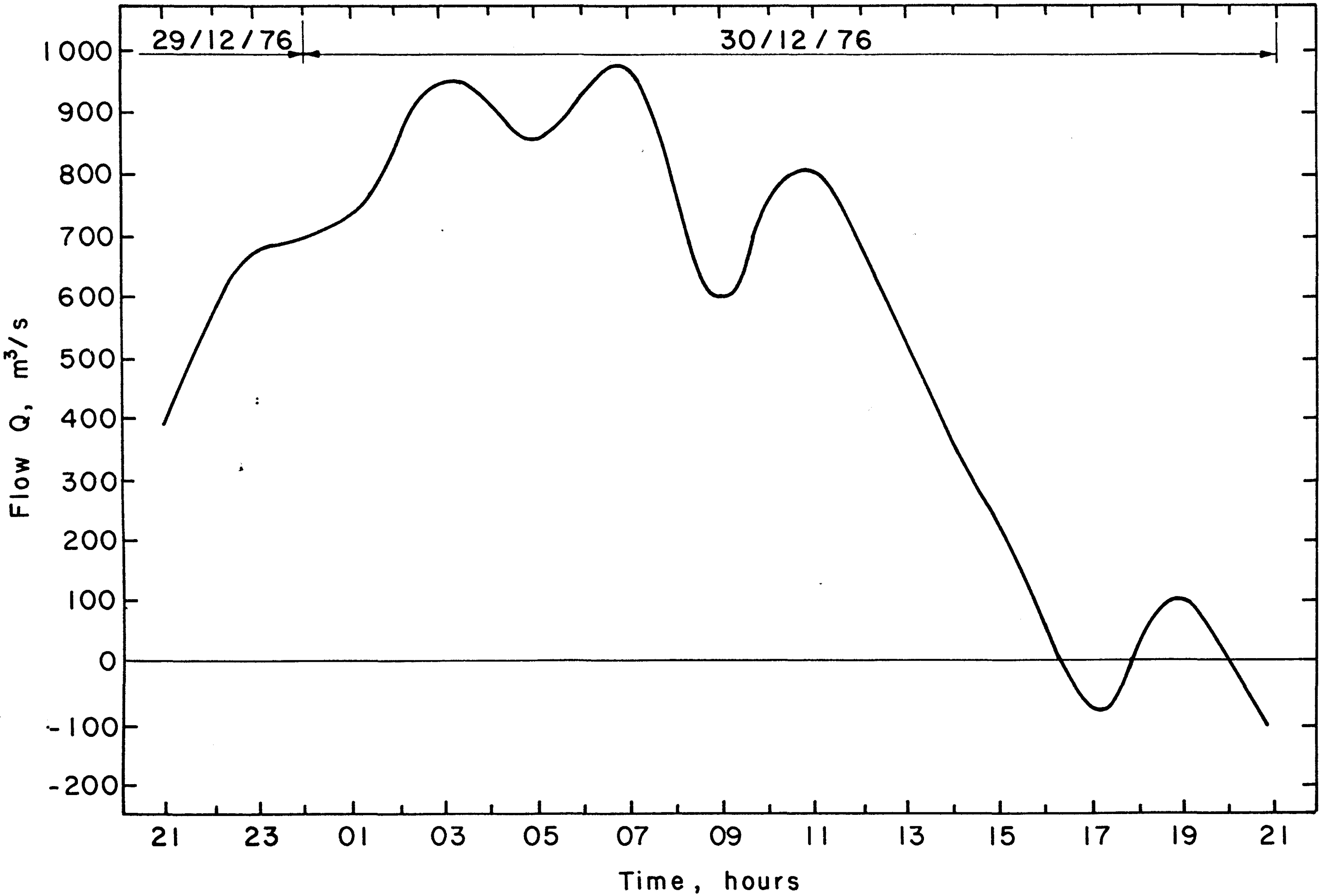


Figure 5.3 Discharge at Section F for December 29, 1976 to December 30, 1976

Figure 5.4. These data were not used to test the model since at this point in time only these two sections had recorded values.

The Gravatai and Sinos rivers carry domestic and industrial pollution to the delta. In the near future the Jacui will bring pollution from a petrochemical complex. The Jacui River has a low level of pollution, most of it coming from agricultural sources.

The water from this Delta was used for water supply, waste dillution, navigation, and recreation. Upstream in the Gravatai and Sinos Rivers, the waste is dumped directly into the rivers without treatment; and downstream near the harbor, water is collected for domestic water supply. Water quality is poor during the summer when the flow is low. Due to the complex behavior of this Delta with its flow inversion, it is difficult to decide where the water supply intake should be located.

A.2 Available data

The Institute of Hydraulic Research of the Federal University of Rio Grande do Sul installed six measuring stations to provide continuous records of stage (Figure 5.1) and others for discontinuous readings. In these stations, levels are recorded five times daily (8am, 10am, 12 noon, 3pm, and 6pm). The reference level used was sea level but the reference elevation of some of these stations has not yet been determined.

The discharges were recorded in seven sections (Figure 5.1). The record of discharge available in those sections is for 24 hours (12noon April 27, 1977 to 12noon of April 28, 1977). The discharges were recorded at time intervals of 3 hours, but conditions were bad and errors

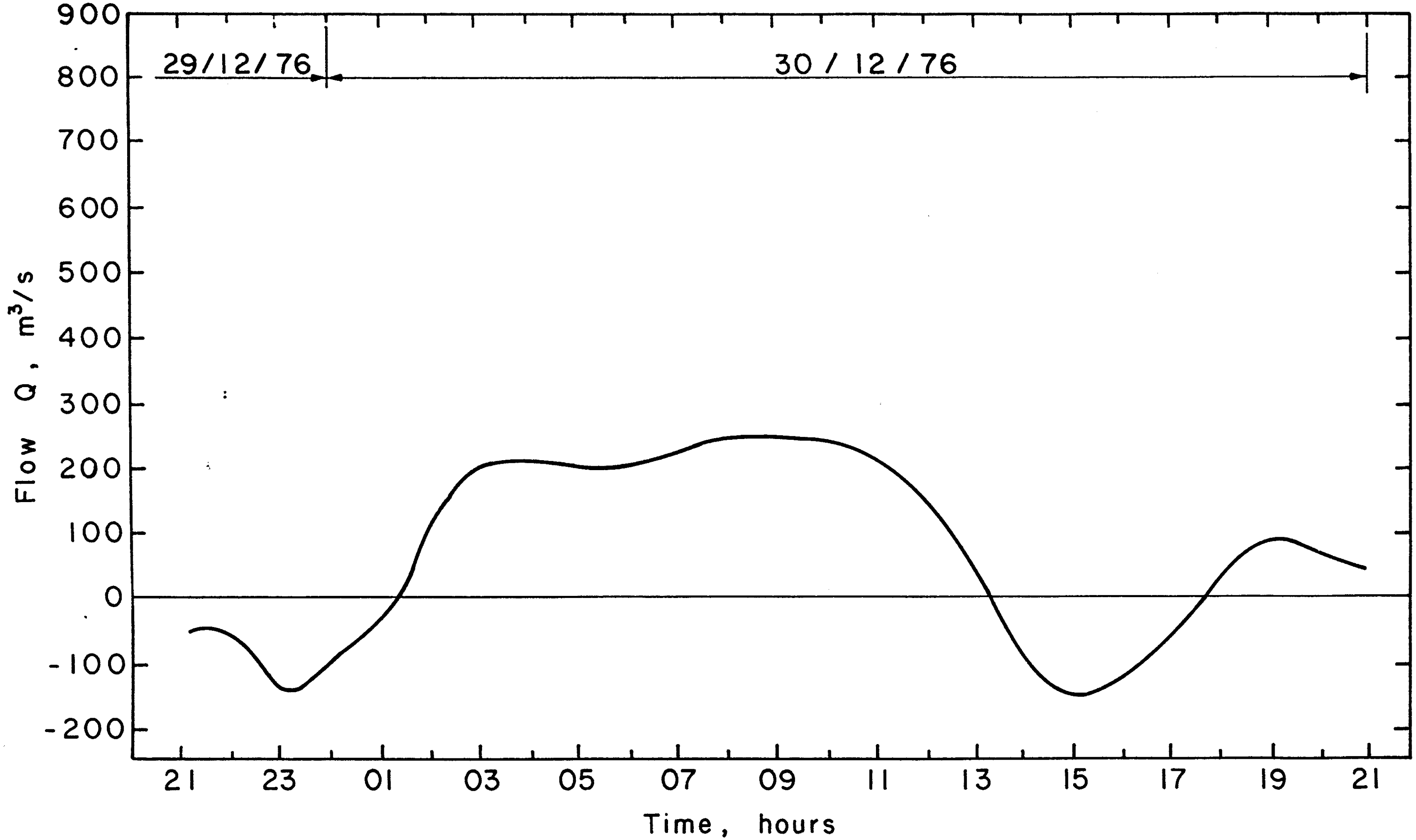


Figure 5.4 Discharge at Section D for December 29, 1976 to December 30, 1976

resulted. Only four or five verticals were used in each section. The important sections were between 800 and 1200 meters wide. Periodically, the vertical position of measurement was difficult to hold since the velocity was high for this size of river. Some records are missing at sections M, J, and R.

The data available from the stages is for April and May of 1977. Section F has a continuous record beginning in February 1976. Sections R, J, and G have only relative levels.

A map with a batimetry measured thirty years ago was available. It was used as a reference to locate the sections for a new batimetry measured in 1977. In most of the sections used by the model a new batimetry was available. In the event that a new batimetry did not exist, the old batimetry was used. Figure 5.5 shows the configuration of the system taking into account the sections. Appendix A contains a table describing each section, giving the physical characteristics used by the model including area, hydraulic radius, width, level, and bottom level. These values were calculated using the coordinates obtained from the maps in a small program that printed and punched the tables in the format used by the simulation program.

B. Hydraulic Simulation

B.1 Systems configuration for the model

Figure 5.5 shows that 64 sections were used in the model to represent the river system. These sections were selected to consider the cross section changes and confluence criteria. However, the distance between the confluences is often short which implies the distance between the sections is small. The distance of sub-reaches Δx are from 460 to 4160 meters long. Table A.1 in

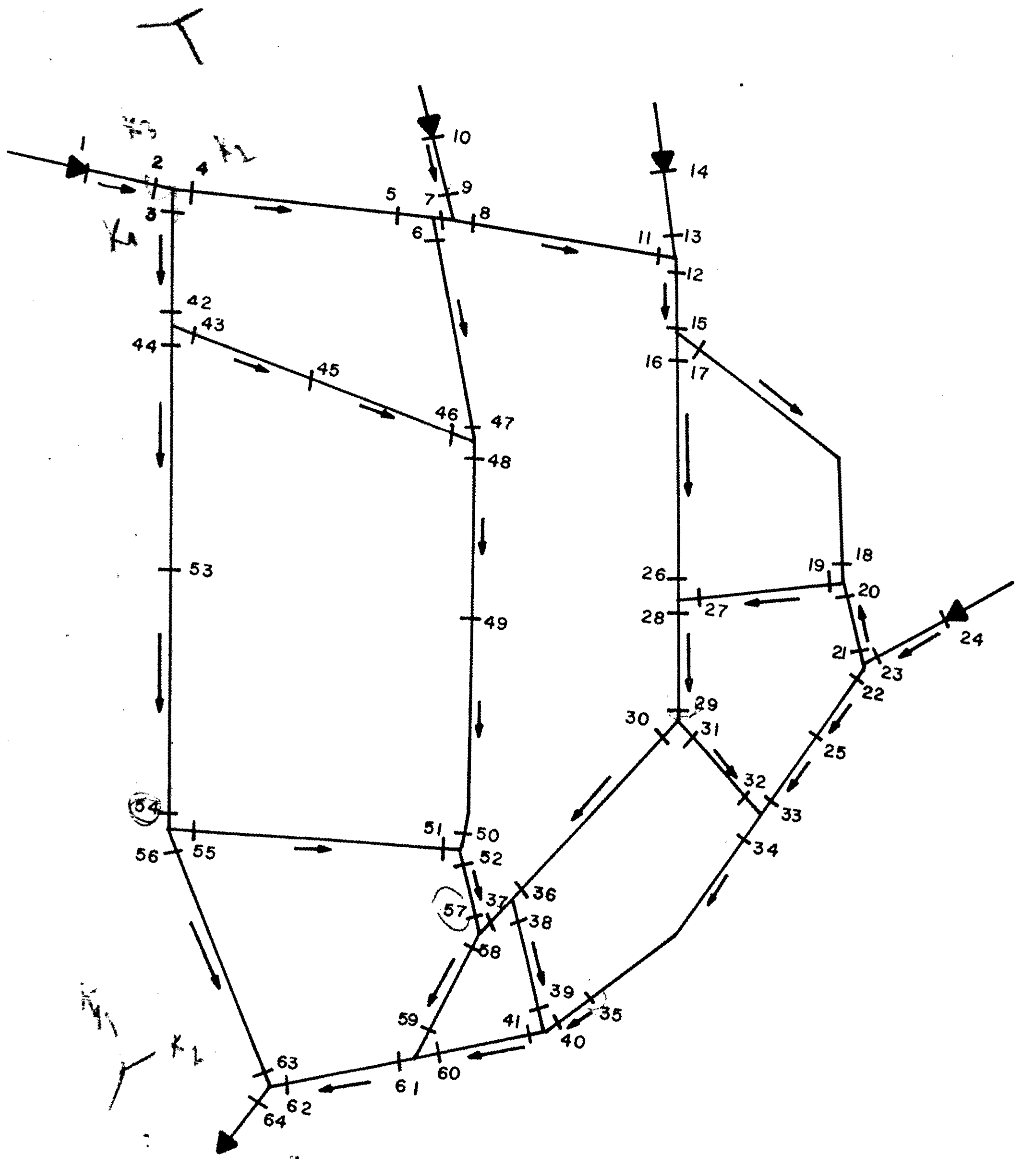


Figure 5.5 Sections of Jacui Delta Used in the Mathematical Model

Appendix A gives the geometry for all 64 sections. The sections that use the old batimetry are 15, 27, 37, and 46. Section 37 may have been dredged during this time but this information was not available. The sections near the harbor are often dredged.

Tables 5.1 and 5.2 give the sections at the reaches and confluences, the spacing, and the code. The batimetry of the storage areas was not available and these internal boundaries could not be used. The small island downstream of sections A and D was examined since the water supply intake could be close to these sections. This calls for a more detailed analysis.

The discharges were recorded at section G' instead of G which would be the best location for the boundary. Since the flow is low compared to sections M and F, estimated discharges were used at section G.

The notation used in the program consider the positive flow in the reaches is always from the upstream specified section to the downstream section. There are two ways to specify the confluences sections (see Figure 3.6 or Table 5.2). Figure 5.5 gives the positive direction used in the program for this system.

The boundaries would be specified at sections M(1), J(10), R(14), G(24), and F(64). The main sections are M and F.

B.2 Model adjustment

Data used for the parameter adjustment was from 12 noon of April 27, 1977 to 12 noon of April 28, 1977.

The levels recorded at the sections in the Jacui Delta are illustrated in Figure 5.6. During this period, variations in the levels were small, almost 10 cm in section M and about 7 cm in the other

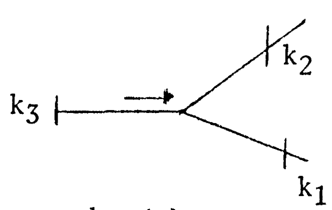
Table 5.1. Distance between the sections in the reaches

Reach Number	Number of Sections Upstream	Downstream	X (m)
1	1	2	1200
2	3	42	1420
3	4	5	2800
4	6	47	4160
5	8	11	2880
6	10	9	1160
7	12	15	800
8	14	13	1250
9	16	26	2600
10	17	18	3660
11	19	27	840
12	21	20	780
13	24	23	470
14	22	25	1040
15	25	33	940
16	28	29	1040
17	30	36	2600
18	31	32	920
19	34	35	1940
20	35	40	720
21	38	39	500
22	41	60	1000
23	43	45	1100
24	44	53	2800
25	45	46	2860
26	48	49	2440
27	49	50	1140
28	52	57	460
29	53	54	2060
30	55	51	1920
31	56	63	1900
32	58	59	1120
33	61	62	1080

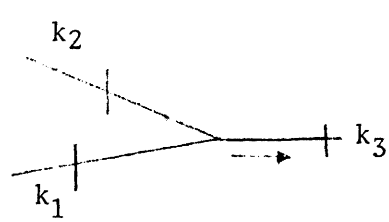
Table 5.2. Confluence sections and the distance between each two sections.

Confluence Number	sections at confluence			Δx		confluence loss α_c	
	k_1	k_2	k_3	$k_1 - k_3$	$k_2 - k_3$	k_1-k_3	k_2-k_3
1	3	4	2	440	420	1.10	1.20
2	6	7	5	360	300	1.10	1.10
3	7	9	-8	140	140	1.00	1.00
4	11	13	-12	280	300	1.00	1.00
5	16	17	15	270	280	1.00	1.05
6	18	20	-19	220	260	1.00	1.30
7	21	22	23	260	280	1.30	1.00
8	26	27	-28	700	680	1.10	1.00
9	30	31	29	540	320	1.00	1.30
10	32	33	-34	540	640	1.30	1.10
11	37	38	36	480	520	1.10	1.00
12	39	40	-41	180	320	1.00	1.00
13	43	44	42	440	340	1.10	1.00
14	46	47	-48	320	380	1.00	1.00
15	50	51	-52	920	660	1.00	1.00
16	55	56	54	460	900	1.05	1.00
17	37	57	-58	120	240	1.00	1.00
18	59	60	-61	340	360	1.00	1.00
19	62	63	-64	1040	1000	1.10	1.10

*confluence code for k_3 is



k_3 (+)



k_3 (-)

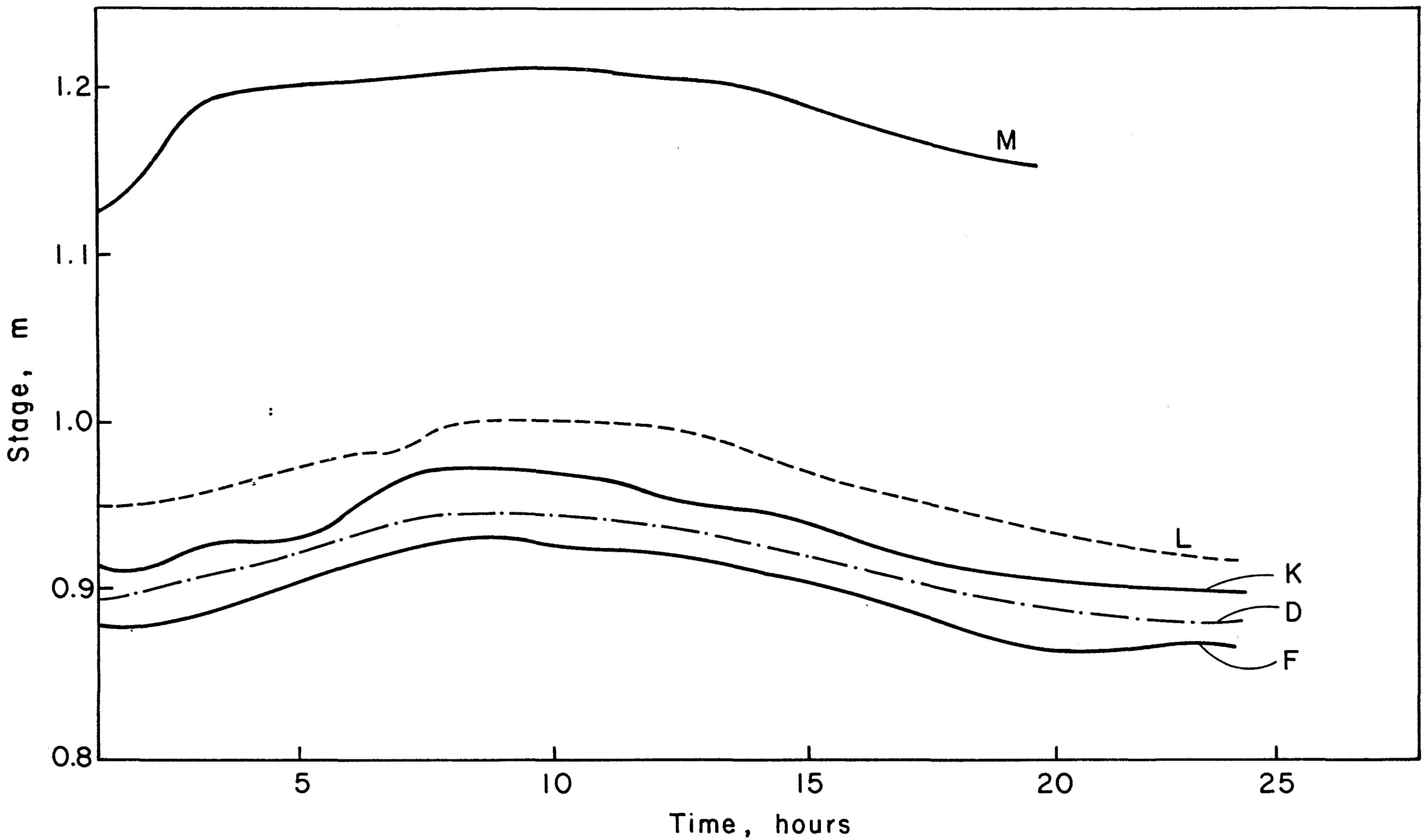


Figure 5.6 Levels Recorded at Different Locations in the Jacui Delta from 12 noon April 27, 1977 to 12 noon April 28, 1977

sections. The flows recorded during the same period were checked to see whether the volume coming into the system was approximately equal to the volume that came out of section F during this period. The following integration was used:

$$\int_{o}^{t_1} (Q_M + Q_J + Q_R + Q_G)\ dt = \int_{o}^{t_1} Q_F\ dt \qquad (5.1)$$

where $t_1 = 24$ hours.

The measured discharges of sections J, R, and G' are shown in Figure 5.7. The discharges from sections M and F are shown in Figure 5.8 and 5.10. The boundary conditions used in this adjustment were the discharges at sections M, J, R, and G and the level was used at section F.

The initial conditions for all sections were calculated by assuming an arbitrary initial condition for all sections and running the program, holding the boundary values constant. After about 30 time steps the steady state condition was reached. These values are given in Table 5.3.

The parameters of the model that are important in the flow division at the confluences and in the adjustment of the discharges and levels at the sections are the cross-section area, hydraulic radius, Manning's roughness coefficient (n), and the loss coefficient at the confluences (α).

Normally the area and hydraulic radius are defined by the data from the maps. The Manning's roughness coefficient has to be estimated for each cross section or, in some situations, for each level in these cross sections. A practical procedure is to record the discharge in a section and also record the level of the section and of another nearby section in order to calculate the water surface slope. Using

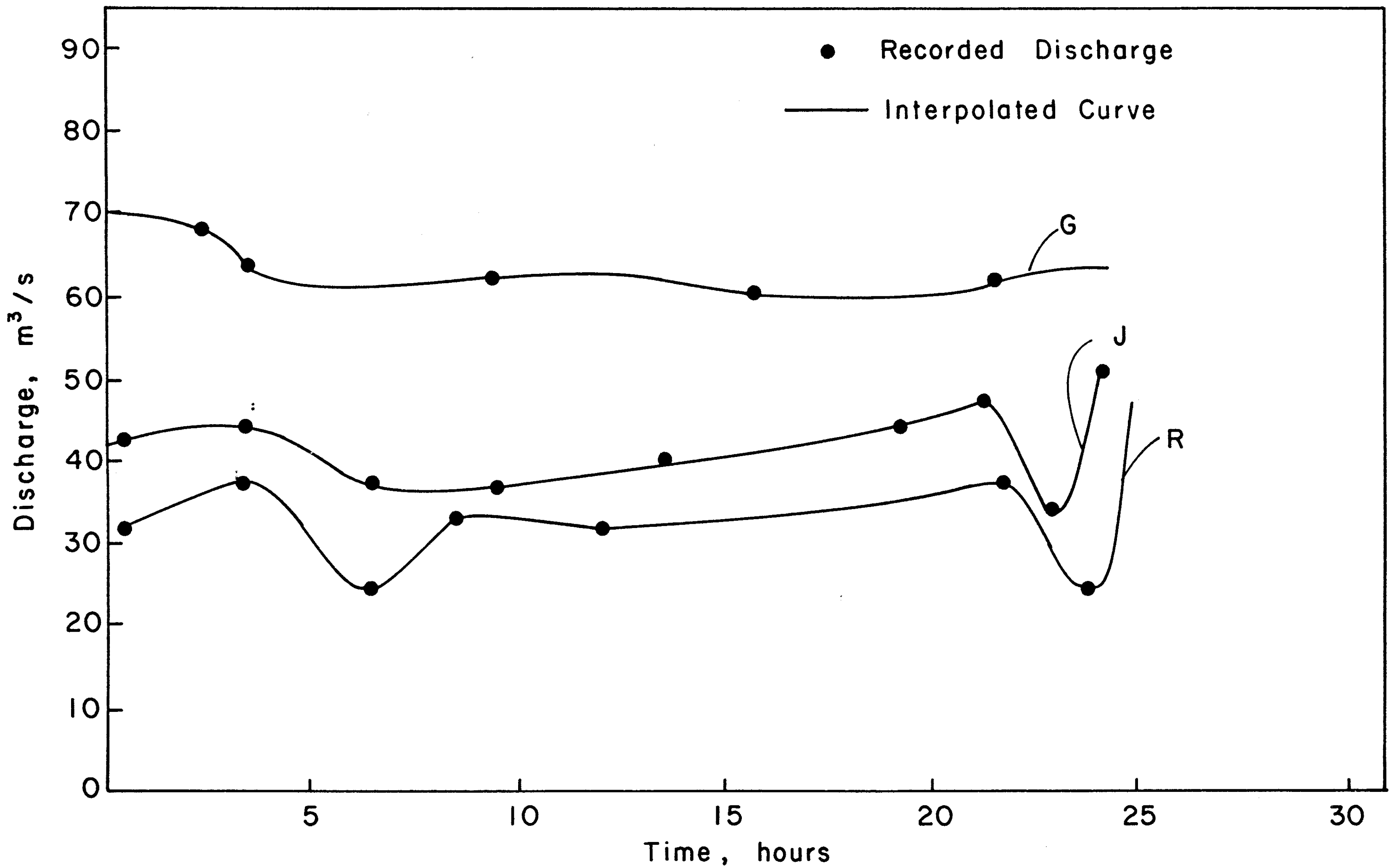


Figure 5.7 Discharges at Sections J, R, and G' From 12 noon April 27, 1977 to 12 noon April 28, 1977

Table 5.3. Initial condition in the sections.

Section Number	Level (meter, sea level)	Discharge (m^3/s)	Section Number	Level (meter, sea level)	Discharge (m^3/s)
1	1.124	3297	33	0.890	90
2	1.000	3297	34	0.885	610
3	1.000	1797	35	0.88	610
4	1.000	1500	36	0.90	1000
5	0.930	1500	37	0.924	200
6	0.915	100	38	0.90	800
7	0.915	1400	39	0.90	800
8	0.91	1450	40	0.88	610
9	0.94	50	41	0.879	1410
10	1.00	43	42	0.993	1800
11	0.91	1450	43	0.98	200
12	0.905	1490	44	0.98	1600
13	0.93	40	45	0.978	200
14	1.00	32	46	0.94	200
15	0.91	1500	47	0.915	100
16	0.91	1400	48	0.90	300
17	0.90	100	49	0.90	300
18	0.89	100	50	0.90	300
19	0.89	80	51	0.894	150
20	0.89	20	52	0.88	450
21	0.885	20	53	0.893	1620
22	0.90	90	54	0.951	1650
23	0.94	70	55	0.894	150
24	0.885	70	56	0.92	1500
25	0.91	90	57	0.876	450
26	0.89	1420	58	0.876	650
27	0.89	80	59	0.876	650
28	0.89	1500	60	0.876	1410
29	0.89	1520	61	0.875	2060
30	0.89	1000	62	0.877	2060
31	0.89	520	63	0.877	1530
32	0.89	520	64	0.877	3590

Equation (3.5) it is possible to estimate n. In the Jacui Delta there are limited data available to estimate these values. The procedure used here adjusted the calculated levels and discharges to the recorded ones.

The roughness coefficient has a large effect on the water surface levels. In a river without a confluence, for the same input hydrograph, when n increases, the level increases for the same discharge. When the river system is similar to that of the Jacui Delta with confluences, the relationship among the variables and this parameter is more complex. When n increases in confluence section and in the respective branch, less flow comes through this branch, and instead of increasing the level it can decrease. The adjustment is more complex when there are many junctions, as is the case in the Jacui Delta.

The procedure here was to adjust the levels and discharges of the main branches using data from sections L(54), D(57), A(35), and K(29). The main division is among sections 2, 3, and 4. The secondary branches could not be adjusted since there was no data and the n values were estimated.

The values of α were used in the adjustment. The variables are not sensitive to these parameters. It has a direct effect on the flow division. Increasing α decreases the flow. These values are listed in Table 5.2.

The values of n were adjusted in the 24 hour period and are listed in Table 5.4. Recorded and calculated values for all sections are shown in Figures 5.8 to 5.12. The differences in the levels were on the order of 1.0 cm in most of the sections. The difference in the discharges was on the order of 8%. Since the errors assumed in the

Table 5.4. Manning's roughness coefficient "n" for each section number.

Section Number	n	Section Number	n
1	0.025	33	0.022
2	0.025	34	0.020
3	0.020	35	0.020
4	0.028	36	0.020
5	0.025	37	0.040
6	0.022	38	0.028
7	0.022	39	0.028
8	0.020	40	0.020
9	0.025	41	0.022
10	0.025	42	0.020
11	0.020	43	0.022
12	0.020	44	0.022
13	0.025	45	0.022
14	0.025	46	0.022
15	0.020	47	0.022
16	0.020	48	0.020
17	0.028	49	0.020
18	0.028	50	0.020
19	0.022	51	0.022
20	0.035	52	0.020
21	0.035	53	0.025
22	0.022	54	0.025
23	0.022	55	0.022
24	0.022	56	0.025
25	0.022	57	0.020
26	0.020	58	0.022
27	0.022	59	0.022
28	0.020	60	0.022
29	0.020	61	0.022
30	0.018	62	0.020
31	0.025	63	0.022
32	0.022	64	0.020

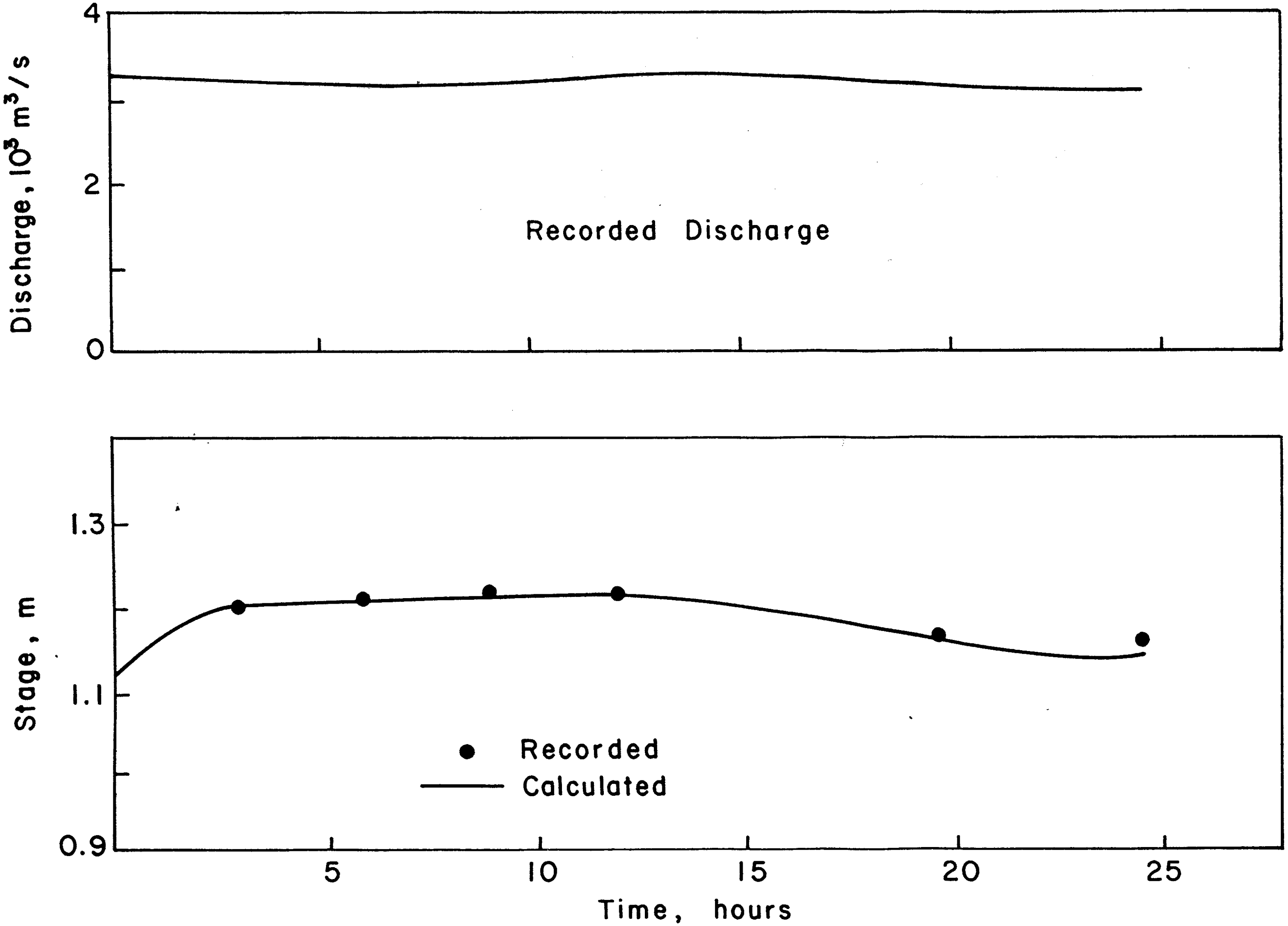


Figure 5.8 Calculated and Recorded Values at Section M

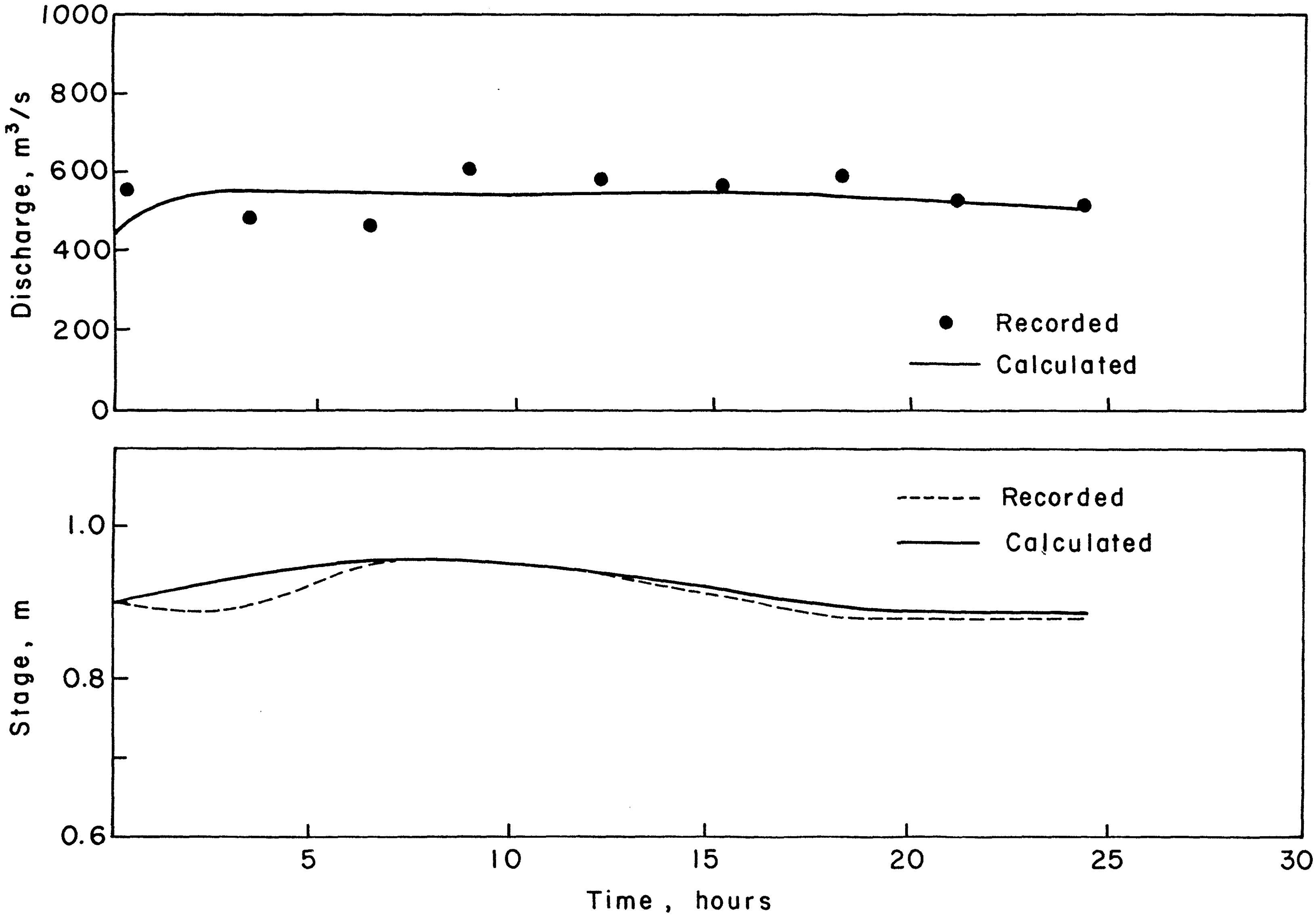


Figure 5.9 Calculated and Recorded Values at Section A

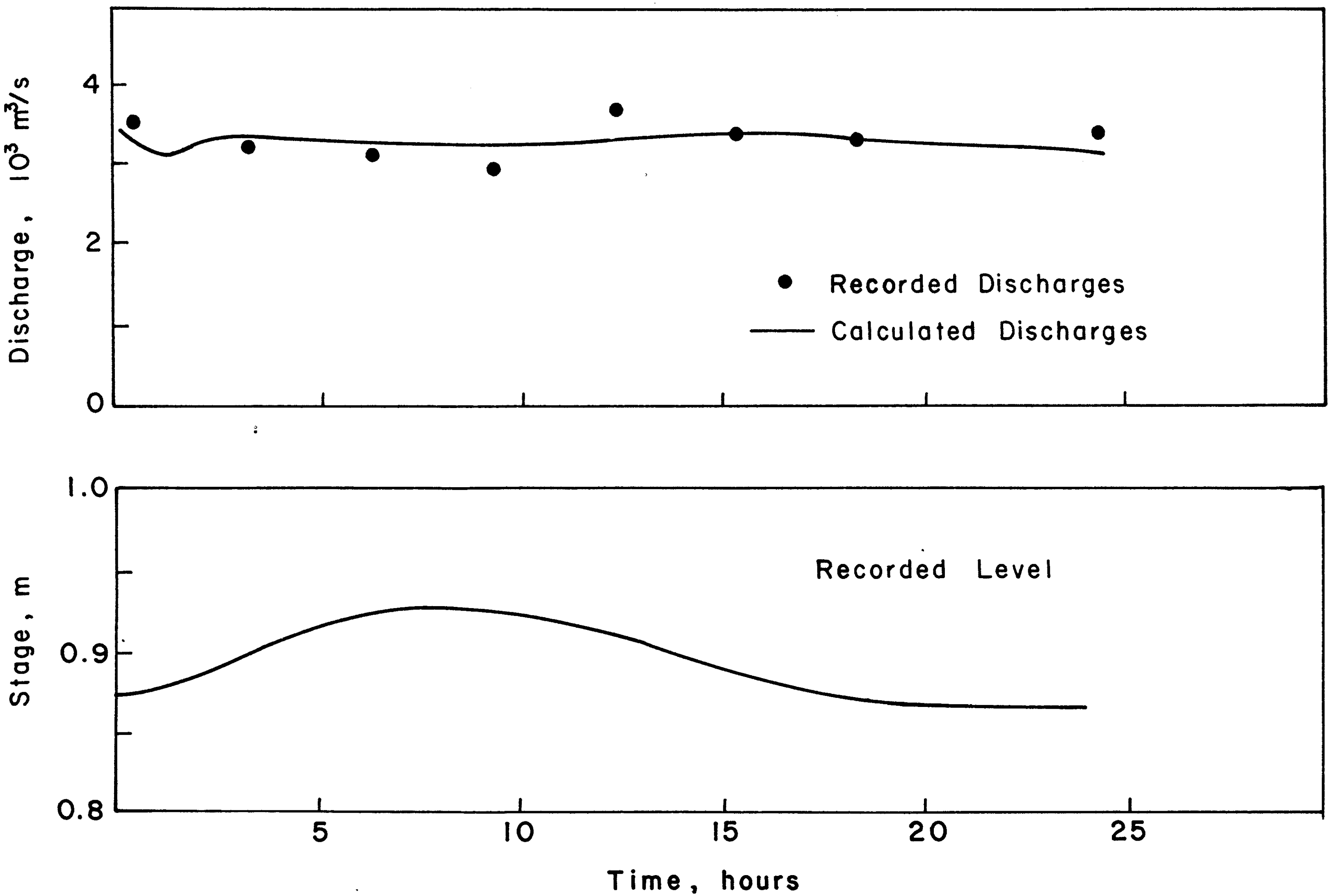


Figure 5.10 Calculated and Recorded Values at Section F

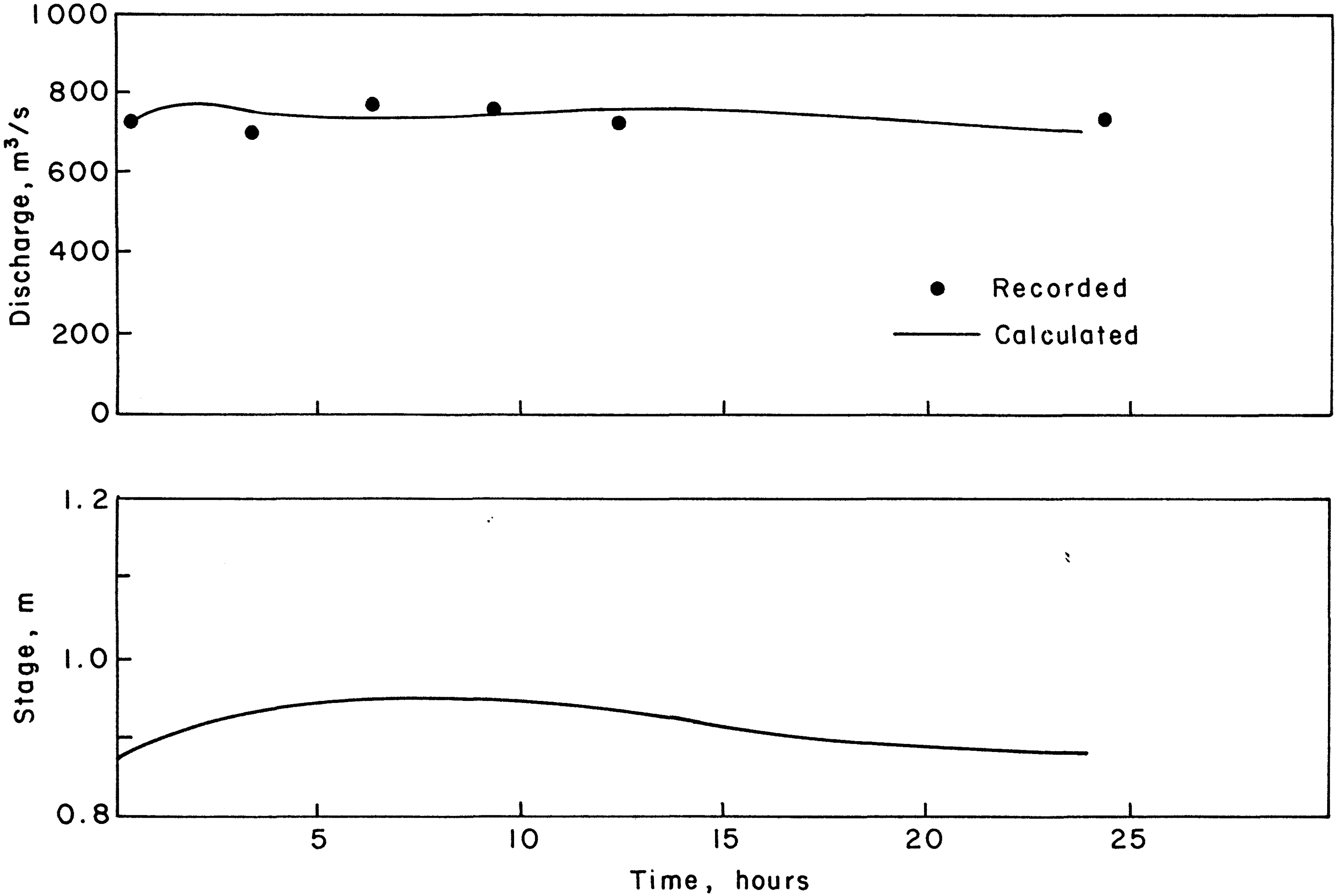


Figure 5.11 Calculated and Recorded Values at Section D

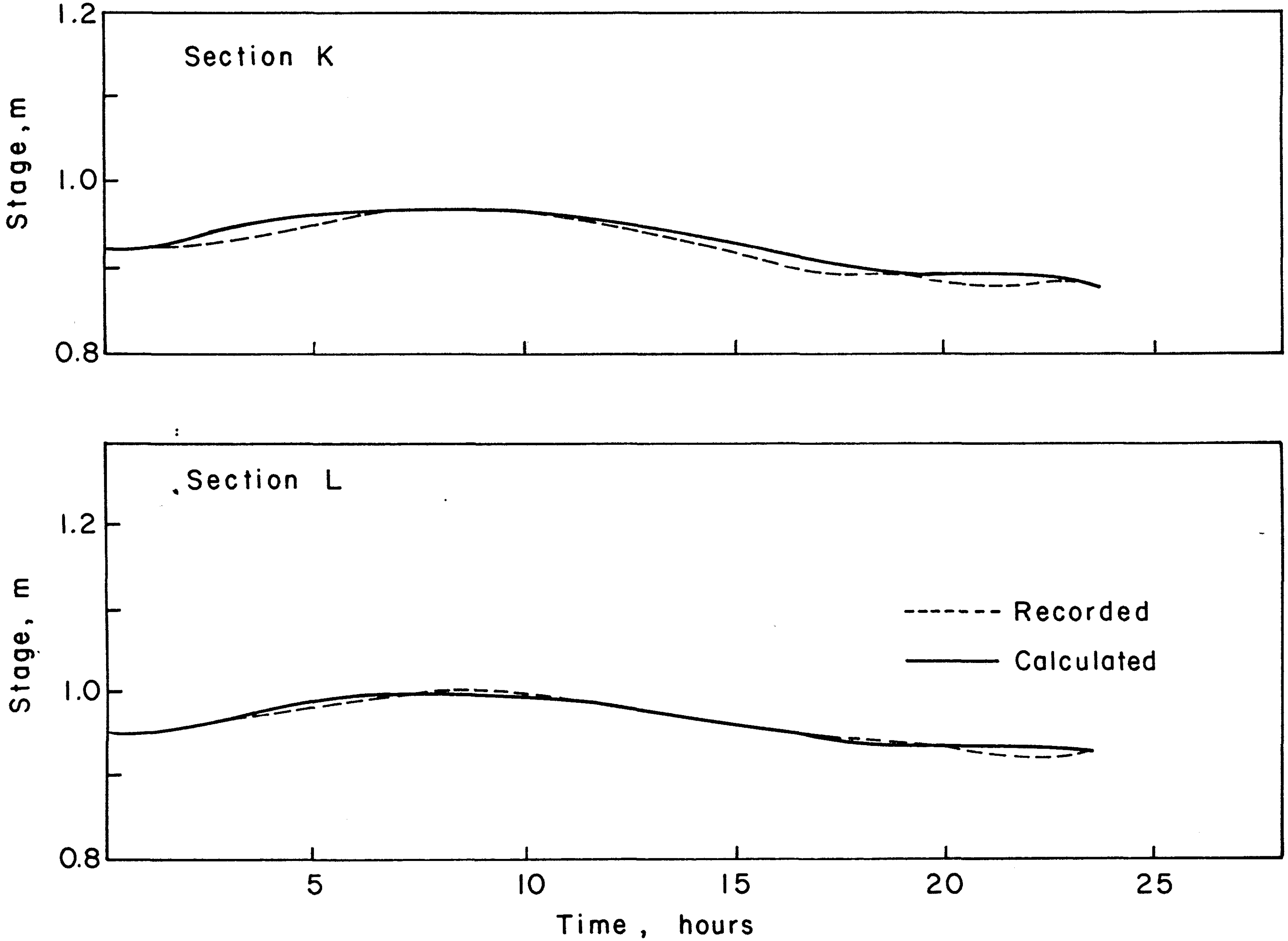


Figure 5.12 Calculated and Recorded Levels at Sections K and L

record of these discharges are greater than that, greater refinement would be a waste of time.

The time step in the adjustment was 20 minutes. When there was a big time step such as 30 minutes or one hour, the loop of the island downstream of sections D and A started to invert the flow direction showing an unstable solution. This also happens when the roughness coefficient is decreased, since the roughness term is dissipative. It can also be due to the great value of the ratio $c\Delta t/\Delta x$.

Unfortunately, the period of recorded discharge did not show a critical situation with inversion flow and great variation in level. The levels during this period were above the seasonal normal. More information needs to be obtained in the secondary branches as most of the pollution is carried by small streams. A period in the dry season must be chosen to record the discharges and levels.

B.3 Verification

After the parameters were found by adjustment using a period of 24 hours, model verification was required to determine whether the parameters were sufficiently reliable for use in another simulation period other than adjustment. A 48-hour period was chosen for verification, April 8 and 9 when the level variation is of about 0.65 m. In this case it was only possible to verify the levels because the discharges were not available.

The configuration, geometry data, parameters, and time step are the same as those used in the model adjustment. The boundaries used were the levels at the sections F and M (Figure 5.13). In the sections J, R, and G, only relative levels were available and to adjust these references would be expensive because a small error in the reference can create unstable results in the discharges. In these sections a constant discharge arriving in the Delta was assumed, and the value used was 30.0 m^3/s. The effect of those boundaries in the downstream section in which one can verify the model are very small.

The initial condition was calculated using the procedure described above in Chapter 4, section B.3. Recorded stages used in the verification were only from sections L and D. Results of section D are shown in Figure 5.13; and the results of section L are shown in Figure 5.14. The recorded stages of April 9, 1977 for section L were considered unreliable and were not used in the verification. The solution of the mathematical model shows good agreement with the recorded values.

C. Water Quality Simulation

C.1 Upstream inflow test

A complete set of data is not available to adjust and verify the concentration distribution in the Jacui Delta. Some tests were designed in order to test the capability of the water quality model. The channel configuration shown in Figure 5.15 was used. The sections in the main channel have a width of 30.0 m; the sections in the branches in between the confluences have a width of 15.0 m. The channel slope is 0.00005 m/m, the Manning coefficient is 0.03, $\Delta x = 1000$ m in the main channel, and $\Delta x = 500$ m between the confluence sections.

The first test examined an upstream input of stormwater runoff with a BOD concentration. The upstream boundary condition for the hydraulic equations is the flow hydrograph shown in Figure 5.16 (section 1). The downstream condition is the rating curve given by Manning's equation. The initial condition used was $Q = 20\ m^3/s$ and $y = 2.0$ m in the main channel and $Q = 10\ m^3/s$ and $y = 2.0$ m in the branches. The

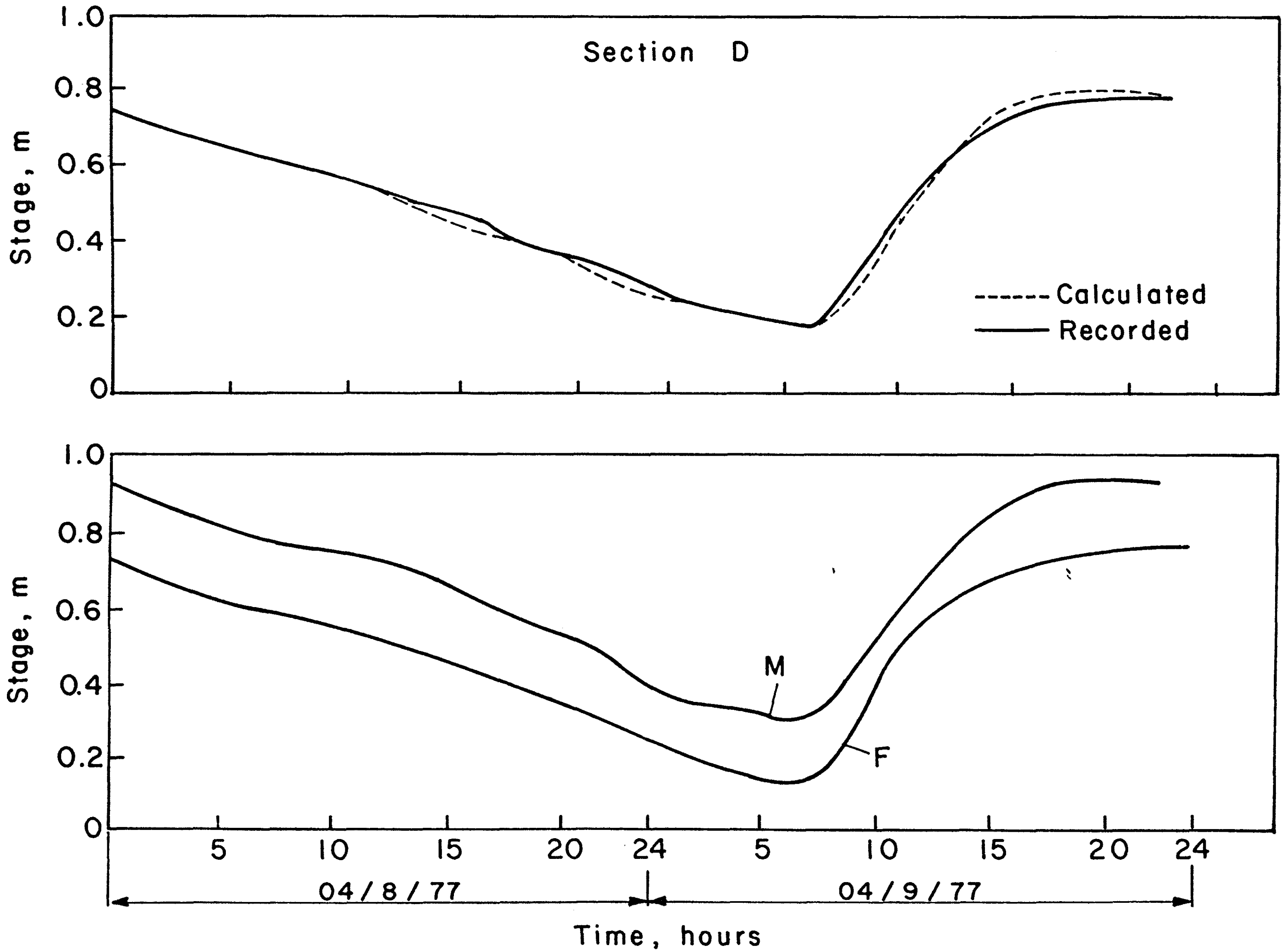


Figure 5.13 Recorded Level at Sections M, F and D and the Calculated Level at Section D

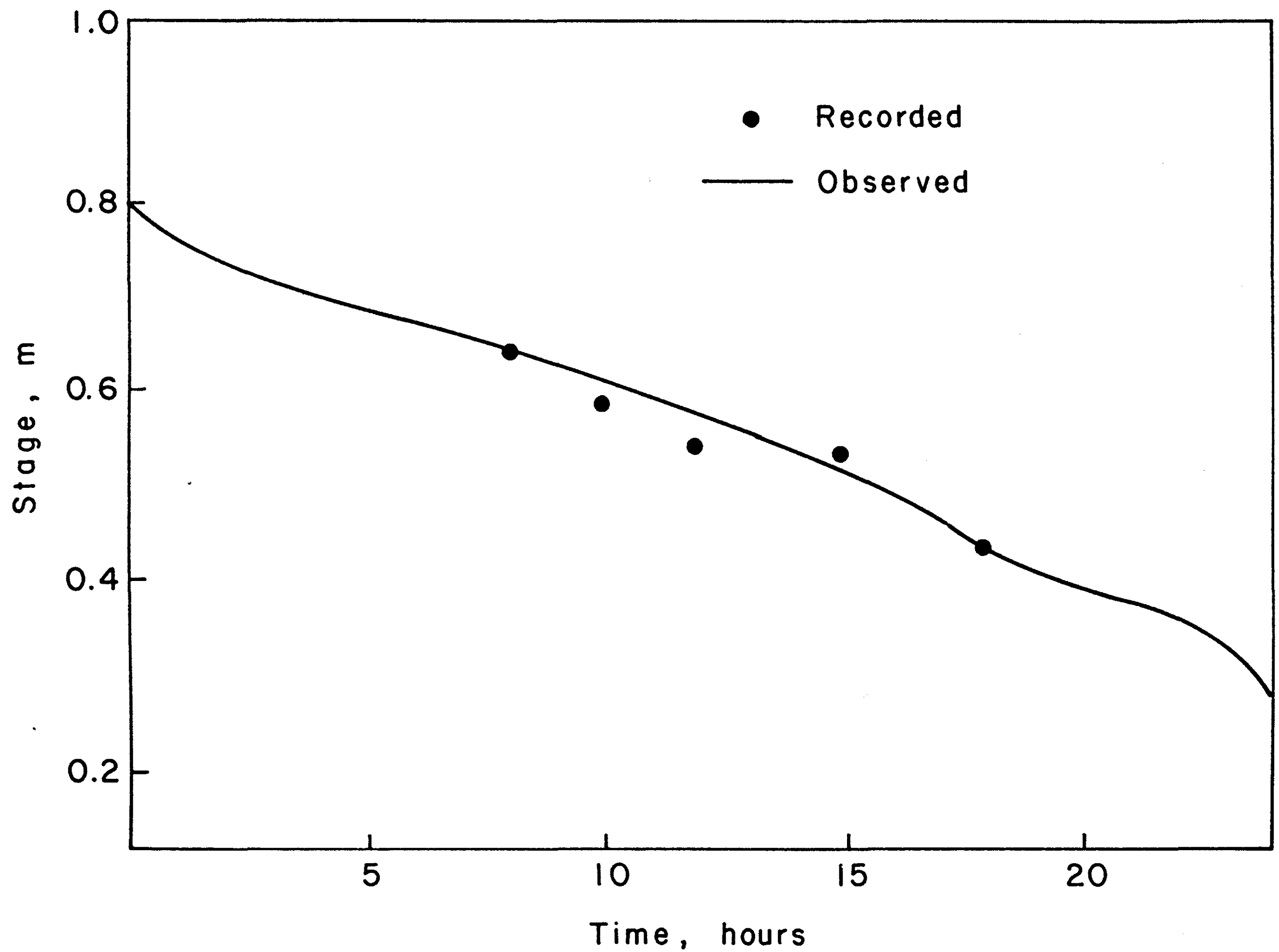


Figure 5.14 Levels Recorded and Calculated at Section L During April 8, 1977

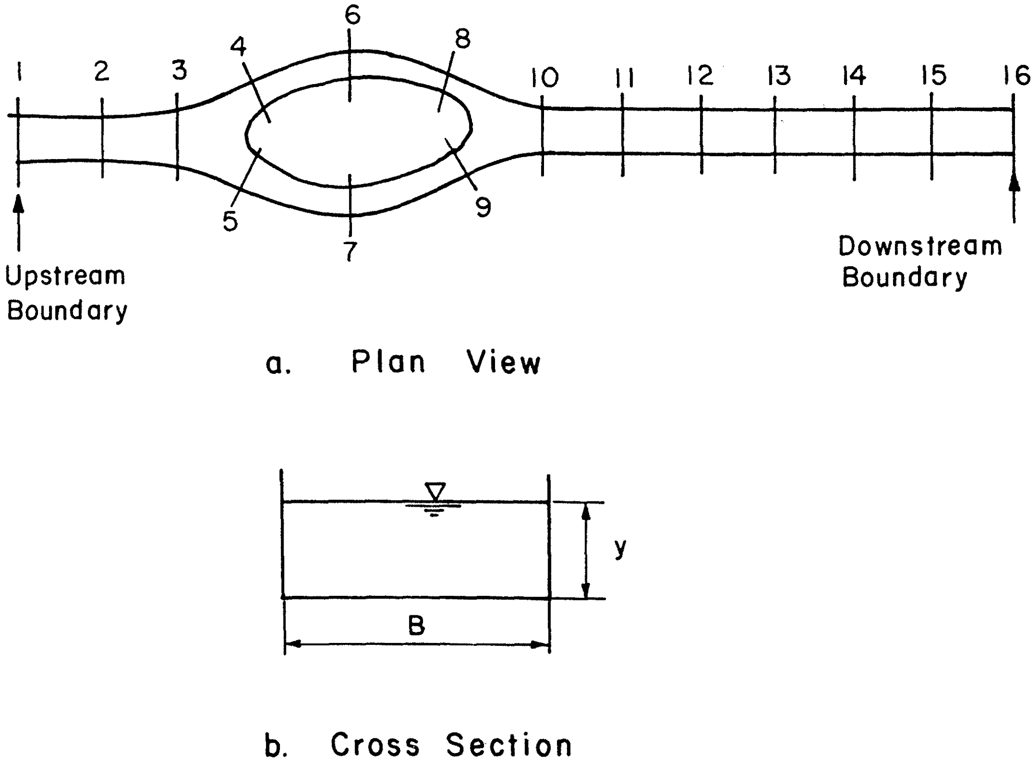


Figure 5.15 Channel System Used in the Upstream and Lateral Inflow Test

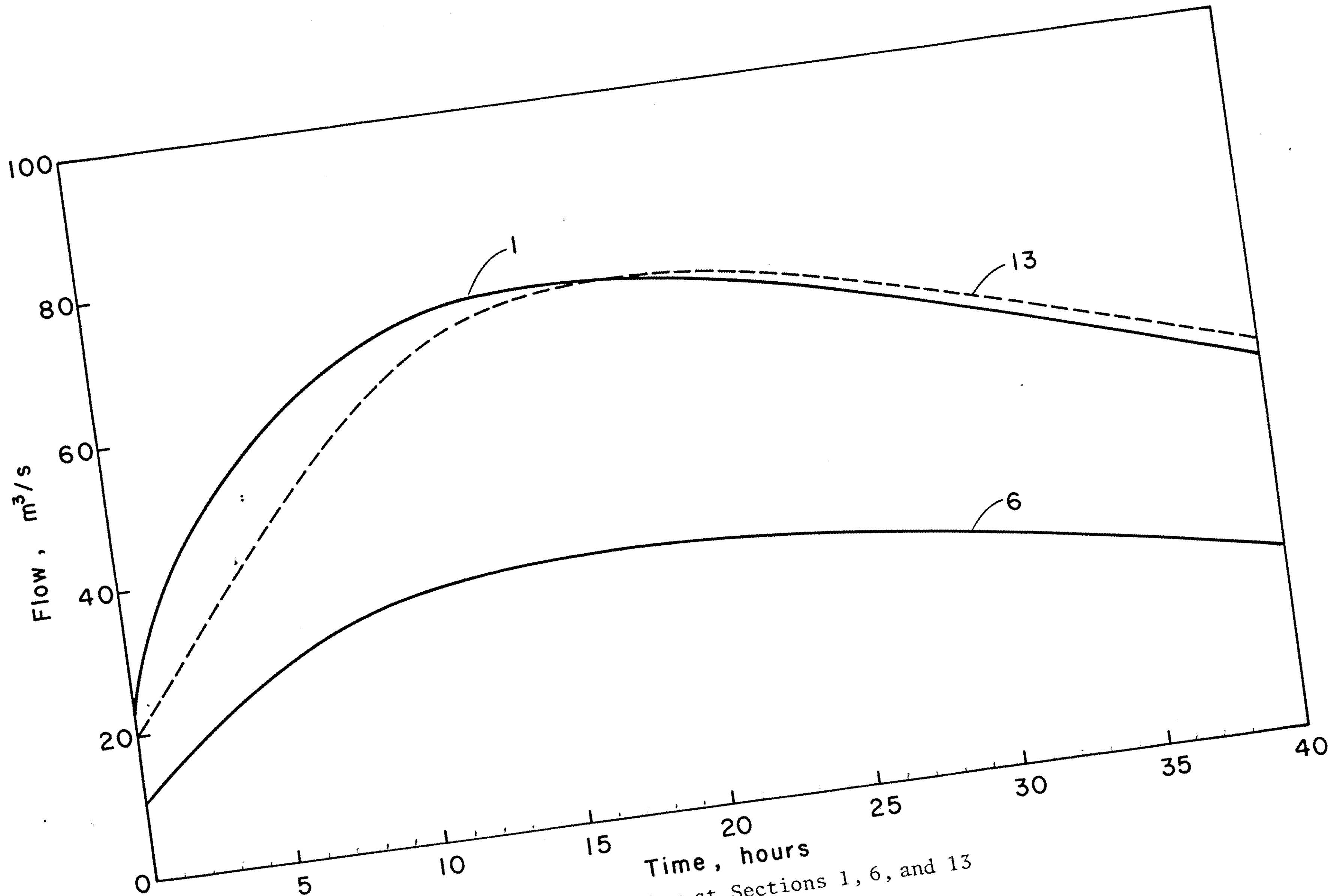


Figure 5.16 Flow at Sections 1, 6, and 13

upstream condition for BOD is shown in Figure 5.17 (section 1). The downstream condition was given by Equation (4.67). The initial condition assumed a concentration of 1 mg/l for all sections. The water quality coefficients are $C_s = 7.8$ mg/l, $K_1 = 0.25$/day, $K_3 = 0$, $K_2 =$ 2.0/day, and $E = 5\ m^2/s$. The parameter of the numerical scheme is $\theta = 0.5$ and the maximum velocity is about $v = 0.7$ m/s.

The stability condition for the time step, using Equation (4.51) is

$$\Delta t \leq 1430 \text{ s}$$

The time step used was 20 minutes or 1200 s.

In Figure 5.16 there are flow hydrographs of sections 1, 6, and 13. The values plotted included those up to time step 40, showing the recession part of the hydrograph did not reach the initial flow of $Q =$ $20\ m^3/s$. Figure 5.17 shows the BOD concentration in mg/l and the function of time for sections 1, 6, and 13. The peak concentration damps from section 1 to 13, was 8-km downstream from 10.5 mg/l to 8.1 mg/l with a lag of 6 hours. Also, the damp of the curve at section 6 was from 10.5 mg/l to 8.9 mg/l with a 3.0 hour lag.

This test shows the formulation used in the model can well represent the hydraulic flood routing and substance transport processes through a channel system with a confluence. A simple system was used to better understand the processes.

C.2 Lateral inflow test

The assumption is to use the same river system as that of Figure 5.15, but with a lateral input between sections 4 and 6. The upstream condition has a constant discharge of 20 m^3/s and the downstream condition has the rating curve given by Manning's equation. The

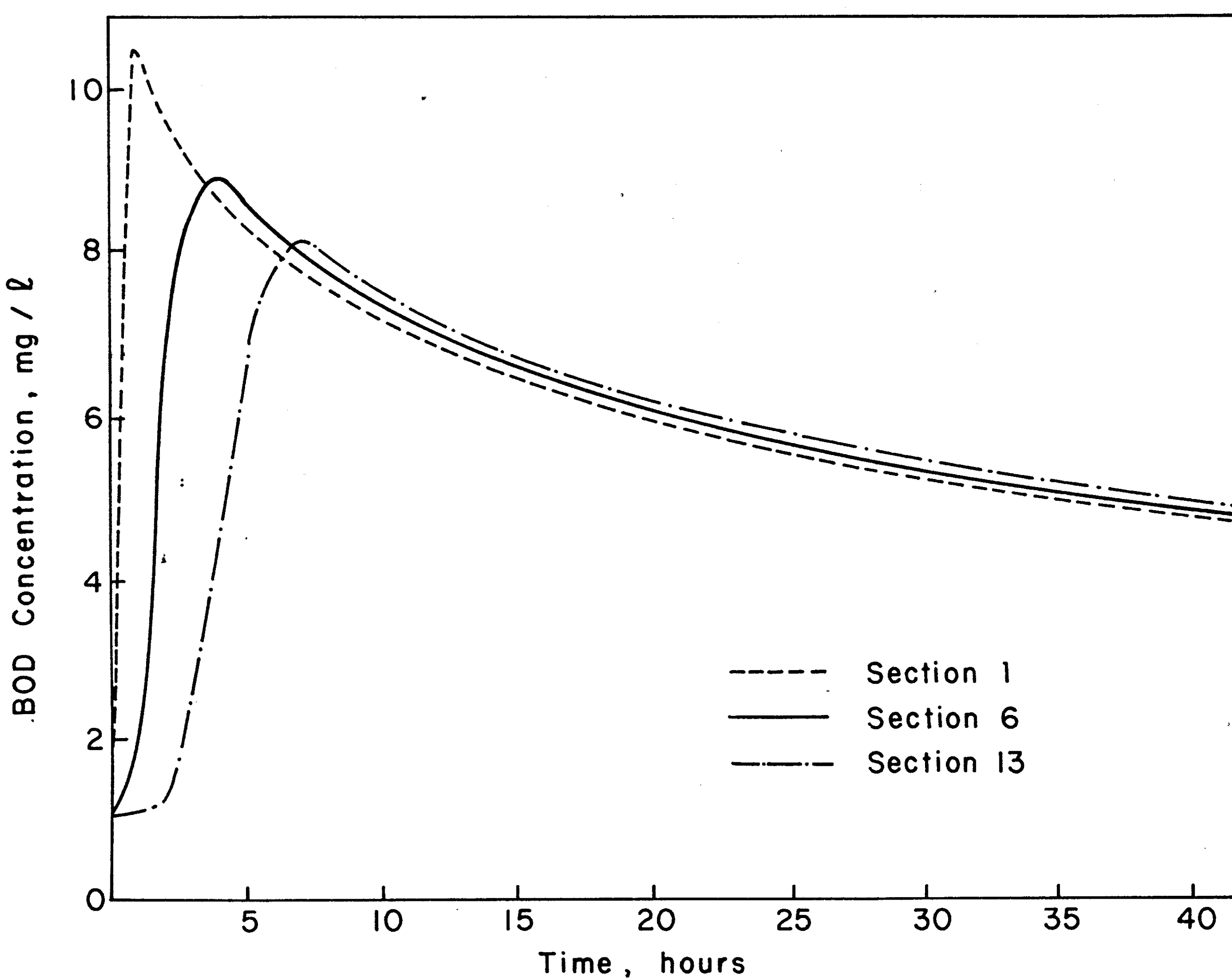


Figure 5.17 BOD Concentration at Sections 1, 6, and 13

initial conditions are the same as in the upstream inflow test. The lateral flow is given in Figure 5.18. The upstream and downstream boundary conditions are assumed by a linear relationship between the concentration and the space given by Equation (4.67). Again, the initial condition for BOD is the same as in the previous test; dissolved oxygen is assumed to be 7.5 mg/l for all sections. The concentration of BOD and DO for the lateral flow are plotted in Figure 5.18. The water quality coefficients are the same as in the previous test.

The maximum velocity is about 0.5 m/s, and the stability condition requires that

$$\Delta t \leq 1950 \text{ s}$$

The time step used in the calculation was twenty minutes.

Figure 5.19 shows the flow hydrograph of some sections. The flow in section 4 decreases with the lateral flow input between 4 and 6. This flow shortage goes to the other branch, increasing the flow at section 7. The flow hydrograph of sections 10 and 16 also are plotted in Figure 5.19. The concentration distribution of BOD and DO in sections 6 and 10 are shown in Figure 5.20 illustrating that time step 5 is critical for BOD at section 6. The damping in the concentration of the substances from sections 6 and 13 is due mainly to the transport in branch 6-8, the mixture with clean water of branch 5-7-9, and transport in the main channel until section 13.

The concentration profile for specific time steps is plotted in Figure 5.21. Time step 5 is critical, showing a high concentration of BOD at section 6 and decreasing suddenly at section 10 due to the mixture with clean water from branch 5-7-9. At time step 8 the maximum concentration is at section 8 and the polluted flow is moving downstream.

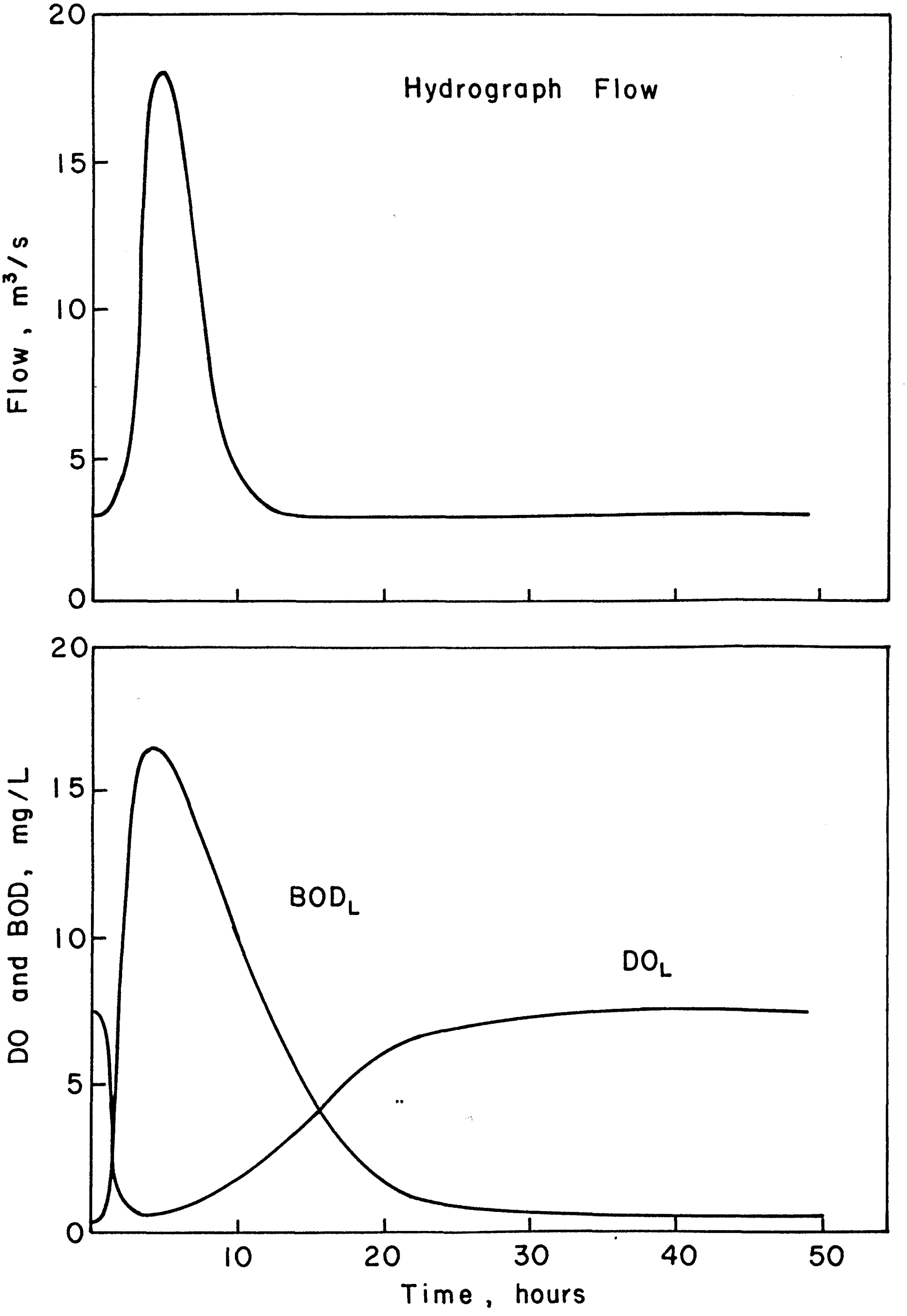


Figure 5.18 Flow, BOD, and DO Concentration in the Lateral Input at Section 6

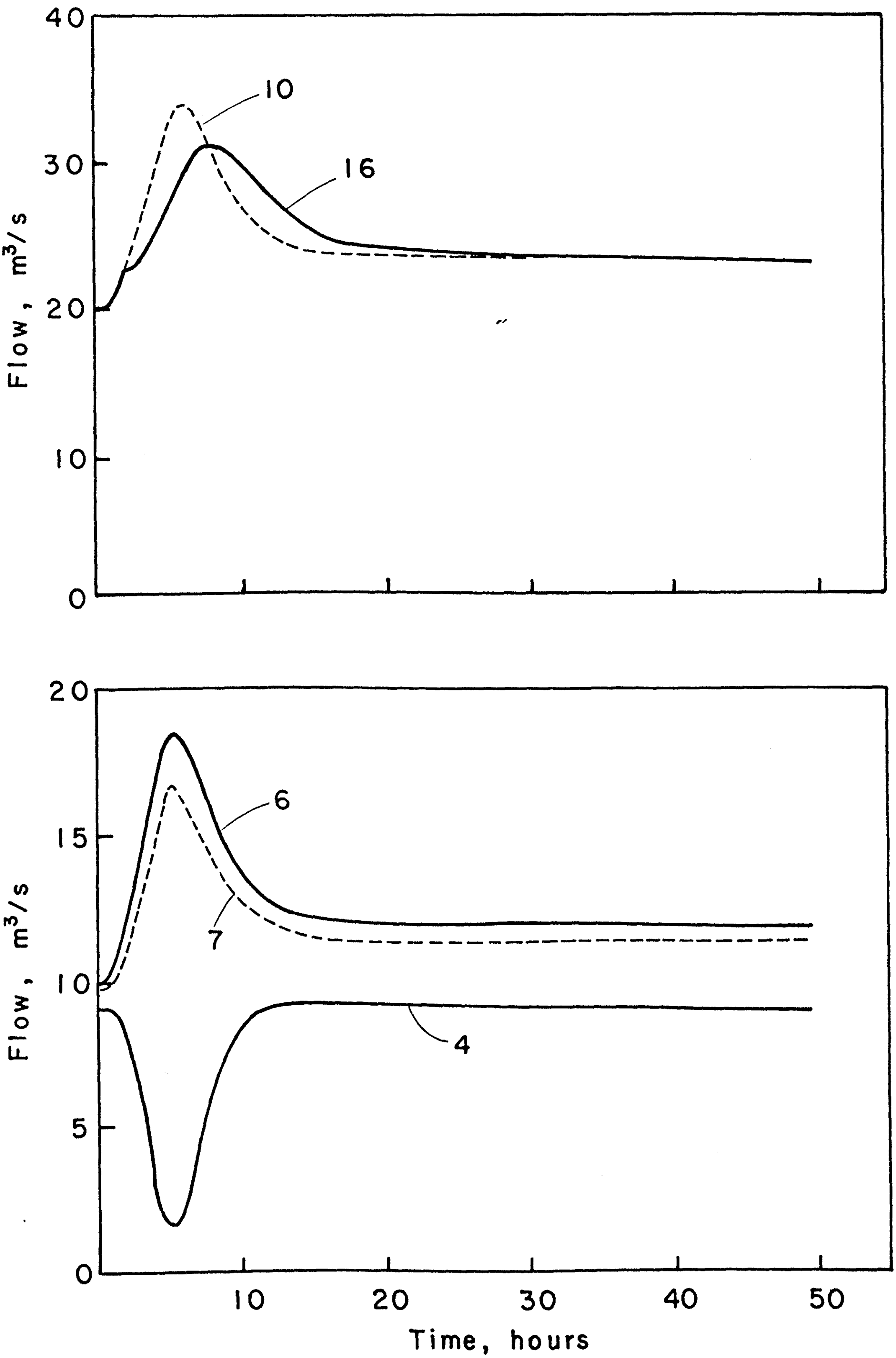


Figure 5.19 Hydrograph Flow at Sections 4, 6, 7, 10, and 16

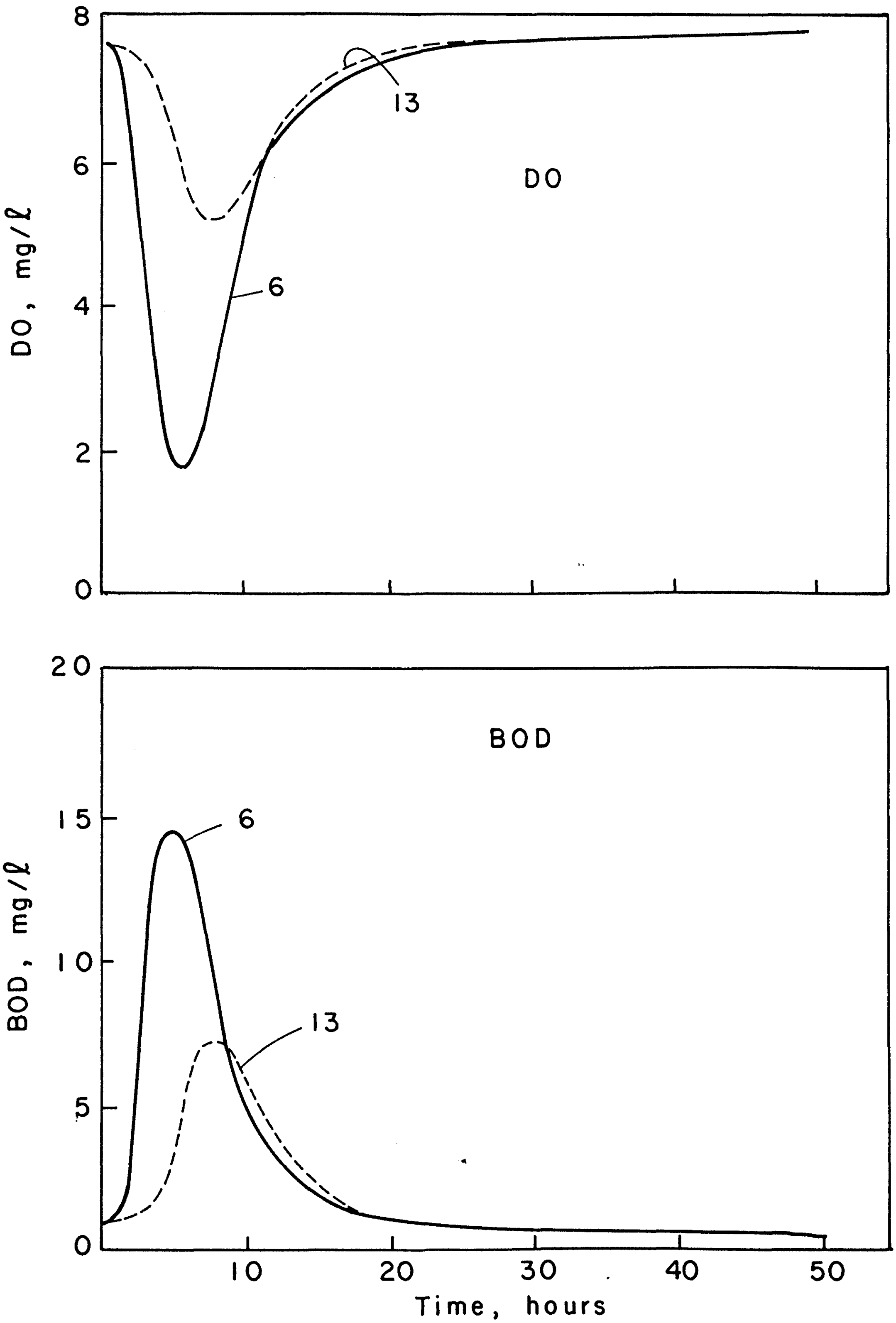


Figure 5.20 Concentration Distribution of BOD and DO at Sections 6 and 13

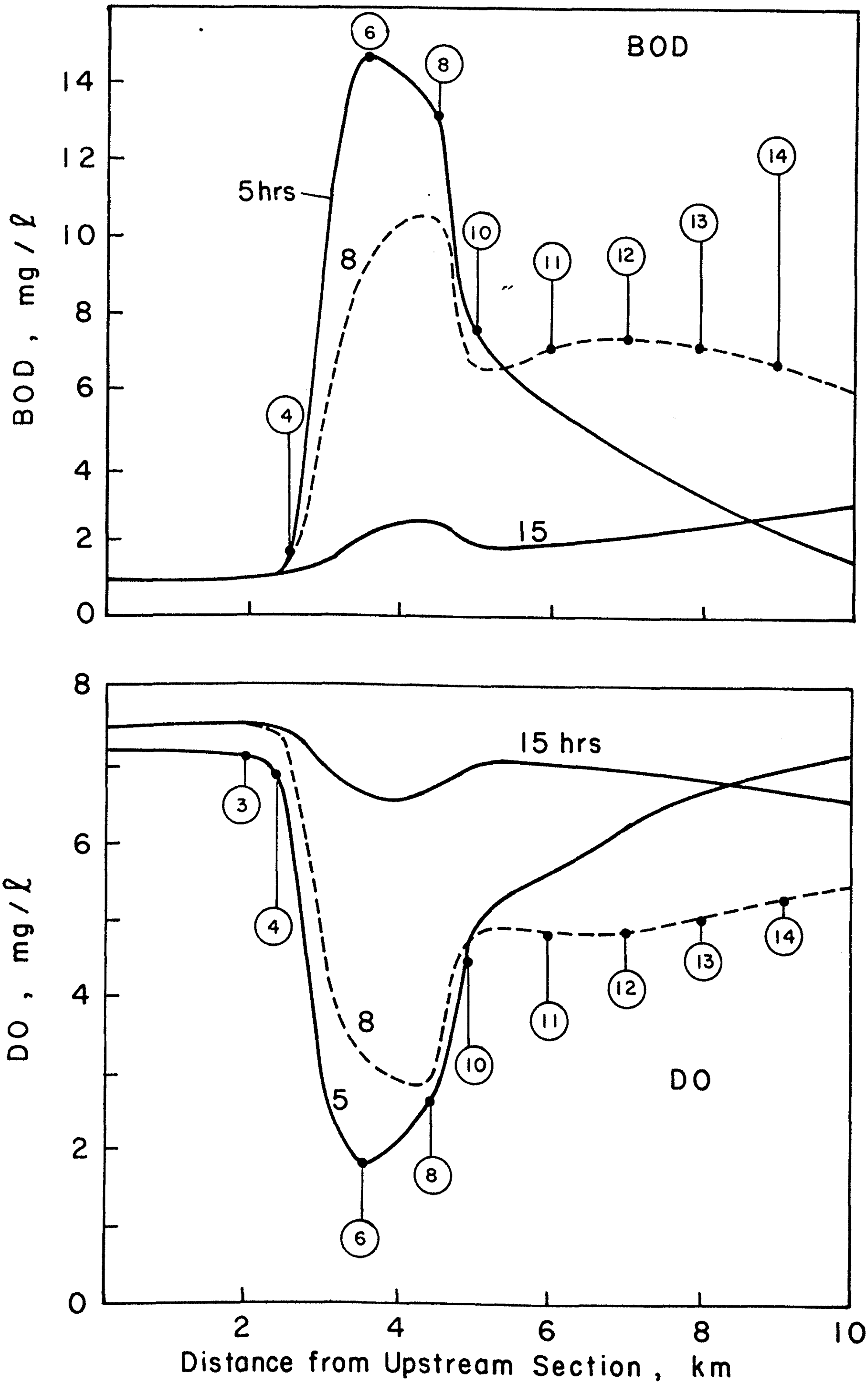


Figure 5.21 BOD and DO Distribution in the Space

After 15 time steps the BOD is low and the DO is high, as the lateral flow does not have more pollution and the river tends to improve its water quality. In branch 5-7-9 the quality remains high without major modification during this lateral inflow of polluted water because the longitudinal dispersion coefficient is small and the velocities are high.

This test shows the important effect of lateral flow input in the flow division in the river concentration downstream of the intake. The objective of both tests were to simulate practical situations in order to show the model capability and check its performance.

CHAPTER VI

MANAGEMENT USE OF THE MODEL

A. General

The model can be used in management analysis to evaluate alternative solutions in river systems. It also can be used to test hydraulic structures and control data measurement.

The hydraulic model can be used for such purposes as the forecasting of flood stages and the suitability of navigation in rivers with islands and tributaries. The water quality model is useful in measuring the impacts of dumping wastes into different sections of the river. The basic requirement of the models is the data. A model cannot be reliable if the data are insufficient or if there are uncertainties in the data.

The Jacui Delta was used in this study to show the utility of the model in management studies. Assuming the purpose of the study was the evaluation of the water quality level, then the biochemical oxygen demand (BOD) and dissolved oxygen (DO) parameters would be used in performing the analysis.

The river geometry data and the roughness coefficient are the same as those discussed in Chapter V. The water quality parameters include:

Longitudinal dispersion coefficient. There is no data concerning this coefficient in the Jacui Delta. Dailey and Harleman (1972) used the modified Taylor equation to predict this coefficient for the hydraulic model of the James River. They used a dye test to verify the longitudinal dispersion coefficient predicted by the modified Taylor equation. They multiplied this predicted coefficient by three to account for channel irregularities. This resulted in a good prediction during high-water slack values, but not as good for low-water slack values. Amein and Galler (1978) determined the dispersion coefficient for the lower Chowan River. They obtained values in the range of 25,000 ft^2/hr (0.65 m^3/s) to 130,000 ft^2/hr (3.36 m^2/s) for no current and no wind. They showed that because of dispersion due to wind and reversed flow, a coefficient of 300,000 ft^2/hr (7.8 m^2/s) is typical of most estuary flow. The Jacui Delta is most irregular. It has wind and reverse flow effects. Therefore, the longitudinal dispersion coefficient used in this management test was 10 m^2/s.

Saturated oxygen concentration. This concentration was calculated by Equation (4.16) with a temperature of 27°C with C_s = 7.8 mg/ℓ.

Decay rate. The decay rate was assumed as 0.1/day based on data collected in the Sinos River.

Reaeration coefficient. The reaeration coefficient predicted by the equations developed for rivers (e.g., O'Connor and Dobbins, 1958) estimated small values due to the small velocities (Amein and Galler, 1978). They used an equation given by Kanishwer (1963) applicable to estuaries based on depth and wind velocity. The wind velocity is not available for the Jacui Delta and this coefficient was assumed as 0.5/day.

Some options were tested with a set of generated boundary conditions. The upstream conditions were assumed constant during the simulation period. The downstream condition at section F was defined by two second-order polynomial equations; one for the positive flow period, and the other for the negative flow period. The positive period was assumed to be 16 hours and the negative period to be 8 hours.

The flow equations are

$$4 \leq t \leq 20 - Q = \frac{Q_1(t - 4)}{4} [1 - \frac{(t - 4)}{16}]$$

$$t \leq 4 \quad - Q = \frac{Q_o(4 - t)}{2} [1 - \frac{(4 - t)}{8}] \qquad (6.1)$$

$$t \geq 20 - Q = \frac{Q_o(t - 20)}{2} [1 - \frac{(t - 20)}{8}]$$

where t is in hours, Q_1 is the maximum positive discharge, and Q_o is the maximum negative discharge in absolute value.

The water volume entering the Delta through sections M, J, R, and G should approximately equal the volume leaving the Delta at section F within a 24 hour period, assuming the same storage. The area given by the second order polynomial equation (Figure 6.1) is

$$S = \frac{Q\, t_2^3}{6(t_1^2 - t_2 t_1)} \qquad (6.2)$$

where Q is the maximum discharge, t_1 is the time of maximum discharge, and t_2 the time of zero discharge as defined in Figure 6.1.

The continuity equation is

$$S_1 - S_0 = (Q_M + Q_J + Q_R + Q_G)24 \qquad (6.3)$$

where S_1 and S_0 are the volume of positive and negative water flow at section F and Q_M, Q_J, Q_R, and Q_G are the flow at sections M, J, R, and G.

Using a total input of 150 m^3/s and $Q_o = - 105\ m^3/s$, the positive flow calculated by Equation (6.3) is $Q_1 = 390\ m^3/s$. The flow calculated by Equation (6.1) is plotted in Figure 6.2.

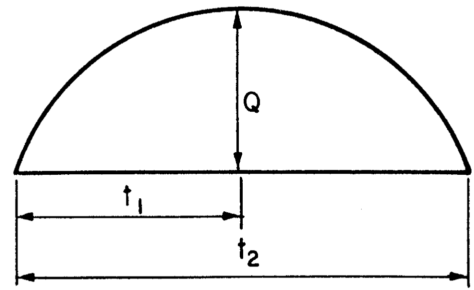


Figure 6.1 Flow Curve for the Positive and Negative Period

B. Options

Option 1

The rivers with the highest levels of pollution were the Gravatai and the Sinos. Based on BOD and DO records, the maximum concentration of BOD in the boundary at the Gravatai River was about 11 mg/l and the minimum DO was about 1 mg/l. In the Sinos River these values were 9 mg/l and 1.5 mg/l, respectively. The concentration at sections M and J was assumed to be 0.5 mg/l for BOD and 7.5 mg/l for DO.

Flow conditions at the boundaries are $Q_M = 100\ m^3/s$; $Q_J = 10\ m^3/S$; $Q_R = 20\ m^3/s$; and $Q_G = 20\ m^3/s$. Figure 6.2 shows the flow hydrograph at section F.

Those values were held constant during a simulation period of 27 hours. The results showed that the concentration of BOD was high near the river source and decreased after mixing with Jacui and Cai water. Figure 6.3 shows the profile of concentration along the harbor up to section F. Curve a shows the profile when the flow at section F was equal to Q_o which is the maximum negative flow absolute value. Curve b shows the profile for a discharge Q_1 at section F where Q_1 is the maximum positive discharge. It can be seen from the figure that

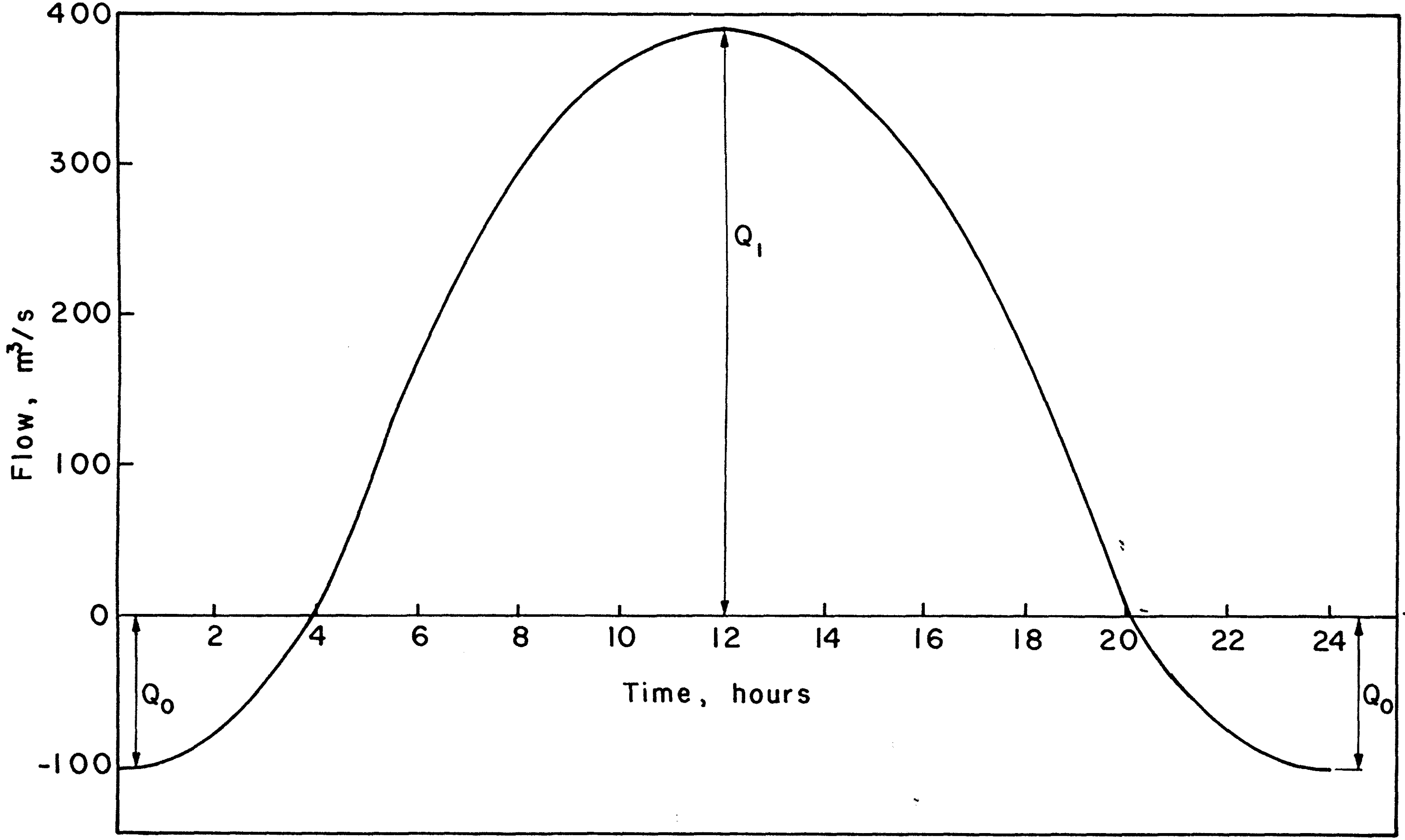


Figure 6.2 Generated Flow at Section F During 24 Hours

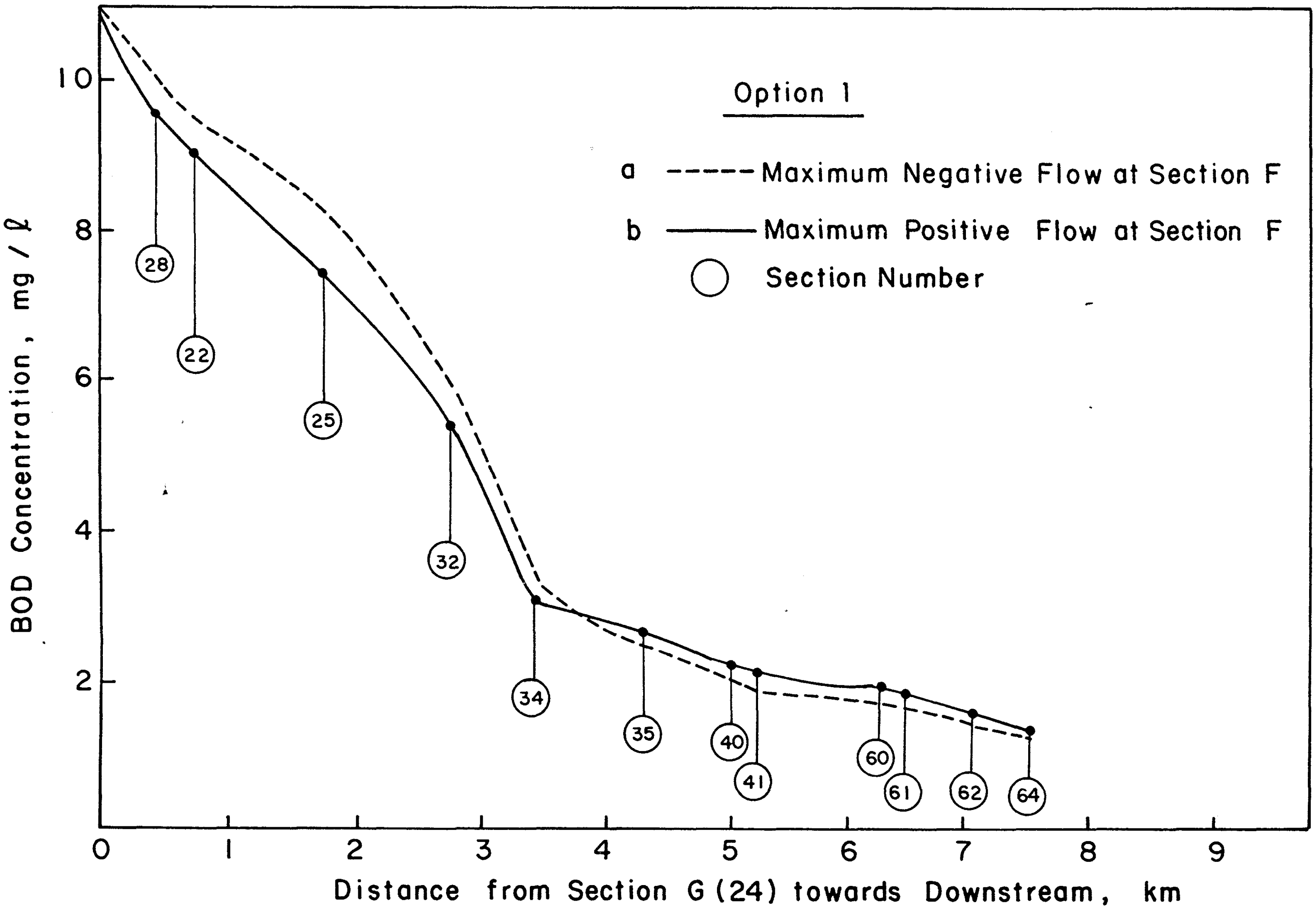


Figure 6.3 BOD Concentration Profile Along the Harbor Above Section F

between sections 33 and 34 there was a sudden decrease in BOD concentration caused by dilution from the confluent river branches. Condition a, as compared to condition b, increases BOD concentration in the sections near the boundary and decreases it downstream. A profile of BOD concentration from section R (14) in the Sinos River toward section 41 downstream, was plotted for the same tested option in Figure 6.4. Again here, the sudden decrease of BOD concentration between sections 13 and 12 was due to a mixing of Sinos water with less polluted water coming from the Jacui and Cai Rivers. At section 28 there was a slight increase of BOD caused by more polluted water coming from the Gravatai River through the loop 21-20-19-27. Also, the small increase in sections 30 and 29 were due to the dispersion effect at junction 32-33-34. The same phenomena occurred again at sections 38 and 39 near the confluence 39-40-41. Figure 6.5 shows those profiles for the DO concentration having a similar pattern.

Option 2

An alternative option was to test for an increase in flow of the polluted rivers (the Gravatai and the Sinos) and a decrease in flow of the Jacui River, assuming the same concentration values. The flow was $Q_M = 70\ m^3/s$; $Q_J = 10\ m^3/s$; $Q_R = 35\ m^3/s$; and $Q_G = 35\ m^3/s$.

Figure 6.6 shows the profiles according to options 1 and 2. The increase in the profiles of option 2 corresponds to the upstream sections G(24) to (34), while the downstream sections caused very little change in the concentration. The sections of branch 48 through 57 were not affected by this pollution and stayed at low levels (0.5 to 0.9).

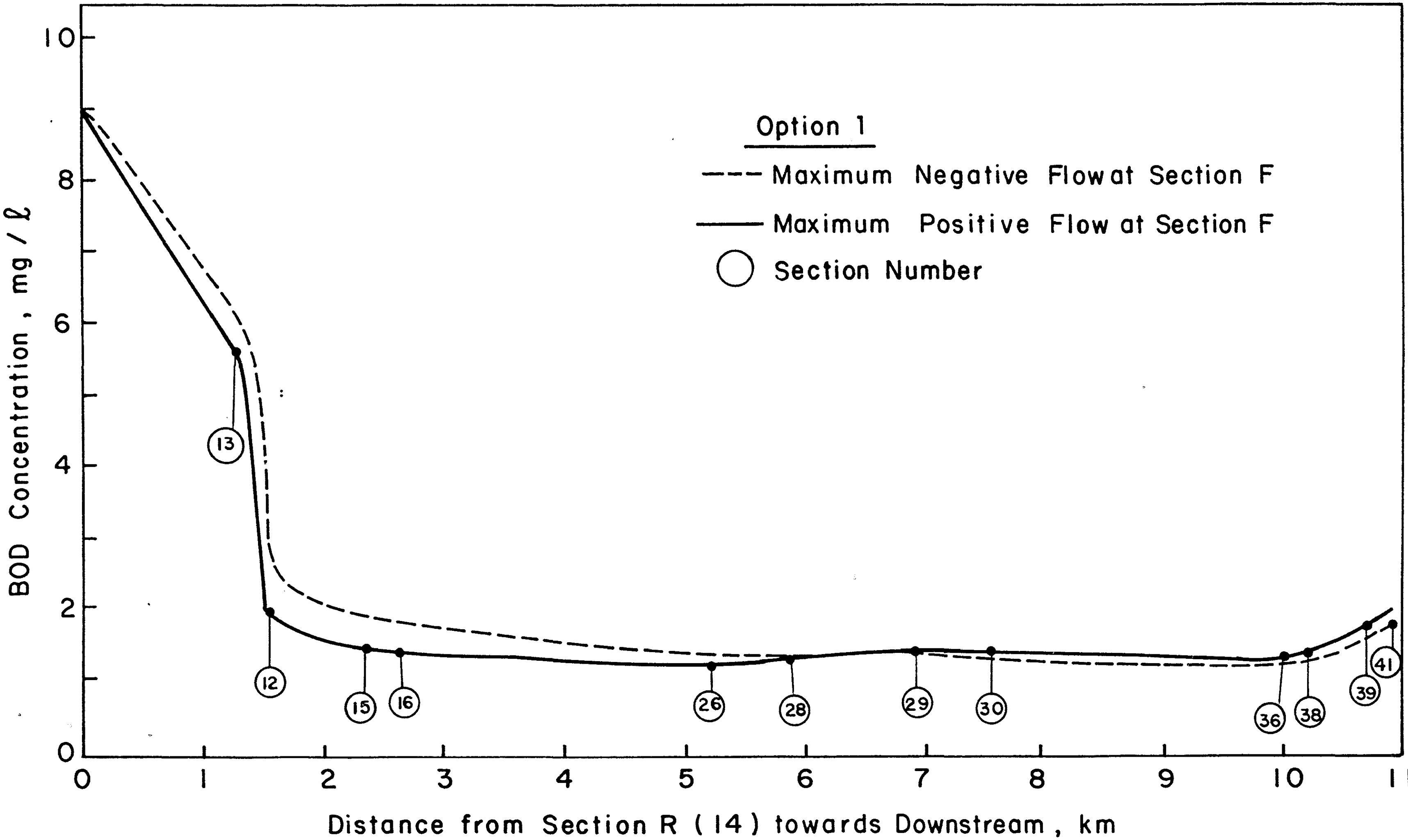
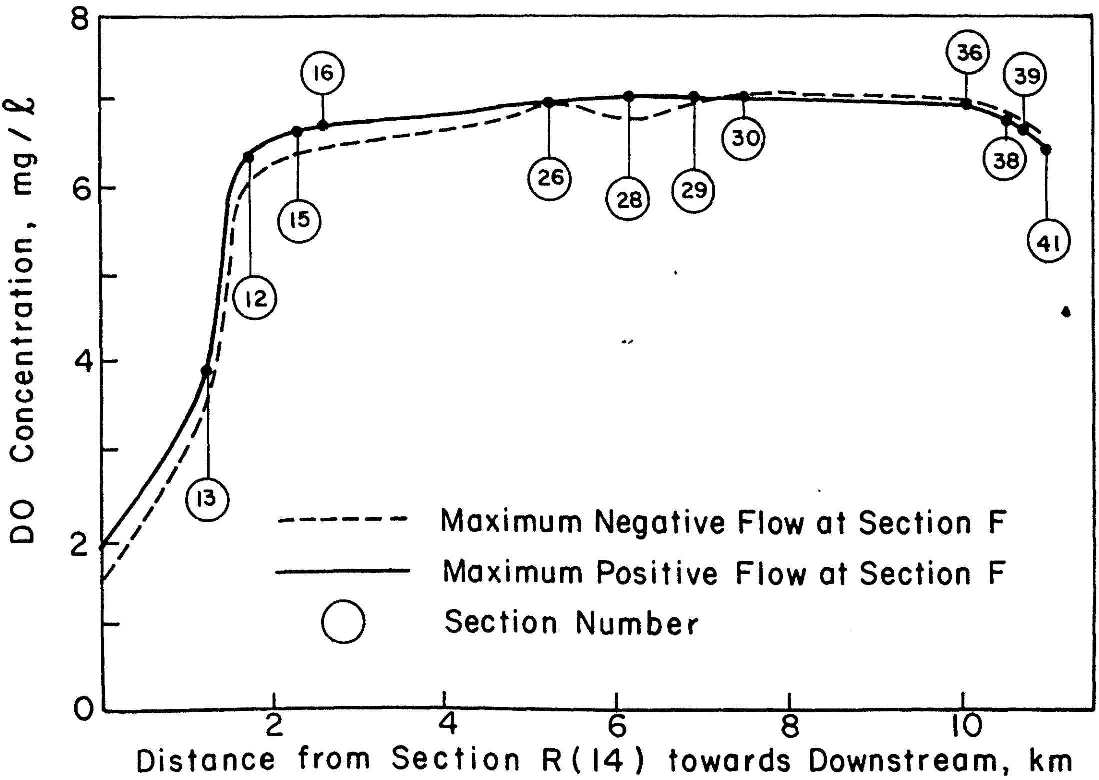


Figure 6.4 BOD Concentration Profile From Sinos River Above Section 41 in Jacui Delta



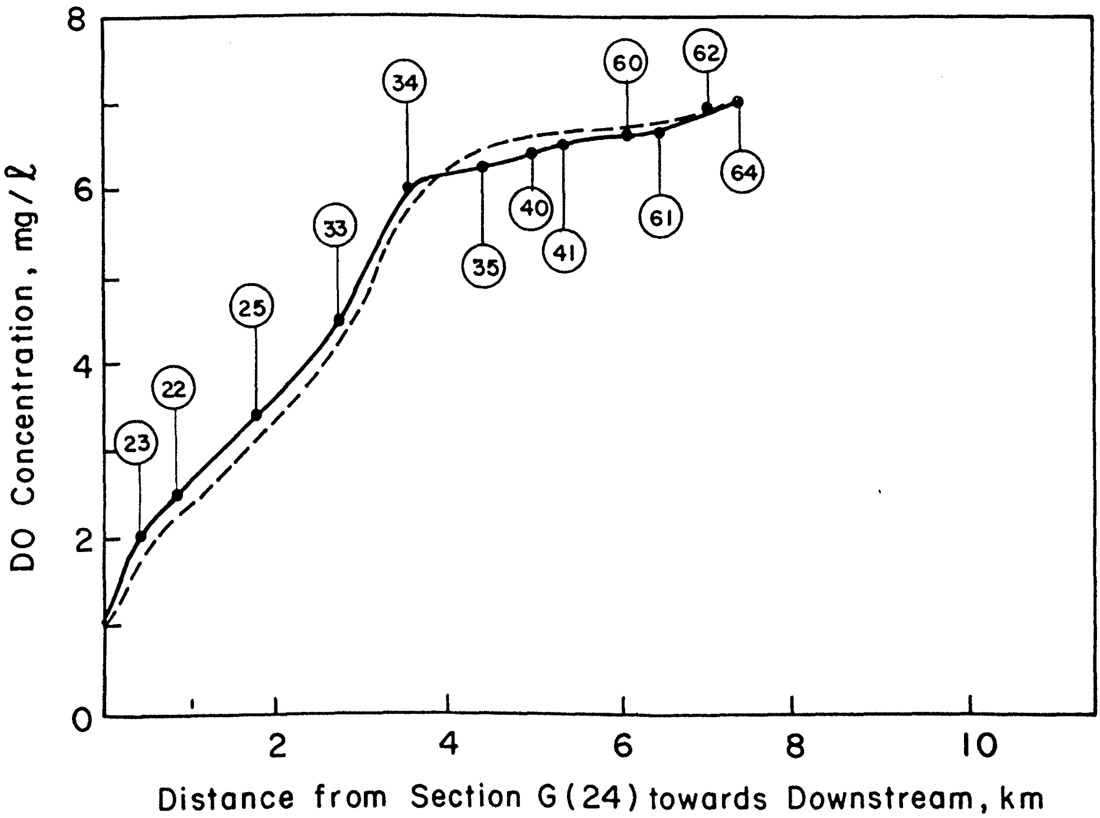


Figure 6.5 DO Profiles in the Jacui Delta

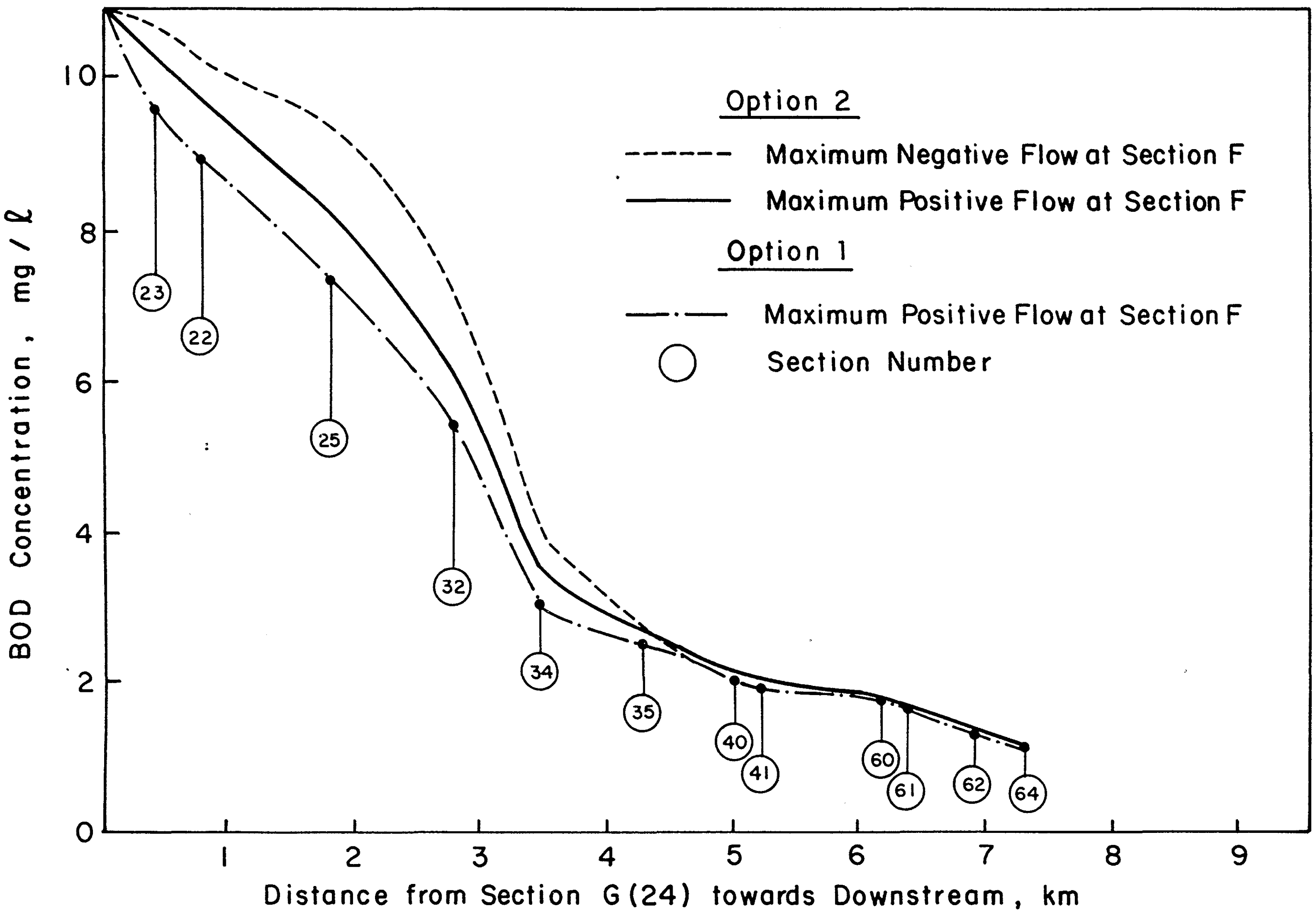


Figure 6.6 BOD Profiles for Options 1 and 2

Option 3

The third option tested for a change in BOD concentration of the Cai River to 9 mg/1 and flow conditions at the boundaries to: Q_M = 90 m^3/s; Q_J = 20 m^3/s; Q_R = 20 m^3/s; and Q_G = 20 m^3/s. This condition primarily affected the branch between sections 8 to 29. There was an increase in BOD concentration before section 28, after which the changes were not significant.

Option 4

This final option tested for a BOD concentration of 4 mg/1 for the Jacui River, maintaining the same flow conditions at the boundary as Option 3. The flow of the Jacui River was the largest, and its effects were important in most downstream sections. The concentration profile from section J(10) to section (41) for all four options tested here is illustrated in Figure 6.7.

C. Discussion

The following discussion is based on two assumptions; that the sources of pollution are restricted to those previously defined and secondly, the generated data are reliable.

The concentration of the rivers entering the Delta mainly affects the concentration of the branches nearby, and usually these values for BOD are high during the period of negative flow. During positive flows or low tide at the downstream boundary there is more flow for dilution of the waste, making it the most convenient period for waste disposal. Table 6.1 lists the maximum concentration value for sections in these options. For instance, the maximum concentration for section 25 was in Option 2 where the polluted discharge from the Gravatai was greater, which had a direct effect in this section. Section 57 had the greatest

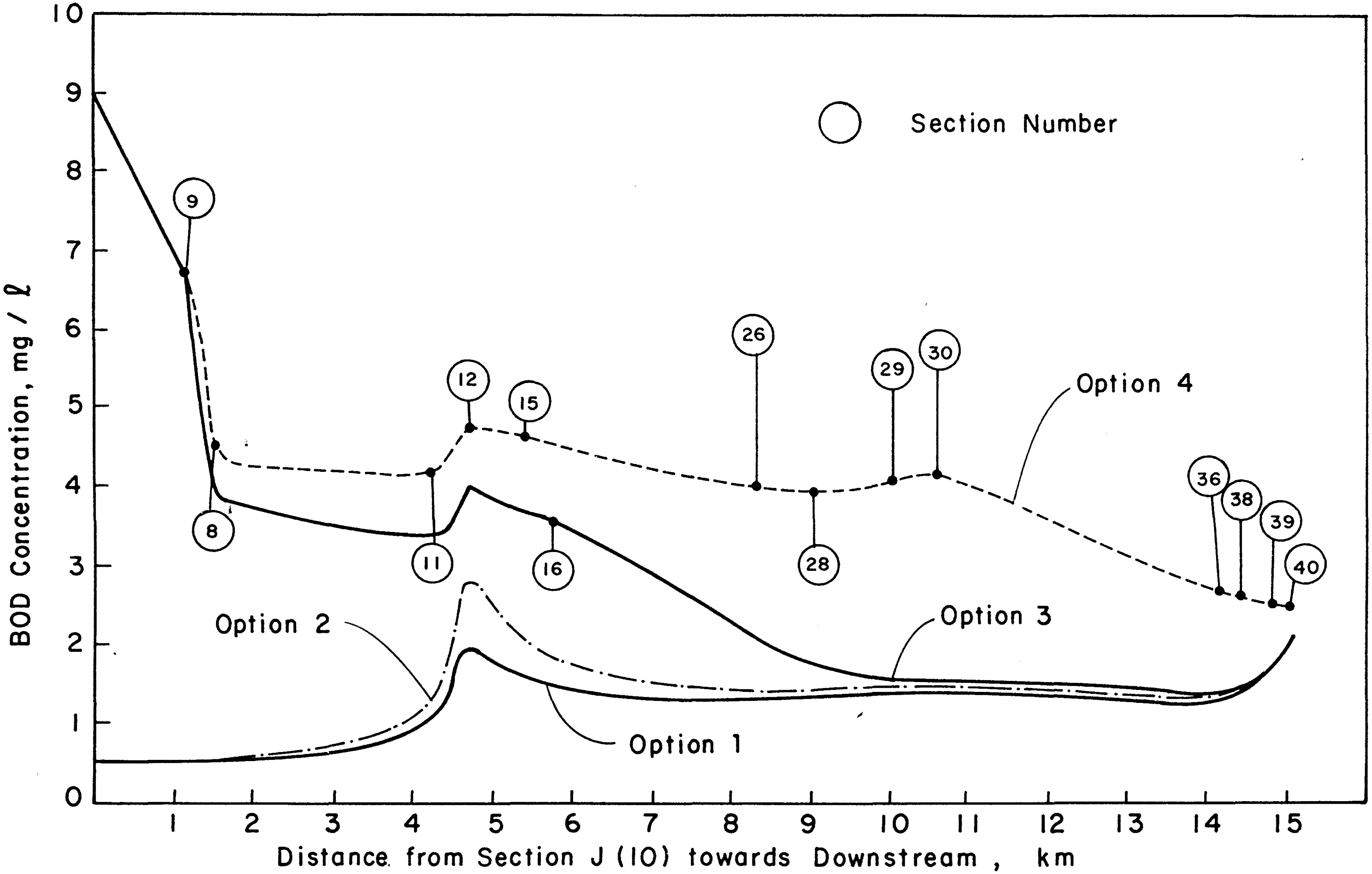


Figure 6.7 BOD Profiles for all Four Options

value in option 4 when the Jacui River had the highest concentration. There was not much change in the other tests. For sections near section F and branches 42-53-56 and 48-50-57, there was no significant change in the concentration when the pollution in the Cai, Sinos, and Gravatai Rivers increased. The Cai River had a major effect on the concentrations of sections between 8 to 28. The Sinos River primarily affected the sections between 12 to 28, and the Gravatai River primarily affected the sections between 22 to 35. The Jacui River is the main source of water in this Delta and if its water quality level decreases, most of the sections of the Delta will also decrease the water quality level.

Table 6.1 Maximum BOD concentration at the sections in each option

Section	Option 1	Option 2	Option 3	Option 4
25	8.38	9.53	8.46	8.58
29	1.55	2.04	2.03	4.00
36	1.38	1.40	1.44	3.05
61	1.72	1.80	1.75	2.33
57	0.92	0.95	0.94	2.00
C*	2.67	4.67	3.27	5.67

C* is the mean concentration of the flow that enters the Jacui Delta.

CHAPTER VII

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A hydraulic and water quality mathematical model for a river network under unsteady flow conditions was developed. A summary of the main features of the model and conclusions related to the study are presented in this chapter.

A. Summary and Conclusions

The following summary conclusions are related to the hydraulic module of the model.

1) The description of the flow behavior was mathematically explained by two one-dimensional partial differential equations called St. Venant equations that were derived from mass and momentum conservation. These equations assumed a uniform variation of the cross-section reach. The sections should be positioned so as to promote as much uniformity in the reach as possible.

2) The continuity and momentum equations under steady flow condition were used at the confluence. A section was defined in each branch near the river confluence. Those equations were used only between those sections at the confluence. The distance from one section to another should be short enough to allow a steady flow condition. The positive flow direction assumed by the model refers to: (i) the upstream section toward the downstream section in the reach, and (ii the notation given by Figure 3.6 at the confluence.

3) A finite difference implicit scheme was developed to solve those equations. The scheme linearized the equations using a Taylor series with first order approximation. The linearization of the flow

equations was valid when changes in the flow variables were small.

4) Boundary conditions should be specified in terms of the type of flow regime. For a subcritical regime one boundary must be specified upstream and another downstream. When the regime is supercritical both conditions can be specified upstream.

Usually, the rating curve was used as a downstream boundary condition in flood problems or where the major effect was from upstream to downstream. This condition can not be used when there are effects from downstream in the boundary section such as in the case of a river sections near the sea or lakes. In this case, a stage hydrograph can be used as the downstream boundary condition.

5) The initial condition can be satisfied by the steady state solution by running about 50 time steps in the computer keeping boundary values constant. These are the boundary values for the initial time step.

6) The criteria used by Price (1974), who approximately chose the time step by the ratio of the space step divided by the kinematic wave speed, is a guide used to get good accuracy in the numerical solution.

7) The method used in solving the system of equations attempted to minimize the computer storage and calculation for the Gauss elimination procedure. Some of the guidelines that proved useful in reducing computer time for the numbering procedure include: (a) The increase in the difference between the section number in the reaches and confluences increased the storage and calculations, (b) the number of the sections should be given in crescent order from upstream towards downstream, and

(c) the best procedure for short loops, such as that of Figure 5.15, is the alternate numbering.procedure.

This method used 1584 storage values and used 1.0 second of central processor time to solve the system of 128 by 128 equations for each time step at the Jacui Delta using a Cyber 172 computer. The total number of sections used was 64.

8) The geometrical data required for the model are: (a) In each section: a table with area, hydraulic radius and top width for each level, the roughness coefficient, and the bottom level of the section; (b) in each reach: the space step, and the upstream and downstream section numbers; (c) in the confluence: the space step, the section numbers, and the positive flow direction; and (d) related to the time variation: time step, number of time steps, boundary values in each time step, and initial conditions.

9) The roughness coefficient of Manning's equation (n) is necessary for each section. The stages are very sensitive to the value of <u>n</u>, given a river reach with a flow hydrograph as boundary input. When one increases the <u>n</u> value the stages also increase. On the other hand, when the boundary is a stage hydrograph, an increase in <u>n</u> will decrease the flow. When the river has confluences and a hydrograph flow as boundary, the effect in the stages due to variation of the roughness coefficients in the branches may not be the same as before. An increase of roughness in a branch decreases its flow, that may result in a stage decrease instead of an increase.

The major parameters in the flow division are the cross-sectional area, the hydraulic radius, and the Manning roughness coefficient. The greater the area and the hydraulic radius, the greater is the flow that goes through the section. Conversely, the greater the n the smaller the flow that goes through the section.

10) The hydraulic model was applied to the Jacui Delta in Brazil. This Delta has the contribution of four rivers with a total watershed of about 100,000 km^2. There are many confluences and storage basins, and there is a harbor located on the eastern side where there is also a large city, Porto Alegre.

Actual data were used to make the adjustment of the roughness of the main branches. The agreement between the recorded and the calculated stages was good; the difference was about 1.0 cm, and the discharge discrepancies were on the order of 8%. Verification of the model was performed using stages for a period of only two days with the stage at section F varying 0.65 m. Two sections could be verified showing good agreement between the observed and the calculated values.

The following summary and conclusions are related to the compound model for water quality.:

1) The description of the variation of a substance in a river at a one-dimensional level was accomplished with the transport equation. It is based on the conservation of mass through a channel reach and utilizes the advection, dispersion, and source and sink terms. The solution of this equation requires knowledge of the hydraulic variables of the river. The model used here was an uncoupled one that first solved the hydraulic equations and then the transport equation for a time step.

2) The source and sink terms for a conservative substance, biochemical oxygen demand, and dissolved oxygen were defined. The longitudinal dispersion coefficient can be estimated by many different

formulas, that are usually a function of the velocity and depth, or it can be determined by a dye test. The reaeration coefficient does not have an equation that always yields a good estimation, but the user may choose between the equation or a constant value during the computer program execution. The decay rate of BOD is usually obtained from laboratory measurements of water samples.

The model was set to simulate a substance that had a first order decay reaction formulation. Other substances can be simulated by defining the proper formulation for the reaction processes in the program.

3) The numerical method used to solve the transport equation was a backward implicit scheme. At the confluences, the equations were defined based on the mass conservation of the transport process. The concentration was defined at the same section as the stage and discharge which was the most convenient procedure.

4) The conservation of mass does not guarantee the accuracy and stability of the numerical scheme. The stability of a backward implicit scheme is given by Equation (4.41). When there was inversion flow, an additional restriction was given by Equation (4.42). This restriction may create problems when the dispersion is not great enough, and the procedure to solve it requires the use of a forward scheme for a negative flow. The accuracy analysis was performed on a linear version of the transport equation by using the Fourier series.

5) Usually two boundary conditions (or more if there are more boundaries) should be specified for the transport equation. The upstream condition can be the concentration as a function of time. The more commonly used boundary condition is the assumption of a linear

relationship between concentration and space. This condition can be used when the concentration gradient is not steep near the boundary. When the source of pollution is a lateral flow this condition can be used in both boundaries. The initial condition also can be determined, as in the hydraulic equations.

6) The model was tested in a channel system with confluences. The two conditions tested were an upstream flow hydrograph and a lateral inflow hydrograph entering within two confluences. Both hydrographs had a degree of pollution described by the concentration of biochemical oxygen demand and dissolved oxygen. The results showed that the processes were well-described.

7) The limitation of the model was in the one-dimensional assumption. The advantages of the model were in handling a broad and complex river system with minimal computer cost, and in using complete partial differential equations in a one-dimensional level, therefore, minimizing the empirical formulations.

B. Recommendations

1) With the development of the bases of the model and its application for specific substances, the major effort in the mathematical solution was made. Further application of this model might include additional substances by defining the mathematical formulation of the reaction processes in order to make the model useful for other purposes. For instance, the nitrification phase of waste oxidation could be added to the model and used in rivers and estuaries where this type of process is important.

2) Utilization of this model in management analysis could be improved by using optimization techniques in the selection of alternatives and decisions.

3) The hydraulic model can be improved to take into account the wind effects on the stage and flow. These effects can be important in some wide branches of a estuary.

4) Management analysis of the Jacui Delta was conducted here to demonstrate the applicability of the model in a specific condition. The main concern with this Delta is the actual determination of the most suitable site for water supply intake for Porto Alegre. Accurate analysis of this purpose would require the use of the coliform parameter for water quality along with more information on the following: (a) location, amount, and distribution in time of the waste inflow in the Delta, (b) concentration of coliforms during the day in the Gravatai and Sinos Rivers at sections R and G, the concentration at sections 35, 38, 37, and 57, which are the alternative sections for the intake, and (c) the longitudinal dispersion coefficient in the important branches. The collection of samples to determine the decay rate of this parameter is also important, and (d) hydraulic data from a critical period for water quality in order to verify the model adjustment and better define the roughness coefficient in some secondary branches.

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APPENDIX A

GEOMETRICAL PROPERTIES OF EACH SECTION IN JACUI DELTA

APPENDIX A

Table A.1. Geometrical properties of each section in Jacui Delta.

	section 1; z_o=-8.38m			section 2; z_o=-8.30m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	4877.5	687.8	7.079	6201.1	1020.0	6.164
0.00	4198.6	666.6	6.288	5296.8	968.5	5.466
-1.00	3549.0	635.5	5.576	4357.3	916.1	4.764
-2.00	2924.8	613.0	4.765	3461.9	874.8	3.955
-3.00	2322.3	593.1	3.912	2605.6	839.6	3.102
-4.00	1742.0	562.9	3.092	1782.3	806.9	2.208
-5.00	1195.3	530.9	2.250	1004.4	723.6	1.308

	section 3; z_o=-12.50m			section 4; z_o=-4.00m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	2562.7	319.4	7.914	2556.3	722.5	3.535
0.00	2249.1	306.8	7.238	1845.4	699.3	2.637
-1.00	1958.5	272.1	7.099	1164.1	666.7	1.745
-2.00	1695.4	255.1	6.555	500.9	543.6	0.921
-3.00	1447.9	239.8	5.961	101.3	263.5	0.384
-4.00	1215.4	226.2	5.310	0	0	0
-5.00	994.4	215.9	4.599	0	0	0

	section 5; z_o=-6.65m			section 6; z_o=-4.40m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	2633.4	620.0	4.234	207.7	59.0	3.300
0.00	2035.8	575.3	3.527	149.8	57.1	2.494
-1.00	1483.7	521.4	2.837	93.4	55.6	1.627
-2.00	1003.2	439.5	2.277	44.9	38.9	1.127
-3.00	604.7	357.6	1.688	12.0	17.9	0.568
-4.00	302.8	232.2	1.302	0.9	4.6	0.197
-5.00	126.0	155.3	0.802	0	0	0

*z_o is the bottom level.

Table A.1 (continued)

	section 7; z_o=-6.00m			section 8; z_o=-6.40m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	-	-	-	3265.7	980.0	3.327
0.00	1870.0	620.0	3.011	2297.0	957.8	2.304
-1.00	1260.0	540.0	2.329	1489.9	625.7	2.376
-2.00	772.0	436.0	1.768	926.1	502.0	1.841
-3.00	393.0	182.0	2.153	647.7	509.7	1.269
-4.00	218.0	168.0	1.295	280.8	244.1	1.148
-5.00	57.0	114.0	0.499	91.4	134.8	0.678

	section 9; z_o=-3.60m			section 10; z_o=-8.70m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	722.3	233.8	3.084	619.8	148.0	4.141
0.00	498.4	213.9	2.326	478.9	135.5	3.499
-1.00	302.8	182.7	1.656	348.0	127.8	2.700
-2.00	138.1	140.0	0.986	222.8	122.6	1.806
-3.00	27.2	81.9	0.332	107.3	99.4	1.075
-4.00	0	0	0	50.8	97.0	0.523
-5.00	-	-	-	2.2	11.8	0.188

	section 11; z_o=-8.70m			section 12; z_o=-8.80m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	3184.1	504.0	6.306	3163.0	616.0	5.123
0.00	2700.0	464.1	5.808	2577.6	554.9	4.634
-1.00	2256.0	424.2	5.309	2061.9	458.3	4.486
-2.00	1845.0	404.4	4.555	1652.1	379.7	4.337
-3.00	1451.5	381.8	3.797	1286.5	351.5	3.647
-4.00	1081.0	359.2	3.007	950.4	317.9	2.978
-5.00	746.6	309.6	2.410	654.6	268.3	2.431

Table A.1 (continued)

	section 13; z_o=-2.80m			section 14; z_o=-6.05m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	487.7	163.0	2.981	486.7	93.0	5.113
0.00	332.5	147.4	2.250	396.1	88.2	4.404
-1.00	192.9	131.8	1.462	310.4	83.4	3.663
-2.00	66.4	120.8	0.550	229.4	78.5	2.886
-3.00	0	0	0	153.3	73.5	2.070
-4.00	-	-	-	86.5	58.3	1.476
-5.00	-	-	-	34.6	45.9	0.751

	section 15; z_o=-8.00m			section 16; z_o=-9.00m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	-	-	-	3095.4	520.0	5.941
0.00	2666.0	640.0	4.162	2592.3	486.0	5.324
-1.00	2100.0	490.0	4.283	2123.3	452.1	4.688
-2.00	1625.0	460.0	3.531	1686.0	422.1	3.990
-3.00	1185.0	420.0	2.820	1281.0	386.0	3.316
-4.00	785.0	380.0	2.065	919.5	336.7	2.726
-5.00	422.5	305.0	1.450	613.3	279.3	2.193

	section 17; z_o=-3.30m			section 18; z_o=-5.95m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	398.3	166.5	2.385	246.8	55.8	4.212
0.00	241.3	145.0	1.660	193.3	51.3	3.605
-1.00	111.4	114.7	0.969	144.4	46.1	3.008
-2.00	24.9	44.9	0.553	101.6	39.4	2.487
-3.00	1.3	8.8	0.149	65.6	32.7	1.944
-4.00	0	0	0	36.2	25.9	1.368
-5.00	-	-	-	13.9	18.7	0.736

Table A.1 (continued)

	section 19; z_o=-3.40m			section 20; z_o=-4.68m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	281.4	96.8	2.889	172.1	60.5	2.773
0.00	191.4	83.2	2.288	119.5	44.7	2.596
-1.00	114.6	71.1	1.604	78.2	39.0	1.958
-2.00	48.9	59.6	0.820	41.9	31.6	1.304
-3.00	5.0	21.4	0.235	15.4	21.4	0.710
-4.00	0	0	0	1.2	7.0	0.172

	section 21; z_o=-4.68m			section 22; z_o=-9.90m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	172.1	60.5	2.773	1325.3	172.0	7.534
0.00	119.5	44.7	2.596	1157.5	163.7	6.917
-1.00	78.2	39.0	1.958	997.1	157.1	6.217
-2.00	41.9	31.6	1.304	843.2	150.8	5.485
-3.00	15.4	21.4	0.710	695.5	144.9	4.716
-4.00	1.2	7.0	0.172	553.3	139.6	3.901
-5.00	0	0	0	416.3	134.4	3.057

	section 23; z_o=-6.90m			section 24; z_o=-4.30m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	575.4	117.0	4.851	524.6	141.8	3.680
0.00	463.7	106.3	4.306	388.4	130.5	2.963
-1.00	362.8	95.6	3.749	263.3	120.7	2.175
-2.00	272.2	86.4	3.115	147.5	108.6	1.357
-3.00	190.0	76.7	2.453	50.3	80.7	0.629
-4.00	119.7	63.9	1.860	2.5	16.5	0.150
-5.00	62.3	51.0	1.215	0	0	0

Table A.1 (continued)

	section 25; z_o=-5.50m			section 26; z_o=-4.00m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	830.7	291.3	2.844	-	-	-
0.00	559.0	248.5	2.243	2220.0	700.0	3.170
-1.00	372.7	143.3	2.590	1550.0	640.0	2.421
-2.00	253.1	99.0	2.542	940.0	580.0	1.621
-3.00	161.2	86.2	1.862	385.0	530.0	0.726
-4.00	79.2	77.2	1.029	-	-	-
-5.00	12.3	49.1	0.250	-	-	-

	section 27; z_o=-2.00m			section 28; z_o=-5.50m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	145.0	100.0	1.448	4004.5	1130.0	3.541
0.00	57.5	75.0	0.766	2917.9	1043.3	2.794
-1.00	0	0	0	1961.1	1021.9	1.918
-2.00	-	-	-	1046.6	628.1	1.338
-3.00	-	-	-	461.3	409.6	1.125
-4.00	-	-	-	175.8	187.3	0.938
-5.00	-	-	-	35.2	99.0	0.355

	section 29; z_o=-5.90m			section 30; z_o=-6.30m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	4100.0	968.0	4.232	1524.3	271.3	5.577
0.00	3165.2	901.3	3.509	1256.9	263.5	4.741
-1.00	2284.7	870.6	2.623	997.3	255.7	3.882
-2.00	1418.9	861.1	1.647	745.5	247.9	2.998
-3.00	687.4	606.9	1.132	501.5	240.1	2.085
-4.00	267.1	293.6	0.899	272.5	200.3	1.247
-5.00	68.3	133.9	0.510	91.0	143.3	0.635

Table A.1 (continued)

	section 31; z_o=-7.10m			section 32; z_o=-10.40m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	2715.0	712.5	3.797	3555.8	559.0	6.341
0.00	2027.9	661.6	3.054	3025.6	501.4	6.014
-1.00	1395.6	554.6	2.506	2545.6	464.8	5.457
-2.00	1009.6	342.8	2.933	2097.4	424.4	4.925
-3.00	685.1	285.8	2.392	1679.2	412.0	4.064
-4.00	438.3	186.5	2.346	1274.7	394.7	3.221
-5.00	258.2	173.8	1.484	823.2	308.2	2.662

	section 33; z_o=-5.20m			section 34; z_o=-5.20m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	1076.5	347.5	3.088	1308.3	280.0	4.600
0.00	745.3	314.9	2.359	1033.0	270.5	3.768
-1.00	451.4	258.3	1.741	777.8	236.6	3.247
-2.00	242.8	113.3	2.127	550.3	218.2	2.297
-3.00	149.6	87.3	1.703	341.4	199.8	1.697
-4.00	65.8	80.3	0.818	150.8	181.3	0.828
-5.00	2.5	24.7	0.100	6.8	49.9	0.136

	section 35; z_o=-11.59m			section 36; z_o=-7.30m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	3189.9	402.0	7.750	1396.5	230.0	6.030
0.00	2788.8	400.2	6.825	1177.2	208.7	5.604
-1.00	2389.4	398.4	5.891	974.4	197.6	4.904
-2.00	2023.1	338.8	5.864	781.4	189.2	4.110
-3.00	1690.9	327.1	5.089	596.4	180.7	3.287
-4.00	1365.8	323.2	4.173	419.9	172.3	2.431
-5.00	1044.6	319.3	3.240	251.9	163.8	1.535

Table A.1 (continued)

	section 37; z_o=-2.00m			section 38; z_o=-5.70m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	-	-	-	1446.3	441.0	3.275
0.00	390.0	260.0	1.500	1014.8	422.1	2.401
-1.00	152.5	215.0	0.709	603.6	382.6	1.576
-2.00	0	0	0	324.6	141.8	2.285
-3.00	-	-	-	195.6	116.1	1.682
-4.00	-	-	-	91.3	93.0	0.981
-5.00	-	-	-	17.2	42.3	0.407

	section 39; z_o=-5.70m			section 40; z_o=-10.40m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	1446.3	441.0	3.275	3195.4	414.8	7.601
0.00	1014.8	422.1	2.401	2784.2	408.5	6.732
-1.00	603.6	382.6	1.576	2378.3	403.4	5.832
-2.00	324.6	141.8	2.285	1980.9	391.3	5.014
-3.00	195.6	116.1	1.682	1594.6	383.7	4.123
-4.00	91.3	93.0	0.981	1212.5	380.6	3.166
-5.00	17.2	42.3	0.407	833.5	377.5	2.199

	section 41; z_o=-6.40m			section 42; z_o=-10.80m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	3581.7	755.0	4.711	3149.6	516.0	6.080
0.00	2828.0	752.3	3.738	2644.7	475.7	5.537
-1.00	2077.1	749.5	2.758	2196.9	438.4	4.991
-2.00	1308.7	736.9	1.769	1797.9	322.7	4.708
-3.00	690.3	337.3	1.938	1455.4	310.4	4.668
-4.00	382.2	284.3	1.340	1156.2	292.7	3.933
-5.00	125.9	224.1	0.561	869.2	281.5	3.076

Table A.1 (continued)

	section 43; z_o=-4.40m			section 44; z_o=-9.00m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	568.3	162.8	3.446	2565.6	464.3	5.504
0.00	407.5	158.8	2.542	2108.3	450.2	4.666
-1.00	254.6	130.7	1.932	1673.6	391.6	4.259
-2.00	139.0	101.8	1.357	1290.0	373.9	3.437
-3.00	53.7	70.6	0.757	955.9	278.1	3.424
-4.00	4.4	21.9	0.200	696.7	241.5	2.875
-5.00	0	0	0	468.1	215.7	2.163

	section 45; z_o=-4.30m			section 46; z_o=-2.00m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	816.1	211.0	3.855	-	-	-
0.00	609.8	201.2	3.023	310.0	220.0	1.408
-1.00	414.1	190.3	2.172	58.9	78.5	0.247
-2.00	230.8	175.5	1.314	0	0	0
-3.00	93.3	97.6	0.955	-	-	-
-4.00	10.5	59.3	0.178	-	-	-

	section 47; z_o=-4.10m			section 48; z_o=-8.0m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	256.5	62.4	3.961	986.1	182.0	5.377
0.00	195.5	59.5	3.196	814.9	160.5	5.037
-1.00	137.4	56.7	2.386	663.7	144.7	4.553
-2.00	83.2	51.8	1.591	524.2	134.5	3.872
-3.00	35.4	43.8	0.803	394.7	124.3	3.159
-4.00	0.7	14.2	0.050	276.0	112.1	2.452
-5.00	0	0	0	171.4	93.9	1.819

Table A.1 (continued)

	section 49; z_o=-2.95m			section 50; z_o=-8.50m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	1700.0	630.0	2.750	1645.7	410.0	4.006
0.00	1150.0	515.0	2.220	1276.8	327.9	3.885
-1.00	650.0	450.0	1.460	976.9	282.1	3.455
-2.00	220.0	410.0	0.550	708.2	255.4	2.767
-3.00	0	0	0	485.4	188.9	2.563
-4.00	-	-	-	327.9	126.2	2.587
-5.00	-	-	-	220.3	95.4	2.299

	section 51; z_o=-3.90m			section 52; z_o=-9.80m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	259.4	90.5	2.843	3077.1	828.0	3.711
0.00	176.6	75.0	2.338	2284.9	756.6	3.015
-1.00	108.6	63.0	1.727	1623.6	573.6	2.825
-2.00	51.9	48.6	1.063	1252.6	307.4	4.061
-3.00	12.8	28.4	0.449	974.6	248.6	3.904
-4.00	0	0	0	750.4	201.7	3.702
-5.00	-	-	-	566.2	173.7	3.244

	section 53; z_o=-6.50m			section 54; z_o=-6.40m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	2391.1	420.8	5.661	2391.1	420.8	5.904
0.00	1974.5	412.5	4.772	1974.5	412.5	5.134
-1.00	1578.8	380.8	4.135	1578.8	380.9	4.417
-2.00	1205.7	367.1	3.279	1205.8	367.1	3.512
-3.00	813.2	306.7	2.647	813.2	306.7	2.587
-4.00	535.9	278.0	1.925	535.9	278.0	1.640
-5.00	278.5	244.5	0.886	278.5	244.5	0.886

Table A.1 (continued)

	section 55; z_o=-2.55m			section 56; z_o=-7.40m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	233.9	102.5	2.266	2079.9	375.0	5.521
0.00	137.5	83.3	1.643	1712.0	360.0	4.726
-1.00	64.4	65.3	0.982	1357.9	348.4	3.882
-2.00	6.9	37.6	0.183	1015.5	332.4	3.044
-3.00	0	0	0	701.1	296.6	2.355
-4.00	-	-	-	438.2	236.5	1.846
-5.00	-	-	-	212.6	217.3	0.976

	section 57; z_o=-8.91m			section 58; z_o=-10.70m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	2920.6	633.0	2.557	3584.2	831.0	4.291
0.00	2552.6	542.1	4.705	2780.9	775.6	3.566
-1.00	2039.6	480.9	4.237	2078.8	549.9	3.179
-2.00	1594.7	408.2	3.904	1455.0	583.7	2.478
-3.00	1207.9	367.7	3.283	991.9	339.2	2.898
-4.00	855.3	336.4	2.541	683.8	275.4	2.460
-5.00	555.6	261.6	2.123	438.5	218.7	1.987

	section 59; z_o=-8.00m			section 60; z_o=-6.20m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	2920.7	680.0	4.290	2971.3	930.0	3.186
0.00	2271.8	617.7	3.674	2065.3	882.1	2.335
-1.00	1681.0	572.4	2.934	1473.4	490.9	2.990
-2.00	1138.6	473.3	2.403	990.6	474.5	2.081
-3.00	743.0	304.4	2.439	524.3	458.2	1.142
-4.00	474.9	231.9	2.046	263.9	341.5	0.772
-5.00	264.9	185.9	1.425	67.9	73.6	0.919

Table A.1 (continued)

	section 61; z_o=-5.90m			section 62; z_o=-5.90m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	5582.5	1140.0	4.884	5582.5	1140.0	4.884
0.00	4450.0	1124.9	3.946	4450.0	1124.9	3.946
-1.00	3332.6	1109.9	2.997	3332.6	1109.9	2.997
-2.00	2277.9	945.8	2.404	2277.9	945.8	2.404
-3.00	1417.9	782.8	1.809	1417.9	782.8	1.809
-4.00	693.2	666.5	1.039	693.2	666.5	1.039

	section 63; z_o=-8.20m			section 64; z_o=-12.92m		
stage m	area m^2	top width m	hyd.rad. m	area m^2	top width m	hyd.rad. m
1.00	2255.4	331.0	6.771	7699.3	1113.0	6.915
0.00	1926.4	326.9	5.863	6648.6	974.7	6.818
-1.00	1601.5	322.9	4.942	5733.4	862.3	6.646
-2.00	1280.6	318.9	4.008	4910.9	790.6	6.209
-3.00	969.8	299.8	3.230	4150.3	731.6	5.670
-4.00	682.7	269.3	2.533	3447.7	673.7	5.115
-5.00	438.9	218.8	2.004	2802.9	613.7	4.565

APPENDIX B

WATER QUALITY COMPUTER PROGRAM

APPENDIX B

WATER QUALITY COMPUTER PROGRAM

B.1 Program Capabilities

The model can be used to simulate the hydraulic and water quality of a river with islands, tributaries, a confluence, and where there are sea or lake effects river flow. The model simulates the following water quality parameters: conservative substance, biochemical oxygen demand, and dissolved oxygen.

The program is based on the St. Venant equations of continuity and momentum conservation and on the transport equation of a substance. These equations assume one-dimensional flow; their solution obtainable through an implicit numerical scheme.

Data required by the program includes:

1) For each cross section: (a) a table for each level with values for area, top width, and hydraulic radius, (b) the beta, roughness, decay, reaeration, and longitudinal dispersion coefficients. Some coefficients can be estimated in the program.

2) For each reach: the space between the sections, section number, and positive flow direction.

3) For each confluence: the distance between the sections, section number, and positive flow direction.

4) Boundary conditions should be specified for the period the calculations are required. The initial level, discharge and concentration for all sections is also required.

The output, for the specified time step and sections gives the depth at time t, the depth at time t + Δt, the depth variation, level discharge at time t, and the discharge at time t + Δt. The output also gives the concentration of a substance in each cross-section.

The program requires about 15,000 words for the arrays. The dimension is set up in 65 sections, 40 reaches, 20 confluences, 5 boundaries, 100 time steps, 10 points per section in the tables, 2 lateral contributions, and 3 substances.

The central processor time spent by the program in each time step for the Jacui Delta is about 2 seconds, which includes the simulation of the hydraulic equations and the transport equation for two substances. This system has 64 sections, 33 reaches, and 19 confluences.

B.2 Routine Description

This computer program has 21 subprograms including 19 subroutines and 2 functions. The general flowchart of the main program is shown in Figure B.1.

Main Program - At the start of the program, input values are read by calling the subroutines INPUT and INPUT1. The subroutines MATRIX and MATRIX1 are called to organize the coefficient matrix. The time step loop solves the hydraulic partial differential equations using COEF1 which calculates the coefficients, SKYLINE which solves the solution of the hydraulic equations, and program WQSIM which solves the transport equation.

Subroutine ARRAY

MATRIX and MATRIX1 are called in this subroutine that computes the position of the matrix coefficients in the one-dimensional array AA(I)

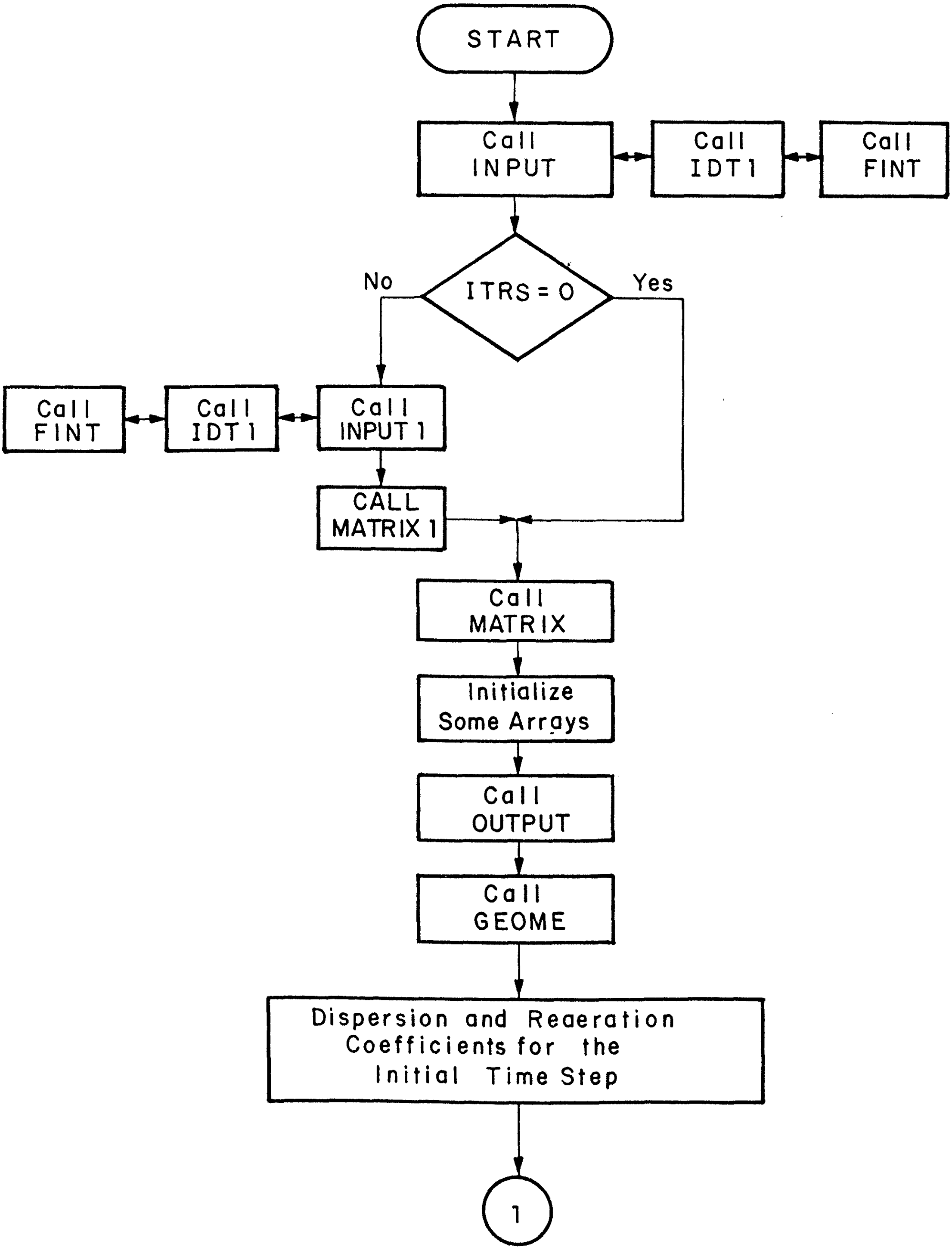

Figure B.1 Flowchart for the Main Computer Program

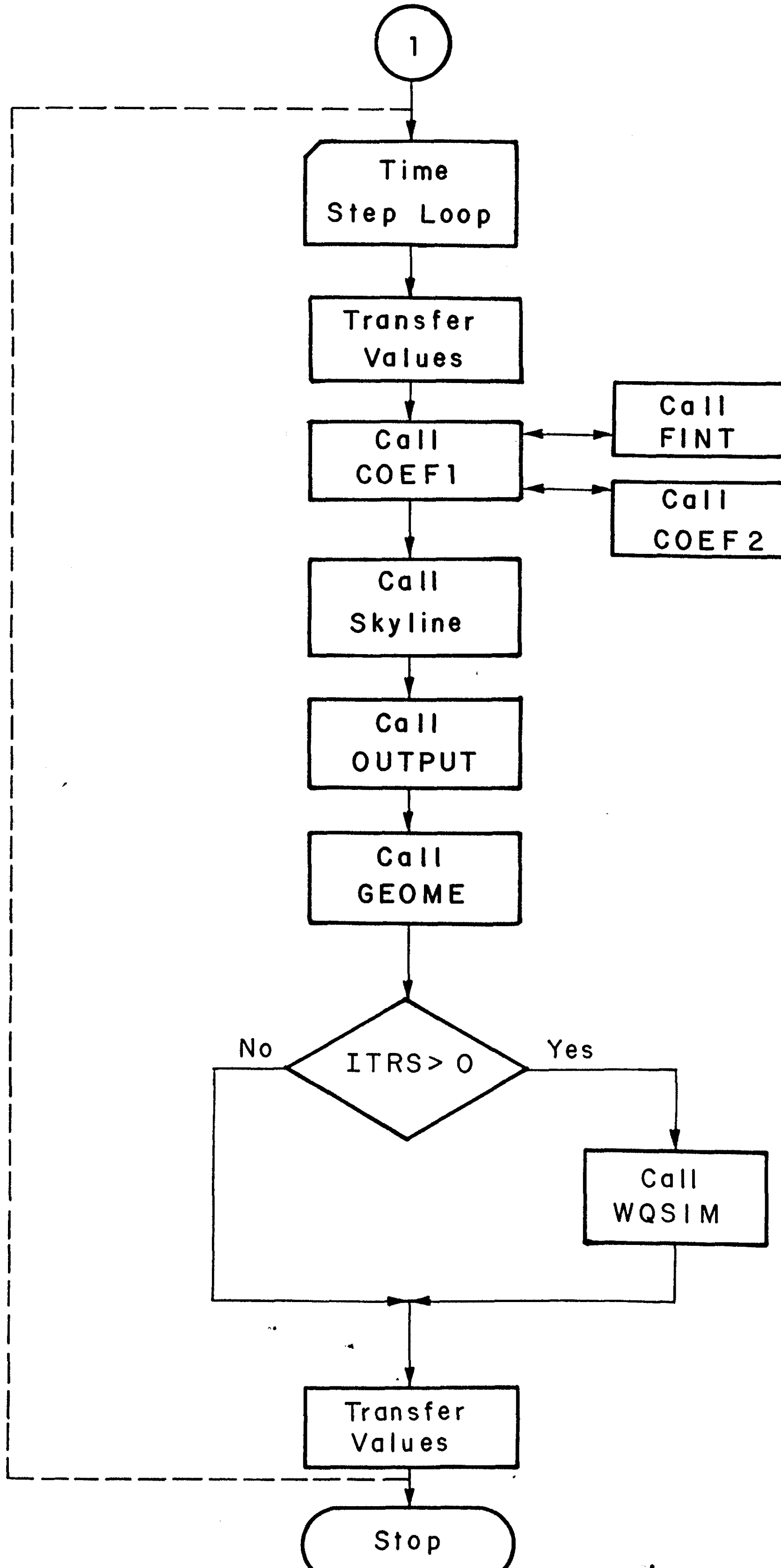

Figure B.1 Flow chart for the Main Computer Program (continued) used in the solution of the system of equations. It also calculates the arrays IR(I), IHIGH(I), and IDIAG(I) used in SKYLINE.

Subroutine BODOD

This subroutine is called in WQSIM and computes the source and sink term for the biochemical oxygen demand (BOD) and dissolved oxygen (DO).

Subroutine COEF1

This subroutine computes the coefficients for the reach equations and boundary condition. If there are confluences this subroutine calls COEF2. It is called at each time step in the main program.

Subroutine COEF2

This subroutine called in COEF1 computes the coefficients of the confluence equations.

Subroutine DISPER

This subroutine computes the longitudinal dispersion coefficient for all sections by a modified Taylor equation at each time step. It is called in WQSIM and in the main program.

Function FINT

This function is called in IDT1 and COEF1 and is used to interpolate a value in a table.

Subroutine GEOME

For all sections this subroutine computes area, hydraulic radius, top width, friction slope, conveyance, and derivative of the conveyance with respect to depth by interpolation in the tables. It is called in the main program.

Subroutine IDT1

When the boundaries and lateral contribution values are not given in the same time spacing as that of the calculations, it can be interpolated linearly by using an option. This subroutine is called in INPUT and INPUT1.

Subroutine INPUT

This subroutine reads such system parameters as the cross-section tables, number of sections, and the roughness coefficients. It also reads the boundary and initial conditions and lateral contribution, and prints the input values as an option. This subroutine is called in the main program.

Subroutine INPUT1

This subroutine reads the water quality coefficients and the initial and the boundary conditions of the transport equation that will be simulated for each parameter. This subroutine is called in the main program.

Subroutine MATRIX

This subroutine is called in the main program and is used at the beginning of the execution to organize the coefficient matrix of the hydraulic equations. In this way it minimizes the number of sparse elements outside of a main diagonal band.

Subroutine MATRIX1

This subroutine performs the same function as MATRIX but for the coefficient matrix of the transport equation. It is called in the main program.

Subroutine OUT

This subroutine prints the concentration for all sections at each time step and is called in WQSIM.

Subroutine OUTPUT

This subroutine prints depth, depth variation in the time step, level, discharge, and discharge variation at each time step. This subroutine gives the option of printing only some specified sections and time steps. This subroutine is called in the main program.

Subroutine REARE

This subroutine computes the reaeration coefficient by the O'Connor and Dobbins equations for all sections in each time step. It is called in BODOD and in the main program.

Subroutine RHS

This subroutine computes the right hand side matrix of the transport equation and it is used when there is more than one parameter to be simulated. It is called in WQSIM.

Subroutine SKYLINE

This subroutine solves the system of equations by the Gauss elimination procedure by a storage scheme described in Section C.2 in Chapter III. This subroutine is called at each time step in the main program and in WQSIM.

Subroutine SUB

This is an auxiliary subroutine used by subroutine MATRIX to give the column position of the coefficients in the matrix. This subroutine is called in MATRIX.

Subroutine SUB2

This is an auxiliary subroutine used by the subroutine MATRIX to give the non-zero coefficients of a specified row of the matrix.

Subroutine TRANSP

This subroutine computes the coefficient matrix and right hand side matrix. It is called in WQSIM.

Subroutine WQSIM

This subroutine is used at each time step to solve the transport equation for as many parameters as required. The flowchart of this subroutine is shown in Figure B.2. It is called at each time step in the main program.

B.3 List of FORTRAN Symbols

A list of the most important variables in the computer program is given in this section.

FORTRAN Variable	Description
A(I)	Area of the cross section I at time t
AA(I)	One-dimensional array that stores the coefficient matrix
ALFA(I)	Coefficient of losses in the confluence
AR(J,I)	Cross-sectional area, table at section I
AT1(I)	Area of cross section I at time $t + \Delta t$
BB(I)	Right hand side matrix of the system of equations
C(I,J)	Concentration at section I of the substance J
CABE(I)	Stores the title that is printed with the input cards of the hydraulic data.
CABE1(I)	Stores the title that is printed with the input cards of the water quality data.

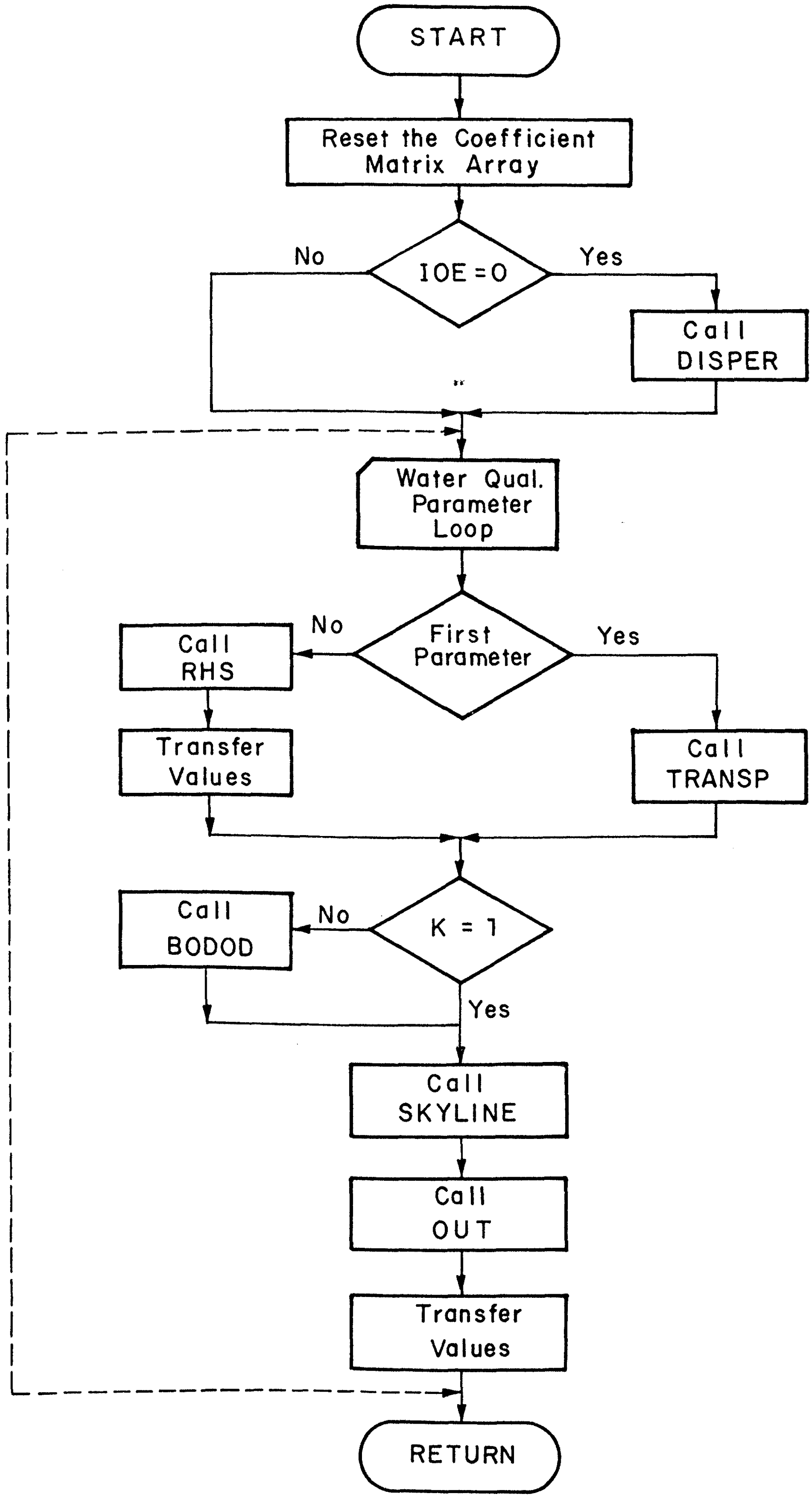

Figure B.2 Flowchart for Subroutine WQSIM

Fortran Variable	Description
CB(I,J,K)	Concentration at boundary J in time step I for the substance K
CK(I)	Conveyance at section I in time t
CKY(I)	Partial derivative of the conveyance with respect to y at section I in time t
CKT1(I)	Conveyance at section I in time t + Δt
CKYT1(I)	Partial derivative of the conveyance with respect to y at section I in time t + Δt
DB(I)	The rate D_b for section I
DT	Time step of the boundary and lateral inflow data
DT1	Calculation ›o of time step
DX(I)	Distance between two sections where I is the upstream section of the reach
DXC(I)	Distance between two sections in the confluence First the distance between J = 1 and J = 3 in NCC (I,J) and after J = 2 and J = 3
E(I)	Longitudinal dispersion coefficient at time t
ET1(I)	Longitudinal dispersion coefficient at time t + Δt
F(I,J)	Manning roughness coefficient, table at the section J
FAF(I,J)	Table of flood area values for section J
G	Gravitational acceleration
HA(I,J)	Depth, table at section J
HF(I)	Depth, table of flood depth at section I
HO(I)	Depth at section I in time step t

FORTRAN Variable	Description
HQB(I,J)	Variable value (depth or discharge) at boundary J in time step I
HT(I)	Depth values from the rating curve table
ICONF	Confluence option ICONF = -1, the program uses Equation (3.9) at the confluence ICONF = 0, the program uses Equation (3.10) at the confluence
IOE	Longitudinal dispersion coefficient option IOE = 0, the program computes E by the subroutine DISPER IOE > 0, the coefficient is given in the input data
IOK2	Reaeration coefficient option IOK2 = 0, the program computes the coefficient by the subroutine REARE. IOK2 > 0, the coefficient is given in the input data
IOP1	Print option IOP1 = 1, the program prints the input data IOP1 = 0, the program does not print the input data
IOP2	Time step option IOP2 = 0, the data time step is equal to the calculation time step. IOP2 = 1, the time steps are not equal
ITRS	The number of water quality parameters that will be simulated
KC1(I)	The constant rate of first order decay at section I

FORTRAN Variable	Description
KC3(I)	The coefficient K_3 for each section I
KTO(I)	Reaeration coefficient at section I in time t
KT1(I)	Reaeration coefficient at section I in time $t + \Delta t$
LD	Input data option
LDT2	Specifies the spacing of the time step to be printed
LOT	Prints sections option
	LOT = 0, prints the values of all sections in each time step
	LOT = N, prints only N sections
IQ(I)	The section number where there is a lateral contribution
LRO	Roughness option
	LRO = 0, one roughness coefficient per section
	LRO = 1, table per section
LUNI	Unit option
	LUNI = 1, metric system
	LUNI = 0, English system
NB(I)	The section number of boundary I. If the number is positive the boundary is the level, negative is the discharge, and when it is zero the condition is a rating curve
NBOUN	Number of boundaries
NBS(I)	Boundary option
	NBS > 0, reads the boundary value in each time step
	NBS < 0, linear relationship between C and X

FORTRAN Variable	Description
NCC(I,J)	The section number of the confluence I(K_1 = NCC(I,1) K_2 = NCC(I,2), K_3 = NCC(I,3)
NCONF	Number of confluences
NP(I)	Number of points of the table for section I
NPF(I)	Number of points of the table in section I for the flood area
NPS(I)	The sections number in which the variables should be printed
NPX	Number of points of the rating curve
NQS	Number of sections with lateral contribution
NREAC	Number of reaches
NSUBS(I)	Specifies the type of section NSUBS(I) = 0, boundary section NSUBS(I) = N, confluence section where N is the confluence number NSUBS(I) = -N, reach section where N is the reach number
NST(I,J)	Upstream (J = 1) and downstream (J = 2) sections of the reach I
NT	Number of time steps
NTRS(I)	The code of each substance I that will be simulated 1 - Conservative 2 - First order decay (BOD) 3 - DO
NUD(I)	The number of the section upstream of the boundary I

FORTRAN Variable	Description
NX	Number of sections
QO(I)	Discharge at section I at time t
QT(I)	Discharge values from the rating curve table
QWL(I,J)	The lateral flow for section LQ(I) in the time step J
R(I)	Hydraulic radius at time step t
RR(J,I)	Hydraulic radius, table in section I
RT1(I)	Hydraulic radius at time step $t + \Delta t$
SF(I)	Friction slope at section I at time step I
SFT1(I)	Friction slope at section I at time step $t + \Delta t$
SUBS(I)	The name of each substance I that will be simulated
TA(J,I)	Top width, table at section I
TE	Temperature
TET	Weighting factor θ
XLA(I)	The rate L_a for each section
ZO(I)	The bottom level of the cross section

```
      PROGRAM UNSWQ(INPUT,OUTPUT)
      REAL KC1,KC3,KT1,KT0
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NF(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LO,LRC,CONST,F(10,65),IOP1,ITRS
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,IT ,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /PRINT/CABE(20),CABE1(20),SUBS(3),NPS(65),LOT,LCT2,IOP2,
     1QAUX(200)
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON /CONC/C(65,3),XC(65),C1(65),NTRS(3),IOE
      COMMON /TRSP/NSUBS(65),NSUC(65),NBS(6),NUD(6),TET,TE,IS1,ICK2
      DATA G/9.81/,CONST/1.486/
C
C   CALL INPUT, READS THE RIVER GEOMETRY AND THE HYDRAULIC CONDITIONS
C
      CALL INPUT
      IF(ITRS.LE.0)GO TO 140
C
C   CALL INPUT1 AND MATRIX1 WHEN A WATER QUALITY SIMULATION IS REQUIRED
C
      CALL INPUT1
      CALL MATRIX1
C
C   CALL MATRIX TO ORGANIZE THE COEFFICIENT MATRIX
C
 140  CALL MATRIX
C
C   INITIALIZATION
C
      CSO=G*DT
  150 DO 160 I=1,NX
      DO 160 J=1,2
  160 QL2(I,J)=0.
      IF(NQS.EQ.0)GO TO 171
      DO 170 I=1,NQS
      L=LQ(I)
  170 QL2(L,1)=QWL(1,I)
 171  DO 175 I=1,NX
      IF(DX(I).EQ.0)GO TO 175
      THETA(I)=DT/DX(I)
  175 AFI(I)=0.
      IT=1
      CALL OUTPUT(ILIN,IFOL)
      CALL GEOME
      IF(ITRS.EQ.0)GO TO 184
      IF(IOE.GT.0)GO TO 181
      CALL DISPER(XI)
      GO TO 179
  181 DO 182 I=1,NX
  182 ET1(I)=E(I)
 179  IF(ICK2.GT.0)GO TO 178
      CALL REARE(QO,HO,XI,NX,ZO,IT)
      GO TO 176
  178 DO 177 I=1,NX
```

```
  177 KT1(I)=KT0(I)
  176 IS1=0
      DO 183 J=1,NX
  183 IS1=IS1+IR1(J)+IHIGH1(J)
  184 IS=0
      DO 185 J=1,NUM
  185 IS=IS+IR(J)+IHIGH(J)
      DO 230 IT=2,NT
C
C   TIME STEP LOOP
C
      DO 186 J=1,IS
 186  AA(J)=0.
      IF(IT.LE.2)GO TO 195
      DO 190 J=1,NX
      A(J)=AT1(J)
      R(J)=RT1(J)
      CK(J)=CKT1(J)
      SF(J)=SFT1(J)
  190 CKY(J)=CKYT1(J)
  195 IF(NQS.EQ.0)GO TO 210
      DO 200 I=1,NQS
      J=LQ(I)
  200 QL2(J,2)=QHL(IT,I)
C
C   CALL COEF1 TO CALCULATE THE COEFFICIENT MATRIX AND THE RIGHT HAND
C   SIDE MATRIX
C
  210 CALL COEF1
C
C   CALL SKYLINE TO SOLVE THE SYSTEM OF EQUATIONS
C
      CALL SKYLINE(AA,BB,NUM,XI,IHIGH,IR,IDIAG)
      IF(LOT2.EQ.0)GO TO 212
      IF((IT-1)/LOT2*LOT2  .NE.IT-1)GO TO 213
C
C   CALL OUTPUT TO PRINT THE HYDRAULIC RESULTS
C
  212 CALL OUTPUT(ILIN,IFOL)
  213 IV=0
C
C   CALL GEOME TO CALCULATE THE GEOMETRIC ELEMENTS FOR THE TIME T + AT
C
      CALL GEOME
C
C   IF ITRS.GT.0 CALL WQSIM TO SIMULATE THE WATER QUALITY
C
      IF(ITRS.GT.0)CALL WQSIM
C
C   TRANSFER THE SOLUTICN FCR THE ARRAYS HO AND QO
C
      DO 220 I=1,NX
      IV=IV+1
      HO(I)=XI(IV)
      IV=IV+1
  220 QO(I)=XI(IV)
      IF(NQS.EQ.0)GO TO 230
      DO 225 I=1,NQS
      J=LQ(I)
  225 QL2(J,1)=QL2(J,2)
  230 CONTINUE
      STOP
      END
```

```
      SUBROUTINE INPUT
C     I--------------------------------------------------------------I
C     I   SUBROUTINE INPUT -- READ THE SYSTEM PARAMETERS,            I
C     I BOUNDARY CONDITIONS, INITIAL CONDITIONS. PRINT THE           I
C     I INPUT AS AN OPTION. THIS SUBROUTINE IS CALLED IN THE MAINI
C     I PROGRAM                                                      I
C     I--------------------------------------------------------------I
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LO,LRO,CONST,F(10,65),IOP1,ITRS
      COMMON /TIME/HC(65),OO(65),HQB(100,5),OHL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /PRINT/CABE(20),CABE1(20),SUBS(3),NPS(65),LCT,LCT2,IOP2,
     1QAUX(200)
      DIMENSION REF(5)
C
C   READ THE SYSTEM INFORMATION
C
      READ 1,CABE
      READ 2,NX,NREAC,NCONF,NBOUN,NOS,IOP1,ICONF,LO,LUNI,LRO,LOT,ITRS
      IF(LOT.NE.0)READ 2,(NPS(I),I=1,LOT)
      NPS(LOT+1)=0
      READ 2,((NST(I,J),J=1,2),I=1,NREAC)
      IF(LUNI.EQ.0)CONST=1.
      NCA=NCONF*2
      IF(NCONF.GT.0)READ 2,((NCC(I,J),J=1,3),I=1,NCONF)
      READ 2,(NB(I),I=1,NBOUN)
      READ 3,(REF(I),I=1,NBOUN)
      IF(NQS.GT.0)READ 2,(LQ(I),I=1,NQS)
      READ 8,(BETA(I),I=1,NX)
      READ 9,(ZO(I),I=1,NX)
      IF(LRO.EQ.0)READ  4,(F(1,I),I=1,NX)
      READ 3,(DX(I),I=1,NX)
      IF(NCONF.GT.0)READ 3,(DXC(I),I=1,NCA)
      IF(NCONF.GT.0)READ 3,(ALFA(I),I=1,NCA)
      READ 2,(NP(I),I=1,NX)
      DO 100 I=1,NX
      L=NP(I)
      IF(L.EQ.0)GO TO 90
      READ 2,NS
      READ 4,  (HA(J,I),J=1,L)
      READ 4,  (AR(J,I),J=1,L)
      READ 4,  (RR(J,I),J=1,L)
      READ 4,  (TA(J,I),J=1,L)
      IF(LRO.NE.0)READ  4,(F(J,I),J=1,L)
      GO TO 100
   90 NP(I)=NP(1)
      L=NP(1)
      DO 95 J=1,L
      AR(J,I)=AR(J,1)
      TA(J,I)=TA(J,1)
      RR(J,I)=RR(J,1)
      HA(J,I)=HA(J,1)
   95 CONTINUE
  100 CONTINUE
      READ 2,(NPF(I),I=1,NX)
      DO 110 I=1,NX
      IF(NPF(I).LE.0)GO TO 110
      L=NPF(I)
      READ 4,  (FAF(J,I),J=1,L)
```

```
      READ 4,   (HF(J,I),J=1,L)
  110 CONTINUE
C
C    PRINT SYSTEM INFCRMATION
C
      IF(IOP1.EQ.0) GO TO 140
      PRINT 5
      PRINT 2,NX,NREAC,NCONF,NBOUN,NQS,IOP1,ICONF,LD,LUNI,LRC,LOT,ITRS
      IF(LOT.NE.0)PRINT 2,(NPS(I),I=1,LOT)
      PRINT 2,((NST(I,J),J=1,2),I=1,NREAC)
      IF(NCONF.GT.0)PRINT 2,((NCC(I,J),J=1,3),I=1,NCONF)
      PRINT 2,(NB(I),I=1,NBOUN)
      PRINT 3,(REF(I),I=1,NBOUN)
      IF(NQS.GT.0)PRINT 2,(LO(I),I=1,NQS)
      PRINT 8,(BETA(I),I=1,NX)
      PRINT 9,(ZO(I),I=1,NX)
      IF(LRO.EQ.0)PRINT 4,(F(1,I),I=1,NX)
      PRINT 3,(DX(I),I=1,NX)
      IF(NCONF.GT.0)PRINT 3,(DXC(I),I=1,NCA)
      IF(NCONF.GT.0)PRINT 3,(ALFA(I),I=1,NCA)
      IF(LUNI.EQ.0)GO TO 117
      DO 115 I=1,NX
  115 DX(I)=DX(I)*5280.
      IF(NCONF.EQ.0)GO TO 117
      DO 116 I=1,NCA
  116 DXC(I)=DXC(I)*5280.
  117 PRINT 2,(NP(I),I=1,NX)
      DO 120 I=1,NX
      PRINT 2,I
      L=NP(I)
      PRINT 4,   (AR(J,I),J=1,L)
      PRINT 4,   (TA(J,I),J=1,L)
      PRINT 4,   (RR(J,I),J=1,L)
      PRINT 4,   (HA(J,I),J=1,L)
      IF(LRO.NE.0)PRINT 4,(F(J,I),J=1,L)
  120 CONTINUE
      PRINT 2,(NPF(I),I=1,NX)
      DO 130 I=1,NX
      IF(NPF(I).LE.0)GO TO 130
      L=NPF(I)
      PRINT 4,   (FAF(J,I),J=1,L)
      PRINT 4,   (HF(J,I),J=1,L)
  130 CONTINUE
  140  READ 6,DT,NT,IOP2,DT1,LDT2
C
C    READ THE INFORMATICN ABOUT TIME VARIATION
C
      DTO=DT
C
C    READ INITIAL CONDITION
C
      READ 3,(HO(I),I=1,NX)
      READ 3,(QO(I),I=1,NX)
      IF(IOP1.EQ.0)GO TO 145
      PRINT 6,DT,NT,IOP2,DT1,LDT2
      PRINT 3,(HO(I),I=1,NX)
      PRINT 3,(QO(I),I=1,NX)
      IF(LD.EQ.0)GO TO 145
      DO 142 I=1,NX
  142 HO(I)=HO(I)-ZO(I)
C
```

```
C   READ THE BOUNDARY CONDITIONS
C
  145 DO 150 J=1,NBOUN
      IF(NB(J).EQ.0)GO TO 148
      READ 3,(HQB(I,J),I=1,NT)
      IF(IOP1.EQ.0)GO TO 149
      PRINT 3,(HQB(I,J),I=1,NT)
  149 IF(NB(J).LT.0)GO TO 150
      KJ=NB(J)
      DO 146 I=1,NT
      IF(LD.NE.0)HQB(I,J)=HQB(I,J)-ZO(KJ)
  146 HQB(I,J)=HQB(I,J)+REF(J)
      GO TO 150
  148 READ 2,NV,NPX
      IF(NV.LT.0)GO TO 150
      READ 3,(QT(I),I=1,NPX)
      READ 3,(HT(I),I=1,NPX)
  150 CONTINUE
      IF(NQS.EQ.0)GO TO 170
C
C   READ LATERAL FLOW CONTRIBUTION
C
      DO 160 J=1,NQS
      READ  3,(QWL(I,J),I=1,NT)
      IF (IOP1.EQ.0) GO TO 160
      PRINT 3,(QWL(I,J),I=1,NT)
 160  CONTINUE
  170 IF(IOP2.EQ.0)GC TO 240
      IFIN=DT*NT/DT1
      QAUX(1)=0.
      DO 180  IK=2,NT
  180 QAUX(IK)=QAUX(IK-1)+DT
      DO 200 J=1,NBOUN
      IF(NB(J).NE.0)CALL IDT1(HQB(1,J),QAUX,NT,DT1,IFIN)
  200 CONTINUE
      IF(NQS.EQ.0) GO TO 230
      DO 220 J=1,NQS
  220 CALL IDT1(QWL(1,J),QAUX,NT,DT1,IFIN)
  230 NT=IFIN
      DT=DT1
      RETURN
  240 DT1=DT
      RETURN
   1  FORMAT(20A4)
    2 FORMAT(16I5)
  3   FORMAT(8F10.2)
 4    FORMAT(8F10.3)
   5  FORMAT(≠1≠,/,10X,≠PRINTING INPUT CARDS≠,///)
  6   FORMAT(F10.0,2I10,F10.0,I10)
    8 FORMAT(16F5.2)
   9  FORMAT(10F8.2)
      END


      SUBROUTINE IDT1(Q,QAUX,NT,DT1,IFIN)
C     I------------------------------------------------------------I
C     I   SUBROUTINE IDT1 IS USED TO CALL FINT TO INTERPCLATE      I
C     I THE BOUNDARY AND LATERAL VALUES WHEN THE TIME STEP OF THE I
C     I DATA IS DIFFERENT FRCM THE TIME STEP OF CALCULATION. IT    I
C     I IS CALLED IN INPUT AND INPUT1                              I
C     I------------------------------------------------------------I
      DIMENSION Q(1),QAUX(1),QAUX2(200)
      ACON=0.
      DO 210 K=1,NT
  210 QAUX2(K)=Q(K)
      DO 220 IA=2,IFIN
      ACON=ACON+DT1
  220 Q(IA)=FINT(QAUX,QAUX2,NT,ACON)
      RETURN
      END
```

```
      SUBROUTINE  INPUT1
C     I-----------------------------------------------------------I
C     I   SUBROUTINE INPUT1 IS USED TO READ THE WATER QUALITY     I
C     I COEFFICIENTS, THE INITIAL AND BOUNDARY CONDITIONS FOR THE I
C     I TRANSPORT EQUATION AND EACH PARAMETER WHICH WILL BE       I
C     I SIMULATED                                                 I
C     I-----------------------------------------------------------I
      REAL KC1,KC3,KT1,KT0
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),Z0(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LR0,CONST,F(10,65),IOP1,ITRS
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON /TIME/H0(65),Q0(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DT0,CS0,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /TRSP/NSUBS(65),NSUC(65),NBS(6),NUD(6),TET,TE,IS1,IOK2
      COMMON /PRINT/CABE(20),CABE1(20),SUBS(3),NPS(65),LOT,LCT2,IOP2,
     1QAUX(200)
      COMMON /CONC/C(65,3),XC(65),C1(65),NTRS(3),IOE
C
C   READ THE WATER QUALITY COEFFICIENTS
C
      READ 1,CABE1
      READ 2,TET,TE,IOE,IOK2,CS
      READ 3,(SUBS(I),I=1,ITRS)
      READ 4,(NTRS(I),I=1,ITRS)
      READ 4,(NBS(I),I=1,NBOUN)
      READ 4,(NUD(I),I=1,NBOUN)
      READ 4,(NSUBS(I),I=1,NX)
      READ 4,(NSUC(I),I=1,NX)
      READ 7,(KC1(I),I=1,NX)
      READ 7,(KC3(I),I=1,NX)
      READ 7,(XLA(I),I=1,NX)
      READ 7,(DB(I),I=1,NX)
      IF(IOK2.GT.0)READ 7,(KT0(I),I=1,NX)
      IF(IOE.GT.0)READ 7,(E(I),I=1,NX)
C
C   READ INITIAL CONDITION FOR EACH PARAMETER
C
      DO 90 J=1,ITRS
   90 READ 7,(C(I,J),I=1,NX)
      NTA=DT1*NT/DT0
C
C   READ BOUNDARY CONDITION FOR EACH PARAMETER
C
      DO 100 I=1,NBOUN
      IF(NBS(I).LE.0)GO TO 100
      DO 95 J=1,ITRS
   95 READ 7,(CB(IT,I,J),IT=1,NTA)
  100 CONTINUE
C
C   READ LATERAL INPUT FOR EACH PARAMETER
C
      IF(NQS.EQ.0)GO TO 109
      DO 105 J=1,ITRS
      DO 105 I=1,NQS
  105 READ 7,(CL(K,I,J),K=1,NTA)
  109 IF(IOP1.EQ.0)GO TO 140
      PRINT 1,CABE1
```

```
      PRINT 2,TET,TE,ICE,IOK2,CS
      PRINT 3,(SUBS(I),I=1,ITRS)
      PRINT 4,(NTRS(I),I=1,ITRS)
      PRINT 4,(NBS(I),I=1,NBOUN)
      PRINT 4,(NUD(I),I=1,NBOUN)
      PRINT 4,(NSUBS(I),I=1,NX)
      PRINT 4,(NSUC(I),I=1,NX)
      PRINT 7,(KC1(I),I=1,NX)
      PRINT 7,(KC3(I),I=1,NX)
      PRINT 7,(XLA(I),I=1,NX)
      PRINT 7,(DB(I),I=1,NX)
      IF(IOK2.GT.0)PRINT 7,(KT0(I),I=1,NX)
      IF(IOE.GT.0)PRINT 7,(E(I),I=1,NX)
C
C   PRINT INITIAL CONDITION FOR EACH PARAMETER
C
      DO 110 J=1,ITRS
  110 PRINT 7,(C(I,J),I=1,NX)
C
C   PRINT BOUNDARY CONDITIONS FOR EACH PARAMETER
C
      DO 120 I=1,NBOUN
C
      IF(NBS(I).LE.0)GO TO 120
      DO 115 J=1,ITRS
  115 PRINT 7,(CB(IT,I,J),IT=1,NTA)
  120 CONTINUE
      IF(NQS.EQ.0)GO TO 140
C
C   PRINT LATERAL INPUT FOR EACH PARAMETER
C
      DO 130 J=1,ITRS
      DO 130 I=1,NQS
  130 PRINT 7,(CL(K,I,J),K=1,NTA)
 140  IF(IOP2.EQ.0)RETURN
C
C   INTERPOLATION WHEN THE TIME STEPS ARE NOT EQUAL
C
      DO 150 K=1,ITRS
      DO 150 J=1,NBOUN
      IF(NBS(J).GT.0)CALL IDT1(CB(1,J,K),QAUX,NTA,DT1,NT)
  150 CONTINUE
      IF(NQS.EQ.0)RETURN
      DO 160 J=1,ITRS
      DO 160 I=1,NQS
  160 CALL IDT1(CL(1,I,J),QAUX,NTA,DT1,NT)
  1   FORMAT(20A4)
  2   FORMAT(2F10.2,2I10,F10.2)
   3  FORMAT(10A8)
   4  FORMAT(16I5)
  7   FORMAT(8F10.2)
      RETURN
      END
```

```
      FUNCTION FINT(X,Y,N,ABC)
C     I-------------------------------------------------------I
C     I   FUNCTION FINT - USED TO INTERPOLATE A VALUE IN A TABEL I
C     I IT IS CALLED IN THE MAIN PROGRAM AND IN THE SUBROUTINE  I
C     I COEF1                                                   I
C     I-------------------------------------------------------I
      DIMENSION X(1),Y(1)
      NM1=N-1
      DO 10 I=2,NM1
      IF(ABC-X(I))20,20,10
 10   CONTINUE
      J=N-1
      I=N
      GO TO 30
  20 J=I-1
  30 FINT=Y(J)+(Y(I)-Y(J))*(ABC-X(J))/(X(I)-X(J))
      RETURN
      END
```

```
      SUBROUTINE OUTPUT(ILIN,IFOL)
C     I--------------------------------------------------------------I
C     I   SUBROUTINE OUTPUT - PRINT THE LEVEL, LEVEL VARIATION INI
C     I TIME, DISCHARGE, AND DISCHARGE VARIATION, IN EACH TIME  I
C     I STEP. THIS SUBROUTINE IS CALLED IN THE MAIN PROGRAM     I
C     I--------------------------------------------------------------I
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRO,CONST,F(10,65),IOP1,ITRS
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,IT ,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /PRINT/CABE(20),CABE1(20),SUBS(3),NPS(65),LOT,LDT2,IOP2,QAUX
     1QAUX(200)
      IF(IT.GT.1)GO TO 100
      IFOL=1
      ILIN=0
  100 IV=0
      NPP=1
C
C   LOOP FOR SECTION VARIATICN
C
      DO 150 I=1,NX
      HZ=HO(I)+ZO(I)
      IF(IT.EQ.1)GO TO 105
      IV=IV+1
      HV=XI(IV   )-HO(I)
      HZ=HZ+HV
      IV=IV+1
      QV=XI(IV   )-QO(I)
  105 IF(LOT.EQ.0)GO TO 101
      IF(NPS(NPP).NE.I)GO TO 150
      NPP=NPP+1
  101 IF(ILIN.GT.0)GO TO 110
C
C   PRINT TITLE IN THE BEGINING OF EACH PAGE
C
      IFOL=IFOL+1
      ILIN=52
      PRINT 300,IFOL
      PRINT 301,CABE
      PRINT 302,DTO,DT1
      PRINT 303
      PRINT 304
      PRINT 303
  110 IF(ILIN.LT.52.AND.I.NE.1)GO TO 120
      PRINT 310,IT
      ILIN=ILIN-2
  120 IF(IT.EQ.1)GO TO 130
C
C   PRINT LEVEL, DEPTH VARIATION, DISCHARGE AND DISCHARGE VARIATION
C
      PRINT 311,I,HO(I),HV,XI(IV-1),HZ,I,QO(I),QV,XI(IV)
      GO TO 140
  130 PRINT 314,I,HO(I),HZ,I,QO(I)
  140 ILIN=ILIN-1
  150 CONTINUE
      RETURN
  300 FORMAT(≠1≠,2X,≠IMPLICIT SCHEME OF UNCOUPLED EQUATIONS≠,40X,≠PAGE≠,
     1I4)
  301 FORMAT(10X,20A4)
  302 FORMAT(10X,≠DT =≠,F5.0,≠ SEC  DT1 =≠,F5.0,≠ SEC≠,/)
  303 FORMAT(1X,9(≠*≠),2X,≠*≠,7(≠-≠),4(≠*≠,10(≠-≠)),≠*≠,7(≠-≠),3(≠*≠,11(
     1≠-≠)),≠*≠)
  304 FORMAT(1X,≠I TIME  I  ISECTIONI  INITIAL I     AY    I  FINAL   I
     1LEVEL   ISECTICNI  INITIAL I     AQ     I  FINAL   I≠,/,1X,≠I STEP
     2I  I          I  DEPTH   I            I  DEPTH   I            I   I O
     3ISCHARGE I            I DISCHARGE I≠)
  311 FORMAT(15X,I3,2X,4(1X,F10.4),3X,I3,2X,3(1X,F11.4))
  310 FORMAT(/,3X,I5)
  314 FORMAT(15X,I3,3X,F10.4,23X,F10.4,3X,I3,2X,F11.4)
      END
```

```
      SUBROUTINE MATRIX
C     I-----------------------------------------------------------I
C     I   SUBROUTINE MATRIX - THIS SUBROUTINE IS USED IN THE      I
C     I BEGINING OF THE EXECUTION TO ORGANIZE THE COEFFICIENTS    I
C     I MATRIX IN THE WAY THAT THE MAIN DIAGONAL HAS NON-ZEROS    I
C     I VALUES. IT IS NEEDED TO SOLVE THE SYSTEM OF EQUATIONS     I
C     I   THIS SUBROUTINE IS CALLED IN THE MAIN PROGRAM           I
C     I-----------------------------------------------------------I
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRC,CONST,F(10,65),IOP1,ITRS
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QHL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,IT ,CB(100,5,3),NPX,CL(100,3,3)
      DIMENSION INC(5)
C
C   STORE IN THE ARRAY ICOL(I,J) THE NUMBER OF THE VARIABLES OF THE
C  BOUNDARY EQUATIONS
C
      DO 60 J=1,NBOUN
      JLIN(J)=J
      IF(NB(J).EQ.0)GO TO 50
      ISU=1
      IF(NB(J).LT.0)ISU=2
      K=(IABS(NB(J))-1)*2+ISU
      ICOL(J,1)=2
      ICOL(J,2)=K
      GO TO 60
   50 ICOL(J,1)=3
      K=(IABS(NV)-1)*2
      ICOL(J,2)=K+1
      ICOL(J,3)=K+2
   60 CONTINUE
      J=NBOUN
C
C   STORE IN THE ARRAY ICOL(I,J) THE NUMBER OF THE VARIABLES OF THE
C  REACH EQUATIONS
C
      DO 80 I=1,NREAC
      CALL SUB(NST(I,1),NST(I,2),J)
   80 CALL SUB(NST(I,1),NST(I,2),J)
      IF(NCONF.EQ.0)GO TO 76
C
C   STORE IN THE ARRAY ICOL(I,J) THE NUMBER OF THE VARIABLES OF
C  CONFLUENCE EQUATIONS
C
      DO 75 I=1,NCONF
      J=J+1
      JLIN(J)=J
      ICOL(J,1)=4
      DO 70 M=1,3
      K=(IABS(NCC(I,M))-1)*2+2
   70 ICOL(J,M+1)=K
      N1=NCC(I,1)
      N2=NCC(I,2)
      N3=IABS(NCC(I,3))
      IF(NCC(I,3).LT.0)GO TO 71
      CALL SUB(N3,N1,J)
```

```
      CALL SUB(N3,N2,J)
      GO TO 75
   71 CALL SUB(N1,N3,J)
      CALL SUB(N2,N3,J)
   75 CONTINUE
C
C   ORGANIZE THE ROWS IN ORDER TO HAVE A NON-ZERO ELEMENT IN THE MAIN
C  DIAGONAL
C
  76  DO 399 I=1,J
  399 ICAUX(I,1)=1
      NUM=J
      DO 120 J=1,NBOUN
      K=ICOL(J,2)
      JLIN(J)=K
      MH=ICOL(J,1)
      DO 100 M=1,MH
  100 ICAUX(K,M)=ICOL(J,M)
      KX=IABS(NB(J))
      NC=1
      IF(NCONF.EQ.0)GO TO 120
      DO 118 I=1,NCONF
      KT=0
      K1=(NCC(I,1)-1)*2+1
      K2=(NCC(I,2)-1)*2+1
      K3=(IABS(NCC(I,3))-1)*2+1
      IF(KX.NE.NCC(I,1))GO TO 105
      IF(NCC(I,3).GT.0)K3=K3+1
      IF(NB(J).GT.0)K1=K1+1
      CALL SUB2(K2+1,K1,K3,I)
      KT=1
  105 IF(KX.NE.NCC(I,2))GO TO 110
      IF(NCC(I,3).GT.0)K3=K3+1
      IF(NB(J).GT.0)K2=K2+1
      CALL SUB2(K1+1,K2,K3,I)
      KT=1
  110 IF(KX.NE.IABS(NCC(I,3)))GO TO 115
      IF(NB(J).GT.0)K3=K3+1
      CALL SUB2(K2+1,K1+1,K3,I)
      KT=1
  115 IF(KT.NE.1)GO TO 118
      INC(NC)=I
      NC=NC+1
  118  CONTINUE
  120 CONTINUE
      INC(NC)=0
      J=NBOUN
      DO 170 I=1,NREAC
      K1=(NST(I,1)-1)*2+2
      K2=(NST(I,2)-1)*2+1
      J=J+1
      IF(ICAUX(K1,1).GT.1)K1=K1-1
      JLIN(J)=K1
      DO 150 M=1,5
  150 ICAUX(K1,M)=ICOL(J,M)
      J=J+1
      IF(ICAUX(K2,1).GT.1)K2=K2+1
      JLIN(J)=K2
      DO 160 M=1,5
  160 ICAUX(K2,M)=ICOL(J,M)
  170 CONTINUE
```

```
      NC=1
      IF(NCONF.EQ.0)GO TO 205
      DO 200 I=1,NCONF
      IF(INC(NC).EQ.I)GO TO 195
      K1=(NCC(I,1)-1)*2+1
      K2=(NCC(I,2)-1)*2+1
      K3=(IABS(NCC(I,3))-1)*2+1
      IF(NCC(I,3).LT.0)GO TO 190
      CALL SUB2(K3+1,K1,K2,I)
      GO TO 200
  190 CALL SUB2(K1+1,K3,K2+1,I)
      GO TO 200
  195 NC=NC+1
  200 CONTINUE
  205 DO 210 I=1,NUM
      L1=ICAUX(I,1)
      DO 210 M=1,L1
  210 ICOL(I,M)=ICAUX(I,M)
      CALL ARRAY(IR,IHIGH,IDIAG,NUM)
      RETURN
      END
```

```
      SUBROUTINE ARRAY(IR,IHIGH,IDIAG,NUM)
C     I-----------------------------------------------------------I
C     I   SUBROUTINE ARRAY COMPUTES THE POSITION OF EACH MATRIX   I
C     I ELEMENT IN THE ONE-DIMENSIONAL ARRAY AA(I) AND STORE THIS I
C     I POSITION IN THE ARRAY ICOL(I,J).  THIS SUBROUTINE COMPUTESI
C     I ALSO THE VALUES FOR THE ARRAYS IR(J), IHIGH(J),IDIAG(J),  I
C     I IDIAG(J)  USED IN SKYLINE. IT IS CALLED IN MATRIX         I
C     I-----------------------------------------------------------I
      COMMON /MAT/ICCL(130,5),JCX(65,5),IHH(130),IRR(130),IDD(130),IHIGH
     11(65),IR1(65),IDIAG1(65),NXM,JLIN(130),ICAUX(130,5),XI(130),AA(
     22000),BB(130),AK(1000),BK(65)
      DIMENSION IR(1),IHIGH(1),IDIAG(1)
      IDIAG(1)=1
      DO 220 K=1,NUM
      IR(K)=0
  220 IHIGH(K)=1
      N1=0
      DO 260 K=1,NUM
      M=ICOL(K,1)
      N1=N1+1
      LM=0
      IDIAG(K)=N1
      DO 250 L=2,M
      K1=ICOL(K,L)
      L1=K1-K
      IF(L1)230,250,240
  230 IF(LM.GT.L1)LM=L1
      IR(K)=IABS(LM)
      GO TO 250
  240 IF(IHIGH(K1).LT.L1+1)IHIGH(K1)=L1+1
  250 CONTINUE
      N1=N1+IHIGH(K)-1+IR(K)
  260 CONTINUE
      DO 310 K=1,NUM
      L=ICOL(K,1)
      DO 300 JJ=2,L
      JDIF=ICOL(K,JJ)-K
      IF(JDIF)270,280,280
  270 JAU =IDIAG(K)+IHIGH(K)-1+IABS(JDIF)
      GO TO 290
  280 KT=JDIF+K
      JAU =IDIAG(KT)+JDIF
  290 ICOL(K,JJ)=JAU
  300 CONTINUE
  310 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SUB2(K1,K2,K3,I)
C     I-----------------------------------------------------------
C     I  THIS SUBROUTINE IS USED TO TRANSFER ONE ROW OF THE       I
C     I ARRAY ICOL TO ICAUX                                       I
C     I-----------------------------------------------------------I
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRC,CONST,F(10,65),IOP1,ITRS
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      DIMENSION K(3)
      K(1)=K1
      K(2)=K2
      K(3)=K3
      KJ=NBOUN+NREAC*2+(I-1)*3
      DO 100 J=1,3
      KJ=KJ+1
      KK=K(J)
      JLIN(KJ)=KK
      MH=ICOL(KJ,1)
      DO 100 M=1,MH
  100 ICAUX(KK,M)=ICCL(KJ,M)
      RETURN
      END
```

```
      SUBROUTINE SUB(N1,N2,J)
C     I-------------------------------------------------------I
C     I   SUBROUTINE SUB - THIS IS AN AUXILIARY SUBROUTINE USED BY I
C     I THE SUBROUTINE MATRIX TO GIVE THE COLUMN POSITION OF    I
C     I THE  COEFFICIENTS IN THE MATRIX                         I
C     I-------------------------------------------------------I
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      J=J+1
      JLIN(J)=J
      ICOL(J,1)=5
      M=2
      NUP=N1
   80 K=(NUP-1)*2+1
      ICOL(J,M)=K
      ICOL(J,M+1)=K+1
      IF(M.EQ.4)RETURN
      NUP=N2
      M=M+2
      GO TO 80
      END
```

```
      SUBROUTINE MATRIX1
C     I-----------------------------------------------------------------I
C     I   SUBROUTINE MATRIX1 IS USED TO ORGANIZE THE COEFFICIENT  I
C     I MATRIX RESULTED FROM THE USE OF THE NUMERICAL SCHEME IN   I
C     I THE TRANSPORT EQUATION. IT IS CALLED IN THE MAIN PROGRAM  I
C     I-----------------------------------------------------------------I
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NFF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRC,CONST,F(10,65),IOP1,ITRS
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TRSP/NSUBS(65),NSUC(65),NBS(6),NUD(6),TET,TE,IS1,IOK2
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      DO 8 I=1,NBOUN
      K=IABS(NB(I))
      IF(K.EQ.0)K=IABS(NV)
      IF(NBS(I).LT.0)GO TO 5
      ICOL(K,1)=2
      ICOL(K,2)=K
      GO TO 8
    5 ICOL(K,1)=4
      ICOL(K,4)=K
      IF(IABS(NBS(I)).NE.1)GO TO 6
      ICOL(K,2)=NUD(I)
      ICOL(K,3)=NSUC(K)
      GO TO 8
    6 KI=NUD(I)
      ICOL(K,2)=NCC(KI,1)
      ICOL(K,3)=NCC(KI,2)
    8 CONTINUE
      DO 50 I=1,NX
      KI=NSUBS(I)
      IF(KI)10,50,20
   10 ICOL(I,1)=4
      KI=IABS(KI)
      ICOL(I,2)=NST(KI,1)
      ICOL(I,3)=I
      ICOL(I,4)=NSUC(I)
      GO TO 50
   20 IF(I.EQ.IABS(NCC(KI,3)))GO TO 40
      ICOL(I,1)=4
      ICOL(I,2)=NSUC(I)
      IF(NSUC(I).LT.0)ICOL(I,2)=IABS(NSUC(I))
      ICOL(I,3)=I
      ICOL(I,4)=IABS(NCC(KI,3))
      GO TO 50
   40 ICOL(I,1)=5
      ICOL(I,2)=NSUC(I)
      ICOL(I,3)=I
      ICOL(I,4)=NCC(KI,1)
      ICOL(I,5)=NCC(KI,2)
   50 CONTINUE
      CALL ARRAY(IR1,IHIGH1,IDIAG1,NX)
      DO 60 I=1,NX
      M=ICOL(I,1)
      DO 60 JK=1,M
   60 JCX(I,JK)=ICOL(I,JK)
      RETURN
      END
```

```
      SUBROUTINE SKYLINE(A,BB,NUM,X, IHIGH,IR,IDIAG)
C     I----------------------------------------------------------I
C     I   SUBROUTINE SKYLINE - SUBROUTINE USED TO SOLVE THE SYS-I
C     ITEM OS EQUATIONS BY THE GAUSS ELIMINATION PROCEDURE.      I
C     I THIS SUBROUTINE IS CALLED IN THE MAIN PROGRAM            I
C     I----------------------------------------------------------I
      DIMENSION A(1),BB(1),X(1),IHIGH(1),IR(1),IDIAG(1)
C
C   J IS THE INDEX LOOP OF THE COLUMN
C
      NN1=NUM-1
      DO 500 J=1,NN1
      L1=IDIAG(J)
      M=J+1
      DO 500 JJ=M,NUM
      JAUX=JJ-J
      IF(IR(JJ).LT.JAUX)GO TO 500
      JAUX=IDIAG(JJ)+IHIGH(JJ)-1+JAUX
      IF(A(JAUX).EQ.0)GO TO 500
      COEF=A(JAUX)/A(L1)
      BB(JJ)=BB(JJ)-BB(J)*COEF
C
C   JJ IS THE LOOP INDEX OF THE ROWS BELOW THE DIAGONAL ELEMENTS
C
      DO 400 JM=M,NUM
      JB=JM-J+1
      IF(JB.GT.IHIGH(JM))GO TO 400
      J3=IDIAG(JM)+JB-1
      IF(JM.LT.JJ)GO TO 300
      J1=JM-JJ+1
      J2=IDIAG(JM)+J1-1
  250 A(J2)=A(J2)-A(J3)*COEF
      GO TO 400
  300 J1=JJ-JM
      J2=IDIAG(JJ)+IHIGH(JJ)-1+J1
      GO TO 250
  400 CONTINUE
  500 CONTINUE
C
C  AT THIS POINT THE COEFFICIENT MATRIX IS A UPPER TRIANGULE MATRIX
C  AND STARTS THE BACKWARDS SUBSTITUTION
      L=IDIAG(NUM)
      X(NUM)=BB(NUM)/A(L)
      DO 700 J=2,NUM
      K=NUM-J+1
      K1=K+1
      DO 600 M=K1,NUM
      JAUX=M-K+1
      IF(JAUX.GT.IHIGH(M))GO TO 600
      J1=IDIAG(M)+JAUX-1
      BB(K)=BB(K)-A(J1)*X(M)
  600 CONTINUE
      J1=IDIAG(K)
      X(K)=BB(K)/A(J1)
  700 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GEOME
C     I-------------------------------------------------------------I
C     I SUBROUTINE GEOME COMPUTES THE GEOMETRIC PROPERTIES OF THE I
C     I CROSS SECTIONS. IT IS CALLED IN THE MAIN PROGRAM           I
C     I-------------------------------------------------------------I
      REAL KC1,KC3,KT1,KT0
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DT0,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRO,CONST,FF(10,65),IOP1,ITRS
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      DO 100 I=1,NX
      L=NP(I)
      IF(ITT.EQ.1)GO TO 70
      IKT=(I-1)*2+1
      QQ=XI(IKT+1)
      H1=XI(IKT)
      GO TO 75
   70 H1=HO(I)
      QQ=QO(I)
   75 IF(LD.NE.0)H1=H1+ZO(I)
C
C   TABLES INTERPOLATION
C
      T(I)=FINT(HA(1,I),TA(1,I),L,H1)
      AREA=FINT(HA(1,I),AR(1,I),L,H1)
      RAIO=FINT(HA(1,I),RR(1,I),L,H1)
      IF(LRO.EQ.0)GO TO 80
      F    =FINT(HA(1,I),FF(1,I),L,H1)
      GO TO 85
   80 F    =FF(1,I)
      DF=0.
   85 IF(NPF(I).EQ.0)GO TO 90
      AFI(I)=FINT(HF(1,I),FAF(1,I),NPF(I),H1)
   90 CONV =(RAIO**(2./3.)*AREA*CONST/F)
      SLOPE=QQ*ABS(QQ)/CONV**2
      HAUX =H1+.01
      RAUX=FINT(HA(1,I),RR(1,I),L,HAUX)
      AUX=FINT(HA(1,I),AR(1,I),L,HAUX)
      DA=(AUX-AREA)/.01
      IF(LRO.NE.0)DF=(FINT(HA(1,I),FF(1,I),L,HAUX)-F)/.01
      DR=(RAUX-RAIO)/.01
      DCONV=CONV*(DA/AREA+(2./3.)*DR/RAIO-DF/F)
C
C   STORE THE CALCULATED VALUES IN THE ARRAYS
C
      IF(ITT.EQ.1)GO TO 95
      AT1(I)=AREA
      RT1(I)=RAIO
      CKT1(I)=CONV
      SFT1(I)=SLOPE
      CKYT1(I)=DCONV
      GO TO 100
   95 A(I)=AREA
      R(I)=RAIO
      CK(I)=CONV
      SF(I)=SLOPE
      CKY(I)=DCONV
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COEF1
      REAL KC1,KC3,KT1,KT0
C     I----------------------------------------------------------I
C     I   SUBROUTINE COEF1 - CALCULATE THE COEFFICIENTS OF ALL  I
C     I EQUATIONS, IT IS CALLED IN THE MAIN PROGRAM IN EACH     I
C     I TIME STEP                                               I
C     I----------------------------------------------------------I
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),Z0(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRO,CONST,F(10,65),IOP1,ITRS
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TIME/H0(65),Q0(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DT0,CS0,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
C
C   BOUNDARY EQUATIONS
C
      DO 101 J=1,NBOUN
      K=JLIN(J)
C
C   SPECIFICATION OF THE BOUNDARIES
C
      IF(NB(J).EQ.0)GO TO 102
      BB(K)=HQB(ITT,J)
      K=ICOL(K,2)
      AA(K)=1.
      GO TO 101
C
C   RATING CURVE CONDITION
C
  102 L1=ICOL(K,2)
      L2=ICOL(K,3)
      NS=IABS(NV)
      AA(L2)=1.
      H1=H0(NS)
      IF(LD.NE.0)H1=H1+Z0(NS)
      IF(NV.LT.0)GO TO 103
      DF=FINT(HT,QT,NPX,HAUX)
      HAUX=H1+.01
      DF1=FINT(HT,QT,NV,HAUX)
      DF1= FINT(HT,QT,NPX,HAUX)
      DF1=(DF1-DF)/.01
      AA(L1)=-DF1
      BB(K)=DF-DF1*H0(NS)
      GO TO 101
  103 L=NP(NS)
      H1=H1+.01
      R2=FINT(HA(1,NS),RR(1,NS),L,H1)
      DR=(R2-R(NS))/.01
      COC=2/3.*DR/R(NS)+T(NS)/A(NS)
      AA(L1)=-Q0(NS)*COC
      BB(K)=Q0(NS)*(1.-H0(NS)*COC)
  101 CONTINUE
C
C   REACH EQUATION
C
```

```
      J1=NBOUN+1
      DO 110 I=1,NREAC
      J2=J1+(I-1)*2 +1
      M=NST(I,1)
      J=NST(I,2)
      CS13=HO(M)+HO(J)
      CS1=T(M)+T(J)+(AFI(M)+AFI(J))/DX(M)
      K=JLIN(J2)
      AXY=(T(J)-T(M))/DX(M)
      L2=ICOL(K,2)
      L3=ICOL(K,3)
      L4=ICOL(K,4)
      L5=ICOL(K,5)
C
C   CONTINUITY EQUATION
C
      AA(L3)=-4.*THETA(M)
      AA(L5)=-AA(L3)
      AA(L2)=CS1
      AA(L4)=CS1
      BB(K)=THETA(M)*2*(QL2(J,1)+QL2(J,2))+CS1*CS13
      V1=QO(M)/A(M)
      V2=QO(J)/A(J)
      CS1=THETA(M)*(V1+V2)
      CS2=THETA(M)*(BETA(M)*V1+BETA(J)*V2)
      CS3=THETA(M)*(BETA(M)*V1*V1*T(M)+BETA(J)*V2*V2*T(J))
      CS4=THETA(M)*G*(A(M)+A(J))
      CS5=2.*CSO*A(M)*SF(M)/CK(M)
      CS6=2.*CSO*A(J)*SF(J)/CK(J)
      CS7=0.
      CS8=THETA(M)*G*(A(M)+A(J))
      CS9=SF(M)/V1*CSO
      CS10=SF(J)/V2*CSO
      CS11=SF(M)*T(M)*CSO
      CS12=SF(J)*T(J)*CSO
      T1=1.+2.*CS9
      J2=J2-1
      K=JLIN(J2)
      L2=ICOL(K,2)
      L3=ICOL(K,3)
      L4=ICOL(K,4)
      L5=ICOL(K,5)
C
C   MOMENTUM EQUATION
C
      AA(L3)=T1-CS2-CS1*BETA(M)
      AA(L3)=T1-CS2-CS1*THETA(M)
      T1=CS4+CS5*CKY(M)-CS11
      AA(L2 )=CS3-T1+CS7
      T1=1.+2.*CS10
      AA(L5 )=T1+CS2+CS1*BETA(J)
      T1=CS4-CS6*CKY(J)+CS12
      AA(L4 )=-CS3+T1+CS7
      T1=QO(M)*(1.+CS9)
      T2=QO(J)*(1.+CS10)
      T3=HO(M)*(CS11-CS5*CKY(M))
      T4=HO(J)*(CS12-CS6*CKY(J))
      T5=CS7*CS13
      T6=DT*(BETA(M)*V1*V1+BETA(J)*V2*V2)*AXY
      BB(K)=T1+T2+T3+T4+T5+T6+CS8*(ZO(M)-ZO(J))
  110 CONTINUE
```

```
      IF(NCONF.EQ.0)RETURN
C
C   CONFLUENCE EQUATION, CALL COEF2
C
      J1=NBOUN+NREAC*2+1
      DO 120 I=1,NCONF
      J2=J1+(I-1)*3
      K=JLIN(J2)
      KM=ICOL(K,2)
      AA(KM)=1.
      KM=ICOL(K,3)
      AA(KM)=1.
      KM=ICOL(K,4)
      AA(KM)=-1.
      BB(K)=0.
      JA=(I-1)*2+1
      K1=NCC(I,1)
      K2=NCC(I,2)
      K3=IABS(NCC(I,3))
      IT=1
      IF(NCC(I,3).LT.0)GO TO 116
      CALL COEF2(K3,K1,IT,DXC(JA),ALFA(JA))
      CALL COEF2(K3,K2,IT,DXC(JA+1),ALFA(JA+1))
      GO TO 120
  116 CALL COEF2(K1,K3,IT,DXC(JA),ALFA(JA))
      IT=0
      CALL COEF2(K2,K3,IT,DXC(JA+1),ALFA(JA+1))
  120 CONTINUE
      RETURN
       END
```

```
      SUBROUTINE COEF2(K1,K3,IT,DX1,ALF)
C     I-----------------------------------------------------------I
C     I   SUBROUTINE COEF2 -  CALCULATE THE COEFFICIENTS  OF     I
C     I THE BIFURCATIONS EQUATIONS. THIS SUBROUTINE IS CALLED IN I
C     I THE SUBROUTINE COEF1                                     I
C     I-----------------------------------------------------------I
      REAL KC1,KC3,KT1,KT0
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QHL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRO,CONST,F(10,65),IOP1,ITRS
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      J2=J2+1
      K=JLIN(J2)
      IF(ICONF.EQ.0)GO TO 115
C
C   WHEN THERE IS NOT LOSS OF ENERGY AT THE CONFLUENCE
C
      P14=0.
      P15=1.
      P17=0.
      P18=-1.
      P20=ZO(K3)-ZO(K1)
      GO TO 118
  115 IF(IT.EQ.0)GO TO 117
C
C   WITH LOSS OF ENERGY AT THE CONFLUENCE
C
      V3=QO(K3)/A(K3)
      P5=V3*V3*T(K3)/A(K3)/G
      P8=SF(K3)/CK(K3)
      P11=HO(K3)+ZO(K3)
      P12=V3/A(K3)/G
      P13=SF(K3)/QO(K3)
      P1P=V3*V3/2./G
  117 V1=QO(K1)/A(K1)
      P3=V1*V1*T(K1)/A(K1)/G
      P6=DX1    *SF(K1)/CK(K1)
      P9=HO(K1)+ZO(K1)+V1*V1/2./G
      P14=V1/A(K1)/G-DX1    *SF(K1)/QO(K1)
      P15=1.-P3+P6*CKY(K1)
      P17=-ALF*P12-DX1*P13
      P18=ALF*P5-1.+DX1*P8*CKY(K3)
      P20=-P9+P11+ALF*P1P+DX1/2.*(SF(K1)+SF(K3))
      P20  =P14*QO(K1)+P15*HO(K1)+P17*QO(K3)+P18*HO(K3)+P20
  118 L2=ICOL(K,2)
      L3=ICOL(K,3)
      L4=ICOL(K,4)
      L5=ICOL(K,5)
      AA(L3 )=P14
      AA(L2 )=P15
      AA(L5 )=P17
      AA(L4 )=P18
      BB(K)=P20
      RETURN
       END
```

```
      SUBROUTINE WQSIM
C     I-------------------------------------------------------------I
C     I SUBROUTINE WQSIM IS CALLED IN EACH TIME STEP TO SOLVE THE I
C     I TRANSPORT EQUATION FOR AS MANY PARAMETER AS IS REQUIRED . I
C     I IT IS CALLED IN THE MAIN PROGRAM                           I
C     I-------------------------------------------------------------I
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LO,LRC,CONST,F(10,65),IOP1,ITRS
      COMMON /MAT/ICCL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TRSP/NSUBS(65),NSUC(65),NBS(6),NUD(6),TET,TE,IS1,IOK2
      COMMON /CONC/ C(65,3),XC(65),C1(65),NTRS(3),IOE
C
C   INITIALIZATION
C
      DO 90 J=1,IS1
      AA(J)=0.
   90 AK(J)=0.
      IF(IOE.EQ.0)CALL DISPER(XI)
C
C   LOOP FOR EACH PARAMETER
C
      DO 100 I=1,ITRS
      K=NTRS(I)
      IF(I.EQ.1)GO TO 30
C
C   CALL RHS TO COMPUTE THE RIGHT HAND SIDE MATRIX
C
      CALL RHS(I,C(1,I))
      DO 20 J=1,IS1
   20 AA(J)=AK(J)
      GO TO 35
C
C   CALL TRANSP TO COMPUTE THE COEFFICIENT MATRIX AND THE RIGHT HAND
C  SIDE MATRIX
C
   30 CALL TRANSP(I,C(1,I))
 35   CONTINUE
      GO TO (50,40,45),K
C
C   CALL BODOD FOR BOD SIMULATION
C
   40 CALL BODOD(K,C(1,I),C1,C1,I)
      GO TO 50
C
C   CALL BODOD FOR DO SIMULATION
C
   45 CALL BODOD(K,C(1,I),C(1,I-1),C1,I)
   50 CONTINUE
C
C   CALL SKYLINE TO SOLVE THE SYSTEM OF EQUATIONS
C
      CALL SKYLINE(AA,BB,NX,XC,IHIGH1,IR1,IDIAG1)
C
C   CALL OUT TO PRINT THE SOLUTION
C
      CALL OUT(I,XC)
      DO 60 J=1,NX
      C1(J)=C(J,I)
   60 C(J,I)=XC(J)
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BODOD(K1,C,C1,C2,K2)
C     I----------------------------------------------------------------I
C     I SUBROUTINE BODOD COMPUTES THE SOURCE AND SINK TERM FOR        I
C     I THE BIOCHEMICAL OXYGEN DEMAND (BOD) AND DISSOLVED OXYGEN      I
C     I (DO) . IT IS CALLED IN WQSIM                                  I
C     I----------------------------------------------------------------I
      REAL KC1,KC3,KT1,KT0
      COMMON /TRSP/NSUBS(65),NSUC(65),NBS(6),NUD(6),TET,TE,IS1,IOK2
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XI(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRO,CONST,F(10,65),IOP1,ITRS
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QHL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      DIMENSION C(1),C1(1),C2(1)
      IF(K1.EQ.3.AND.IOK2.EQ.0)CALL REARE(QO,HO,XI,NX,ZO,ITT)
      K=0
      KK=1
      DO 100 I=1,NX
      CT=0.
      KI=NSUBS(I)
      IF(KI.EQ.0)GO TO 100
      KM=IDIAG1(I)
      IF(KK.GT.NQS)GO TO 35
      IF(LQ(KK).NE.I)GO TO 35
C
C     LATERAL CONTRIBUTION
C
      CT=(CL(ITT-1,KK,K2)*QL2(I,1)*(1.-TET)+CL(ITT,KK,K2)*TET*QL2(I,2))/
     1DX(I)
      KK=KK+1
   35 IF(K1.EQ.3)GO TO 45
C
C   BOD
C
      CONS=(KC1(I)+KC3(I))/86400.
      AA(KM)=AA(KM)+TET*CONS*AT1(I)
      BB(I)=BB(I)-(1.-TET)*CONS*A(I)*C(I)+XLA(I)*(TET*(AT1(I)-A(I))+A(I)
     1)+CT
      GO TO 100
   45 AA(KM)=AA(KM)+KT1(I)*AT1(I)*TET/86400.
C
C   DO
C
      BB(I)=BB(I)+TET/86400.*(-KC1(I)*C1(I)+KT1(I)*CS)*AT1(I)+(1.-TET)/
     186400.*(-KC1(I)*C2(I)+KT0(I)*(CS-C(I)))*A(I)+CT+DB(I)*(TET*(AT1(I)
     1-A(I))+A(I))
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE REARE(QO,HO,XJ,NX,ZO,ITT)
C     I--------------------------------------------------------------I
C     I   SUBROUTINE REARE COMPUTES THE REAERATION COEFFICIENT       I
C     I BY O' CONNOR AND DOBBINS EQUATION FOR ALL SECTIONS IN        I
C     I EACH TIME STEP                                               I
C     I--------------------------------------------------------------I
      REAL KC1,KC3,KT1,KT0
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON/TRSP/NSUBS(65),NSUC(65),NBS(6),NUD(6),TET,TE,IS1,IOK2
      DIMENSION XJ(1),ZO(1),QO(1),HO(1)
      DM=0.00192*1.04**(TE-20.)
      CS=14.652-.41022*TE+7.991E-3*TE*TE-7.7774E-5*TE*TE*TE
      IF(ITT.EQ.1)GO TO 15
      DO 10 I=1,NX
      KT0(I)=KT1(I)
      M=(I-1)*2+2
      VW=ABS(XJ(M))/AT1(I)/0.3048
      HW=ABS(XJ(M-1))/.3048
   10 KT1(I)=SQRT(DM*VW)/HW**1.5/2.303
      RETURN
   15 DO 20 I=1,NX
      VW=ABS(QO(I))/A(I)/.3048
      HW=(HO(I)-ZO(I))/.3048
      KT1(I)=SQRT(DM*VW)/HW**1.5/2.303
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DISPER(XJ)
C     I--------------------------------------------------------------I
C     I SUBROUTINE DISPER COMPUTES THE LONGITUDINAL DISPERSION       I
C     I COEFFICIENT BY A MODIFIED TAYLOR EQUATION FOR ALL SECTIONS
C     I IN EACH TIME STEP. IT IS CALLED IN WQSIM AND IN THE MAIN    I
C     I PROGRAM                                                      I
C     I--------------------------------------------------------------I
      REAL KT1,KT0,KC1,KC3
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LO,LRO,CONST,F(10,65),IOP1,ITRS
      DIMENSION XJ(1)
      IF(ITT.EQ.1)GO TO 15
      DO 10 I=1,NX
      E(I)=ET1(I)
      M=(I-1)*2+2
      VW=XJ(M)/AT1(I)
      ET1(I)=8.72*F(1,I)*ABS(VW)*RT1(I)**(5./6.)
   10 CONTINUE
      RETURN
   15 DO 20 I=1,NX
      ET1(I)=8.72*F(1,I)*ABS(QO(I)/A(I))*R(I)**(5./6.)
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUT(K,C)
C     I--------------------------------------------------------------I
C     I  SUBROUTINE OUT IS USED TO PRINT THE CONCENTRATION FOR ALLI
C     I SECTIONS IN EACH TIME STEP. IT IS CALLED IN WQSIM          I
C     I--------------------------------------------------------------I
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRO,CONST,F(10,65),IOP1,ITRS
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /PRINT/CABE(20),CABE1(20),SUBS(3),NPS(65),LOT,LOT2,IOP2,
     1QAUX(200)
      DIMENSION C(1)
      IF(ITT.EQ.2)PRINT 1,CABE1
      IF(K.EQ.1)PRINT 2,ITT
      PRINT 3,SUBS(K)
      PRINT 4,(I,C(I),I=1,NX)
 1    FORMAT(10X,20A4)
  2   FORMAT(//,10X,≠TIME STEP≠,I7)
  3   FORMAT(/,10X,≠SIMULATED PARAMETER≠,A8,3X,≠(MG/L)≠)
  4   FORMAT(8(I5,F10.3))
      RETURN
      END
```

```
      SUBROUTINE RHS(J,C)
      REAL KC1,KC3,KT1,KT0
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRO,CONST,F(10,65),IOP1,ITRS
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /MAT/ICOL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XJ(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TRSP/NSUBS(65),NSUC(65),NBS(6),NUD(6),TET,TE,IS1,IOK2
C     I-----------------------------------------------------------------I
C     I   SUBROUTINE RHS COMPUTES THE RIGHT HAND SIDE  MATRIX .         I
C     I IT IS USED WHEN THERE ARE MORE THAN ONE PARAMETER TO BE         I
C     I SIMULATED. IT IS CALLED IN WQSIM.                               I
C     I-----------------------------------------------------------------I
      DIMENSION C(1)
      AET(X1,X2,X3,X4)=(X1*X2+X3*X4)
C
C   BOUNDARY EQUATIONS
C
      DO 10 I=1,NBOUN
      K=IABS(NB(I))
      IF(NB(I).EQ.0)K=IABS(NV)
      IF(NBS(I).LE.0)GO TO 5
      BB(K)=CB(ITT,I,J)
      GO TO 10
 5    IF(IABS(NBS(I)).EQ.1)GO TO 7
      KI=NUD(I)
      MK=(KI-1)*2+1
      AX1=DXC(MK)
      AX2=DXC(MK+1)
      I1=NCC(KI,1)
      I2=NCC(KI,2)
      AX=(AX1+AX2)/2.
      C4=A(I2)*E(I2)*2
      C3=A(I1)*E(I1)*2
      AX=AX*500.
      BB(K)=A(K)*C(K)/DT+(1.-TET)*((-C3*(C(K)-C(I1))/AX1-C4*(C(K)-C(I2))
     1/AX2)/AX-(QO(I1)*(C(K)-C(I1))/AX1+QO(I2)*(C(K)-C(I2))/AX2))
      GO TO 10
  7   BB(K)=0.
  10  CONTINUE
C
C   LOOP OF THE SECTIONS
C
      DO 100 I=1,NX
      KI=NSUBS(I)
      IF(KI.EQ.0)GO TO 100
      K2=NSUC(I)
      IF(K2.LT.0)GO TO 15
      C2=AET(A(I),E(I),A(K2),E(K2))
   15 IF(KI.GT.0)GO TO 20
C
C   REACH EQAUTION
C
      KI=IABS(KI)
```

```
      K1=NST(KI,1)
      DXM=DX(I)+DX(K1)
      BB(I)=A(I)/DT*C(I)+(1-TET)*((C2*(C(K2)-C(I))/DX(I)-AET(A(I),E(I),A
     1(K1),E(K1))*(C(I)-C(K1))/DX(K1))/DXM-(QO(I )*C(I )-QO(K1)*C(K1))/D
     1X(K1))
      GO TO 100
   20 IF(I.EQ.IABS(NCC(KI,3)))GO TO 40
C
C    CONFLUENCE EQUATIONS
C
      M3=(KI-1)*2+2
      AX=DXC(M3)
      IF(NCC(KI,1).EQ.I)AX=DXC(M3-1)
      K3=IABS(NCC(KI,3))
      C3=A(I)*E(I)*2
      IF(NCC(KI,3).LT.0)GO TO 30
      IF(K2.LT.0)GO TO 25
      AX1=AX+DX(I)
      AX2=DX(I)
      GO TO 27
  25  C2=A(I)*E(I)*2
      K2=IABS(K2)
      KAUX=NSUBS(K2)
      MN=(KAUX-1)*2+1
      AX2=DXC(MN)
      IF(NCC(KAUX,1).NE.I )AX2=DXC(MN+1)
      AX1=AX+AX2
   27 BB(I)=A(I)/DT*C(I)+(1.-TET)*((C2*(C(K2)-C(I))/AX2   -C3*(C(I)-C(K3)
     1)/AX)/AX1-QO(I)*(C(I)-C(K3))/AX)
      GO TO 100
   30 AX1=AX+DX(K2)
      BB(I)=A(I)/DT*C(I)+(1.-TET)*((-C2*(C(I)-C(K2))/DX(K2)+C3*(C(K3)-C(
     1I))/AX)/AX1-(QO(I)*C(I)-QO(K2)*C(K2))/DX(K2))
      GO TO 100
   40 MK1=(KI-1)*2+1
      AX=(DXC(MK1)+DXC(MK1+1))/2.
      AX1=DXC(MK1)
      AX2=DXC(MK1+1)
      KK1=NCC(KI,1)
      KK2=NCC(KI,2)
      C3=A(KK1)*E(KK1)*2
      C4=A(KK2)*E(KK2)*2
      IF(NCC(KI,3).LT.0)GO TO 50
      AX=AX+DX(K2)
      BB(I)=A(I)/DT*C(I)+(1.-TET)*(((C(KK1)-C(I))/AX1*C3+(C(KK2)-C(I))/A
     1X2*C4-(C(I)-C(K2))*C2/DX(K2))/AX-(QO(I)*C(I)-QO(K2)*C(K2))/DX(K2))
      GO TO 100
   50 AX=AX+DX(I)
      BB(I)=A(I)*C(I)/DT+(1.-TET)*(((C(K2)-C(I))*C2/DX(I)-C3*(C(I)-C(KK1
     1))/AX1-C4*(C(I)-C(KK2))/AX2)/AX-(QO(KK1)*(C(I)-C(KK1))/AX1+QO(KK2)
     2*(C(I)-C(KK2))/AX2))
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRANSP(J,C)
C     I-------------------------------------------------------------I
C     I    SUBROUTINE TRANSP  COMPUTES THE COEFFICIENT MATRIX AND I
C     I THE RIGHT HAND SIDE MATRIX. IT IS CALLED IN WQSIM          I
C     I-------------------------------------------------------------I
      REAL KT1,KT0,KC1,KC3
      COMMON /PAR1/NX,NREAC,NCONF,NBOUN,NST(40,2),NCC(20,3),NB(5),ZO(65)
     1,DX(65),DXC(65),NP(65),AR(10,65),RR(10,65),HA(10,65),FAF(65,5),HF(
     165,5),LQ(30),NPF(65),TA(10,65),ALFA(65),AFI(65),BETA(65),THETA(65)
     1,NQS,ICONF,LD,LRO,CONST,F(10,65),IOP1,ITRS
      COMMON /PP/V3,P5,P8,P12,P13,P1P,J2,T(65),A(65),R(65),CK(65),CKY(65
     1),SF(65),AT1(65),RT1(65),CKT1(65),CKYT1(65),SFT1(65),E(65),ET1(65)
     2,KT1(65),KT0(65),KC1(65),KC3(65),CS,XLA(65),DB(65)
      COMMON /TIME/HO(65),QO(65),HQB(100,5),QWL(100,2),QL2(65,2),DT,
     1DT1,DTO,CSO,G,NT,QT(30),HT(30),NV,ITT,CB(100,5,3),NPX,CL(100,3,3)
      COMMON /MAT/ICCL(130,5),JCX(65,5),IHIGH(130),IR(130),IDIAG(130),
     1IHIGH1(65),IR1(65),IDIAG1(65),NUM,JLIN(130),ICAUX(130,5),XJ(130),
     2AA(2000),BB(130),AK(1000),BK(65)
      COMMON /TRSP/NSUBS(65),NSUC(65),NBS(6),NUD(6),TET,TE,IS1,ICK2
      DIMENSION C(1)
      AET(X1,X2,X3,X4)=(X1*X2+X3*X4)
C
C   BOUNDARY EQUATIONS
C
      DO 10 I=1,NBOUN
      K=IABS(NB(I))
      IF(NB(I).EQ.0)K=IABS(NV)
      KM=JCX(K,2)
      IF(NBS(I).LE.0)GO TO 5
      AA(KM)=1.
      BB(K)=CB(ITT,I,J)
      GO TO 10
    5 KM2=JCX(K,3)
      KM3=JCX(K,4)
      IF(IABS(NBS(I)).EQ.1)GO TO 7
      KI=NUD(I)
      MK=(KI-1)*2+1
      AX1=DXC(MK)
      AX2=DXC(MK+1)
      I1=NCC(KI,1)
      I2=NCC(KI,2)
      AX=(AX1+AX2)/2.
      C4=A(I2)*E(I2)*2
      C3=A(I1)*E(I1)*2
      AX=AX+500.
      M1=(I1-1)*2+1
      M3=(I2-1)*2+1
      CC4=AT1(I2)*ET1(I2)*2
      CC3=AT1(I1)*ET1(I1)*2
      AA(KM)=-TET/AX1*(XJ(M1)+CC3/AX)
      AA(KM2)=-TET/AX2*(XJ(M3)+CC4/AX)
      AA(KM3)=AT1(K)/DT+TET*((CC3/AX1+CC4/AX2)/AX+XJ(M1)/AX1+XJ(M3)/AX2)
      BB(K)=A(K)*C(K)/DT+(1.-TET)*((-C3*(C(K)-C(I1))/AX1-C4*(C(K)-C(I2))
     1/AX2)/AX-(QO(I1)*(C(K)-C(I1))/AX1+QO(I2)*(C(K)-C(I2))/AX2))
      GO TO 10
    7 AA(KM)=1.
      AA(KM2)=-2.
      AA(KM3)=1.
      BB(K)=0.
   10 CONTINUE
C
```

```
C     SECTIONA LOOP
C
      DO 100 I=1,NX
      KI=NSUBS(I)
      IF(KI.EQ.0)GO TO 100
      KM=JCX(I,2)
      K2=NSUC(I)
      IF(K2.LT.0)GO TO 15
      C2=AET(A(I),E(I),A(K2),E(K2))
      CC2=AET(AT1(I),ET1(I),AT1(K2),ET1(K2))
C
C     REACH EQUATION
C
   15 M2=(K2-1)*2+2
      M=(I-1)*2+2
      IF(KI.GT.0)GO TO 20
      KI=IABS(KI)
      K1=NST(KI,1)
      M1=(K1-1)*2+2
      CC1=AET(AT1(I),ET1(I),AT1(K1),ET1(K1))
      DXM=DX(I)+DX(K1)
      AA(KM)=-TET/DX(K1)*(XJ(M1)+CC1/DXM)
      KM=JCX(I,3)
      AA(KM)=AT1(I)/DT+TET*((CC2/DX(I)+CC1/DX(K1))/DXM+XJ(M)/DX(K1))
      KM=JCX(I,4)
      AA(KM)=-TET*CC2/(DX(I)*DXM)
      BB(I)=A(I)/DT*C(I)+(1-TET)*((C2*(C(K2)-C(I))/DX(I)-AET(A(I),E(I),A
     1(K1),E(K1))*(C(I)-C(K1))/DX(K1))/DXM-(QO(I )*C(I )-QO(K1)*C(K1))/D
     1X(K1))
      GO TO 100
   20 IF(I.EQ.IABS(NCC(KI,3)))GO TO 40
C
C   CONFLUENCE EQUATIONS
C
      K3=IABS(NCC(KI,3))
      M3=(KI-1)*2+2
      AX=DXC(M3)
      IF(NCC(KI,1).EQ.I)AX=DXC(M3-1)
      C3=A(I)*E(I)*2
      CC3=AT1(I)*ET1(I)*2
      IF(NCC(KI,3).LT.0)GO TO 30
      IF(K2.LT.0)GO TO 25
      AX1=AX+DX(I)
      AX2=DX(I)
      GO TO 27
  25  C2=A(I)*E(I)*2
      CC2=AT1(I)*ET1(I)*2
      K2=IABS(K2)
      KAUX=NSUBS(K2)
      MN=(KAUX-1)*2+1
      AX2=DXC(MN)
      IF(NCC(KAUX,1).NE.I )AX2=DXC(MN+1)
      AX1=AX+AX2
   27 BB(I)=A(I)/DT*C(I)+(1.-TET)*((C2*(C(K2)-C(I))/AX2  -C3*(C(I)-C(K3)
     -1)/AX)/AX1-QO(I)*(C(I)-C(K3))/AX)
      AA(KM)=-TET*CC2/AX1/AX2
      KM=JCX(I,3)
      AA(KM)=AT1(I)/DT+TET*((CC2/AX2  +CC3/AX)/AX1+XJ(M)/AX)
      KM=JCX(I,4)
      AA(KM)=-TET/AX*(XJ(M)+CC3/AX1)
      GO TO 100
```

```
 30 AX1=AX+DX(K2)
    AA(KM)=-TET/DX(K2)*(XJ(M2)+CC2/AX1)
    KM=JCX(I,3)
    AA(KM)=AT1(I)/DT+TET*((CC2/DX(K2)+CC3/AX)/AX1+XJ(M)/DX(K2))
    KM=JCX(I,4)
     AA(KM)=-TET*CC3/AX1/AX
    BB(I)=A(I)/DT*C(I)+(1.-TET)*((-C2*(C(I)-C(K2))/DX(K2)+C3*(C(K3)-C(
   1I))/AX)/AX1-(QO(I)*C(I)-QO(K2)*C(K2))/DX(K2))
    GO TO 100
 40 MK1=(KI-1)*2+1
    AX=(DXC(MK1)+DXC(MK1+1))/2.
    AX1=DXC(MK1)
    AX2=DXC(MK1+1)
    KK1=NCC(KI,1)
    KK2=NCC(KI,2)
    M1=(KK1-1)*2+2
    M3=(KK2-1)*2+2
    C4=A(KK2)*E(KK2)*2
    C3=A(KK1)*E(KK1)*2
    CC3=AT1(KK1)*ET1(KK1)*2
    CC4=AT1(KK2)*ET1(KK2)*2
    IF(NCC(KI,3).LT.0)GO TO 50
    AX=AX+DX(K2)
    AA(KM)=-TET/DX(K2)*(XJ(M2)+CC2/AX)
    KM=JCX(I,3)
    AA(KM)=AT1(I)/DT+TET*((CC2/DX(K2)+CC3/AX1+CC4/AX2)/AX+XJ(M)/DX(K2)
   1)
    KM=JCX(I,4)
    AA(KM)=-TET*CC3/AX/AX1
    KM=JCX(I,5)
    AA(KM)=-TET*CC4/AX/AX2
    BB(I)=A(I)/DT*C(I)+(1.-TET)*(((C(KK1)-C(I))/AX1*C3+(C(KK2)-C(I))/A
   1X2*C4-(C(I)-C(K2))*C2/DX(K2))/AX-(QO(I)*C(I)-QO(K2)*C(K2))/DX(K2))
    GO TO 100
 50 AX=AX+DX(I)
    AA(KM)=-TET*CC2/AX/DX(I)
    KM=JCX(I,3)
    AA(KM)=AT1(I)/DT+TET*((CC2/DX(I)+CC3/AX1+CC4/AX2)/AX+XJ(M1)/AX1+XJ
   1(M3)/AX2)
    KM=JCX(I,4)
    AA(KM)=-TET/AX1*(XJ(M1)+CC3/AX)
    KM=JCX(I,5)
    AA(KM)=-TET/AX2*(XJ(M3)+CC4/AX)
    BB(I)=A(I)*C(I)/DT+(1.-TET)*(((C(K2)-C(I))*C2/DX(I)-C3*(C(I)-C(KK1
   1))/AX1-C4*(C(I)-C(KK2))/AX2)/AX-(QO(KK1)*(C(I)-C(KK1))/AX1+QO(KK2)
   2*(C(I)-C(KK2))/AX2))
100 CONTINUE
    DO 110 M=1,IS1
110 AK(M)=AA(M)
    RETURN
    END
```

Typical output from the program UNSWQ:

```
          PRINTING INPUT CARDS

  16    12     2     2     0     1     0     0     0     0     0     1
   1     2     2     3     4     6     5     7     6     8     7     9    10    11    11    12
  12    13    13    14    14    15    15    16
   4     5     3     8     9   -10
  -1     0
    0.00        0.00
1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00
  .55        .50        .45        .43        .43        .38        .38        .33        .33        .30
  .25        .20        .15        .10        .05       0.00
    .030       .030       .030       .030       .030       .030       .030       .030
    .030       .030       .030       .030       .030       .030       .030       .030
 1000.00    1000.00    1000.00    1000.00    1000.00    1000.00    1000.00    1000.00
 1000.00    1000.00    1000.00    1000.00    1000.00    1000.00    1000.00    1000.00
  500.00     500.00     500.00     500.00
    1.00       1.00       1.00       1.00
   4     4     4     4     4     4     4     4     4     4     4     4     4     4     4     4
   1
   0.000     30.000     60.000     90.000
  30.000     30.000     30.000     30.000
   0.000       .938      1.765      2.500
   0.000      1.000      2.000      3.000
   2
   0.000     30.000     60.000     90.000
  30.000     30.000     30.000     30.000
   0.000       .938      1.765      2.500
   0.000      1.000      2.000      3.000
   3
   0.000     30.000     60.000     90.000
  30.000     30.000     30.000     30.000
   0.000       .938      1.765      2.500
   0.000      1.000      2.000      3.000
   4
   0.000     15.000     30.000     45.000
  15.000     15.000     15.000     15.000
   0.000       .882      1.579      2.143
   0.000      1.000      2.000      3.000
   5
   0.000     15.000     30.000     45.000
  15.000     15.000     15.000     15.000
   0.000       .882      1.579      2.143
   0.000      1.000      2.000      3.000
   6
   0.000     15.000     30.000     45.000
  15.000     15.000     15.000     15.000
   0.000       .882      1.579      2.143
   0.000      1.000      2.000      3.000
   7
   0.000     15.000     30.000     45.000
  15.000     15.000     15.000     15.000
   0.000       .882      1.579      2.143
   0.000      1.000      2.000      3.000
   8
   0.000     15.000     30.000     45.000
  15.000     15.000     15.000     15.000
   0.000       .882      1.579      2.143
   0.000      1.000      2.000      3.000
   9
   0.000     15.000     30.000     45.000
  15.000     15.000     15.000     15.000
```

Typical output (continued)

```
10
  0.000    30.000    60.000    90.000
 30.000    30.000    30.000    30.000
  0.000      .938     1.765     2.500
  0.000     1.000     2.000     3.000
11
  0.000    30.000    60.000    90.000
 30.000    30.000    30.000    30.000
  0.000      .938     1.765     2.500
  0.000     1.000     2.000     3.000
12
  0.000    30.000    60.000    90.000
 30.000    30.000    30.000    30.000
  0.000      .938     1.765     2.500
  0.000     1.000     2.000     3.000
13
  0.000    30.000    60.000    90.000
 30.000    30.000    30.000    30.000
  0.000      .938     1.765     2.500
  0.000     1.000     2.000     3.000
14
  0.000    30.000    60.000    90.000
 30.000    30.000    30.000    30.000
  0.000      .938     1.765     2.500
  0.000     1.000     2.000     3.000
15
  0.000    30.000    60.000    90.000
 30.000    30.000    30.000    30.000
  0.000      .938     1.765     2.500
  0.000     1.000     2.000     3.000
16
  0.000    30.000    60.000    90.000
 30.000    30.000    30.000    30.000
  0.000      .938     1.765     2.500
  0.000     1.000     2.000     3.000
 0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
  3600.         8         1     1200.         0
   2.00      2.00      2.00      2.00      2.00      2.00      2.00      2.00
   2.00      2.00      2.00      2.00      2.00      2.00      2.00      2.00
  20.00     20.00     20.00     10.00     10.00     10.00     10.00     10.00
  10.00     20.00     20.00     20.00     20.00     20.00     20.00     20.00
  27.10     33.50     39.30     44.50     49.10     53.20     56.80     60.10
TEST BOD  AND DO
    .50     25.00         1         1      7.80
BOD
 2
 1   -1
 0   15
 0   -1    1    1    1   -3   -4    2    2    2   -7   -8   -9  -10  -11    0
 0    3    2    6    7    8    9    6    7   11   12   13   14   15   16   14
    .25       .25       .25       .25       .25       .25       .25       .25
    .25       .25       .25       .25       .25       .25       .25       .25
   0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
   0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
   0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
   0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
   0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
   0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00
   2.00      2.00      2.00      2.00      2.00      2.00      2.00      2.00
   2.00      2.00      2.00      2.00      2.00      2.00      2.00      2.00
   5.00      5.00      5.00      5.00      5.00      5.00      5.00      5.00
   5.00      5.00      5.00      5.00      5.00      5.00      5.00      5.00
   1.00      1.00      1.00      1.00      1.00      1.00      1.00      1.00
   1.00      1.00      1.00      1.00      1.00      1.00      1.00      1.00
```

IMPLICIT SCHEME OF UNCOUPLED EQUATIONS PAGE 2
TEST CONFLUENCE SYSTEM
DT =3600. SEC DT1 =1200. SEC

TIME STEP	ISECTION	INITIAL DEPTH	AY	FINAL DEPTH	LEVEL	ISECTION	INITIAL DISCHARGE	AQ	FINAL DISCHARGE
	1	2.0000			2.5500	1	20.0000		
	2	2.0000			2.5000	2	20.0000		
	3	2.0000			2.4500	3	20.0000		
	4	2.0000			2.4250	4	10.0000		
	5	2.0000			2.4250	5	10.0000		
	6	2.0000			2.3750	6	10.0000		
	7	2.0000			2.3750	7	10.0000		
	8	2.0000			2.3250	8	10.0000		
	9	2.0000			2.3250	9	10.0000		
	10	2.0000			2.3000	10	20.0000		
	11	2.0000			2.2500	11	20.0000		
	12	2.0000			2.2000	12	20.0000		
	13	2.0000			2.1500	13	20.0000		
	14	2.0000			2.1000	14	20.0000		
	15	2.0000			2.0500	15	20.0000		
	16	2.0000			2.0000	16	20.0000		
	1	2.0000	.1273	2.1273	2.6773	1	20.0000	9.2333	29.2333
	2	2.0000	.0942	2.0942	2.5942	2	20.0000	6.4654	26.4654
	3	2.0000	.0736	2.0736	2.5236	3	20.0000	4.3679	24.3679
	4	2.0000	.0648	2.0648	2.4898	4	10.0000	2.1839	12.1839
	5	2.0000	.0648	2.0648	2.4898	5	10.0000	2.1839	12.1839
	6	2.0000	.0421	2.0421	2.4171	6	10.0000	1.5157	11.5157
	7	2.0000	.0421	2.0421	2.4171	7	10.0000	1.5157	11.5157
	8	2.0000	.0250	2.0250	2.3500	8	10.0000	1.0963	11.0963
	9	2.0000	.0250	2.0250	2.3500	9	10.0000	1.0963	11.0963
	10	2.0000	.0200	2.0200	2.3200	10	20.0000	2.1926	22.1926
	11	2.0000	.0149	2.0149	2.2649	11	20.0000	1.7566	21.7566
	12	2.0000	.0118	2.0118	2.2118	12	20.0000	1.4233	21.4233
	13	2.0000	.0103	2.0103	2.1603	13	20.0000	1.1473	21.1473
	14	2.0000	.0104	2.0104	2.1104	14	20.0000	.8884	20.8884
	15	2.0000	.0122	2.0122	2.0622	15	20.0000	.6058	20.6058
	16	2.0000	.0162	2.0162	2.0162	16	20.0000	.2513	20.2513

TEST BOD AND DO

TIME STEP 2

SIMULATED PARAMETER BOD (MG/L)

1	11.633	2	3.364	3	1.477	4	1.131	5	1.131	6	1.017	7	1.017	8	.997
9	.997	10	.990	11	.993	12	.994	13	.994	14	.994	15	.993	16	.995

APPENDIX C

SOLUTION OF THE TRANSPORT EQUATION WITH CONSTANT COEFFICIENTS

APPENDIX C

SOLUTION OF THE TRANSPORT EQUATION WITH CONSTANT COEFFICIENTS

The transport equation is

$$\frac{\partial C}{\partial t} + \frac{v\partial C}{\partial x} = E\,\frac{\partial^2 C}{\partial x^2} \qquad (C.1)$$

using the following numerical scheme

$$\frac{\partial C}{\partial t} = \frac{C_i^{t+1} - C_i^t}{\Delta t}$$

$$\frac{\partial C}{\partial x} = \left[\frac{\alpha(C_{i+1} - C_i)}{\Delta x} + \frac{\beta(C_i - C_{i-1})}{\Delta x}\right]^{t+\theta} \qquad (C.2)$$

$$\frac{\partial^2 C}{\partial x^2} = \frac{1}{\Delta x}\left[\frac{(C_{i+1} - C_i)}{\Delta x} - \frac{(C_i - C_{i-1})}{\Delta x}\right]^{t+\theta}$$

Substituting the numerical scheme Equation (C.2) in (C.1) yields

$$\frac{C_i^{t+1} - C_i^t}{\Delta t} + \frac{v}{\Delta x}\left[\alpha(C_{i+1} - C_i) + \beta(C_i - C_{i-1})\right]^{t+\theta} = \frac{E}{\Delta x^2}$$

$$[C_{i+1} - 2C_i + C_{i-1}]^{t+\theta}$$

then

$$A_i C_{i-1}^{t+1} + B_i C_i^{t+1} + D_i C_{i+1}^{t+1} = E_i \qquad (C.3)$$

where

$$A_i = -\,\theta\left[\frac{\beta v \Delta t}{\Delta x} + \frac{E\Delta t}{\Delta x}\right]$$

$$B_i = 1 + \theta\left[\frac{v\Delta t}{\Delta x}(\beta - \alpha) + \frac{2E\Delta t}{\Delta x^2}\right]$$

$$D_i = \theta\left[\frac{v\Delta t}{\Delta x} - \frac{E\Delta t}{\Delta x^2}\right]$$

$$E_i = C_i^t - (1 - \theta)\Delta t \; \{v[\alpha(C_{i+1}^t - C_i^t) + \beta(C_i^t - C_{i-1}^t)] - \\ - \frac{E}{\Delta x^2}(C_{i+1}^t - 2C_i^t + C_{i-1}^t)\}$$

The values of α and β are used to define the scheme:

$\alpha = 1$ and $\beta = 0$	forward
$\alpha = 1$ and $\beta = 1$	central
$\alpha = 0$ and $\beta = 0$	backward

In a reach with N sections the Equation (C.3) is applied to the sections 2, 3, ...N-1. There are N unknowns and N-2 equations. Conditions at the boundaries give two more equations that result in a system of equations of N x N. The system of equations is

$$\underline{F} \cdot \underline{C} = \underline{E} \tag{C.4}$$

where

$$\underline{F} = \begin{vmatrix} B_1 & D_1 & & & & & \\ & A_2 & B_2 & D_2 & & & \\ & . & . & . & . & . & \\ & & & & A_{N-1} & B_{N-1} & D_{N-1} \\ & & & & & A_N & B_N \end{vmatrix}$$

$$\underline{C} = \begin{vmatrix} C_1 \\ C_2 \\ C_3 \\ . \\ . \\ . \\ . \\ C_{N-1} \\ C_N \end{vmatrix} \qquad \underline{E} = \begin{vmatrix} E_1 \\ E_2 \\ E_3 \\ . \\ . \\ . \\ . \\ E_{N-1} \\ E_N \end{vmatrix}$$

The coefficient matrix $\underline{F}$ is a tridiagonal matrix and to solve the system of equations the Thomas algorithm can be applied. This algorithm is

$$B_i = B_i - \frac{A_i D_{i-1}}{B_{i-1}} \qquad \text{for } i = 2, 3, \ldots N$$

$$E_i = E_i - \frac{A_i E_{i-1}}{B_i}$$

then

$$C_N = E_N / B_N$$

$$C_i = \frac{(E_i - C_{i-1} D_i)}{B_i} \qquad \text{for } i = N-1, \ldots 2, 1$$

The program TRANS solves the transport equation with constant coefficients by the numerical scheme of Equation (C.2). The description of the input variables is in the computer program listing on the next pages.

```
      PROGRAM TRANS(INPUT,OUTPUT)
C---------------------------------------------------------------------C
C       THE PROGRAM TRANS WAS PREPARED TO SOLVE THE ONE DIMENSICN     C
C    TRANSPORT EQUATION WITH CONSTANT COEFFICIENTS BY A FINITE        C
C    DIFFERENCE METHOD                                                C
C       PROGRAMED BY CARLOS TUCCI                                     C
C                                                                     C
C       INPUT DATA                                                    C
C CARD A  NX      - NUMBER OF SECTIONS                                C
C         NT      -NUMBER OF TIME STEPS                               C
C         IOP     - NUMBER OF PARAMETERS TO BE SIMULATED              C
C         IOP2    -   =0 DO NOT PRINT INPUT DATA                      C
C                     =1 PRINTS                                       C
C         I03     -   =1 CONSTANT BOUNDARY CONCENTRATION              C
C                     =0   C(T) IN THE BOUNDARY                      +3
C         IOP4    -   =N WHERE ≥ IS THE SPACING OF TIME STEPS IN WHICH C
C                 THE CONCENTRATIONS SHOULD BE PRINTED
C                 FORMAT(8I5)                                         C
C         WHEN NX=0 THE PROGRAM STOPS THE EXECUTION. USE A BLANK CARD C
C    TO STOP.                                                         C
C                                                                     C
C CARD B  V       - VELOCITY                                          C
C         E       - LONGITUDINAL DISPERSION COEFFICIENT               C
C         XK1     - COEFFICIENT K1(BOD REACTION RATE)                 C
C         XK3     - COEFFICIENT K3                                    C
C         XK2 REAERATION COEFFICIENT K2                               C
C         AT      - TIME STEP                                         C
C         CS      - SATURATED CONCENTRATION                           C
C         XLA     - RATE LA                                           C
C         DB      - RATE DB                                           C
C         TET     - INTEGRATION PARAMETER TETA                        C
C                   TETA=0  EXPLICIT                                  C
C                   TETA=0.5  CENTRAL IMPLICIT                        C
C                   TETA=1. FULLY IMPLICIT                            C
C                   FORMAT(10F8.2)                                    C
C         USE THE SAME SYSTEM OF UNITS FOR THOSE VARIABLES            C
C                                                                     C
C CARD C  ALFA AND BETA - DEFINE THE NUMERICAL SCHEME IN THE SPACE    C
C                         ALFA=1, BETA=0  FORWARD                     C
C                         ALFA=1,BETA=1 CENTRAL                       C
C                         ALFA=0, BETA=1  BACKWARD                    C
C                    FORMAT(10F8.2)                                   C
C                                                                     C
C CARD D  DX(I)   - SPACING BETWEEN THE SECTIONS, I IS THE UPSTREAM   C
C                   SECTION     , FORMAT(10F8.2)                      C
C                                                                     C
C CARD E  NS(I)   - THE CODE OF THE PARAMETER TO BE SIMULATED         C
C                 =1 CONSERVATIVE                                     C
C                 =2 BOD                                              C
C                 =3 DO                                               C
C                   FORMAT(8I5)                                       C
C                                                                     C
C CARD F  CI(I,J)- FOR EACH SUBSTANCE J THE CONCENTRATION IN EACH     C
C                 SECTION I AT THE INITIAL TIME STEP , FORMAT(10F8.2) C
C                                                                     C
C CARD G  WHEN I03=0                                                  C
C         CB(IT,M)- FOR EACH SUBSTANCE M THE PROGRAM READS THE BOUNDARYC
C                 CONCENTRATION FOR EACH TIME STEP.  FORMAT(10F8.2)   C
C         WHEN IOE=1                                                  C
C         CB1(I) -  THE CONSTANT CONCENTRATION FOR EACH SUBSTANCE  I  C
C                 FORMAT(10F8.2)                                      C
```

```
      VV=(CI(I+1,M)-CI(I,M))*ALFA
      VB=( CI(I,M)-CI(I-1,M))*BETA
      D(I)=CI(I,M)-(1.-TET)*(AA*(VV+VB)/(ALFA*DX(I)+BETA*DX(I-1))-E*AT/(
     1DX(I)+DX(I-1))*((CI(I+1,M)-CI(I,M))/DX(I)-(CI(I,M)-CI(I-1,M))/DX(I
     2-1)))
  150 CONTINUE
      IF(M1-2)200,160,180
  160 DO 170 I=2,NN
      BK(I)=BK(I)+CX*TET*AT
      D(I)=D(I)+((1.-TET)*(-CX*CI(I,M))+XLA)*AT
  170 CONTINUE
      GO TO 200
  180 DO 190 I=2,NN
      BK(I)=BK(I)+TET*CX1*AT
      D(I)=D(I)+(CX1*CS-TET*CX2*CI(I,M-1)-(1-TET)*(CX2*C1(I)+CX1*CI(I,M)
     1)-DB)*AT
  190 CONTINUE
  200 CALL MATRIX(AK,BK,CK,D,X,NX)
      IF(IT/IOP4*IOP4.NE.IT)GO TO 205
      PRINT 4,NS(M)
      PRINT 5,(K,X(K),K=1,NX)
  205 DO 210 J=1,NX
      C1(J)=CI(J,M)
  210 CI(J,M)=X(J)
  220 CONTINUE
      GO TO 90
    1 FORMAT(8I5)
    2 FORMAT(10F8.2)
    3 FORMAT(//,5X,≠TIME STEP≠,I5)
    4 FORMAT(5X,≠PARAMETER≠,I5)
    5 FORMAT(8(I3,F10.3))
    6 FORMAT(1H1)
      END


      SUBROUTINE MATRIX(A,B,C,D,X,NX)
      DOUBLE PRECISION A,B,C,D,X
      DIMENSION A(1),B(1),C(1),D(1),X(1)
      NN=NX-1
      DO 100 I=2,NN
      B(I)=B(I)-A(I)/B(I-1)*C(I-1)
      D(I)=D(I)-A(I)/B(I-1)*D(I-1)
  100 CONTINUE
      BN=-2.-C(NX-2)/B(NX-2)
      DD=-D(NX-2)/B(NX-2)
      CN=1.-BN*C(NX-1)/B(NX-1)
      X(NX)=(DD-BN*D(NX-1)/B(NX-1))/CN
      DO 120 I=1,NN
      N=NX-I
  120 X(N)=(D(N)-C(N)*X(N+1))/B(N)
      RETURN
      END
```

Typical output from the program TRANS

```
 40    50       2     1     1     10
  5.00     1.50     .25     0.00     .50     .01     9.00     0.00     0.00     .50
  0.00     1.00
   .50      .50     .50      .50     .50     .50      .50      .50      .50     .50
   .50      .50     .50      .50     .50     .50      .50      .50      .50     .50
   .50      .50     .50      .50     .50     .50      .50      .50      .50     .50
   .50      .50     .50      .50     .50     .50      .50      .50      .50
  2     3
  0.00     0.00    0.00     0.00    0.00    0.00     0.00     0.00     0.00    0.00
  0.00     0.00    0.00     0.00    0.00    0.00     0.00     0.00     0.00    0.00
  0.00     0.00    0.00     0.00    0.00    0.00     0.00     0.00     0.00    0.00
  0.00     0.00    0.00     0.00    0.00    0.00     0.00     0.00     0.00    0.00
  9.00     9.00    9.00     9.00    9.00    9.00     9.00     9.00     9.00    9.00
  9.00     9.00    9.00     9.00    9.00    9.00     9.00     9.00     9.00    9.00
  9.00     9.00    9.00     9.00    9.00    9.00     9.00     9.00     9.00    9.00
  9.00     9.00    9.00     9.00    9.00    9.00     9.00     9.00     9.00    9.00
 10.00     8.73
```

Typical output (continued)

```
    TIME STEP     10
    PARAMETER      2
 1   10.000  2    6.369  3    2.934  4    1.037  5     .297  6     .072  7     .015  8     .003
 9     .000 10     .000 11     .000 12     .000 13     .000 14     .000 15     .000 16     .000
17     .000 18     .000 19     .000 20     .000 21     .000 22     .000 23     .000 24     .000
25     .000 26     .000 27     .000 28     .000 29     .000 30     .000 31     .000 32     .000
33     .000 34     .000 35     .000 36     .000 37     .000 38     .000 39     .000 40    -.000
    PARAMETER      3
 1    8.728  2    8.777  3    8.885  4    8.957  5    8.987  6    8.997  7    8.999  8    9.000
 9    9.000 10    9.000 11    9.000 12    9.000 13    9.000 14    9.000 15    9.000 16    9.000
17    9.000 18    9.000 19    9.000 20    9.000 21    9.000 22    9.000 23    9.000 24    9.000
25    9.000 26    9.000 27    9.000 28    9.000 29    9.000 30    9.000 31    9.000 32    9.000
33    9.000 34    9.000 35    9.000 36    9.000 37    9.000 38    9.000 39    9.000 40    9.000

    TIME STEP     20
    PARAMETER      2
 1   10.000  2    8.380  3    6.029  4    3.683  5    1.922  6     .869  7     .345  8     .122
 9     .039 10     .011 11     .003 12     .001 13     .000 14     .000 15     .000 16     .000
17     .000 18     .000 19     .000 20     .000 21     .000 22     .000 23     .000 24     .000
25     .000 26     .000 27     .000 28     .000 29     .000 30     .000 31     .000 32     .000
33     .000 34     .000 35     .000 36     .000 37     .000 38     .000 39     .000 40    -.000
    PARAMETER      3
 1    8.728  2    8.662  3    8.706  4    8.800  5    8.838  6    8.947  7    8.975  8    8.992
 9    8.997 10    8.999 11    9.000 12    9.000 13    9.000 14    9.000 15    9.000 16    9.000
17    9.000 18    9.000 19    9.000 20    9.000 21    9.000 22    9.000 23    9.000 24    9.000
25    9.000 26    9.000 27    9.000 28    9.000 29    9.000 30    9.000 31    9.000 32    9.000
33    9.000 34    9.000 35    9.000 36    9.000 37    9.000 38    9.000 39    9.000 40    9.000

    TIME STEP     30
    PARAMETER      2
 1   10.000  2    9.084  3    7.617  4    5.782  5    3.937  6    2.403  7    1.319  8     .655
 9     .296 10     .123 11     .047 12     .017 13     .006 14     .002 15     .000 16     .000
17     .000 18     .000 19     .000 20     .000 21     .000 22     .000 23     .000 24     .000
25     .000 26     .000 27     .000 28     .000 29     .000 30     .000 31     .000 32     .000
33     .000 34     .000 35     .000 36     .000 37     .000 38     .000 39     .000 40    -.000
    PARAMETER      3
 1    8.728  2    8.665  3    8.576  4    8.626  5    8.722  6    8.820  7    8.897  8    8.947
 9    8.975 10    8.990 11    8.996 12    8.999 13    9.000 14    9.000 15    9.000 16    9.000
17    9.000 18    9.000 19    9.000 20    9.000 21    9.000 22    9.000 23    9.000 24    9.000
25    9.000 26    9.000 27    9.000 28    9.000 29    9.000 30    9.000 31    9.000 32    9.000
33    9.000 34    9.000 35    9.000 36    9.000 37    9.000 38    9.000 39    9.000 40    9.000
```