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Article

Investigating Metropolitan Hierarchies through a Spatially Explicit (Local) Approach

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Abstract: Assuming a non-neutral impact of space, an explicit assessment of metropolitan hierarchies based on local regression models produces a refined description of population settlement patterns and processes over time. We used Geographically Weighted Regressions (GWR) to provide an enriched interpretation of the density gradient in Greece, estimating a spatially explicit rank–size relationship inspired by Zipf's law. The empirical results of the GWR models quantified the adherence of real data (municipal population density as a predictor of metropolitan hierarchy) to the operational assumptions of the rank–size relationship. Local deviations from its prediction were explained considering the peculiarity of the metropolitan cycle (1961–2011) in the country. Although preliminary and exploratory, these findings decomposed representative population dynamics in two stages of the cycle (namely urbanization, 1961–1991, and suburbanization, 1991–2011). Being in line with earlier studies, this timing allowed a geographical interpretation of the evolution of a particularly complex metropolitan system with intense (urban) primacy and a weak level of rural development over a sufficiently long time interval. Introducing a spatially explicit estimation of the rank–size relationship at detailed territorial resolutions provided an original contribution to regional science, covering broad geographical scales.

Keywords: population dynamics; spatial divides; Zipf's law; indicators; Mediterranean Europe

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1. Introduction

Metropolitan growth and socioeconomic disparities are pivotal issues in urban studies [1–4]. In this perspective, population dynamics are recognized as one of the most powerful engines of local development, in both advanced economies and emerging countries worldwide [5–7]. Multiple factors such as (i) the globalization of economic activities, (ii) accelerated structural change toward advanced services, (iii) a more intense cycle of building activity, and (iv) international migration, intrinsically affect the spatial distribution of population [8–10]. While regarded as informative of urban trends, comparative analyses of population dynamics over sufficiently long time intervals [11] and disaggregated geographical scales [12] are rather infrequent, even in advanced economies. Results of diachronic and comparative investigations of long-term settlement patterns based on population distribution over small areas are particularly intriguing in advanced world macro-regions [13–15], being linked with complex development paths [16]. Especially in Europe, individual countries (and even single regions) often display diverging development paths because of distinctive social grounds, economic structures, historical, cultural, religious and political/institutional backgrounds [17–19]. In this perspective, a spatial analysis of population indicators may reveal new patterns of regional divides [20–22], an intriguing issue in geography and regional studies [23–26].

Being an appropriate variable describing the recent evolution of European regions, local-scale population dynamics anticipate complex socioeconomic patterns of change [27–29] that may bring new models of metropolitan growth and decline [30–32]. In this context, earlier studies focused on the most evident stages of the metropolitan cycle, including compact urbanization with population concentration and dispersed suburbanization with low-density settlement expansion [33–35]. Less efforts have been devoted to investigating the long-term evolution of metropolitan hierarchies considering together central and peripheral locations [36–38]. This gradient outlines a spatial dimension depending on multiple factors of accessibility that interact among centers and peripheries [39], namely, the network in which central locations participate to a broader system of towns and villages [40]. Space, therefore, remains a substantial dimension of metropolitan hierarchies, which needs an explicit analysis of its impact on the density gradient evolving over time [41].

While showing more or less evident deviations from normality, the statistical distribution of social phenomena (basically, events correlated to human activities) can be more easily framed adopting non-linear specifications, such as hyperbolic equations that can be inspired to the class of relationships referring to Zipf's law [42–44]. This is an empirical law that describes the frequency of an event, that is part of a complete set of occurrences, in relation with its rank, namely the decreasing order position calculated with respect to the frequency of that event [45–47]. The results predicted with this law have been verified in real datasets; one of these cases is the distribution of a resident population (namely population size or density) along a density gradient (cities, towns, villages) within a sufficiently wide geographical coverage, namely a continent, a country, or a broad region [48–50]. While the empirical fields of Zipf's law are relatively varied and broadly conceived, the comparative and diachronic analysis of metropolitan hierarchies within a given country is a renowned application of this approach [51–53].

Despite the intrinsic importance of this analysis' dimension, the spatial aspect of the diachronic evolution of metropolitan hierarchies was demised even in recent studies that have investigated urban systems (and density gradients) worldwide adopting the rank–size relationship [54–56]. In other words, the empirical verification of Zipf-like relationships was carried out, to the best of our knowledge, as a spatially implicit phenomenon, with geographical structures assumed as neutral with respect to the gradient from large cities to medium-size towns and rural villages [57]. Assuming space as a non-neutral dimension of a given metropolitan hierarchy, this study hypothesizes deviations from a Zipf's law over time as a characteristic feature of the investigated social system [58]. Having made the spatial dimension explicit through local regressions that augment the global estimate based on canonical (or generalized) models, these deviations, estimated at a local scale, indicate the net impact of space on the rank–size relationship, highlighting where spatial effects are more or less intense [59]. These findings allow an accurate analysis of economic factors at the base of such impacts.

When defining local population trends, a spatially explicit (local) analysis of density gradients based on Geographically Weighted Regressions contributes to informing strategic planning and regional policies for a more coordinated urban growth [60–62]. This knowledge may promote a sustainable and spatially balanced system of cities tuned finely with the original structure of settlements, accessibility, and latent networking/interaction factors [63–65] that are partially demised in spatially implicit models. Based on these premises, our paper is organized as follows: Section 2 describes the study area, data sources, indicators, and the methodology adopted. Section 3 delineates the most relevant results of the econometric analysis. Section 4 debates the relevance of the main findings in light of the recent literature on regional science and applied economics. Section 5 concludes the study with some policy considerations and recommendations for future works.

2. Methodology

2.1. Study Area

Our study considered the whole statutory coverage of Greece (nearly 130 thousand km^2) as the investigated area (Figure 1). The LAU (Local Administrative Units) level adopted by Eurostat (the Statistical Office of the European Commission) in the Nomenclature of Territorial Statistical Units (NUTS), was chosen as the elementary spatial domain. This level includes a total of 1033 municipalities ('dimoi' and 'koinotites' in Greek) regarded as a suitable analysis unit when investigating spatial patterns of population and economic activities reflective of basic geographical gradients [66]. Earlier studies have extensively documented the appropriateness of using a detailed spatial domain such as municipalities in empirical studies of local development, economic geography, and urbanization [26,38,40]. As a matter of fact, local administrative units depict the intrinsic geography of Greece likely better than more aggregate spatial domains, identifying (i) the major urban nodes (e.g., Athens, Thessaloniki, Iraklion), (ii) dynamic coastal areas, densely settled islands in both the Ionian and Aegean Sea, and more accessible lowlands attracting tourists and the temporary/permanent population [24], as well as (iii) internal areas exposed to land abandonment, depopulation, and economic marginality because of poor accessibility and social fragility due to their peripheral location [39]. More than 30% of the Greek population gravitate around metropolitan Athens, the capital city [67], evidencing a well-consolidated urban primacy since World War II [37].

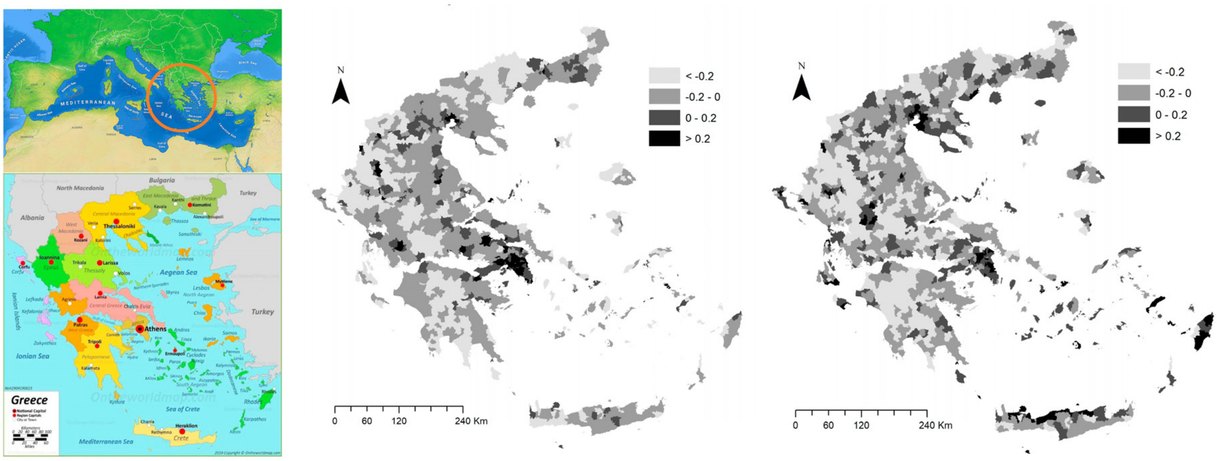


Figure 1. Maps illustrating the location of the study area (Greece, within the orange circle) in the Mediterranean basin (**upper left**); the administrative geography of Greece (**lower left**); and the annual population growth rate in 1961–1971 (**middle**) and 2001–2011 (**right**).

2.2. Data and Variables

This study exploited a complete and homogenized database released by Eurostat and reporting the total population, derived from national demographic censuses carried out every 10 years at each LAU unit (municipalities, see above) separately for 6 points in time (1961, 1971, 1981, 1991, 2001, and 2011). Since LAUs were subjected to minor changes over long observation times [68], Eurostat disseminated a homogenized list of spatial units and boundaries for cross-region and cross-country comparisons [69]. Within the activities of the Geographical Information System of the Commission (GISCO, https://ec.europa.eu/eurostat/web/gisco/overview accessed on 26 July 2023), Eurostat released the list of spatial units and their related boundaries in a geo-spatial format (shapefile). Technical details for data collection and handling, homogenization, the imputation of lacking data (if any) and the derivation of geo-spatial information were provided by the website mentioned above. Based on the original dataset, two variables were extracted from the total population data, namely (i) population size, i.e., the absolute number of inhabitants in each municipality at a given time (log-transformed), and (ii) demographic density, i.e., the ratio of the resident population in a total municipal area (km^2, log-transformed) [12]. The municipal area was automatically calculated from the shapefile of municipal boundaries released by Eurostat and mentioned above [70].

2.3. Statistical Analysis

Our analysis was organized in three steps aimed at preparing data to the subsequent inferential and econometric treatment [51]. After a preliminary step based on descriptive statistics, parametric correlations aimed to define the optimal descriptor of the metropolitan hierarchy in the case of Greece [61]. A cross-section analysis was subsequently run comparing the results of spatially implicit, standard regressions (taken as a benchmark of the rank–size relationship) with those from a spatially explicit (local) model, having municipalities as the elementary analysis' domain [71]. The first descriptive analysis' step included a study of the statistical distribution of the population size across Greek municipalities (n = 1033), investigated using metrics of (i) central tendency/dispersion (average, standard error, minimum and maximum) and (ii) ranking/form (median, skewness, kurtosis, 25th and 75th percentile) [8]. These metrics were calculated separately for each observation year (1961, 1971, 1981, 1991, 2001, 2011) and provided a preliminary description of the target variable, which was in turn mapped at the same spatial scale using the shapefile provided by Eurostat and mentioned above [72].

The rank–size relationship characteristic of the total population and demographic density was initially estimated in both sign and intensity using a pair-wise correlation analysis with parametric (Pearson product-moment) coefficients of both linear (i.e., native) and log-transformed variables (namely, municipal rank and population size or density). Pearson coefficients range from 1 (the highest positive correlation between two variables) to -1 (the highest negative correlation between two variables), with 0 indicating uncorrelated variables [26]. Significant pair-wise correlations were tested at $p < 0.05$ after Bonferroni's correction for multiple comparisons [62]. Assuming that Zipf's law holds true in the present dataset, positive correlation coefficients were expected between the tested variables (municipal rank vs. population size or density). Moreover, results of this analysis indicated the most appropriate variable (population size or density) for the subsequent econometric model [42]. Selecting the most appropriate demographic variable for econometric analysis was an indirect result of this investigation, with relevance for urban studies and regional analysis [25], since earlier studies used both population size and density as target variables [31], with an incomplete discussion of the research implications of a concurrent use of such regressors [40].

Following the empirical evidence derived from correlation analysis, we incorporated the spatial structure of the metropolitan hierarchy in an econometric specification that considered population density (inhabitants/km^2) as a function of the municipal rank [73]. More specifically, we interpreted the bivariate relationship between population density (log-transformed) of the i-th municipality and the rank (log-transformed) of the same i-th domain within the urban-rural hierarchy of Greek municipalities, evaluating the impact of rank on density through a regression coefficient (slope). The average effect of independent (external) factors (namely, the overall level of the process irrespective of the impact of the predictor) was estimated with another regression coefficient (intercept). In other words, the form of the size distribution of municipalities was assumed as following a Pareto distribution based on $\log(y) = \log(A) - \alpha\log(x)$ where x is a particular population size (population density in our case), y is the number of municipalities with populations greater than x (resulting from the municipal rank), and A and α are constants both assuming positive values. These coefficients, respectively, reflect the density of the largest city (A) and the linear distribution of city sizes when $\alpha = 1$ [43–45].

Regression models were first estimated using Ordinary Least Squares (OLS), as well as the Reduced Major Axis (RMA) and Moving Average (MA) regression generalizations aimed at reducing model's errors in a spatially implicit (global) framework [74]. RMA and MA provided an alternative (spatially implicit and global) strategy to the standard estimation run using OLS, with the final aim of determining best-fit models with minimal estimation errors. Assumptions for RMA and MA estimation were the same of those traditionally formulated for any general linear model estimated with OLS techniques. Regression intercept quantified the density of the largest city in the sample and the slope coefficient verified the working hypothesis of a linear distribution of city sizes (observed with a slope around 1, in line with the main assumption of Zipf's law [43–45]. The calculation of standard errors for the slope and intercept assumes the normal distribution of residuals and independence between the variables and the variance of residuals. RMA/MA fitting, standard error estimation and slope comparison were according to Warton et al. [75].

The empirical results of these models were subsequently compared with a spatially explicit (local estimation) strategy [66] based on Geographically Weighted Regressions (GWRs) and adopting the same specification presented above, i.e., testing population density (log) as a function of the municipal rank (log). GWRs estimate regression parameters at each location using weighted least squares, each coefficient in the model being a function of space [76]. The weights for the estimation of local regression models were derived from a bi-square nearest-neighbor kernel function, a common specification weighting the observations closer to the locations more intensively [77]. Following this specification, the optimal bandwidth value was estimated automatically and maintained as stable along the whole time interval at 125.8 km. Regressions were estimated separately for each phase of the metropolitan cycle [78], adopting a ten-year schedule, as illustrated above. The model's goodness of fit was assessed using both global and local R^2 coefficients. Maps with the spatial distribution of local parameters were provided for the intercept, predictor slope coefficients, R^2 values, and the standardized residuals of each model. Taken as a summary outcome of local regressions [79], an additional map illustrating the spatial distribution of slope-to-intercept (local) ratios was finally proposed and discussed [13]. This parameter estimated the implicit strength of the relationship between the two variables [5] by standardizing the gross impact of the predictor (slope) with the average (local) level of the process independent of the impact of the predictor itself (intercept).

3. Results

3.1. Descriptive Statistics

Table 1 illustrates some descriptive measures of the statistical distribution of the resident population in the 1033 municipalities of Greece during the six years of study. A progressive increase in the resident population was observed in the area: at the beginning of the observation period (1961), the gap between the average and the maximum value per municipality ranged between 400 inhabitants/km^2 and 19,800 inhabitants/km^2. In 2011, the absolute range was considerably higher because of the increase in the maximum density value (nearly 30,000 inhabitants/km^2), that was compensated only in part with a coherent increase in the average value (nearly 700 inhabitants/km^2). At all observation years, the median value was found different from the average value, highlighting a structural asymmetry that rose over time. The 25th percentile of the statistical distribution of the resident population showed a slight decrease (from 27 to 17 inhabitants/km^2), and contrasts with a more evident increase in the 75th percentile (from 76 to 93 inhabitants/km^2). This result testifies an increase in the urban component of the hierarchy (i.e., rising population in urban municipalities) at the expense of low-density (basically rural) areas, where a systematic population decrease was observed (as a result of land abandonment and the consequent rural exodus to central locations). Despite the strong statistical asymmetry, the morphological indexes of skewness and kurtosis showed a slow convergence towards a more balanced distribution over time.

Table 1. Descriptive statistics of population density (inhabitants/km^2) in Greek municipalities by year.

Variable	1961	1971	1981	1991	2001	2011
Mean	374	486	612	650	693	695
Max	19,809	24,650	36,781	34,431	34,106	29,976
Min	0	0	0	1	1	1
Std. error	52	67	84	84	85	82
Median	42	37	37	38	40	35
25th pctile	27	22	21	22	22	17
75th pctile	76	70	75	80	91	93
Skewness	7.2	6.7	6.9	6.4	6.0	5.5
Kurtosis	59.5	51.4	59.9	49.7	44.8	36.2

3.2. Correlation Analysis

Table 2 shows the results of a linear correlation analysis (based on parametric Bravais–Pearson moment-product coefficients) that investigates the strength of the rank–size relationship in 1033 municipalities of Greece. We compared separately (i) two variables (population size and density), and (ii) two metrics (a traditional logarithmic specification and a purely linear specification). The linear specification was systematically less efficient than the traditional logarithmic specification. In addition, the population density performed much better than the absolute population size for both specifications. Based on these premises, the subsequent analysis was run using the logarithmic specification of population density as the input variable.

Table 2. Correlation coefficients (Pearson) assessing the rank–size relationship in Greece (n = 1033 municipalities) with linear and logarithmic forms, by year.

Year	Total Population		Population Density	
	Linear	Log	Linear	Log
1961	−0.909	−0.843	−0.842	−0.962
1971	−0.921	−0.877	−0.849	−0.967
1981	−0.922	−0.887	−0.859	−0.971
1991	−0.927	−0.894	−0.870	−0.974
2001	−0.934	−0.904	−0.879	−0.978
2011	−0.947	−0.908	−0.900	−0.977

3.3. City-Size Distribution and the Metropolitan Hierarchy in Greece

Table 3 illustrates the most representative trends over time in the global econometric estimates (logarithmic specification) for population density at Greek municipalities using three techniques (OLS, RMA, and MA) and, by comparison, the global goodness of fit of the related (spatially explicit) GWR model. For all observation years, the density–rank relationship assumed a systematically high goodness-of-fit (adjusted $R^2 > 0.9$), despite the wide and possibly heterogeneous sample size (n = 1033 municipalities). Global R^2 coefficients increased over time, reaching 0.95 in 2011 from an initial value of 0.93 (1961). OLS estimates of the regression intercept showed rising values over time (from 5.23 to 6.39). The slope of the estimated relationship was negative and increased over time (from -1.35 to -1.81); the ratio between slope and intercept also grew over time (from -0.259 in 1961 to -0.283 in 2011). The estimates obtained with models other than the OLS (namely, RMA and MA) confirmed these results, while reducing the regression coefficient's error. On the basis of the same logarithmic specification, the comparative scrutiny of the adjusted R^2 values of the GWR model documented the superiority of a spatially explicit model compared with spatially implicit approaches. The overall improvement in the adjusted R^2 was 0.8% in 1961 and 0.5% in 2011. For 2011, the spatially explicit model captured approximately 96% of the overall variability in the data series, highlighting the importance of an explicit evaluation of spatial structures when assessing metropolitan hierarchies.

Table 3. Results of a global regression estimating the metropolitan hierarchy in Greece (n = 1033 municipalities) by year (see text for abbreviations; estimate ± standard error).

Parameter	OLS	RMA	MA
1961			
Slope	−1.352 ± 0.012	−1.405 ± 0.012	−1.423 ± 0.012
Intercept	5.227 ± 0.031	5.364 ± 0.001	5.411 ± 0.001
Adjusted-R^2	0.926		
Global R^2 (GWR)	0.934		
1971			
Slope	−1.500 ± 0.012	−1.550 ± 0.012	−1.572 ± 0.013
Intercept	5.567 ± 0.032	5.698 ± 0.001	5.754 ± 0.001
Adjusted-R^2	0.936		
Global R^2 (GWR)	0.942		
1981			
Slope	−1.606 ± 0.012	−1.655 ± 0.012	−1.678 ± 0.013
Intercept	5.857 ± 0.032	5.983 ± 0.001	6.043 ± 0.001
Adjusted-R^2	0.942		
Global R^2 (GWR)	0.946		
1991			
Slope	−1.650 ± 0.012	−1.694 ± 0.012	−1.715 ± 0.012
Intercept	6.001 ± 0.031	6.113 ± 0.001	6.169 ± 0.001
Adjusted-R^2	0.950		
Global R^2 (GWR)	0.954		
2001			
Slope	−1.690 ± 0.011	−1.729 ± 0.011	−1.749 ± 0.012
Intercept	6.135 ± 0.030	6.235 ± 0.001	6.286 ± 0.001
Adjusted-R^2	0.956		
Global R^2 (GWR)	0.959		
2011			
Slope	−1.806 ± 0.012	−1.850 ± 0.012	−1.874 ± 0.013
Intercept	6.387 ± 0.033	6.500 ± 0.001	6.564 ± 0.001
Adjusted-R^2	0.953		
Global R^2 (GWR)	0.958		

3.4. *Deriving a Spatially Explicit City–Size Relationship for Greece*

Figures 2–6 showed the results of local regression models estimating the metropolitan hierarchy in Greece as a spatially explicit phenomenon, producing a local estimate of the intercept, slope, and adjusted R^2, as well as the estimation error for each municipality. More specifically, Figure 2 illustrates the spatial distribution of the locally adjusted R^2. Values of the adjusted R^2 close to 1 indicate a close adherence with the predictions of Zipf's law. The highest values of this index were associated with Central Greece, the Peloponnese, and the Aegean islands. The area with high R^2 values increased over time, embracing the whole Peloponnese, Crete, and other districts of Central Greece (e.g., Thessaly). This macro-region corresponded with the districts of the oldest settlements in classical Greece and the actual metropolitan structure close to Athens, confirming the existence of a mono-centric model for Greek settlements gravitating on Attica. The model performed better in 2011 (the highest R^2 in the available time series) following the dense and compact urbanization characteristic of recent Athens' development. The subsequent suburbanization did not seem to have altered this mono-centric model, contributing in turn to consolidate it further. The metropolitan hierarchy characteristic of this settlement seems to follow better than other regional settlements (e.g., Macedonia region with Thessaloniki urban center) that likely consolidated more recently. As an indirect confirmation, the regression fit of the metropolitan hierarchy in peripheral regions was relatively less satisfactory: in 1961, local $R^2 < 0.85$ were recorded in Thrace and North-Western Greece (Epirus). However, areas with $R^2 < 0.85$ decreased over time. Among Greek regions, only Thrace remained associated with $R^2 < 0.9$ in 2011, possibly confirming the peripheral profile of this region reflecting a slight deviation from the predictions of Zipf's law.

Figure 2. Results of a local regression (spatially adjusted R^2 coefficients) estimating the metropolitan hierarchy of Greece (n = 1033 municipalities) by year.

The demographic and urban evolution of Greece led to a particularly homogeneous density gradient. In line with the general assumptions of the GWR model, the regression residuals showed a random spatial distribution. Residual values higher than +2 or −2 concentrated in peripheral municipalities of North-Western Greece (Epirus) or North-Eastern Greece (Thrace), confirming the economic marginality of such contexts (Figure 3).

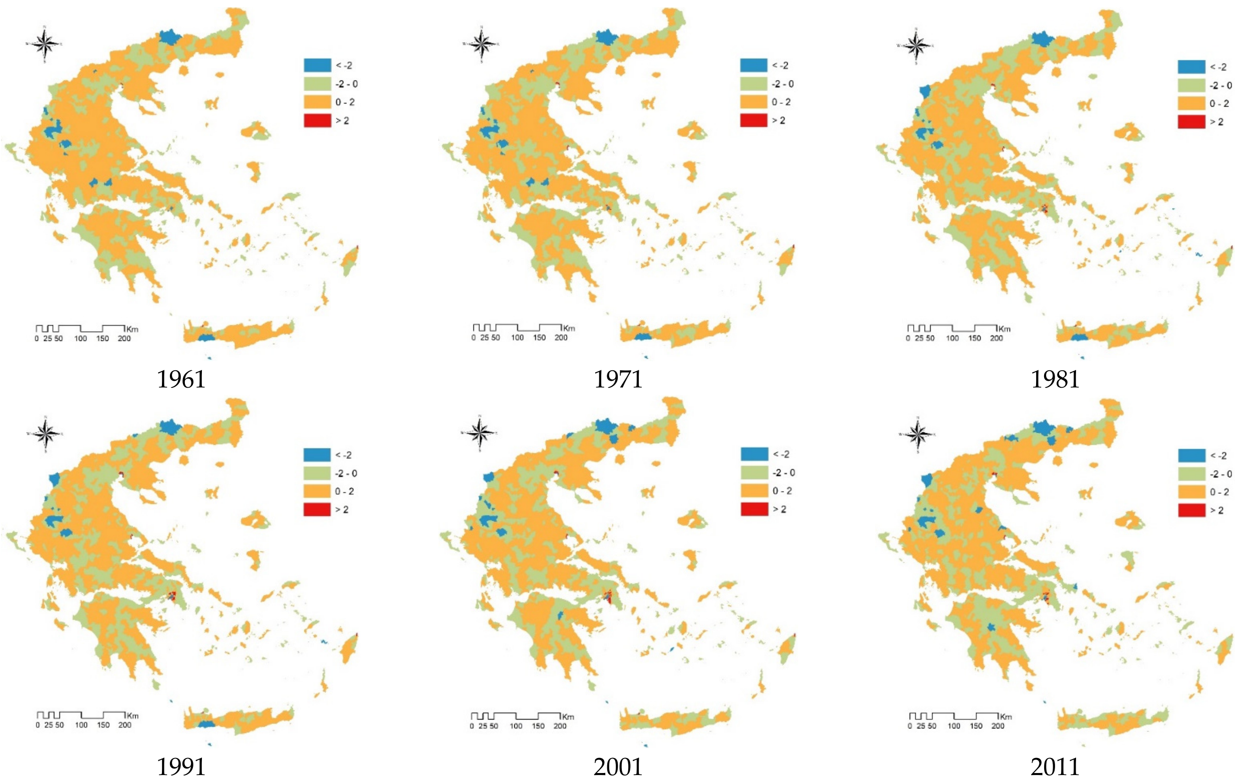


Figure 3. Results of a local regression (standardized model's residuals) estimating the metropolitan hierarchy of Greece (n = 1033 municipalities) by year.

Estimating Local Regression Parameters

Figure 4 illustrates the spatial distribution of slope coefficients estimated using local regression models separately for the six years of study, from 1961 to 2011. Slope coefficients assumed systematically negative local values around the global mean (between −1.3 and −1.8) and highlighted the existence of a highly dynamic metropolitan system. In other words, resident population along the density gradient displayed a more heterogeneous spatial distribution at the end of the study period, concentrating in particularly accessible and dense areas. The highest slope coefficient values were observed in metropolitan Athens, in line with earlier results. This trend was particularly evident between 1961 and 1991, while decreasing progressively in the following two decades (2001 and 2011). In these recent decades, the highest (local) slope coefficients concentrated in peripheral areas such as North-Western Greece, with expanding urban nodes such as Ioannina, Preveza, Arta, and Agrinio. This development path benefited from an improved infrastructural network, namely the construction of the Egnatia road connecting Igoumenitsa (the most accessible port from Italy) to Thessaloniki and the highway from Igoumenitsa to Athens, which took advantage of the new Rio-Antirio bridge over the Strait of Patras. Combined with more latent phenomena of metropolitan suburbanization responding to urban concentration (1961–1991), generalized accessibility gains led to a redistribution of the metropolitan hierarchy. Northern Greece was characterized by lower values of the slope coefficient since 1971. This result highlights the greater stability of the urban system gravitating around Thessaloniki (the second Greek city) and belonging to the administrative region of Macedonia.

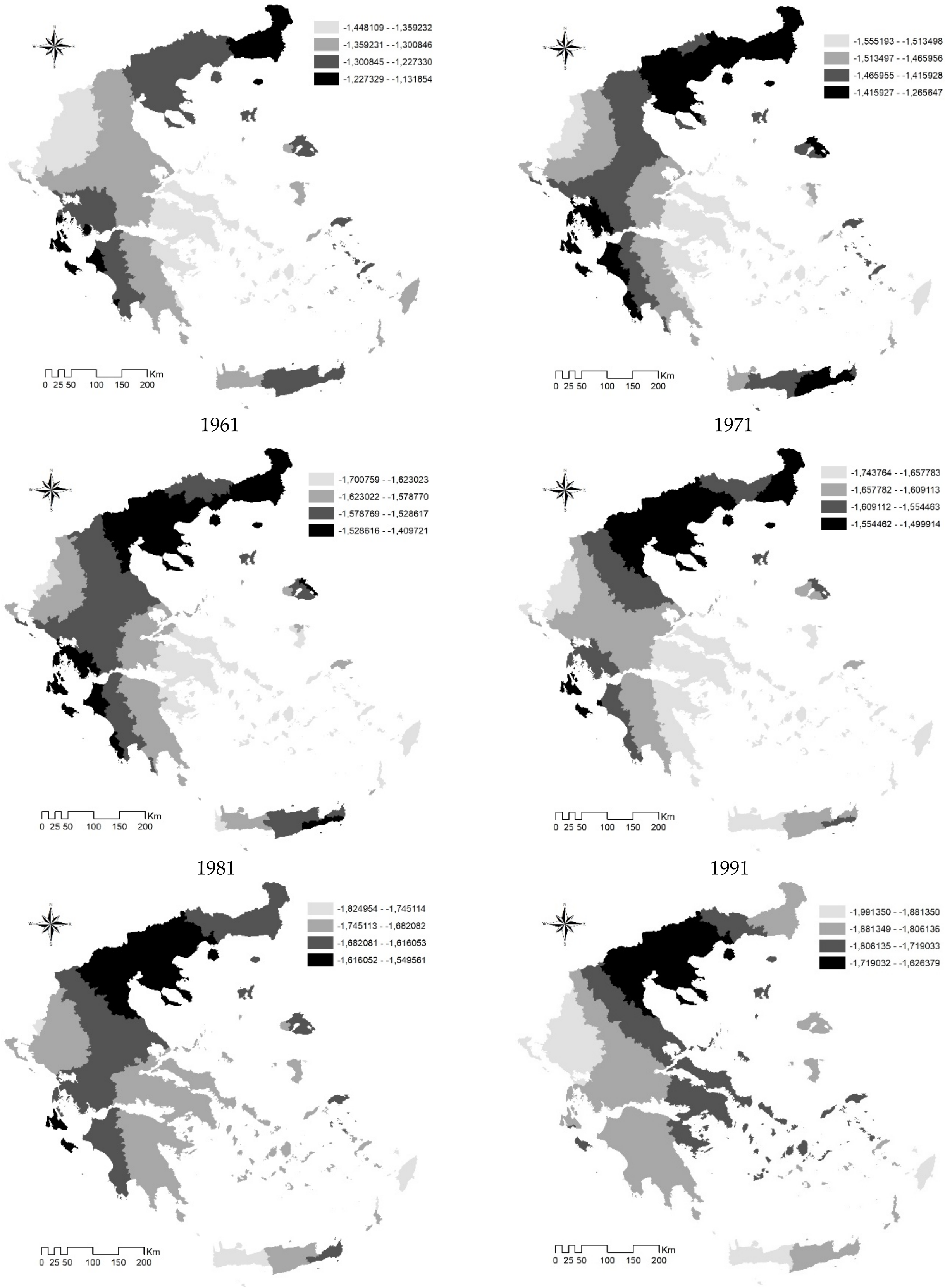
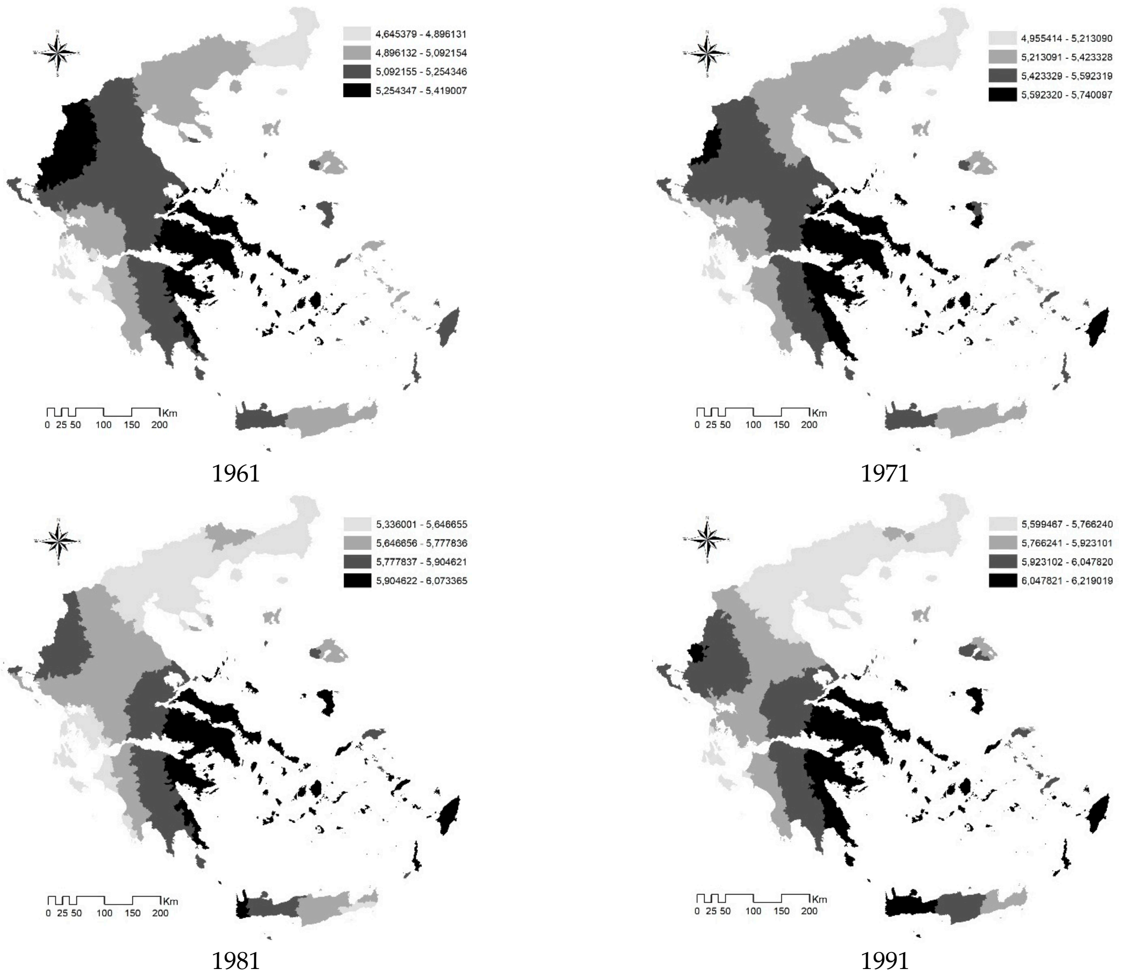


Figure 4. Results of a local regression (slope coefficients) estimating the metropolitan hierarchy of Greece (n = 1033 municipalities) by year.

Figure 5 illustrates the spatial distribution of local intercepts calculated by GWR. In line with what has been observed for local slope coefficients, the highest values of local intercepts concentrated in metropolitan Athens and in the peripheral region of Epirus, namely, the areas with a peculiar rank–size relationship with respect to the Greek average. This result can be explained by the peculiar characteristics of both areas, the former having evident infrastructural endowments and agglomeration/scale potential (higher than the country average), and the latter displaying rural attributes and latent peripheral conditions. This structure slowly diverged over time: metropolitan Athens realized its maximum intercept coefficient in 1981, in coincidence with the period of maximum urban concentration in Greece (in the 1980s, more than 35% of the Greek population resided in Attica). Conversely, the highest intercept coefficient in 2011 was observed in Epirus and in the Western part of the island of Crete, following relevant changes in the spatial structure of human settlement in Greece. This result testifies the progressive rebalancing of settlement structures and population dynamics along the density gradient in the country since 1991. Since the 1980s, these dynamics have coincided with the end of compact urbanization, a persistent, intense, and characteristic stage of the metropolitan cycle in Greece, leaving the ground to low-density suburbanization.



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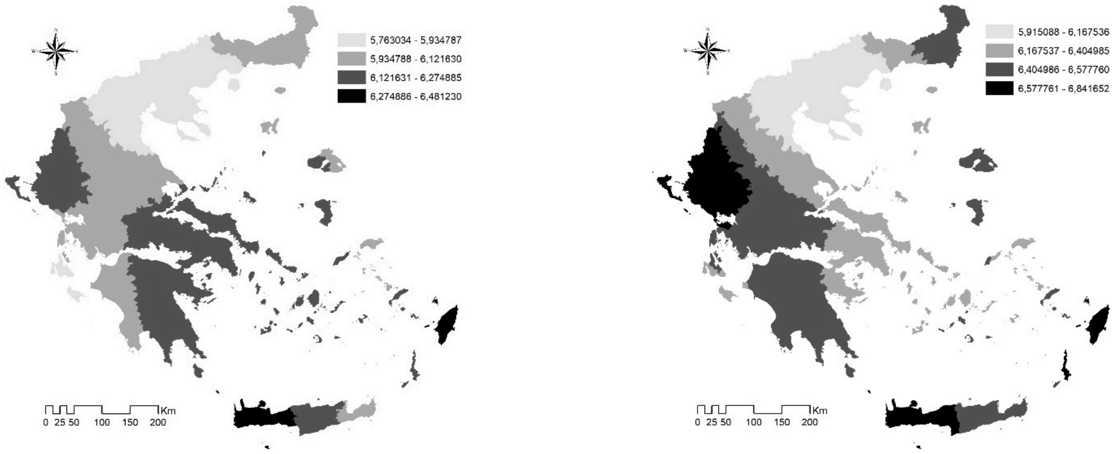

Figure 5. Results of a local regression (intercept coefficients) estimating the metropolitan hierarchy of Greece (n = 1033 municipalities) by year.

Changes over time in density gradient were finally quantified considering the ratio of (local) slope coefficients to (local) intercepts (Figure 6). In line with the empirical results already described above, the highest values were observed in the most peripheral regions (particularly North-Western Greece) which, however, has achieved fast (relative) population gains in recent years, being more aligned with the city–size relationship found at the national level. On the Ionian side, the region between Ioannina to the North and Agrinio to the South was the best example of such dynamics. The lowest values identified the metropolitan area gravitating around Thessaloniki and including most of the municipalities in Central Macedonia. This system was relatively stable—squeezed between the dynamism of peripheral areas and the gigantism of metropolitan Athens.

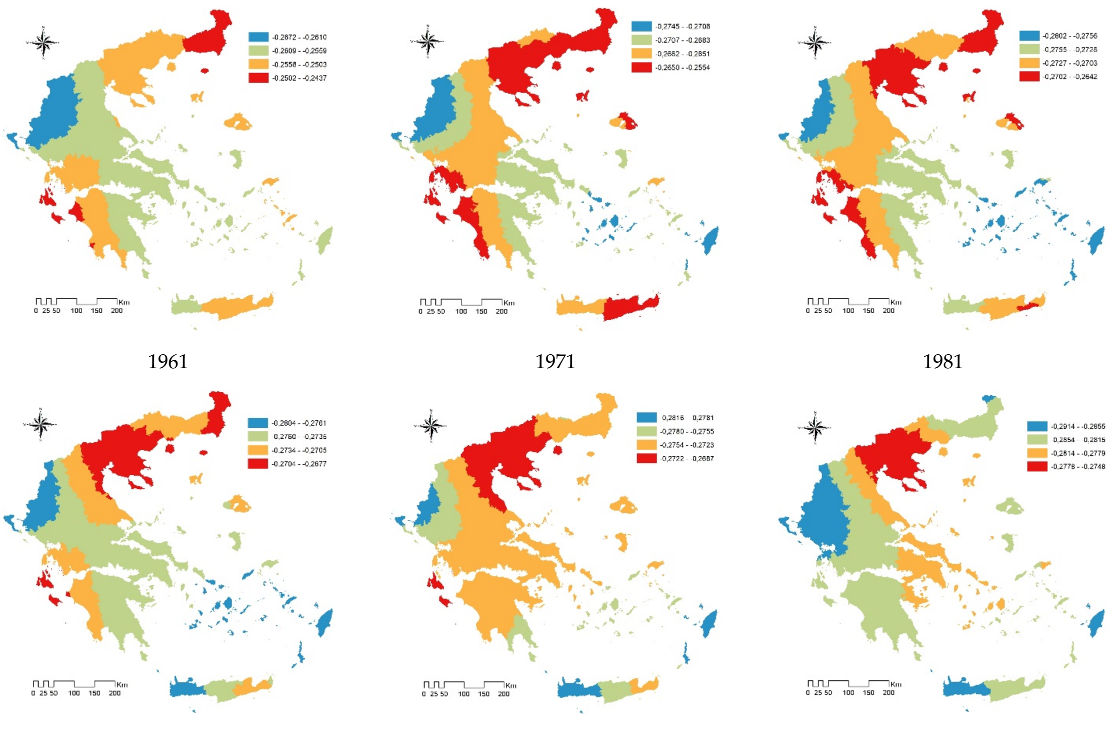

Figure 6. Results of a local regression (slope-to-intercept ratios) estimating the metropolitan hierarchy of Greece (n = 1033 municipalities) by year.

4. Discussion

The empirical approach presented in this study offers conceptual and methodological advancements to urban studies, with reference to the analysis of metropolitan hierarchies [80]. In line with similar studies carried out in other European countries [81], the work assessed the evolution over time in the metropolitan hierarchy of Greece at a fine geographical scale (municipalities). As a novel contribution to regional science, we adopted a spatially explicit, analytical framework derived from classical studies of metropolitan hierarchies [82], including the empirical verification of Zipf's law applied to diachronic investigations of urban systems worldwide. The literature applications of the city–size rule are available for both advanced and emerging countries [83]. From an operational point of view, the empirical results of such analyses demonstrated how the city–size relationship fits the metropolitan hierarchy appropriately in any investigated system [43–45], either considering urban systems (namely, cities and metropolitan areas), or including the entire density gradient (from urban centers to small rural municipalities with a low population density). From this perspective, making Zipf's assumptions spatially explicit is an original contribution to the Applied Economics literature.

Together with such important aspects, our findings demonstrate how population density—instead of total population size—is the most appropriate variable for testing the city–size relationship [84–86]. Being intrinsically standardized to a space unit (municipal area), and reflecting—at least indirectly—economic agglomeration and scale processes [78], the use of population density as the dependent variable in the city–rank relationship seems to provide a relevant contribution to the empirical analysis of metropolitan hierarchies [87]. In this perspective, the use of total population size seems to be sub-optimal [88], being dependent on the choice of political dimension (e.g., municipalities, districts, provinces) and scale [80], because administrative borders are intrinsically variable within the same region, e.g., between urban and rural areas in the same country [89–91] and, even more clearly, between countries [92–94]. One possible methodological development for future studies is the improvement in econometric approaches based on an enlarged availability of population grid data, which may reduce intrinsic heterogeneity in the use of administrative (polygonal) boundaries. For Europe, population grid data are becoming increasingly popular in recent years, despite the availability of time series computed on regular lattices seemingly still restricted, at least as far as official statistics are concerned.

4.1. Exploring the Spatial Dimension of Metropolitan Hierarchies

In addition to the empirical verification, earlier studies have tried to explain the motivations underlying the city–size rule in metropolitan areas [55]. One of the most appropriate explanations concerning the impact of the economic factors of agglomeration and scale along the metropolitan gradient [95], the conceptual and operational intersection with Christaller central location theory and its generalizations, as well as the effect of centralized (e.g., mono-centric), decentralized (e.g., polycentric), or mixed spatial organizations on settlement spatial patterns [96–98]. At the same time, earlier works based on a mathematical rationale and numerical simulations have shown the existence of a significant rank–size in random data [99–101]. These results suggest how power laws describing the relationship between size and rank of a given phenomenon are a statistical feature of all data series [102]. Therefore, the empirical verification of such a law does not necessarily require an economic explanation and seems to be independent from the specific phenomenon investigated [46,57,60].

Metropolitan hierarchies, however, have an eminent spatial dimension. In fact, population distribution and the morphological structure of settlements follow non-automatic dynamics (i.e., dependent on their intrinsic demographic dimension or economic strength) derived from spatial interaction [45], accessibility [103], and the networks in which urban centers participate [104–106]. Space, therefore, remains a substantial dimension underlying the rank–size relationship, thus needing an explicit impact analysis [59]. As an innovative contribution to urban science [107], our study assumes space as a non-neutral dimension of the city-rank rule and assesses it explicitly by integrating global estimates (based on canonical or generalized regression models) with local estimates derived from spatial regressions [54]. Having made explicit the anisotropic effect of settlement location and their geographical structure, namely proximity or distance, our approach interprets the deviations over time from the city–rank rule as a characteristic feature of the investigated metropolitan system [58]. In other words, these deviations, estimated at a local scale, highlight the net impact of space on the rank–size relationship, highlighting where spatial effects are more or less intense, and, thus, suggesting a particularly accurate analysis of economic factors at the base of such impacts [50–52].

4.2. From Global to Local Econometric Models

While the estimation and comparison of the global regression coefficients of the city–size rule (using linear, logarithmic, or generalized specifications) still feed the debate (both empirical and methodological) in regional science [108–110], (i) sources of heterogeneity in real-world data [111], (ii) spatially anisotropic effects [48], and (ii) the importance of changing time structures [57], require additional investigation. In this perspective, our study provides a first application to some of these research questions confirming, on the one hand, the topicality of the rank–size rule in an explicit analysis of metropolitan evolution and, on the other hand, the need of theoretical generalizations faced with latent (e.g., space–time) statistical structures that may affect the model's precision [105]. Being aware of the descriptive and exploratory aim of our study, further research should clarify the possible impact of the spatial support used in this paper (namely, municipalities) with respect to other candidate solutions (e.g., regular grids) [56].

On the one hand, regular lattices have been recognized as a possible solution to the eventual Modifiable Area Unit Problem (MAUP), which is intrinsic in the use of polygonal, administrative units such as local municipalities [112]. On the other hand, administrative units were traditionally used in most of the empirical tests of Zipf's law, and are particularly appropriate when long time intervals are extensively investigated, with the main target variable (namely, population size) coming from (historical) demographic censuses [59]. Especially in Europe, these data were released at municipal domains more than one century ago [60], while the availability of population grid data from official statistics (e.g., provided by Eurostat) is, at least up to now, rather limited to the last one to two decades [113].

Assuming the strengths of our paper in the (sufficiently long) time interval of investigation, the intrinsic focus on spatially non-neutral deviations from the Zipfian behavior (thus justifying a spatially explicit approach to the analysis of metropolitan hierarchies) rather than on the Zipf's α exponent (as is typical of the most traditional studies of the rank–size rule), may justify the adoption of a polygonal, administrative domain like the municipalities, as clearly documented in Calderín-Ojeda [114] for France. The use of local municipalities provides an indirect, additional advantage to the study of metropolitan hierarchies, namely, the opportunity to investigate the whole of the density gradient in a country, from the larger to the smaller settlement [15]. This approach seems to be particularly appropriate when selecting population density (and not the total population size) as the target variable, removing (or at least containing) the intrinsic effect of the differential land area inherent in the municipal structure of almost every country [38]. Considering the whole density hierarchy (namely, the whole territorial coverage of a given country, without excluding settlements below a given threshold), makes the spatially explicit analysis of Zipf's law a reasonable and coherent issue in regional studies [76].

4.3. Local Econometrics and the Zipf's Law

While preliminary, descriptive and exploratory in their scope, the results of GWR models seem to confirm the appropriateness of our assumptions. Going beyond the simple estimation of Zipf's coefficients, the overall improvement in the goodness-of-fit of spatial regression models compared with global, spatial implicit models suggests that peri-urbanization—a well-known phenomenon in urban studies—may affect the formation and consolidation of any metropolitan hierarchy [81]. The use of a joint coefficient (slope-to-intercept) estimated from GWR may provide further information and gives room to a refined discussion of the spatial structure of data. More specifically, it contributes to explain some systematic deviations of the empirical data from Zipf's law, especially at the tails of the hierarchy (e.g., in urban conditions). Spatial departures from the expected behavior may be, therefore, explained with a refined investigation of the role of economic agglomeration and scale, despite other social forces meriting a specific analysis [76]. In this perspective, moving from spatially implicit to spatially explicit econometric techniques when testing Zipf's law is an appropriate solution, assuming spatially asymmetric departures from the standard behavior [115].

In the earlier literature, going beyond the vast spectrum of global models estimating Zipf's law and generalizing the traditional power specification at the base of Pareto's law, quantile regression—a well-known and widely used econometric technique—proved to be appropriate when identifying important deviations from Zipf's law in specific portions of the statistical distribution, or for specific city sizes (for instance, in the top-level rank of the metropolitan hierarchy) [52]. However, quantile regression in these studies was mainly adopted in a spatially implicit framework, implying a homogeneous behavior of the metropolitan hierarchy across space (i.e., depending exclusively on the city rank, and not on the spatial relationship between cities) [53]. Assuming spatially heterogeneous deviations from Zipf's law, a local spatial regression was preferred to a global or mixed (quantile) approaches to verify the relevance of any spatially asymmetric departure to the standard rank–size rule [54]. In this direction, further research should to clarify the impact of some important choices when developing geographically weighted regressions with the aim at testing the adherence of any metropolitan hierarchy to Zipf's law [78]. One important aspect is the selection of the optimal bandwidth adopted to estimate local GWR parameters [13,19]. For instance, the use of fixed or adaptive Kernel approaches, and their impact on parameter estimations, should be carefully investigated in theory and practice [116]. The use of spatially explicit, local techniques going beyond a regressive/econometric approaches, such as the geographically weighted summary statistics [117] that include spatially explicit, local correlation coefficients, can be particularly appropriate in this perspective, with the aim of providing a preliminary, descriptive analysis of local heterogeneity [115]. Unraveling local heterogeneity on the base of exploratory approaches based on correlation inference may represent an effective procedure contributing to the delineation of novel exploratory approaches and explicative (e.g., econometric or statistic) rationales to the (spatially explicit) analysis of metropolitan hierarchies worldwide.

5. Conclusions

Considering the impact of non-neutral spatial structures, this study demonstrated how the econometric estimation of the rank–size rule based on local regression models produces an augmented description of metropolitan hierarchies. Based on these premises, our work estimated a local and spatially adjusted city–size relationship over time, providing a local, dynamic interpretation of metropolitan systems in Greece focusing on the spatially non-neutral departure from the expected behavior. While descriptive and exploratory in its aims, our study documented the specific contribution of local regressions in a refined analysis of the adherence of the empirical data to the operational assumptions of the city–size rule. Empirical results also provided insights into the specific investigation of local deviations from city–size predictions. This allowed an indirect decomposition of representative population dynamics in at least two different phases of the metropolitan cycle in Greece (e.g., urbanization, 1961–1991, and suburbanization, 1991–2011) with diverging social contexts and economic forces at the base of the density gradient. Being regarded as an original contribution to regional science, this timing was also in line with the main results from earlier studies, and provides a geographical interpretation of the evolution of metropolitan systems based on the rank–size rule. From a methodological point of view, future research should consider more tightly new or refined methodologies (i) addressing the increasing (local) heterogeneity in real-world geo-spatial data, (ii) exploring spatially anisotropic effects, and (iii) evaluating changes in time structures. As mentioned above, further studies should also clarify the impact of spatial support (e.g., polygons vs. grids) and the selection of optimal estimation bandwidth in local regression models.

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